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Abstract

We construct a general equilibrium model with home production where consumers choose how to spend their off-market time using market consumption purchases. The time-intensities and productivities of different home production activities determine the degree to which variation in income and relative market prices affects both the composition of expenditure and market labor hours per worker. When accounting for time to consume, homothetic utility functions can still generate non-linear expansion paths as wages increase. For the United States substitution effects due to relative price changes dominate income effects from wage growth in contributing to the rise in the services share and the fall in hours per worker. Quality improvements to goods and services have roughly kept pace with each other, so that changes to sectoral production efficiencies are the primary driver of relative price variation.

Keywords: household production, labor-leisure, time use, aggregate consumption, structural change, technical change, services, goods
JEL Classification: D13, E2, O3

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1 Introduction

When considering how households use market purchases, complementarities exist between the consumption of these purchases and non-work time. In frameworks with only one consumption commodity and elastic labor supply, complementarities between leisure and consumption are explicitly considered. However, this is often not the case in models where consumers derive utility from multiple consumption commodities. Under this premise we explore the fundamental question as to why household consumption allocations vary in relative prices and income.

Demand for different market purchases depends ultimately on how households spend time using their purchases in various home production activities. Gary Becker recognized this in his seminal paper on home production, "A Theory of the Allocation of Time" (Becker 1965). In a model where households choose both market purchases and how to allocate time toward their consumption, both the relative productivities and labor intensities of different home production processes determine the responsiveness of the consumption allocation to relative prices and income. Using such a model here, we provide a micro-foundational explanation rooted in home production for why demand is sensitive to price and income changes. Specifically, we provide a novel expanation for why the share of spending devoted to services has risen in developed economies.[1]

The Beckerian model yields both theoretical and empirical results that have causal implications for the rise in the services share of United States (U.S.) consumption expenditure and the gradual decline in hours per worker. Using a flexible framework that allows for, but need not, exhibit non-linear expansion paths as incomes rise, we show that differences in sectoral productivity growth rates are the primary drivers of structural change. Further, if consumption tasks are complementary with off-market time in different ways, market hours may vary in relative sectoral prices. We estimate that substitution effects driven by variation in the relative market price of goods to services have caused a re-allocation of off-market time toward different tasks where different types of market commodities are used. As will be discussed, this phenomenon has also contributed to the deceleration of the decline in labor hours per worker since the mid-1970s.

The framework presented here can also help answer broader questions pertaining to how consumers adjust their behavior and spending in response to the introduction of new technologies. In particular, we explore the degree to which technological advance-

[1]While non-homothetic preferences are sufficient to explain variation in the expenditure basket due to rising incomes, they are not necessary. A homothetic preference structures that allows for differential complementarities between consumption and off-market time can also match the data.

ment has helped drive rising services consumption in the last half of the twentieth century. As Gordon (2016) discusses, most innovative labor-saving in-home appliances, such as electric laundry machines, refrigerators, and vacuum cleaners, were already in many American homes by 1950. Major technological advances in the last half of the twentieth century that most drastically affected consumer time utilization were in the realms of communication and entertainment (Gordon 2016). These encompass many products that are classified under the NIPA "services" umbrella. An open question is to what degree improvements to the consumer experience of using these new services helped contribute to the relative rise in services demand. Through the lens of our model we provide an answer to this question. Specifically, we estimate the degree to which the value-added to various off-market activities of using services grew faster or slower than that of physical, manufactured goods.

Moving forward, we will place our work in context with the extensive literature on structural change, home production, and consumer time use. We will then round out the introduction by defining some terms we use throughout the paper in order to distinguish where our proposed mechanism yields results that depart from the literature. Later in the paper, after presenting and analyzing a stylized version of a Beckerian model, we perform a quantitative assessment of the degree to which rising wages versus relative price variation are responsible for the rising services share since 1948.

1.1 Literature

Our work intersects with several broad strands of literature, namely those dealing with the structural rise of the services sector, technological change, home production, off-market time use, and the decline in labor hours per worker. Here, we place the paper in context with others that grapple with these topics.

Rising Services Share and Structural Change: The literature generally posits two primary theories as to why the services share of spending has risen in developed economies. One explanation is that, as personal income has grown, so has demand for services consumption. Since the relative price of goods to services has fallen, then it must be that income effects play a role in driving up the relative demand of services (Caselli and Coleman 2001; Kongsamut, Rebelo, and Xie 2001; Matsuyama 2009; Herrendorf, Rogerson, and Valentinyi 2013; Uy, Yi, and Zhang 2013; Boppart 2014; Comin, Lashkari, and Mestieri 2015; Kehoe, Ruhl, and Steinberg 2018). Non-homothetic preferences are sufficient to justify this theory, but they are not necessary if the consumption of different types of products, say goods and services, is linked with separate off-market time-utilization decisions in different ways.

A partially-overlapping explanation for structural change posits that differentials in capital deepening, human capital productivity, and/or total factor productivity (TFP) growth leads to differences in sectoral growth rates and thus variation in relative prices and expenditure (Caselli and Coleman 2001; Ngai and Pissarides 2007; Acemoglu and Guerrieri 2008; Buera and Kaboski 2012; Autor and Dorn 2013; Comin, Lashkari, and Mestieri 2015; Herrendorf, Herrington, and Valentinyi 2015; Porzio, Rossi, and Santangelo 2020).[2] Such mechanisms can explain the rise in the share of production devoted to technologically-advanced products even with homothetic preferences.

We follow guidance in Buera and Kaboski (2009) who advocate for home production models to match the structural change data. In this regard our work most closely aligns with Ngai and Pissarides (2008), though we allow for goods and services consumption to each be complementary with off-market time in different ways. Ngai and Pissarides (2008) focus primarily on how the decline in hours worked per employee can be attributed to differentials in technological growth between home and market sectors. A secondary result of their home production formulation in which off-market time is divided between in-home labor and leisure is that as $t \rightarrow \infty$ all market hours are eventually devoted toward services. Over time the services sector eventually dominates manufacturing and agriculture due to differentials in technological change.

The main result featured in this paper departs from that of Ngai and Pissarides (2008) and others in two important ways. First, we use a stylized model with elastic time-use and multiple off-market time-use choices that are each complementary with consumption in different ways to explicitly show that income effects can be generated *regardless* of differences in technological growth between sectors or between market output and in-home production. Second, the first result holds even if preferences are homothetic.

Finally, note that much of the literature on structural change considers reasons for the decline in the sectoral share of agriculture and the contemporaneous rise in manufacturing (Caselli and Coleman 2001; Kongsamut, Rebelo, and Xie 2001; Herrendorf, Rogerson, and Valentinyi 2013; Uy, Yi, and Zhang 2013; Comin, Lashkari, and Mestieri 2015; Herrendorf, Herrington, and Valentinyi 2015; Porzio, Rossi, and Santangelo 2020). The mechanisms proposed to explain early transitions into an industrial society are almost identical to those used to explain the post-industrial transitions from manufacturing to services: non-homothetic preferences, different sectoral rates of technological change and

[2] In a recent working paper Porzio, Rossi, and Santangelo (2020) use a two-sector model (agriculture and non-agriculture) to show that human capital deepening has led to a decline in agriculture's share of labor globally. While their paper does not consider the subsequent late-twentieth century shift in labor hours from manufacturing to services in advanced economies, their mechanism is generalizable to such a setting.

capital and human capital deepening. In this paper we will refer to "structural change" in the context of an already-developed economy transitioning from making and purchasing physical manufactured goods to services.

Home Production: The term "home production" is used to characterize a wide range of phenomena explained by models with various features. We will distinguish here between home production formulations where time use is considered directly complementary to market purchases versus those where time and market purchases are not directly combined to produce a home good. The former camp of papers generally assumes that a particular type of market purchase, say consumer durables or goods, is combined with time to yield final consumption (Becker 1965; Bernanke 1985; Greenwood and Hercowitz 1991; McGrattan, Rogerson, and Wright 1993; Greenwood, Rogerson, and Wright 1995; Rupert, Rogerson, and Wright 1995; Gomme, Kydland, and Rupert 2001; Greenwood, Seshadri, and Yorukoglu 2005; Goolsbee and Klenow 2006; Ngai and Pissarides 2008; Bridgman, Duernecker, and Herrendorf 2018; Fang, Hannusch, and Silos 2020). In most of these models, however, consumption of services is generally not associated with a corresponding time allocation decision, as in the original Beckerian formulation.[3] Rather, only physical goods are considered home production inputs. In the latter camp of home production papers, market purchases or inventories of consumer durables are featured as inputs into some technological process that does not admit time but often features an exogenous productivity component (Gronau 1977; Graham and Green 1984; Benhabib, Rogerson, and Wright 1991; Ingram, Kocherlakota, and Savin 1997; Boerma and Karabarbounis 2019). Our formulation is related more to the former camp than the latter, though we allow both for time-use complementarities and exogenous changes to in-home productivities. Our findings suggest that using services are time intensive, so the flexible Beckerian framework that allows for both goods and services to be complementary to different time-use decisions is important.

Allocation of Off-market Time: Papers on household time use typically make an effort to distinguish between time engaged in market work, work in the home (think doing chores) or human capital accumulation, and leisure activities (King, Plosser, and Rebelo 1988; Lucas Jr. 1988; Ríos-Rull 1993; Perli and Sakellaris 1998; Aguiar and Hurst 2007; Ramey and Francis 2009; Ramey 2009; Bridgman, Duernecker, and Herrendorf 2018; Kopytov, Roussanov, and Taschereau-Dumouchel 2020). The Beckerian home-production framework does not require a distinction between in-home work and leisure. This is because, regardless of whether the off-market activities are themselves laborious or relaxing,

[3] A recent exception is the working paper by Fang, Hannusch, and Silos (2020), which most closely follows the Beckerian framework in the manner that we also employ in this paper.

Becker's premise is that consumers must spend time using any market purchase in order to derive utility from its consumption. This is true regardless of whether the consumer decides to use a market purchase for a mundane household chore or a more pleasurable, relaxing leisure activity. We thus do not find it necessary to make a distinction between time devoted toward work in the home versus leisure. The reason for this is that either type of activity can be associated with either goods or services consumption, and we categorize time-use based on the type of consumption with which it is complementary.

Decline in Market Hours: The decline in market hours per worker in the U.S. and other developed economies is well-established, though theories explaining the decline are wide-ranging (Barro 1984; Ngai and Pissarides 2008; Ramey and Francis 2009; Mankiw 2010; Gordon 2016; Jones 2016; Aguiar et al. 2017; Boerma and Karabarbounis 2019; Boppart and Krusell 2020; Fenton and Koenig 2020; Kopytov, Roussanov, and Taschereau-Dumouchel 2020). Naturally, if a key mechanism affects the total allocation of off-market time, through say home production or consumption/leisure complementarities as in Ngai and Pissarides (2008), Boppart and Krusell (2020), and Kopytov, Roussanov, and Taschereau-Dumouchel (2020), market hours would be impacted as well. Recent work by Boppart and Krusell (2020) do not attempt to rationalize why labor hours have fallen, but outline the parameter constraints under which a standard, separable consumption/leisure utility function, like that proposed in MaCurdy (1981) can reconcile the decline.

In a recent working paper Kopytov, Roussanov, and Taschereau-Dumouchel (2020) take an alternative approach, proposing a theory that the decline in market hours can be attributed to the declining implicit price of leisure activities due to technological advancements and quality improvements of recreational goods and entertainment services. Their results suggest that further study is warranted regarding the link between time-allocation and consumption when accounting for the particular kinds of products being consumed and used. One of our aims is to understand the effects of such linkages in detail.

1.2 Definitions

Throughout this paper we will make several references to terms that help provide context for our theoretical and empirical results. In this section we define those terms and briefly discuss the context in which we will refer to them. It is particularly necessary to succinctly define what constitute "substitution" and "income" effects because, in the literature, these terms are used to describe various phenomena which manifest themselves in often model-dependent ways. For example, what constitutes an income effect in a model with inelastic time-use may actually be masking an underlying substitution effect which a richer model could capture.

Classic Substitution Effect: changes in the distribution of total real consumption across multiple commodities as resulting from a change in relative prices, holding the utility level fixed.

Classic Income Effect: changes in the consumption allocation, possibly including the distribution of expenditure, due either to variation in income or price inflation. This comprises a parallel shift in the budget set and can result both from variation in income and prices, since changes to prices may affect the overall affordability of the current bundle.

Pure Income Effect: the phenomenon by which variation in income, either cross-sectionally, over time, or both, affects the relative consumption of goods and services, holding prices fixed. In a model where consumers choose amongst multiple consumption commodities but supply labor inelastically, pure income effects are equivalent to classic income effects. In a model with elastic labor, pure income effects can determine the distribution of expenditure across different commodities either through classic substitution- or classic income-effect channels, since wages simultaneously comprise the price of off-market time and affect income. Heretofore, the structural change literature documenting the rise in the services share of U.S. expenditure has mostly considered models where the pure income effect and classic income effect are synonymous. Without elastic time use non-homothetic preferences are one way to generate the observed changes in relative consumption.[4]

Inferiority: the phenomenon by which demand for either market purchases or off-market time-use declines in absolute terms due to an absolute increase in income.

Classic c/ℓ Model: a classic consumption/leisure model with a single consumption commodity and a single, elastic leisure choice. In such models, a rise in wages induces both classic income and substitution effects, where one or the other may dominate, depending on preferences.

Classic c/ℓ Income Effect: rising wages lead to less time working and more time spent engaging in leisure, though on the whole, rising wages shift the budget constraint out and consumers experience a more preferable (c, ℓ) bundle. In a model with multiple consumption commodities and multiple off-market time-utilization decisions, consumers may increase the time they spend on certain activities more than others. If different activities are complementary with different market commodities in different ways, we will show that the re-allocation of time can also affect the allocation of expenditure.

[4]For example, Kongsamut, Rebelo, and Xie (2001), Matsuyama (2009), and Herrendorf, Rogerson, and Valentinyi (2013) use variations of Geary (1950) and Stone (1954) preferences with non-zero subsistence terms, Boppart (2014) uses a more flexible PIGL specification from Muellbauer (1975, 1976), while Foellmi and Zweimuller (2008) construct a quadratic utility function that yields non-linear Engel curves.

Classic c/ℓ Substitution Effect: rising wages are associated with an increase in the opportunity cost (the price) of leisure, so consumers substitute leisure time for additional consumption which they fund by working more. In this situation leisure is an inferior good, since income rises but leisure time falls. In a model with multiple off-market time-utilization decisions, some time-use choices may be inferior while others may still be normal even though the classic c/ℓ substitution effect dominates.

2 Model Economy with Beckerian Home Production

Time is discrete and indexed by t. There is a unit-mass of infinite-lived households $i \in [0,1]$, each of which consists of N_t members. The population grows at rate g_{Nt}. Households buy goods q_{igt} and services q_{ist} on the market at prices P_{gt} and P_{st}. They also allocate off-market time to two tasks n_{igt} and n_{ist} and supply labor ℓ_{it} earning wages $w_{it} = \eta_{it} w_t$, where η_{it} is a household-specific labor productivity and w_t is the average, economy-wide wage-per-hour-worked. Households have final utility over the outputs c_{ijt} of home production activities $j \in \{g, s\}$, each associated with a separate market purchase. Households can also save by investing i_{it} in market capital k_{it}, which is assumed non-negative in the initial period $k_{i0} \geq 0$, depreciates at rate δ, and yields net return r_t.

There are three representative firms, each of which separately produce goods Q_{gt}, services Q_{st}, and investment capital I_t. Capital letters will denote aggregates. The producers of goods and services utilize capital K_{jt} and labor L_{jt} as inputs in Hicks-neutral Cobb-Douglas production technologies: $Q_{jt} = A_{jt} K_{jt}^{\alpha_j} L_{jt}^{1-\alpha_j}$. As in Acemoglu and Guerrieri (2008), we allow the intensity of capital α_j to vary across sectors. As in Ngai and Pissarides (2007), Acemoglu and Guerrieri (2008), and Boppart (2014) we allow the total factor productivities (TFP) to differ across sectors as well. These will fluctuate according to stochastic processes we parameterize in Section 5. Note that when labor is inelastically supplied and $\alpha_j = \alpha$ for all sectors, the ratio of total factor productivities is just the inverse of the price ratio (Herrendorf, Rogerson, and Valentinyi 2014, 2018). The investment producer only uses capital K_{It}, transforming it one-to-one to investment, $I_t = K_{It}$. Finally, the economy is assumed to be closed with all prices generated endogenously to support market clearing of goods, services, capital, and labor. In the forthcoming exposition we will focus on the household's decision process in detail, demonstrating how wage variation impacts the consumption allocation generally for models with multiple, elastic off-market time use decisions and consumption, regardless of whether preferences are homothetic.

2.1 Households

The household decision process takes a Becker (1965) form. Consumers derive periodic flow utility $u(c_{igt}, c_{ist})$ directly from the outputs of two home production processes. We denote these outputs by c_{ijt}, indexing them by the type of market commodity with which they are associated $j \in \{g, s\}$. Preferences are time-separable with β governing the degree of time preference:

$$U_i = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_{igt}, c_{ist}) \tag{1}$$

Expectations are over sequences of future prices, which are affected by real fluctuations in firm TFP's, and the productivities of home production processes, which we now describe.

Market purchases q_{ijt} along with off-market time n_{ijt} are inputs into home production processes or activities that yield final consumption. Letting f_j be some constant returns to scale production function, final consumption is produced using time and either goods or services according to

$$c_{ijt} = z_{ijt} f_j(q_{ijt}, n_{ijt}), \qquad \forall j \in \{g, s\} \tag{2}$$

z_{ijt} allows for variation over time in the TFP of home process j. Consumers thus simultaneously allocate their off-market time toward two in-home activities, each of which are associated with utilizing either market goods q_{igt} or market services q_{ist}.

Let $\overline{n}$ denote the total time available to the household, and assume that all households face the same time constraints. Their total time allocation must satisfy $\ell_{it} + n_{igt} + n_{ist} \leq \overline{n}$. Capital investments evolve according to $k_{i,t+1} \leq k_{it}(1-\delta) + i_{it}$. The standard budget constraint with market expenditure and investment on the left and labor plus capital income on the right is $\sum_{j \in \{g,s\}} P_{jt} q_{ijt} + i_{it} \leq w_{it}\ell_{it} + r_t k_{it}$. Letting $R_t = 1 - \delta + r_t$ be the gross rate of return on capital investments, we substitute out household labor supply and flow investment using the time allocation constraint and the law of motion for capital investments to write a dynamic version of the Becker (1965) budget constraint

$$\sum_{j \in \{g,s\}} (P_{jt} q_{ijt} + w_{it} n_{ijt}) \leq w_{it}\overline{n} + R_t k_{it} - k_{i,t+1} \tag{3}$$

Consumers thus choose $\{q_{igt}, q_{ist}, n_{igt}, n_{ist}, k_{it+1}\}_{t=0}^{\infty}$ to maximize (1) subject to (2) and (3).

Preferences over c_{ijt} and the structure of f_j both matter in determining the composition of the market basket. That is, the degree to which the composition of the market basket responds to wage or relative price variation depends on both the elasticity of substitution between final consumption, c_{igt} and c_{ist}, and the relative time-intensities of the different home production processes. This is true regardless of variability in z_{ijt} between processes and over time. It can also be true even if f_j and u are all homogeneous of degree one, corresponding to homothetic preferences. We will demonstrate this using fairly standard parametric forms in Section 3.

2.2 Income and Substitution Effects

Let us now briefly provide intuition by examining model-implied income and substitution effects in the context of classic consumer theory. For such an analysis we require the dual problem (EMP) associated with the utility-maximization problem (UMP) described above. As Blundell and Macurdy (1999) point out, the objective function for the EMP in models featuring at least one off-market time-utilization decision and elastic labor is just the left-hand side of (3). Further, the marginal rates of substitution for the model's control variables along with the budget constraint contain all necessary information to relate off-market time-utilization decisions to market consumption. In our Beckerian framework with two market commodities, the control variables are q_{igt}, q_{ist}, n_{igt}, and n_{ist}, while $k_{i,t+1}$ is a dynamic choice variable. Denoting the right hand side of (3) by y_{it}, it is clear that this object is endogenous, a fact which makes estimating substitution and income elasticities difficult but does not preclude us from discussing their theoretical implications.

Let superscript m index the Marshallian demand functions derived by solving the UMP. Let superscript h index the Hicksian demand functions derived by solving the EMP, where total expenditure is equal to the endogenous variable y_{it}. Marshallian demands are functions of consumption prices, wages, and full income, which is the value of income if all time were devoted to labor. We write the Marshallian demands as $q^m_{ijt}(P_{gt}, P_{st}, w_{it}, y_{it})$. With elastic time use, just as in a standard c/ℓ model, the Hicksian is a function both of market prices and wages, $q^h_{ijt}(P_{gt}, P_{st}, w_{it}, \overline{u}_{it})$, since the opportunity cost of engaging in home production activities is w_{it}. Thus, w_{it} is both the wage and a price.

In classic consumer theory it is assumed that the vector of prices faced by the consumer when making purchasing decisions is of the same cardinality as the vector of those decisions. Off-market time utilization is effectively a purchase decision: the consumer gives up a share of his possible income he could have earned working in exchange for more time. In the standard c/ℓ model, there are thus two prices — one for each of the two purchasing decisions, c and ℓ. But with multiple off-market time-use decisions each weighted in the budget constraint by the same price, the cardinality of the price vector is less than the cardinality of the quantity vector, which includes time. Indeed, in Beckerian models the price vector is constrained to be one plus the size of the vector of market purchase prices, while the number of time-utilization decisions may grow as much as the modeler sees fit. Thus, Beckerian models do not conform to a fundamental assumption underlying classic consumer theory: if the left hand side of the budget constraint contains $\mathcal{M}$ decisions then the corresponding vector of prices also has dimension $\mathcal{M}$.[5] In our case the consumer faces four effective purchase decisions — q_{igt}, q_{ist}, n_{igt}, and n_{ist} — but only three prices — P_{gt}, P_{st}, and w_{it}.

Let $e_{it}(P_{gt}, P_{st}, w_{it}, \overline{u}_{it})$ be the expenditure function associated with the consumer's EMP. Since the prices of n_{igt} and n_{ist} are constrained to be identical, the model's version of Shepherd's Lemma is slightly different than the standard version.

Lemma 1. Shepherd's Lemma for off-market time use and wages is

$$n^h_{igt} + n^h_{ist} = \frac{\partial e_{it}}{\partial w_{it}}$$

All proofs are presented in Appendix A. Lemma 1 follows directly from the fact that n_{igt} and n_{ist} always have the same price but are separate decisions which will differ from each other due strictly to the structures of f_g, f_s, and u.

The classical Shepherd's Lemma breaks here simply because the price set is smaller than the choice set. This has implications for the terms of the cross-price responsiveness of market commodities q_{ijt} to wages w_{it}. Lemma 2 characterizes the Slutsky equation describing the responsiveness of market consumption to wage variation.

Lemma 2. The Slutsky equations describing the responsiveness of demand q_{ijt} to wages w_{it} are

$$\frac{\partial q^m_{ijt}}{\partial w_{it}} = \frac{\partial q^h_{ijt}}{\partial w_{it}} - \frac{\partial q^m_{ijt}}{\partial y_{it}}(n_{igt} + n_{ist}), \qquad \forall j \in \{g, s\}$$

This expression simply encodes cross-price responsiveness, where the price is the opportunity cost of off-market time utilization which is just w_{it}.

Notice that if q_{ijt} is observed in the data to increase as wages rise, then Lemma 2 says that the substitution effect, not the income effect, *must be dominating.* Note, though, that Lemma 2 does not say anything about whether the classic c/ℓ income or substitution effect dominates. This is because demand may be linked in complicated ways to the separate off-market time-use decisions. For example, if q_{ijt} and n_{ijt} are strong complements

[5] See, for example, Chapter 2.D of Mas-Colell, Winston, and Green (1995).

and c_{igt} and c_{ist} are strong substitutes, then possibly $\frac{\partial q^m_{ijt}}{\partial w_{it}} < 0$. In such a case q_{ijt} would appear in the data to be an inferior good if labor income rises. We demonstrate this for an explicit parameterization in Section 3.1.

It is clear that q_{igt} and q_{ist} may respond to wage variation in different ways even if all of f_j and u are homothetic. This is because the effect of w_{it} on q_{ijt} is both an income effect and a substitution effect. In a model with inelastic labor, rising wages can only cause budget shares to vary through the pure-income-effect channel. In the next section, amongst the many model features we explore, we show that under the Beckerian framework demand can vary in wages even if u composed with f_j yields a homothetic preference structure.

3 Comparative Statics for Household Decisions

To illustrate the important theoretical implications of the model, we engage in several comparative statics using fairly conventional parameterizations for home production and utility. The aim is to show that when consumers face multiple market-purchase decisions each complementary to a separate off-market time-use decision, even a homothetic preference structure can generate non-linear Engel curves. Thus, in this section we focus only on household decisions in a static environment with no savings mechanism.[6] The household receives income only from supplying labor, $w\ell$. Assume there is no variation in home-production productivities so that $z_j = 1$ for all j. We will focus on how household time-use and market purchases are affected by relative price and wage variation.

For these exercises only, consider Cobb-Douglas home production functions, with process-specific output elasticities ω_j. These functions are $f_j(q_j, n_j) = q_j^{\omega_j} n_j^{1-\omega_j}$. As $\omega_j \to 1$, transforming q_j to final consumption requires less and less time. As $\omega_j \to 0$, consuming q_j is increasingly time intensive. Consider constant elasticity of substitution (CES) utility over final consumption: $u(c_g, c_s) = \left(c_g^{\rho} + c_s^{\rho}\right)^{\frac{1}{\rho}}$. When $\rho \in (0, 1)$ c_g and c_s are gross substitutes, and when $\rho < 0$ they are gross complements. The composite utility function $u\big(f_g(q_g, n_g), f_s(q_s, n_s)\big)$ is homothetic in both market quantities and time, but since w is both income and the price of off-market time, expenditure shares will be affected by its variation.

Under this home-production parameterization, the infra-marginal rate of substitution between market purchases and time for activity j is

$$\frac{n_j \omega_j}{q_j(1-\omega_j)} = \frac{P_j}{w} \tag{4}$$

[6] We drop subscripts i and t in this section only.

Note that (4) encodes equilibrium off-market time use as an implicit function of market purchases. We can thus use (4) to replace instances of n_j from the marginal rate of substitution for market goods and services and instances of q_j from the marginal rate of substitution between the two choices for off-market time use to derive expressions for relative market consumption and relative off-market time use as functions of prices and wages:

$$\left(\frac{q_g}{q_s}\right) = \left[\frac{\omega_s[(1-\omega_s)/\omega_s]^{(1-\omega_s)\rho}}{\omega_g[(1-\omega_g)/\omega_g]^{(1-\omega_g)\rho}}\right]^{\frac{1}{\rho-1}} P_g^{\frac{1-\rho+\rho\omega_g}{\rho-1}} P_s^{\frac{1-\rho+\rho\omega_s}{1-\rho}} w^{\frac{\rho(\omega_s-\omega_g)}{\rho-1}} \tag{5}$$

$$\left(\frac{n_g}{n_s}\right) = \left[\frac{(1-\omega_s)[\omega_s/(1-\omega_s)]^{\rho\omega_s}}{(1-\omega_g)[\omega_g/(1-\omega_g)]^{\rho\omega_g}}\right]^{\frac{1}{\rho-1}} P_g^{\frac{\rho\omega_g}{\rho-1}} P_s^{\frac{\rho\omega_s}{1-\rho}} w^{\frac{\rho(\omega_s-\omega_g)}{\rho-1}} \tag{6}$$

Note that these represent ratios of Marshallian demands, though we ignore the m superscript for notational simplicity. Since Marshallian demand functions are homogeneous of degree zero in prices and total income, their ratios are also homogeneous degree zero in prices, so that relative demand is aggregate-inflation neutral.

3.1 Income and Substitution Effects from Wage Variation

Relative market purchases will vary in wages as long as $\rho \neq 0$ and $\omega_s \neq \omega_g$, that is when utility (not home production) is *not* Cobb-Douglas and the home production processes associated with the consumption of goods and services have different time intensities. Whether the ratio of goods to services consumption rises or falls in w will depend on whether the outputs of home production are complements or substitutes and whether services or goods are more time intensive. The same goes for relative time use. Labor supply responsiveness to wage variation will depend on the elasticity of substitution, $\frac{1}{1-\rho}$, for the home production outputs.

For this section only, assume P_g and P_s are fixed, and consider the responsiveness of consumer choices to wages in the context of the two-good, static economy with CES utility and Cobb-Douglas home production. Propositions 1 through 3 characterize the responsiveness of Marshallian labor supply and demands for market goods and off-market time to wage variation.

Proposition 1. In a two-good, static economy with CES utility and Cobb-Douglas home production, the intensive margin of labor varies in wages as follows:

i. If the outputs of home production are substitutes so that $\rho \in (0, 1)$, ℓ is increasing in w and the classic c/ℓ substitution effect dominates.

ii. If the outputs of home production are complements so that $\rho < 0$, ℓ is decreasing in w and the classic c/ℓ income effect dominates.

Proposition 2. Relative market purchases and off-market time use vary in wages as follows:

i. If $\rho \in (0, 1)$ then market purchases and time use for the more time-intensive task fall relative to the less time-intensive task as w rises.

ii. If $\rho < 0$ then market purchases and time use for the more time-intensive task rise relative to the less time-intensive task as w rises.

Proposition 3. Marshallian demands for off-market time respond to wage increases as follows:

i. If $\rho \in (0, 1)$ then time devoted to the more time-intensive task is inferior.

ii. If $\rho < 0$ then time devoted to the less time-intensive task is inferior.

We now illustrate how the classic c/ℓ substitution and income effects are related to the inferiority of certain off-market time-use decisions under our parameterization. Consider the case where $\rho \in (0, 1)$, so c_g and c_s are gross substitutes. Suppose that goods are more time intensive, so that $\omega_s > \omega_g$. By Proposition 1, as $w \uparrow$, $\ell \uparrow$, which implies total off-market time $\overline{n} - \ell$ falls. Thus, as $w \uparrow$, employing Propositions 1 and 3, note that since the total change in off-market time is $\mathrm{d}(\overline{n} - \ell) = \mathrm{d}n_g + \mathrm{d}n_s$ then it must be that $\mathrm{d}n_s < |\mathrm{d}n_g|$ since $\mathrm{d}n_s > 0$ and $\mathrm{d}n_g < 0$. It follows that the substitution effect driving n_g down must be dominating the income effect driving n_s up. This explains why when $\rho \in (0, 1)$ the classic c/ℓ substitution effect dominates. When $\rho < 0$ the logic is the same, though the signs of total changes are the opposite: $|\mathrm{d}n_s| < \mathrm{d}n_g$ which implies that the income effect associated with increasing n_g dominates the substitution effect associated with decreasing n_s, as $w \uparrow$.

Proposition 4. Marshallian demands for market purchases respond to wage increases as follows:

i. If $\rho \in (0, 1)$ then the market purchase associated with the less time-intensive process is normal, but the market purchase associated with the more time-intensive process may, but need not, be inferior for certain prices and parameter combinations.

ii. If $\rho < 0$ then all market purchases are normal.

Here, we discuss how relative consumption q_g/q_s is impacted by classic c/ℓ substitution and income effects. Differences in time-use complementarities across the different home production processes play an important role. Indeed, if $\omega_s = \omega_g$ then clearly relative consumption is independent of wages, which can be seen by inspecting (6). Consider the case when goods are more time intensive, so that $\omega_s > \omega_g$. In this parameterization Proposition 4 states that Marshallian demand for services is always normal, though for goods it may be inferior if $\rho \in (0, 1)$. The fact that the Beckerian model, even under a parameterization using conventional functional forms, can yield inferior market commodities has been little explored.[7] In our version, inferiority is more likely as $\rho \to 1$: that is, if the outputs of home production are strongly substitutable. When $\rho \in (0, 1)$ time-spent using goods (more time-intensive) is always inferior by Proposition 3. When q_g is also inferior, home production complementarities induce negative Hicksian substitution effects on q_g as w rises. These substitution effects, driven by the gross substitutability between c_g and c_s, dominate the positive impact of rising income, and so q_g manifests as an inferior good, while services consumption and time use increase.

When $\rho < 0$ the outputs of home production are gross complements, which eliminates the possibility that either market purchase may manifest as inferior. This is because, even though n_s falls as w rises, c_g and c_s co-move together since they are gross complements. Thus, to make up for declining services time, consumers still increase services consumption, perhaps by purchasing more valuable, higher quality services: think about substituting bus travel for faster air travel, for example. q_g and q_s both rise due to wage increases, though q_g rises faster as Proposition 2 states. While in the case of $\rho \in (0, 1)$, the classic c/ℓ substitution effect may make both q_g and n_g manifest as inferior, when $\rho < 0$ the classic c/ℓ income effect works as would be expected: total income rises, total off-market time rises, and total consumption rise, with all components of the consumption vector increasing.

3.2 Income and Substitution Effects from Relative Price Variation

Generally, variation in the relative market price of goods to services P_g/P_s can induce both classic income and substitution effects. In this model it can also induce classic c/ℓ income and substitution effects: relative price variation affects relative market consumption which in turn affects relative off-market time utilization via home production com-

[7] Benhabib, Rogerson, and Wright (1991) discuss conditions under which leisure time is inferior, and Hymer and Resnick (1969) show that under certain conditions the activities themselves can be inferior, but to our knowledge nobody has used Beckerian models to address the possible inferiority of measured market purchases.

plementarities, which in turn affects the intensive margin of labor. The following propositions will outline these mechanics.

Assume w/P_s is fixed, so that w and P_s inflate at the same rate. For illustration consider the responsiveness of consumer choices to decreases in the relative price P_g/P_s in the context of the two-good, static economy with CES utility and Cobb-Douglas home production. It is sufficient to also assume $\omega_s > \omega_g$, so that goods are more time intensive.

Proposition 5. Relative market purchases and off-market time use vary in the relative price of market purchases as follows:

i. If $\rho \in (0, 1)$ then market purchases and time use for the more time-intensive task rise relative to the less time-intensive task as the more time-intensive task becomes cheaper.

ii. If $\rho < 0$ then market purchases for the more time-intensive task rise relative to the less time-intensive task, but time use for the more time-intensive task relative to the less time-intensive task falls as the more time-intensive task becomes cheaper.

Proposition 6. Marshallian demands for market purchases vary in relative prices as follows:

i. If $\rho \in (0, 1)$, consumption of the less time-intensive market purchase falls while consumption of the more time-intensive purchase rises as the more time-intensive task becomes cheaper.

ii. If $\rho < 0$, consumption of both market purchases rises as the more time-intensive task becomes cheaper.

Proposition 7. Marshallian demands for off-market time vary in the relative price of market purchases as follows:

i. If $\rho \in (0, 1)$, off-market time use for the less time-intensive task falls and time use for the more time-intensive task rises as the more time-intensive task becomes cheaper.

ii. If $\rho < 0$, off-market time use for the less time-intensive task rises and time use for the more time-intensive task falls as the more time-intensive task becomes cheaper.

Proposition 8. Marshallian labor supply varies in the relative price of market purchases as follows:

i. If $\rho \in (0, 1)$, ℓ falls as the more time-intensive task becomes cheaper. Relative price variation thus induces a classic c/ℓ income effect which dominates.

ii. If $\rho < 0$, ℓ rises as the more time-intensive task becomes cheaper. Relative price variation thus induces a classic c/ℓ substitution effect which dominates.

Unlike with straight wage variation, when relative prices change relative market consumption and off-market time use can move in opposite directions if the outputs of home production are gross complements, $\rho < 0$. As more time-intensive tasks become cheaper, consumers may buy relatively more market inputs associated with those tasks but spend relatively more time engaged with less time-intensive activities. Indeed, if the outputs of home production are gross complements, then a decline in the relative price of the more time-intensive market purchase will lead to increased consumption across the board, though consumption of the more time-intensive commodity, whose price is falling, increases faster. This induces a substitution effect, where off-market time flows away from the more time-intensive task to the less time-intensive one as a result of gross complementarities in the preference structure. Further, the consumer's desire for more of each market commodity induces what manifests as a classic c/ℓ substitution effect as ℓ rises.

When c_g and c_s are gross substitutes so $\rho \in (0, 1)$, both off-market time and market consumption covary in the same manner. If more time-intensive market purchases become cheaper, consumers substitute both expenditure and time toward such purchases and away from the less time-intensive, but more expensive, commodities. ℓ also declines which is actually a result of the Hicksian substitution effect brought on by a declining relative price: as the consumption basket has become more affordable, the consumer now need not work as much as before. He is thus better off buying more of the time-intensive commodity and spending more time using that commodity.

The mechanics outlined here thus show that the linkages between off-market time utilization, labor supply, and market demand decisions can lead to rather complex comovements of observables even under a fairly standard preference structure.

4 Empirical Regularities

In this section we discuss several trends in both long-run, aggregate U.S. consumption expenditure and labor-hours data and dis-aggregated spending and time use data. We draw aggregate non-durable and services data from the National Income and Product Accounts (NIPA). Consumer durables service flows and firms' capital utilization by sector are taken from the Bureau of Economic Analysis' (BEA) Fixed Asset Tables. Aggregate capital and labor income data, as well as sector-specific aggregate labor hours are taken from NIPA. All aggregate data are at annual frequencies from 1948-2019. Micro expenditure and time-use data are drawn from the Bureau of Labor Statistics' (BLS) Consumer

Expenditure Survey (CEX) from 1984-2018 and American Time Use Survey (ATUS) from 2003-2019. Household-level wage data are from the annual March release of the Current Population Survey (CPS).[8] For details on how we build out our household-level and sectoral data series, see Online Technical Appendix A.

4.1 Aggregate U.S. Expenditure Data

Our quantitative exercises operate on several well-established long-run trends in U.S. economic activity from 1948-2019: the decline in the aggregate nominal consumption value of goods to services X_{gt}/X_{st}, the decline in aggregate relative goods to services prices P_{gt}/P_{st}, and the rise in labor income per hour, w_t. Both the signs and magnitudes of changes to spending, quantity, and price indices depend on the degree to which we account for the presence of consumer durables in the various goods series. Since durable service flows are a non-trivial part of aggregate goods consumption, the failure to properly account for how consumers use accumulated durables in their everyday activities can lead to different estimates as to what degree wage and relative-price effects have contributed to structural change.

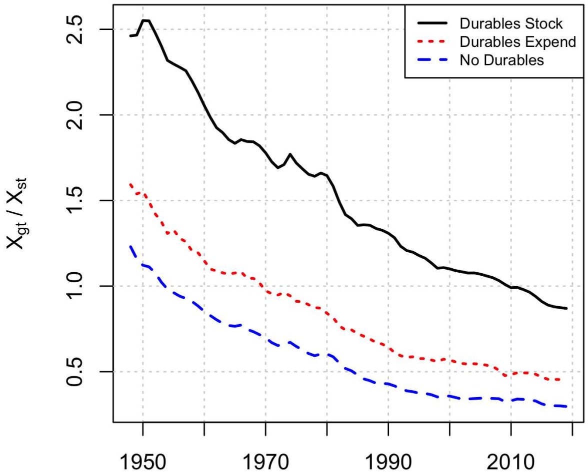
Assuming the nominal value of the service flows of durables is equal to the aggregate resale value of all durables presently in utilization, the main goods expenditure series we construct will be the sum of non-durable expenditure and the nominal value of all consumer durables. Goods prices are adjusted to accommodate this new series. The details of how we construct spending, price, and quantity series are described in Online Technical Appendix A.1.

Aggregate wages are constructed by dividing total labor compensation by total hours worked using NIPA Table 2.1 and Tables 6.9B, 6.9C, and 6.9D. For total labor compensation we sum "Compensation of employees" and "Proprietors' income with inventory valuation and capital consumption adjustments." For hours per full-time equivalent worker per day, we divide total hours by total full-time equivalent workers from NIPA Tables 6.5B, 6.5C, and 6.5D.[9]

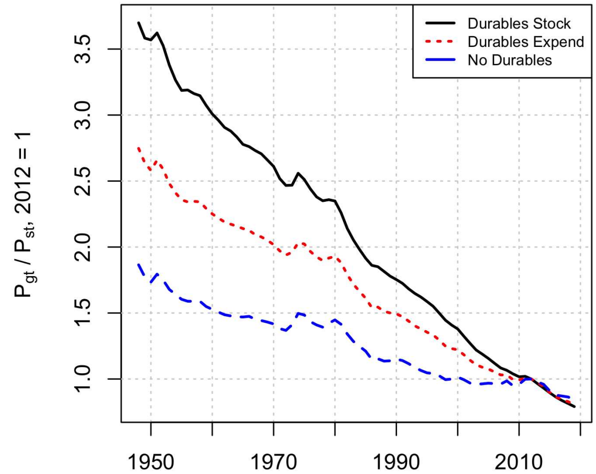
Figure 1 presents the aggregate data series of interest. Several facts stand out. First, spending and price ratios decline together, while wages rise, providing a preliminary suggestion that classic substitution effects may be weak. This is because if the classic sub-

[8] At the time this paper was written a preliminary update to the CEX for 2019 had been released, but we found undocumented changes to the dataset with respect to how certain expenditures were classified. We await a reply to our correspondence with the BLS before incorporating the 2019 data into our analysis.

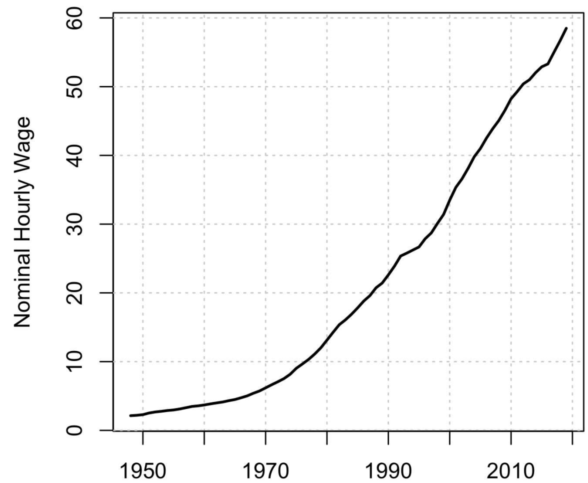
[9] According to the BEA, total full-time equivalent workers are computed by dividing total labor hours by average hours for full-time workers only: $\left(\sum_i \ell_{it}\right)/\left(\frac{1}{\#\text{full-time}} \sum_i \ell_{it} \mathbf{1}\{i \text{ is full-time}\}\right)$.



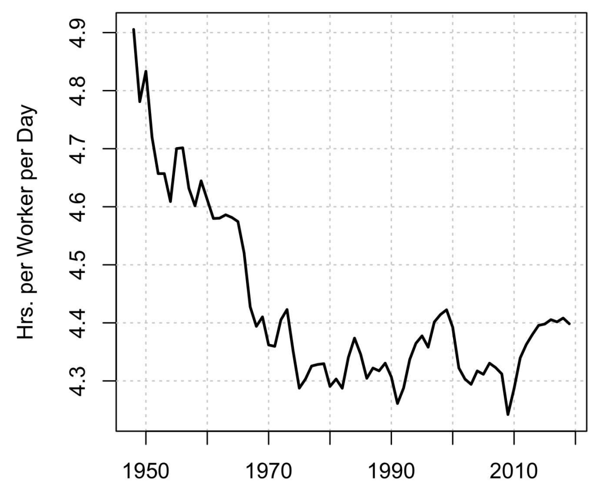
(a) Expenditure Ratio



(b) Price Index Ratio



(c) Nominal Wages



(d) Labor Hours

Figure 1: Here we present the evolution of several long-run aggregate data series. The ratio of the aggregate nominal value of final goods to services consumption is in (a), the relative chain-weighted price of goods to services where 2012 = 1 is (b), average nominal labor income per hour worked, including proprietors' incomes with inventory and capital consumption adjustments, is in (c), and total hours worked per day for each effective full-time worker is in (d). In (a) and (b) we show three data series each constructed to include different measures of consumer durables. The "Durables Stock" plots (solid black line) include the entire stock of existing consumer durables in the goods series. The "Durables Expend" plots (dotted red line) include only new investment in durables. The "No Durables" plots (dashed blue line) only include non-durables in the goods series. All series are annual, 1948-2019.

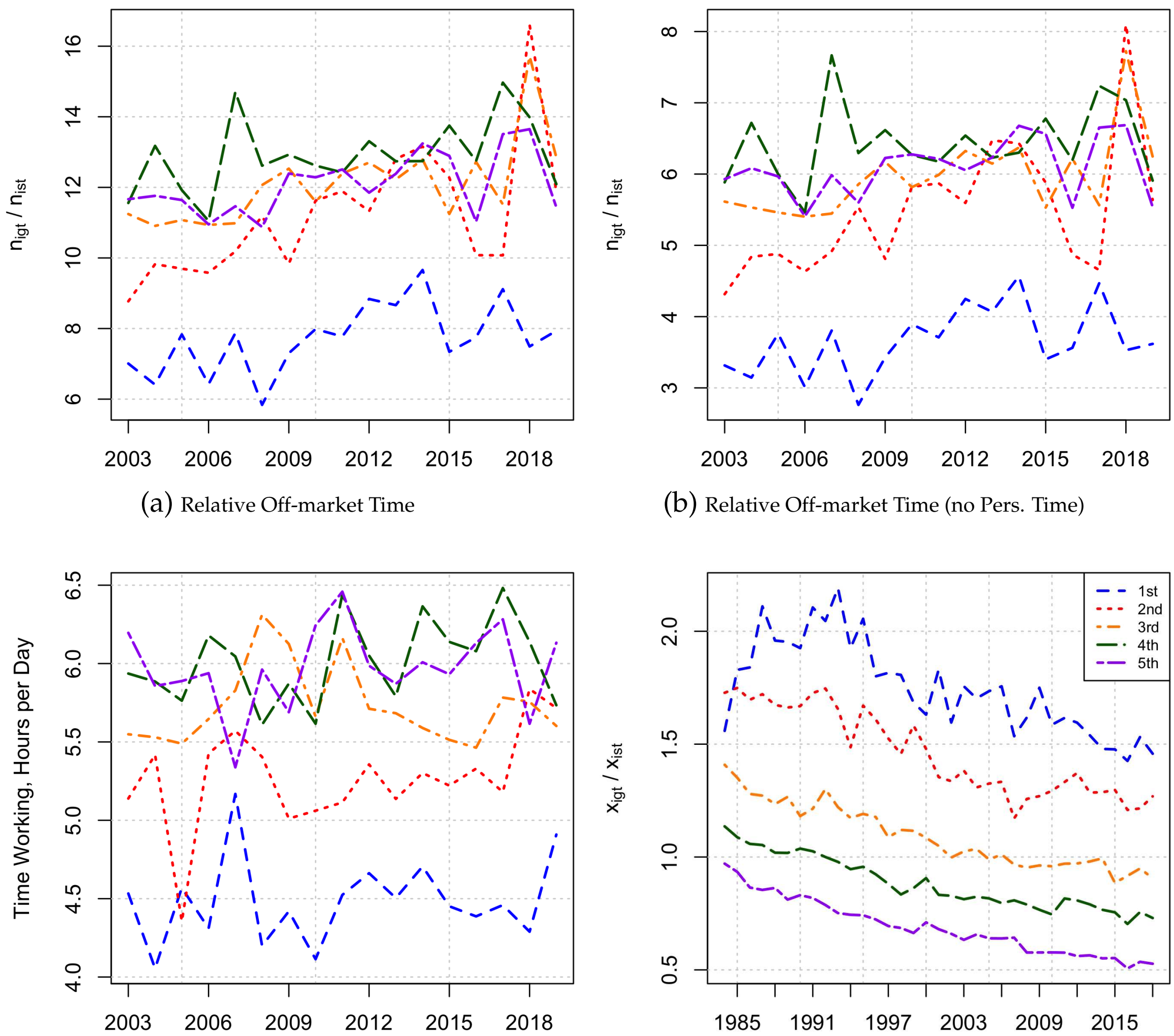
stitution effect were at play, we would expect to see opposite co-movement of relative prices and relative expenditure. Second, focussing on just (a), (b), and (c) it is obvious that if hourly wage gains are highly correlated with income gains overall, including gains from capital income, then models with inelastic labor and non-homothetic preferences will match the expenditure series. In such models the rise in the services share of spending will be explained by pure income effects that are highly correlated with wage gains. Third, in simple c/ℓ models with elastic labor, it appears the classic c/ℓ income effect would dominate the classic c/ℓ substitution effect at least over the period 1948-1970. After 1970, average hours per worker per day are relatively flat, so that c/ℓ income effects may not be as strong as they were during mid-century. In our structural estimations we will attempt to assess the degree to which these various theoretical mechanisms have contributed to structural change.

4.2 Time-use and Expenditure in Micro Data

In this section we examine cross-sectional variation in market demand and time use amongst different consumers at different income levels. We match the CEX summary cross-tabs by income quintile to the ATUS, which includes the annual March CPS wage data. Then, to construct separate spending series for goods and services, we roughly match CEX spending and ATUS activities to the detailed expenditure categorizations in NIPA Table 2.3.5 — spending by "major type of product." For the CEX classification, we apply the same classification rubric as in Boppart (2014). The ATUS time-use classification details are in Online Technical Appendix A.2.

The ATUS dataset provides a convenient way to distinguish between activities associated with using goods versus services. As an example, the dataset contains both a variable that presents the time an individual respondent spent "Interior cleaning" and a separate variable that presents the time that same individual respondent spent "Using interior cleaning services." Many tasks within the survey are classified in this manner. Continuing with the interior cleaning example, a researcher can reasonably assume that an individual engaged in "Interior cleaning" is using his time along with goods like soaps, brushes, vacuums, dusters, etc. to accomplish the task of cleaning, while one engaged in "Using interior cleaning services" could reasonably be thought to be spending time monitoring a maid or housekeeper whom he pays to perform cleaning services. While this is just one example, for certain tasks the survey structure makes it easy to establish whether they are complementary to using market goods or complementary to using services.

Not all tasks are so easily classifiable. For example the survey does not distinguish between traveling by car in one's own personal vehicle versus traveling by plane, bus,



(a) Relative Off-market Time

(b) Relative Off-market Time (no Pers. Time)

(c) Labor Hours

(d) Expenditure Ratio

Figure 2: In panel (a) we present the ratio of off-market goods to services time utilization from ATUS by income quintile, where personal-care time, including time spent sleeping, is categorized as goods utilization. In panel (b) we present the same ratio, except we exclude all activities in the personal care-time category except those associated with using shampoos, soaps, and personal hygiene products. Panel (c) shows total hours worked per day by income quintile from ATUS. Panel (d) features the ratio of goods to services expenditure from CEX. ATUS runs from 2003-2019 while the CEX runs from 1984-2018. The legend denoting which color and line type scheme correspond to which income quintile is included in the Expenditure Ratio plot.

train, or rental car. The former would require the consumer to purchase gasoline which is classified as a good in the NIPA data, while the latter activities would be classified as services consumption. Given this particular inconsistency, in conjunction with the rather short length of the ATUS time series, we highlight how relative time use has evolved under our particular classification rubric but only use these series to construct prior estimates for hyper-parameters that we can then feed to our structural estimation routine which operates only on consumption data. Later, as a robustness check, we will compare the out-of-sample fit of our model's predictions for off-market time use against the ATUS data.

Figure 2 presents a breakdown by income quintile of observed time use and spending behavior from the micro data. Lower income consumers spend relatively more time using services than goods, tend to work less, and spend a larger fraction of their disposable income on goods compared to higher income consumers. While all workers spend, on average, more time using services than goods, it appears that the patterns of time use have not changed much since the ATUS was started in 2003. From Figure 2d, however, it is clear that relative goods to services expenditure has declined in a consistent manner for all consumers in all income quintiles. To compare the breakdown by income quintile with aggregates, use the dotted red line in Figure 1a as a reference point since new durables expenditure is included in the CEX measures but not the exact value of durable assets owned.

Finally, for a preliminary assessment of the relationship in data between labor hours and relative prices, we regressed labor hours ℓ_{it} on the hourly wage w_{it} and the relative price P_{gt}/P_{st}, where i indexes all respondents in the ATUS dataset, not just income quintiles. A one unit rise in the relative price P_{gt}/P_{st} corresponds to a fall in labor hours per day of ≈ -0.396 (10% significance level), while increases in wages correspond to an increase in hours per day of ≈ 0.020 (1% significance level). Such a regression is obviously not causal, so we will not discuss the results here further. The main takeaway is that work time is correlated with market prices in ways that suggest home production complementarities are at play. As will be seen, a Beckerian model can rationalize such relationships.

5 Quantitative Model and Estimation

We consider several estimation specifications for the structural model. In one set of specifications we estimate the household's model using aggregate expenditure data only, both with and without accounting for price endogeneity. When accounting for general equilibrium effects we include the goods and services firms' marginal products of labor in the system of estimating equations. We also estimate the dis-aggregated model in both general and partial equilibrium with synthetic expenditure and wage series indexed by income quintile. Our estimation procedures take a Bayesian approach, as we target the posterior distribution of structural parameters conditional upon observed data. Estimating this distribution requires computing an intractable integral, which we accomplish using Hamiltonian Monte Carlo (HMC) integration techniques.[10]

5.1 Structural Estimating Equations

For our quantitative exercises, we consider a more flexible CES parameterization for home production: $f_j(q_{ijt}, n_{ijt}) = \left(\omega_j q_{ijt}^{\nu_j} + (1-\omega_j) n_{ijt}^{\nu_j}\right)^{\frac{1}{\nu_j}}$. Preferences over final consumption are assumed to take the same CES form as in Section 3.[11] Composing home production functions featuring Hicks-neutral home productivities with preferences for final consumption, we get the nested CES structure:

$$u\left(c_{igt}(q_{igt}, n_{igt}), c_{ist}(q_{ist}, n_{ist})\right) = \left(\sum_{j\in\{g,s\}} z_{ijt}^{\rho}\left(\omega_j q_{ijt}^{\nu_j} + (1-\omega_j) n_{ijt}^{\nu_j}\right)^{\frac{\rho}{\nu_j}}\right)^{\frac{1}{\rho}} \tag{7}$$

After solving (1) using this preference structure, we get the marginal rate of substitution for market inputs q_{igt} and q_{ist}:

$$\left(\frac{c_{igt}}{c_{ist}}\right)^{\rho-1} \frac{z_{igt}\omega_g q_{igt}^{\nu_g-1}}{z_{ist}\omega_s q_{ist}^{\nu_s-1}} \left(\omega_g q_{igt}^{\nu_g} + (1-\omega_g) n_{igt}^{\nu_g}\right)^{\frac{1-\nu_g}{\nu_g}} \left(\omega_s q_{ist}^{\nu_s} + (1-\omega_s) n_{ist}^{\nu_s}\right)^{\frac{\nu_s-1}{\nu_s}} = \frac{P_{gt}}{P_{st}} \tag{8}$$

and the marginal rate of substitution for off-market time-utilization decisions:

$$\left(\frac{c_{igt}}{c_{ist}}\right)^{\rho-1} \frac{z_{igt}(1-\omega_g) n_{igt}^{\nu_g-1}}{z_{ist}(1-\omega_s) n_{ist}^{\nu_s-1}} \left(\omega_g q_{igt}^{\nu_g} + (1-\omega_g) n_{igt}^{\nu_g}\right)^{\frac{1-\nu_g}{\nu_g}} \left(\omega_s q_{ist}^{\nu_s} + (1-\omega_s) n_{ist}^{\nu_s}\right)^{\frac{\nu_s-1}{\nu_s}} = 1 \tag{9}$$

[10]We describe HMC integration techniques in Online Technical Appendix B.1. For detailed explanations of HMC techniques see Neal (2011), Betancourt and Stein (2011), and Gelman et al. (2013b, 2013a).

[11]We choose to estimate the model using this parameterization so that elasticities of substitution between q_{ijt} and n_{ijt} are allowed to vary across processes. While this parameterization is indeed more flexible, it still yields a homothetic composite utility function.

There are four possible versions of the Euler equation describing consumption dynamics, each of which must simultaneously hold in equilibrium:

$$\begin{aligned}
&\left(\sum_{j\in\{g,s\}} c_{ijt}^{\rho}\right)^{\frac{1-\rho}{\rho}} c_{ikt}^{\rho-1}\left(\omega_k q_{ikt}^{\nu_k}+(1-\omega_k)n_{ikt}^{\nu_k}\right)^{\frac{1-\nu_k}{\nu_k}} z_{ikt}\omega_k q_{ikt}^{\nu_k-1} \\
&\quad=\beta\,\mathbb{E}_t\Bigg\{R_{t+1}\left(\sum_{j\in\{g,s\}} c_{im,t+1}^{\rho}\right)^{\frac{1-\rho}{\rho}} c_{im,t+1}^{\rho-1} \\
&\quad\times\left(\omega_m q_{im,t+1}^{\nu_m}+(1-\omega_m)n_{im,t+1}^{\nu_m}\right)^{\frac{1-\nu_m}{\nu_m}} z_{im,t+1}\omega_m q_{im,t+1}^{\nu_m-1}\Bigg\}, \qquad \forall k,m\in\{g,s\}
\end{aligned} \tag{10}$$

Using the infra-marginal rate of substitution between off-market time and market inputs for process j, we can write n_{ijt} as an implicit function of q_{ijt}:

$$n_{ijt}(q_{ijt})=q_{ijt}\left[\frac{w_{it}\omega_j}{P_{jt}(1-\omega_j)}\right]^{\frac{1}{\nu_j-1}}, \qquad \forall j\in\{g,s\} \tag{11}$$

Note that we possess time-use data only for the period 2003-2019. Lacking time-use data for earlier years, we can use (11) to substitute out instances of n_{ijt} in (8) and/or (10), allowing us to estimate the full model for the post-war years using only aggregate consumption data from 1948-2019 and CEX data from 1984-2018.

To recover the household's structural parameters we will focus on estimating (8). Relative home productivities (z_{igt}/z_{ist}) comprise the sole stochastic component of (8), while the Euler equations in (10) depend on productivity levels, not just relative productivities. Ultimately, we want to build the likelihood function around fluctuations in structural productivities without introducing additional model or measurement errors. Such a choice, however, comes with tradeoffs, namely that, depending on the home-productivity normalization we choose, (8) and (10) constitute a stochastically singular system. This is because, up to normalization, knowing the relative productivities means that we can back out a time series of productivity levels from one of the Euler equations so that it identically holds. Given the stochastic singularity, we thus choose to estimate the model using (8) while treating relative home productivities as the residual.

Issues pertaining to stochastic singularity arise in other applications involving estimation of dynamic stochastic general equilibrium (DSGE). The canonical example of a stochastically-singular system is the stochastic growth model where output, consumption, and investment are all co-integrated and driven by a single, structural shock — TFP. To overcome the problem of having more endogenous variables than shocks, Komunjer and Ng (2011) point out that in practice it is common to either add measurement errors to the system or drop equations containing certain endogenous variables. The decision to exclude observables and equations in stochastically singular systems is usually motivated by the need to reduce computational complexity, rather than economic considerations (Qu 2018). This is because for most DSGE models that deal with aggregates, the data series are easily obtained. This is not the case, however, in our application where quality off-market time-use data series are required: the ATUS data only extends back to 2003, and as we document in Section 4.2, there are issues with the way the ATUS survey may align with NIPA-categorized consumption activities. Further, the first-order conditions that describe households' decisions are many and each are highly non-linear, especially the Euler equations. While most DSGE models can be easily linearized for estimation, our parameterization does not admit a convenient linearization. For these reasons, we confront stochastic singularity by forming the likelihood around (8), dropping (9) and (10) from the system of estimating equations for now.

While direct estimation of DSGE models is particularly attractive for those engaged in out-of-sample forecasting, moment-based calibration techniques remain the gold standard for models whose primary purpose is an assessment of theory. Our methodological approach demonstrates that estimation techniques can also be used to assess theory. Specifically, we can exploit the fact that our estimating system is stochastically singular to assess model performance. That is, we estimate the model's parameters using a subset of data associated with the general equilibrium variables. We then test model performance by using the model's equilibrium conditions and estimated parameters to simulate data series that were not targeted in the estimation, specifically $\widehat{\ell}_{it}$ and $\widehat{n_{igt}/n_{ist}}$.[12] We can then use standard statistical methods to test the hypotheses that the model-simulated data series are equal to actual data, which we do in Sections 5.4.2 and 5.5.1.

Now, to arrive at an estimating equation for the household's parameters, we substitute out time use from (8). After collecting like terms we get an implicit expression, featuring only relative market quantities, that is consistent with the model's equilibrium:

$$\begin{aligned}\left(\frac{z_{igt}}{z_{ist}}\right)^{\rho}\left(\frac{q_{igt}}{q_{ist}}\right)^{\rho-1}\left(\frac{\omega_g}{\omega_s}\right)\left(\omega_g+(1-\omega_g)\left[\frac{w_{it}\omega_g}{P_{gt}(1-\omega_g)}\right]^{\frac{\nu_g}{\nu_g-1}}\right)^{\frac{\rho-\nu_g}{\nu_g}}&\\ \times\left(\omega_s+(1-\omega_s)\left[\frac{w_{it}\omega_s}{P_{st}(1-\omega_s)}\right]^{\frac{\nu_s}{\nu_s-1}}\right)^{\frac{\nu_s-\rho}{\nu_s}}&=\frac{P_{gt}}{P_{st}}\end{aligned} \tag{12}$$

[12]Hats are used to denote simulated data.

We can multiply both sides of (12) by $(P_{gt}/P_{st})^{\rho-1}$, so that the model can be estimated directly using relative expenditure data x_{igt}/x_{ist}. Then, to isolate z_{igt}/z_{ist}, so that relative productivities may be treated as a structural residual, we exponentiate both sides by $1/\rho$, take logs of both sides, and rearrange to get the expression:

$$\begin{aligned}
&\frac{1-\rho}{\rho}\ln\left(\frac{x_{igt}}{x_{ist}}\right) - \frac{1}{\rho}\ln\left(\frac{\omega_g}{\omega_s}\right) + \ln\left(\frac{P_{gt}}{P_{st}}\right) \\
&\quad - \frac{\rho-\nu_g}{\rho\nu_g}\ln\left(\omega_g + (1-\omega_g)\left[\frac{w_{it}\omega_g}{P_{gt}(1-\omega_g)}\right]^{\frac{\nu_g}{\nu_g-1}}\right) \\
&\quad + \frac{\rho-\nu_s}{\rho\nu_s}\ln\left(\omega_s + (1-\omega_s)\left[\frac{w_{it}\omega_s}{P_{st}(1-\omega_s)}\right]^{\frac{\nu_s}{\nu_s-1}}\right) = \ln\left(\frac{z_{igt}}{z_{ist}}\right)
\end{aligned} \tag{13}$$

Assume the log-ratio $\xi^1_{it} = \ln z_{igt} - \ln z_{ist}$ is first-difference stationary, and let Δ be the one-period, backwards first-difference operator. Define the residual term $\epsilon^1_{it} = \Delta\xi^1_{it} = \xi^1_{it} - \xi^1_{i,t-1}$, which is assumed mean zero. Taking first-differences of (13), we arrive at an estimating equation for household consumption decisions consistent with equilibrium utility maximization:

$$\begin{aligned}
&\frac{1-\rho}{\rho}\Delta\ln\left(\frac{x_{igt}}{x_{ist}}\right) + \Delta\ln\left(\frac{P_{gt}}{P_{st}}\right) - \frac{\rho-\nu_g}{\rho\nu_g}\ln\left(\omega_g + (1-\omega_g)\left[\frac{w_{it}\omega_g}{P_{gt}(1-\omega_g)}\right]^{\frac{\nu_g}{\nu_g-1}}\right) \\
&\quad + \frac{\rho-\nu_g}{\rho\nu_g}\ln\left(\omega_g + (1-\omega_g)\left[\frac{w_{i,t-1}\omega_g}{P_{g,t-1}(1-\omega_g)}\right]^{\frac{\nu_g}{\nu_g-1}}\right) \\
&\quad + \frac{\rho-\nu_s}{\rho\nu_s}\ln\left(\omega_s + (1-\omega_s)\left[\frac{w_{it}\omega_s}{P_{st}(1-\omega_s)}\right]^{\frac{\nu_s}{\nu_s-1}}\right) \\
&\quad - \frac{\rho-\nu_s}{\rho\nu_s}\ln\left(\omega_s + (1-\omega_s)\left[\frac{w_{i,t-1}\omega_s}{P_{s,t-1}(1-\omega_s)}\right]^{\frac{\nu_s}{\nu_s-1}}\right) = \epsilon^1_{it}
\end{aligned} \tag{14}$$

Goods and services producing firms have Cobb-Douglas technologies and face the same input prices w_t and r_t. Equilibrium sectoral capital and labor inputs must satisfy:

$$\begin{aligned}
A_{jt}(1-\alpha_j)\left(\frac{K_{jt}}{L_{jt}}\right)^{\alpha_j} &= \frac{w_t}{P_{jt}}, \quad \forall j \in \{g,s\} \\
A_{jt}\alpha_j\left(\frac{K_{jt}}{L_{jt}}\right)^{\alpha_j-1} &= \frac{r_t}{P_{jt}}, \quad \forall j \in \{g,s\}
\end{aligned} \tag{15}$$

If A_{jt} is the only residual term to the econometrician in the above two equations, then these equations, again, constitute a stochastically singular system. Absent introducing model or measurement error, we ignore one of the equations and estimate α_j using the other. We choose to focus on the marginal product of labor (MPL) conditions because we seek to also understand how household labor supply has been affected by rising wages.

Take logs of (15) and define the log-TFP term $\xi^2_{jt} = \ln A_{jt}$, which we assume is first-difference stationary. The residual term $\epsilon^2_{jt} = \Delta\xi^2_{jt} = \xi^2_{jt} - \xi^2_{j,t-1}$ is assumed mean zero. After taking first-differences, isolating ϵ^2_{jt}, and rearranging, we get the estimating equations on firms' equilibrium conditions:

$$\Delta \ln\left(\frac{w_t}{P_{jt}}\right) - \alpha_j \Delta \ln\left(\frac{K_{jt}}{L_{jt}}\right) = \epsilon^2_{jt}, \qquad \forall j \in \{g, s\} \tag{16}$$

Note that the market prices P_{jt} associated with the firms' conditions must only take into consideration the value of *new* durables being manufactured and sold in the period, since the firms do not sell vintage durables back to households, but rather households trade them amongst themselves.

Under our assumption that there exists a unit-mass of households in the economy, each household is a price-taker, so households do not consider how their decisions impact market prices, including wages. Suppose that $\mathbb{E}\left[\epsilon^1_{it} \mid \Delta P_{gt}, \Delta P_{st}, \Delta w_{it}\right] = 0$ and $\mathbb{E}\left[\epsilon^1_{it}\epsilon^2_{jt} \mid \Delta P_{gt}, \Delta P_{st}, \Delta w_{it}\right] = 0, \forall i, j$. Note that the assumption that households exist on a continuum allows for replacement of agent-level variables with aggregates. When only aggregates are considered, replace ϵ^1_{it} with ϵ^1_t and w_{it} with w_t, labor income per hour. Under these orthogonality assumptions prices are uncorrelated with first-differenced household relative productivities. Thus, home productivities and aggregate TFP's do not covary. In such a case (14) can be consistently estimated on its own. We will consider several specifications where (14) is estimated on its own and where we also estimate the system including (14) and both of (16) simultaneously, allowing for $\mathrm{Cov}(\epsilon^1_{it}, \epsilon^2_{jt}) \neq 0, \forall i, j$. We can then test this orthogonality condition with either aggregate or micro data.

5.2 Prior Distributional Assumptions

Let $\boldsymbol{\epsilon}_{it} = [\epsilon^1_{it}, \epsilon^2_{gt}, \epsilon^2_{st}]^\top$. We assume

$$\boldsymbol{\epsilon}_{it} \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\Sigma}), \qquad \forall i$$

with $\mathbb{E}[\epsilon^1_{it}\epsilon^1_{i',t}] = 0$ for $i \neq i'$. If households are indexed by wealth or income, this amounts to saying that households with access to different resources face idiosyncratic shocks to home production, though the variance of such shocks is constrained to be the same across

Table 1: Prior Distributional Assumptions

Household Parameters	*Firm Parameters*
$-\rho \sim \text{Lognormal}\big(-\frac{1}{2}, 1\big)$, if $\rho < 0$ or $\rho \sim \text{Beta}(1,1)$, if $\rho \in (0,1)$ $\omega_j \sim \text{Beta}(1, \underline{\omega}_j)$ $-\nu_j \sim \text{Lognormal}(\underline{\nu}_j, 1)$	$\alpha_j \sim \text{Beta}(10, \underline{\alpha}_j)$
Likelihood Variance/Covariance	
When it is assumed $\text{Cov}(\epsilon^1_{it}, \epsilon^2_{jt}) = 0$, $\frac{1}{\sigma^2_1} \sim \text{Gamma}(2,4)^a$ Otherwise, $\text{chol}(\boldsymbol{\Sigma}) = \text{diag}(\boldsymbol{\chi}) \cdot \Xi$, with $\chi_k \sim \text{Cauchy}_{(0,\infty)}(0,2)$, and $\Xi \sim \text{LKJ}(2)$.[b]	

[a] When using gamma distributions we use the shape/rate parameterization: $\text{Gamma}(a,b) = \frac{b^a}{\Gamma(a)} x^{a-1} e^{-bx}$

[b] See Lewandowski, Kurowicka, and Joe (2009) for detailed derivation and discussion regarding the properties of the LKJ distribution.

households. Let σ^2_1 be the first term on the diagonal of $\boldsymbol{\Sigma}$. When assuming $\text{Cov}(\epsilon^1_{it}, \epsilon^2_{jt}) = 0$, we estimate only the households' equation and let $\epsilon^1_{it} \sim \mathcal{N}(0, \sigma^2_1)$.

Table 1 defines the prior distributions imposed on the model parameters targeted in our estimation routine. We define $\underline{\omega}_j$, $\underline{\nu}_j$, and $\underline{\alpha}_j$ in the next section. ν_j are assumed negative because our prior estimates suggest that consumption and time-use are complementary inputs to the production processes f_j. ρ can be either positive or negative, depending on the specification, though our robustness assessments suggest that $\rho < 0$, so that the outputs of home production are gross complements. For easy computation of the variance/covariance matrix we impose priors on a Cholesky factorization of $\boldsymbol{\Sigma}$ using half-Cauchy and LKJ distributions.

5.3 Retrieving Hyper-parameter Estimates from Data

We want to allow our limited series of time-use data to inform our prior distributional assumptions for the home production parameters ω_j and ν_j. To do so, we run the following regressions of (11) using OLS on a panel of CEX and ATUS data with NIPA prices from 2003-2018. The regression listed here operates on time-use data with personal care time removed from n_{igt} to be consistent with the convention in macroeconomic analysis. We assume independent residuals, $\overline{\epsilon}_{ijt}$, over time and across households for each $j \in \{g, s\}$:

$$\ln P_{jt} n_{ijt} - \ln x_{ijt} = \frac{1}{\nu_j - 1} \ln\left(\frac{\omega_j}{1-\omega_j}\right) + \frac{1}{\nu_j - 1} \ln\left(\frac{w_{it}}{P_{jt}}\right) + \overline{\epsilon}_{ijt} \tag{17}$$

These regressions generate prior estimates for the CES substitution elasticities that suggest time and market inputs are gross complements in both home production processes: $\widehat{\nu}_g = -2.187$ and $\widehat{\nu}_s = -1.527$. Preliminary estimates of the weights associated with market inputs in the two processes are rather small: $\widehat{\omega}_g = 0.015$ and $\widehat{\omega}_s = 0.002$.[13] We want $\widehat{\nu}_j$ to correspond to the mean of the log-normal distribution for $-\nu_j$, giving location hyper-parameter estimates of $\widehat{\underline{\nu}}_g = 0.283$ and $\widehat{\underline{\nu}}_s = -0.077$. Centering the Beta distributions for ω_j around these prior estimates, we get hyper-parameter estimates of $\widehat{\underline{\omega}}_g = 65.307$ and $\widehat{\underline{\omega}}_s = 423.262$.

To get prior estimates of $\underline{\alpha}_j$, we run OLS on (16) taking $\Delta \ln\left(\frac{w_t}{P_{jt}}\right)$ as the dependent variable, where w_t is labor income per hour. We construct the time series K_{jt}/L_{jt} for each j using the BEA Fixed Asset Tables and NIPA data for sectoral labor hours.[14] These regressions give estimates of $\widehat{\alpha}_g = 0.185$ and $\widehat{\alpha}_s = 0.089$. Again, placing the mean of the Beta distributions around these values, the shape hyper-parameters are $\widehat{\underline{\alpha}}_g = 44.028$ and $\widehat{\underline{\alpha}}_s = 102.180$.

5.4 Model Estimates with Aggregate Data

The HMC integration procedure yields estimates of the posterior distribution of model parameters given aggregate data, which we present in Table 2, assuming $\rho < 0$ and using data accounting for the presence of consumer durable service flows.[15] In all cases, whenever we just estimate the household's parameters or whenever we estimate the system of simultaneous equations including firms' marginal products of labor, we run the estimation routine twice, restricting $\rho \in (0, 1)$ or $\rho < 0$. Our preferred specifications are those where c_{gt} and c_{st} are gross complements, i.e. $\rho < 0$. This is because $\rho < 0$ implies

[13] Including personal care time in n_{igt} only slightly changes the prior estimates: $\widehat{\nu}_g = -2.145$, $\widehat{\nu}_s = -1.527$, $\widehat{\omega}_g = 0.001$, and $\widehat{\omega}_s = 0.002$.

[14] Capital and labor income data are taken from NIPA Table 2.1. Labor income is the sum of "Compensation of employees" and "Proprietors' income with inventory valuation and capital consumption adjustments." Capital income is the sum of "Rental income of persons with capital consumption adjustment" and "Personal income receipts on assets." The capital level is computed by using BEA Fixed Assets Tables 3.1 and 3.2. Note that we use the net stock of capital assets, which the BEA adjusts for depreciation. We categorize capital and labor by sector to best match the detailed consumption expenditure categories in NIPA Table 2.4.5. Detailed spreadsheets containing our sectoral categorization rubric are available in this paper's supplementary materials at the author's website: https://www.npretnar.com/research.

[15] Online Technical Appendix B.2 features parameter estimates for data that includes only new durables expenditure and no service flows.

Table 2: HMC Posterior Distribution, 1948-2019, Agg. Data Incl. Durables, $\rho < 0$

Simultaneous Equations

	Mean	S.D.	2.5%	25%	50%	75%	97.5%
ρ	-24.7260	14.7090	-61.6961	-29.8038	-21.1438	-15.3274	-8.7977
ν_g	-1.0621	0.4604	-2.1119	-1.2315	-0.9717	-0.7796	-0.5072
ν_s	-3.0301	2.8352	-11.0880	-3.5280	-2.2514	-1.4085	-0.5520
ω_g	0.0158	0.0152	0.0004	0.0049	0.0116	0.0216	0.0550
ω_s	0.0025	0.0024	0.0001	0.0008	0.0018	0.0035	0.0090
α_g	0.1260	0.0306	0.0716	0.1045	0.1239	0.1456	0.1907
α_s	0.0942	0.0214	0.0545	0.0794	0.0936	0.1080	0.1376
σ_1^2	0.0002	0.0000	0.0001	0.0002	0.0002	0.0002	0.0003
σ_g^2	0.0010	0.0002	0.0007	0.0009	0.0010	0.0011	0.0014
σ_s^2	0.0003	0.0000	0.0002	0.0002	0.0003	0.0003	0.0004
σ_{1g}	0.0000	0.0001	-0.0002	-0.0000	0.0000	0.0001	0.0003
σ_{1s}	0.0000	0.0001	-0.0001	0.0000	0.0000	0.0001	0.0002
σ_{gs}	0.0003	0.0001	0.0002	0.0003	0.0003	0.0004	0.0005
$V(\mathcal{P})^a$	663.4594	2.7878	657.2541	661.7949	663.8329	665.4722	667.9289

Household MRS Only

	Mean	S.D.	2.5%	25%	50%	75%	97.5%
ρ	-1.4670	1.6246	-5.3131	-1.7408	-1.0101	-0.6275	-0.2878
ν_g	-2.1933	2.7917	-9.1023	-2.6161	-1.3341	-0.7320	-0.2126
ν_s	-1.5646	2.0884	-6.8562	-1.8879	-0.9331	-0.4831	-0.1333
ω_g	0.0148	0.0138	0.0005	0.0045	0.0107	0.0210	0.0499
ω_s	0.0023	0.0024	0.0001	0.0007	0.0017	0.0032	0.0086
σ_1^2	0.1161	0.0200	0.0835	0.1020	0.1138	0.1278	0.1615
$V(\mathcal{P})^a$	26.7491	1.7700	22.5420	25.7657	27.1224	28.0502	29.1780

[a] $V(\mathcal{P})$ is the log posterior density of parameters, $\mathcal{P}$.

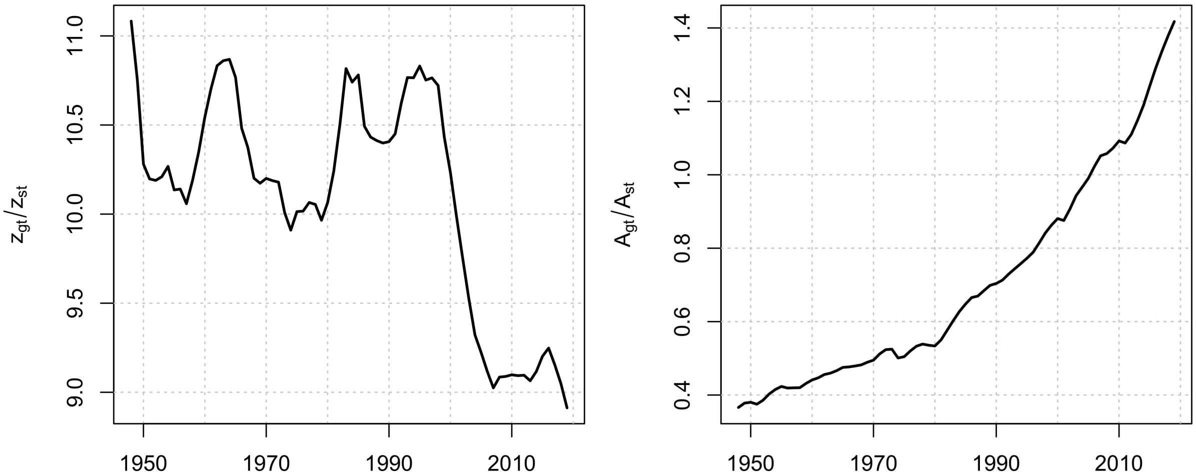
c/ℓ income effects dominate and thus ℓ_t falls as w_t rises. This is consistent with what we observe over time in aggregate data as can be seen in Figures 1c and 1d. Indeed, as will be discussed in Section 5.4.2 the substitutes model fitted to the expenditure condition in (14) cannot predict the observed decline in aggregate hours per worker, while the complements model can. Thus, we only present posterior distribution estimates taken from the estimation procedures where $\rho < 0$.

Services are slightly more time-intensive which can be seen by noting $\omega_g > \omega_s$ on average. Time is substantially more complementary with market consumption for services than goods, which can be seen by noting that $\nu_s << \nu_g$ when the simultaneous equations model is considered. Further, estimates of σ_{1g} and σ_{1s} suggest that we cannot reject the hypothesis that $\mathrm{Cov}(\epsilon_t^1, \epsilon_{jt}^2) = 0, \forall j \in \{g, s\}$. While the orthogonality condition may hold, information is lost when only estimating the household condition, which our robustness checks will reveal. Specifically, elasticity estimates are significantly affected by not accounting for price endogeneity, which can be seen by comparing the estimates for ρ, ν_g, and ν_s when MPL conditions are excluded from the estimation (Household MRS Only) with the simultaneous equations model in Table 2. This impacts the model's ability to predict labor hours.

5.4.1 Estimated Home and Sectoral Technological Change

Figure 3 compares the evolution of estimated in-home relative productivities versus sectoral relative productivities. In both panels we show the time series of estimated posterior means. Focussing first on Figure 3a we observe that the relative returns to in-home activities from using goods versus services has fallen since the late 1990s, despite the fact that goods are more efficiently produced over this period, as can be seen by noting the perpetual increase in A_{gt}/A_{st}. Thus, value-added to the consumer for services has improved relative to goods despite them becoming relatively less efficient to produce. Indeed, since 1997 we observe a 17.2% increase in the in-home efficiency of services relative to goods versus a 74% decline in the estimated efficiency of producing services relative to goods over this same period. This perhaps explains why consumers are willing to pay a relatively higher price for services: they get relatively more value out of them.

The analysis here is thus part of an emerging literature in economics that considers changes to the in-home efficiency of using certain types of products (see for example Boerma and Karabarbounis (2019), Fenton and Koenig (2020), and Kopytov, Roussanov, and Taschereau-Dumouchel (2020)). As Fenton and Koenig (2020) point out, the quality-adjusted price of a television has fallen by over 1000% since the 1950s. Kopytov, Roussanov, and Taschereau-Dumouchel (2020) highlight this result along with the proliferation of new content streaming and production services like Netflix and Amazon Prime Video to argue that both the choice set of off-market activities and the ability of consumers to enjoy those choices has expanded dramatically. The increase in entertainment content, counted as services, along with consumers' ability to enjoy it, is one factor that Kopytov, Roussanov, and Taschereau-Dumouchel (2020) attribute to declining labor market participation and labor hours. The point being made, however, has broader implications, which our model can help clarify: the often unmeasured returns to consumption can affect the ways individuals choose how to spend their time.



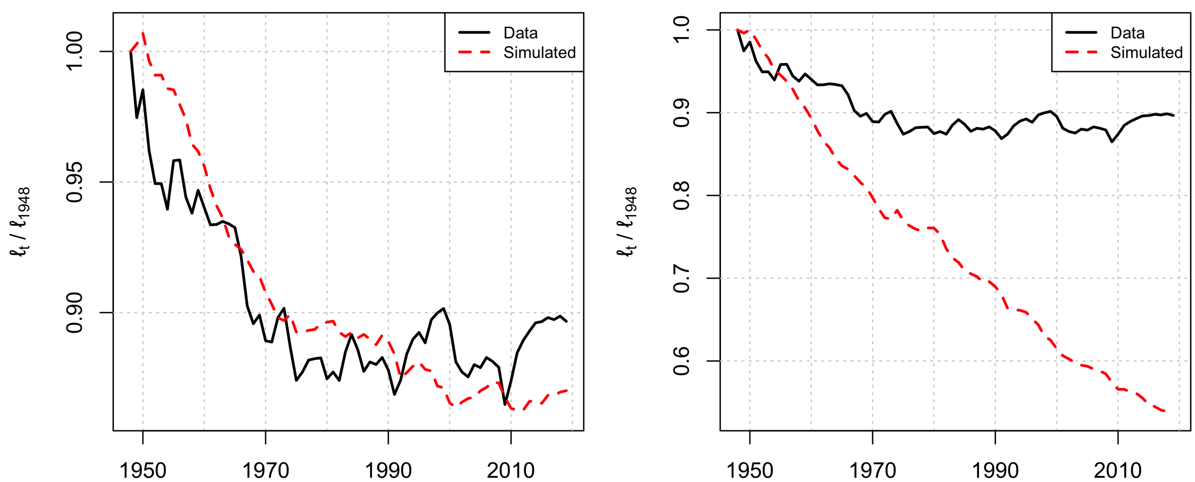
(a) Relative Home Productivities (b) Relative Sectoral Productivities

Figure 3: In this figure we present the posterior means of relative in-home productivities (a) and relative sectoral productivities (b) from the gross complements model with simultaneous equations.

Theory suggests that as P_{gt}/P_{st} declines, the sectoral TFP ratio A_{gt}/A_{st} should rise (Ngai and Pissarides 2007; Herrendorf, Rogerson, and Valentinyi 2014). As Ngai and Pissarides (2007) show, rising sectoral TFP ratios can cause a decline in prices even if preferences are homothetic. Indeed, when sectoral production functions are Cobb-Douglas with $\alpha_j = \alpha$, $P_{gt}/P_{st} = (A_{gt}/A_{st})^{-1}$ (Herrendorf, Rogerson, and Valentinyi 2014). Note that the posterior distributions of α_g and α_s contain substantial overlap, suggesting we may not be able to reject the hypothesis that $\alpha_g = \alpha_s$. We estimate Pearson's correlation coefficient between observed relative prices and the posterior mean of estimated relative TFP, finding a correlation of -0.936, suggesting that sectoral capital-intensities are very similar. Thus, rather than differential sectoral capital-deepening, as proposed by Acemoglu and Guerrieri (2008) as a cause of structural change, our findings suggest that differential sectoral-productivity growth is the primary cause of changing relative prices and thus the rising services share. This result thus conforms with those in Ngai and Pissarides (2007) and Kehoe, Ruhl, and Steinberg (2018) but also leaves little room for preference-based causes of structural change.

5.4.2 Robustness Checks with Aggregate Data

In calibration exercises common practice is to simulate the model to match specific moments in the data, then check those simulations against data moments that were not used in the initial fitting procedure. We can perform a similar exercise in our likelihood-based approach by comparing posterior simulations against data series associated with the equilibrium conditions that were left out of the estimation procedure due to stochastic singularities. Specifically, we assess how our aggregate estimation performs with respect to predicting labor hours and off-market time use.



(a) Simultaneous Equations (b) Household MRS Only

Figure 4: Here, we compare the simulated fit of normalized ℓ_t relative to data when the parameters are estimated under a likelihood function formed around the MRS condition featuring relative expenditure. In panel (a) we show the fit of the simultaneous equations model, and panel (b) features the fit of the partial-equilibrium model. In each figure the dashed red line corresponds to the mean of the posterior distribution of the simulated data conditional upon parameters.

Model-implied Aggregate Labor Hours: Since the classic c/ℓ income effect appears in aggregate data to dominate for labor hours decisions, we want to ensure that our model estimates are consistent with this fact. Note that the household's Marshallian labor supply can be written as a function of prices, wages, capital income net of savings which we denote by $\widetilde{y}_t$, and relative home productivities: $\ell_t(P_{gt}, P_{st}, w_t, \widetilde{y}_t, z_{gt}/z_{st})$.[16]

To test our model's performance with respect to predicting how household labor supply evolves over time, we simulate $\widehat{\ell}_t$ using productivities and parameters estimated with

[16]For derivation of the rather cumbersome parameterized version of this expression see Online Technical Appendix B.4.

the relative expenditure equation in (14). We perform these simulations for both the simultaneous equations model and household-only model when $\rho < 0$. Figure 4 compares trend decline in the posterior mean of simulated $\widehat{\ell}_t$ against data from 1948-2019, normalized so that both series are unity in 1948. The simultaneous equations model is a better fit for the non-targeted labor hours data with $\rho < 0$. Table 3 provides t-tests of the hypotheses that the period-t means of the posterior distribution of $\widehat{\ell}_t$ are the same as data. We cannot reject the hypotheses, suggesting a good fit. Finally, when simulating $\widehat{\ell}_t$ in the gross substitutes models with $\rho \in (0, 1)$ and simultaneous equations, the value hits the lower bound of zero in almost every period. Taken together these results thus support our preference for the simultaneous equations model with $\rho < 0$.

Table 3: Robustness Checks with Aggregate Data, Fit of Non-targeted Data Series, $\rho < 0$

	Simultaneous Equations		*Household MRS Only*	
Variable	S.E.[a]	Test Statistic	S.E.[a]	Test Statistic
ℓ_t	0.324	0.009	0.329	-3.55×10^{-4}
n_{gt}/n_{st}	9.668	-3.376×10^{-5}	99.913	-2.151×10^{-4}

Let M denote the total number of atomic draws (epochs) from the posterior HMC sampler, and let m index these draws. Suppose υ_t is the targeted data series and $\widehat{\upsilon}_t^m$ is a single epoch of the posterior sampler. For each t we compute the distribution of $\widetilde{\upsilon}_t^m = (\widehat{\upsilon}_t^m - \upsilon_t)/SE(\widehat{\upsilon}_t^m)/\sqrt{M}$, where $\widetilde{\upsilon}_t^m$ represents the weighted deviation of the simulated draw from the actual data. We test the hypothesis $H_0 : \overline{\widetilde{\upsilon}_t^m} = 0$, i.e. the mean of $\widetilde{\upsilon}_t^m$ is zero. Since a separate hypothesis test is conducted for each period t, here we average over the test statistics associated with those hypothesis tests over time. This statistic technically follows a Student's-t distribution with $M - 1$ degrees of freedom, where $M = 4000$, so the standard normal distribution is a sufficient benchmark for comparison.

[a] This is the average standard error over time: $\frac{1}{T}\sum_t SE(\widehat{\upsilon}_t^m)$.

Model-implied Relative Off-market Time Use, 2003-2019: We use the weighted average of our limited 2003-2019 off-market time use data series to perform a similar model fit assessment as above. Given posterior parameter and productivity estimates from the gross complements models, we simulate predicted $\widehat{n_{gt}/n_{st}}$ from the MRS condition for off-market time use in (9). We perform this exercise for both of the complements models and the substitutes model with simultaneous equations. The posterior distribution of implied relative off-market time use from the simultaneous equations model with $\rho \in (0, 1)$ has infinite variance, suggesting further the assumption that $\rho \in (0, 1)$ is incorrect. To understand how well both models with $\rho < 0$ fit the off-market time-use data, we use

t-tests to compare the posterior distributions against data each period from 2003-2019. The results of these tests are presented in Table 3, alongside the results of the same tests performed for $\widehat{\ell}_t$. As with labor supply, we cannot reject the hypotheses that the posterior means of simulated relative off-market time-use are equal to data.

Table 4: HMC Posterior Distribution, 1984-2019, CEX Micro Data w/o Durable Service Flows, $\rho < 0$

			Simultaneous Equations				
	Mean	S.D.	2.5%	25%	50%	75%	97.5%
ρ	-46.7432	23.8167	-107.7348	-56.1772	-41.0156	-30.6278	-18.6442
ν_g	-3.8061	3.8098	-14.2313	-4.6886	-2.7453	-1.5762	-0.3732
ν_s	-2.0590	1.6454	-6.1130	-2.3627	-1.7351	-1.3129	-0.3232
ω_g	0.0161	0.0153	0.0005	0.0049	0.0113	0.0224	0.0567
ω_s	0.0026	0.0024	0.0001	0.0008	0.0018	0.0035	0.0090
α_g	0.1170	0.0215	0.0766	0.1021	0.1169	0.1309	0.1603
α_s	0.0613	0.0121	0.0387	0.0530	0.0611	0.0693	0.0856
σ_1^2	0.0028	0.0003	0.0022	0.0026	0.0028	0.0030	0.0034
σ_g^2	0.0009	0.0001	0.0007	0.0008	0.0009	0.0010	0.0011
σ_s^2	0.0009	0.0001	0.0007	0.0008	0.0009	0.0010	0.0011
σ_{1g}	-0.0003	0.0002	-0.0005	-0.0004	-0.0003	-0.0002	0.0002
σ_{1s}	-0.0001	0.0001	-0.0002	-0.0001	-0.0001	-0.0001	0.0000
σ_{gs}	0.0003	0.0000	0.0002	0.0002	0.0003	0.0003	0.0003
$V(\mathcal{P})$	1517.0406	2.6204	1511.1848	1515.4457	1517.3573	1518.9858	1521.2151
			Households' MRS Only				
	Mean	S.D.	2.5%	25%	50%	75%	97.5%
ρ	-6.4679	4.6315	-18.5461	-7.6833	-5.1964	-3.6192	-2.0329
ν_g	-2.1646	2.6734	-9.1634	-2.5728	-1.3410	-0.7310	-0.2280
ν_s	-1.5896	1.8463	-6.6186	-1.8625	-1.0300	-0.5673	-0.1715
ω_g	0.0154	0.0148	0.0005	0.0046	0.0108	0.0213	0.0558
ω_s	0.0023	0.0023	0.0001	0.0007	0.0017	0.0032	0.0083
σ_1^2	0.0513	0.0056	0.0416	0.0473	0.0508	0.0546	0.0633
$V(\mathcal{P})$	151.6211	1.8424	147.4082	150.6639	151.9786	152.9598	154.1043

5.5 Model Estimates with Micro Data

In this section we estimate the household-only model using CEX spending ratios for the income quintiles as described in Section 4.2. Households are heterogeneous in wages w_{it} and their relative productivities z_{igt}/z_{ist}. The first-differenced model error ϵ^1_{it} is heterogeneous across households, though the variance of the error distribution is the same across all households.

Table 4 presents the posterior distribution estimates for the micro-data model with $\rho < 0$.[17] As with the household-only model with aggregate data featured in Table 2, note that $\nu_g < \nu_s$, which differs from the general equilibrium, simultaneous equations model with aggregate data. This is true in both the simultaneous equations and household-only models with micro data, so that we predict stronger complementarities between consumption and off-market time use for goods using micro data. Note that this could be a by-product of the fact that we do not have micro data on durables service flows, so that the prices and spending series with which we are estimating the model do not correspond exactly to those presented in the aggregate analysis. Indeed, when comparing Table 4 to parameter estimates from the aggregate model without durables service flows featured in Online Technical Appendix B.2, $\nu_g < \nu_s$ as here. For goods, complementarities between time and market commodities are stronger when durables are not accounted for. This suggests including durables service flows is important to accurately estimate structural elasticities.

5.5.1 Robustness Checks with Micro Data

We seek two assessments: the time-use fits by income quintile of the simultaneous equations model with parameters estimated from aggregate data and those same heterogeneous time-use fits with parameters estimated from the simultaneous equations model on micro data. Neither ℓ_{it} nor n_{igt}/n_{ist} are targeted in the likelihood functions. If predicted labor hours and/or off-market time-use by income quintile are closer to observed data under the aggregate parameterization, this helps support our preference for the aggregate fit when moving on to counterfactuals.

Table 5 shows estimated t-statistics and standard errors for posterior means of $\widehat{\ell}_{it}$ and $\widehat{n_{igt}/n_{ist}}$ against data. We find that there is little difference between the fit of the two models when comparing to micro data. The aggregate-estimated parameters applied to micro

[17] Given the robustness assessments from aggregate data suggest $\rho \in (0, 1)$ is implausible, we do not run estimation routines on micro-data under such a restriction.

data provide a good fit for both ℓ_{it} and n_{igt}/n_{ist}, as does the micro-estimated model.[18]

Table 5: Robustness Checks with Micro Data, Fit of Non-targeted Data Series, $\rho < 0$

	Aggregate Parameters[a]				*Micro Parameters*[b]			
	ℓ_{it}		n_{igt}/n_{ist}		ℓ_{it}		n_{igt}/n_{ist}	
Quintile	S.E.	Test Statistic	S.E.	Test Statistic	S.E.	Test Statistic	S.E.	Test Statistic
1^{st}	0.393	0.012	1.720	-0.025	0.359	0.025	1.331	-0.034
2^{nd}	0.359	0.011	1.217	-0.062	0.355	0.020	0.985	-0.078
3^{rd}	0.324	0.009	0.850	-0.101	0.333	0.015	0.703	-0.124
4^{th}	0.289	0.008	0.645	-0.147	0.293	0.011	0.545	-0.176
5^{th}	0.232	0.007	0.439	-0.209	0.210	0.007	0.383	-0.241

This table compares posterior simulated $\widehat{\ell}_{it}$ and $\widehat{n_{igt}/n_{ist}}$ against data, where $\widehat{\ell}_{it}$ is simulated from 1984-2019 and $\widehat{n_{igt}/n_{ist}}$ is simulated from 2003-2019 to match-up against CPS and ATUS data respectively.

[a] Parameters are taken from the simultaneous equations model estimated with aggregate data, excluding durables service flows. These estimates are featured in Online Technical Appendix B.2. Relative productivities are re-computed to fit CEX x_{igt}/x_{ist} for all income quintiles.

[b] Parameters and relative productivities are taken from the simultaneous equations model estimated with micro data.

6 Counterfactual Simulations

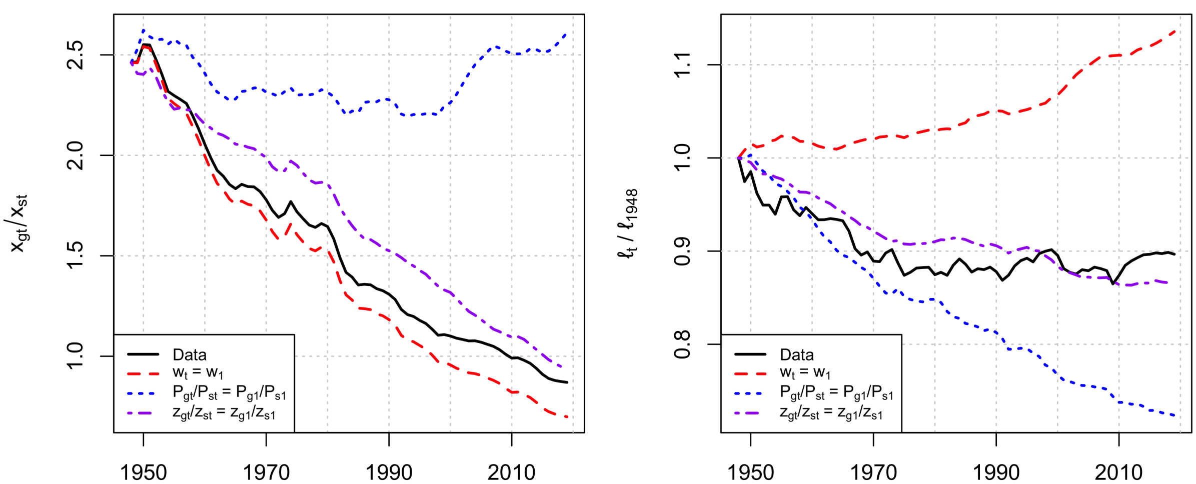
6.1 Implications from Aggregate Data

One goal of this paper is to better understand how wage growth and relative price changes affect aggregate demand allocations and labor hours when accounting for home production complementarities with off-market time. In this section we consider three counterfactual simulations using parameters estimated from the aggregate gross complements model with simultaneous equations. First, we fix aggregate wages at their 1948 levels $w_t = w_{1948}$, while allowing non-inflationary relative rices P_{gt}/P_{st} to evolve according to data. Second, we allow w_t to evolve as observed but fix relative prices $P_{gt}/P_{st} =$

[18]Note that in simulating ℓ_{it} we require an estimate of capital income net of savings $\widetilde{y}_{it}$ per household. Lacking such estimates we simulated ℓ_{it} for all income quintiles both with $\widetilde{y}_t$ per-capita computed from macro data and $\widetilde{y}_t = 0$. The results do not change, as ℓ_{it} appears not to be very sensitive to variation in $\widetilde{y}_{it}$.

$P_{g,1948}/P_{s,1948}$. In each of these scenarios we simulate the counterfactual series of x_{gt}/x_{st} and ℓ_t using the implied relative productivities as estimated from the model residuals. Third and finally, we assume relative home productivities remain fixed so that $z_{gt}/z_{st} = z_{g,1948}/z_{s,1948}$. These counterfactual exercises are partial equilibrium in nature and designed to isolate the effects of wage growth, relative price variation, and relative in-home productivity variation on aggregate demand. The goal of these exercises is to succinctly quantify which channel has exhibited the strongest influence on structural change.

Figure 5 presents the posterior means of our counterfactual simulations for relative expenditure and the trend in labor hours per effective full-time worker. Data are featured using black lines. The main takeaway is that relative price effects dominate the effects of wage growth in determining the long-run decline in x_{gt}/x_{st}. This can be seen by noting that in the case where wages are allowed to grow but relative prices are held fixed at their 1948 level, counterfactual relative spending (dashed blue line) barely declines at all. The counterfactual results regarding labor hours are somewhat mixed, as relative price declines and wage growth seem to have been competing with each other in the context of our model.



(a) Relative Expenditure

(b) Labor Hours Per Worker

Figure 5: Here we present data series for aggregate x_{gt}/x_{st} and ℓ_t against the posterior means of our counterfactual simulations. The difference between dashed blue and black lines corresponds to the substitution effect from relative price variation. The difference between the dashed red and black lines corresponds to the simultaneous income and substitution effects from wage variation. The difference between the dashed purple and black lines represents the effect of evolving home productivities.

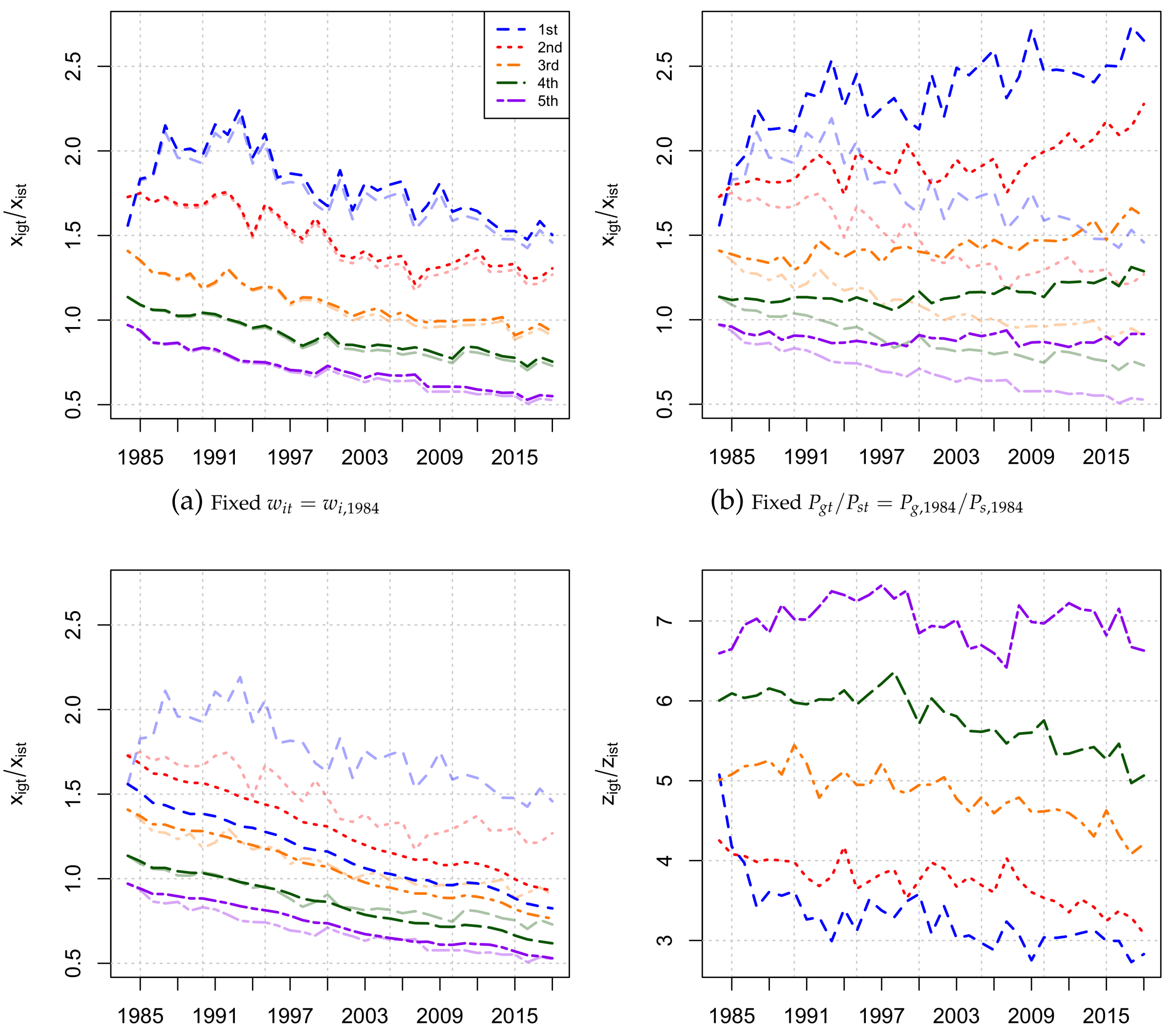
For detailed intuition, consider first the fixed wage scenario corresponding to the dashed red lines. When looking at the red lines we are observing how relative price and home-productivity variation have affected long-run outcomes, and theoretical intuition corresponds to that discussed in Section 3.2. From panel (b) it is clear that classic c/ℓ income effects have helped drive down labor hours since counterfactual ℓ_t rises instead of falls when wages do not grow. When w_t is fixed, the rise in ℓ_t is driven by falling P_{gt}/P_{st}, and we are thus observing the classic c/ℓ substitution effect in relative prices. Clearly, the classic c/ℓ income effect has dominated the classic c/ℓ substitution effect since the data actually fall, so wage variation has stronger influence over labor hours than relative price variation. Nonetheless, the partial correlation between P_{gt}/P_{st} and ℓ_t appears to be non-zero. By our parameter estimates $\nu_g > \nu_s$ and $\omega_g > \omega_s$ suggesting services are both more time-intensive and exhibit stronger complementarities with off-market time. Thus, as P_{gt}/P_{st} falls the market commodity associated with the more time-intensive task is becoming relatively more expensive, so that following the intuition from Section 3.2, n_{st} is rising but n_{gt} falls more than n_{st} rises. Consumers then increase q_{gt} and decrease q_{st} but make up for the declines in q_{st} by spending more time using services: think of switching from higher quality to lower quality services.

Now suppose w_t evolves as observed and relative prices remain fixed at their 1948 level. This simulation corresponds to the theoretical intuition discussed in Section 3.1 and is featured in the dashed blue lines. Note that the classic c/ℓ income effect is dominating, sending ℓ_t down. Time devoted toward services consumption rises, but x_{gt}/x_{st} changes very little, despite the fact that off-market time is being re-allocated to different tasks. The rise in w_t induces increased consumption, but strong gross complementarities between c_{gt} and c_{st} appear to dominate differences between the underlying home production processes. This is because when relative prices are constant, re-allocations are driven by differences in the strength of off-market time complementarities between the different processes. If gross complementarities are stronger, the time allocation will respond more to wage variation than the expenditure allocation, which is what we observe here.

Finally, note that fixing relative productivities has little affect on outcomes. Relative expenditure would have continued to decline, while labor hours would have still exhibited their partial, sideways J-shaped pattern. As an example, consider the increase in television quality captured by z_{gt} alongside the availability of new streaming entertainment content captured by z_{st}. Strong gross complementarities suggest that consumers prefer that quality improvement to both televisions and content roughly keep pace with each other. That is, demand for new higher-quality services depends on the availability of new higher-quality goods.

These results suggest that neither pure income effects nor changes to relative in-home productivities are responsible for the rising services share of consumption expenditure.

What we observe instead is that if wages had remained fixed at their 1948 level, the services share would have risen faster than the data, in contrast with other arguments in the literature. Declining relative prices are the primary cause of the rising services expenditure share.



(a) Fixed $w_{it} = w_{i,1984}$

(b) Fixed $P_{gt}/P_{st} = P_{g,1984}/P_{s,1984}$

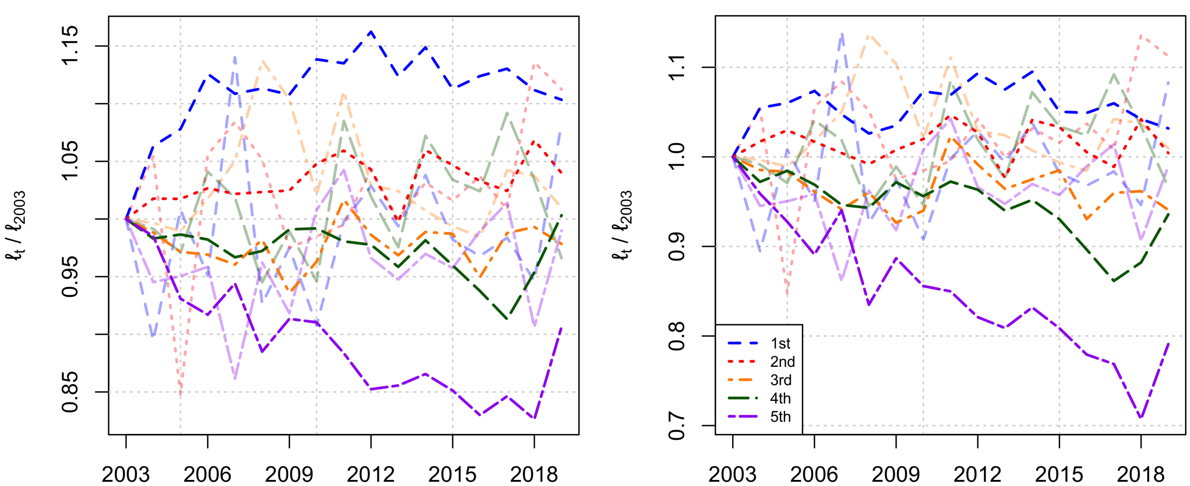
(c) Fixed $z_{igt}/z_{ist} = z_{ig,1984}/z_{is,1984}$

(d) z_{igt}/z_{ist}

Figure 6: In panels (a) through (c) we show the posterior means of counterfactual spending series in bold against their data counterparts in the same color and line-type scheme though faded in the background.

6.2 Implications from Micro Data

We draw the same conclusions regarding the causes of structural change when using the parameter and productivity estimates from micro data. Figure 6 shows the counterfactual series in bold against the corresponding data series in faded contrast, along with heterogeneous posterior means of z_{igt}/z_{ist}. Notice there is little difference between x_{igt}/x_{ist} for all i when holding wages fixed in panel (a). Relative price variation, shown in panel (b), again appears to have contributed to the rise in services share more than wage variation. In panel (c) we present the effect of holding z_{igt}/z_{ist} fixed, while the raw relative home productivities for heterogeneous agents are presented in panel (d). Notice from panel (d) that z_{igt}/z_{ist} are fairly flat, and so it is not surprising that holding z_{igt}/z_{ist} fixed at its estimated 1984 level has little impact on the trends of the spending series.



(a) Fixed $w_{it} = w_{i,1984}$

(b) Fixed $P_{gt}/P_{st} = P_{g,1984}/P_{s,1984}$

Figure 7: Here, we only present counterfactual variation in ℓ_{it} for the fixed wage and fixed relative-price cases. The posterior means of simulated counterfactual ℓ_{it} are normalized so that $\ell_{i,1984} = 1$.

Perhaps more interesting is the responsiveness to wage and relative price variation of the intensive margin of labor across the income distribution. Figure 7 shows that high-income households are more sensitive to wage and relative price variation. In panel (a) we observe the effect of wage variation on ℓ_{it}, and in panel (b) we observe the effect of relative price variation on ℓ_{it}. The counterfactuals suggest that the classic c/ℓ substitution effect dominates for high-income workers but the classic c/ℓ income effect dominates for low-income workers. This is because in panel (a) we observe low-income workers working more hours had their wages stayed at 1984 levels, while high-income workers would have worked less hours, suggesting that wage growth places upward pressure on high-income workers' labor time.

The effect of relative prices on ℓ_{it} has similar variability across the income distribution. Recall that relative price variation is also associated with classic c/ℓ income and substitution effects due to differentials in the time-use intensities of off-market activities. In panel (b) had the goods-to-services price ratio remained fixed at its 1984 level, high-income workers would have worked less, as consumption would not have been substituted away from services to goods. Since their income is rising the classic c/ℓ income effect dominates here as the substitution effect from relative price variation is turned off. We see little change in ℓ_{it} from data for the first quintile, suggesting that lower income consumers' labor supply is less sensitive to relative price variation.

7 Conclusion

We have shown that accounting for differential time-use complementarities in the consumption decision process can impact economic inference and thus conclusions regarding causality. This is especially true when considering which mechanisms are most responsible for the structural evolution of the U.S. economy from one previously dominated by the consumption of manufactured goods to today's service economy. The results presented here call into question the notion that rising incomes are responsible for changing tastes. Rather, the increase in the services share of expenditure appears to be a consequence of efficiency gains in goods production that have driven down relative prices and also driven workers seeking labor income toward the services sector.

While our results here utilize the limited time-use data that is available, this paper should encourage the stewards of data collection to continue measuring the time-utilization decisions of consumers. A longer horizon of time-use data that easily matches to consumption activities can be used in the future to help validate our results here. Indeed, we are encumbered by the relative shortness of the ATUS data series which limits analysis to the period since 2003, missing much of the major structural transformation that took place throughout the 70s, 80s, and 90s. Of course a longer time-use panel is valuable for many additional research questions as well, so the BLS should ensure the survey continues annually and its structure is relatively consistent over time.

The relationship between time-use and consumption lends itself to exploring many questions at the frontier of our field. Some software services companies like Google and Facebook offer base-level products for free but their revenues, via advertisements, depend on consumers choosing to spend time and engage with their software. Similarly, the COVID-19 pandemic has ushered in major changes with respect to the way we communicate with each other, an activity that is now directly associated with the utilization of a particular market service. What value do these services provide to the household beyond what is measurable from input and output data? How has aggregate welfare been affected by the proliferation of new services? To explore such questions require models with rich, off-market time-utilization structures, since the time-utilization component is such an important part of the consumption activities associated with these products. We thus hope that our work encourages future exploration of these interesting questions and future utilization of the classic, but durable, Beckerian model of home production.

A Proofs

Lemma 1. Shepherd's Lemma for off-market time use and wages is

$$n^h_{igt} + n^h_{ist} = \frac{\partial e_{it}}{\partial w_{it}}$$

Proof. Note that

$$e_{it}(P_{gt}, P_{st}, w_{it}, \overline{u}_{it}) = P_{gt}q^h_{igt} + P_{st}q^h_{ist} + w_{it}n^h_{igt} + w_{it}n^h_{ist} \tag{A.1}$$

where n^h_{ijt} are the Hicksian demands for off-market time use in process j. Differentiating (A.1) in w_{it} we get

$$\frac{\partial e_{it}}{\partial w_{it}} = P_{gt}\frac{\partial q^h_{igt}}{\partial w_{it}} + P_{st}\frac{\partial q^h_{ist}}{\partial w_{it}} + w_{it}\frac{\partial n^h_{igt}}{\partial w_{it}} w_{it}\frac{\partial n^h_{ist}}{\partial w_{it}} + n^h_{igt} + n^h_{ist} \tag{A.2}$$

Letting λ_{it} be the multiplier on the budget constraint, we replace prices with the first-order conditions from the UMP, where we set $w_{it} = \frac{\partial u}{\partial n^h_{ijt}}\frac{1}{\lambda_{it}}$, next to its corresponding Hicksian partial derivative:

$$\frac{\partial e_{it}}{\partial w_{it}} = \frac{\partial u}{\partial q^h_{igt}}\frac{1}{\lambda_{it}}\frac{\partial q^h_{igt}}{\partial w_{it}} + \frac{\partial u}{\partial q^h_{ist}}\frac{1}{\lambda_{it}}\frac{\partial q^h_{ist}}{\partial w_{it}} + \frac{\partial u}{\partial n^h_{ist}}\frac{1}{\lambda_{it}}\frac{\partial n^h_{igt}}{\partial w_{it}} + \frac{\partial u}{\partial n^h_{ist}}\frac{1}{\lambda_{it}}\frac{\partial n^h_{ist}}{\partial w_{it}} + n^h_{igt} + n^h_{ist} \tag{A.3}$$

By the fact that $u(q^h_{igt}, q^h_{ist}, n^h_{igt}, n^h_{ist}) = \overline{u}_{it}$ holds for all prices including wages, and given the Hicksian demand functions minimize the Lagrangian for the EMP:

$$\frac{\partial u}{\partial q^h_{igt}}\frac{1}{\lambda_{it}}\frac{\partial q^h_{igt}}{\partial w_{it}} + \frac{\partial u}{\partial q^h_{ist}}\frac{1}{\lambda_{it}}\frac{\partial q^h_{ist}}{\partial w_{it}} + \frac{\partial u}{\partial n^h_{ist}}\frac{1}{\lambda_{it}}\frac{\partial n^h_{igt}}{\partial w_{it}} + \frac{\partial u}{\partial n^h_{ist}}\frac{1}{\lambda_{it}}\frac{\partial n^h_{ist}}{\partial w_{it}} = 0 \tag{A.4}$$

□

Lemma 2. The Slutsky equations describing the responsiveness of demand q_{ijt} to wages w_{it} are

$$\frac{\partial q^m_{ijt}}{\partial w_{it}} = \frac{\partial q^h_{ijt}}{\partial w_{it}} - \frac{\partial q^m_{ijt}}{\partial y_{it}}(n_{igt} + n_{ist}), \qquad \forall j \in \{g, s\}$$

Proof. The proof is the standard one, where the version of Shepherd's Lemma used is that of Lemma 1. Note that

$$q^m_{ijt}\big(P_{gt}, P_{st}, w_{it}, e_{it}(P_{gt}, P_{st}, w_{it}, \overline{u}_{it})\big) = q^h_{ijt}(P_{gt}, P_{st}, w_{it}, \overline{u}_{it}) \tag{A.5}$$

and totally differentiate it to get

$$\frac{\partial q^m}{\partial w_{it}} + \frac{\partial q^m}{\partial y_{it}}\frac{\partial e_{it}}{\partial w_{it}} \tag{A.6}$$

Use Lemma 1 to replace $\frac{\partial e_{it}}{\partial w_{it}}$ then rearrange to get the result. □

Lemma 3. In a two-good, static economy with CES utility and Cobb-Douglas home production, suppose $\omega_s > \omega_g$. Then

$$\phi^n = \left[\frac{(1-\omega_s)[\omega_s/(1-\omega_s)]^{\rho\omega_s}}{(1-\omega_g)[\omega_g/(1-\omega_g)]^{\rho\omega_g}}\right]^{\frac{1}{\rho-1}} > \left[\frac{\omega_s[(1-\omega_s)/\omega_s]^{(1-\omega_s)\rho}}{\omega_g[(1-\omega_g)/\omega_g]^{(1-\omega_g)\rho}}\right]^{\frac{1}{\rho-1}} = \phi^q$$

Proof. Start with $1 > \omega_s > \omega_g > 0$.

$$\Rightarrow \quad \frac{\omega_g}{\omega_s} < \frac{1-\omega_g}{1-\omega_s} \tag{A.7}$$

Since $1-\rho > 0$ for all $\rho < 1$:

$$\begin{aligned}
&\Rightarrow \quad \left(\frac{\omega_g}{1-\omega_g}\right)^{1-\rho-\rho\omega_g+\rho\omega_g} < \left(\frac{\omega_s}{1-\omega_s}\right)^{1-\rho-\rho\omega_s+\rho\omega_s} && \text{(A.8)}\\
&\Leftrightarrow \quad \left(\frac{\omega_g}{1-\omega_g}\right)^{1-\rho\omega_g}\left(\frac{\omega_g}{1-\omega_g}\right)^{(\omega_g-1)\rho} < \left(\frac{\omega_s}{1-\omega_s}\right)^{1-\rho\omega_s}\left(\frac{\omega_s}{1-\omega_s}\right)^{(\omega_s-1)\rho} && \text{(A.9)}\\
&\Leftrightarrow \quad \left(\frac{\omega_g}{1-\omega_g}\right)\left(\frac{\omega_g}{1-\omega_g}\right)^{-\rho\omega_g}\left(\frac{\omega_g}{1-\omega_g}\right)^{(\omega_g-1)\rho} \\
&\qquad < \left(\frac{\omega_s}{1-\omega_s}\right)\left(\frac{\omega_s}{1-\omega_s}\right)^{-\rho\omega_s}\left(\frac{\omega_s}{1-\omega_s}\right)^{(\omega_s-1)\rho} && \text{(A.10)}\\
&\Leftrightarrow \quad \left[\frac{(1-\omega_s)[\omega_s/(1-\omega_s)]^{\rho\omega_s}}{(1-\omega_g)[\omega_g/(1-\omega_g)]^{\rho\omega_g}}\right] < \left[\frac{\omega_s[(1-\omega_s)/\omega_s]^{(1-\omega_s)\rho}}{\omega_g[(1-\omega_g)/\omega_g]^{(1-\omega_g)\rho}}\right] && \text{(A.11)}
\end{aligned}$$

Since $\frac{1}{\rho-1} < 0$ for all $\rho < 1$:

$$\left[\frac{(1-\omega_s)[\omega_s/(1-\omega_s)]^{\rho\omega_s}}{(1-\omega_g)[\omega_g/(1-\omega_g)]^{\rho\omega_g}}\right]^{\frac{1}{\rho-1}} > \left[\frac{\omega_s[(1-\omega_s)/\omega_s]^{(1-\omega_s)\rho}}{\omega_g[(1-\omega_g)/\omega_g]^{(1-\omega_g)\rho}}\right]^{\frac{1}{\rho-1}} \tag{A.12}$$

Thus, $\phi^n > \phi^q$. □

Lemma 4. In a two-good, static economy with CES utility and Cobb-Douglas home production, suppose $\omega_s > \omega_g$. For $\omega_s < \omega_g$ just exchange indices. Define the function

$$\Upsilon^n(P_g, P_s, w) = \phi^n P_g^{\frac{\rho\omega_g}{\rho-1}} P_s^{\frac{\rho\omega_s}{1-\rho}} w^{\frac{\rho(\omega_s-\omega_g)}{\rho-1}}$$

i. If $\rho \in (0,1)$ then Υ^n is decreasing in w.

ii. If $\rho < 0$ then Υ^n is increasing in w.

Proof. Note that

$$\frac{\partial \Upsilon^n}{\partial w} = \Upsilon^n(P_g, P_s, w)\left(\frac{\rho(\omega_s-\omega_g)}{\rho-1}\right)\left(\frac{1}{w}\right) \tag{A.13}$$

Clearly, $\Upsilon^n > 0$ and $w > 0$ always, so the sign of $\frac{\partial \Upsilon^n}{\partial w}$ hinges on the term $\frac{\rho(\omega_s-\omega_g)}{\rho-1}$.

i. If $\rho \in (0,1)$, $\rho - 1 < 0$ and $\rho(\omega_s - \omega_g) > 0$, so $\frac{\rho(\omega_s-\omega_g)}{\rho-1} < 0$.

ii. If $\rho < 0$, $\rho - 1 < 0$ and $\rho(\omega_s - \omega_g) < 0$, so $\frac{\rho(\omega_s-\omega_g)}{\rho-1} > 0$.

□

Lemma 5. In a two-good, static economy with CES utility and Cobb-Douglas home production, the Marshallian off-market time-utilization functions are

$$\begin{aligned}
n_s(P_g, P_s, w) &= \overline{n}\left[\Upsilon^n(P_g, P_s, w)\left(1 + \frac{\phi^q \omega_s}{\phi^n(1-\omega_s)}\right) + \frac{\omega_s}{1-\omega_s} + 1\right]^{-1}\\
n_g(P_g, P_s, w) &= \Upsilon^n(P_g, P_s, w) n_s(P_g, P_s, w)
\end{aligned}$$

Proof. Define the function

$$\Upsilon^q(P_g, P_s, w) = \phi^q P_g^{\frac{1-\rho+\rho\omega_g}{\rho-1}} P_s^{\frac{1-\rho+\rho\omega_s}{1-\rho}} w^{\frac{\rho(\omega_s-\omega_g)}{\rho-1}} \tag{A.14}$$

$$\Rightarrow \quad \Upsilon^q(P_g, P_s, w) = \frac{\phi^q}{\phi^n} \Upsilon^n(P_g, P_s, w)\left(\frac{P_s}{P_g}\right) \tag{A.15}$$

Dropping dependencies on prices and wages, we have the following implicit functions using (4), (5), and (6) from the main text:

$$q_j(n_j) = \left(\frac{\omega_j}{1-\omega_j}\right)\left(\frac{w}{P_j}\right) n_j \tag{A.16}$$

$$q_g(q_s) = \Upsilon^q q_s \tag{A.17}$$

$$n_g(n_s) = \Upsilon^n n_s \tag{A.18}$$

Starting with the derivation of n_s, note that $q_g\big(q_s(n_s)\big) = \Upsilon^q\left(\frac{\omega_s}{1-\omega_s}\right)\left(\frac{w}{P_s}\right)n_s$. Using the Beckerian budget constraint, $P_g q_q + P_s q_s + w(n_g + n_s) = \overline{n}$, we can substitute out q_g, q_s, and n_g using the implicit functions derived here, divide by w, substitute Υ^q for the expression in (A.15), and then rearrange to get the expression for $n_s(P_g, P_s, w)$. The expression for $n_g(P_g, P_s, w)$ follows directly from the relative off-market time use expression in (6). □

Corollary 1. In the two-good, static economy with CES utility and Cobb-Douglas home production, the Marshallian demand functions for market services and goods are

$$q_s(P_g, P_s, w) = w\overline{n}\left[P_s \frac{\phi^q}{\phi^n}\Upsilon^n(P_g, P_s, w)\left(1 + \frac{1-\omega_s}{\omega_s}\right) + P_s + P_s\left(\frac{1-\omega_s}{\omega_s}\right)\right]^{-1}$$

$$q_g(P_g, P_s, w) = \frac{\phi^q}{\phi^n}\Upsilon^n(P_g, P_s, w)\left(\frac{P_s}{P_g}\right) q_s(P_g, P_s, w)$$

Proof. Given the first-order conditions from the utility-maximization problem and using the definitions of Υ^n and Υ^q from Lemmas 4 and 5, we can write

$$q_g(q_s) = \Upsilon^q(P_g, P_s, w) q_s \tag{A.19}$$

$$n_s(q_s) = \left(\frac{P_s}{w}\right)\left(\frac{1-\omega_s}{\omega_s}\right) q_s \tag{A.20}$$

$$n_g\big(n_s(q_s)\big) = \Upsilon^n(P_g, P_s, w)\left(\frac{P_s}{w}\right)\left(\frac{1-\omega_s}{\omega_s}\right) q_s \tag{A.21}$$

Then we can plug the above objects into the Beckerian budget constraint to get:

$$P_g \Upsilon^q q_s + P_s q_s + P_s \Upsilon^n\left(\frac{1-\omega_s}{\omega_s}\right) q_s + P_s\left(\frac{1-\omega_s}{\omega_s}\right) q_s = w\overline{n} \tag{A.22}$$

Isolate q_s to get the result.

For q_g just plug in $q_s(P_g, P_s, w)$ to (A.19), substitute (A.15) for Υ^q, and the result is attained. □

Lemma 6. In a two-good, static economy with CES utility and Cobb-Douglas home production, the Marshallian labor supply function is

$$\ell(P_g, P_s, w) = \overline{n} - \Upsilon^n(P_g, P_s, w) n_s(P_g, P_s, w) - n_s(P_g, P_s, w)$$

Proof. From Lemmas 4 and 5 note that

$$n_g(P_g, P_s, w) = \Upsilon^n(P_g, P_s, w) n_s(P_g, P_s, w) \tag{A.23}$$

Using the time use constraint:

$$\ell(P_g, P_s, w) = \overline{n} - n_g(P_g, P_s, w) - n_s(P_g, P_s, w) \tag{A.24}$$
$$\Rightarrow \quad \ell(P_g, P_s, w) = \overline{n} - \Upsilon^n(P_g, P_s, w) n_s(P_g, P_s, w) - n_s(P_g, P_s, w) \tag{A.25}$$

□

Proposition 1. Fix prices P_g and P_s. In a two-good, static economy with CES utility and Cobb-Douglas home production, the intensive margin of labor varies in wages as follows:

i. If the outputs of home production are substitutes so that $\rho \in (0,1)$, ℓ is increasing in w and the classic c/ℓ substitution effect dominates.

ii. If the outputs of home production are complements so that $\rho < 0$, ℓ is decreasing in w and the classic c/ℓ income effect dominates.

Proof. Assume $\omega_s > \omega_g$, so that home production using goods is more time-intensive. For $\omega_s < \omega_g$ just exchange indices. Let $w' > w > 0$. We will prove each case separately. For this proof we will ignore the dependency of Υ^n on prices P_g and P_s to reduce notational clutter.

i. Suppose $\rho \in (0,1)$. Start with the fact that $\Upsilon^n(w') < \Upsilon^n(w)$ as shown in Lemma 4. Denote the constants $a = 1 + \frac{\phi^q \omega_s}{\phi^n (1-\omega_s)}$ and $b = 1 + \frac{\omega_s}{1-\omega_s}$. Since, by Lemma 3, $\phi^n > \phi^q$ then $(b-a) > 0$ which implies

$$\Upsilon^n(w')(b-a) < \Upsilon^n(w)(b-a) \tag{A.26}$$
$$\Leftrightarrow \quad \Upsilon^n(w)\Upsilon^n(w')a + \Upsilon^n(w')(b-a) + b < \Upsilon^n(w)\Upsilon^n(w')a + \Upsilon^n(w)(b-a) + b \tag{A.27}$$
$$\begin{aligned} \Leftrightarrow \quad & \Upsilon^n(w)\Upsilon^n(w')a + \Upsilon^n(w')b + \Upsilon^n(w)a + b \\ & < \Upsilon^n(w)\Upsilon^n(w')a + \Upsilon^n(w)b + \Upsilon^n(w')a + b \end{aligned} \tag{A.28}$$
$$\Leftrightarrow \quad \big(\Upsilon^n(w)a + b\big)\big(\Upsilon^n(w') + 1\big) < \big(\Upsilon^n(w')a + b\big)\big(\Upsilon^n(w) + 1\big) \tag{A.29}$$
$$\Leftrightarrow \quad \overline{n}\left(\frac{\Upsilon^n(w') + 1}{\Upsilon^n(w')a + b}\right) < \left(\frac{\Upsilon^n(w) + 1}{\Upsilon^n(w)a + b}\right)\overline{n} \tag{A.30}$$

Since $n_s(w) = \frac{\overline{n}}{\Upsilon^n(w)a+b}$ by Lemma 5

$$\Upsilon^n(w')n_s(w') + n_s(w') < \Upsilon^n(w)n_s(w) + n_s(w) \tag{A.31}$$
$$\Leftrightarrow \quad \overline{n} - \Upsilon^n(w')n_s(w') - n_s(w') > \overline{n} - \Upsilon^n(w)n_s(w) - n_s(w) \tag{A.32}$$

Since $\ell(w) = \overline{n} - \Upsilon^n(w)n_s(w) - n_s(w)$ by Lemma 6, then $\ell(w') > \ell(w)$.

ii. Suppose $\rho < 0$. Note that $\Upsilon^n(w') > \Upsilon^n(w)$ when $\rho < 0$ by Lemma 4, but $\phi^n > \phi^q$ by Lemma 3, so just flip the inequalities from case (i).

□

Proposition 2. Relative market purchases and off-market time use vary in wages as follows:

i. If $\rho \in (0,1)$ then market purchases and time use for the more time-intensive task fall relative to the less time-intensive task as w rises.

ii. If $\rho < 0$ then market purchases and time use for the more time-intensive task rise relative to the less time-intensive task as w rises.

Proof. Assume throughout the proof that $\omega_s > \omega_g$. For $\omega_s < \omega_g$ just exchange indices. Note that:

$$\frac{\partial(q_g/q_s)}{\partial w} = \Upsilon^q \frac{\rho(\omega_s - \omega_g)}{\rho - 1}\left(\frac{1}{w}\right) \tag{A.33}$$
$$\frac{\partial(n_g/n_s)}{\partial w} = \Upsilon^n \frac{\rho(\omega_s - \omega_g)}{\rho - 1}\left(\frac{1}{w}\right) \tag{A.34}$$

where $\Upsilon^q, \Upsilon^n > 0$ always.

i. Suppose $\rho \in (0,1)$. Clearly $\frac{\rho(\omega_s - \omega_g)}{\rho - 1} < 0$ so relative consumption and relative time use both fall.

ii. Suppose $\rho < 0$. Then $\frac{\rho(\omega_2 - \omega_1)}{\rho - 1} > 0$ so relative consumption and relative time use both rise.

Since g is assumed more time-intensive, this completes the proof. $\square$

Corollary 2. If the more time-intensive market commodity is more expensive than the less time-intensive commodity, and the relative price of the two exceeds the ratio $\frac{\phi^q}{\phi^n}$ then relative time use changes faster than relative consumption in response to wage increases.

Proof. Assume throughout the proof that $\omega_s > \omega_g$. For $\omega_s < \omega_g$ just exchange indices. Since g is more time intensive, $P_g > P_s$. Further, we are given:

$$\frac{P_g}{P_s} > \frac{\phi^q}{\phi^n} \tag{A.35}$$

By inspecting Proposition 2, it is clear that $\frac{n_g}{n_s}$ will change more than $\frac{q_g}{q_s}$ if and only if $\Upsilon^q < \Upsilon^n$. Starting with (A.35):

$$\Leftrightarrow \quad \phi^q P_g^{-1} < \phi^n P_s^{-1} \tag{A.36}$$

$$\Leftrightarrow \quad \phi^q P_g^{\frac{1-\rho+\rho\omega_g-\rho\omega_g}{\rho-1}} w^{\frac{\rho(\omega_s-\omega_g)}{\rho-1}} < \phi^n P_s^{\frac{1-\rho+\rho\omega_s-\rho\omega_s}{\rho-1}} w^{\frac{\rho(\omega_s-\omega_g)}{\rho-1}} \tag{A.37}$$

$$\Leftrightarrow \quad \phi^q P_g^{\frac{1-\rho+\rho\omega_g}{\rho-1}} P_s^{\frac{1-\rho+\rho\omega_s}{1-\rho}} w^{\frac{\rho(\omega_s-\omega_g)}{\rho-1}} < \phi^n P_g^{\frac{\rho\omega_g}{\rho-1}} P_s^{\frac{\rho\omega_s}{1-\rho}} w^{\frac{\rho(\omega_s-\omega_g)}{\rho-1}} \tag{A.38}$$

$$\Leftrightarrow \quad \Upsilon^q < \Upsilon^n \tag{A.39}$$

$\square$

Proposition 3. Marshallian demands for off-market time respond to wage increases as follows:

i. If $\rho \in (0,1)$ then time devoted to the more time-intensive task is inferior.

ii. If $\rho < 0$ then time devoted to the less time-intensive task is inferior.

Proof. Inferiority amounts to showing that Marshallian demand for time in category j is decreasing in $w\overline{n}$, which is total income. Since $\overline{n}$ is fixed we can simply show that demand for time is decreasing in w. Dropping dependencies on P_g and P_s for notational convenience, from Lemma 5, note that

$$\frac{\partial n_s}{\partial w} = -\frac{\left[n_s(w)\right]^2}{\overline{n}} \left(1 + \frac{\phi^q \omega_s}{\phi^n (1-\omega_s)}\right) \frac{\partial \Upsilon^n}{\partial w} \tag{A.40}$$

Suppose $\omega_s > \omega_g$ so process g is more time intensive.

i. Suppose $\rho \in (0,1)$. Then $\frac{\partial \Upsilon^n}{\partial w} < 0$ by Lemma 4, and $\frac{\partial n_s}{\partial w} > 0$. By Proposition 1 since ℓ is increasing in w, $\overline{n} - \ell$ is decreasing in w, which implies that n_g is decreasing in w since n_s is increasing. Thus, n_g is inferior.

ii. Suppose $\rho < 0$. Then $\frac{\partial \Upsilon^n}{\partial w} > 0$ by Lemma 4, and $\frac{\partial n_s}{\partial w} < 0$. By Proposition 1 since ℓ is decreasing in w, $\overline{n} - \ell$ is increasing in w, but n_s falls as w rises, so n_g must be increasing in w.

$\square$

Proposition 4. Marshallian demands for market purchases respond to wage increases as follows:

i. If $\rho \in (0,1)$ then the market purchase associated with the less time-intensive process is normal, but the market purchase associated with the more time-intensive process may, but need not, be inferior for certain prices and parameter combinations.

ii. If $\rho < 0$ then all market purchases are normal.

Proof. Differentiating the Marshallian demand for services in w from Corollary 1, we get:

$$\frac{\partial q_s}{\partial w} = \left(\frac{q_s}{w}\right)\left[1 - \left(\frac{\rho(\omega_s - \omega_g)}{\rho - 1}\right)\underbrace{\frac{P_s\frac{\phi^q}{\phi^n}\Upsilon^n + P_s\frac{\phi^q(1-\omega_g)}{\phi^n\omega_g}\Upsilon^n}{P_s\frac{\phi^q}{\phi^n}\Upsilon^n + P_s\frac{\phi^q(1-\omega_g)}{\phi^n\omega_g}\Upsilon^n + P_s + \frac{1-\omega_s}{\omega_s}P_s}}_{\kappa}\right] \tag{A.41}$$

First, suppose that $\omega_s > \omega_g$, so that goods are more time intensive. Note that $0 < \kappa < 1$, clearly. When $\rho \in (0,1)$, $\frac{\rho(\omega_s-\omega_g)}{\rho-1} < 0$, so the second term in (A.41) is positive. Thus $\frac{\partial q_s}{\partial w} > 0$, always.

Now, suppose without loss of generality, $\omega_s < \omega_g$, so that now services are more time intensive. We need only show that there exists one combination of parameters and prices, such that $\frac{\partial q_s}{\partial w} < 0$, so that the market purchase associated with the more time intensive task is inferior. Consider $P_g = P_s = w = \overline{n} = 1$, $\omega_s = 0.2$, $\omega_g = 0.75$, and $\rho = 0.9$. In this case $\frac{\partial q_s}{\partial w} \approx -0.102 < 0$. Thus, market purchases associated with the more time-intensive task are inferior. To show they need not be inferior, consider the same parameterization, except now let $\rho = 0.2$. Then $\frac{\partial q_s}{\partial w} \approx 0.094 > 0$.

ii. Again, first suppose that $\omega_s > \omega_g$, so that goods are more time intensive. Since $\frac{\rho(\omega_s-\omega_g)}{\rho-1} > 0$ and $0 < \kappa < 1$ always, it is sufficient to show that $\frac{\rho(\omega_s-\omega_g)}{\rho-1} < 1$ always. Note that since $0 < \omega_s - \omega_g < 1$ then $|\rho(\omega_s - \omega_g)| < |\rho - 1|$ which implies $\frac{\rho(\omega_s-\omega_g)}{\rho-1} < 1$.

Now suppose services are more time intensive so that $\omega_s < \omega_g$. Then $\frac{\rho(\omega_s-\omega_g)}{\rho-1} < 0$ and the result is proven.

□

Lemma 7. Since $\Upsilon^n(P_g, P_s, w)$ is homogeneous of degree 0, we can equivalently write the function with two relative-price arguments: $\widehat{\Upsilon}^n\left(\frac{P_g}{P_s}, \frac{w}{P_s}\right) = \Upsilon^n(P_g, P_s, w)$. Assume $\frac{w}{P_s}$ is fixed, so that w and P_s have the same rate of inflation. $\widehat{\Upsilon}^n$ varies in $\frac{P_g}{P_s}$ as follows:

i. If $\rho \in (0,1)$, $\widehat{\Upsilon}^n$ is decreasing in $\frac{P_g}{P_s}$.

ii. If $\rho < 0$, $\widehat{\Upsilon}^n$ is increasing in $\frac{P_g}{P_s}$.

Proof. First, note that

$$\Upsilon^n(P_g, P_s, w) = \phi^n P_g^{\frac{\rho\omega_g}{\rho-1}} P_s^{\frac{\rho\omega_s}{1-\rho}} w^{\frac{\rho(\omega_s-\omega_g)}{\rho-1}} = \phi^n\left(\frac{P_g}{P_s}\right)^{\frac{\rho\omega_g}{\rho-1}}\left(\frac{w}{P_s}\right)^{\frac{\rho(\omega_s-\omega_g)}{\rho-1}} = \widehat{\Upsilon}^n(P_g/P_s, w/P_s) \tag{A.42}$$

Differentiating $\widehat{\Upsilon}^n$ in P_g/P_s:

$$\frac{\partial\widehat{\Upsilon}^n}{\partial(P_g/P_s)} = \widehat{\Upsilon}^n\left(\frac{\rho\omega_g}{\rho-1}\right)\left(\frac{P_s}{P_g}\right) \tag{A.43}$$

which is clearly < 0 if $\rho \in (0,1)$ and > 0 if $\rho < 0$. □

Proposition 5. Relative market purchases and off-market time use vary in the relative price of market purchases as follows:

i. If $\rho \in (0,1)$ then market purchases and time use for the more time-intensive task rise relative to the less time-intensive task as the more time-intensive task becomes cheaper.

ii. If $\rho < 0$ then market purchases for the more time-intensive task rise relative to the less time-intensive task, but time use for the more time-intensive task relative to the less time-intensive task falls as the more time-intensive task becomes cheaper.

Proof. By Lemma 7, $\Upsilon^n(P_g, P_s, w)$ is homogeneous of degree 0. By the fact that $\Upsilon^q = \frac{\phi^q}{\phi^n}\Upsilon^n\left(\frac{P_s}{P_g}\right)$ it is also homogeneous of degree 0. Therefore, we can rewrite the relative demand and time use functions as follows:

$$\left(\frac{q_g}{q_s}\right) = \frac{\phi^q}{\phi^n}\widehat{\Upsilon}^n\left(\frac{P_g}{P_s}, \frac{w}{P_s}\right)\left(\frac{P_s}{P_g}\right) \tag{A.44}$$

$$\left(\frac{n_g}{n_s}\right) = \widehat{\Upsilon}^n\left(\frac{P_g}{P_s}, \frac{w}{P_s}\right) \tag{A.45}$$

Suppose without loss of generality $\omega_s > \omega_g$, so g is more time intensive. If this were not the case, just exchange indices. By Lemma 7, as P_g/P_s falls n_g/n_s rises when $\rho \in (0,1)$ and falls when $\rho < 0$. Relative quantities vary in P_g/P_s as follows:

$$\frac{\partial(q_g/q_s)}{\partial(P_g/P_s)} = \frac{\phi^q}{\phi^n}\left(\frac{P_s}{P_g}\right)\left[\frac{\partial\widehat{\Upsilon}^n}{\partial(P_g/P_s)} - \widehat{\Upsilon}^n\left(\frac{P_s}{P_g}\right)\right] \tag{A.46}$$

As long as the second term is < 0 then q_g/q_s will rise as P_g/P_s falls. When $\rho \in (0,1)$ this is clearly true since $\widehat{\Upsilon}^n(P_s/P_g) > 0$. Note that

$$\frac{\partial\widehat{\Upsilon}^n}{\partial(P_g/P_s)} - \widehat{\Upsilon}^n\left(\frac{P_s}{P_g}\right) = \widehat{\Upsilon}^n\left(\frac{P_s}{P_g}\right)\left[\frac{\rho\omega_g}{\rho-1} - 1\right] \tag{A.47}$$

which is < 0 as long as

$$\frac{\rho\omega_g}{\rho-1} < 1 \tag{A.48}$$

$$\Leftrightarrow \quad \frac{\rho\omega_g}{1-\rho} > -1 \tag{A.49}$$

$$\Leftrightarrow \quad \rho\omega_g > \rho - 1 \tag{A.50}$$

$$\Leftrightarrow \quad \omega_g < \frac{\rho-1}{\rho} \tag{A.51}$$

Since $\frac{\rho-1}{\rho} > 1$ this is always true. Thus q_g/q_s is declining in P_g/P_s always. □

Proposition 6. Marshallian demands for market purchases vary in relative prices as follows:

i. If $\rho \in (0,1)$, consumption of the less time-intensive market purchase falls while consumption of the more time-intensive purchase rises as the more time-intensive task becomes cheaper.

ii. If $\rho < 0$, consumption of both market purchases rises as the more time-intensive task becomes cheaper.

Proof. Suppose without loss of generality $\omega_s > \omega_g$, so g is more time intensive. If this were not the case, just exchange all indices. By the fact that they are classic Marshallian demand functions, $q_s(P_g, P_s, w)$ and $q_g(P_s, P_g, w)$ are homogeneous of degree 0. We can thus write $\widehat{q}_j(P_g/P_s, w/P_s) = q_j(P_g, P_s, w)$ for all $j \in \{g, s\}$. Referring back to Corollary 1 and Lemma 7, note that:

$$\frac{\partial\widehat{q}_s}{\partial(P_g/P_s)} = -(\widehat{q}_s)^2\left(\frac{P_s}{\overline{n}w}\right)\left(\frac{\phi^q}{\phi^n}\right)\left(\frac{1}{\omega_s}\right)\frac{\partial\widehat{\Upsilon}^n}{\partial(P_g/P_s)} \tag{A.52}$$

Since $\frac{\partial\widehat{\Upsilon}^n}{\partial(P_g/P_s)} < 0$ when $\rho \in (0,1)$, q_s falls as P_g/P_s falls, and the classic substitution effect appears to dominate for services consumption. This is only true when final goods are gross substitutes. When final goods are gross complements, the classic income effect dominates and $\widehat{q}_s$ rises as P_g/P_s falls, again by Lemma 7.

For q_g:

$$\begin{aligned}
\frac{\partial\widehat{q}_g}{\partial(P_g/P_s)} &= \left(\frac{\phi^q}{\phi^n}\right)\left[-\left(\frac{P_s}{P_g}\right)^2\widehat{\Upsilon}^n\widehat{q}_s + \left(\frac{P_s}{P_g}\right)\frac{\partial\widehat{\Upsilon}^n}{\partial(P_g/P_s)}\widehat{q}_s + \left(\frac{P_s}{P_g}\right)\widehat{\Upsilon}^n\frac{\partial\widehat{q}_s}{\partial(P_g/P_s)}\right] \\
&= \left(\frac{\phi^q}{\phi^n}\right)\left(\frac{P_s}{P_g}\right)\left[\widehat{\Upsilon}^n\widehat{q}_s\left(\frac{P_s}{P_g}\right)\left(\frac{\rho\omega_g}{\rho-1} - 1\right) + \widehat{\Upsilon}^n\frac{\partial\widehat{q}_s}{\partial(P_g/P_s)}\right] \\
&= \left(\frac{\phi^q}{\phi^n}\right)\left(\frac{P_s}{P_g}\right)^2\widehat{\Upsilon}^n\widehat{q}_s\left[\left(\frac{\rho\omega_g}{\rho-1} - 1\right) - \underbrace{\frac{\widehat{\Upsilon}^n\frac{\phi^q}{\phi^n\omega_s}}{\widehat{\Upsilon}^n\frac{\phi^q}{\phi^n\omega_s} + \frac{1}{\omega_s}}}_{\kappa}\left(\frac{\rho\omega_g}{\rho-1}\right)\right]
\end{aligned} \tag{A.53}$$

where the sign hinges on the term.

$$\frac{\rho\omega_g}{\rho-1}(1-\kappa) - 1 \tag{A.54}$$

$0 < \kappa < 1$ always. Note that

$$\frac{\rho\omega_g}{\rho - 1}(1-\kappa) < 1 \tag{A.55}$$

$$\Leftrightarrow \quad \frac{\rho\omega_g}{\rho - 1}(1-\kappa) > -1 \tag{A.56}$$

$$\Leftrightarrow \quad \rho\omega_g(1-\kappa) > \rho - 1 \tag{A.57}$$

When $\rho \in (0,1)$ the left side is positive and the right side is negative so the inequality holds. When $\rho < 0$, note that $|\rho\omega_g(1-\kappa)| < |\rho - 1|$, and the inequality holds. □

Proposition 7. Marshallian demands for off-market time vary in the relative price of market purchases as follows:

i. If $\rho \in (0,1)$, off-market time use for the less time-intensive task falls and time use for the more time-intensive task rises as the more time-intensive task becomes cheaper.

ii. If $\rho < 0$, off-market time use for the less time-intensive task rises and time use for the more time-intensive task falls as the more time-intensive task becomes cheaper.

Proof. Suppose without loss of generality $\omega_s > \omega_g$, so g is more time intensive. If this were not the case, just exchange all indices. By Lemma 5 prices only enter $n_s(P_g, P_s, w)$ via $\Upsilon^n(P_g, P_s, w)$. Since Υ^n is homogeneous of degree 0 by Lemma 7, then n_s is also homogeneous of degree 0. Thus we can write $n_s(P_g, P_s, w) = \widehat{n}_s(P_g/P_s, w/P_s)$. Differentiating in P_g/P_s:

$$\frac{\partial \widehat{n}_s}{\partial(P_g/P_s)} = -\frac{(\widehat{n}_s)^2}{\overline{n}}\left(1 + \frac{\phi^q\omega_s}{\phi^n(1-\omega_s)}\right)\frac{\partial\widehat{\Upsilon}^n}{\partial(P_g/P_s)} \tag{A.58}$$

Note that the sign of (A.58) hinges solely on the sign of $\frac{\partial\widehat{\Upsilon}^n}{\partial(P_g/P_s)}$. By Lemma 7, it thus follows that the level of n_s falls as P_g/P_s falls when $\rho \in (0,1)$ and it rises when $\rho < 0$.

For n_g note that

$$\frac{\partial \widehat{n}_g}{\partial(P_g/P_s)} = \frac{\partial\widehat{\Upsilon}^n}{\partial(P_g/P_s)}\widehat{n}_s + \widehat{\Upsilon^n}\frac{\partial \widehat{n}_s}{\partial(P_g/P_s)} \tag{A.59}$$

$$= \frac{\partial\widehat{\Upsilon}^n}{\partial(P_g/P_s)}\widehat{n}_s\left[1 - \widehat{\Upsilon^n}\left(1 + \frac{\phi^q\omega_s}{\phi^n(1-\omega_s)}\right)\left(\widehat{\Upsilon^n}\left(1 + \frac{\phi^q\omega_s}{\phi^n(1-\omega_s)}\right) + \frac{\omega_s}{1-\omega_s} + 1\right)^{-1}\right] \tag{A.60}$$

Note that the second term in (A.60) is > 0 always. Thus, the sign of $\frac{\partial\widehat{\Upsilon}^n}{\partial(P_g/P_s)}$ also governs how $\widehat{n}_g$ varies in P_g/P_s. When P_g/P_s falls and $\rho \in (0,1)$, $\widehat{n}_g$ rises. When P_g/P_s falls and $\rho < 0$, $\widehat{n}_g$ falls. □

Proposition 8. Marshallian labor supply varies in the relative price of market purchases as follows:

i. If $\rho \in (0,1)$, ℓ falls as the more time-intensive task becomes cheaper. Relative price variation thus induces a classic c/ℓ income effect which dominates.

ii. If $\rho < 0$, ℓ rises as the more time-intensive task becomes cheaper. Relative price variation thus induces a classic c/ℓ substitution effect which dominates.

Proof. Suppose without loss of generality $\omega_s > \omega_g$, so g is more time intensive. If this were not the case, just exchange all indices.

Note that the Marshallian labor supply function described in Lemma 6 is homogeneous of degree 0 and can be written:

$$\ell(P_g/P_s, w/P_s) = \overline{n} - \widehat{\Upsilon}^n(P_g/P_s, w/P_s)\widehat{n}_s(P_g/P_s, w/P_s) - \widehat{n}_s(P_g/P_s, w/P_s) \tag{A.61}$$

$$\Rightarrow \quad \frac{\partial \widehat{\ell}}{\partial(P_g/P_s)} = -\frac{\partial \widehat{\Upsilon}^n}{\partial(P_g/P_s)}\widehat{n}_s - \widehat{\Upsilon}^n \frac{\partial \widehat{n}_s}{\partial(P_g/P_s)} - \frac{\partial \widehat{n}_s}{\partial(P_g/P_s)} \tag{A.62}$$

$$= -\frac{\partial \widehat{\Upsilon}^n}{\partial(P_g/P_s)}\widehat{n}_s - \frac{\partial \widehat{n}_s}{\partial(P_g/P_s)}(\widehat{\Upsilon}^n + 1) \tag{A.63}$$

$$= -\widehat{\Upsilon}^n \left(\frac{\rho \omega_g}{\rho - 1}\right)\left(\frac{P_s}{P_g}\right)\widehat{n}_s + \frac{(\widehat{n}_s)^2}{\overline{n}}\left(1 + \frac{\phi^q \omega_s}{\phi^n(1-\omega_s)}\right)\frac{\partial \widehat{\Upsilon}^n}{\partial(P_g/P_s)}(\widehat{\Upsilon}^n + 1) \tag{A.64}$$

$$\begin{aligned} &= -\widehat{\Upsilon}^n \left(\frac{\rho \omega_g}{\rho - 1}\right)\left(\frac{P_s}{P_g}\right)\widehat{n}_s \\ &\qquad + \frac{(\widehat{n}_s)^2}{\overline{n}}\left(1 + \frac{\phi^q \omega_s}{\phi^n(1-\omega_s)}\right)\widehat{\Upsilon}^n\left(\frac{\rho \omega_g}{\rho - 1}\right)\left(\frac{P_s}{P_g}\right)(\widehat{\Upsilon}^n + 1) \\ &= \widehat{\Upsilon}^n \left(\frac{\rho \omega_g}{\rho - 1}\right)\left(\frac{P_s}{P_g}\right)\widehat{n}_s \underbrace{\left(\frac{\widehat{\Upsilon}^n\left(1 + \frac{\phi^q \omega_s}{\phi^n(1-\omega_s)}\right) + 1 + \frac{\phi^q \omega_s}{\phi^n(1-\omega_s)}}{\widehat{\Upsilon}^n\left(1 + \frac{\phi^q \omega_s}{\phi^n(1-\omega_s)}\right) + 1 + \frac{\omega_s}{1-\omega_s}} - 1\right)}_{\kappa} \end{aligned} \tag{A.65}$$

Note that $\kappa < 0$ always since $\phi^n > \phi^q$ by Lemma 3. Thus the sign of the derivative is governed by $\frac{\rho\omega_g}{\rho-1}$ which is < 0 when $\rho \in (0,1)$ and > 0 when $\rho < 0$. It thus follows that as P_g/P_s falls, ℓ falls when $\rho \in (0,1)$ and ℓ rises when $\rho < 0$. $\square$

References

Acemoglu, Daron, and Veronica Guerrieri. 2008. "Capital Deepening and Nonbalanced Economic Growth". *Journal of Political Economy* 116 (3). (Cit. on pp. 3, 7, 31).

Aguiar, Mark, and Erik Hurst. 2007. "Life-Cycle Prices and Production". *The American Economic Review* 97 (5). (Cit. on p. 4).

Aguiar, Mark, et al. 2017. "Leisure Luxuries and the Labor Supply of Young Men". *NBER Working Paper #23552*. (Cit. on p. 5).

Autor, David, and David Dorn. 2013. "The Growth of Low-Skill Service Jobs and the Polarization of the US Labor Market". *The American Economic Review* 103 (5): 1553–1597. (Cit. on p. 3).

Barro, Robert. 1984. *Macroeconomics*. Cambridge, MA: MIT Press. (Cit. on p. 5).

Becker, Gary. 1965. "A Theory of the Allocation of Time". *The Economic Journal* 75 (299). (Cit. on pp. 1, 4, 8).

Benhabib, Jess, Richard Rogerson, and Randall Wright. 1991. "Homework in Macroeconomics: Household Production and Aggregate Fluctuations". *Journal of Political Economy* 99 (6). (Cit. on pp. 4, 14).

Bernanke, Ben. 1985. "Adjustment Costs, Durables, and Aggregate Consumption". *Journal of Monetary Economics* 15:41–68. (Cit. on p. 4).

Betancourt, Michael, and Leo Stein. 2011. "The Geometry of Hamiltonian Monte Carlo". *arXiv:1112.4118*. (Cit. on p. 22).

Blundell, Richard, and Thomas Macurdy. 1999. "Labor Supply: A Review of Alternative Approaches". Chap. 27 in *Handbook of Labor Economics*, ed. by O. Ashenfelter and D. Card, vol. 3. Elsevier Science. (Cit. on p. 9).

Boerma, Job, and Loukas Karabarbounis. 2019. "Inferring Inequality with Home Production". *Working Paper*. (Cit. on pp. 4, 5, 30).

Boppart, Timo. 2014. "Structural Change and the Kaldor Facts in a Growth Model with Relative Price Effects and Non-Gorman Preferences". *Econometrica* 82 (6). (Cit. on pp. 2, 6, 7, 19).

Boppart, Timo, and Per Krusell. 2020. "Labor Supply in the Past, Present, and Future: A Balanced-Growth Perspective". *Journal of Political Economy* 128 (1): 118–157. (Cit. on p. 5).

Bridgman, Benjamin, George Duernecker, and Berthold Herrendorf. 2018. "Structural transformation, marketization, and household production around the world". *Journal of Development Economics* 133:102–126. (Cit. on p. 4).

Buera, Francisco, and Joseph Kaboski. 2009. "Can Traditional Theories of Structural Change Fit the Data?" *Journal of the European Economic Association* 7 (2-3): 469–477. (Cit. on p. 3).

— . 2012. "The Rise of the Service Economy". *The American Economic Review* 102 (6). (Cit. on p. 3).

Caselli, Francesco, and Wilbur Coleman. 2001. "The U.S. Structural Transformation and Regional Convergence: A Reinterpretation". *Journal of Political Economy* 109 (3). (Cit. on pp. 2, 3).

Comin, Diego, Danial Lashkari, and Martí Mestieri. 2015. "Structural Change with Long-run Income and Price Effects". *NBER Working Paper No. 21595*. (Cit. on pp. 2, 3).

Fang, Lei, Anne Hannusch, and Pedro Silos. 2020. "Bundling Time and Goods: Implications for Hours Dispersion". *Working Paper*. (Cit. on p. 4).

Fenton, George, and Felix Koenig. 2020. *Labor Supply and Innovation in Entertainment: Evidence from the US TV Rollout*. Tech. rep. Working Paper. (Cit. on pp. 5, 30).

Foellmi, Reto, and Josef Zweimuller. 2008. "Structural change, Engel's consumption cycles and Kaldor's facts of economic growth". *Journal of Monetary Economics* 55. (Cit. on p. 6).

Geary, R.C. 1950. "A Note on "A Constant-Utility Index of the Cost of Living"". *The Review of Economic Studies* 18 (1). (Cit. on p. 6).

Gelman, Andrew, et al. 2013a. "Computation in R and Stan". Chap. Appendix C in *Bayesian Data Analysis*, Third. Chapman & Hall. (Cit. on p. 22).

— . 2013b. "Computationally efficient Markov chain simulation". Chap. 12 in *Bayesian Data Analysis*, Third. Chapman & Hall. (Cit. on p. 22).

Gomme, Paul, Finn Kydland, and Peter Rupert. 2001. "Home Production Meets Time to Build". *Journal of Political Economy* 109 (5). (Cit. on p. 4).

Goolsbee, Austan, and Peter Klenow. 2006. "Valuing consumer products by the time spent using them: An application to the Internet". *American Economic Review* 96 (2): 108–113. (Cit. on p. 4).

Gordon, Robert. 2016. *The Rise and Fall of American Growth*. Princeton University Press. (Cit. on pp. 2, 5).

Graham, John, and Carole Green. 1984. "Estimating the Parameters of a Household Production Function with Joint Products". *The Review of Economics and Statistics* 66 (2). (Cit. on p. 4).

Greenwood, Jeremy, and Zvi Hercowitz. 1991. "The Allocation of Capital and Time over the Business Cycle". *Journal of Political Economy* 99 (6): 1188–1214. (Cit. on p. 4).

Greenwood, Jeremy, Richard Rogerson, and Randall Wright. 1995. "Household Production in Real Business Cycle Theory". In *Frontiers of Business Cycle Research*, ed. by Thomas Cooley, 157–174. Princeton: Princeton University Press. (Cit. on p. 4).

Greenwood, Jeremy, Ananth Seshadri, and Mehmet Yorukoglu. 2005. "Engines of Liberation". *Review of Economic Studies* 72:109–133. (Cit. on p. 4).

Gronau, Reuben. 1977. "Leisure, Home Production, and Work – the Theory of the Allocation of Time Revisited". *Journal of Political Economy* 85 (6). (Cit. on p. 4).

Herrendorf, Berthold, Christopher Herrington, and Ákos Valentinyi. 2015. "Sectoral Technology and Structural Transformation". *American Economic Journal: Macroeconomics* 7 (4): 104–133. (Cit. on p. 3).

Herrendorf, Berthold, Richard Rogerson, and Ákos Valentinyi. 2014. "Growth and structural transformation". In *Handbook of economic growth*, 2:855–941. Elsevier. (Cit. on pp. 7, 31).

— . 2018. *Structural change in investment and consumption: A unified approach*. Tech. rep. National Bureau of Economic Research. (Cit. on p. 7).

— . 2013. "Two Perspectives on Preferences and Structural Transformation". *American Economic Review* 103 (7). (Cit. on pp. 2, 3, 6).

Hymer, Stephen, and Stephen Resnick. 1969. "A Model of an Agrarian Economy with Nonagricultural Activities". *The American Economic Review* 59 (4): 493–506. (Cit. on p. 14).

Ingram, Beth, Narayana Kocherlakota, and N.E. Savin. 1997. "Using theory for measurement: An analysis of the cyclical behavior of home production". *Journal of Monetary Economics* 40:435–456. (Cit. on p. 4).

Jones, Charles. 2016. "The Facts of Economic Growth". In *Handbook of Macroeconomics*, ed. by John Taylor and Harald Uhlig, vol. 2. Amsterdam: North-Holland. (Cit. on p. 5).

Kehoe, Timothy, Kim Ruhl, and Joseph Steinberg. 2018. "Global Imbalances and Structural Change in the United States". *Journal of Political Economy* 126 (1): 761–796. (Cit. on pp. 2, 31).

King, Robert, Charles Plosser, and Sergio Rebelo. 1988. "Production, Growth and Business Cycles I. The Basic Neoclassical Model". *Journal of Monetary Economics* 21:195–232. (Cit. on p. 4).

Komunjer, Ivana, and Serena Ng. 2011. "Dynamic Identification of Dynamic Stochastic General Equilibrium Models". *Econometrica* 79 (6): 1995–2032. (Cit. on p. 23).

Kongsamut, Piyabha, Sergio Rebelo, and Danyang Xie. 2001. "Beyond Balanced Growth". *Review of Economic Studies* 68:869–882. (Cit. on pp. 2, 3, 6).

Kopytov, Alexandr, Nikolai Roussanov, and Mathieu Taschereau-Dumouchel. 2020. "Cheap Thrills: the Price of Leisure and the Global Decline in Work Hours". *Working Paper*. (Cit. on pp. 4, 5, 30).

Lewandowski, Daniel, Dorota Kurowicka, and Harry Joe. 2009. "Generating random correlation matrices based on vines and extended onion method". *Journal of Multivariate Analysis* 100 (9). (Cit. on p. 27).

Lucas Jr., Robert. 1988. "On the Mechanics of Economic Development". *Journal of Monetary Economics* 22:3–42. (Cit. on p. 4).

MaCurdy, Thomas. 1981. "An Empirical Model of Labor Supply in a Life-Cycle Setting". *Journal of Political Economy* 89 (6): 1059–1085. (Cit. on p. 5).

Mankiw, Gregory. 2010. *Macroeconomics*. 7th ed. New York: Worth. (Cit. on p. 5).

Mas-Colell, Andreu, Michael Winston, and Jerry Green. 1995. *Microeconomic Theory*. New York: Oxford University Press. (Cit. on p. 10).

Matsuyama, Kiminori. 2009. "Structural Change in an Interdependent World: A Global View of Manufacturing Decline". *Journal of the European Economic Association* 7 (2-3): 478–486. (Cit. on pp. 2, 6).

McGrattan, Ellen, Richard Rogerson, and Randall Wright. 1993. "Household Production and Taxation in the Stochastic Growth Model". *Federal Reserve Bank of Minneapolis: Staff Report #166*. (Cit. on p. 4).

Muellbauer, John. 1975. "Aggregation, Income Distribution, and Consumer Demand". *Review of Economic Studies* 62 (4): 526–542. (Cit. on p. 6).

— . 1976. "Community Preferences and the Representative Consumer". *Econometrica* 44 (5): 979–999. (Cit. on p. 6).

Neal, Radford. 2011. "MCMC using Hamiltonian dynamics". Chap. 5 in *Handbook of Markov Chain Monte Carlo*, ed. by Steve Brooks et al. Chapman & Hall. (Cit. on p. 22).

Ngai, Rachel, and Christopher Pissarides. 2007. "Structural Change in a Multisector Model of Growth". *American Economic Review* 97 (1). (Cit. on pp. 3, 7, 31).

— . 2008. "Trends in hours and economic growth". *Review of Economic Dynamics* 11:239–256. (Cit. on pp. 3–5).

Perli, Roberto, and Plutarchos Sakellaris. 1998. "Human capital formation and business cycle persistence". *Journal of Monetary Economics* 42:67–92. (Cit. on p. 4).

Porzio, Tommaso, Federico Rossi, and Gabriella Santangelo. 2020. "The Human Side or Structural Transformation". *Warwick Economics Research Papers, No. 1297*. (Cit. on p. 3).

Qu, Zhongjun. 2018. "A Composite Likelihood Framework for Analyzing Singular DSGE Models". *The Review of Economics and Statistics* 100 (5): 916–932. (Cit. on p. 24).

Ramey, Valerie. 2009. "Time Spent in Home Production in the Twentieth-Century United States: New Estimates from Old Data". *The Journal of Economic History* 69 (1): 1–47. (Cit. on p. 4).

Ramey, Valerie, and Neville Francis. 2009. "A Century of Work and Leisure". *American Economic Journal: Macroeconomics* 1 (2): 189–224. (Cit. on pp. 4, 5).

Ríos-Rull, José-Víctor. 1993. "Working in the Market, Working at Home, and the Acquisition of Skills: A General Equilibrium Approach". *The American Economic Review* 83 (4): 893–907. (Cit. on p. 4).

Rupert, Peter, Richard Rogerson, and Randall Wright. 1995. "Estimating Substitution Elasticities in Household Production Models". *Economic Theory* 6 (1). (Cit. on p. 4).

Stone, Richard. 1954. "Linear Expenditure Systems and Demand Analysis: An Application to the Pattern of British Demand". *The Economic Journal* 64 (255). (Cit. on p. 6).

Uy, Timothy, Kei-Mu Yi, and Jing Zhang. 2013. "Structural change in an open economy". *Journal of Monetary Economics* 60:667–682. (Cit. on pp. 2, 3).