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Central University of Finance and Economics, University College London

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Engaging Central Banks in Climate Change?

The Mix of Monetary and Climate Policy

Chuanqi Chen¹, Dongyang Pan², Raimund Bleischwitz³, Zhigang Huang⁴

Abstract

Given the recent debate on central banks’ role under climate change, this research theoretically investigates the mix of monetary and climate policy and provides some insights for central banks who are considering their engagement in the climate change issue. The “climate-augmented” monetary policy is pioneeringly proposed and studied. We build an extended Environmental Dynamic Stochastic General Equilibrium (E-DSGE) model as the method. By this model, we find the following results. First, the making process of monetary policy should consider the existing climate policy and environmental regulation. Second, the coefficients in traditional monetary policy can be better set to enhance welfare when climate policy is given. This provides a way to optimise the policy mix. Third, if a typical form climate target is augmented into the monetary policy rule, a dilemma could be created. This means that it has some risks for central banks to care for the climate proactively by using the narrow monetary policy. At the current stage, central banks could and should use other measures to help the climate and the financial stability.

Keywords: Central Bank, Climate Change, Monetary Policy, Climate Policy, E-DSGE

¹ PhD Candidate, School of Finance, Central University of Finance and Economics & Research Fellow, International Institute of Green Finance (中央财经大学绿色金融国际研究院)
² Corresponding author. PhD Candidate, Institute for Sustainable Resources, University College London & Research Fellow, International Institute of Green Finance, Email: d.pan.17@ucl.ac.uk
³ Professor, Institute for Sustainable Resources, University College London
⁴ Professor, School of Finance, Central University of Finance and Economics
1. Introduction

Should central banks engage in the climate change issue? In 2015, a report published by the Bank of England\(^5\) proposed that climate change could pose a risk to financial stability and economic development. Since then, and especially after the signing of the Paris Agreement, climate change and the broader environmental issue have become a factor that central banks are called on to consider. By forming the Network of Central Banks and Supervisors for Greening the Financial System (NGFS) in 2017 and the International Platform on Sustainable Finance (IPSF) in 2019, many central banks are starting to investigate ways to manage risks from climate change and to support a green economic transition. For instance, China’s central bank pioneered the field by supporting “green finance” via monetary policy in 2018\(^6\). This can be viewed as a kind of “climate-augmented” monetary policy, or “green monetary policy”. However, these arguments and actions do not mean that it is totally justifiable for central banks to engage in the climate change issue without condition. Some experts worry that such engagement could deviate central banks’ market neutrality and overburden their policy tools (violate the Tinbergen Rule). The momentum in policy practice and the debate on the feasibility of the engagement naturally raise the need of research on the monetary policy under climate change considerations.

In academia, the exacerbated climate change and environmental challenge has brought new waves of research in the “environmental macroeconomics” (Hassler et al., 2016). Since 2010, some theoretical frameworks have been founded and applied to assess how environmental risks and relevant policies could affect the macro-economy.


The “Environmental Dynamic Stochastic General Equilibrium (E-DSGE)” model has been newly developed as a mainstream method. Angelopoulos (2010), Fischer and Springborn (2011), Heutel (2012), Golosov et al. (2014), Doda (2014), Annicchiarico and DiDio (2015), and Dissou and Karnizova (2016) investigated relationships between greenhouse gas (GHG)/pollutant emissions and business cycles by setting GHG/pollutant as an externality in the economy and determined how environmental policies influence either fluctuation or economic growth. Other researchers have studied the effect of weather on economic volatility. Chen (2014) built a model with weather shocks embedded and found that it had good explanatory power for China’s business cycle. Gallic and Vermandel (2019) found that weather shocks account for a very significant proportion of economic volatility in the long run. Of those policy-related studies, two regimes of environmental policy, namely cap-and-trade (permitting) and taxing, are the main subjects of focus. For example, Golosov et al. (2014) tried to find the optimal level of taxing fossil fuels. Dissou and Karnizova (2016) compared the different implications of reducing CO2 emissions with carbon permits and carbon taxes in place.

At first glance, monetary policy and environmental issues are seemingly unrelated. However, such traditional notion starts changing. According to the above research, environment factors and policies are proven to influence either the fluctuation or the growth of the economy, which is exactly what monetary policy cares about. Hence, some researchers have started to investigate the role of central banks and monetary policy under climate change. Pioneering discussions, including Haavio (2010), Campiglio (2016), Ma (2017), McKibbin et al. (2017), and Bolton et al. (2020), have qualitatively explained the linking mechanism between monetary policy and climate change. Particularly, Krogstrup and Oman (2019) point out that the mix of macroeconomic and financial policies for climate change mitigation needs further investigation.

Quantitatively, Annicchiarico and DiDio (2017) were the first to use an E-DSGE
model to study the mix of monetary and climate policy. They compared three specific mixes and showed that the optimal monetary policies should be tightened slightly when GHG emissions are considered. Economides and Xepapadeas (2018) compared monetary policy both with and without considering climate change in the model and found that the reaction of monetary policy to economic shocks will be affected by climate change. Punzi (2019) introduced borrowing constraints and heterogeneous production sectors into the model to investigate green financing activity and found that only the differentiated capital requirement policy can sustain green financing. Huang and Punzi (2019) incorporated financial friction, according to Bernanke et al. (1999), and found that environmental regulations can accelerate the risks that the financial system faces. Chan (2020) introduced environmental targeting carbon taxation, fiscal, and monetary policies and compared their different effects in terms of improving the environment and welfare.

These scholars can be regarded to have started a new discussion on monetary policy and the environment. However, because of the growing global enthusiasm on sustainability, central banks are expected to respond to more concerns about this issue. It includes the macroeconomic and financial stability implications of climate change, the risks of stranded assets, the relationship between monetary policy and both climate change and climate policy, how to encourage green finance, the cost and benefit of “green monetary policy”, and many other aspects. Many specific concerns have not been touched upon by previous works.

In this research, we aim to investigate the relationship between and the mix of monetary and climate policy and provide some insights for central banks who are

considering their engagement in the climate change issue. We will answer three new
and relevant questions: (1) Whether and how monetary policy is influenced by climate
policy? (2) Whether and how monetary policy can be improved when the climate policy
is considered in the framework of analysis and whether there is an optimal monetary
policy? and (3) Should a central bank adopt a “climate-augmented” monetary policy or
use monetary policy to care for the climate proactively? By answering these questions,
we can understand how monetary policy can coordinate with climate policy and some
mechanisms of the policy mixing. The above research topic and questions mainly
extend that of Annicchiarico and DiDio (2017). We also extend it by working on more
research objects: more kinds of policy mixes that are closer to the real-world policy, not
only the mixes that contain Ramsey optimised policy.

Our method for research is an extended E-DSGE model. The basic DSGE setting
is in line with the standard New Keynesian framework. The basic “Environmental”
features are introduced following Annicchiarico and DiDio (2017) by incorporating the
GHG emissions from production, their negative externality on productivity, and the
climate policy that controls emissions, i.e., cap-and-trade or carbon tax. To consider the
environmental module in a more comprehensive way, we also introduce some novel
environmental features into the model: the concealed emissions, the potential penalty
for them, and the effectiveness of enforcement of such penalty. These concealed
emission-related features are omitted by traditional E-DSGE models, but actually
common in the reality and found to be nontrivial in the model economy.

Based on the E-DSGE model, we first mix monetary policy [of Taylor rule type
(Taylor, 1993), which is a close approximation of the real-world] with different types
of climate policy and compare these different mixes to see if climate policy can
influence monetary policy. The impulse responses of major economic and
environmental variables to shocks and the conditional welfare and consumption
equivalents are calculated. The results show that when monetary policy is mixed with
different types of climate policy under different effectiveness of environmental
regulation, its dynamic changes. Therefore, the making process of monetary policy should consider the existing climate policy and environmental regulation.

We then explore a traditional way to improve the mix of monetary and climate policy. This is to optimise the coefficients in the Taylor rule of monetary policy. The results show that the coefficients can always be better set to enhance welfare when a certain regime of climate policy is considered in the framework. If the cost-push shock is dominant in the economy, optimal coefficients exist. Both the climate policy regime and the effectiveness of environmental regulation can affect the value of the optimal coefficients.

Finally, we propose to improve the policy mix by introducing a radically “climate-augmented” monetary policy, which can help determine whether it is good for a central bank to use monetary policy to care for the climate proactively. This is to introduce an emission gap target into the Taylor rule of monetary policy. The results show that the welfare of the economy can be enhanced when monetary policy is augmented by the new target and the coefficient of the target is set in a specific interval. However, under some circumstances, such monetary policy will create a dilemma for central banks. This indicates a risk if we directly use the narrow monetary policy to care for the climate.

The novelty of this research lies in three aspects. First, the research topic. Besides being among the first within the emerging discussion and modelling work on monetary policy in the context of climate change, this research investigates three important questions (see above) that are newly emerging in policy making and pioneeringly studies the “‘climate-augmented’ monetary policy” in a formal model. This help answer questions raised by the NGFS. Second, the research scope. Extending Annicchiarico and DiDi (2017) who considered either monetary or climate policy as Ramsey type in the policy mix, we work on mixes with both the two policies non-Ramsey optimised, which can better represent the real-world. Third, the research method. The traditional E-DSGE model is firstly enriched with concealed emission-related features so that its environmental module is more comprehensive and closer to the reality.
The paper proceeds as follows. Section 2 describes the extended E-DSGE model. Section 3 compares the mixes of monetary policy with different climate policies. Section 4 investigates the optimisation of policy mixes. Sections 5 concludes.

2. Model

We construct an extended E-DSGE model based on the New Keynesian framework. GHG emissions from production, their negative externality on productivity, and environmental policies that control emissions are introduced following Annicchiarico and DiDio (2017). Innovatively, concealed (illegal) emissions, the potential penalty for them, and the effectiveness of enforcement of such penalty are set into the model by extending the enterprise sector and environmental authority. This is to depict the reality that in many countries the environmental regulation is not very strict, and firms have some space to emit more than the legal level.

The introduction of concealed emission-related features enriches the traditional E-DSGE models, making it more comprehensive and closer to the reality. Such introduction is also found to be nontrivial for answering our research questions. The potential penalty for concealed emissions can be regarded as another dimension of environmental regulation, in addition to the traditional climate policy (carbon tax or cap-and-trade).

2.1 Household

A representative household maximises its expected lifetime utility, which is determined by consumption $C_t$ and labour $L_t$ and has the form of

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t S_t \left( \ln C_t - \mu_t \frac{L_t^{1+\eta}}{1+\eta} \right) \right\}$$

(1)

where $0 < \beta < 1$ is the discount factor, $\eta \geq 0$ is the inverse of the elasticity of labour supply, and $\mu_t > 0$ is the coefficient of disutility of labour. $S_t$ represents the
stochastic shocks of time-preference, which follows $\ln S_t = \rho_S \ln S_{t-1} + (1 - \rho_S) \ln S + e_{S,t}$ to evolve, where $0 < \rho_S < 1$ and $e_{S,t} \sim \text{i.i.d.} \mathcal{N}(0, \sigma^2_S)$.

The budget constraint of the household is

$$P_tC_t + R_t^{-1}B_{t+1} = B_t + W_tL_t + D_t + P_tT_t$$

(2)

where $P_t$ is the price of final good, $B_t$ and $B_{t+1}$ are the nominal quantity of riskless bonds at period $t$ and $t + 1$, $R_t$ is the riskless interest rate of the bonds which is determined by the central bank, $W_t$ is the nominal wage of labour, $D_t$ denotes the nominal dividend derived from enterprises, and $T_t$ is the lump-sum transfer from government.

At the optimum we have the following first-order conditions

$$\beta R_t \mathbb{E}_t \left[ \frac{S_{t+1}}{S_t} \frac{C_t}{C_{t+1}} \frac{1}{\Pi_{t+1}} \right] = 1$$

(3)

$$L^\eta_t = \frac{W_t}{\mu_t P_t C_t}$$

(4)

where $\Pi_{t+1} = P_{t+1}/P_t$ is the inflation of period $t + 1$. Equation (1) is the Euler equation, and equation (2) is the labour supply equation.

2.2 Enterprise and the Environment

Consistent with the standard New Keynesian framework, the enterprise sector is formed by final good and intermediate good producers. The final good $Y_t$ is produced by competitive firms using the Constant Elasticity of Substitution (CES) technology

$$Y_t = \left[ \int_0^1 Y_{j,t}^{\theta_t-1} \frac{\theta_t}{\theta_t-1} dj \right]^{\theta_t}$$

(5)

where $Y_{j,t}$ denotes the intermediate goods produced by monopolistically competitive firms, and the subscript $j \in [0,1]$ denotes the intermediate good firms of a continuum. $\theta_t > 1$ is the elasticity of substitution and is also a stochastic process that describes the cost-push shock (Smets and Wouters (2003)). It follows $\ln \theta_t = \rho_{\theta} \ln \theta_{t-1} + (1 - \rho_{\theta}) \ln \theta + e_{\theta,t}$ with $0 < \rho_{\theta} < 1$ and $e_{\theta,t} \sim \text{i.i.d.} \mathcal{N}(0, \sigma^2_{\theta})$.

Final good producers maximise their profit, which is determined by
The first-order condition yields the demand function for intermediate goods
\[ Y_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\theta_t} Y_t \]  
(7)

and
\[ P_t = \left[ \int_0^1 P_{j,t}^{1-\theta_t} dj \right]^{\frac{1}{1-\theta_t}} \]  
(8)

which implies that the price of final good \( P_t \) is also the price level.

A typical intermediate good firm has a production function
\[ Y_{j,t} = A_t A_t L_{j,t} \]  
(9)

where \( A_t \) is the total factor productivity (TFP) factor or technology that follows a stochastic process \( \ln A_t = \rho_A \ln A_{t-1} + (1 - \rho_A) \ln A + e_{A,t} \), in which \( 0 < \rho_A < 1 \) and \( e_{A,t} \) is a random variable following \( N(0, \sigma_A^2) \). Following Golosov et al. (2014), \( A_t \) is a damage coefficient that describes the negative externality of GHG emissions on productivity (TFP damage coefficient). It is the pivot linking the economy and the environment. \( A_t \) is determined by the stock of emissions following
\[ A_t = e^{-\chi(M_t - \bar{M})} \]  
(10)

where \( M_t \) is the stock of emissions of period \( t \), \( \bar{M} \) is the level before the industrial revolution, and \( \chi > 0 \) measures the intensity of negative externality.

According to Heutel (2012), GHG emissions are a by-product of the production process. The original emissions from production are \( Z_{j,t}^{ori} \) which is proportional (measured by \( \varphi \)) to the volume of output of intermediate firms
\[ Z_{j,t}^{ori} = \varphi Y_{j,t} \]  
(11)

To dispose of the original emissions, a firm has three channels to use and trade-off: abate emission, emit legally and pay tax, and conceal emission. A firm can choose to abate a percentage of \( U_{t,j} \) \( (0 \leq U_{t,j} \leq 1) \) of the original emissions which will bring a marginal increasing cost of \( \phi_1 U_{j,t} \phi_2 Y_{j,t} \), where \( \phi_1 = \phi_1' \varphi > 0 \) and \( \phi_2 > 1 \) are cost.
coefficients. A firm can also choose to legally emit some original emissions. This requires a firm to pay a carbon tax or buy an emission permit in the cap-and-trade system (depending on the climate policy regime) at a price $p_{Z,t}$ for every unit of GHG emissions.

The novelty of our model is the introduction of **concealed emitting channel and related environmental regulation.** Normally, a government or environmental authority cannot detect every source of pollution. So, firms have some space to emit secretly and provide artificially low legal emission data, making their real emission higher than the legal level which they have either paid tax or bought a permit. The secret or concealed emissions will save some costs for either emission abating or legal emitting. Meanwhile, the concealed emissions are subject to potential fine. Although a government may not be able to spot every concealed emission, they usually have some degree of regulation on such emissions and will pose some costs (most commonly penalty or prosecution) to the emitters spotted. A recent example of the concealed emission and the related regulation is the Volkswagen emissions scandal in 2015. The Volkswagen company concealed their cars’ excessive emissions by technical manipulation for years. It was detected by chance and then the company has faced a huge amount of fine by governments.

To abstract the above, we assume that firms (as a whole) have the concealed emitting channel to dispose of the original emissions; the government spots the concealed emissions with a certain probability (the lower, the weaker the effectiveness of environmental regulation on emissions). If spotted, the government penalises the firm with a certain amount of fine (the fewer, the weaker the effectiveness). To model this, we assume that a firm faces an expected fine volume that equals to $\frac{\psi}{2} V_{t,j} \phi Y_{j,t}$, where $\phi Y_{j,t} = Z_{j,t}^{ori}$ is the original emissions; $0 \leq V_{t,j} \leq 1$ is the proportion of concealed emissions in the original emissions; $\psi > 0$ is defined as the “Effectiveness of Enforcement of Environmental Regulation” (EOEER), which is proportional to the
probability of the government spotting concealed emissions and the amount of the fine
for every unit of concealed emissions. Using the \( \frac{\psi}{2} V_{t,j}^2 \phi Y_{j,t} \) term as the volume of the
fine is derived from a simple intuition: the more concealed emissions that are emitted
and spotted or the more effective the enforcement of environmental regulation, then the
greater the fine. \( V_{t,j} \) is quadratic in the term to describe that the total amount of fine is
marginally increasing with regard to \( V_{t,j} \) — the more that a firm emits concealingly,
the easier are the emissions to be spotted. A number \( \frac{1}{2} \) is put into the term to simplify
calculation.

The introduction of concealed emissions and EOEER relaxes the hidden
assumption of the perfect effectiveness of environmental regulation in most previous
E-DSGE models and makes the environmental regulation in our study more
comprehensive and closer to the reality. Such introduction is nontrivial for answering
our specific research questions, as we will show that the differences in EOEER will
make the regimes of climate policy either more similar or more different and further
influence the dynamics of financial and economic variables (see Subsection 3.3). The
potential penalty for concealed emissions can be regarded as another dimension of
environmental regulation, in addition to the traditional climate policy (carbon tax or
cap-and-trade).

The three channels by which firms can dispose of their original emissions, namely
emission abating, legally emitting, and concealingly emitting, have now all been
explained. This is helpful for illuminating the following variables. The real emission
\( Z_{j,t}^{\text{real}} \) is the amount of GHG that is really emitted to the atmosphere and can be
monitored by the government. It equates to the original emissions \( Z_{j,t}^{\text{ori}} \) minus the
abated emissions \( Z_{j,t}^{\text{abate}} = U_{t,j} \phi Y_{j,t} \). The claimed emissions \( Z_{j,t}^{\text{claimed}} \) is the amount
of GHG emissions that a firm reports to the government concealing its concealed
emissions \( Z_{j,t}^{\text{concealed}} \). It is the amount of legal emissions \( Z_{j,t}^{\text{legal}} \) and also the amount
of tax or permit that a firm needs to either pay or buy ($p_{Z,t}Z_{j,t}^{\text{legal}}$). It equals to the real emissions minus the concealed emissions. Accordingly, we have

$$Z_{j,t}^{\text{real}} = Z_{j,t}^{\text{ori}} - Z_{j,t}^{\text{abate}} = (1 - U_{t,j})\varphi Y_{j,t} = Z_{j,t}^{\text{legal}} + Z_{j,t}^{\text{illegal}} \quad (12)$$

$$Z_{j,t}^{\text{claimed}} = Z_{j,t}^{\text{real}} - Z_{j,t}^{\text{illegal}} = (1 - U_{t,j} - V_{t,j})\varphi Y_{j,t} = Z_{j,t}^{\text{legal}} \quad (13)$$

The above relationship is illustrated in Figure 1.

Figure 1: The relationship among emission variables

Considering the cost of disposing of emissions via the three channels and the sticky pricing assumption in the standard New Keynesian framework (Rotemberg, 1982), the objective of an intermediate firm is to maximise

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \prod_{0,t} \left[ \frac{P_{j,t}}{P_t} Y_{j,t} - TC_{j,t} - \gamma \left( \frac{P_{j,t}}{P_{j,t-1}} - 1 \right)^2 Y_t \right] \right\} \quad (14)$$

which is subject to

$$TC_{j,t} = W_{t}L_{j,t} + \prod_{1,t} Y_{j,t} + \prod_{Z,t}(1 - U_{j,t} - V_{j,t})\varphi Y_{j,t} + \psi^2 Y_{j,t} \varphi Y_{j,t} \quad (15)$$

where $\prod_{0,t} = \beta^t \frac{C_0}{C_t}$ is the stochastic discount factor.

The above settings and assumptions yield the following first-order conditions (more details in Appendix)

$$\left( 1 - \theta_t \right) - \gamma(\Pi_t - 1)\Pi_t + \beta \gamma \mathbb{E}_t \left[ \frac{C_t}{C_{t+1}} (\Pi_{t+1} - 1)\Pi_{t+1} \frac{Y_{t+1}}{Y_t} \right] + \theta_t MC_t = 0 \quad (16)$$
\[ MC_t = \frac{W_t}{A_t \Delta t} + \phi_1 U_t^{\phi_2} + p_{Z,t}(1 - U_t - V_t)\varphi + \frac{\psi}{2} V_t^2 \varphi \] (17)

\[ p_{Z,t} = \frac{1}{\varphi} \phi_3 U_t^{\phi_2 - 1} \] (18)

\[ V_t = \frac{1}{\psi \varphi} \phi_1 \phi_2 U_t^{\phi_2 - 1} = \frac{p_{Z,t}}{\psi} \] (19)

where \( MC_t \) is the marginal cost of production, \( \gamma > 0 \) is the price adjusting cost coefficient, and \( \Pi_t = \frac{P_t}{P_{t-1}} \) denotes inflation. Equation (16) is the New Keynesian Phillips Curve.

2.3 Monetary and Environmental Authorities

The monetary policy authority (central bank) decides the nominal interest rate following a traditional Taylor rule

\[ R_t = \left( \frac{\Pi_t}{\Pi} \right)^{\rho_{\Pi}} \left( \frac{Y_t}{Y_{t-1}} \right)^{\rho_{\gamma}} \] (20)

where \( Y_{t-1} \) is the natural output without price stickiness, \( R \) and \( \Pi \) are the steady state of nominal interest rate and inflation, and \( \rho_{\Pi} \) and \( \rho_{\gamma} \) are the intensity coefficients for targeting on inflation and output gap, respectively. The Taylor rule type monetary policy is a closer approximate of the real-world than the Ramsey monetary policy. We do not consider the latter in this research.

The environmental authority decides the climate policy regime. In this research we analyse two major regimes: cap-and-trade (CA regime) and carbon tax (TX regime).

Under the CA regime, the environmental authority sets an emission cap \( Z_{t, ca}^{cap} \) and sells emission permits to the market at a price decided by the market competition. In equilibrium, the total legal emissions \( Z_{t, legal}^{cap} \) equates to \( Z_{t, ca}^{cap} \). Under the TX regime, the authority sets a fixed carbon tax level for every unit of legal emissions. The authority does not set a ceiling for total legal emissions. We also include climate policy regimes of no control on emissions (NO regime) and of Ramsey optimal control (RM regime) in the following analysis, but mainly for benchmarking and comparison purpose.
Besides choosing the climate policy regime, the environmental authority fines firms if concealed emissions are spotted. The earnings of the authority, including the income from selling emission permits or levying a carbon tax and from the fines are transferred to households directly.

### 2.4 Market Clearing and Aggregation

In equilibrium, we have the market clearing condition

\[ Y_t = C_t + \phi_1 U_t^{\phi_2} Y_t + \frac{\phi}{2} (\Pi_t - 1)^2 Y_t \]  

(21)

Following Rotemberg (1982), we assume that all the firms are symmetrical. So, the gross variables share the same form of expressions with individual variables. The total production function is

\[ Y_t = \Lambda_t A_t L_t \]  

(22)

The totalities of emissions are

\[ Z_t^{legal} = \int_0^1 Z_{ij,t}^{legal} dj = (1 - U_t - V_t)\varphi Y_t \]  

(23)

\[ Z_t^{real} = \int_0^1 Z_{ij,t}^{real} dj = (1 - U_t)\varphi Y_t \]  

(24)

The total transfer is

\[ T_t = p_{Z,t} Z_t^{legal} + \frac{\psi}{2} \nu_t^2 \varphi Y_t \]  

(25)

The total stock of emissions is

\[ M_t = (1 - \delta_M)M_{t-1} + Z_t^{real} + \bar{Z} \]  

(26)

where \( \bar{Z} \) is the emissions from nature without human influence, and \( 0 < \delta_M < 1 \) is the natural rate of decay of GHG stock.

### 2.5 Calibration

We calibrate the parameters as follows and list them in Table 1. Following Gali (2015), the discount factor \( \beta \) is set as 0.99, the elasticity of substitution in steady state \( \theta \) is set as 6, and the inverse of the Frisch elasticity \( \eta \) is set as 1. The adjusting cost coefficient \( \gamma \), which measures price stickiness, is set as 58.25 so that the stickiness has...
a duration of three quarters when it is converted into Calvo pricing. The disutility coefficient of labour $\mu_L$ is set as 24.9983 so that the steady state of labour is 0.2 without monopoly. Following tradition, the persistent coefficients of shocks (including TFP shock, preference shock, and cost-push shock) are set as 0.9, and the Taylor-rule elasticities (coefficients) of monetary policy $\rho_\Pi$ and $\rho_Y$ are set as 1.5 and 0.5, respectively, in Section 3. Following Annicchiarico and DiDio (2017), the scale coefficient of abatement cost $\phi_1$ is set as 0.185, and the elasticity $\phi_2$ is set as 2.8.

The parameter determining the damage caused by emissions on output $\chi$ is set as 0.000457. Following Heutel (2012), the decay rate of emission stock $\delta_M$ is set as 0.0021. Following Xu et al. (2016), the coefficient measuring the original emissions per unit of output $\varphi$ is set as 0.601. As for the EOEER $\psi$, according to the proportion of the “environmental penalties” collected by the government in total GDP in China, which is approximately 0.01%, the $\psi$ should be approximately 0.45. This is within the magnitude of 0.1 to 1. For comparison purposes, we need to set a large $\psi$ and a small $\psi$. Considering the magnitude, the benchmark of $\psi$ (in Subsection 3.1 and 3.2) is set as 1, which is the upper bound of the magnitude, and the value describing a relative ineffective regulation is set as 0.1 (in Subsection 3.3), which is the lower bound.

Table 1: Calibrated values of the parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$ Discount factor</td>
<td>0.99</td>
<td>$\beta = \frac{1}{1+\rho}$, where risk-free (pure time preference) discount rate $\rho \approx 1%$</td>
</tr>
<tr>
<td>$\eta$ Inverse of the Frisch elasticity,</td>
<td>1</td>
<td>Literature</td>
</tr>
<tr>
<td>$\mu_L$ Disutility coefficient of labour</td>
<td>24.9983</td>
<td>Steady labour time is 0.2 under fully competition market</td>
</tr>
<tr>
<td>$\theta$ Elasticity of substitution in steady state</td>
<td>6</td>
<td>Literature</td>
</tr>
<tr>
<td>$\gamma$ Adjusting cost coefficient of sticky</td>
<td>58.25</td>
<td>Literature</td>
</tr>
</tbody>
</table>

### 3. The Mixes of Monetary Policy with Different Climate Policies

In this section, we mix the monetary policy with four different types of climate policies: cap-and-trade, carbon tax, no control (with climate policy absent), and Ramsey optimal, and compare the mixes in terms of differences in fluctuation and welfare. We also consider the differences brought by the (in)effectiveness of enforcement of environmental regulation. The comparison in this section will show whether and how the monetary policy will vary when the type of climate policy and the effectiveness of environmental regulation are different. This is an extension of Annicchiarico and Di Dio (2017), also a prerequisite for optimising the policy mixes in Section 4.

#### 3.1 Fluctuation Comparison

Annicchiarico and Di Dio (2017) started investigating the mixes of monetary...
policy and climate policy by considering one policy as the Ramsey type and the other as varying types. They showed that key macroeconomic variables, including labour, emissions, interest rate, and inflation, respond differently to a productivity shock when the policy type differs. Their work is an inspiring start on such issue, meanwhile, can be extended or improved in some respects. First, at least one policy was assumed as the Ramsey type in any mix they studied. This type of policy is the ideal optimisation but difficult to carry out directly in reality. The mix that purely consists of practically realisable policies is not studied. So, such real-world practical policy mixes can be further investigated. Second, the potential ineffectiveness of environmental regulation that could change the dynamics of the economy can be considered additionally. This relaxes the hidden assumption of the perfect effectiveness of environmental regulation. Third, the regimes with “no climate policy” and “Ramsey climate policy” can be introduced into the comparison to serve as benchmarks.

We still compare the response of key macroeconomic variables to the productivity shock, but extend the work of Annicchiarico and DiDio (2017) by including the mixes of Taylor rule type monetary policy with four different types of climate policy (constituting four regimes) with consideration of the EOEER. The four types of climate policy include cap-and-trade, carbon tax, no control and Ramsey optimal (see Appendix for equations). The first three and the Taylor rule monetary policy are all commonly implemented in the real-world. In this subsection, we compare the fluctuation of the economy in different regimes via impulse response analysis. To be specific, we give a 1% positive TFP shock and then find the dynamics of economic variables. Here, the EOEER $\psi$ is set as 1 as a benchmark. The values of tax level and emission target are set so that all regimes (except for the NO regime\(^9\)) share the same steady state with the case of Ramsey.

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\(^9\) The No Control regime is equivalent to a TX regime with a tax level at 0. This makes the steady state different and predefined.
The results of impulse response analysis (absolute deviation from steady states) are shown in Figure 2. It can be found that the responses of endogenous variables to the shock have different paths under the four different regimes. For economic and monetary variables, output under the CA regime increases by less than under the RM regime, whereas output under the TX regime increases by more than under the RM regime. The TFP damage coefficient ($\lambda_t$), inflation, and the resulting interest rate under the CA regime drop less than under the RM regime, whereas under the TX regime the negative changes are larger than is the case under the RM regime. For environmental related variables, abatement, concealed emissions, and emission price under the CA regime rise by more than under the RM regime, whereas, under the TX regime they either change less than under the RM regime or do not change. Legal emissions and real emissions under the TX regime increase by more than under the RM regime, whereas, under the CA regime, real emissions rise by less than under the RM regime, and legal emissions do not change.

The differences between regimes (note the scales of the y-axes) are not large, because the environmental-related disruption and costs (for abatement, emissions, and fines) are relatively small under current parameters. The differences could be more significant in the future if the climate change problem becomes more serious. Since it could aggravate the external shock (e.g., severer weather extremes) and increase the emission-related costs.

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10 The standard deviation of $\lambda_t$ is less than 0.00027 under the CA and TX regimes. The proportion of environmental-related costs to output (GDP) at steady state is less than 0.7%.
To understand the mechanism behind the differences of the changes, we first need to understand that after a positive TFP shock, emission prices and real emissions will rise under the RM regime. When the shock happens, every unit of output will have a lower cost. This decreases the price level and increases the demand. An increased demand causes an increased supply or output. When the level of output increases, the original emissions from production also increase. This can cause a higher marginal damage to TFP, so the Ramsey optimization requires a higher rate of abatement \( U_t \). According to equation (18), the emission price \( p_{Z,t} \) also needs to be higher simultaneously under the RM regime. To dispose of the extra original emissions from production under the RM regime, firms will be arranged to use all three channels — namely abating, legally emitting, and concealedly emitting — as all the channels have
an increasing marginal cost for society. Hence, abatement, legal emissions, and concealed emissions will all rise. As a result, **real emissions, which equates to the sum of legal and concealed emissions, will also rise** under the RM regime.

Then, the differences between the CA and TX regimes can be explained. Under the TX regime (and the NO regime), the **emission prices** (for legal emissions) are fixed at the carbon tax level (or 0), irrespective of how much firms emit. After a shock, this is lower than the Ramsey optimal (increased) emission price. The relative lower emission price has three implications: (1) On **output**. As the emission price is fixed, its marginal level is also fixed and equates to the tax level. At optimum, the costs of all three channels for disposing of the original emissions from production share this same marginal level. The costs for disposing of every unit of emissions via concealed emitting and abatement are marginal increasing; hence, the average cost of these two channels is lower than the tax level. Given that the tax level is lower than is the Ramsey optimal emission price, the average cost for disposing of every unit of emission via all three channels is less than is the case under the RM regime. When the unit emission cost is lower, the price level decreases, which causes a higher demand for production output. So, it is higher than is the case under the RM regime. (2) On **real emissions** and the **TFP damage coefficient**. The relatively lower costs of disposing of legal and concealed emissions allow real emissions, which is the sum of legal and concealed emissions, to rise by more than is the case under the RM regime. Real emissions accumulate into emission stock and directly decrease the TFP damage coefficient (N.B., it is negative). Therefore, the TFP damage coefficient drops by more than it does under the RM regime. (3) On **legal emissions, abatement, and concealed emissions**. With a lower emission price, the legal emissions increase by more than they would under the RM regime. When relatively more original emissions from production are disposed of via the legal emitting channel, a lesser amount of emissions need to be disposed of via the other two channels, namely abating and concealed emitting. This causes the abatement and concealed emissions to increase by less than is the case under the RM
regime. (4) On inflation and interest rate. A lower than RM regime emission price causes a lower marginal cost of production, and then a lower inflation and lower interest rate in succession. Hence, both the change in inflation and the change in interest rate are lower than their changes under the RM regime.

Under the CA regime, the mechanism of change is the antithesis of that under the TX regime. The legal emissions volume is fixed at a target, so it is lower than the new Ramsey optimal (increased) legal emissions’ level. After the shock and the rise of original emissions, the concealed emitting and abatement channels need to dispose of more emissions than is the case under the RM regime. This leads to higher marginal disposing costs of these two channels. At optimum, the costs of all three channels for disposing of the original emissions share a same marginal level, hence the emission price (for legal emissions) rises higher than is the case under the RM regime. The higher than RM regime emission price (which is opposite to the lower than RM regime price under the TX regime) has implications for the endogenous variable that are exactly antithetical to those under the TX regime. Therefore, there are differences in the changes between the CA and TX regimes. Meanwhile, we can say that there exists a “price level-offsetting” effect in the CA regime that can better stabilise the economy when a shock happens. This is because the fixed legal emission volume causes a higher/lower price for disposing of emissions and offsets the lowering/heightening price level (and also attenuates monetary policy). Under the TX regime, the fixed carbon price does not have such a function.

In general, the above analysis shows that when monetary policy is mixed with different climate policies, the monetary policy itself (interest rate) and the effect of the policies on the economy (other endogenous variables) will differ in facing a TPF shock. Under the TX regime, the monetary policy (interest rate) is strengthened compared with under the RM regime and, meanwhile, the TX regime-type climate policy is looser than is the RM regime-type (real emissions too high and abatement too low). Conversely, under the CA regime the monetary policy is weakened; the CA regime-type climate
The above analysis conveys two key messages: (1) The cap-and-trade regime of climate policy could offset the price fluctuation after shocks and become an attenuator for monetary policy. (2) The making process of monetary policy should consider the existing regime of climate policy, as the dynamic of monetary policy is influenced by the selection of climate policy.

3.2 Welfare Comparison

To further investigate the above policy mixes, we compare the welfare of the four regimes in addition to the above fluctuation analysis. This will help us find which of the four mixes are better and which are worse.

In the comparison, we maintain all parameters, including the coefficients in the Taylor rule and the EOEER, fixed. We set the steady states of the CA and TX regimes equal to that of the RM regime. The steady state of the NO regime comes from the \( p_{Z,t} = 0 \) case of the TX regime. So, the differences in welfare between the CA, TX, and RM regimes are due only to the difference in regime. We follow the welfare criterion of Mendicino and Pescatori (2007) and calculate the conditional welfare of individuals. The expression is

\[
W_j = \mathbb{E}_t \sum_{m=0}^{\infty} \beta^m \left( \ln c_{j,t+m} - \mu_L \frac{l_{j,t+m}^{1+\eta}}{1+\eta} \right) \tag{27}
\]

where \( W_j \) is the conditional welfare, and \( j = \{\text{NO, TX, CA, RM}\} \) means the four regimes of climate policy: no control, carbon tax, cap-and-trade, and Ramsey optimal.

To show results more intuitive, we also calculate the consumption equivalent (CE) of each case. CE is the additional fraction of consumption that households under no policy can obtain if a certain policy is introduced for them. Let

\[
W'_j = \mathbb{E}_t \sum_{m=0}^{\infty} \beta^m \left[ \ln \left(1 + CE_j'\right) c_{NO,t+m} - \mu_L \frac{l_{NO,t+m}^{1+\eta}}{1+\eta} \right] \tag{28}
\]

we have
\[ CE_{j'} = \exp\{(1 - \beta)(W_{j'} - W_{NO})\} - 1 \]  

where \( j' = \{TX, CA, RM\} \) represents a certain regime of climate policy.

The welfare of all four regimes and the corresponding CEs are shown in Table 2.

**Table 2: Welfare and Consumption Equivalents of the four regimes**

<table>
<thead>
<tr>
<th></th>
<th>Welfare</th>
<th>CE</th>
</tr>
</thead>
<tbody>
<tr>
<td>NO</td>
<td>-59.469</td>
<td>0</td>
</tr>
<tr>
<td>TX</td>
<td>-58.583</td>
<td>0.0088972</td>
</tr>
<tr>
<td>CA</td>
<td>-58.585</td>
<td>0.0088727</td>
</tr>
<tr>
<td>RM</td>
<td>-58.566</td>
<td>0.0090715</td>
</tr>
</tbody>
</table>

We can find

\[ W_{RM} > W_{TX} > W_{CA} > W_{NO} \]  

and

\[ CE_{RM} > CE_{TX} > CE_{CA} > CE_{NO} \]

Specifically: (1) Any regime with a climate policy has better welfare than has the NO regime, as any climate policy can somehow reduce emissions, and so does its externality. (2) The RM regime has the highest welfare and CE of all the regimes. This is the nature of Ramsey policy. (3) The TX regime is a little better than is the CA regime in terms of welfare and CE; however, the differences between them are not big.

In terms of the welfare standard, the TX regime tends to be a better choice among the three real-world implementable regimes (CA, TX, and NO) when a TFP shock happens. However, sensitivity analysis indicates that it is not always the best choice. We find that either when the parameter EOEER is small enough or when the shock is changed to demand-type, the result \( W_{TX} > W_{CA} \) and \( CE_{TX} > CE_{CA} \) will reverse to \( W_{TX} < W_{CA} \) and \( CE_{TX} < CE_{CA}. \) Hence, among the three real-world implementable regimes, no one is always dominant over others regardless of parameters and shocks, in terms of the welfare standard.
3.3 The Role of Environment Regulatory Effectiveness

This section investigates whether the effectiveness of enforcement of environmental regulation, in addition to the choice of climate policy type, will also affect the economy and the monetary policy.

To do this, we set a lower effectiveness parameter $\psi$ equal to 0.1. This is a much smaller value than the benchmark case in Subsection 3.1, where $\psi = 1$. The small value means that the environmental regulation is less effective. In Figure 3, we show the fluctuation of economy following the same method as in Subsection 3.1. It needs to be noted that the units of some of the vertical axes in Figure 2 and Figure 3 are different. Then, we compare the results in Figure 2 ($\psi = 1$) and in Figure 3 ($\psi = 0.1$) to identify any differences arising from the effectiveness of enforcement of environmental regulation.

It can be found, for variables apart from legal and concealed emissions, that when the effectiveness is lower the differences of fluctuation between the CA and TX regimes become smaller — mainly because the variables’ paths under the CA regime are more approximate to the paths under the TX regime. Under the TX regime, legal emissions change by more than is the case when environmental regulation is more effective. Under the CA regime, concealed emissions change more. This makes the mixes with different regimes of climate policy become more similar to each other.
Figure 3: The dynamics of endogenous variables after a 1% positive TFP shock under different regimes (EOEER=0.1)

The pivotal reason for the diminishing differences between regimes is that the less effective enforcement of environmental regulation gives firms more space to dispose of their emissions via the concealed emitting channel and the “price level-offsetting” effect in the CA regime is weakened. When $\psi$ is lower, the unit cost for concealed emissions and the total cost for disposing of every unit of original emissions will decrease. This allows the steady state share of concealed emissions in original emissions (i.e. $V_t$) and original emissions to increase. After a TFP shock under the TX regime, concealed emissions rise by more than is the case with higher $\psi$ because of the increased steady state $V_t$. The path of abatement is almost unchanged because the extra original emissions after a shock do not change significantly, and the share of abatement for disposing of every unit of original emissions (i.e., $U_t$) is not changed according to...
equation (18), which does not include $\psi$. Neither does the path of real emissions, whose share is $1 - U_t$, change significantly, for the same reason. The legal emissions rise by less because their share in disposing of every unit of original emissions $1 - U_t - V_t$ is reduced due to an increased $V_t$. The paths of inflation and interest rate are almost unchanged due to a fixed $p_{z,t}$ under the TX regime.

After a TFP shock under the CA regime, $p_{z,t}$ increases by less than is the case when $\psi$ is higher, as the cost for concealed emissions rises by less.\(^{11}\) The “price level-offsetting” effect is weakened. This brings more similar changes in the paths of inflation and the interest rate. Illegal emissions rise by more than is the case with a higher $\psi$ for the same reason under the TX regime. Abatement increases by less as more original emissions are disposed of via the concealed emitting channel. Real emissions rise by more because the concealed emissions increase by more and the legal emissions are fixed under the CA regime.

In addition to the fluctuation analysis, we also calculate and compare the welfare of each regime after the EOEER is changed to 0.1. We find that the order of welfare and the consumption equivalent comparison will change to $W_{ET} > W_{TX}$ and $CE_{CA} > CE_{TX}$. The reason is that consumption, as one of the determinants of welfare, increases by more under the CA regime than under the TX regime. A lower $\psi$ brings a lower cost for concealed emissions. Under the CA regime this also brings a lower $p_{z,t}$. Then, the price level decreases and demand, production output, and consumption increase. However, under the TX regime, $p_{z,t}$ is fixed, and, hence, the price level decreases by less than is the case under CA. Then, consumption does not rise by so much.\(^{12}\) The

\(^{11}\) There is a marginal increasing cost for concealed emissions $\frac{\psi^2}{2} \phi Y_{t,j}$. When $\psi$ is lower, the steady state cost for concealed emissions is lower. Hence, the cost for concealed emissions rises less here. Meanwhile, the three channels for disposing of original pollution have the same marginal cost (a natural result of economic optimisation); hence $p_{z,t}$ equals the cost for concealed emissions.

\(^{12}\) The fluctuation of price also influences welfare, according to Rotemberg (1982). However, the result here means that the influence of consumption on welfare is stronger.
output under the CA regime rises more than it does under the TX regime, after a shock, which makes the output gap under the CA regime relatively smaller and the welfare larger.

The above analysis shows that the ineffectiveness of enforcement of environmental regulation will make climate policy less effective and that different regimes become more similar. This implies that the difference in the fluctuation of economy and monetary policy between regimes will also change due to the differentiation of EOEER. Therefore, in addition to the regime of climate policy, the EOEER also needs to be considered when designing monetary policy. Otherwise, the dynamics of monetary policy and its effect on the economy will be somewhat different (too strong or too weak) from what is envisaged with only considering the regime of climate policy. Another implication is that, when making monetary policy, developed countries should consider the existing regime of climate policy more carefully than developing countries, as their effectiveness of environmental regulation is often better and the differences between regimes are more significant.

4. The Optimisation of Policy Mixes

From Subsection 3.2, it can be found that, among the three real-world implementable regimes of policy mix (CA, TX, and NO), no one is always dominant over others, in terms of the welfare standard. In this section, we propose to improve or “optimise” these regimes respectively. The first way is to optimise policy coefficients in the traditional Taylor rule of monetary policy. The second and also a novel way is to introduce a radically “climate-augmented” monetary policy. This is to include the emission gap target into the Taylor rule of monetary policy. We will try to find the best coefficient for the new target and determine whether this inclusion can become a desirable practice. The results will give an answer to central banks’ question of “whether it is good for the monetary authority to proactively care for the climate”.

27
4.1 Optimisation in the Traditional Monetary Policy

The Ramsey optimal monetary policy, which has been investigated by Annicchiarico and DiDio (2017), constitutes the ideally optimal policy mix. However, as this kind of policy assumes that all endogenous variables in the economy can be controlled and adjusted by the authority, it is difficult for policy makers to carry out in reality. We do not work more on it here. For real-world implementable climate policy regimes (CA, TX, and NO), Subsection 3.2 showed that no one is always dominant.

In this subsection, our way to improve or to “optimise” the policy mix is to first choose a certain regime that is real-world implementable, then optimise the policy coefficients in them. To do this, we have three potential options. The first is to give a fixed strength of climate policy and optimise the coefficients in the Taylor rule of monetary policy ($\rho_\gamma$ and $\rho_\Pi$). The second is to fix the monetary policy coefficients and optimise the climate policy strength. The third is to optimise the climate strength and the monetary coefficients simultaneously. We choose the first method because this research is on the angle of central banks. The second method is on the angle of environmental regulator. The third approach is more comprehensive but is also more complex and difficult for policy makers to coordinate and carry out.

To calculate, we first combine different values of monetary policy coefficients with different types of climate policy (CA or TX\(^{13}\)) under different EOEER and shocks. Shocks include TFP, cost-push, and preference shocks, considering that these three can cover both supply- and demand-side shocks. Then, we derive the welfare and CE of every combination. The policy coefficients $\rho_\Pi$ and $\rho_\gamma$ that maximise the welfare and CE of a certain combination of climate policy, EOEER, and shock, if exist, is the optimised policy coefficients for it. For simplicity, we only consider the regimes that can solve the model with a unique solution.

We find that under a cost-push shock (a positive $\theta_t$ shock), there exist optimal

\(^{13}\) We do not incorporate the NO regime as Subsection 3.2 showed that it is always an inferior one.
monetary policy coefficients for every climate policy and EOEER, as shown in Table 3. This means that if the cost-push shock is dominant in the economy, the central bank has the best choice of coefficients in the Taylor rule of monetary policy, when climate policy and EOEER are given.

Table 3: Optimal policy coefficients in the Taylor rule of monetary policy under different climate policies and EOEER (cost-push shock)

<table>
<thead>
<tr>
<th>( \varphi ) (EOEER)</th>
<th>Cap-and-Trade</th>
<th>Carbon Tax</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \rho_\pi )</td>
<td>( \rho_Y )</td>
</tr>
<tr>
<td>0.1</td>
<td>3.2335</td>
<td>0.4573</td>
</tr>
<tr>
<td>0.5</td>
<td>2.8024</td>
<td>0.4573</td>
</tr>
<tr>
<td>1</td>
<td>2.6819</td>
<td>0.4589</td>
</tr>
<tr>
<td>10</td>
<td>2.5549</td>
<td>0.4619</td>
</tr>
<tr>
<td>100</td>
<td>2.5418</td>
<td>0.4624</td>
</tr>
</tbody>
</table>

Table 3 shows that \( \rho_Y \) does not vary significantly across climate policy regimes; however, \( \rho_{\pi} \) is always larger under the TX regime than under the CA regime. This is because the emission price in the CA regime changes when a shock happens. When a cost-push shock (a positive \( \theta_t \) shock) happens, the price level becomes lower, which increases demand, production output, and emissions. The higher emissions then lead to an increase in the price for disposing of emissions under the CA regime (see Subsection 3.1 for details). Hence, the price level under the TX regime (which is fixed) is relatively lower than is the case under the CA regime. To suppress deflation, a stronger \( \rho_{\pi} \) is needed. This again shows the “price level-offsetting” effect in the CA regime and the basic mechanism that differentiates the two climate regimes. Table 3 also shows that across different EOEER, only \( \rho_{\pi} \) under the CA regime goes lower significantly when EOEER increases. This is because a higher EOEER pushes up the cost for concealed emissions and increases the demand for legal emissions. Under the CA regime, the
emission permit price $p_{Z,t}$ increases more, offsetting the decrease of price level more after the cost-push shock. So, the strength of inflation targeting, $\rho_{\Pi}$, could be eased.

Under TFP or preference shocks, we find that the welfare and CE become higher when $\rho_{\pi}$ and $\rho_{Y}$ become larger. This is a common result of the New-Keynesian model. However, this means there are no optimal values of $\rho_{\pi}$ and $\rho_{Y}$ if the ranges of the coefficients are not limited and a TFP (or preference) shock is dominant in the economy.

To summarise, we find that when climate policy is considered in the framework, the monetary policy can always be improved by adjusting the Taylor rule coefficients. If a cost-push shock is dominant in the economy, optimal coefficients exist. Both the climate policy regime and the EOEER can affect the value of the optimal coefficients. At this point, we can report that when the existing climate policy is brought into the framework of the central bank’s policy making, at least three things can be considered to improve the monetary policy: the type (regime) of climate policy, the EOEER, and the coefficient in the Taylor rule of monetary policy.

### 4.2 The “Climate-Augmented” Monetary Policy

In this subsection, we propose a radical way to improve the traditional policy mixes. This is to change the form of the Taylor rule of monetary policy by incorporating the emission gap target into it and create a so called “climate-augmented” monetary policy. We will search the best coefficient for the new target and determine whether this introduction can become a good practice. This will give an answer to central banks’ question of “whether it is good for the monetary authority to proactively care for the climate”.

Our method is to add the emission gap as the third target into the traditional inflation and output gap targeting Taylor rule. The emission gap is the relative deviation of current real emissions to the ideal real emissions (we use the steady state real
emissions calculated under Ramsey optimal climate policy\textsuperscript{14}). It is an analogue to the inflation and output gap target and is a typical form. The new form of the Taylor rule is

\[
R_t \to R = \left( \frac{\Pi_t}{\Pi} \right)^{\rho_{\Pi}} \left( \frac{Y_t}{Y_t^{na}} \right)^{\rho_Y} \left( \frac{Z_{t-1}}{Z} \right)^{\rho_Z}
\]

(32)

where $Y_t^{na}$ is the natural output without nominal price stickiness, $R$, $\Pi$, and $Z$ are the steady states of nominal interest rate, inflation rate, and real emissions, respectively. $Z_{t-1}$ represents the current real emissions. We assume that the authority uses $Z_{t-1}$, not $Z_t$, to represent the current real emissions since real emissions includes the concealed emissions which often cannot be detected during the period of policy making (period $t$). The emission gap target is not a replication of the output gap target as we use the real emissions in it, not the original emissions who are proportional to output. Real emissions incorporate abatement and is the ultimate factor that influences the environment and, thus, can directly reflect the climate objective. $\rho_Z$ is the intensity coefficients for targeting on the emission gap. This new form of Taylor rule makes the monetary policy proactively care for the climate.

Then, we set the strength of the traditional target of monetary policy (i.e. the inflation and the output coefficient) as fixed: $\rho_Y = 0.5$ and $\rho_{\Pi} = 1.5$, and calculate welfare values of the economy with different $\rho_Z$ and different shocks.\textsuperscript{15} $\rho_Z$ takes every value in the interval that can produce a unique solution for the equilibrium. Common shocks (TFP, cost-push, and preference) that cover both supply- and demand-side shocks are introduced respectively. Under a same shock, if the welfare with a $\rho_Z$ is higher than is the welfare with $\rho_Z = 0$, a $\rho_Z$ that can improve the policy mix is found. As $\rho_Y$ and $\rho_{\Pi}$ are fixed and allowing $\rho_Z$ to change is introducing a new

\textsuperscript{14} A more intuitive “ideal real emissions” is the carbon budget measured against the 1.5°C (or lower) target. However, the calculation requires some reliable data in natural science which is currently unavailable.

\textsuperscript{15} A more comprehensive method is to simultaneously optimise the three targets. We do not do it in this research as our method is enough when we give the condition “the strength of the traditional Taylor rule target of monetary policy is given” in the conclusion.
dimension for optimisation, there must be some $\rho_Z$ that can improve the welfare. It will serve as a supplement of the potentially either over-strong or over-weak $\rho_Y$ and $\rho_\Pi$.

Applying the above method, we can find the intervals of $\rho_Z$ that can improve the welfare, as well as the values of $\rho_Z$ that can enhance the welfare at the greatest extent (define as “the best value of $\rho_Z$”) under different regimes and different shocks. Results using parameters calibrated in Subsection 2.5 shown in Table 4. When the TFP or cost-push shock is dominant, the best $\rho_Z$ is negative in both climate regimes. When the preference shock is dominant, the best $\rho_Z$ lies in the right boundary of possible values, which means that the higher the $\rho_Z$, the higher the welfare.

Table 4: The interval of $\rho_Z$ that can improve welfare and the best $\rho_Z$ under different climate policies and shocks (original price stickiness)

<table>
<thead>
<tr>
<th>Shock</th>
<th>Cap-and-Trade</th>
<th>Carbon Tax</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Interval</td>
<td>Best</td>
</tr>
<tr>
<td>TFP shock</td>
<td>(-0.866, 0)</td>
<td>-0.453</td>
</tr>
<tr>
<td>Cost-push shock</td>
<td>(-0.509, 0)</td>
<td>-0.261</td>
</tr>
<tr>
<td>Preference shock</td>
<td></td>
<td>The higher the better</td>
</tr>
</tbody>
</table>

However, sensitivity analysis shows that under TFP or cost-push shock, the best $\rho_Z$ can also be positive under different parameter values. For example, if the price stickiness parameter $\gamma$ is large enough [e.g., 10 times larger, which is roughly in line with Gertler et al. (2019)], the best $\rho_Z$ becomes positive under both regimes with a cost-push shock, as shown in Table 5.
Table 5: The interval of $\rho_Z$ that can improve welfare and best $\rho_Z$ under different climate policies and shocks (price stickiness 10 times larger)

<table>
<thead>
<tr>
<th>Shock</th>
<th>Cap-and-Trade</th>
<th>Carbon Tax</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Interval</td>
<td>Best</td>
</tr>
<tr>
<td>TFP shock</td>
<td>(-0.934, 0)</td>
<td>-0.508</td>
</tr>
<tr>
<td>Cost-push shock</td>
<td>(0, 1.342)</td>
<td>0.602</td>
</tr>
<tr>
<td>Preference shock</td>
<td>The higher the better</td>
<td></td>
</tr>
</tbody>
</table>

We must point out that when the interval of $\rho_Z$ that can improve welfare is negative, there is a dilemma between the welfare objective and the environmental objective. Suppose a positive TFP or cost-push shock happens, then the emission gap is positive due to the lower price level, higher output, and higher emissions. With a negative $\rho_Z$, a lower interest rate will be derived, which encourages demand and production, fulfilling the welfare objective. However, the higher production causes higher emissions, which is adverse to the environmental objective. On the contrary, if we change the $\rho_Z$ to a positive value to realise the environmental objective (emission gap), then it deviates from the interval that can improve welfare. Failing to enhance welfare is incompatible with the fundamental purpose of a central bank. This is the potential dilemma that emerges to a central bank if they add the emission gap target into the traditional monetary policy.

The above analysis gives an answer to the question “whether a central bank should adopt ‘climate-augmented’ (emission gap targeting) monetary policy” or “whether it is good for the monetary authority to proactively care for the climate”. If the interval of the new target’s coefficient ($\rho_Z$) that can improve welfare consists of a positive part, it is good to do so by adding the emission gap target into the Taylor rule of monetary policy and setting the targeting coefficient as a value in the positive interval. If the interval consists of only negative values, it is not good to add the emission gap target.
Based on the above results and the real-world circumstance, we do not suggest central banks to add the new climate target (the emission gap target) into the Taylor rule of monetary policy without further reviews. Considering that the welfare improving interval of $\rho_z$ is not fixed and is determined by many uncertain factors including deep parameters, the regime of climate policy, and the type of shock, a central bank cannot assure that the climate augmented Taylor rule monetary policy always does not bring the dilemma between the welfare and the environmental objective. Meanwhile, many central banks in the real-world are already overburdened with multiple targets other than price stability and employment.

This subsection shows that, when the strength of the traditional Taylor rule target of monetary policy is given, incorporating the emission gap target into the rule and setting the coefficient of the new target in a specific interval can improve the policy mix in terms of the welfare standard. The best value of the coefficient for emission targeting is found under different situations (given the coefficients for inflation and output gap targeting fixed). However, under some circumstances, this radically “climate-augmented” monetary policy will create a dilemma between the welfare and the environmental objectives, making it less valuable of recommendation for central banks to adopt without further reviews.

**4.3 A Discussion**

Although the “climate-augmented” (emission gap targeting) monetary policy is found to be controversial above, it does not mean that this kind of monetary policy is useless from other points of view. The DSGE model is used mainly for fluctuation analysis, so the conclusions are based on short-term standards. Climate change can be characterized as a long-term challenge for mankind. Considering that “climate-augmented” monetary policy of certain forms can limit emission and reduce future climate risks, it could become a preferable choice for policy makers in the long-run.
From the modelling prospective, the reasons include: First, the steady state welfare could be higher if emission is limited. This can compensate for the welfare loss shown in the fluctuation analysis. Second, a lower climate risk increases economic stability and decreases welfare loss brought by fluctuation.

The above results neither means that central banks should not proactively care for the climate by measures other than the narrow monetary policy (interest rate). Climate change can bring physical and transition risks to firms so that can cause financial and economic instability. Safeguarding financial and economic stability is a major mandate of most central banks. They could use macroprudential and other regulatory policy tools, such as environmental stress testing and green asset purchase, and play a coordinating role among regulators and the market to fulfil this mandate in facing climate change.

5. Conclusion

In this paper, we have studied the relationship between and the mix of monetary and climate policy. By using an Environmental Dynamic Stochastic General Equilibrium (E-DSGE) model augmented with a range of emissions including what we call concealed emissions and related regulations, we have compared the mixes of Taylor rule-based monetary policy with different climate policies to find whether and how climate policy will influence monetary policy; this paper optimised the coefficients in the monetary policy rule under certain climate policies; and proposed a “climate-augmented” monetary policy and investigated if and when it can be a good choice for the central bank. All these provide insights for central banks who are considering their engagement in the climate change issue.

The main findings consist of three parts. First, the dynamics of monetary policy and the economy are influenced by the selection of regimes of climate policy and the effectiveness of enforcement of environmental regulation (EOEER). The pivotal reason is that the cap-and-trade regime can offset the price fluctuation after shocks, whereas
the carbon tax regime cannot. The effectiveness of environmental regulation also plays a role, as it can make climate policy less effective by providing more space for concealed emissions. Therefore, the making process of monetary policy should consider the existing climate policy and environmental regulation. Developed countries should consider the climate policy more carefully than do the developing ones.

Second, the coefficients in the traditional Taylor rule of monetary policy can always be better set to enhance welfare when a certain regime of climate policy is considered in the economy. If the cost-push shock is dominant in the economy, optimal coefficients exist. Both the climate policy regime and the effectiveness of environmental regulation can affect the value of the optimal coefficients. We can summarise from the above that, under the framework with climate factors, at least three aspects can be considered to improve the monetary policy: the type (regime) of climate policy, the effectiveness of enforcement of environmental regulation, and the coefficients of the inflation and output gap targets in the Taylor rule of monetary policy.

Third, the welfare of the economy can be enhanced by adding the target of emission gap into the rule of monetary policy and setting the coefficient of the new target in a specific interval, when the strength of the traditional Taylor rule target of monetary policy is given. The best value of the coefficient for targeting can be found under different scenarios. However, under some circumstances, this radically “climate-augmented” (emission gap targeting) monetary policy is likely to create a dilemma between the welfare and the environmental objectives. If we do not want central banks to take the risk of such dilemma, it is better not to introduce the climate target into the monetary policy rule without further reviews. Central banks could and should use measures other than the narrow monetary policy (interest rate) to proactively care for the climate.

The above findings give insights to the initial question of this paper “Should central banks engage in the climate change issue?” — The making process of monetary policy should consider the existing climate policy; otherwise, the dynamic of monetary policy
and its effect on the economy will be different from what is originally envisaged. However, it is not recommended for central banks to add the climate (emission gap) target into the narrow monetary policy at the current stage, as this may create a dilemma for them.

This research can be extended in several aspects. For example: (1) Set the EOEER as a shock to study the “transition risk” brought by climate change and the tightening process of environmental regulation (e.g., China’s environmental inspection). (2) Set a dynamic rule (e.g., the Taylor rule) for climate policy. (3) Improve the form of climate target in the monetary policy rule (e.g., use an ideal real emission that is in line with the 1.5°C climate target). (4) Introduce more types of shocks (e.g., climate change shock after the tipping point). (5) Introduce more financial fractions and constraints (e.g. zero lower bound of interest rate) to describe the role of monetary policy more precisely. (6) Along with the monetary policy, introduce and study more policy tools and measures that central banks can use to mitigate climate risk and support the green economic transition [e.g., identifying green financing and differentiating reserve rate requirements, re-lending and collateral requirements (Pan, 2019), green asset purchase and credit guidance]. (7) Find whether the three-target “climate-augmented” monetary policy is better than the traditional two-target policy when all the Taylor rule coefficients in them are simultaneously optimised.
Appendix

Derivation of the New Keynesian Phillips Curve

The maximisation problem of firm \( j \) is

\[
V_0 = \max \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \Omega_{0,t} \left[ \frac{P_{j,t}}{P_t} Y_{j,t} - TC_{j,t} - \frac{\gamma}{2} \left( \frac{P_{j,t}}{P_{j,t-1}} - 1 \right)^2 Y_t \right] \right\}
\]

s.t.

\[
TC_{j,t} = \frac{W_t}{P_t} L_{j,t} + \phi_1 U_{j,t}^{\phi_2} Y_{j,t} + p_{Z,t} (1 - U_{j,t} - V_{j,t}) \varphi Y_{j,t} + \frac{\psi}{2} V_{j,t}^2 \varphi Y_{j,t}
\]

\[
Y_{j,t} = \Lambda_t A_t L_{j,t}
\]

\[
Y_{j,t} = \left( \frac{P_{j,t}}{P_t} \right)^{-\theta_t} Y_t
\]

We can rewrite the objective function by the Bellman Equation as

\[
V_t = \max \left\{ \frac{P_{j,t}}{P_t} Y_{j,t} - TC_{j,t} - \frac{\gamma}{2} \left( \frac{P_{j,t}}{P_{j,t-1}} - 1 \right)^2 Y_t + \mathbb{E}_t \Omega_{t,t+1} V_{t+1} \right\}
\]

which yields the Lagrangian function as

\[
\mathcal{L}_t = \frac{P_{j,t}}{P_t} Y_{j,t} - \left[ \frac{W_t}{P_t} Y_{j,t} + \phi_1 U_{j,t}^{\phi_2} Y_{j,t} + p_{Z,t} (1 - U_{j,t} - V_{j,t}) \varphi Y_{j,t} + \frac{\psi}{2} V_{j,t}^2 \varphi Y_{j,t} \right]
\]

\[
- \frac{\gamma}{2} \left( \frac{P_{j,t}}{P_{j,t-1}} - 1 \right)^2 Y_t + \mathbb{E}_t \left[ \Omega_{t,t+1} V_{t+1} \right] + \lambda_t \left[ \left( \frac{P_{j,t}}{P_t} \right)^{-\theta_t} Y_t - Y_{j,t} \right]
\]

where \( \Omega_{t,t+1} = \beta \frac{c_t}{c_{t+1}} \) is the stochastic discount factor. So, we can obtain the FOC for

\( U_{j,t} \) and \( V_{j,t} \)

\[
p_{Z,t} = \frac{\phi_1 \phi_2}{\varphi} U_{j,t}^{\phi_2-1}
\]

\[
V_{j,t} = \frac{p_{Z,t}}{\psi}
\]

and derive

\[
MC_{j,t} = \frac{W_t}{P_t} \frac{1}{\Lambda_t A_t} + \phi_1 U_{j,t}^{\phi_2} + p_{Z,t} (1 - U_{j,t} - V_{j,t}) \varphi + \frac{\psi}{2} V_{j,t}^2 \varphi
\]

The FOCs for \( P_{j,t} \) and \( Y_{j,t} \) derive

\[
1 - \theta_t - \gamma \left( \frac{P_{j,t}}{P_{j,t-1}} - 1 \right) \frac{P_{j,t}}{P_{j,t-1}} + \beta \gamma \mathbb{E}_t \left[ \left( \frac{P_{j,t+1}}{P_{j,t}} - 1 \right) \frac{P_{j,t+1}}{P_{j,t}} \frac{C_t}{C_{t+1}} Y_{t+1} \right] + \theta_t MC_{j,t} = 0
\]
Taylor Rule Monetary Policy Mix Cap-and-Trade Climate Policy

\[
\beta R_t E_t \left[ \frac{C_t}{C_{t+1} \Pi_{t+1}} \right] = 1 \\
(1 - \theta_t) - \gamma (\Pi_t - 1) \Pi_t + \beta R_t E_t \left[ \frac{C_t}{C_{t+1}} \frac{(\Pi_{t+1} - 1) \Pi_{t+1}}{Y_t} \frac{Y_{t+1}}{Y_t} \right] + \theta_t MC_t = 0 \\
MC_t = \frac{W_t}{\Lambda_t A_t P_t} + \phi_1 U_t^\phi_2 + p_{Z,t} (1 - U_t - v_t) \phi + \frac{\psi}{2} v_t^2 \phi \\
L_t^\eta = \frac{W_t}{\mu_t P_t C_t} \\
Y_t = C_t + \phi_1 U_t^{\phi_2} Y_t + \frac{\psi}{2} (\Pi_t - 1)^2 Y_t \\
Z = (1 - U_t - v_t) \phi Y_t + \bar{Z} \\
M_t = (1 - \delta_M) M_{t-1} + (1 - U_t) \phi Y_t + \bar{Z} \\
Y_t = \Lambda_t A_t L_t \\
p_{Z,t} = \frac{1}{\phi} \phi_1 \phi_2 U_t^{\phi_2 - 1} \\
v_t = \frac{1}{\psi \phi} \phi_1 \phi_2 U_t^{\phi_2 - 1} = \frac{p_{Z,t}}{\psi} \\
\frac{R_t}{R} = \left( \frac{\Pi_t}{\Pi} \right)^{\rho_n} \left( \frac{Y_t}{Y_t^{\eta \theta}} \right)^{\rho_Y} \\
\]

Taylor Rule Monetary Policy Mix Carbon Tax Climate Policy

\[
\beta R_t E_t \left[ \frac{C_t}{C_{t+1} \Pi_{t+1}} \right] = 1 \\
(1 - \theta_t) - \gamma (\Pi_t - 1) \Pi_t + \beta R_t E_t \left[ \frac{C_t}{C_{t+1}} \frac{(\Pi_{t+1} - 1) \Pi_{t+1}}{Y_t} \frac{Y_{t+1}}{Y_t} \right] + \theta_t MC_t = 0 \\
MC_t = \frac{W_t}{\Lambda_t A_t P_t} + \phi_1 U_t^\phi_2 + p_{Z,t} (1 - U - v) \phi + \frac{\psi}{2} v^2 \phi \\
L_t^\eta = \frac{W_t}{\mu_t P_t C_t} \\
Y_t = C_t + \phi_1 U_t^{\phi_2} Y_t + \frac{\psi}{2} (\Pi_t - 1)^2 Y_t \\
M_t = (1 - \delta_M) M_{t-1} + (1 - U) \phi Y_t + \bar{Z} \\
Y_t = \Lambda_t A_t L_t \\
p_{Z} = \frac{1}{\phi} \phi_1 \phi_2 U_t^{\phi_2 - 1} \\
v = \frac{1}{\psi \phi} \phi_1 \phi_2 U_t^{\phi_2 - 1} = \frac{p_{Z}}{\psi} \\
\frac{R_t}{R} = \left( \frac{\Pi_t}{\Pi} \right)^{\rho_n} \left( \frac{Y_t}{Y_t^{\eta \theta}} \right)^{\rho_Y} \\
\]

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Taylor Rule Monetary Policy Mix No Control Climate Policy

No control policy is a special case of the carbon tax policy with \( p_Z = 0 \). The equation system is all the same as with the “Taylor Rule Monetary Policy Mix Carbon Tax Climate Policy” except that \( p_Z \) is set as 0.

Taylor Rule Monetary Policy Mix Ramsey Optimal Climate Policy

\[
\begin{align*}
\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t & \left( \ln C_t - \mu_L L_t^{1+\eta} \right) \\
\beta R_t \mathbb{E}_t \left[ \frac{C_t}{C_{t+1} \Pi_{t+1}} \right] & = 1 \\
(1 - \theta_t) - \gamma (\Pi_t - 1) \Pi_t + \beta \gamma \mathbb{E}_t \left[ \frac{C_t}{C_{t+1}} (\Pi_{t+1} - 1) \Pi_{t+1} \frac{Y_{t+1}}{Y_t} \right] + \theta_t MC_t & = 0
\end{align*}
\]

s. t.

\[
MC_t = \frac{W_t}{\lambda_t A_t P_t} + \phi_1 U_t^\phi_2 + p_Z (1 - U_t - v_t) \phi + \frac{\psi}{2} v_t^2 \phi
\]

\[
L_t = \frac{W_t}{\mu_L P_t C_t}
\]

\[
Y_t = C_t + \phi_1 U_t^\phi_2 Y_t + \frac{\gamma}{2} (\Pi_t - 1)^2 Y_t
\]

\[
M_t = (1 - \delta_M) M_{t-1} + (1 - U_t) \phi Y_t + \bar{Z}
\]

\[
Y_t = \lambda_t A_t L_t
\]

\[
R_t = \left( \frac{\Pi_t}{\Pi} \right)^{\rho_n} \left( \frac{Y_t}{Y_t^{\text{na}}} \right)^{\rho_Y}
\]


Reference


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