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Uncertainty and sentiments in asset prices

Michele Berardi
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Abstract

In this paper, I propose a novel way to model sentiments in asset prices. Under this new representation, sentiments, or animal spirits, are sparked by exogenous shocks to beliefs, but feed on the uncertainty generated by imperfect information. Sentiments cause expectations to deviate from optimal, information-based estimates of fundamental values and their magnitude depends on the amount of uncertainty in such estimates. The higher the uncertainty, the larger the scope for psychological attitudes to affect expectations.

Armed with this framework, I investigate the role of uncertainty on the transmission channel from sentiment shocks to prices in a market with imperfect information and Bayesian agents. The main result that emerges is that the source of noise generating uncertainty, whether fundamental or informational shocks, has important consequences for the effect of sentiments. Specifically, while more informational noise always amplifies the impact of psychological shocks on prices, more fundamental noise can actually reduce such impact, depending on the elasticity of sentiments to uncertainty. This result implies that, for example, noise traders in stock markets can actually reduce the relevance of animal spirits for asset prices.

Key words: information; uncertainty; sentiments; Bayesian learning; financial markets.

JEL classification: D83; D84; G14.

1 Introduction

The impact of agents' psychological attitudes, or sentiments, on economic outcomes has long attracted the interest of economists. Notable early contributions recognizing the role played by sentiments in determining economic activity are [Pigou (1927)], who discussed psychological causes of industrial fluctuations, and [Keynes (1936)], who famously coined the term "animal spirits" to describe the impact of emotions and "a spontaneous urge to action" on the economic actions of agents.

While these early works lacked formal rigor, more recent contributions have tried to make more precise the exact nature of sentiments and the transmission mechanism from them to economic outcomes. One approach put forward is to model sentiments as shocks to higher order beliefs, i.e., beliefs about other agents' beliefs, as done in [Angeletos and La'O (2009)] and [Angeletos et al (2018)]. Alternatively, sentiments could be considered as a primitive of a model, as proposed by [Farmer (2012)], who uses animal spirits, in the form of arbitrary beliefs, to select among multiple equilibria in the market. From an empirical perspective, sentiments have been captured as exogenous shocks to expectations, as in [Milani (2011)] and [Milani (2017)].

In this paper, instead, I suggest that sentiments should be linked to the amount of uncertainty in the economy, as specified by the relevant information structure. Under this novel way to model, and understand, sentiments, animal spirits feed on the uncertainty that derives from the imperfect information of agents when forecasting relevant variables. Without uncertainty, there could be no sentiments, and the greater is the uncertainty, the larger is the scope for agents' psychological attitudes to impact on their expectations.

With perfect information and no uncertainty, consumers could not over- or under-estimate their future consumption, investors could not be pessimistic or optimistic about future profits conditions and workers and unions could not possibly misjudge future labour market conditions. It is the uncertainty arising from imperfect information that opens the door to subjective beliefs, and the larger the uncertainty, the more scope there is for stronger psychological attitudes (both positive and negative).

In terms of financial markets, the focus of this paper, if traders had complete information and knew exactly the fundamental value of an asset, that is, the stream of cash flows that it entitles to, there could be no psychological attitudes affecting beliefs about its value: no-one could be optimistic or pessimistic about something it is known for sure. Animal spirits in financial market are made possible by the uncertainty surrounding the estimated value of an asset, which allows agents to include subjective elements in their beliefs.

Building on these simple considerations, I model sentiments as a combination of an exogenous shock to beliefs, something similar to a sunspot, and uncertainty. It is uncertainty that allows exogenous psychological shocks to affect beliefs, and through them, prices. Formalizing a link between uncertainty, due to imperfect (or noisy) information, and sentiments, makes it possible to derive implications for the impact of psychological attitudes on asset prices in relation to different sources of noise in the market.

In order to model uncertainty, I draw on the literature about noisy rational expectations equilibria, in particular on the early works by [Grossman and Stiglitz (1980)], [Hellwig (1980)] and [Diamond and Verrecchia (1981)], extending that class of models to include sentiments. Agents use the information they have efficiently, as Bayesian learners, but include in their beliefs an "animal spirits" component. The larger is the uncertainty around the value of the asset, the more scope there is for psychological attitudes to affect beliefs. Establishing a formal link between uncertainty and sentiments allows then for an investigation of the propagation mechanism from sentiments to prices, in relation to different sources of uncertainty.

The idea that imperfect information can lead to sentiments is not new. For example, [Benhabib et al (2015)] propose a model where imperfect information in forecasting demand by firms means that consumer sentiments can matter in determining equilibrium aggregate supply. The new contribution of this paper, though, is to lay out and investigate the relationship between sentiments and the structure of information, in particular with respect to the source of noise on the market.

The main question this paper addresses then is the following: given the assumed amplified role of uncertainty on sentiments, would an increase in noise always increase the impact of psychological attitudes on prices? The surprising answer is that no, increased uncertainty does not necessarily lead to a greater impact of sentiments on prices. In particular, informational and non-informational noise impacts differently on the transmission channel from sentiment shocks to prices: while more informational noise always amplifies psychological shocks, and thus increases the volatility of prices due to sentiments, more fundamental noise can actually reduce the impact of psychological shocks on prices, depending on the elasticity of sentiments to uncertainty.

The elasticity of sentiments to uncertainty is crucial in shaping the response of prices to psychological shocks, as it interacts with the relative weights agents put on different sources of information under Bayesian updating. Sentiments enter prices through beliefs, and prices enter beliefs as a source of information: the equilibrium relationship between prices and sentiments depends on the amplification mechanism of psychological shocks through uncertainty.

An increase in non-informational, or fundamental, noise reduces the precision of prices as signals relative to that of exogenous information, thus decreasing the Bayesian weight on prices. An elasticity of sentiments to uncertainty smaller than one means that sentiments respond less than proportionally to increases in noise so in this case the reduced weight on prices in the signal extraction process more than compensates for the amplification of sentiments through increased uncertainty: the net effect is a dampened response of prices to sentiments.

With informational noise, instead, the weight on prices increases relative to the weight on exogenous information when noise increases; at the same time, overall uncertainty increases, so both effects lead to a stronger effect of psychological shocks on prices, irrespective of the elasticity of sentiments to uncertainty.

These results are important in understanding the possible impact of sentiments on asset prices, as they highlight the role of uncertainty as a mediating factor. They also provide a useful framework for empirical work aiming at identifying sentiments in observed stock prices, establishing formal links between variations in the amount of noise on the market from different sources and the impact of sentiments, which could be exploited for identification purposes.

2 Asset prices with sentiments

A one period risky asset is traded on the market and pays an unknown fixed dividend θ at liquidation.

Agents receive private noisy information about such dividend, in the form of the signal

$$x^i = \theta + v^i,$$

where the noise in private information $v^i \sim N(0, \sigma_v^2)$. I will refer to v^i as informational noise, as it represents solely the accuracy of agents' exogenous signal.

Agents form beliefs about the dividend using such exogenous information and prices, optimally weighted using Bayesian theory. Prices are useful here as a source of information for the dividend because of imperfect private information, as they reflect the information of other traders.

The key assumption of this paper is that agents' beliefs about the dividend are also affected by a sentiment component, to be specified below.

Agents are assumed to be risk neutral.[1] Given individual agents' beliefs (denoted by $\hat{\theta}^i$), and assuming no discount within period, prices are thus determined by the noisy equilibrium condition

$$p = \int^i \hat{\theta}^i di + \varepsilon. \tag{1}$$

The stochastic term $\varepsilon \sim N(0, \sigma_\varepsilon^2)$ prevents prices from being fully revealing, and could be thought of as a shock in the exogenous supply of shares. It is sometimes justified in terms of noise traders, agents whose demand is purely random and unrelated to any source of information in the model. It is a source of uncertainty which I will refer to as fundamental, or non-informational, noise: while it affects the accuracy of the endogenous information, its nature does not depend on the informational structure assumed in the market and it derives instead from fundamental elements.

Beliefs are formed according to

$$\hat{\theta}^i = \hat{\theta}_I^i + S \tag{2}$$
$$\hat{\theta}_I^i = \alpha x^i + (1-\alpha)\, p, \tag{3}$$

where S represents the sentiments component in beliefs, in addition to the rational, information-based element $\hat{\theta}_I^i$ derived through Bayesian theory.

The parameter α, which represents the optimal Bayesian weight on private versus public information, is given by the relative precision of the two signals:

$$\alpha = \frac{\sigma_p^2}{\sigma_v^2 + \sigma_p^2}. \tag{4}$$

The level of uncertainty in information, denoted σ_I^2, is measured by the variance of agents' information-based estimates of the fundamental:

$$\sigma_I^2 \equiv Var\left(\hat{\theta}_I^i\right) = \alpha^2 \sigma_v^2 + (1-\alpha)^2 \sigma_p^2. \tag{5}$$

[1] In the Appendix, I derive prices with risk averse agents: it can be seen that the sunspot component in the price equation has the same form as the one derived under risk averse agents, so all results in this paper carry through to a market with risk averse traders.

Combining (4) and (5) leads to

$$\sigma_I^2 = \frac{\sigma_p^2 \sigma_v^2}{\sigma_v^2 + \sigma_p^2} = \alpha \sigma_v^2. \tag{6}$$

In order to capture the amplifying effect of uncertainty on sentiments, I model sentiments as a combination of uncertainty and psychological attitudes, taken as exogenous. I thus posit

$$S = h\left(\sigma_I^2, s\right), \tag{7}$$

where s is a zero mean sentiments shock, with $s \sim N(0, \sigma_s^2)$. Positive values of s could be interpreted as optimism, as they make beliefs of the value of the asset higher than their information-based valuation (that is, its fundamental value, as far as agents can establish given the information available in the market); negative values of s would instead capture pessimism among agents. S thus represents uncertainty-amplified sentiments, which modify the information-based element of expectations to determine agents' beliefs. The assumption that $E(s) = 0$ ensures that sentiments do not introduce any systematic bias in prices, so it is optimal for agents to use prices, together with private exogenous information, in deriving the fundamental.

One would expect the function h to satisfy the following restrictions: $h_s^{'} > 0, h_{\sigma_I^2}^{'} > 0$, meaning that both higher uncertainty and larger psychological shocks (levels of optimism or pessimism) lead to larger sentiments. Specifically, I assume the following flexible form for the function h, parameterized by $\gamma \geqslant 0$:

$$h\left(\sigma_I^2, s\right) = \left(\sigma_I^2\right)^{\gamma} s. \tag{8}$$

The parameter γ measures the elasticity of sentiments to uncertainty, $\frac{dS}{d\sigma_I^2}\frac{\sigma_I^2}{S}$, allowing for different degrees of responsiveness. If $\gamma = 0$, sentiments are unaffected by uncertainty, so they reduce down to an exogenous shock to beliefs. For $\gamma > 0$, instead, sentiments grow with uncertainty: if $\gamma < 1$, such growth is less than proportional, limiting the impact of exogenous psychological shocks on beliefs; if $\gamma > 1$, instead, sentiments grow more than proportionately with uncertainty and even a small amount of optimism can potentially have a large impact on beliefs if uncertainty is high. Fig. (1) provides a visual representation of the relationship between sentiments and uncertainty for three different levels of γ: 0.5, 1 and 1.5. Values of γ between 0 and 1 would seem most reasonable, though values of 1 or above could characterize episodes of heightened animal spirits in the market. I remain agnostic in this work about

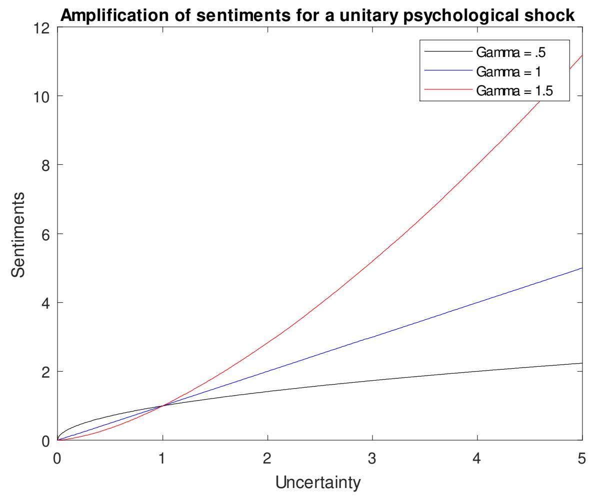

Figure 1: Amplification of sentiments with uncertainty. Figure drawn for a positive unitary s shock.

appropriate values for γ and instead focus on deriving the implications of different values for the impact of sentiments on prices.

Starting from (1) and using (2), (3), (7) and (8) leads to the price equation

$$p = \theta + \alpha^{-1}\varepsilon + \alpha^{-1}\left(\sigma_I^2\right)^{\gamma} s. \tag{9}$$

Substituting (4) and (5) into it, gives

$$p = \theta + \frac{\sigma_v^2 + \sigma_p^2}{\sigma_p^2}\varepsilon + \left(\frac{\sigma_p^2}{\sigma_v^2 + \sigma_p^2}\right)^{\gamma-1}\left(\sigma_v^2\right)^{\gamma} s, \tag{10}$$

from which one can derive an implicit equation for the price variance

$$\sigma_p^2 = \left(\frac{\sigma_v^2 + \sigma_p^2}{\sigma_p^2}\right)^2 \sigma_\varepsilon^2 + \left(\frac{\sigma_p^2}{\sigma_v^2 + \sigma_p^2}\right)^{2(\gamma-1)}\left(\sigma_v^2\right)^{2\gamma}\sigma_s^2. \tag{11}$$

While it is not possible in general to solve analytically for σ_p^2 as a function of the exogenous parameters of the model, $\sigma_v^2, \sigma_\varepsilon^2, \sigma_s^2$ and γ, one can use the implicit function theorem to derive some properties of prices and their variance. Of course, one can also solve for σ_p^2 numerically. It can be shown that, for $\gamma = 1$, the discriminant of the ensuing third order polynomial is always negative, for any parameterization of $\sigma_v^2, \sigma_\varepsilon^2, \sigma_s^2$ and γ, which means that there is always only one real root. I will use such root as initial condition in the search routine for finding a positive real root when $\gamma \neq 1$.

3 Uncertainty and sentiments

The main aim of this paper is to analyze the relationship between uncertainty and sentiments, and how they interplay. In particular, I am interested in analyzing the effect of sentiments on prices and how it varies with changes in the amount of uncertainty in the market.

Uncertainty in this model comes from two sources. The first is the aggregate shock ε, which affects prices directly. It can be interpreted as an exogenous shock to the supply of shares, for example because of noise traders. Even though this shock has informational consequences, since it affects the precision of the endogenous information in prices, I refer to it as a fundamental, or non-informational shock, as it originates from real sources rather than representing pure noise in information. The second type of shocks, v, is instead a purely informational shock, as it represents noise in private exogenous information.

I start by defining the transmission channel from sentiments shocks to prices as

$$\Delta \equiv \frac{dp}{ds} = \alpha^{\gamma-1}\left(\sigma_v^2\right)^{\gamma}. \tag{12}$$

I am thus interested in how such channel varies with changes in the amount of uncertainty on the market. Specifically

$$\frac{d\Delta}{d\sigma_\varepsilon^2} = (\gamma-1)\,\alpha^{\gamma-2}\left(\sigma_v^2\right)^{\gamma}\frac{d\alpha}{d\sigma_\varepsilon^2} \tag{13}$$

captures how the sentiments channel varies with the amount of non-informational, or fundamental, noise; and

$$\frac{d\Delta}{d\sigma_v^2} = (\gamma-1)\,\alpha^{\gamma-2}\left(\sigma_v^2\right)^{\gamma}\frac{d\alpha}{d\sigma_v^2} + \gamma\alpha^{\gamma-1}\left(\sigma_v^2\right)^{\gamma-1} \tag{14}$$

captures how the sentiments channel varies with the amount of informational noise.

3.1 A special case

In the special case where the elasticity of sentiments to uncertainty is equal to one, that is $\gamma = 1$, it is straightforward to establish that $\frac{d\Delta}{d\sigma_\varepsilon^2} = 0$ and $\frac{d\Delta}{d\sigma_v^2} > 0$, since $\Delta = \sigma_v^2$. In this case, thus:

1. Changes in uncertainty from fundamental, or non-informational, noise affect prices directly but do not affect the sentiments channel. This result is due to the fact that, as the variance of prices increases (and thus the overall uncertainty σ_I^2, since $\frac{\delta\sigma_I^2}{\delta\sigma_p^2} > 0$),

agents put less wight on prices in the signal extraction process (that is, α increases): when $\gamma = 1$, the two effects cancel out, and overall $\Delta = \sigma_v^2$ remains constant. An aggregate non-informational shock (for example, from supply or noise traders) does not create room for sentiments to propagate on prices.

2. Changes in uncertainty from informational noise, instead, affect prices both through the informational component and the sentiments channel. In this case, in fact, the reduction of precision in exogenous information increases the Bayesian weight on prices, thus making room for sentiments to affect prices.

These results already highlight the difference in scope between informational and non-informational noise in affecting sentiments, though they refer to the special case of unitary elasticity of sentiments to uncertainty. In general, with $\gamma \neq 1$, things are less straightforward and require a bit more analysis.

3.2 Non-informational uncertainty

I start first by considering non-informational noise: in particular, I will analyze how changes in uncertainty coming from this component affect the sentiments channel. Such feature is captured by $\frac{d\Delta}{d\sigma_\varepsilon^2}$ from (13). Clearly, two elements determine the sign of such derivative: whether $\gamma \lesseqgtr 1$, and the sign of $\frac{d\alpha}{d\sigma_\varepsilon^2}$. This last component captures the effect of a change in the amount of non-informational noise on the Bayesian weight on private information and it can be decomposed as

$$\frac{d\alpha}{d\sigma_\varepsilon^2} = \frac{\delta\alpha}{\delta\sigma_p^2}\frac{\delta\sigma_p^2}{\delta\sigma_\varepsilon^2}.$$

The first term is always positive, as it represents the change in the weight on private information from an increase in the variance of public information (prices):

$$\frac{\delta\alpha}{\delta\sigma_p^2} = \frac{\sigma_v^2}{\left(\sigma_v^2 + \sigma_p^2\right)^2}.$$

The second term represents the change in the variance of prices from an increase in the variance of fundamental, non-informational noise: it is also always positive, as it can be checked numerically. Since it is not possible to derive an explicit and tractable solution for σ_p^2, one can use the implicit function theorem to derive

$$\frac{\delta\sigma_p^2}{\delta\sigma_\varepsilon^2} = -\frac{\delta f/\delta\sigma_\varepsilon^2}{\delta f/\delta\sigma_p^2}, \tag{15}$$

where, from (11),

$$f \equiv \sigma_p^2 - \left(\frac{\sigma_v^2 + \sigma_p^2}{\sigma_p^2}\right)^2 \sigma_\varepsilon^2 - \alpha^{2(\gamma-1)} \left(\sigma_v^2\right)^{2\gamma} \sigma_s^2. \tag{16}$$

The numerator in (15) is always negative, since

$$\frac{\delta f}{\delta \sigma_\varepsilon^2} = -\left(\frac{\sigma_v^2 + \sigma_p^2}{\sigma_p^2}\right)^2.$$

The denominator is given by

$$\frac{\delta f}{\delta \sigma_p^2} = 1 + 2\frac{\left(\sigma_v^2 + \sigma_p^2\right)\sigma_v^2 \sigma_\varepsilon^2}{\left(\sigma_p^2\right)^3} - 2\frac{(\gamma - 1)\left(\sigma_v^2\right)^{2\gamma+1}\sigma_s^2 \left(\sigma_p^2\right)^{2\gamma-3}}{\left(\sigma_v^2 + \sigma_p^2\right)^{2\gamma-1}}. \tag{17}$$

First, if $\gamma \leq 1$, all three terms in (17) are positive and clearly $\frac{\delta f}{\delta \sigma_p^2} > 0$. This covers the cases where sentiments respond proportionally or less than proportionally to uncertainty. Also, with no sentiments, i.e., for $\sigma_s^2 = 0$, the last term in (17) becomes zero and the remaining terms are both positive, so again $\frac{\delta f}{\delta \sigma_p^2} > 0$.

For the case with $\gamma > 1$ and $\sigma_s^2 > 0$, instead, the sign of the expression in (17) is more difficult to establish analytically, since the last term is negative. From a superficial look at the expression, it might seem that there could be parameterizations for which the third term in (17) dominates the other two and the whole expression becomes negative, which would imply that $\frac{\delta \sigma_p^2}{\delta \sigma_\varepsilon^2} < 0$. This, though, is not the case. To properly assess whether $\frac{\delta f}{\delta \sigma_p^2} > 0$, one needs to numerically solve (11) for σ_p^2 and substitute the solution value into (17): numerical results show that, for any sensible parameterization (i.e., for $\sigma_v^2, \sigma_\varepsilon^2, \sigma_s^2, \gamma > 0$), $\frac{\delta f}{\delta \sigma_p^2} > 0$. To this end, it can be useful to rewrite (17) as

$$\frac{\delta f}{\delta \sigma_p^2} = 1 + 2(1 - \alpha)\left[1 + \gamma\left(\frac{\sigma_\varepsilon^2}{\sigma_p^2 \alpha^2} - 1\right)\right]. \tag{18}$$

Fig. (2) shows the value of this expression for different values of γ, from 0.01 to 5, fixing $\sigma_v^2, \sigma_\varepsilon^2, \sigma_s^2$ (these are all fixed equal to one in the picture, but similar results hold for any sensible values of these parameters).

Having established that $\frac{\delta f}{\delta \sigma_p^2} > 0$ and $\frac{\delta f}{\delta \sigma_\varepsilon^2} < 0$, it follows that $\frac{\delta \sigma_p^2}{\delta \sigma_\varepsilon^2} > 0$: an increase in the volatility of the fundamental shock always increases the volatility of prices. This makes intuitive sense, as an increase in the exogenous noise in prices is bound to increase their volatility; moreover, as said above, the result is easily proved if there are no sentiments, and there is no reason to expect that an additional sentiment component would reverse the effect.

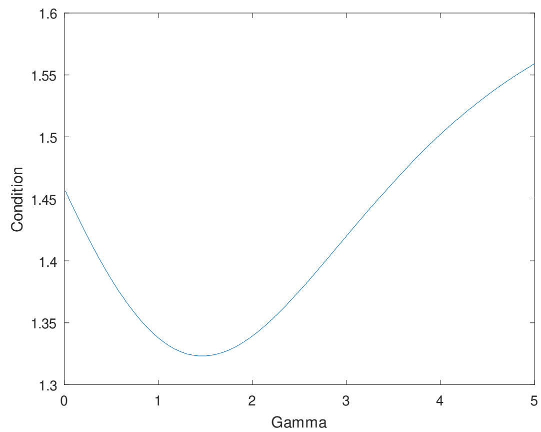

Figure 2: Values of $\frac{\delta f}{\delta \sigma_p^2}$ for different values of γ, with $(\sigma_v^2, \sigma_\varepsilon^2, \sigma_s^2) = (1, 1, 1)$.

This result, in turn, together with the fact that $\frac{\delta \alpha}{\delta \sigma_p^2} > 0$, implies that $\frac{d\alpha}{d\sigma_\varepsilon^2} > 0$ in (13), which then implies that $\frac{d\Delta}{d\sigma_\varepsilon^2}$ is negative if $\gamma < 1$. An increase in non-informational uncertainty can reduce the sentiments channel, thus dampening the effect of sentiments on prices.

Proposition 1 *For $\gamma < 1$, $\frac{d\Delta}{d\sigma_\varepsilon^2} < 0$: when the elasticity of sentiments to uncertainty is less than one, an increase in the amount of fundamental uncertainty dampens the impact of sentiments on prices. For $\gamma = 1$, $\frac{d\Delta}{d\sigma_\varepsilon^2} = 0$: when such elasticity is equal to one, the effect of sentiments on prices does not depend on the amount of fundamental uncertainty in the market. For $\gamma > 1$, $\frac{d\Delta}{d\sigma_\varepsilon^2} > 0$: when the elasticity is greater than one, an increase in the amount of fundamental uncertainty increases the effect of sentiments on prices.*

For example, for $\gamma = 0.5$ sentiments grow with the square root of uncertainty: in this case, an increase in non-informational uncertainty decreases the impact of psychological shocks on prices.

It must be noted that the price volatility still increases with σ_ε^2, but the impact of sentiments on prices decreases. A larger σ_ε^2 increases the variance of prices but not that of exogenous information, causing the Bayesian weight on prices to decrease. At the same time, the increased overall uncertainty allows for stronger sentiments, modulated by γ. If $\gamma < 1$, the amplification of uncertainty in the sentiments channel is restrained, so the net effect from, on one side, amplified sentiments from greater uncertainty and, on the other side, reduced role of prices (and thus sentiments) as Bayesian signal, leads to an overall reduction in the impact of sentiments on prices.

3.3 Informational uncertainty

The other source of uncertainty in the model, from the private signal, gives rise to informational uncertainty and its effect on the sentiments channel is captured by the derivative $\frac{d\Delta}{d\sigma_v^2}$ from 14. There are two terms in the expression: while the second one is always positive, the sign of the first term depends on whether $\gamma \gtreqless 1$ and on the sign of $\frac{d\alpha}{d\sigma_v^2}$. This last element represents the effect of a change in the amount of informational noise on the Bayesian weight on private information and it can be decomposed into

$$\frac{d\alpha}{d\sigma_v^2} = \frac{\delta\alpha}{\delta\sigma_v^2} + \frac{\delta\alpha}{\delta\sigma_p^2}\frac{\delta\sigma_p^2}{\delta\sigma_v^2}. \tag{19}$$

The first term represents the direct effect, and it is always negative:

$$\frac{\delta\alpha}{\delta\sigma_v^2} = \frac{-\sigma_p^2}{\left(\sigma_v^2 + \sigma_p^2\right)^2}.$$

The second part represents the indirect effect through the variance of prices and its sign depends on the combined effect of $\frac{\delta\alpha}{\delta\sigma_p^2}$, which is positive (established above), and $\frac{\delta\sigma_p^2}{\delta\sigma_v^2}$:

$$\frac{\delta\sigma_p^2}{\delta\sigma_v^2} = -\frac{\delta f/\delta\sigma_v^2}{\delta f/\delta\sigma_p^2}.$$

It was already established before that $\delta f/\delta\sigma_p^2 > 0$. As for the numerator,

$$\frac{\delta f}{\delta\sigma_v^2} = -2\frac{\sigma_\varepsilon^2}{\alpha\sigma_p^2} - 2\sigma_s^2\alpha^{2(\gamma-1)}\left(\sigma_v^2\right)^{2\gamma}\left(\gamma - \frac{\gamma-1}{\sigma_v^2+\sigma_p^2}\right):$$

this is clearly always negative, and therefore $\frac{\delta\sigma_p^2}{\delta\sigma_v^2} > 0$ and $\frac{\delta\alpha}{\delta\sigma_p^2}\frac{\delta\sigma_p^2}{\delta\sigma_v^2} > 0$.

From (19), thus, the sign of $\frac{d\alpha}{d\sigma_v^2}$ depends on which of the two effects of σ_v^2 on α prevail, the negative direct one (from the decrease in the precision of private information, $\frac{\delta\alpha}{\delta\sigma_v^2} < 0$) or the positive indirect one (as the increase in noise in exogenous information also increases the volatility of prices, $\frac{\delta\alpha}{\delta\sigma_p^2}\frac{\delta\sigma_p^2}{\delta\sigma_v^2} > 0$). As the direct effect of σ_v^2 on α necessarily dominates the indirect effect through σ_p^2, the whole term $\frac{d\alpha}{d\sigma_v^2} < 0$. This result is confirmed numerically, and supports the intuition that a decrease in the precision of private information should reduce the Bayesian weight on such signal.

Finally, from (14):

$$\frac{d\Delta}{d\sigma_v^2} = (\gamma - 1)\,\alpha^{\gamma-2}\left(\sigma_v^2\right)^{\gamma}\frac{d\alpha}{d\sigma_v^2} + \gamma\alpha^{\gamma-1}\left(\sigma_v^2\right)^{\gamma-1}.$$

Numerical results show that $\frac{d\Delta}{d\sigma_v^2}$ is always positive: it is clearly the case for $\gamma < 1$, as then both terms are positive; but even for $\gamma > 1$, the (positive) second term always prevails on the (negative) first one. An increase in informational noise always amplifies the sentiments channel, enhancing the effect of psychological shocks on prices.

Proposition 2 *Irrespective of the elasticity of sentiments to uncertainty, an increase in the amount of informational uncertainty always increases the impact of psychological shocks on prices.*

Informational uncertainty, thus, differs considerably from non-informational uncertainty when it comes to sentiments and their impact on prices. Informational uncertainty always creates space for sentiments to impact on prices, as it simultaneously increases both overall uncertainty and the Bayesian weight on prices, reinforcing the endogenous transmission from sentiments to prices.

3.4 Discussion

In a setting where uncertainty amplifies sentiments, informational and non-informational noise can have different effects on the impact of psychological shocks on prices. In particular, when the elasticity of sentiments to uncertainty is less than one, non-informational noise dampens the impact of sentiments on prices, contrary to what one might expect.

Fig. (3) shows the value of the sentiments channel, Δ, for three different values of γ, smaller, equal and greater than one, as a function of fundamental noise (σ_ε^2) and informational noise (σ_v^2). The graphs visually confirm results derived above: larger informational noise always increases Δ, irrespective of the value of γ; non-informational noise, instead, has differential effects on Δ depending on the value of γ: Δ decreases with σ_ε^2 for $\gamma < 1$, it is constant for $\gamma = 1$, and it increases with σ_ε^2 for $\gamma > 1$.

The finding that the volatility of the fundamental shock has a negative relation with the sentiments channel for $\gamma < 1$ implies that, for example, an increase in noise trading can actually reduce the relevance of psychological attitudes in stock markets. As prices become more volatile from increased noise trading, rational agents rely less on prices as an indicator for the value of an asset, dampening the effect of sentiments on those prices. The overall

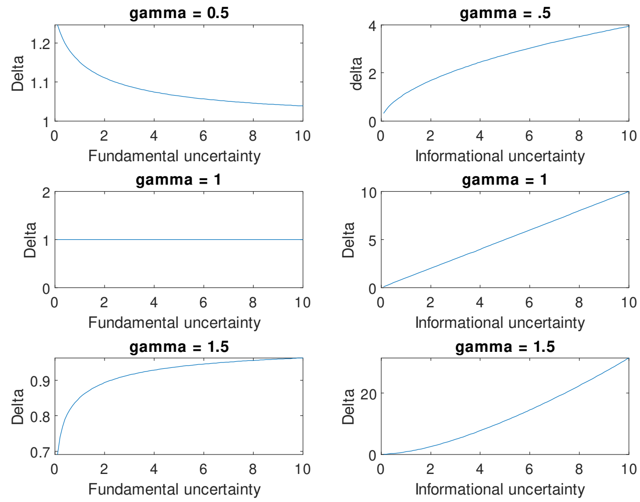

Figure 3: Relationship between uncertainty and Δ for different values of γ and different sources of noise. All other variances set equal to one.

variance of prices still increases, but the part of such variance attributable to sentiments decreases. There is thus a substitution between fundamental noise and sentiments noise in prices.

4 Concluding remarks

What is the best way to model sentiments in an economy? In this paper, I propose a framework that retains the exogeneity of psychological shocks (similar to sunspots) but links the effect of such shocks on the market to the uncertainty coming from imperfect information. This modelling choice allows the effect of sentiments to be endogenous, though the shock that sparks them is exogenous. It captures the intuitive feature that only in the presence of uncertainty can agents entertain subjective beliefs on economic variables that include psychological elements not based on information, i.e., sentiments, or animal spirits.

The ensuing sentiments channel, the mechanism through which psychological attitudes affect stock prices, displays a non-trivial relationship with the level of uncertainty in the market when agents are Bayesian and use prices as a source of information. An increase in noise can amplify or dampen the impact of exogenous psychological shocks on prices depending on the nature of the shock, whether fundamental or informational, and on the elasticity of sentiments to uncertainty, whether larger or smaller than one. These findings imply that periods of high uncertainty are not necessarily periods of high animal spirits on financial markets. It is instead important to identify the source of uncertainty in order to be able to understand its interaction with psychological attitudes.

An important advantage of the proposed modelling strategy, compared to other ways of representing sentiments, is that it generates a direct link between uncertainty and the effect of psychological attitudes on economic outcomes. In a dynamic setting with time-varying uncertainty, this could generate waves of optimism or pessimism, which, sparked by exogenous shocks, propagate endogenously through changes in information accuracy over time.

I leave it for future work to embed such mechanism in more fully fledged macroeconomic and financial models. While this work only considers sentiments in a financial market, the same modelling strategy can be employed to capture sentiments in any market where the uncertainty of agents' estimates (about, e.g., future demand, productivity or any other element of the economy) creates room for psychological attitudes to affect beliefs.

5 Appendix

5.1 Risk averse agents

I derive here the equivalent of the pricing equation (9) for risk averse agents. It will be shown that the sentiments channel is the same as the one derived in the model with risk neutral agents presented in the main text.

If agents' preferences are represented by a CARA utility function and all shocks are normally distributed (see [Hellwig (1980)] for a derivation), one gets that the demand for shares for the generic agent i is given by

$$k^i = \frac{\hat{\theta}^i - p}{\lambda \sigma_T^2},$$

where λ is the coefficient of risk aversion and

$$\sigma_T^2 \equiv Var\left(\hat{\theta}^i\right) = \sigma_I^2 + \left(\sigma_I^2\right)^{2\gamma} \sigma_s^2$$

represents the total error variance.

Aggregating across agents and assuming a stochastic supply $\varepsilon \sim N\left(0, \sigma_\varepsilon^2\right)$ gives the market equilibrium condition

$$\begin{aligned} \int_i k^i di &= \varepsilon \\ \frac{\alpha(\theta - p) + (\sigma_I^2)^\gamma s}{\lambda \sigma_T^2} &= \varepsilon \end{aligned}$$

and thus the pricing equation

$$p = \theta - \alpha^{-1} \lambda \sigma_T^2 \varepsilon + \alpha^{-1} \left(\sigma_I^2\right)^\gamma s. \tag{20}$$

It can be seen that the pricing equation (20) is of the same form as (9). In particular, the sentiments channel, $\alpha^{-1} (\sigma_I^2)^\gamma$, is exactly the same as the one derived for risk-neutral agents, so all the analysis pertaining to this element carries through to a setting with risk averse agents.

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