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Abstract

We study interactions with uncertainty about demand sensitivity. In our solution concept (1) firms choose seemingly-optimal strategies given the level of sophistication of their data analytics, and (2) the levels of sophistication form best responses to one another. Under the ensuing equilibrium firms underestimate price elasticities and overestimate advertising effectiveness, as observed empirically. The misestimates cause firms to set prices too high and to over-advertise. In games with strategic complements (substitutes), profits Pareto dominate (are dominated by) those of the Nash equilibrium. Applying the model to team production games explains the prevalence of overconfidence among entrepreneurs and salespeople.

Keywords: Advertising, pricing, data analytics, strategic distortion, strategic complements, indirect evolutionary approach. **JEL Classification**: C73 , D43, M37.

1 Introduction

Researchers often assume that better measurement and accurate estimations of the sensitivity of demand allow firms to improve their advertising and pricing decisions. Arriving at such accurate estimations requires careful experimental techniques or sophisticated econometric

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methods that correct for the endogeneity of decision variables in the empirically observed data (see, e.g., Blake, Nosko, and Tadelis 2015; Shapiro, Hitsch, and Tuchman 2019; Gordon, Zettelmeyer, Bhargava, and Chapsky 2019; Sinkinson and Starc 2019 who estimate advertising effectiveness, and Berry, 1994; Nevo, 2001; Alé-Chilet and Atal, 2020 who estimate price elasticities).

Despite the emphasis on precision and unbiasedness by researchers, many companies have been slow to adopt these techniques (Rao and Simonov, 2019), often questioning the benefit of causal inference and precise measurement. This reluctance to measure effectiveness precisely is often attributed to implementation difficulties, lack of knowledge and cognitive limitations by decision makers, or moral hazard (Berman, 2018; Frederik and Martijn, 2019). Empirically, we often observe that firm advertising budget allocations are consistent with over-estimation of advertising effectiveness (see, e.g., Blake, Nosko, and Tadelis, 2015; Lewis and Rao, 2015; Golden and Horton, 2020), while pricing decisions are consistent with under-estimation of price elasticities (see, e.g., Besanko, Gupta, and Jain 1998; Chintagunta, Dubé, and Goh 2005; Villas-Boas and Winer 1999; see also Hansen, Misra, and Pai, 2020 who demonstrate that common AI pricing techniques induce "too-high" prices).

In this paper we challenge the assumption that better estimates are always beneficial for firms. Our results show that in many cases firms are better off with biased, less precise, measurements because of strategic considerations in oligopolistic markets. In equilibrium firms will converge to biased measurements because their profits are maximized when they act on these measurements. Moreover, the directions of the biases, as predicted in our model, fit the empirically observed behavior of firms well.

Highlights of the Model We present a model where the payoffs of competing players (firms) each depend on her actions and on her demand, where the demand depends on the actions of all players. The players do not know the demand function, but can select actions and observe the realized demand. The game has two stages. In stage 1 each player hires a (possibly biased) analyst to estimate the sensitivity of demand. An analyst may under- or over-estimate the sensitivity of demand. In stage 2 each player chooses an action taking the estimate as the true value.

Our solution concept, called a Naive Analytics Equilibrium (NAE), is a profile of analysts' biases and a profile of actions, such that (1) each action is a perceived best-reply to the opponents' actions, given the biased estimation, and (2) each bias is a best-reply to the opponents' biases in the sense that if a player deviates to another bias this leads to a new second-stage equilibrium, in which the deviator's (real) profit is weakly lower than the original equilibrium payoff. The first-stage best-replying is interpreted as the result of a gradual process in which firms hire and fire analysts from a heterogeneous pool, and each firm is more likely to fire its analyst if its profit is low.

Summary of Results Our model is general enough to capture price competition with differentiated goods (where the goods can be either substitutes or complements), as well as advertising competition (where the advertising budget of one firm has either a positive or a negative externality on the competitors' demand). Our results show that firms hire biased analysts in any naive analytics equilibrium, and that the direction of the bias is consistent with the empirically observed behavior of firms: in price competition firms hire analysts that under-estimate price elasticities, and in advertising competition firms hire analysts who over-estimate the effectiveness of advertising.

We also show that there is a Pareto-domination relation between the naive analytics equilibrium and the Nash equilibrium (of the game without biases), where its direction depends on the type of strategic complementarity. In a game with strategic complements (i.e., price competition with differentiated goods) the naive analytics equilibrium Pareto dominates the Nash equilibrium, while the opposite holds in a game with strategic substitutes (i.e., advertising competition with negative externalities). The intuition is that in a game with strategic complements (resp., substitutes), each player hires a naive analyst that induces a biased best reply in the direction that benefits (resp., harms) the opponents. This is so because these biases have a strategic advantage of inducing the opponents to change their strategies in the same (resp., opposite) direction, which is beneficial to the player.

Next, we analyze a standard functional form in each type of competition, and show that each price/advertising competition admits a unique naive analytics equilibrium, and that the competing firms choose an equal level of biasedness that depends on the parameters of the competition.

Finally, we demonstrate that our model can be applied in more general settings. Specifically, in Section 5.3 we apply the model to a game of team production with strategic complementarity (see, e.g., Holmstrom, 1982; Cooper and John, 1988). We present two testable predictions in this setup: (1) players are overconfident in the sense of overestimating their ability to contribute to the team's output, and (2) players contribute more than in the (unbiased) Nash equilibrium. These predictions are consistent with the observable behavior of entrepreneurs and salespeople, who often exhibit overconfidence.

Related Literature and Contribution From a theoretical aspect, our methodology of studying a two-stage auxiliary game where each firm is first endowed with a biased analyst and then chooses her pricing/advertising level given results of the analysis is closely related to the literature on delegation (e.g., Fershtman and Judd, 1987; Fershtman and Kalai, 1997; Dufwenberg and Güth, 1999; Fershtman and Gneezy, 2001)[1]. The delegation literature shows that in price competition, firm owners would design incentives for managers that encourage the managers to maximize profits as if the marginal costs are higher than their true value (see, in particular, Fershtman and Judd, 1987, p. 938).

Our model contributes to this literature but also differs from it in a few important aspects. First, in our setup the incentives of all agents are aligned and are based solely on the firm's profit. A deviation of the firm from profit-maximizing behavior is due to (non-intentional) naive analytics, rather than due to explicitly distorting the compensation of the firm's manager. Our novel mechanism is qualitatively different (as it relies on biased estimations rather than different incentives), and it induces testable predictions and policy implications which are different than those induced by delegation (as further discussed in Remark 1). Second an important merit of our model is its generalizability to a wide variety of phenomena and its applicability to wide class of games. The concept of biased estimation of sensitivity of demand can be applied in many seemingly-unrelated setups (e.g., price competition, advertising competition, and team production), while yielding sharp results

[1]See also the related literature on the "indirect evolutionary approach" (e.g., Güth and Yaari, 1992; Heifetz and Segev, 2004; Dekel, Ely, and Yilankaya, 2007; Heifetz, Shannon, and Spiegel, 2007; Herold and Kuzmics, 2009; Heller and Winter, 2020).

about the direction of the observed biases as well as their magnitudes.

Our research is also related to solution concepts that represent agents with misconceptions (e.g., conjectural equilibrium (Battigalli and Guaitoli, 1997; Esponda, 2013), self-confirming equilibrium (Fudenberg and Levine, 1993), analogy-based expectation equilibrium (Jehiel, 2005), cursed equilibrium (Eyster and Rabin, 2005; Antler and Bachi, 2019), coarse reasoning and categorization (Azrieli, 2009, 2010; Steiner and Stewart, 2015; Heller and Winter, 2016), Berk-Nash equilibrium (Esponda and Pouzo, 2016), rational inattention (Steiner, Stewart, and Matějka, 2017), causal misconceptions (Spiegler, 2017, 2019), and noisy belief equilibrium (Friedman, 2018). These equilibrium notions have been helpful in understanding strategic behavior in various setups, and yet they pose a conceptual challenge: why do players not eventually learn to correct their misconceptions? Much of the literature presenting such models points to cognitive limitations as the source of this rigidity. Our model and analysis offer an additional perspective to this issue by suggesting that misperceptions, such as naive analytics, may yield a strategic advantage and are likely to emerge in equilibrium. In this sense our approach can be viewed as providing a tool to explain why some misconceptions persist while others do not.

Structure Section 2 presents a motivating example. In Section 3 we describe our model and solution concept. Our main results are presented in Section 4. Section 5 analyzes three applications: price competition, advertising competition, and team-production game. The main text contains proof sketches and formal proofs are relegated to the appendix.

2 Motivating Example

Consider two firms $i \in \{1,2\}$ each selling a product with price $x_i \in \mathbb{R}_+$. The products are substitute goods. The demand of firm $i \in \{1,2\}$ at day t is given by:

$$q_{it}\left(x_i, x_{-i}\right) = \max\left(20 - x_i + 0.8x_{-i} + z_{it}, 0\right), \quad \text{with } z_{it} \sim \begin{cases} \epsilon & 0.5 \\ -\epsilon & 0.5, \end{cases}$$

where $-i$ denotes the other firm. That is, the expected demand follows Bertrand competition with differentiated goods, and the daily demand of each firm has a random i.i.d demand shock, with value either ϵ or $-\epsilon$ with equal probability. We assume that the marginal costs are zero, which implies that the profit of each firm is given by its revenue $\pi_{it}(x_i, q_{it}) = x_i \cdot q_{it}$.

The firm managers do not know their demand functions, and they hire analysts to estimate the sensitivity of demand to price, in order to find the optimal price. The analyst at each firm asks the firm's employees to experiment for a couple of weeks with offering a discount of Δx in some of the days, and uses the average change in demand Δq between days with and without the discount to estimate the elasticity of demand.

Importantly, the firm's employees do not choose the days with discounts uniformly. The employees observe in each morning a signal that reveals the demand shock (say, the daily weather), and they implement discounts on days of low demand, possibly due to the employees having more free time in these days to deal with posting the discounted price.

There are two types of analysts: naive and sophisticated. A naive analyst does not monitor in which days the employees choose to give a discount, and he implicitly assumes in his econometric analysis that the environment is the same in the days with discounts as in those without discounts. In contrast, sophisticated analysts either monitor the discount decisions to enforce uniform distribution of discounts, or correct the correlation between the weather and discounts in their econometric analysis (e.g., by controlling for the weather).

A sophisticated analyst correctly estimates the mean change in demand

$$\Delta q = (20 - x_i + 0.8x_{-i}) - (20 - (x_i - \Delta x) + 0.8x_{-i}) = -\Delta x,$$

and thus he accurately estimates the elasticity of demand

$$\eta_i = -\frac{x_i}{q_i}\frac{\Delta q}{\Delta x} = -\frac{x_i}{q_i}\frac{(-\Delta x)}{\Delta x} = \frac{x_i}{q_i}.$$

In contrast, a naive analyst under-estimates the mean change in demand to be:

$$\Delta q_{\text{sloppy}} = (20 - x_i + 0.8x_{-i} + \epsilon) - (20 - (x_i - \Delta x) + 0.8x_{-i} - \epsilon) = -\Delta x + 2\epsilon,$$

and thus under-estimates the elasticity of demand to be

$$\eta_{i,\text{naive}} = \frac{x_i}{q_i} \frac{(\Delta x - 2\epsilon)}{\Delta x} \equiv \frac{x_i}{q_i} \alpha_i.$$

Assume, for example, that the parameters Δx and ϵ are such that $\alpha_i = \frac{(\Delta x - 2\epsilon)}{\Delta x} = 0.6$ (which is the optimal level of naïveté as analyzed in Section 5.1). If each firm adjusts prices according to the estimated elasticity (i.e., slightly increasing the price if the estimated elasticity is more than 1, and slightly decreasing the price if less than 1), then the prices converge to a unique equilibrium in which the estimated elasticity of each firm is equal to one. Table 1 presents the prices, demands, and profits as a function of the type of analyst hired by each firm (the calculations are a special case of the analysis of Section 5.1).

Table 1: Equilibrium prices, demands and profits as a function of the analysts' types

Prices		
$\alpha_1 \setminus \alpha_2$	1	0.6
1	17, 17	19, 22
0.6	22, 19	25,25

Demands		
$\alpha_1 \setminus \alpha_2$	1	0.6
1	17, 17	19, 13
0.6	13, 19	15,15

Profits		
$\alpha_1 \setminus \alpha_2$	1	0.6
1	277, 277	351, 287
0.6	287, 351	375, 375

Observe that each firm's profit increases when the firm hires a naive analyst, and decreases when it hires a sophisticated analyst (regardless of the identity of the analyst hired by the competing firms). The intuition is that a naive analyst induces a firm to under-estimate the elasticity of demand, and as a result, to raise prices. This has a beneficial indirect strategic effect of inducing the competitor to increase prices as well. It turns out that the positive indirect effect outweighs the negative direct effect. Thus, if firms occasionally replace their analysts based on their annual profits (i.e., they are more likely to fire an analyst the lower the profit is), then the firms are likely to converge to both hiring naive analysts. This would induce both firms to choose higher prices and have higher profits relative to the Nash equilibrium prices arising with sophisticated analysts.

Our formal results show that these insights hold in a general model. Specifically, we show that in a large class of strategic interactions (incorporating both price competition and advertising competition, as well as both strategic complements and strategic substitutes) players (i.e., firms) choose to hire naive analysts. These naive analysts under-estimate elasticity of demand in price competition, while they over-estimate the effectiveness of advertising in advertising competition. The strategic choices of firms in the equilibria induced by the presence of naive analysts are in the direction that induces a beneficial strategic effect. Finally, we show that the equilibrium induced by naive analysts Pareto dominates the Nash equilibrium with sophisticated analysts in a game with strategic complements, while it is Pareto dominated by the Nash equilibrium in a game with strategic substitutes.

3 Model and Solution Concept

In this section we introduce an analytics game in which competing firms hire analysts to estimate the sensitivity of demand, which is then used to determine the strategic choices of the firm; importantly, the demand of each firm is also affected by the strategic choices of its competitors. Next we present our solution concept of a naive analytics equilibrium.

3.1 Underlying Game

An *analytics game* is a two stage game in which each of $N = \{1, 2, ..., n\}$ players (firms) hire an analyst who estimates the sensitivity of demand in the first stage and then make a strategic choice that affects demand in the second stage. We first describe the structure of the second stage, which we call the *underlying game* and denote by $G = (N, X, q, \pi)$. In the underlying game each firm $i \in N$ makes a strategic choice $x_i \in X_i$ that affects the demands and the profits of all firms, where $X_i \subseteq \mathbb{R}$ is a (possibly unbounded) interval of feasible choices of firm i. Let $X = \prod_{i \in N} X_i$ be the Cartesian product of these intervals.

Let $-i \equiv N \backslash \{i\}$ denote all firms except firm i and $-ij \equiv N \backslash \{i, j\}$ denote all firms except i and j. Let (x'_i, x_{-i}) denote the strategy profile, in which player i plays strategy x'_i, while all other players play according to the profile x_{-i} (and we apply a similar notation for x_{-ij}). Let $q_i(x)$ denote the demand of firm i. The (true) payoff, or profit, of each firm $i \in N$, denoted by $\pi_i(x_i, q_i(x))$, depends on the firm's demand $q_i(x)$ and its strategic choice x_i. We assume that the demand functions $q_i(x)$ and payoff functions $\pi_i(x_i, q_i)$ of all firms are continuously twice differentiable in all parameters. We further assume that the profit is increasing in demand, i.e., $\frac{\partial \pi_i(x_i, q_i)}{\partial q_i} > 0$ for any $x_i \in X_i$ and any q_i.

We assume that the payoff function of each player is unimodal.

Assumption 1 (Unimodality). *For each player i and each profile $x_{-i} \in X_{-i}$, there exists $x_i^* \in X_i$ such that $\frac{d\pi_i(x_i,x_{-i})}{\partial x_i} > 0$ for any $x_i < x_i^*$ and $\frac{d\pi_i(x_i,x_{-i})}{dx_i} < 0$ for any $x_i > x_i^*$.*

Unimodality implies that any opponents' profile $x_{-i} \in X_{-i}$ has a unique best-reply, which we denote by $BR_i(x_{-i}) = \text{argmax}_{x_i \in X_i}(\pi_i(x_i, x_{-i}))$.

Next we assume that the player's strategy influences the demand and the payoff (given a fixed demand) in opposing directions. In the motivating example the strategic choice is the price, which increases the profit per product sold, while decreasing the firm's demand.

Assumption 2 (Opposing effects). $\frac{\partial\pi_i(x_i,q_i)}{\partial x_i} \cdot \frac{\partial q_i(x)}{\partial x_i} < 0$ *for any $x_i \in X_i$ and any q_i.*

Let $Int(X_i)$ denote the interior of X_i. A necessary (and due to unimodality, also sufficient) condition for a strategy $x_i \in Int(X_i)$ to be a best reply to the opponents' strategy profile is that it satisfies the following first-order condition:

$$\frac{d\pi_i(x_i, q_i(x))}{dx_i} = \underbrace{\frac{\partial\pi_i}{\partial x_i}}_{(i)} + \underbrace{\frac{\partial\pi_i}{\partial q_i}}_{(ii)} \cdot \underbrace{\frac{\partial q_i}{\partial x_i}}_{(iii)} = 0 \tag{3.1}$$

Sections 5.1–5.2 present two applications of this model. The first application generalizes the motivating example of price competition. The second application is for advertising competition where the strategic choice of each firm is its advertising spending.

3.2 First-Stage Choice of Analysts

In this subsection we describe the first stage of the analytics game, in which each firm chooses an analyst to estimate its demand.

In order to maximize their profits when choosing x_i, firms need to know or estimate the impact of their actions on their profits. We assume that each firm knows (or correctly estimates): (i) the direct effect of its strategy on its profit $\frac{\partial\pi_i}{\partial x_i}$; and (ii) the effect of the firm's demand on its profit $\frac{\partial\pi_i}{\partial q_i}$. In contrast, we assume it is difficult for the firm to estimate (iii) the response of its demand to marginal changes in its strategy, i.e., to estimate $\frac{\partial q_i}{\partial x_i}$. For example, during price competition firms know how their product's prices affect their profit margins and how demand affects profit, but might not know how sensitive consumers might be to price changes. Similarly, in advertising competition firms know how increasing advertising spending affects their bottom line costs, but might not know the impact of their advertising on demand. Each firm therefore hires an analyst in the first stage who is tasked with estimating $\frac{\partial q_i}{\partial x_i}$.

Let $A \subseteq \mathbb{R}_{++}$ denote the interval of feasible biases of analysts, and we assume that A includes an open interval around 1. Analysts are characterized by a bias $\alpha_i \in A \subseteq \mathbb{R}_{++}$ that causes them to estimate the marginal effect of the strategy x_i on demand q_i as $\alpha_i \frac{\partial q_i}{\partial x_i}$ instead of $\frac{\partial q_i}{\partial x_i}$. We denote the bias profile of all analysts by $\alpha = (\alpha_1, \ldots, \alpha_n)$. A sophisticated analyst is unbiased, while naive analysts have $\alpha_i \neq 1$. Consequently, an α_i-analyst induces the firm to choose a strategy x_i that solves the biased first-order condition

$$\frac{\partial \pi_i}{\partial x_i} + \frac{\partial \pi_i}{\partial q_i} \cdot \alpha_i \cdot \frac{\partial q_i}{\partial x_i} = 0$$

instead of the unbiased condition in (3.1).

There are many reasons why analysts might be biased. One example is inadvertently creating endogenous correlation between the firm's strategy and demand without taking this correlation into account in the analysis. If, as in the motivating example, a firm sets lower sale prices on days of low demand and higher regular prices on days of high demand, estimating price elasticities using the resulting data will show that consumers are less price sensitive than they actually are. Another example is when firms set their advertising budgets differently in specific times such as before holidays, or weekends. This would create correlation in the levels of advertising with those of competitors. Ignoring this correlation during analysis may lead to a biased estimate of advertising effectiveness. We present micro-foundations for biased analytics towards the end of Section 5.1 (price competition) and Section 5.2 (advertising).

3.3 RIDE and α-Equilibrium

In what follows we define the ratio of indirect effect to direct effect (RIDE), and use this notion to define an equilibrium of the second-stage, given the analysts' biased profile.

The ratio of the indirect marginal effect to the direct marginal effect (henceforth, *RIDE*) of a firm's strategy on profit is defined as follows:

$$\text{RIDE}_i(x) \equiv -\frac{\frac{\partial \pi_i}{\partial q_i(x)} \cdot \frac{\partial q_i}{\partial x_i}}{\frac{\partial \pi_i}{\partial x_i}} \tag{3.2}$$

Assumption 2 (opposing effects) implies that $\text{RIDE}_i(x)$ is positive and well-defined. When a firm changes its strategy, it influences its profit through two channels: the direct effect on profit $\frac{\partial \pi_i}{\partial x_i}$ and the indirect effect through the influence on the demand $\frac{\partial \pi_i}{\partial q_i(x)} \cdot \frac{\partial q_i}{\partial x_i}$. RIDE_i measures the ratio between the indirect effect and the direct effect. We note that RIDE_i is a unitless measure, and that it coincides with the elasticity of demand $|\eta_{q_i,x_i}|$ when the firm's profit is given by the multiplication $\pi_i(x) = x_i \cdot q_i(x)$, as in the the motivating example (see Example 1 in Section 4.1). If the profit function can be written as the difference between revenues R and (demand-independent) costs C, as in $\pi_i(x) = R(q_i(x)) - C(x_i)$, then RIDE_i is the ratio of marginal revenues to marginal costs. This is the case, for example, in advertising competition where advertising affects revenue only through demand (Section 5.2).

Using the definition of $RIDE_i$ we observe that the standard (interior) Nash equilibrium solution to (3.1) is equivalent to solving:

$$0 = \frac{d\pi_i(x_i, q_i(x))}{dx_i} = \frac{\partial \pi_i}{\partial x_i} + \frac{\partial \pi_i}{\partial q_i} \cdot \frac{\partial q_i}{\partial x_i} \;\Leftrightarrow\; \text{RIDE}_i(x) = 1.$$

When analysts may be biased, we define an α-*equilibrium* as a strategy profile such that each firm's biased first order condition holds:

$$\frac{\partial \pi_i}{\partial x_i} + \frac{\partial \pi_i}{\partial q_i} \cdot \left(\alpha_i \cdot \frac{\partial q_i}{\partial x_i}\right) = 0 \;\Leftrightarrow\; \text{RIDE}_i(x) = \frac{1}{\alpha_i}. \tag{3.3}$$

Definition 1 (α – Equilibrium)**.** Fix a biasedness profile $\alpha \in A^n$. A strategy profile x is an α-*equilibrium* if $\text{RIDE}_i(x) = \frac{1}{\alpha_i}$ for each player i.

An implication of biasedness is that an analyst with $\alpha_i < 1$ will cause the firm to set a strategy x_i that creates a relatively larger indirect effect, while when $\alpha_i > 1$, the analyst will cause the firm to set x_i to have a relatively larger direct effect when comparing to the (unbiased) profit-maximizing value of x_i.

3.4 Naive Analytics Equilibrium (NAE)

In what follows we define our main solution concept of a naive analytics equilibrium. To simplify the definition and exposition, we assume that the underlying game G has a unique α-equilibrium for every biasedness profile α.

Assumption 3 (Uniqueness)**.** *For each $\alpha \in A^n$ there exists a unique α-equilibrium, which we denote by $x(\alpha) \in X$.*

Assumption 3 is satisfied in various economic applications, including price and advertising competitions presented in Sections 5.1 and 5.2. Due to the unimodality assumption the unique $\vec{1}$-equilibrium is the unique Nash equilibrium of the underlying game, which we denote by $x^{NE} \equiv x\left(\vec{1}\right)$. In Appendix B we present conditions on the RIDEs of the players that imply the existence of a unique α-equilibrium.

Assumption (3) allows us to define the underlying-game's payoff of a biasedness profile $\alpha \in \mathbb{R_+}^n$ as $\tilde{\pi}_i(\alpha) \equiv \pi_i(x(\alpha))$, which is the payoff of firm i when all firms follow the unique α-equilibrium. In particular, $\tilde{\pi}_i\left(\vec{1}\right)$ is the payoff of player i in the unique Nash equilibrium of the underlying game.

A naive analytics equilibrium is a bias profile and a strategy profile, where (1) the strategy profile is an α-equilibrium, and (2) each bias is a best-reply to the opponents' bias profile (i.e., a unilateral deviation to another bias would induce a new α-equilibrium with a lower payoff to the deviator). Formally,

Definition 2. A *naive analytics equilibrium* is a pair (α^*, x^*), where:

1. $x^* \in X^n$ is the α^*-equilibrium of the underlying game G (i.e., $x^* = x(\alpha^*)$).

2. $\tilde{\pi}_i\left(\alpha_i^*, \alpha_{-i}^*\right) \geq \tilde{\pi}\left(\alpha_i', {\alpha^*}_{-i}\right)$ for each player i and each $\alpha_i' \in A$.

3.5 Interpretation of Naive Analytics Equilibrium

We do not interpret the equilibrium behavior in the first-stage as the result of an explicit maximization of sophisticated firms who know the demand function and calculate the optimal α-s for their analysts. Conversely, we think of the firms as having substantial uncertainty about the demand function and its dependence on the behavior of various competitors. Due to this uncertainty, the firms hire analysts to estimate the sensitivity of demand. Occasionally (say, at the end of each year) firms consider replacing the current analyst with a new one (say, with a new random value of α_i), and a firm is more likely to do so the lower its profit is. Gradually, such a process would induce the firms to converge to hiring most of the time analysts with values of α that are best replies to each other, and thus constitute a naive analytics equilibrium (α, x).

It is important to note that the observed data does not contradict the optimality of the strategic choices of the firms or the correctness of the estimations of the sensitivity of demand of the analysts. Consider, for example, a naive analytics equilibrium (α, x) in the price competition described in Section 2. A firm that wishes to confirm that its price is indeed optimal (i.e., that it maximizes its profit given the demand) is likely to experiment with temporary changes in prices to see its influence on demand. Under the arguably plausible assumption that the analysis of such an experiment will be done with the same level of sophistication as the one leading to (α, x), the firm's conclusion from the experiment would be that the sensitivity of demand is exactly as estimated by the firm's naive analyst, and that the firm's equilibrium strategy is optimal (e.g., it induces elasticity of -1 in the motivating example, and thus maximizes the firm's profit). Moreover, a firm in a naive analytics equilibrium that will insist on running a properly randomized experiment and acting on the "proper" estimate will experience a decrease in profits.

Remark 1 (Delegation interpretation). An alternative interpretation of our model (which we do not use in the paper) is of delegation. Let $\pi_i^{\alpha_i} : X \to \mathbb{R}$ be a subjective payoff function such that maximizing $\pi_i^{\alpha_i}$ with an unbiased estimation is equivalent to maximizing π_i with a biased estimation of α_i, i.e., for any strategy profile $x \in X$

$$\mathrm{RID}E_i\left(x\right) = \frac{1}{\alpha_i} \text{ iff } \pi_i^{\alpha_i}\left(x\right) = \mathrm{argmax}_{x_i' \in X_i} \pi_i^{\alpha_i}\left(x_i', x_{-i}\right).$$

Let $\Pi_i^A = \{\pi_i^{\alpha_i} | \alpha_i \in A\}$ be the set of all such subjective payoff function. One can reinterpret a naive analytics equilibrium as an equilibrium of a delegation game (Fershtman and Judd, 1987) in which in the first stage each firm's owner simultaneously chooses a payment scheme to its manager, which induces the manager with a subjective payoff function in Π_i^A. In the second stage the managers play a Nash equilibrium of the game induced by the subjective payoffs. Although, both interpretations (namely, naive analytics and delegation) yield the same prediction about the equilibrium strategy profile, they differ in other testable predictions. For concreteness, we focus the comparison for price competition (as in the motivating example). The delegation interpretation predicts firms to correctly estimate the elasticity of demand and to pay managers a payoff that increases in the firm profit and decreases in the firm's sales (see, Fershtman and Judd, 1987, p. 938). It is seldom observed that firms encourage managers to decrease the firm's sales. The naive analytics interpretation predicts that firms will hire naive analysts that over-estimate elasticity of demand, with a manager's payoff scheme that depends on the firm's profit (and is not a decreasing function of the firm's sales). As mentioned elsewhere in the paper, we believe this latter prediction is more consistent with the empirically observed behavior of firms.

4 Characteristics of NAE with Monotone Derivatives

We answer 3 questions about firms in a naive analytics equilibrium: (1) what is their direction of deviation from an unbiased best reply to the opponents' strategies (Section 4.2), (2) when do they under or over estimate the sensitivity of demand through biased analytics (Section 4.3), and (3) when do they achieve payoffs that Pareto dominate the Nash equilibrium (Section 4.3). Our results rely on assumptions of monotone derivatives, which are presented in Section 4.1. In Section 4.4 we show a Stackelberg-leader representation of our results, which will be helpful in the applications in Section 5. A summary of the results is presented in Table 2 in the end of the section.

4.1 Monotone Derivatives and Strategic Complementarity

Our next assumption requires the externality of a player's strategy on her opponent's payoff $\frac{d\pi_i}{dx_j}$ to be monotone (i.e., either always positive or always negative).

Assumption 4. ***Monotone payoff externalities:*** *$\frac{d\pi_i(x)}{dx_j}$ are either all positive or all* *negative for every $i \neq j \in N$ and every $x \in X$.*

Note that Assumption 4 is equivalent to requiring that demand externalities are monotone (i.e., $\frac{dq_i(x)}{dx_j}$ are either all positive or all negative) due to our Section 3.1's assumptions: (1) an opponent's strategy influences a player's payoff only through the player's demand, and (2) player's payoff is increasing in her demand.

Next we require that the externality of a player on her opponent's RIDE is monotone.

Assumption 5. ***Monotone RIDE externalities**: $\frac{d(RIDE_i(x))}{dx_j}$ are either all positive or all negative for every $i \neq j \in N$ and every $x \in X$.*

Finally, we require that the RIDE of a player is monotone in her own strategy.

Assumption 6'. ***Monotone RIDE**:* $\frac{d(\mathrm{RIDE}_i(x))}{dx_i}$ are either all positive or all negative for every $i \in N$ and every $x \in X$.

The following simple observation shows that the payoffs are unimodal iff the RIDE externality $\frac{d(\mathrm{RIDE}_i(x))}{dx_i}$ has the same sign as the player's direct effect on her payoff.

Claim 1. Let $G = (N, X, q, \pi)$ be an underlying game that satisfies Assumption 2 (opposing effects) and Assumption 6'. Assume that for each player i and each opponents' profile x_{-i}, there is $x_i^* \in X_i$ such that $\mathrm{RIDE}_i(x_i^*, x_{-i}) = 1$.[2] Then G satisfies Assumption 1 (unimodality) iff $\frac{d(\mathrm{RIDE}_i)}{dx_i} \cdot \frac{\partial \pi_i}{\partial x_i} > 0$ for every $i \in N$ and every $x \in X$.

Sketch of proof (proof is in Appx. A.2). In this sketch we show that if $\frac{d(\mathrm{RIDE}_i)}{dx_i}, \frac{\partial \pi_i}{\partial x_i} > 0$ then the payoffs are unimodal (the remaining cases are analogous). Observe that $\frac{d\pi_i}{dx_i} = 0$ (resp., $\frac{d\pi_i}{dx_i} < 0$, $\frac{d\pi_i}{dx_i} > 0$) iff $\frac{\partial \pi_i}{\partial x_i} = \left|\frac{\partial \pi_i}{\partial q_i} \cdot \frac{\partial q_i}{\partial x_i}\right|$ (resp., $\frac{\partial \pi_i}{\partial x_i} < \left|\frac{\partial \pi_i}{\partial q_i} \cdot \frac{\partial q_i}{\partial x_i}\right|$, $\frac{\partial \pi_i}{\partial x_i} > \left|\frac{\partial \pi_i}{\partial q_i} \cdot \frac{\partial q_i}{\partial x_i}\right|$), which holds iff $\mathrm{RIDE}_i(x) = 1$ (resp., $RIDE_i(x) > 1$, $RIDE_i(x) < 1$). This latter equality (resp., inequality) holds iff $x_i = x_i^*$ (resp., $x_i > x_i^*$, $x_i < x_i^*$), which implies unimodality. □

Claim 1 allows us to replace both Assumption 6' (monotone RIDE) and Assumption 1 (unimodality) with the following assumption that the RIDE is monotone and in the direction equivalent to unimodality. Formally[3]

[2] The assumption that there is x_i^* s.t. $\mathrm{RIDE}_i(x_i^*, x_{-i}) = 1$ can be omitted if one assumes compact X_i-s.
[3] Appx. B presents conditions on the RIDE derivatives that imply unique α-equilibrium (Asm. 3).

Assumption 6. ***Unimodal monotone RIDE***: $\frac{d(RIDE_i)}{dx_i}$ *has the same sign as* $\frac{\partial \pi_i(x_i,q_i)}{\partial x_i}$ *(i.e.,* $\frac{d(RIDE_i)}{dx_i} \cdot \frac{\partial \pi_i(x_i,q_i)}{\partial x_i} > 0$*) for every* $i \in N$ *and every* $x \in X$*.*

Assumptions 4–6 are satisfied in many economic applications (including price competition and advertising competition, as detailed in the Sections 5.1–5.2).

Example 1 (Motivating example revisited)**.** Recall that in the motivating example the profit of firm i is $\pi_i = x_i \cdot q_i$ and its expected demand is $q_i = 20 - x_i + 0.8x_j$. Observe that the RIDE coincides with the elasticity of demand:

$$\text{RIDE}_i = -\frac{\frac{\partial \pi_i}{\partial q_i(x)} \cdot \frac{\partial q_i}{\partial x_i}}{\frac{\partial \pi_i}{\partial x_i}} = -\frac{x_i \cdot (-1)}{q_i} = \frac{x_i}{q_i} = \left|\eta_{q_i,x_i}\right|.$$

Further observe that the payoff externalities are positive:

$$\frac{d\pi_i}{dx_j} = \frac{x_i d\left(20 - x_i + 0.8x_j\right)}{dx_j} = 0.8x_i > 0,$$

and the RIDE externalities and RIDE derivative are negative and positive, respectively:

$$\frac{d\text{RIDE}_i}{dx_j} = \frac{d}{dx_j}\left(\frac{x_i}{q_i}\right) = -\frac{0.8x_i}{q_i^2} < 0, \qquad \frac{d\text{RIDE}_i}{dx_i} = \frac{d}{dx_i}\left(\frac{x_i}{q_i}\right) = \frac{q_i + x_i}{q_i^2} > 0.$$

Assumptions 4–6 map to eight combinations on the directions of the derivatives. Effectively, these eight combinations define four unique classes of games since relabeling strategies as their negative values (i.e., replacing x_i with $-x_i$) results in essentially the same game with opposite signs to each of three monotone derivatives (see, Fact 1 in Appendix A.3).

Next we show that any game with monotone derivatives satisfies either strategic complementarity or strategic substitutability. A game has strategic complements (resp., substitutes) if the players' decisions reinforce (resp., offset) one another in the sense that increasing a player's strategy increases (resp., decreases) the opponents' best replies. Formally,[4]

Definition 3. A game with a best-reply function has *strategic complements* (*resp., strategic substitutes*) if $BR_j\left(x_i', x_{-ij}\right) > BR_j\left(x_i, x_{-ij}\right)$ (resp., $BR_j\left(x_i', x_{-ij}\right) < BR_j\left(x_i, x_{-ij}\right)$) for

[4] Our definition of strategic complementarity/substitutability in term of the best-reply function follows Monaco and Sabarwal (2016). It is essentially equivalent in our setup to the commonly-used alternative definitions of increasing differences and the sign of the cross derivative (see, e.g., Bulow, Geanakoplos, and Klemperer, 1985).

each players $i, j \in N$, strategy profile x and strategy $x'_i > x_i$.

The following observation shows that any game with monotone derivatives has either strategic complements (if the two RIDE derivatives have the opposite signs) or strategic substitutes (if the two RIDE derivatives have the same sign). Formally,

Claim 2. Let G be an underlying game that satisfies Assumptions 2 and 4–6. Then:

1. The game has strategic complements iff $\frac{d(\mathrm{RIDE}_j)}{dx_j} \cdot \frac{d(\mathrm{RIDE}_j)}{dx_i} < 0$.

2. The game has strategic substitutes iff $\frac{d(\mathrm{RIDE}_j)}{dx_j} \cdot \frac{d(\mathrm{RIDE}_j)}{dx_i} > 0$.

Proof sketch for the motivating example in which $\frac{d(RIDE_j)}{dx_i} < 0 < \frac{d(RIDE_j)}{dx_j}$ *(proof in Appx. A.4).*

Let $x'_i > x_i$. Negative RIDE externalities imply that $RIDE_j(x'_i, x_{-ij}) < RIDE_j(x_i, x_{-ij})$, which, in turn, implies (due to positive RIDE derivative) that $BR_j(x'_i, x_{-ij}) > BR_j(x_i, x_{-ij})$, which shows that G has strategic complements . □

4.2 Direction of Commitment and Under/Over Replying in G

In this subsection we show that the direction in which agents deviate from an unbiased best reply to the opponents' strategies is the one that induces a beneficial reply by its opponents.

We say that a player benefits from an upward commitment if increasing a firm's own strategy induces best-replying opponents to change their strategies in the direction that is beneficial to the firm. That is, in a game with positive payoff externalities a firm's decision to increase its strategy would elicit competitors to pick a higher strategy, while in a game with negative externalities, increasing a firm's own strategy would induce competitors to decrease their strategy. Formally:

Definition 4. A game with monotone payoff externalities has a *beneficial upward (resp., downward) commitment* iff either:

1. the game has positive (resp., negative) payoff externalities and $BR_j(x'_i, x_{-ij}) > BR_j(x_{-j})$ for each $i, j \in N$, each $x \in X$ and each $x'_i > x_i$, or

2. the game has negative (resp., positive) payoff externalities and $BR_j(x'_i, x_{-ij}) < BR_j(x_{-j})$ for each $i, j \in N$, each $x \in X$ and each $x'_i > x_i$.

In a game with beneficial upward commitment, if firm i were a Stackelberg leader it would deviate from the simultaneous-game Nash equilibrium towards a higher strategy, since the competing firms will reciprocate with a deviation in the direction that benefits firm i. Similarly a game with beneficial downward commitment would entice a Stackelberg-leading firm to deviate from the simultaneous-game Nash equilibrium towards a lower strategy.

Fix a strategy profile x. We say that player i *under-replies* if her strategy is lower than the unbiased ($\alpha_i = 1$) best reply to the opponents' strategies x_{-i}, i.e., if $x_i < BR_i(x_{-i})$. We say that player i *over-replies* if $x_i > BR_i(x_{-i})$. Our first result shows that in any naive analytics equilibrium all players differ from (unbiased) best replying in the direction of beneficial commitment:

Proposition 1. *Let G be an underlying game satisfying Assumptions 2–5. All agents:*

1. over reply in any naive analytics equilibrium if G has a beneficial upward commitment;

2. under reply in any NAE if G has a beneficial downward commitment.

Sketch of proof. If any player i differs from the (unbiased) best replying in the direction opposite of beneficial commitment, then this cannot be a naive analytics equilibrium. This is so because a deviation of player i to bias α_i slightly closer to 1 must increase the deviator's payoff (contradicting the profile being a NAE) because the deviation yields both a direct advantage (the new strategy is closer to best-replying) and a strategic advantage (the new strategy has shifted in the direction that yields a beneficial commitment). □

Next, we characterize the direction of beneficial commitment in terms of the number of positive derivatives out of the three monotone derivatives of Assumptions 4–5. Specifically, the direction of beneficial commitment is upwards if the number of positive derivatives is even, while it is downwards if the number of positive derivatives is odd:

Claim 3. Let G be an underlying game that satisfies Assumptions 2 and 4–5. Then G:

1. has a beneficial upward commitment iff the number of positive derivatives is even;

2. has a beneficial downward commitment iff the number of positive derivatives is odd.

Proof sketch for the motivating example ($\frac{d(RIDE_i)}{dx_j} < 0 < \frac{d(RIDE_i)}{dx_i}, \frac{d(\pi_i)}{dx_j}$)*; proof is in Appx. A.6.* If a player increases her strategy, it decreases the opponents' RIDEs (due to negative RIDE externalities). Due to increasing RIDE derivative, it implies that all opponents $j \neq i$ have to increase their own strategies in order to maintain $\text{RIDE}_j = \frac{1}{\alpha_j}$ in the new naive analytics equilibrium. The change of the opponents' strategies increases player i's payoff due to having positive externalities, which implies that the game has a beneficial downward commitment. □

4.3 Direction of Analytics Bias and Equilibrium Payoffs

Next we characterize the condition for analysts to either over estimate or under estimate the sensitivity of demand in any naive analytics equilibrium. Specifically, we show that

1. All agents overestimate the sensitivity of demand (i.e., $\alpha_i > 1$ for each player i) iff both the RIDE externality and payoff externality have the same sign.

2. All agents underestimate the sensitivity of demand (i.e., $\alpha_i < 1$ for each player i) iff the RIDE externality and payoff externality have different signs.

Proposition 2. *Let G be an underlying game that satisfies Assumptions 2–5. Let (α^*, x^*) be a naive analytics equilibrium, and let $i \in N$. Then:*

1. $\alpha_i^* > 1$ *if* $\frac{d(RIDE_j)}{dx_i} \cdot \frac{d\pi_j}{dx_i} > 0$.

2. $\alpha_i^* < 1$ *if* $\frac{d(RIDE_j)}{dx_i} \cdot \frac{d\pi_j}{dx_i} < 0$.

Sketch of proof for the motivating example (in which $\frac{d(RIDE_i)}{dx_j} < 0 < \frac{d(RIDE_i)}{dx_i}, \frac{d(\pi_i)}{dx_j}$*).*
By Claim 3 the game has a beneficial commitment advantage. By Proposition 1 all agents over reply in any naive analytics equilibrium (α^*, x^*) (i.e., $x_i^* > BR\left(x_{-i}^*\right)$). Due to the RIDE derivative being positive ($\frac{d(\text{RIDE}_i)}{dx_i} > 0$), it implies that $\frac{1}{\alpha_i^*} = \text{RIDE}_i\left(x^*\right) > 1 \Rightarrow \alpha_i^* < 1$. □

Finally, we show that any naive analytics equilibrium of any game with strategic complements Pareto improves over the Nash equilibrium payoffs of the underlying game. Moreover, the converse is true for symmetric equilibria of games with strategic substitutes. Any symmetric naive analytics equilibrium of any game with strategic substitutes (which admits a symmetric Nash equilibrium) is Pareto dominated by the Nash equilibrium of the underlying game.

Definition 5. Strategy profile x is symmetric if $x_i = x_j$ for any pair of players $i, j \in N$.

Proposition 3. *Let G be an underlying game that satisfies Assumptions 2–5. Let (α^*, x^*) be a naive analytics equilibrium.*

1. If G has strategic complements, then $x_i^* > x_i^{NE}$ and $\pi_i(x^*) > \pi_i\left(x^{NE}\right)$ for each $i \in N$.
2. If G has strategic substitutes, and x^* and x^{NE} are symmetric profiles, then $x_i^* < x_i^{NE}$ and $\pi_i(x^*) < \pi_i\left(x^{NE}\right)$ for each player $i \in N$.

Sketch of proof for the motivating example (in which $\frac{d(RIDE_i)}{dx_j} < 0 < \frac{d(RIDE_i)}{dx_i}, \frac{d(\pi_i)}{dx_j}$).
The fact that (α^*, x^*) is a naive analytics equilibrium implies that

$$\pi_i(x^*) = \pi_i(x(\alpha^*)) \geq \pi_i\left(x\left(1, \alpha_{-i}^*\right)\right).$$

The fact that $x_i\left(1, \alpha_{-i}^*\right)$ is the unbiased best reply of player i implies that

$$\pi_i\left(x\left(1, \alpha_{-i}^*\right)\right) > \pi_i\left(x_i^{NE}, x_{-i}\left(1, \alpha_{-i}^*\right)\right).$$

Due to Proposition 1 all players over reply in (α^*, x^*) (i.e., $x_i^* > BR_i\left(x_{-i}^*\right)$). This implies that all players $j \neq i$ over reply also in $x\left(1, \alpha_{-i}^*\right)$ (because they have the same values of α_j^* in both profiles). In games with strategic complements this observation implies that $x_j\left(1, \alpha_{-i}^*\right) > x_j^{NE}$ for each player $j \in N$ (as formalized in Lemma 3 in Appendix A.8).[5] Finally, The fact that the game has positive payoff externalities implies that

$$\pi_i\left(x_i^{NE}, x_{-i}\left(1, \alpha_{-i}^*\right)\right) > \pi_i\left(x^{NE}\right).$$

Combining the three inequalities we obtain $\pi_i(x^*) > \pi_i\left(x^{NE}\right)$. □

Table 2 summarizes the results presented so far in Section 4.

[5] Games with strategic substitutes do not have a property analogous to Lemma 3. Due to this our statement of Pareto domination for games with strategic substitutes holds only for symmetric profiles.

Table 2: Summary of Results Under Assumptions 1–5

Applications (Sections 5.1–5.3)	Payoff Extr. $\frac{d\Pi_i}{dx_j}$	RIDE extr. $\frac{d(RIDE_i)}{dx_j}$	RIDE deriv. $\frac{d(RIDE_i)}{dx_i}$	Direction of beneficial commitment	Under / Over estimation	Strategic substitutes/ complements
Price competition w. subs. goods (motiv. example)	+	-	+	Beneficial upward commitment	$\alpha_i^* < 1$ (under)	Strategic complements: $x_i^* > x_i^{NE} \ \ \forall i$ NAE Pareto dominates NE.
Advertising with positive exter. & team production	+	+	-		$\alpha_i^* > 1$ (over)	
Price competition w comp. goods	-	+	+	over reply in any NAE.	$\alpha_i^* < 1$ (under)	Str. Substitutes: Sym. NE Pareto dominates symmetric NAE
Advertising with negative extr.	-	-	-		$\alpha_i^* > 1$ (over)	

4.4 Stackelberg-Leader Representation

An interesting representation of the NAE, which will prove useful in the applications in the next section, is to characterize it as an α-equilibrium in which each player plays her (unbiased) optimal Stackelberg-Leader strategy. The representation holds in a general setup, without relying on the assumptions of monotone derivatives of Section 4.1.

We begin by defining an α_{-i}-equilibrium given a fixed action of player i.

Definition 6. Fix player $i \in N$, strategy $x_i \in X_i$, and a bias profile of the remaining players $\alpha_{-i} \in A^{n-1}$. A profile of the remaining players $x_{-i} \in X_{-i}$ is an α_{-i}-equilibrium if $\text{RIDE}_j\left(x\right) = \frac{1}{\alpha_j}$ for each player $j \neq i$.

We assume that each strategy x_i induces a unique α_{-i}-equilibrium.

Assumption 3'. ***(Adapted uniqueness)*** For each player $i \in N$, strategy $x_i \in X_i$, and bias profile $\alpha_{-i} \in A^{n-1}$, there exists a unique α_{-i}-equilibrium, denoted by $x_{-i}\left(x_i, \alpha_{-i}\right) \in X_{-i}$.

Next we define $X_i^{SL}\left(\alpha_{-i}\right)$ as the set of optimal strategies of an (unbiased) Stackelberg-leader player i who faces opponents with bias profile α_{-i} (when the set of biases is restricted, i.e., $A \neq \mathbb{R}_{++}$,we restrict the feasible Stackelberg-leader strategies to those for which the multiplicative inverse of the induced RIDE is in A).

Definition 7. Let G be an underlying game satisfying assumptions 1, 2 and 3' with set of of feasible biases A . Player i's *Stackelberg-leader strategy* against bias profile $\alpha_{-i} \in A^{n-1}$ is:

$$X_i^{SL}(\alpha_{-i}) = \text{argmax}_{\left\{x_i \in X_i | \text{RIDE}_i^{-1}(x_i, x_{-i}(x_i, \alpha_{-i})) \in A\right\}} \pi_i\left(x_i, x_{-i}\left(x_i, \alpha_{-i}\right)\right).$$

Next we characterize a naive analytics equilibrium as an α-equilibrium in which everyone plays Stackelberg-leader strategies.

Claim 4. Let G be an underlying game that satisfies Assumptions 1–3 and 3'. The pair $(\alpha^*, x(\alpha^*))$ is a naive analytics equilibrium iff $x_i(\alpha^*) \in X_i^{SL}(\alpha_{-i})$ for each player $i \in N$. Moreover, if $x_i(\alpha^*) \in Int(X_i)$ and $\alpha_i \in Int(A_i)$, then

$$\alpha_i^* - 1 = \sum_{j \neq i} \frac{\mathrm{d}x_j\left(x_i, \alpha_j^*\right)}{dx_i} \cdot \frac{\frac{\partial q_i}{\partial x_j}}{\frac{\partial q_i}{\partial x_i}}.$$

Sketch of proof; proof in Appx. A.1. If $x_i(\alpha^*)$ is (resp., is not) a Stackelberg-leader strategy in $(\alpha^*, x(\alpha^*))$, then there does not (resp., does) exist a bias α'_i that induces an (α'_i, α_{-i})-equilibrium where player i plays a Stackelberg-leader strategy and gains a payoff higher than in $x(\alpha^*)$. The "moreover" part is implied by substituting the FOC in the definition of α^*-equilibrium (namely, $0 = \frac{\partial \pi_i}{\partial x_i} + \frac{\partial \pi_i}{\partial q_i} \cdot \alpha_i \cdot \frac{\partial q_i}{\partial x_i}$) in the FOC of a Stackelberg-leader strategy

$$0 = \frac{d\pi_i\left(x_i, x_{-i}\left(x_i, \alpha_j^*\right)\right)}{dx_i} = \frac{\partial \pi_i}{\partial x_i} + \frac{\partial \pi_i}{\partial q_i} \cdot \frac{\partial q_i}{\partial x_i} + \sum_{j \neq i} \frac{\mathrm{d}x_j\left(x_i, \alpha_j^*\right)}{dx_i} \cdot \frac{\partial q_i}{\partial x_j} \cdot \frac{\partial \pi_i}{\partial q_i}.$$

□

5 Applications

We present three applications of our model and solution concept: price competition, advertising competition, and team production. For each application we derive the predicted deviation of the analytics bias and the predicted levels of strategies and payoffs compared to the Nash equilibrium. We further provide micro-foundations for the processes that cause the predicted bias in analytics.

5.1 Price Competition

Our first application generalizes the motivating example of Section 2.

Underlying Game The underlying game $G_p = (N = \{1, 2\}, X, q, \pi)$ is a price competition between two firms. The demand of each firm $i \in \{1, 2\}$ is the following linear function:[6]

$$q_i(x) = a_i - b_i x_i + c_i \cdot x_{-i},$$

where $c_i \cdot c_{-i}, a_i, b_i > 0$, and $|c_i| < b_i$ for each player i. The sign of c_i (which coincides with sign of c_{-i}) determines whether the sold goods are substitutes ($c_i > 0$ as in the standard differentiated Bertrand competition) or complements ($c_i < 0$ as in a case of two stores that sell complementary goods, such as kitchen appliances and cooking ingredients, or in the case of of adjacent stores that sell unrelated goods, but a price decrease in one attracts more customers that also visit the neighboring store). The inequality $|c_i| < b_i$ constrains the cross-elasticity parameters to be sufficiently small relative to the own-elasticity parameters. If $c_i < 0$ then we further require an additional upper bound on the the cross-elasticity:

$$|c_i| < \frac{a_i}{a_{-i}} b_{-i} \ \forall i \in \{1, 2\}. \tag{5.1}$$

Each seller i sets a price $x_i \in X_i$, where $X_i = \mathbb{R}_+$ if $c_i > 0$ and $X_i = \left[0, \frac{a_i}{b_i}\right]$ if $c_i < 0$. Limiting the maximum price to $\frac{a_i}{b_i}$ is without loss of generality because setting a higher price implies that the seller's demand cannot be positive. Finally, the profit of each firm is given by $\pi_i(x_i, q_i) = q_i(x) \cdot x_i$. This profit function corresponds to constant marginal costs, which have been normalized to zero.

The game G_p has strategic complements if $c_i > 0$ and strategic substitutes if $c_i < 0$. This can be observed directly from a simple analysis of the payoff function, and is immediately implied by Proposition 5 and Claim 2.

[6] All of our results remain the same if one adapts the demand function to be non-negative (as is commonly done in models of price competitions), i.e., $q_i(x) = \max(a_i - b_i x_i + c_i \cdot x_{-i}, 0)$, and adapts Assumptions 2–6 by allowing the various monotone derivatives to be equal to zero when the firm's demand is equal to zero.

Naive Analytics Equilibrium It is simple to show that G_p satisfies all the assumptions of the general model.

Claim 5. Price competition game G_p satisfies Assumptions 2–6 and 3' (WRT an unrestricted set of biases $A = \mathbb{R}_{++}$). Moreover, the RIDE derivative is always positive, and the sign of the payoff (resp., RIDE) externalities is the same as (resp., opposite of) the sign of c_i.

Our next result shows that price competition admits a unique naive analytics equilibrium in which both players under-estimate the elasticity of demand in the same way (despite the game being asymmetric). The prices in this NAE are higher than in the Nash equilibrium. The equilibrium Pareto dominates the Nash equilibrium if the game has strategic complements ($c_i > 0$), while the Nash equilibrium Pareto dominates the NAE if the game has strategic substitutes ($c_i < 0$).

Corollary 1. *G_p admits a unique naive analytics equilibrium (α^*, x^*) satisfying:*

1. *Symmetric under-estimation of elasticity of demand:* $\alpha_1^* = \alpha_2^* = \sqrt{1 - \frac{c_i c_{-i}}{b_i b_{-i}}} \in (0, 1)$.
2. *Prices are higher than the Nash equilibrium prices:* $x_i(\alpha^*) > x_i\left(\overrightarrow{1}\right)$.
3. *Pareto dominance relative to the Nash equilibrium:*
   $c_i > 0 \Rightarrow \tilde{\pi}_i(\alpha^*) > \tilde{\pi}_i\left(\overrightarrow{1}\right)$, *and* $c_i < 0 \Rightarrow \tilde{\pi}_i(\alpha^*) < \tilde{\pi}_i\left(\overrightarrow{1}\right)$.

Sketch of proof; proof of part (1) in Appendix A.10. It is simple to see that $\frac{dx_j\left(x_i,\alpha_j^*\right)}{dx_i} = \frac{c_j}{b_j\left(1+\alpha_j^*\right)}$, $\frac{\partial q_i}{\partial x_j} = c_i$, $\frac{\partial q_i}{\partial x_j} = -b_i$. Claim 4 implies that x_i^* must satisfy

$$\alpha_i^* - 1 = \frac{\mathrm{d}x_j\left(x_i, \alpha_j^*\right)}{dx_i} \cdot \frac{\frac{\partial q_i}{\partial x_j}}{\frac{\partial q_i}{\partial x_i}} = \frac{c_i \cdot c_j}{b_i \cdot b_j\left(1+\alpha_j^*\right)} \Leftrightarrow \left(1+\alpha_j^*\right)\left(1-\alpha_i^*\right) = \frac{c_i \cdot c_j}{b_i \cdot b_j}. \tag{5.2}$$

Observe that the RHS of (5.2) remains the same when swapping i and j. This implies that α_j^* and α_i^* must be equal, and that $\alpha_1^* = \alpha_2^* = \sqrt{1 - \frac{c_i c_{-i}}{b_i b_{-i}}}$, which proves part (1). Parts (2)–(3) are immediately implied by $\alpha_1^* = \alpha_2^*$, Claim 5, and Proposition 3. □

Remark 2 (Oligopoly). Our results can be extended from duopoly to oligopoly ($n > 2$ firms). Specifically, one can show that also when the number of firms is larger than two, a unique naive analytics equilibrium (α^*, x^*) with symmetric biases exist, and it has similar qualitative properties as in Proposition 1.

Micro-Foundations for $\alpha_i < 1$ Next we discuss two plausible mechanisms that can induce naive analysts to unintentionally under-estimate the elasticity of demand:

1. The first mechanism has been illustrated in the motivating example (Section 2), and is described here again briefly. Suppose the daily demand of each firm has a random component for which the firm employees observe an informative signal. For example, the employees observe the weather forecast which is correlated with the realized demand. If the employees prefer to set lower sale prices on days with low demand, e.g., because they have more free time to change prices, then naive analysts who allow the employees to choose the days for price discounts would induce a correlation between low demand days and low prices. As a result, they will under-estimate of the elasticity of demand. A sophisticated analyst who closely ensures that price discounts are set uniformly at random, or accurately controls for the weather forecast in his econometric analysis, would yield an accurate estimation of the elasticity.

2. The second mechanism is formalized in Appx. C.1, and is presented here briefly. Employees of competing stores might also choose the same days for price discounts. One examples is setting discounts by season (holidays) or for specific days of the week (weekends). Another example is if the levels of their inventory is correlated and discounts are given when the inventory level is high. This correlation in prices would induce naive analysts who allow employees to set the days with price discounts to under-estimate the price elasticity, because in days with low prices the competitor is more likely to provide a discount as well, making the response to price discounts seem less strong.

Implications The results of Claim 1 fit the direction of demand elasticity bias when not controlling for price endogeneity, as in, e.g., Table 1 of Berry (1994) and Table 2 of Villas-Boas and Winer (1999). It is commonly assumed in empirical research that firms slowly converge to the correct optimal (best-response) pricing, and that inconsistencies with empirical results, such as the appearance that firms are pricing on the inelastic portion of the demand curve are due to an incorrect econometric analysis by the researcher. Our results provide an alternative explanation to such an assumption—firms in a naive analytics equilibrium would believe that they are pricing optimally, and having all firms price on the inelastic portion of the demand curves would be to their benefit.

5.2 Advertising Competition

In this subsection we present the second application of our model, advertising competition. Research that estimates the effectiveness of advertising often showed that extra care is required to arrive at non-biased estimates, and that not correcting for these biases often results in overestimating advertising effectiveness (Lodish, Abraham, Kalmenson, Livelsberger, Lubetkin, Richardson, and Stevens, 1995; Blake, Nosko, and Tadelis, 2015; Gordon, Zettelmeyer, Bhargava, and Chapsky, 2019; Shapiro, Hitsch, and Tuchman, 2019). In the second application of our model we analyze a duopoly competition in advertising to understand why firms might benefit from naive analytics that overestimates the effectiveness of advertising.

Underlying Game The underlying game $G_a = (N = \{1,2\}, X, q, \pi)$ describes an advertising competition among two firms. Firm $i \in \{1,2\}$ sells a product with exogenous profit margins $p_i > 0$ per unit sold. The expected demand of each firm depends on the advertising budget $x_i \in X_i$ of both firms:

$$q_i = a_i + b_i \cdot \sqrt{x_i} + c_i \cdot \sqrt{x_i \cdot x_{-i}}, \tag{5.3}$$

where $a_i, b_i, c_i \cdot c_{-i} > 0$. When $c_i > 0$ the feasible budget set is unrestricted ($X_i = \mathbb{R}_+$). When $c_i < 0$, we restrict the maximum budget to be $M_i = (p_i \cdot b_i)^2$ (i.e., $X_i = [0, M_i]$). This restriction is without loss of generality as it can be shown that no firm will select $x_i > M_i$, since for budgets above M_i the marginal revenue from increasing advertising is below its marginal cost, regardless of the opponent's strategy.

In this market a firm's own advertising increases demand for its own product (b_i is positive), but the competitor's advertising may affect the level of the increase. When $c_i > 0$ the total category demand increases when the competitor advertises, which increases the effect of a firm's own advertising. When $c_i < 0$ the effect of a firm's own advertising decreases in the competitor's advertising due to competition (e.g., over the same customers). We require the sign of c_i and c_{-i} to coincide. Positive c_i-s might correspond to a new category of goods, in which advertising attracts attention to the category. Negative c_i-s might correspond to a mature category in which advertising mainly causes consumers to switch among competing goods. The payoff of player i is given by

$$\pi_i\left(x_i, x_{-i}\right) = q_i\left(x_i, x_{-i}\right) \cdot p_i - x_i = p_i \cdot \left(a_i + b_i \cdot \sqrt{x_i} + c_i \cdot \sqrt{x_i \cdot x_{-i}}\right) - x_i. \tag{5.4}$$

We require the advertising externalities to be sufficiently small: $|c_i| \in \left(0, \frac{1}{p_i}\right)$, which implies well-behaved interior Nash equilibrium. Further, if $c_i < 0$ we assume

$$|c_i| < \frac{b_i}{b_{-i} \cdot p_{-i}}. \tag{5.5}$$

Naive Analytics Equilibrium Observe that the RIDE in advertising competition is a linear transformation of the return on investment (ROI), which is often used as a measure of advertising effectiveness (Blake, Nosko, and Tadelis, 2015; Lewis and Rao, 2015):

$$\mathrm{RIDE}_i\left(x\right) = -\frac{\frac{\partial \pi_i}{\partial q_i(x)} \cdot \frac{\partial q_i}{\partial x_i}}{\frac{\partial \pi_i}{\partial x_i}} = \frac{p_i}{1} \cdot \frac{\partial q_i}{\partial x_i} = \frac{p_i}{2\sqrt{x_i}}\left(b_i + c_i\sqrt{x_{-i}}\right) = \frac{1}{2}\left(\underbrace{\frac{\pi_i(x_i, x_{-i}) - \pi_i(0, x_{-i})}{x_i}}_{ROI} + 1\right). \tag{5.6}$$

It is simple to show that G_a satisfies all the assumptions of the general model.

Claim 6. The price competition game G_a satisfies Assumptions 2–5 and Assumption 3' with respect to the feasible set of biases $A = (0, 2]$. Moreover, the RIDE derivative is negative, and the signs of the payoff externalities and RIDE externalities coincide with c_i's sign.

Our next result shows that advertising competition admits a unique naive analytics equilibrium in which both players over-estimate the effectiveness of advertising in the same way (despite the game being asymmetric). Both firms spend more on advertising than in the Nash equilibrium. The NAE Pareto dominates the Nash equilibrium if advertising expenditures are strategic complements ($c_i > 0$), while the NAE is Pareto dominated by the Nash equilibrium if advertising expenditures are strategic substitutes.

Corollary 2. *G_a admits a unique naive analytics equilibrium (α^*, x^*) satisfying:*

1. *Symmetric over-estimation of effectiveness: $\alpha_1^* = \alpha_2^* = \frac{2}{1+\sqrt{1-c_1c_2p_1p_2}} \in (1,2)$.*

2. *Advertising budgets are higher than the Nash equilibrium budgets: $x_i(\alpha^*) > x_i\left(\overrightarrow{1}\right)$.*

3. *Pareto dominance relative to the Nash equilibrium:*
   $c_i > 0 \Rightarrow \tilde{\pi}_i(\alpha^*) > \tilde{\pi}_i\left(\overrightarrow{1}\right)$, *and* $c_i < 0 \Rightarrow \tilde{\pi}_i(\alpha^*) < \tilde{\pi}_i\left(\overrightarrow{1}\right)$.

Sketch of proof; proof of part (1) in Appendix. A.12. It is simple to see that $\frac{dx_j\left(x_i,\alpha_j^*\right)}{dx_i} = \frac{\sqrt{x_j}}{\sqrt{x_i}}\alpha_j^*\frac{p_j}{2}c_j$, $\frac{\partial q_i}{\partial x_j} = \frac{c_i\sqrt{x_i}}{2\sqrt{x_j}}$, $\frac{\partial q_i}{\partial x_j} = \frac{1}{2\alpha_i^*\frac{p_i}{2}}$. Claim 4 implies that x_i^* must satisfy

$$\alpha_i^* - 1 = \frac{\sqrt{x_j}}{\sqrt{x_i}}\alpha_j^*\frac{p_j}{2}c_j\frac{c_i\sqrt{x_i}2\alpha_i^*\frac{p_i}{2}}{2\sqrt{x_j}} = \alpha_j^*\alpha_i^*\frac{p_ip_jc_jc_i}{4}. \tag{5.7}$$

Observe that the RHS of (5.7) remains the same when swapping i and j. This implies that α_j^* and α_i^* must be equal. The resulting one-variable quadratic equation has a unique solution satisfying $\alpha_i^* < 2$, which is $\alpha_1^* = \alpha_2^* = \frac{2}{1+\sqrt{1-c_1c_2p_1p_2}}$, proving part (1). Parts (2)–(3) are immediate implications of $\alpha_1^* = \alpha_2^*$, Claim 6, and Proposition 3. □

Micro-Foundations for $\alpha_i > 1$ We conclude this section by providing micro-foundational examples of cases that would cause firms to overestimate their advertising effectiveness:

1. Similarly to price competition, correlation between advertising budgets of firms with positive externalities would cause an overestimate of ad effectiveness. If firms choose to increase advertising budgets during the holidays, or just prior to weekends, they will observe an increase in demand beyond the effects of their own advertising. This correlation will create an overestimate of advertising elasticity.

2. When online advertising is purchased on social media platforms, such as on Facebook, the advertiser provides the advertising platform a budget and a target metric. The platform's algorithm then targets consumers in order to maximize the target metric under the budget constraint. One common such metric is sales or purchases, and a strategy to maximize this metric is to show ads to likely buyers of the product, or to past purchasers of the product. Under such a strategy, an analysis that compares

the purchase rates of people that have seen ads to those that have not seen ads will overestimate the effectiveness of advertising (Berman, 2018).

3. If firms respond to decreased demand by increasing their advertising budgets in the next time period, and if demand is noisy, a standard "regression to the mean" argument implies that demand is likely to increase in the next period regardless of the additional advertising budget. Failing to take this into account would lead to overestimation of the advertising effectiveness, as we formally present in Appendix. C.2.

5.3 Team Production and Overconfidence

Thus far we have interpreted $q(x)$ as market demand and $\alpha_i \neq 1$ as bias due to naive analytics. We now demonstrate that our model applies in more general settings. Specifically, we apply the model to an underlying game of team production with strategic complementarity. Team production is common in partnerships and other input games (see, e.g., Holmstrom, 1982; Cooper and John, 1988; Heller and Sturrock, 2020). Examples include sales force members who are compensated based on the performance of the joint sales of a team, and entrepreneurs who receive a share of the exit value of a startup. It is often observed that entrepreneurial firms are founded by teams of overconfident founders (Astebro, Herz, Nanda, and Weber, 2014; Hayward, Shepherd, and Griffin, 2006). Taking this perspective, we interpret x_i as the contribution of each team member, and $q(x)$ as the value created by the team. This analogy directly leads to interpreting $\alpha_i \neq 1$ as a bias player i has when evaluating their contribution to the value created by the team, which can be seen as a measure of *confidence*. We show that in any naive analytics equilibrium all agents are overconfident in the sense of overestimating their ability to contribute to the team's output (i.e., having $\alpha_i > 1$). In the case of entrepreneurship, for example, much of the past research explained overconfidence as necessary to overcome risk aversion and tackle uncertainty, that is, as a response to the entrepreneurial environment which is external to the firm.[7] Our results provide a novel foundation for the tendency of people (and, in particular, entrepreneurs) to be overconfident in the sense of overestimating one's ability. We show that when skills are complementary,

[7] See also Heller (2014) who demonstrates that overconfidence of entrepreneurs can help an investor in diversifying aggregate risk.

overconfidence contributes to increased team efficiency, and is a response to the internal firm environment. The results provide a novel explanation to why investors might prefer to invest in overconfident startup founders, and why managers might prefer to hire overconfident sales people.

Underlying Game We describe a team production game G_t with strategic complements. Consider two players ($N = \{1, 2\}$), each choosing how much effort $x_i \in X_i \equiv \mathbb{R}_+$ to exert in a joint project. The project yields all agents a value of $q(x)$, where q is twice continuously differentiable in $\mathbb{R}^2_{++}$, strictly increasing, strictly concave, and supermodular (i.e., satisfies strategic complementarity) with respect to its two parameters (i.e., $\frac{\partial q(x)}{\partial x_i} > 0$, $\frac{\partial^2 q(x)}{\partial x_i^2} < 0$, $\frac{\partial^2 q(x)}{\partial x_1 x_2} > 0$ for any $x \in X$ and any $i \in \{1, 2\}$). The payoff of each player i is equal to the project's value minus her effort: $\pi_i(x) = q(x) - x_i$.

We assume that the marginal contribution of effort is sufficiently large if efforts are small, and it is sufficiently small if efforts are large. Formally:

Assumption 7. *For each $\alpha \in \mathbb{R}_{++}$, there exist strategy profiles $\underline{x} \leq \overline{x}$, such that $\frac{\partial q(\overline{x})}{\partial x_i} \leq \alpha \leq \frac{\partial q(\underline{x})}{\partial x_i}$ for each player i.*

Finally, we assume that the Hessian determinant of $q(x)$ never changes its sign (i.e., it is never equal to zero). Formally:

Assumption 8. ***Monotone Hessian determinant****: $\frac{\partial^2 q(x)}{\partial x_1^2} \frac{\partial^2 q(x)}{\partial x_2^2} \neq \left(\frac{\partial^2 q(x)}{\partial x_1 x_2}\right)^2$ $\forall x \in X$.*

We interpret Assumption 8 as having a monotone relation between concavity and supermodularity, i.e., either the amount of concavity is always larger than the amount of supermodularity (i.e., $\frac{\partial^2 q(x)}{\partial x_1^2} \frac{\partial^2 q(x)}{\partial x_2^2} > \left(\frac{\partial^2 q(x)}{\partial x_1 x_2}\right)^2$ for every $x \in X$), or the amount of concavity is always smaller than the amount of supermodularity (i.e., $\frac{\partial^2 q(x)}{\partial x_1^2} \frac{\partial^2 q(x)}{\partial x_2^2} < \left(\frac{\partial^2 q(x)}{\partial x_1 x_2}\right)^2$ $\forall x \in X$).

Example 2 (Cobb-Douglas production)**.** The Cobb-Douglas production function $q(x) = x_1^{\beta_1} x_2^{\beta_2}$ satisfies Assumptions 7 and 8 if $\beta_1 + \beta_2 < 1$:

$$\frac{\partial q(\tilde{x}, \tilde{x})}{\partial x_i} = \frac{\beta_i}{\tilde{x}^{1-\beta_i-\beta_{-i}}} \Rightarrow \lim_{\tilde{x} \to 0} \frac{\partial q(\tilde{x}, \tilde{x})}{\partial x_i} = \infty, \ \lim_{\tilde{x} \to \infty} \frac{\partial q(\tilde{x}, \tilde{x})}{\partial x_i} = 0,$$

$$\frac{\partial^2 q(x)}{\partial x_i^2} = -(1-\beta_i)\beta_i \frac{x_{-i}^{\beta_{-i}}}{x_i^{2-\beta_i}} < 0, \quad \frac{\partial^2 q(x)}{\partial x_1 x_2} = \frac{\beta_1 \beta_2}{x_1^{1-\beta_1} x_2^{1-\beta_2}} > 0.$$

Naive Analytics Equilibrium The RIDE is equal to the agent's marginal contribution to the project:

$$\text{RIDE}_i(x) = -\frac{\frac{\partial \pi_i}{\partial q(x)} \cdot \frac{\partial q}{\partial x_i}}{\frac{\partial \pi_i}{\partial x_i}} = -\frac{1 \cdot \frac{\partial q}{\partial x_i}}{-1} = \frac{\partial q}{\partial x_i}. \tag{5.8}$$

Further, observe that the underlying team-production game satisfies Assumptions 1–5.

Claim 7. The team-production game G_t satisfies Assumptions 1–5 with respect to an unrestricted set of biases $A = \mathbb{R}_{++}$. Moreover, the payoff and RIDE externalities are positive, while the RIDE self-derivative is negative.

The results of Section 4 and Claim 7 immediately imply that in any naive analytics equilibrium both players: experience overconfidence (i.e., $\alpha_i^* > 1$), exert more effort than in the Nash equilibrium and obtain a better payoff than in the Nash equilibrium.

Corollary 3. *In any naive analytics equilibrium* (α^*, x^*) *of* G_t *:*

1. *Both players over-estimate their influence on the joint project, i.e.,* $\alpha_i^* > 1\ \forall i$.
2. *Both players exert more efforts than in the Nash equilibrium, i.e.,* $x_i^* > x_i^{NE}\ \forall i$.
3. *The NAE Pareto dominates the Nash equilibrium.*

6 Conclusion

Naive analytics equilibrium can be used to analyze such games where players have uncertainty about the indirect impact of their actions on their payoffs, and allows players to use biased data analytics to estimate this impact. This scenario is common in economic applications such as price competition, advertising competition and team production.

The predictions of our results are consistent with commonly observed behaviors of firms and teams. In equilibrium, players are predicted to converge to biased estimates in the direction that causes their opponents to respond in a beneficial manner. In pricing competition, players are better off if they perceive consumers to be less price elastic than they actually are, which is a possible interpretation of observed firm pricing if they do not correct for price endogeneity in their econometric analysis. In advertising competition, it is observed that firms often overestimate the response to their advertising and over-advertise, as predicted by our results. These deviations from unbiased estimates cause deviations from the Nash equilibrium that can be beneficial or detrimental to players. When games have strategic complements, players will choose strategies that deviate from the Nash equilibrium in the direction that benefits the opponents, and their equilibrium payoffs will dominate those of the Nash equilibrium. The converse is true for games with strategic substitutes.

The results of our analysis provide testable empirical predictions about the direction and magnitudes of the biases. In particular, the analysis predicts that different firms within a market will have similar level of biasedness, while the level of biasedness will differ across markets. Initial evidence for this phenomenon is observed in Table 2 of Villas-Boas and Winer (1999). Another prediction is that biasedness away from $\alpha^* = 1$ will disappear as the number of players grows, or in a monopolistic market. These predictions could be tested in empirical data as well as serve as a basis for analysis about the adoption and sophistication of analytics in various industries. Further, our results may bring to question some of the assumptions used in practice when performing counterfactual analysis to estimate welfare and assess the impact of regulatory policy. In these analyses, it is often assumed that firms correctly perceive their economic environment and that any observed inconsistency with this assumption may be due to unobserved factors by researchers. However, as we have shown, in a naive analytics equilibrium firms will be profit maximizing if they misperceive their environment. One would expect the conclusions from a counterfactual analysis that utilizes the standard assumptions to be biased if firms are indeed playing an analytics game.

A second implication of our results is for research that focuses on biases in decision making from non-causal inferential methods. The research implicitly assumes that focusing on causality and more precise estimates are better for firm performance, which often translates to normative recommendation about firm practices (see, e.g., Siroker and Koomen (2013) and Thomke (2020) on A/B testing). Our results point to the conclusion that firms may be better off with opting for more naive heuristics, which are indeed quite popular because they are easy to implement. This may suggest that normative recommendations for deploying more sophisticated analytics capabilities should be made with caution in competitive environments.

Finally, there are two natural extensions to our work which we leave for the future. First, our analysis focused on the case in which the underlying game admits a unique α-equilibrium for each bias profile α, while extending the results to games with multiple equilibria is desired. Second, the model we derived assumed that the direction of monotone derivatives of players are the same, i.e., one type of derivative is either increasing or decreasing for all players. A natural question to ask is under what conditions the results of the characteristics of the naive analytics equilibrium we described extend to games where the monotone derivatives are mixed in directions among players.

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Online Appendices

A Formal Proofs

A.1 Proof of Claim 4 (Stackelberg-Leader Representation)

Assume to the contrary that $(\alpha^*, x(\alpha^*))$ is a naive analytics equilibrium and $x_i(\alpha^*) \notin X_i^{SL}(\alpha_{-i})$ for player i. Let $x_i^{SL} \in X_i^{SL}$ be an optimal Stackelberg-leader strategy of player i. Let $\alpha_i' = \frac{1}{\text{RIDE}_i\left(x_i^{SL}, x_{-i}\left(x_i, \alpha_{-i}^*\right)\right)}$. Then:

$$\tilde{\pi}_i\left(\alpha_i', \alpha_{-i}^*\right) = \pi_i\left(\left(x_i^{SL}, x_{-i}\left(x_i, \alpha_{-i}^*\right)\right)\right) > \pi_i\left(x_i\left(\alpha^*\right), x_{-i}\left(\alpha^*\right)\right) = \pi_i\left(\alpha^*\right),$$

and we get a contradiction to $(\alpha^*, x(\alpha^*))$ being a NAE (where the first equality is due to Assumption 3 (uniqueness) and the inequality is due to the definition of $X_i^{SL}(\alpha_{-i})$).

Next assume to the contrary that $x_i(\alpha^*) \in X_i^{SL}(\alpha_{-i})$ for each player $i \in N$ and $(\alpha^*, x(\alpha^*))$ is not a naive analytics equilibrium. This implies that there is player i and bias α_i' such that $\tilde{\pi}_i\left(\alpha_i', \alpha_{-i}^*\right) > \tilde{\pi}_i(\alpha^*)$. Let $\tilde{x}_i = x_i\left(\alpha_i', \alpha_{-i}^*\right)$. Observe that

$$\pi_i\left(\tilde{x}_i, x_{-i}\left(\tilde{x}_i, \alpha_{-i}^*\right)\right) = \tilde{\pi}_i\left(\alpha_i', \alpha_{-i}^*\right) > \tilde{\pi}_i\left(\alpha^*\right) = \pi_i\left(x_i\left(\alpha^*\right), x_{-i}\left(x_i\left(\alpha^*\right), \alpha_{-i}^*\right)\right),$$

which contradicts the assumption that $x_i(\alpha^*) \in X_i^{SL}(\alpha_{-i})$.

Next we prove the "moreover" part. The fact that $x_i(\alpha^*) \in Int(X_i)$ is an optimal Stackelberg-leader strategy implies that it satisfies the following first order condition:

$$0 = \frac{d\pi_i\left(x_i, x_{-i}\left(x_i, \alpha_j^*\right)\right)}{dx_i} = \frac{\partial \pi_i}{\partial x_i} + \frac{\partial \pi_i}{\partial q_i} \cdot \frac{\partial q_i}{\partial x_i} + \sum_{j \neq i} \frac{\mathrm{d}x_j\left(x_i, \alpha_j^*\right)}{dx_i} \cdot \frac{\partial q_i}{\partial x_j} \cdot \frac{\partial \pi_i}{\partial q_i}. \tag{A.1}$$

Substituting $0 = \frac{\partial \pi_i}{\partial x_i} + \frac{\partial \pi_i}{\partial q_i} \cdot \alpha_i \cdot \frac{\partial q_i}{\partial x_i}$ (implied by $x(\alpha^*)$ being an α^*-equilibrium) in A.1 yields:

$$0 = (1 - \alpha_i^*) \cdot \frac{\partial \pi_i}{\partial q_i} \cdot \frac{\partial q_i}{\partial x_i} + \sum_{j \neq i} \frac{\mathrm{d}x_j\left(x_i, \alpha_j^*\right)}{dx_i} \cdot \frac{\partial q_i}{\partial x_j} \cdot \frac{\partial \pi_i}{\partial q_i} \Leftrightarrow \alpha_i^* - 1 = \sum_{j \neq i} \frac{\mathrm{d}x_j\left(x_i, \alpha_j^*\right)}{dx_i} \cdot \frac{\frac{\partial q_i}{\partial x_j}}{\frac{\partial q_i}{\partial x_i}}. \tag{A.2}$$

A.2 Proof of Claim 1 (Unimodal Monotone RIDE)

Fix player i and profile x_{-i}. Let $x_i^* \in X_i$ be the strategy satisfying $RIDE_i(x_i^*, x_{-i}) = 1$. Assume first that $\frac{\partial \pi_i}{\partial x_i} > 0$. Observe that $\frac{d\pi_i}{dx_i} = 0$ (resp., $\frac{d\pi_i}{dx_i} < 0$, $\frac{d\pi_i}{dx_i} > 0$) iff $\frac{\partial \pi_i}{\partial x_i} = \left|\frac{\partial \pi_i}{\partial q_i} \cdot \frac{\partial q_i}{\partial x_i}\right|$ (resp., $\frac{\partial \pi_i}{\partial x_i} < \left|\frac{\partial \pi_i}{\partial q_i} \cdot \frac{\partial q_i}{\partial x_i}\right|$, $\frac{\partial \pi_i}{\partial x_i} > \left|\frac{\partial \pi_i}{\partial q_i} \cdot \frac{\partial q_i}{\partial x_i}\right|$), which holds iff $RIDE_i(x) = 1$ (resp., $RIDE_i(x) > 1$, $RIDE_i(x) < 1$). If $\frac{d(RIDE_i(x))}{dx_i} > 0$ then this latter equality (resp., inequality) holds iff $x_i = x_i^*$ (resp., $x_i > x_i^*$, $x_i < x_i^*$), which implies unimodality. If $\frac{d(RIDE_i(x))}{dx_i} < 0$, then $RIDE_i(x) = 1$ (resp., $RIDE_i(x) > 1$, $RIDE_i(x) < 1$) holds iff $x_i = x_i^*$ (resp., $x_i < x_i^*$, $x_i > x_i^*$), which violates unimodality.

Next assume first that $\frac{\partial \pi_i}{\partial x_i} < 0$. Observe that $\frac{d\pi_i}{dx_i} = 0$ (resp., $\frac{d\pi_i}{dx_i} < 0$, $\frac{d\pi_i}{dx_i} > 0$) iff $\left|\frac{\partial \pi_i}{\partial x_i}\right| = \frac{\partial \pi_i}{\partial q_i} \cdot \frac{\partial q_i}{\partial x_i}$ (resp., $\left|\frac{\partial \pi_i}{\partial x_i}\right| > \frac{\partial \pi_i}{\partial q_i} \cdot \frac{\partial q_i}{\partial x_i}$, $\left|\frac{\partial \pi_i}{\partial x_i}\right| < \frac{\partial \pi_i}{\partial q_i} \cdot \frac{\partial q_i}{\partial x_i}$), which holds iff $RIDE_i(x) = 1$ (resp., $RIDE_i(x) < 1$, $RIDE_i(x) > 1$). If $\frac{d(RIDE_i(x))}{dx_i} < 0$ then this latter equality (resp., inequality) holds iff $x_i = x_i^*$ (resp., $x_i > x_i^*$, $x_i < x_i^*$), which implies unimodality. If $\frac{d(RIDE_i(x))}{dx_i} > 0$, then $RIDE_i(x) = 1$ (resp., $RIDE_i(x) > 1$, $RIDE_i(x) < 1$) holds iff $x_i = x_i^*$ (resp., $x_i < x_i^*$, $x_i > x_i^*$), which violates unimodality.

A.3 Fact on Relabeling Strategies ($x_i \to -x_i$)

Fact 1. *Let $G = (N, X, q, \pi)$ be an underlying game that satisfies Assumptions 1–5. Let $G' = (N, X', q', \pi')$ be the same game after relabeling the strategies $x_i \to -x_i$, i.e., $X_i' = -X_i = \{x_i' \in \mathbb{R} | -x_i' \in X_i\}$, $q_i'(x_1, ..., x_n) = q_i(-x_1, ..., -x_n)$ and $\pi_i'(x_i, q_i) = \pi_i(-x_i, q_i)$. Then G' satisfies Assumptions 1–5 and the sign of each of its three monotone derivatives is the opposite of the respective sign in G, i.e., for each players $i \neq j$*

$$\frac{d\pi_i(x)}{dx_j} > 0 \Leftrightarrow \frac{d\pi_i'(x)}{dx_j} < 0, \quad \frac{d(RIDE_i(x))}{dx_i} > 0 \Leftrightarrow \frac{d(RIDE_i'(x))}{dx_i} < 0,$$

$$AND\ \frac{d(RIDE_i(x))}{dx_j} > 0 \Leftrightarrow \frac{d(RIDE_i'(x))}{dx_j} < 0.$$

A.4 Proof of Claim 2 (Strategic Substitutes/Complements)

In what follows we prove the "if" parts in both parts of the claim. The "only if" parts are immediately implied from the "if" parts due to $\frac{d(RIDE_j)}{dx_j} \cdot \frac{d(RIDE_j)}{dx_i} < 0$ and $\frac{d(RIDE_j)}{dx_j} \cdot \frac{d(RIDE_j)}{dx_i} > 0$

being mutually exclusive and exhaustive cases in our setup (due to Assumptions 4–5).

Fix $i, j \in N$, strategy profile x and strategy $x'_i > x_i$. Assume first that $\frac{d(RIDE_j)}{dx_i} > 0$. This implies that $RIDE_j(x'_i, x_{-ij}) > RIDE_j(x_i, x_{-ij})$. This, in turn, implies that $BR_j(x'_i, x_{-ij}) > BR_j(x_i, x_{-ij})$ (resp., $BR_j(x'_i, x_{-ij}) < BR_j(x_i, x_{-ij})$) if $\frac{d(RIDE_j)}{dx_j} < 0$ (resp., $\frac{d(RIDE_j)}{dx_j} > 0$), which implies that the game has strategic complements (resp., strategic substitutes). Next, assume that $\frac{d(RIDE_j)}{dx_i} < 0$. This implies that $RIDE_j(x'_i, x_{-ij}) < RIDE_j(x_i, x_{-ij})$. This, in turn, implies that $BR_j(x'_i, x_{-ij}) > BR_j(x_i, x_{-ij})$ (resp., $BR_j(x'_i, x_{-ij}) < BR_j(x_i, x_{-ij})$) if $\frac{d(RIDE_j)}{dx_j} > 0$ (resp., $\frac{d(RIDE_j)}{dx_j} < 0$), which implies that the game has strategic complements (resp., strategic substitutes).

A.5 Proof of Proposition 1(Over/Under Reply)

Let (α^*, x^*) be a naive analytics equilibrium in a game with beneficial upward commitment (the proof for the case of beneficial downward commitment is analogous). Assume to the contrary that there is player i for which $x^*_i < BR_i\left(x^*_{-i}\right)$. Consider a first-stage deviation of player i to bias α'_i sufficiently close to α^*_i that induces him to increase its second-stage $\left(\alpha'_i, \alpha^*_{-i}\right)$-equilibrium strategy from x^*_i to x'_i such that $x^*_i < x'_i < BR_i(x^*_{-i})$. That is, $\alpha'_i = \alpha^*_i + \epsilon$ if the RIDE derivative is negative and $\alpha'_i = \alpha^*_i - \epsilon$ if the RIDE derivative is positive for a sufficiently small $\epsilon > 0$. Then in the new naive analytics equilibrium $(\alpha', x') \equiv \left(\left(\alpha'_i, \alpha^*_{-i}\right), x'\right)$, all other players adjust their strategies, x'_j in the direction that benefits player i due to the game having a beneficial upward commitment (i.e., when the payoff externalities are positive $x'_j > x^*_j$ and when negative $x'_j < x^*_j$). This, in turn, implies that $\pi_i(\alpha', x') > \pi_i(\alpha^*, x^*)$ because player i gains both from increasing his own x_i closer to his best-reply and from the others changing their x'_j in the direction in its favor due to the underlying game having a beneficial upward commitment.

A.6 Proof of Claim 3 (Beneficial Commitment)

The following simple lemma will be helpful in the proof of Claim 3.

Lemma 1. *Fix $i \neq j \in N$, $x \in \mathbb{R}^n_+$. Assume that $RIDE_j(x_i, x_j, x_{-ij}) = RIDE_j\left(x'_i, x'_j, x_{-ij}\right)$ with $x_i \neq x'_i$ and $x_j \neq x'_j$. Then $(x_i - x'_i) \cdot \left(x_j - x'_j\right) < 0$ iff the signs of the RIDE derivative* *and RIDE externalities coincide.*

Proof. We prove the lemma for one of the four possible cases in which the signs of the RIDE derivative and RIDE externalities are both positive (the analogous arguments in the remaining three cases is omitted for brevity). If $(x_i - x_i') \cdot \left(x_j - x_j'\right) > 0$, then either:
(I) $x_i' > x_i$ and $x_j' > x_j$ implying that $RIDE_j\left(x_i, x_j, x_{-ij}\right) < RIDE_j\left(x_i', x_j', x_{-ij}\right)$, or
(II) $x_i' < x_i$ and $x_j' < x_j$ implying that $RIDE_j\left(x_i, x_j, x_{-ij}\right) > RIDE_j\left(x_i', x_j', x_{-ij}\right)$.
Thus, $RIDE_j\left(x_i, x_j, x_{-ij}\right) = RIDE_j\left(x_i', x_j', x_{-ij}\right)$ implies that $(x_i - x_i') \cdot \left(x_j - x_j'\right) < 0$. □

Next we prove Claim 3. Due to Lemma 1, the equality

$$RIDE_j\left(x_i, BR_j\left(x_i, x_{-ij}\right), x_{-ij}\right) = RIDE_j\left(x_i', BR_j\left(x_i', x_{-ij}\right), x_{-ij}\right) = 1$$

implies that $(x_i - x_i') \cdot \left(BR_j\left(x_i, x_{-ij}\right) - BR_j\left(x_i', x_{-ij}\right)\right) < 0$ iff the signs of the RIDE derivative and RIDE externalities coincide.

Assume that the game has positive payoff externalities. Then, player i would benefit from a best-replying player j increasing her play, which would happen when player i decreases (resp., increases) her strategy when the signs of the RIDE derivative and RIDE externalities coincide (resp., are different). This implies that the game has beneficial upward commitment iff exactly one of the signs of the RIDE derivative/externalities is negative.

Next, assume that the game has negative payoff externalities. Then, player i would benefit from a best-replying player j decreasing her play, which would happen when player i increases (resp., decreases) her strategy when the signs of the RIDE derivative and RIDE externalities coincide (resp., are different). This implies that the game has beneficial upward commitment iff either both or none of the signs of the RIDE derivative/externalities is negative. Combining the above argument implies that the game has beneficial upward commitment iff the number of positive derivatives (among all three monotone derivative/externalities) is even.

A.7 Proof of Proposition 2 ($\alpha_i^* > 1$ or $\alpha_i^* < 1$)

Assume first that the game has a beneficial upward commitment. By Proposition 1 $x_i^* > BR\left(x_{-i}^*\right)$. This implies that $RIDE\left(x_i^*\right) > 1$ iff $\frac{d(RIDE_i)}{dx_i} > 0$. The fact that x^* is an α^*-equilibrium implies that $RIDE\left(x_i^*\right) = \frac{1}{\alpha_i^*}$, which, in turn, implies that $\alpha_i^* < 1$ iff $\frac{d(RIDE_i)}{dx_i} > 0$. By Proposition 3 in a game with upward commitment benefit $\frac{d(RIDE_i)}{dx_i} > 0$ iff $\frac{d(RIDE_i)}{dx_j} \cdot \frac{d\pi_i}{dx_j} < 0$ (due to the number of positive derivative being even). This shows that $\alpha_i^* < 1$ iff $\frac{d(RIDE_i)}{dx_j} \cdot \frac{d\pi_i}{dx_j} < 0$.

Next, assume that the game has a beneficial downward commitment. By Claim 1 $x_i^* < BR\left(x_{-i}^*\right)$. This implies that $RIDE\left(x_i^*\right) < 1$ iff $\frac{d(RIDE_i)}{dx_i} > 0$. The fact that x^* is an α^*-equilibrium implies that $RIDE\left(x_i^*\right) = \frac{1}{\alpha_i^*}$, which, in turn, implies that $\alpha_i^* > 1$ iff $\frac{d(RIDE_i)}{dx_i} > 0$. By Proposition 3 $\frac{d(RIDE_i)}{dx_i} > 0$ iff $\frac{d(RIDE_i)}{dx_j} \cdot \frac{d\pi_i}{dx_j} > 0$ (due to the number of positive derivative being odd). This shows that $\alpha_i^* > 1$ iff $\frac{d(RIDE_i)}{dx_j} \cdot \frac{d\pi_i}{dx_j} > 0$.

A.8 Proof of Proposition 3 (Pareto domination of NAE and NE)

The following two lemmas will be helpful in the proof of Proposition 3. Lemma 2 shows that whether player i is over replying in an α-equilibrium depends only on her own biasedness parameter α_i.

Lemma 2. *Let G be a game satisfying Assumptions 1–5. Player i over replies in an α-equilibrium iff she over replies in an $\left(\alpha_i, \alpha'_{-i}\right)$-equilibrium for any $\alpha \in \mathbb{R}_+^n$ and $\alpha'_{-i} \in \mathbb{R}_+^n$.*

Proof. Observe that

$$RIDE_i\left(x\left(\alpha\right)\right) = RIDE_i\left(x\left(\left(\alpha_i, \alpha'_{-i}\right)\right)\right) = \frac{1}{\alpha_i}, \text{ and}$$

$$RIDE_i\left(BR_i\left(x_{-i}\left(\alpha_{-i}\right)\right), x_{-i}\left(\alpha\right)\right) = RIDE_i\left(BR_i\left(x_{-i}\left(\alpha_i, \alpha'_{-i}\right)\right), x_{-i}\left(\alpha_i, \alpha'_{-i}\right)\right) = 1.$$

Player i can over reply only if $\alpha_i \neq 1$. There are four exhaustive cases, in all of which player i over-replying behavior is the same in the α-equilibrium and in the $\left(\alpha_i, \alpha'_{-i}\right)$-equilibrium:

1. If $\frac{dRIDE_i(x)}{dx_i} > 0$ and $\alpha_i < 1$, then $RIDE_i\left(x\left(\alpha\right)\right) = RIDE_i\left(x\left(\left(\alpha_i, \alpha'_{-i}\right)\right)\right) = \frac{1}{\alpha_i} > 1$ implying that $x_i\left(\alpha\right) > BR_i\left(x_{-i}\left(\alpha_{-i}\right)\right)$ and $x_i\left(\alpha_i, \alpha'_{-i}\right) > BR_i\left(x_{-i}\left(\alpha_i, \alpha'_{-i}\right)\right)$, i.e.,

Player i over replies in both the α-equilibrium and the $\left(\alpha_i, \alpha'_{-i}\right)$-equilibrium.

By an analogous argument:

2. If $\frac{dRIDE_i(x)}{dx_i} < 0$ and $\alpha_i > 1$, then Player i over replies in both biased equilibria.
3. If $\frac{dRIDE_i(x)}{dx_i} < 0$ and $\alpha_i < 1$, then Player i under replies in both biased equilibria.
4. If $\frac{dRIDE_i(x)}{dx_i} > 0$ and $\alpha_i > 1$, then Player i under replies in both biased equilibria.

□

Lemma 3 (which is a standard result) shows that in games with strategic complements if all agents over (resp., under) reply to each other, then they all must play strategies above (resp., below) their Nash equilibrium strategies. Formally,

Lemma 3. *Let G be a game with strategic complements, concave payoffs (Assumption 1) and a unique Nash equilibrium x^{NE}. Let x^* be a strategy profile.*

1. *If $x_i^* \geq BR_i\left(x_{-i}^*\right)$ for each player $i \in N$ with strict inequality for at least one player, then $x_i^* > x_i^{NE}$ for each player $i \in N$.*
2. *If $x_i^* \leq BR_i\left(x_{-i}^*\right)$ for each player $i \in N$ with strict inequality for at least one player, then $x_i^* < x_i^{NE}$ for each player $i \in N$.*

Proof.

1. We begin by showing the weak inequality $x_i^* \geq x_i^{NE}$ for each player $i \in N$. Assume to the contrary that there exists player j for which $x_j^* < x_j^{NE}$. Consider an auxiliary game G^R similar to G except that each player i is restricted to choose a strategy up to x_i^*. Due to the concavity, the game G^R admits a pure Nash equilibrium, which we denote by x^{RE}. Note that $x^{RE} \neq x^{NE}$ because $x_j^{RE} \leq x_j^* < x_j^{NE}$. The fact that x^{NE} is a unique equilibrium in G implies that x^{RE} cannot be an equilibrium of G. This implies (due to the concave payoffs) that there must exist player k for which $x_k^{RE} = x_k^*$ and $x_k^* < BR_k\left(x_{-k}^{RE}\right)$. The fact that $x_i^{RE} \leq x_i^*$ for each player i and the assumption that the game has strategic complements jointly imply that $x_k^* \geq BR_k\left(x_{-k}^*\right) \geq BR_k\left(x_{-k}^{RE}\right)$ and we get a contradiction.

Next, we want to show the strict inequality $x_i^* > x_i^{NE}$ for each player $i \in N$. Observe that $x^* \neq x^{NE}$ because there exists a player who (strictly) over replies. This implies that there exists player j for which $x_j^* > x_j^{NE}$. The fact that the game has strategic complements imply that

$$x_i^* \geq BR_i\left(x_{-i}^*\right) > BR_i\left(x_{-i}^{NE}\right) = x_i^{NE},$$

which completes the proof.

2. The proof of part (2) is analogous to part (1), and is omitted for brevity.

□

Next we prove Proposition 3 by relying on the above lemmas.

1. Assume that the game has a beneficial upward commitment. Due to Proposition 1 all players over reply in x^* (i.e., $x_i^* > BR_i\left(x_{-i}^*\right)$ for each player $i \in N$). The fact that (α^*, x^*) is a naive analytics equilibrium implies that

$$\pi_i\left(x^*\right) = \pi_i\left(x\left(\alpha^*\right)\right) \geq \pi_i\left(x\left(1, \alpha_{-i}^*\right)\right).$$

Next observe that because $x_i\left(1, \alpha_{-i}^*\right) = BR_i\left(x_{-i}\left(1, \alpha_{-i}^*\right)\right)$, then

$$\pi_i\left(x\left(1, \alpha_{-i}^*\right)\right) > \pi_i\left(x_i^{NE}, x_{-i}\left(1, \alpha_{-i}^*\right)\right)$$

Further observe that player i plays a best reply in $x\left(1, \alpha_{-i}^*\right)$ (because he is unbiased), while each other player $j \neq i$ over replies in $x\left(1, \alpha_{-i}^*\right)$ because she has over replied in $x\left(\alpha^*\right)$ and she has the same value of α_j^* in both naive analytics equilibria (this observation is formalized in Lemma 2). In games with strategic complements this observation implies that $x_j\left(1, \alpha_{-i}^*\right) > x_j^{NE}$ for each player $j \in N$ (as proven in Lemma 3). Due to Claim 2 exactly one of the RIDE derivatives is positive. As Claim 3 implies that the total number of positive derivatives is even, it implies that the remaining derivative is positive, i.e., that the game has positive payoff externalities, which implies that

$$\pi_i\left(x_i^{NE}, x_{-i}\left(1, \alpha_{-i}^*\right)\right) > \pi_i\left(x^{NE}\right)$$

Combining the three inequalities we obtain $\pi_i(x^*) > \pi_i\left(x^{NE}\right)$.

The proof for the case in which the game has beneficial downward commitment is analogous and omitted for brevity (here and in part 2 below).

2. Assume that the game has upward commitment benefit. Due to Proposition 1 all players over reply in x^* (i.e., $x_i^* > BR_i\left(x_{-i}^*\right)$). In symmetric profiles $x_i^* > BR_i\left(x_{-i}^*\right)$ iff $x_i^* > x_i^{NE}$ because

$$x_i^* > x_i^{NE} \Leftrightarrow x_{-i}^* > x_{-i}^{NE} \Leftrightarrow BR_i\left(x_{-i}^*\right) < BR_i\left(x_{-i}^{NE}\right) = x_i^{NE} < x_i^*,$$

where the the first iff is due to the strategy profile being symmetric, and the second iff is due to the game having strategic substitutes. Due to Claim 2 either zero or two the RIDE derivatives are positive. As Claim 3 implies that the total number of positive derivatives is even, it implies that the remaining derivative is negative, i.e., that the game has negative payoff externalities, which implies that

$$\pi_i(x^*) < \pi_i\left(x_i^*, x_{-i}^{NE}\right) < \pi_i\left(x^{NE}\right).$$

A.9 Proof of Claim 5 (Price Competition ⇒ Assumptions 2–6)

We begin by showing that the RIDE coincides with the elasticity of demand:

$$\text{RIDE}_i(x) = -\frac{\frac{\partial \pi_i}{\partial q_i} \cdot \frac{\partial q_i}{\partial x_i}}{\frac{\partial \pi_i}{\partial x_i}} = -\frac{x_i \cdot \frac{\partial q_i}{\partial x_i}}{q_i(x)} = \left|\eta_{x_i, q_i}\right| = \frac{x_i \cdot b_i}{q_i(x)} \tag{A.3}$$

Next we show that G_p satisfies Assumptions 2–6.

- Assumption 2 (opposing payoffs): $\frac{\partial \pi_i}{\partial x_i} \cdot \frac{\partial q_i}{\partial x_i} = q_i \cdot (-b_i) < 0$.
- Assumption 3 (unique α-equilibrium): Strategy profile x is an an α-Equilibrium iff for

each player i

$$\frac{1}{\alpha_i} = \text{RIDE}_i\left(x\right) = \frac{x_i \cdot b_i}{q_i\left(x\right)} = \frac{x_i \cdot b_i}{a_i - b_i x_i + c_i \cdot x_{-i}} \Leftrightarrow x_i = \frac{a_i + c_i \cdot x_{-i}}{b_i\left(1+\alpha_i\right)}.$$

Substituting $x_{-i} = \frac{a_{-i}+c_{-i}\cdot x_i}{b_{-i}(1+\alpha_{-i})}$ and rearranging yields the unique α-Equilibrium $x\left(\alpha\right)$:

$$x_i\left(\alpha\right) = \frac{a_i b_{-i}\left(1+\alpha_{-i}\right) + c_i a_{-i}}{b_i b_{-i}\left(1+\alpha_i\right)\left(1+\alpha_{-i}\right) - c_i c_{-i}}. \tag{A.4}$$

Observe that the numerator of A.4 is positive due to (5.1) and the denominator is positive due to the assumption of $|c_i| < b_i$. This implies that $x\left(\alpha\right)$ is a well-defined positive price profile.

- Assumption 3':

- Assumption 4 (monotone payoff externalities with the same sign as c_i):

$$\frac{d\pi_i\left(x\right)}{dx_{-i}} = \frac{d}{dx_{-i}}\left(x_i \cdot q_i\left(x\right)\right) = \frac{d}{dx_{-i}}\left(x_i \cdot \left(a_i - b_i x_i + c_i \cdot x_{-i}\right)\right) = c_i.$$

- Assumption 5 (monotone RIDE externalities with the opposite sign of c_i):

$$\frac{d\text{RIDE}_i\left(x\right)}{dx_{-i}} = \frac{d}{dx_{-i}}\left(\frac{x_i \cdot b_i}{q_i\left(x\right)}\right) = \frac{d}{dx_{-i}}\left(\frac{x_i \cdot b_i}{a_i - b_i x_i + c_i \cdot x_{-i}}\right) = -\frac{x_i \cdot b_i}{\left(a_i - b_i x_i + c_i \cdot x_{-i}\right)^2} \cdot c_i.$$

- Assumption 6 (increasing RIDE, which has the same sign as $\frac{\partial \pi_i}{\partial x_i} = q_i > 0$):

$$\frac{d\text{RIDE}_i\left(x\right)}{dx_i} = \frac{d}{dx_i}\left(\frac{x_i \cdot b_i}{q_i\left(x\right)}\right) = \frac{d}{dx_i}\left(\frac{x_i \cdot b_i}{a_i - b_i x_i + c_i \cdot x_{-i}}\right) = \frac{b_i \cdot \left(a_i + c_i x_{-i}\right)}{\left(q_i\left(x\right)\right)^2} > 0,$$

where the last inequality is immediate if $c_i > 0$, and it is implied by $x_{-i} \leq M_{-i} = \frac{a_{-i}}{b_{-i}} < \frac{a_i}{|c_i|}$ (where the last inequality is due to (5.1)) if $c_i < 0$.

A.10 Proof of Part (1) of Claim 1 (Price Competition)

It is simple to see that the biased best-reply of player j to strategy x_i is

$$x_j\left(x_i, \alpha_j^*\right) = \frac{a_j + c_j x_i}{b_j\left(1+\alpha_j^*\right)} \;\Rightarrow\; \frac{dx_j\left(x_i, \alpha_i^*\right)}{dx_i} = \frac{c_j}{b_j\left(1+\alpha_j^*\right)}.$$

Clearly any NAE (α^*, x^*) must have positive prices, which implies that $x_i^* \in Int\left(X_i\right)$ for each player i. Claim 4 implies that x_i^* must satisfy

$$\alpha_i^* - 1 = \frac{\mathrm{d}x_j\left(x_i, \alpha_j^*\right)}{dx_i} \cdot \frac{\frac{\partial q_i}{\partial x_j}}{\frac{\partial q_i}{\partial x_i}}. \tag{A.5}$$

Substituting $\frac{dx_j\left(x_i,\alpha_i^*\right)}{dx_i} = \frac{c_j}{b_j\left(1+\alpha_j^*\right)}$, $\frac{\partial q_i}{\partial x_j} = c_i$, $\frac{\partial q_i}{\partial x_i} = -b_i$ yields:

$$1-\alpha_i^* = \frac{c_i \cdot c_j}{b_i \cdot b_j\left(1+\alpha_j^*\right)} \Leftrightarrow \left(1+\alpha_j^*\right)\left(1-\alpha_i^*\right) = \frac{c_i \cdot c_j}{b_i \cdot b_j}. \tag{A.6}$$

Observe that the RHS of (A.6) remains the same when swapping i and j. This implies that α_j^* and α_i^* must be equal, which, in turn, implies that

$$1-\left(\alpha_i^*\right)^2 = \frac{c_i \cdot c_j}{b_i \cdot b_j} \Rightarrow \alpha_1^* = \alpha_2^* = \sqrt{1-\frac{c_i c_{-i}}{b_i b_{-i}}}.$$

A.11 Proof of Claim 6 (Advertising Competition ⇒ Asm. 2–6)

- Assumption 1 (concave payoffs):

$$\frac{d\pi_i\left(x\right)}{dx_i} = \frac{\partial \pi_i}{\partial q_i\left(x\right)} \cdot \frac{\partial q_i}{\partial x_i} + \frac{\partial \pi_i}{\partial x_i} = \frac{p_i}{2\sqrt{x_i}}\left(b_i + c_i\sqrt{x_{-i}}\right) - 1$$

$$\frac{d^2\pi_i\left(x\right)}{dx_i^2} = -\frac{1}{4}\frac{p_i}{x_i^{3/2}}\left(b_i + c_i\sqrt{x_{-i}}\right) < 0,$$

where the last inequality is immediate if $c_i > 0$, and it is implied by the assumptions that $\sqrt{x_{-i}} \leq p_{-i}b_{-i}$ and $|c_i| < \frac{b_i}{b_{-i}\cdot p_{-i}}$ if $c_i < 0$.

- Assumption 3 (unique α-equilibrium): Strategy profile x is an α-Equilibrium iff for each player i

$$\frac{1}{\alpha_i} = \text{RIDE}_i\left(x\right) = \frac{p_i}{2\sqrt{x_i}}\left(b_i + c_i\sqrt{x_{-i}}\right) \Leftrightarrow \sqrt{x_i} = \alpha_i \frac{p_i}{2}\left(b_i + c_i\sqrt{x_{-i}}\right),$$

where if $c_i < 0$ then $b_i + c_i\sqrt{x_{-i}} > 0$ due to the assumptions $|c_i| < \frac{b_i}{b_{-i} \cdot p_{-i}}$ and $x_{-i} \leq \left(p_{-i}b_{-i}\right)^2$, which implies that $x_i\left(x_{-i}\right)$ is well-defined and positive for any x_{-i}. Substituting $\sqrt{x_{-i}} = \alpha_{-i}\frac{p_{-i}}{2}\left(b_{-i} + c_{-i}\sqrt{x_i}\right)$ and rearranging yields the unique α-Equilibrium:

$$\sqrt{x_i\left(\alpha\right)} = \left(\frac{\alpha_i p_i\left(2b_i + \alpha_{-i}b_{-i}c_i p_{-i}\right)}{4 - \alpha_i\alpha_{-i}c_i c_{-i}p_i p_{-i}}\right), \tag{A.7}$$

where the numerator is positive when $c_i < 0$ due to the assumptions $|c_i| < \frac{b_i}{b_{-i} \cdot p_{-i}}$ and $\alpha_{-i} \leq 2$, and the denominator is positive due to the assumptions that $\alpha_i \leq 2$ and $c_i < \frac{1}{p_i}$. This implies that $x\left(\alpha\right)$ is a well-defined positive advertising budget profile.

- Assumption 3':

- Assumption 4 (monotone payoff externalities with the same sign as c_i):

$$\frac{d\pi_i\left(x\right)}{dx_{-i}} = p_i \cdot \frac{dq_i\left(x\right)}{dx_{-i}} = \frac{p_i \cdot \sqrt{x_i}}{2 \cdot \sqrt{x_{-i}}}c_i.$$

- Assumption 6 (negative RIDE):

$$\frac{d\text{RIDE}_i\left(x\right)}{dx_i} = \frac{d}{dx_i}\left(\frac{p_i}{2\sqrt{x_i}}\left(b_i + c_i\sqrt{x_{-i}}\right)\right) = -\frac{p_i}{4x_i^{3/2}}\left(b_i + c_i\sqrt{x_{-i}}\right) < 0$$

where if $c_i < 0$ then $b_i + c_i\sqrt{x_{-i}} > 0$ due to the assumptions $|c_i| < \frac{b_i}{b_{-i} \cdot p_{-i}}$ and $x_{-i} \leq \left(p_{-i}b_{-i}\right)^2$.

- Assumption 4 (monotone RIDE externalities with the same sign as c_i):

$$\frac{d\text{RIDE}_i\left(x\right)}{dx_{-i}} = \frac{d}{dx_{-i}}\left(\frac{p_i}{2\sqrt{x_i}}\left(b_i + c_i\sqrt{x_{-i}}\right)\right) = \frac{p_i}{4\sqrt{x_i x_{-i}}} \cdot c_i.$$

A.12 Proof of Part (1) Claim 2 (Adverting Competition)

In the proof of Claim 6 we have shown that $\sqrt{x_j} = \alpha_j^* \frac{p_j}{2}\left(b_j + c_j\sqrt{x_i}\right)$, which implies:

$$x_j\left(x_i, \alpha_j^*\right) = \left(\alpha_j \frac{p_j}{2}\left(b_j + c_j\sqrt{x_i}\right)\right)^2 \Rightarrow \frac{dx_j\left(x_i, \alpha_i^*\right)}{dx_i} = \frac{\sqrt{x_j}}{\sqrt{x_i}}\alpha_j \frac{p_j}{2} c_j$$

Clearly any NAE (α^*, x^*) must have positive advertising budgets (as otherwise the firm makes no profit), which implies that $x_i^* \in Int\left(X_i\right)$ for each player i. Claim 4 implies that x_i^* must satisfy

$$\alpha_i^* - 1 = \frac{\mathrm{d}x_j\left(x_i, \alpha_j^*\right)}{dx_i} \cdot \frac{\frac{\partial q_i}{\partial x_j}}{\frac{\partial q_i}{\partial x_i}}. \tag{A.8}$$

Substituting $\frac{dx_j\left(x_i,\alpha_i^*\right)}{dx_i} = \frac{\sqrt{x_j}}{\sqrt{x_i}}\alpha_j^* \frac{p_j}{2} c_j$, $\frac{\partial q_i}{\partial x_j} = \frac{c_i\sqrt{x_i}}{2\sqrt{x_j}}$, $\frac{\partial q_i}{\partial x_i} = \frac{b_i + c_i\sqrt{x_j}}{2\sqrt{x_i}} = \frac{1}{2\alpha_i^* \frac{p_i}{2}}$ yields:

$$1 - \alpha_i^* = \frac{\sqrt{x_j}}{\sqrt{x_i}}\alpha_j^* \frac{p_j}{2} c_j \frac{c_i\sqrt{x_i}2\alpha_i^*\frac{p_i}{2}}{2\sqrt{x_j}} = \alpha_j^*\alpha_i^* \frac{p_i p_j c_j c_i}{4}. \tag{A.9}$$

Observe that the RHS of (A.9) remains the same when swapping i and j. This implies that α_j^* and α_i^* must be equal, which, in turn, implies that

$$\frac{1 - \alpha_i^*}{\left(\alpha_i^*\right)^2} = \frac{p_i p_j c_j c_i}{4} \Rightarrow \alpha_1^* = \alpha_2^* = \frac{2}{1 + \sqrt{1 - c_1 c_2 p_1 p_2}} \text{ or } \frac{2}{1 - \sqrt{1 - c_1 c_2 p_1 p_2}}.$$

Because $\alpha_i \in (0, 2)$, the unique solution is $\alpha_1^* = \alpha_2^* = \frac{2}{1+\sqrt{1-c_1 c_2 p_1 p_2}}$.

A.13 Proof of Claim 7 (Team Production)

1. Assumption 1 (concave payoffs):

$$\frac{d^2\pi_i}{dx_i^2} = \frac{d^2}{dx_i^2}\left(q\left(x\right) - x_i\right) = \frac{d^2 q\left(x\right)}{dx_i^2} < 0$$

   due to q being concave..

2. Assumption 3 (unique α-equilibrium): Strategy profile x is an an α-Equilibrium iff for

each player i

$$\frac{1}{\alpha_i} = \text{RIDE}_i\left(x\right) = \frac{\partial q}{\partial x_i}.$$

Let $\epsilon = \max\left(\frac{1}{\alpha_1}, \frac{1}{\alpha_2}\right)$TBD**:** (1) Supermodularity implies that NAE exists. (2) the other two assumptions imply an interior solution. (3) showing that monotone Hessian determinant implies uniqueness.

3. Assumption 4 (positive payoff externalities):

$$\frac{d\pi_i\left(x\right)}{dx_{-i}} = \frac{d}{dx_{-i}}\left(q\left(x\right) - x_i\right) = \frac{d}{dx_{-i}}\left(q\left(x\right)\right) > 0$$

due to q being increasing in x_{-i}.

4. Assumption 6 (negative RIDE self-derivative):

$$\frac{d\text{RIDE}_i\left(x\right)}{dx_i} = \frac{d}{dx_i}\left(\frac{\partial q}{\partial x_i}\right) = \frac{d^2 q}{dx_i^2} < 0$$

due to q being concave in x_i.

5. Assumption 4 (positive RIDE externalities):

$$\frac{d\text{RIDE}_i\left(x\right)}{dx_{-i}} = \frac{d}{dx_{i-}}\left(\frac{\partial q}{\partial x_i}\right) > 0$$

due to q being supermodular.

B Conditions for Unique α-equilibrium (Asm. 3)

In this appendix we present conditions on the RIDE that imply existence and uniqueness of an α-equilibrium for any α (Assumption 3).

Our first result presents a necessary and sufficient condition for the existence of α-equilibrium. The condition requires that there are two strategy profiles , where one profile is weakly higher than the other, such that for each of these profiles, each player has a strategy that yields him a RIDE of α_i. The standard proof relies on applying Brouwer fixed-point theorem.

Claim 8. Let G be an underlying game that satisfies Assumptions 4–5. Let $\alpha \in A^n$. Then the game admits an α-equilibrium iff there exist strategy profiles $\underline{x} \leq \overline{x}$ (i.e., $\underline{x}_i \leq \overline{x}_i$ for each player i), such that for each player i, there exist strategies $\underline{x}'_i, \overline{x}'_i \in [\underline{x}_i, \overline{x}_i]$ that satisfy

$$RIDE_i\left(\underline{x}'_i, \underline{x}_{-i}\right) = RIDE_i\left(\overline{x}'_i, \overline{x}_{-i}\right) = \alpha_i.$$

Proof. "If side": Let $\bar{X} = \{x \in X | \underline{x} \leq x \leq \overline{x}\}$ be the compact and convex subset of profiles between $\underline{x}$ and $\overline{x}$. Define $g : \bar{X} \rightarrow \bar{X}$ as follows

$$g_i\left(x\right) = \left\{x'_i \in \bar{X} | \mathrm{RID}E_i\left(x'_i, x_{-i}\right) = \alpha_i\right\}.$$

Assumptions 4–5 imply that there exists a unique $x'_i \in [\underline{x}'_i, \overline{x}'_i]$ such that $\mathrm{RID}E_i\left(x'_i, x_{-i}\right) = \alpha_i, g_i\left(x\right)$, which implies that g is a well-defined function. The fact that π and q are both twice continuously differentiable implies that g is continuous. Brouwer fixed-point theorem implies g admits a fixed point, which must be an α-equilibrium.

"Only if side": If there exist an α-equilibrium x^*, then taking $\underline{x} = \overline{x} = x^*$ satisfies the condition of the claim. □

Next we present a sufficient condition for the uniqueness of α-equilibrium. For each $x \in X$, let $J\left(x\right)$ be the $n \times n$ Jacobian matrix of partial derivatives of $RIDE$ at x: $J_{ij}\left(x\right) = \frac{d(RIDE_i)}{dx_j}$.

Definition 8. $J\left(x\right)$ is *uniformly directional* if for each $v \in \mathbb{R}^n$, there exists $r \in \mathbb{R}^n$ such that $v \cdot J\left(x\right) \cdot r > 0$ for any $x \in X$.

Observe that if either $J\left(x\right)$ is positive-definite for all x or negative-definite for all x, then it is uniformly directional (where $r = v$ for the positive-definite case, and $r = -v$ for the negative-definite case). Our next result shows that uniform directionality implies uniqueness of α-equilbirium.

Claim 9. Let G be an underlying game that satisfies Assumptions 4–5. Assume that the Jacobian $J\left(x\right)$ is uniformly directional. Let $\alpha \in A^n$ be a bias profile. Then there exists at most one α-equilibrium.

Proof. Assume to the contrary that there exists $x' \neq x'' \in X$ such that $RIDE\left(x'\right) = RIDE\left(x''\right) = \alpha$. Let $v = x'' - x'$. By uniform directionality there exists $r \in \mathbb{R}^n$ such that $v \cdot J\left(x\right) \cdot r > 0$ for any $x \in X$. Let $\hat{X} = \{x \in X | x'_i \leq x_i \leq x''_i \, or \, x''_i \leq x_i \leq {x'}_i\}$be the subset of X between x' and x''. By the compactness of $\hat{X}$, there exist $\delta > 0$ such that $v \cdot J\left(x\right) \cdot r > \delta$ for any $x \in \hat{X}$. The fact that π and q are twice continuously differentiable implies that for any $\epsilon > 0$ there exists n such that

$$\left|\left(RIDE\left(x''\right) - \left(RIDE\left(x''\right) + \sum_{k=1}^{n}\left(\frac{x'' - x'}{n}\right) \cdot J\left(\frac{n-k}{n}x' + \frac{k}{n}x''\right)\right)\right) \cdot r\right| < \epsilon.$$

The fact that $RIDE\left(x'\right) = RIDE\left(x''\right)$ implies that

$$\epsilon > \frac{1}{n}\left|\sum_{k=1}^{n}\left(x'' - x'\right) \cdot J\left(\frac{n-k}{n}x' + \frac{k}{n}x''\right) \cdot r\right| = \frac{1}{n}\left|\sum_{k=1}^{n}\delta\right| = \delta,$$

and we get a contradiction for a sufficiently small ϵ. □

In the two-player case we present a simpler sufficient condition for uniqueness, namely that the determinant of the Jacobian never changes its sign, which is equivalent to requiring that the product of the cross-RIDE derivatives is either always larger, or always smaller, than the product of the self-RIDE derivatives.

Claim 10. Let G be an underlying two-player game that satisfies Assumptions 4–5. Assume that

$$\left|J\left(x\right)\right| \equiv \frac{dRIDE_1\left(x\right)}{dx_1} \cdot \frac{dRIDE_2\left(x\right)}{dx_2} - \frac{dRIDE_2\left(x\right)}{dx_1} \cdot \frac{dRIDE_1\left(x\right)}{dx_2} \neq 0$$

for any $x \in X$. Then there exists at most one α-equilibrium.

Proof. Assume to the contrary that there exist x', x'' such that $RIDE\left(x'\right) = RIDE\left(x''\right)$. Assume WLOG that $x'_1 < x''_1$. For each $x_1 \in [x'_1, x''_1]$, let $f\left(x_1\right) \in X_2$ be the unique profile that satisfies $RIDE_1\left(x_1, f\left(x_1\right)\right)$ (uniqueness is implied by Assumption 6 of RIDE monotonicity). In particular it must be that $x'_2 = f\left(x'_1\right)$ and $x''_2 = f\left(x''_1\right)$. In what follows we show that $RIDE_2\left(x_1, f\left(x_1\right)\right)$ is strictly monotone in x_1, which contradicts $RIDE\left(x'\right) = RIDE\left(x''\right)$.

Observe that the definition of $f(x_1)$ implies that

$$0 = \frac{dRIDE_1(x_1, f(x_1))}{dx_1} = J_{11}(x_1, f(x_1)) + J_{12}(x_1, f(x_1)) f'(x_1) \Leftrightarrow f'(x_1) = \frac{-J_{11}(x_1, f(x_1))}{J_{12}(x_1, f(x_1))}.$$

Next we calculate the derivative

$$\frac{dRIDE_2(x_1, f(x_1))}{dx_1} = J_{21}(x_1, f(x_1)) + J_{22}(x_1, f(x_1)) f'(x_1) =$$

$$J_{21}(x_1, f(x_1)) - J_{22}(x_1, f(x_1)) \frac{J_{11}(x_1, f(x_1))}{J_{12}(x_1, f(x_1))} = -\frac{|J(x_1, f(x_1))|}{J_{12}(x_1, f(x_1))}.$$

$J_{12}(x)$ never changes its sign due to Assumption 5. This implies that if the determinant $|J(x)|$ never changes its sign, then $RIDE_2(x_1, f(x_1))$ is strictly monotone in x_1. □

C Microfoundations for biased estimation

C.1 Biased Price Competition Elasticity Estimates res $\alpha_i < 1$

Suppose the analysts hired by each of the two firms decide to experiment with prices to find the price elasticity of demand by alternating between a high price (p_H) and a low price (p_L), setting a low price (discount) μ_L-share of the time. The experiment can be characterized by a level of sloppiness $\gamma_i \in [0, 1]$. In a fraction γ_i of the time, the analyst doesn't monitor the firm's employees and does not carefully supervise that the employees choose the discount times uniformly at random. Hence, it is possible, for example, that the firm's employees will implement discounts on days of low demand, possibly due to the employees having more free time in these days to deal with posting different prices. In the rest of the time ($1 - \gamma_i$ fraction), the analyst verifies that the prices are set randomly. Consequently, when either analyst sets prices uniformly at random, there will not be correlation between the firm's prices. This happens $1 - \gamma_1\gamma_2$ fraction of the time. In the remaining $\gamma_1\gamma_2$ fraction of the time, there might be correlation between the firm's prices, which we denote by ρ. The joint distribution of prices conditional on the correlation ρ and the fractions γ_1, γ_2 is described in Table 3.

	p_L	p_H
p_L	$\mu_{LL} = \mu_L^2 + \mu_L(1-\mu_L)\gamma_1\gamma_2\rho$	$\mu_{LH} = \mu_L(1-\mu_L)(1-\gamma_1\gamma_2\rho)$
p_H	$\mu_{HL} = \mu_L(1-\mu_L)(1-\gamma_1\gamma_2\rho)$	$\mu_{HH} = (1-\mu_L)^2 + \mu_L(1-\mu_L)\gamma_1\gamma_2\rho$

Table 3: Joint distribution of prices with correlation ρ and the fractions γ_1, γ_2

When calculating the price elasticity of demand to decide how to change prices, the analyst calculates:

$$\eta_i = -\frac{\frac{\Delta Q_i}{\overline{Q_i}}}{\frac{\Delta P_i}{\overline{P_i}}} \tag{C.1}$$

Where ΔQ_i is the difference in average demand between high priced and low priced periods, $\overline{Q_i}$ is the average realized demand, $\Delta P_i = p_H - p_L$ is the difference in price between high and low price periods, and $\overline{P_i} = \mu_L p_L + (1-\mu_L)p_H$ is the average price set by the firm.

The demand observed by firm i when setting price p_i and when its competitor sets a price p_{-i} is $Q_i(p_i, p_{-i}) = a_i - b_i p_i + c_i p_{-i}$.

Using the joint probabilities in Table 3, we find that $\overline{Q_i} = a - (b-c)\left(\mu_L p_L + (1-\mu_L)\, p_H\right)$ and $\Delta Q_i = -\left(p_H - p_L\right)\left(b - c\gamma_1\gamma_2\rho\right)$.

Plugging into (C.1), firm i will estimate its price elasticity as:

$$\eta_i = \frac{\left(b - c\gamma_1\gamma_2\rho\right)\left(\mu_L p_L + p_H\left(1-\mu_L\right)\right)}{a - \left(b-c\right)\left(\mu_L p_L + p_H\left(1-\mu_L\right)\right)}, \tag{C.2}$$

while the true elasticity is $\eta_i^T = \frac{b(\mu_L p_L + p_H(1-\mu_L))}{a-(b+c)(\mu_L p_L + p_H(1-\mu_L))}$. Hence the analyst will estimate the firm's price elasticity as being lower than η_i^T when $c > 0$ and $\rho > 0$.

C.2 Biased Advertising Effectiveness Estimates Result in $\alpha_i > 1$

Assume the firm's sales at time t behave according to the linear model $sales_t = \mu + x_t + \epsilon_t$ where μ is the average sales, x_t is the level of advertising, that can be x_L or x_H with $x_H > x_L \geq 0$, and ϵ_t is demand shock which is distributed i.i.d $\mathcal{N}(0,1)$. In this model, the true effect of advertising, $\frac{d(sales_t)}{dx_t}$ equals 1.

The firm has a sales target μ and its advertising strategy is to increase advertising to $x_{t+1} = x_H$ if sales fall below μ at time t, i.e., if $sales_t < \mu$, and otherwise set $x_{t+1} = x_L$.

To estimate the effect of advertising, the firm looks at the difference in sales when advertising is increased or decreased (otherwise the change cannot be attributed to changes in advertising) and takes the average to calculate

$$\frac{\mathbb{E}[\Delta sales]}{\Delta x} = \frac{\frac{\mathbb{E}[sales_{t+1}-sales_t|x_{t+1}=x_H,x_t=x_L]}{x_H-x_L} + \frac{\mathbb{E}[sales_{t+1}-sales_t|x_{t+1}=x_L,x_t=x_H]}{x_L-x_H}}{2} \tag{C.3}$$

More sophisticated approaches can take into account a weighted average of these estimates and also take into account the baseline sales when advertising does not change.

The lefthand part of the summand equals:

$$\begin{aligned}\frac{\mathbb{E}[sales_{t+1} - sales_t|x_{t+1} = x_H, x_t = x_L]}{x_H - x_L} &= \frac{\mu + x_H + \mathbb{E}[\epsilon_{t+1}] - (\mu + x_L + \mathbb{E}[\epsilon_t|sales_t < \mu])}{x_H - x_L} \\ &= = \frac{\mu + x_H - \left(\mu + x_L - \frac{\phi(-x_L)}{\Phi(-x_L)}\right)}{x_H - x_L} = 1 + \frac{\frac{\phi(-x_L)}{\Phi(-x_L)}}{x_H - x_L} > 1\end{aligned}$$

where $\phi(\cdot)$ is the standard Normal pdf and $\Phi(\cdot)$ its cdf. The righthand part equals:

$$\begin{aligned}\frac{\mathbb{E}[sales_{t+1} - sales_t|x_{t+1} = x_L, x_t = x_H]}{x_L - x_H} &= \frac{\mu + x_L + \mathbb{E}[\epsilon_{t+1}] - (\mu + x_H + \mathbb{E}[\epsilon_t|sales_t \geq \mu])}{x_L - x_H} \\ &= \frac{\mu + x_L - \left(\mu + x_H - \frac{\phi(-x_H)}{1-\Phi(-x_H)}\right)}{x_L - x_H} = 1 - \frac{\frac{\phi(x_H)}{\Phi(x_H)}}{x_H - x_L} < 1\end{aligned}$$

Because $\frac{\phi(x)}{\Phi(x)}$ is decreasing in x, the sum in the numerator of (C.3) is larger than 2, which results in the firm overestimating the effectiveness of its advertising to be more than 1.