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Abstract

Paying negative interest rates on central bank digital currency (CBDC) becomes increasingly relevant to monetary operations, since several major central banks have been actively exploring both negative interest rate policy and CBDC after the Great Recession. This paper provides a formal analysis to evaluate the macroeconomic impact of negative interest rates on CBDC through the lens of a neoclassical general equilibrium model with monetary aggregates. In the benchmark model, agents have access to two types of assets: CBDC and productive capital. The demand for digital currency is motivated by a liquidity constraint. I show that paying negative interest on CBDC induces agents to save less and consume more via a substitution effect. A drop in savings in turn causes a fall in capital investment, subsequent output, and real money balances. To clear the money market, the price level increases. I then extend the model to include government bonds which deliver a positive return. This allows me to study a non-trivial portfolio effect: when the government pays a negative interest rate on CBDC, the tax on agents' capital spending increases, inducing a decrease in capital investment and an increase in government bonds in agents' portfolio. Such a policy causes a drop in investment and output. However, there is a transitory decline in the price level due to a "flight to quality".

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"After all, even today when the door to negative rates is cracked only slightly ajar, several major central banks (including the Bank of Japan and the European Central Bank) have already shoved a foot through. Thus it is important to think about phasing out cash and developing negative interest rate policy in an integrative fashion...

For the foreseeable future, however, the best system is one in which a government issued currency is the unit of account, though of course it will eventually morph into a fully electronic one."

Kenneth S. Rogoff (2016) in The Curse of Cash

1 Introduction

One of the great challenges for central banks is the zero lower bound (ZLB) on nominal interest rates. The logic is simple: since physical cash serves as a store of value and guarantees a nominal return of zero, agents would be unwilling to lend at any lower rates, making zero as the effective interest rate floor. This key constraint results in a notable asymmetry in the implementation of central bank policy.[1] When the economy is booming and inflation threatens to rise to undesirable levels, the central bank can tighten monetary policy by raising the official interest rate. When the economy is in a severe recession and inflation is direly needed, the policy rate simply cannot be lowered below zero to provide an appropriate degree of monetary accommodation. If a further stimulus is desired, the monetary authority may have to deploy unconventional monetary policy tools, such as quantitative easing (QE) and forward guidance. But many researchers are rightly concerned that these unorthodox measures are poor substitutes for plain vanilla interest rate policy and might well have more side-effects (Rogoff, 2017a).

Macroeconomists have proposed ways to finesse the zero lower bound, and the most direct approach is to abolish paper currency and pay negative interest rates on central bank electronic liabilities, such as central bank digital currency (CBDC) (see Buiter,

[1]The ZLB constraint is also the key assumption behind a large literature in macroeconomics, for early contributions, see, for example, Krugman (1998) and Eggertsson and Woodford (2003).

2009; Fischer, 2016; Rogoff, 2015, 2016, 2017b).[2] In recent decades, the increasing marginalization of cash in legal transactions and the technological advances in payment systems have created a smooth path towards this policy. Rogoff (2015) has made it clear that there are two important drawbacks to paper currency: first, its existence creates the artifact of the zero lower bound on the nominal interest rate; second, it can help facilitate crime and tax evasion.[3] He argues that it would be appropriate to take a more proactive strategy for phasing out the use of paper currency.[4] In fact, in Sweden, the embracement of technological developments in payment systems has led to an absolute decline in the amount of cash in circulation, making Sweden close to becoming a *cashless* society.

Recent experience from the aftermath of the Great Recession has shown that paying negative interest rates on central bank liabilities is also possible. Since mid-2014, central banks in Switzerland, Sweden, Denmark, Japan, and the Euro Area have tiptoed into negative rate territory for the first time in history, going as far as -0.75 percent in Switzerland for instance (Bech and Malkhozov, 2016; Eggertsson et al., 2017). The implementation of negative policy rates represents a new monetary policy tool. In doing so, central banks typically set the interest on reserves (IOR) to negative levels, and this appears to have no major changes to central bank monetary frameworks (see Bech and Malkhozov, 2016).[5] Since then, there has been a growing interest in evaluating the effects of negative interest rates, both theoretically and empirically, mainly on the banking sector (see Rognlie, 2016; Eggertsson et al., 2017; Basten and Mariathasan,

[2] Note that there are alternative measures to mitigate the constraints associated with the ZLB, such as taxing cash to pay negative interest on currency (Buiter, 2009) and raising the inflation target (Fischer, 2016).

[3] Apart from the two drawbacks, Berentsen and Schär (2016) also note that the use of cash is inefficient and significantly more expensive than electronic payments from a technological perspective.

[4] Rogoff (2016) explicitly states that "phasing out paper currency is arguably the simplest and most elegant approach to clearing the path for central banks to invoke unfettered negative interest rate policies should they bump up against the 'zero lower bound' on interest rates."

[5] Of course, there is nothing impractical at all about paying negative interest on electronic currency.

2018; Ulate, 2019; Arseneau, 2020; Lopez et al., 2020). It is important to note, however, such investigations are only concerned with one type of central bank liabilities (i.e., bank reserves).[6] Given that only designated financial institutions (mostly commercial banks) can hold reserves, the key of the studies lies on how banks intermediate the transmission of monetary policy.

In the meanwhile, as recent technological advancements in cryptographic and distributed ledger techniques (DLT) have made digital currencies widely accessible, central banks around the world are now actively exploring the possibility of issuing their own digital currencies.[7] Although the idea of providing greater access to digital forms of central bank liabilities is not entirely new, dating at least back to Tobin (1985, 1987), recent discussions on issuing a central bank digital currency (CBDC) have been motivated by a number of factors.[8] These include: (i) interest in technological innovations for the financial sector; (ii) the emergence of new entrants into payment services and intermediation; (iii) declining use of physical currency in a few countries; (iv) increasing attention to private digital tokens (e.g., Bitcoin and Ethereum) (see BIS, 2018).[9]

[6] Currently, there are two types of central bank money. One is physical currency (notes and coins), which is anonymous and made accessible to everyone; another one is central bank reserves, which are in electronic form and accessible to certain financial institutions only.

[7] For example, the Swedish Riksbank and the People's Bank of China are expected to decide on initiating CBDCs in the near term (Agur et al., 2020), the Bank of England is conducting a multi-year investigation (Bordo and Levin, 2017), and the Bank of Japan has reported on the legal issues regarding CBDCs (Fernández-Villaverde et al., 2020). Barontini and Holden (2019) provide a survey of 63 central banks, which shows a majority are collaboratively looking at the implications of a CBDC and many have reached a stage of considering practical issues. One year on, the survey has been re-run (Boar et al., 2020). Boar et al. (2020) find that a significant minority of central banks representing a fifth of the world's population look likely to issue a CBDC in the next few years.

[8] There are also skeptics who warn against the introduction of a CBDC. Some frequently-made arguments are: such a policy would cause a reduction in bank credit, with negative implications for growth; the issuance of a CBDC could encourage depositor runs and threaten financial stability. However, Brunnermeier and Niepelt (2019) claim that these arguments are questionable. They show that, with a strong institutional commitment, a transfer of funds from deposit to CBDC accounts would give rise to an automatic substitution of one type of bank funding (deposits) by another one (central bank funding)–the issuance of a CBDC would simply render the central bank's implicit lender-of-last-resort guarantee explicit. Thus, introducing a CBDC need not imply a credit crunch nor undermine financial stability.

[9] Noted by Berentsen and Schär (2016), another benefit for issuing a CBDC is that it can satisfy

Note that, although CBDC can be used to refer to a number of concepts, it is envisioned by most to be a new form of central bank issued, account-based, electronic currency, which is made accessible to the general public (see Barrdear and Kumhof, 2016; Berentsen and Schär, 2016; Keister and Sanches, 2019; Andolfatto, 2020).[10] In this regard, introducing CBDCs allows "Reserves for All" (Niepelt, 2020) or "Central Banking for All" (Fernández-Villaverde et al., 2020).

There is little doubt that the introduction of *both* a CBDC and negative interest rate policy represents important innovations in the history of central banking. The implementation of such macroeconomic policies is likely to have substantial effects on economic activities. It also raises important questions. What if central banks start to pay negative interest on CBDC? What is the economic impact of such a policy? In particular, how would paying negative interest on CBDC affect consumption, investment, output, and the price level? Given the fact that several major economies around the globe are actively exploring both negative interest rate policies and a CBDC, paying a negative interest rate on CBDC is becoming increasingly relevant.[11] At a macroeconomic level, however, the implications of such a policy are not understood. To the best of my knowledge, this paper is the first to provide a formal evaluation of negative interest rates on CBDC.

In this paper, I evaluate the macroeconomic impact of negative interest rates on central bank digital currency (CBDC) through the lens of a neoclassical general equilibrium model with monetary aggregates. I use an overlapping generations (OLG) model with fiat money, similar to Wallace (1980) and Smith (1991). In the benchmark model,

the population's need for virtual money without facing counterparty risk.

[10] As noted above, central bank electronic money has already existed (i.e., reserves), but it is generally restricted to financial institutions with which the central bank interacts.

[11] There has been no country which paid negative interest rates on CBDC, yet. In this sense, this paper provides a normative, instead of positive, investigation. However, for countries which are intensively examining both the effects of negative interest rates and a CBDC, such as Sweden, paying negative interest on CBDC may not seem to be far off.

agents hold two types of assets: CBDC and productive capital. To motivate a demand for government money in a simple and direct way, I assume agents are subject to a binding liquidity requirement (or a reserve requirement) such that they are required to hold a certain amount of CBDC that is proportional to their capital holdings. The key mechanism in this environment is a *substitution effect*. If the government pays negative interest on CBDC, agents would reduce their savings due to a fall in the anticipated rate of return, and choose to consume more (i.e., current consumption increases). However, as savings decrease, so does investment, which results in a fall in subsequent output. In addition, this policy measure causes an increase in the price level. This is because a reduction in savings implies a smaller demand for government money. Since the stock of CBDC is fixed in this modelling environment, the price level has to increase to clear the money market. Finally, I show that paying negative interest rates on CBDC can be detrimental for all agents from a welfare perspective. The initial old are hurt as real money balances fall. Other generations are also worse off since negative interest payments on CBDC represent a distortionary tax to savings.

Then, I extend the model to include another important type of public money, namely government debt.[12] With the existence of positive-yielding government bonds, I show that a non-trivial *portfolio effect* emerges. As the government pays negative interest rates on CBDC, the tax on capital spending increases, which induces agents to hold more bonds and less capital in their portfolio. The drop in capital investment in turn causes a decline in output in the subsequent period. Interestingly, although the demand for CBDC falls, the total holdings of government money actually *increase*, due to a "flight to quality" (i.e., government bonds). To clear the money market, the price level *drops*, causing deflationary pressures. Taken together, both cases show that there are costs and benefits to negative interest payments on CBDC. One the one hand, such

[12]Note that if one considers a consolidated monetary/fiscal authority, government bonds can also be viewed as public money.

a policy could boost consumption via agent's consumption/saving decision and induce inflationary pressures if inflation is direly needed by central banks. On the other hand, paying a negative interest rate on CBDC can cause a decrease in capital investment and GDP, either from a substitution effect or a portfolio effect. In addition, it is possible that such a policy measure induces a decline in the temporary price level if the total demand for public money increases. Finally, the results indicate that the payment of negative interest on CBDC does not seem to have clear welfare justifications.

Related Literature. This paper relates closely to two important branches of monetary economics. One is a growing and evolving literature on CBDC. Barrdear and Kumhof (2016) introduce a CBDC into a quantitative DSGE model to estimate its impact on GDP. Their simulations show that the initiation of CBDC through purchasing government bonds can lower the real interest rate and thereby increase output. Quantitatively, they find that CBDC issuance of 30% of GDP could permanently raise GDP by as much as 3%. Davoodalhosseini (2018) studies the role of a CBDC as a new monetary instrument. His focus is on how the introduction of a digital currency affects the use of cash and on its implications for monetary policy. Agur et al. (2020) study the optimal design of a CBDC in an environment where agents sort into physical cash, CBDC, and bank deposits based on their preferences over anonymity and security. They find that a deposit-like CBDC causes an increase in deposit and loan rates, and a contraction in bank credit and output, whereas a cash-like CBDC may lead to the disappearance of cash.

Keister and Sanches (2019) also study a cash-like digital currency, but with a focus on the implications of a CBDC that competes with bank deposits. In their model, banks are financially constrained, and the liquidity premium on bank deposits affects the level of aggregate investment. They show that while a CBDC promotes efficiency in exchange, it also crowds out banks' deposits, raises banks' funding costs, and decreases investment. Similarly, Chiu et al. (2019) develop a micro-founded general equilibrium model and evaluate whether the issuance of a CBDC is likely to increase banks' funding costs and cause disintermediation in banks. However, they find that when banks have market powers in the deposit market, the introduction of a CBDC can compel banks to raise the deposit rate and expand intermediation and output. Andolfatto (2020) also studies the effect of a CBDC on bank intermediation. In an environment with a monopoly bank, he shows that introducing a CBDC affects the interest rate on deposits, but not the bank lending rate or the level of investment. Overall, the results of aforementioned studies are mixed regarding the role of CBDC issuance on financial intermediation. However, none of the papers studies the effects of negative interest payments on CBDC. And unlike the above studies, this paper focuses on the macroeconomic impact of negative interest rates on CBDC, not on money demand or bank intermediation.

Another strand of literature examines the effects of negative interest rate policies. Rognlie (2016) studies optimal monetary policy in a continuous New Keynesian model where agents hold both physical cash and government bonds. He finds that there exists a trade-off: negative rates on bonds help stabilize aggregate demand, but at the cost of an inefficient subsidy to paper currency. His results show that when the economy is in a slump, negative rate policies are always optimal to some degree and that breaking the ZLB with negative rates brings significant welfare improvements. Eggertsson et al. (2017) examines the proposition that negative interest rates are expansionary in a DSGE model with a banking sector. Using aggregate and bank-level data, they document a collapse in pass-through to deposit and lending rates once the IOR rate becomes negative.[13] Their results show that once the policy rate turns negative, the usual transmission mechanism of monetary policy breaks down. In addition, because

[13]Bech and Malkhozov (2016) also note that retail deposit rates have remained insulated in response to negative policy rates.

a negative interest rate on reserves reduces bank profits, the total effect on aggregate output can be contractionary. Contrary to their results, Ulate (2019) investigates the same question but finds that negative rates can stimulate the economy by lowering the rates that commercial banks charge on loans, while they can also erode bank profitability by squeezing deposit spreads. Yet, there is no consensus on the effects of negative interest rate policies.[14] Different from these contributions, the focus of this paper is on the macroeconomic impact of negative rates, in a "reserves for all" economy where the role of a CBDC is carefully examined. Thus, this study forms a bridge between the two strands of literature. The novel contribution of this paper is to provide a formal investigation into the effects of negative interest payments on CBDC.

The rest of the paper is organized as follows. In Section 2, I lay out the physical structure of the benchmark economy–individual preferences, demographics, and technologies. I also derive the equilibrium of the baseline model. Section 3 evaluates the economic implications of negative interest rates on CBDC. In Section 4, I extend the model to include government debt, and discuss the impact of such a policy via a portfolio effect. Section 5 offers concluding remarks.

[14] Note that there is also a growing empirical literature that studies the impact of negative rates on commercial banks. For example, Basten and Mariathasan (2018) analyze the effect of negative interest rates on banks, using detailed supervisory information from Switzerland. They find that more affected banks reduce costly reserves and bond financing while maintaining non-negative deposit rates. Banks' portfolio rebalancing implies relatively more lending, compared to an earlier rate cut within positive territory. Arseneau (2020) uses supervisory data to examine bank-level expectations regarding the impact of negative rates on profitability through net interest margins. He finds that the most significant channel of adverse exposure comes from the pass-through of negative rates to short-term liquid assets held on the balance sheet. On the liability side, banks that rely more heavily on short-term wholesale funding may benefit from a reduction in funding costs. Lopez et al. (2020) explore the impact of negative policy rates on banks using data on 5200 banks from 27 advanced European and Asian countries, 2010–2017. Their results show that banks offset interest income losses under negative rates with lower deposit expenses and gains in non-interest income. Banks also increase their lending activity and raise their share of deposit funding in response to a negative interest rate.

2 The benchmark model

I base my analysis on a neoclassical general equilibrium model with fiat money *à la* Wallace (1980) and Smith (1991).[15] The model economy is populated by two-period-lived overlapping generations (OLG). Time is indexed by $t = 1, 2, ..., \infty$. Within each period t, two generations coexist–those in the first period of their life (the "young") and those in the last period of their life (the "old"). There is a set of young agents born in each period $t \geq 1$, all of whom are identical. In the initial period, $t = 1$, there is also a set of initial old agents called *the initial old*, who live for one period. All generations have equal number of N agents.

There exist two assets in the economy–central bank digital currency (CBDC) and productive capital.[16] For ease of exposition, I assume, at $t = 1$, a constant *per capita* stock of CBDC of $\bar{m} = 1$ is created by the government and passed to the initial old.[17] The initial old want as much of consumption good in period 1 as possible, and their welfare is monotonically increasing in consumption. Young agents have the twice continuously differentiable utility function: $U(c_1) + V(c_2)$, where c_j stands for age j consumption; $j = 1, 2$. It is further assumed that $U^{'} > 0, V^{'} > 0$, and $U^{''} \leq 0, V^{''} < 0$. To ensure an interior solution, $U'(0)$ and $V'(0)$ are assumed infinite. Each young

[15] Central bank digital currency (CBDC) is fiat money. Noted by Wallace (1980), there are two widely accepted defining characteristics of fiat money: *inconvertibility* and *intrinsic uselessness*. Inconvertibility means that the issuer does not promise to convert the money into anything else. Intrinsic uselessness means that fiat money is never wanted for its won sake; it is not legitimate to take fiat money to be an argument of anyone's utility function or of any production function. In addition, what differs CBDC from private cryptocurrencies, such as Bitcoin and Ethereum (both are inconvertible and intrinsically useless), is that CBDC is valid as legal tender and thus issued by monetary authorities.

[16] That is, there is only one type of monetary asset in the benchmark economy. Government bonds with a positive net return are later introduced in Section 4.

[17] One can view that the initial old open digital accounts at the central bank and receive the "gift" of the initial stock of CBDC. This is *de facto* "helicopter money". However, whether agents hold funds electronically in CBDC accounts at the central bank or in specially designated accounts at supervised depository institutions (e.g., commercial banks) is irrelevant to my analysis. In reality, it is likely that a stock of CBDC is firstly injected to a small part of the population, e.g., government employees or project contractors, and then it gets circulated among the whole population.

agent has an endowment $y > 0$ of consumption good when young, and agents have no endowment of the good when old.[18] A unit of the consumption good at t that is not consumed may be used to create a unit of capital. The problem facing agents is how to finance consumption when old.

Savings of each young agent take the form of CBDC and capital investment, which offer different rates of return. When invested, the consumption good returns $x > 1$ units of the consumption good in period $t + 1$ for each unit invested in period t. As previously mentioned, the initial old are endowed with $\bar{m}$ units of CBDC.[19] The young can trade a unit of the consumption good for p_t units of CBDC, that is, CBDC serves as *a unit of account*. In period $t + 1$, in the absence of negative interest payments on CBDC, the old (the young of period t) trade each unit of CBDC for $1/p_{t+1}$ units of the consumption good. Thus, the rate of return on CBDC held by the old is p_t/p_{t+1}. With negative interest rates on CBDC, however, I assume CBDC earns the real return $(1 + \rho)$, where $\rho \in (-1, 0)$ is a new monetary policy tool and denotes the net negative interest rate for each unit of digital currency held in CBDC accounts.[20] Note that for generations born at $t > 1$, agents store part of their savings in CBDC when young, which is carried over when old. In period $t + 1$, the old (born in period t) exchange digital currency for consumption good from the young (born in period $t + 1$). Thus, CBDC is also *a store of value* and serves as *a medium of exchange*, and it is made accessible to all generations.

By construction, CBDC is dominated in rate of return by storage capital, i.e., $(1 + \rho) < x$. It is important to note that, technically, CBDC could lose its value in

[18] Alternatively, one can think of the young as being endowed with a unit of labor or time that can be transformed into y units of the consumption good.

[19] Of course, the stock of CBDC can be assumed to grow (or fall), following some quantity rule, which is also a monetary policy instrument. Given the focus of analysis is on the economic impact of negative interest rates on CBDC, I do not allow the stock of fiat money to vary in the benchmark economy.

[20] While the scheme just described has the government setting the real return on CBDC, it is easy to verify that having the government set the nominal returns has the same consequences.

this modelling environment. Suppose that all young agents are identical and have the same productive storage, but that they cannot store their own goods. Therefore each young agent at t deposits some resources to a "bank", which intermediates storage of the good. The bank issues paper IOUs to the depositors and promises them to pay x units of the consumption good in period $t+1$ for every unit of the good deposited in period t. These privately created paper IOUs serve as *inside* money, with a positive net return $x-1>0$. If agents prefer inside private money to outside public money (i.e., CBDC), fiat money will lose its value, and prices would have to be expressed in some other unit of account.[21] In this case, if the government still wishes to maintain fiat money as a unit of account, it must force people to hold CBDC. There are many ways for the government to shore up the demand for fiat money. Most directly, it can simply require agents to hold a certain amount of CBDC.[22] In this "reserves for all" economy (see Niepelt, 2020), I assume all young agents face the legal requirement of holding CBDC, a *pro rata* share of their capital investment.[23]

Let k_t denote the quantity of the good invested by young agents at t, and let m_t denote their holdings of real CBDC balances. All young agents are required by law to hold a certain amount of CBDC, $m_t \geq \gamma k_t$, where $\gamma \in [0,1]$ can be interpreted as liquidity requirements or reserve requirements. Throughout the focus will be on equilibria in which this legal restriction is binding. In every period after the initial period, holding CBDC earns negative net interest for each unit held in required CBDC. Assume a lump-sum subsidy of $\tau_t \geq 0$ units of the consumption good is distributed to each old agent at $t>1$.

[21] Notice that the old in period t cannot credibly issue inside money as they die at the end of period $t+1$.

[22] It is in the same spirit of the legal requirement discussed by Champ and Freeman (1990) and Smith (1991).

[23] Note that in reality the government does impose regulations (e.g., reserve requirements and liquidity requirements) that explicitly require agents (mostly commercial banks) who have access to government digital currency (i.e., central bank reserves), to hold fiat money.

2.1 Behavior of young agents

Taking government subsidies $\{\tau_t\}$ as given, young agents at each date t choose m_t and $k_t(k_t \geq 0)$ to maximize $U(c_1) + V(c_2)$ subject to:

$$c_1 + k_t + m_t = y, \tag{1}$$

$$c_2 = xk_t + (1+\rho)m_t + \tau_{t+1}, \tag{2}$$

$$m_t \geq \gamma k_t. \tag{3}$$

Note that since the focus is on equilibria where equation (3) is binding, this problem can be transformed as follows.[24] Let Q_t denote total savings of a young agent at t, so that $Q_t = (1+\gamma)k_t$, let $\phi = (1+\gamma)^{-1}$, and let $R \equiv \phi x + (1-\phi)(1+\rho)$. Then ϕ is the weight placed on capital in each young agent's portfolio, and R is the appropriately weighted return on this portfolio faced by a young agent at t. Each young agent then chooses his personal savings $Q_t \in [0, y]$ to maximize $U(y - Q_t) + V(RQ_t + \tau_{t+1})$. The first-order condition for this "consumption-saving" problem is:

$$U^{'}(y - Q_t) = RV^{'}(RQ_t + \tau_{t+1}). \tag{4}$$

The above equation also implicitly implies a savings function $Q_t = f(R)$, to be discussed in detail below. In addition, note that in the absence of negative interest payments, these digital money earns the (gross) real return p_t/p_{t+1}. Hence *per capita*

[24]Given that $(1+\rho) < x$, as previously assumed, equation (3) is in fact binding, that is, agents will not hold extra CBDC in their accounts since the return on fiat money is lower than that on productive capital.

government interest revenues at $t+1$ are $(1-\phi)[(p_t/p_{t+1})-(1+\rho)]Q_t$. Following the assumption that interest revenues are appropriated for subsidies τ_{t+1} to the old, government budget balance requires that:

$$\tau_{t+1} = (1-\phi)[(p_t/p_{t+1})-(1+\rho)]Q_t. \tag{5}$$

2.2 Equilibrium

In what follows, I restrict attention to deterministic stationary allocations. A stationary equilibrium is a vector (Q, m, p) that depends on a policy vector (ρ, τ), satisfying equations (4), (5), $m_t = \gamma k_t = [\gamma/(1+\gamma)]Q_t$, and the money market clearing condition:

$$\bar{m} = m_t p_t = 1, \tag{6}$$

for all $t \geq 1$.[25] Note that equation (6) determines the price level in the economy. That is, the price level can still be well-defined in this *cashless* economy. The money market clearing condition also helps us to understand the transitory changes in the price level. For instance, it directly follows from equation (6) that in response to a fall in the demand for CBDC (i.e., a lower m_t), the price level has to increase to clear the money market, which in turn induces inflationary pressures.

Substitution of (5) into (4) gives the equilibrium law of motion for Q_t:

$$U^{'}(y-Q_t) = RV^{'}[\phi x Q_t + (1-\phi)Q_{t+1}], \tag{7}$$

where $Q_{t+1}/Q_t = m_{t+1}/m_t = p_t/p_{t+1}$ has been used in equation (7). It is important to note that in equilibrium, the *wealth effect* of the decreased rate of return on private savings is exactly offset by the increased subsidies,

[25] Note that $\rho \in (-1, 0)$ is exogenously set by the government.

$$c_2 = \phi x Q + (1-\phi)(1+\rho)Q - \rho(1-\phi)Q = [\phi x + (1-\phi)]Q, \tag{8}$$

leaving only a *substitution effect.*

Note that the locus defined by equation (7) has a slope:

$$\frac{dQ_{t+1}}{dQ_t} = \frac{-U'' - R\phi x V''}{R(1-\phi)V''} < 0. \tag{9}$$

And there exists a unique (non-zero) monetary steady state value for Q_t, denoted by Q^*, satisfying:

$$U^{'}(y - Q^*) = RV^{'}[(\phi x + 1 - \phi)Q^*]. \tag{10}$$

Next, I turn to check the stability of steady state equilibrium. From equation (9), this steady state equilibrium is asymptotically stable iff:

$$\frac{dQ_{t+1}}{dQ_t} \mid_{Q_t = Q^*} = \frac{-U^{''}(y - Q^*) - R\phi x V^{''}[(\phi x + 1 - \phi)Q^*]}{R(1-\phi)V^{''}[(\phi x + 1 - \phi)Q^*]} > -1. \tag{11}$$

It is easy to verify that under the assumption that equation (11) holds, i.e., $-1 < \frac{dQ_{t+1}}{dQ_t} \mid_{Q_t = Q^*} < 0$, there exists one possible equilibrium time path for $\{Q_t\}$, and the equilibrium is asymptotically stable.[26] Note that if the above condition is not satisfied, i.e., $\frac{dQ_{t+1}}{dQ_t} \mid_{Q_t = Q^*} \leq -1$, the steady state equilibrium Q^* is unstable. In addition, while a unique steady state Q^* can exist, there can also exist a continuum of (non-zero) monetary equilibria that oscillate as they approach Q^*. Given that $Q_{t+1}/Q_t = p_t/p_{t+1}$, this implies paying negative interest on CBDC allows for oscillations in the price level. Thus, negative interest payments on digital money could open an economy to the

[26] One may refer to Smith (1991) for a similar discussion on the stability properties of the equilibrium.

possibility of "excessive fluctuations" in the price level. However, the focus of discussion will be on deterministic stationary equilibria that arise when negative interest is paid on CBDC.

3 Economic impact of negative rates on CBDC

The focus of my analysis is to evaluate the macroeconomic impact of negative interest payments on CBDC. Let variables with a ($\hat{}$) denote the steady state equilibrium in the absence of negative interest on CBDC. Recall that equilibrium values of variables with negative net interest payments on CBDC are marked by a ($*$). As shown below, the key mechanism is agent's consumption/saving decision, I shall start with young agents' saving behavior. A savings function is $Q_t = f(R)$, from equation (10),

$$f'(R) = \frac{V'(c_2)}{-U''(c_1) - R[\phi x + (1-\phi)]V''(c_2)} > 0, \forall R \tag{12}$$

which leads to the following lemma.

Lemma 1. *Equilibrium savings is an increasing function of the real gross return R. Given that $R \equiv \phi x + (1-\phi)(1+\rho)$, this implies savings is also an increasing function of interest paid on CBDC (i.e., ρ).*

Proof. See above discussions.

Also, It is immediately clear that with negative interest payments on central bank digital currency, i.e., $\rho < 0$, agents reduce their savings.[27]

Proposition 1. *If $\phi < 1$ (or equivalently $\gamma > 0$, so there is a legal requirement for holding CBDC), $R^* < \hat{R}$ and $Q^* < \hat{Q}$.*

Proof. The proof of $R^* < \hat{R}$ can be obtained from Lemma 1. I now turn to prove $Q^* < \hat{Q}$. When CBDC earns no interest, it is easy to show that there exists a unique

[27] As shown before, I have also assumed that $\rho > -1$, so that CBDC yields a positive real return.

steady state value for Q_t, satisfying:

$$U^{'}(y - \hat{Q}) = (\phi x + 1 - \phi)V^{'}[(\phi x + 1 - \phi)\hat{Q}]. \tag{13}$$

Suppose to the contrary that $Q^* \geq \hat{Q}$. Then from (13) and (10), $U^{'}(y-\hat{Q}) = (\phi x+1-\phi)V^{'}[(\phi x+1-\phi)\hat{Q}] > [\phi x+(1-\phi)(1+\rho)]V^{'}[(\phi x+1-\phi)Q^*] = RV^{'}[(\phi x+1-\phi)Q^*] = U^{'}(y-Q^*)$. But given the assumption that $U^{'} > 0, U^{''} \leq 0$, this would imply $Q^* < \hat{Q}$, leading to a contradiction. **Q.E.D.**

One frequently-made argument for launching a negative interest rate on CBDC is to overcome the effective zero lower bound (ZLB) and thus further stimulate aggregate consumption in response to severe adverse shocks (see, for example, Bordo and Levin, 2017). As such, policymakers would be able to provide an appropriate degree of monetary accommodation without relying on unconventional central bank policies, such as quantitative easing (QE) and forward guidance. This result is also obtained in our benchmark economy in which a simple *substitution effect* prevails.

Proposition 2. *If* $\phi < 1$, $c_1^* > \hat{c}_1$.

Proof. Recall that $c_1^* + Q^* = y = \hat{c}_1 + \hat{Q}$. Following the result of Proposition 1, the proof is then immediate.

The intuition behind this proposition is fairly straightforward. In this perfect foresight economy, as young agents face a lower perceived rate of return on their savings when the government pays negative interest on CBDC, their consumption/saving decision induces them substitute savings for consumption, leading to a decrease in personal savings and an increase in current consumption. Note that the wealth effect of the decreased real rate of return on CBDC does not prevail, as it is completely offset by the increased government subsidies, as discussed in equation (8).

Following Freeman and Huffman (1991), total real output (or GDP) at t equals the total endowment ($N_t y$) plus output generated by capital investment that was created in the previous period. Because in period $t-1$ each individual created k_{t-1} units of capital and there were N_{t-1} of those individuals, real GDP in period t equals:

$$GDP_t = Ny + Nxk_{t-1}. \tag{14}$$

Proposition 3. *The payment of negative interest on CBDC causes a drop in real output in the steady state, i.e.,* $GDP^* < \widehat{GDP}$.

Proof. Note that $Q^* = (1+\lambda)k^*$ and $\hat{Q} = (1+\lambda)\hat{k}$, $k^* = \phi Q^*$ and $\hat{k} = \phi\hat{Q}$. Following the result of Proposition 1 $Q^* < \hat{Q}$, one gets $k^* < \hat{k}$. From (14), as N, y, x are all exogenously given, real output in the steady state depends solely on the level of capital stock. Since capital investment falls in the regime of negative rates on CBDC, real output falls as well, i.e., $GDP^* < \widehat{GDP}$. **Q.E.D.**

By and large, the effects of negative interest payments on the real side of the economy can be summarized as follows. Paying negative interest on central bank digital money induces agents to consume more and save less via a substitution effect, which on one hand boost current consumption, but on the other hand puts a drag on real output due to reduced capital investment. Note that in this environment, monetary policy has real effects on the macroeconomy. And this happens without any nominal rigidities as commonly assumed in the New Keynesian literature. Another important task of my analysis is to evaluate the impact of this new monetary policy instrument on the price level, which I turn to discuss.

Proposition 4. *Negative interest payments on CBDC cause an increase in the price level, i.e.,* $p^* > \hat{p}$.

Proof. In this model, real money balances are given by $m^* = (1-\phi)Q^*$ and $\hat{m} = (1-\phi)\hat{Q}$. Since $Q^* < \hat{Q}$, following the result of Proposition 1, one immediately gets $m^* < \hat{m}$. In addition, the money market clearing condition requires that: $m^*p^* = \bar{m} = \hat{m}\hat{p}$. This in turn implies $p^* > \hat{p}$, which completes the proof. **Q.E.D.**

The above result may seem unsurprising since one would expect that lowing interest rates (i.e., monetary policy loosening) induces inflationary pressures, even though the interest rate paid on fiat money goes into negative territory. Indeed, this is the result that one would typically obtain from the canonical New Keynesian model with sticky prices or wages (see, for example, Clarida et al., 1999). However, the underlying mechanism is quite different in this model. In this economy the stock of CBDC is fixed, paying negative interest on government money lowers agent's desired savings.[28] In response, real money balances, as a *pro rata* share of total savings, also fall, and there is a decline in the real demand for government money. As a result, the price level has to increase to clear the money market. It is also interesting to note that the results of Proposition 3 and Proposition 4 indicate there is a price-output *anomaly* in this benchmark economy. That is, exogenous variation in the real return on CBDC can generate a business cycle with a *countercyclical* price-level.

Next, I explore the welfare implications of negative interest payments on CBDC and restrict attention to the steady state utility. Clearly, the initial old are made worse off by this policy, due to a decline in real money balances. All generations except the initial old are also worse off when the government pays negative interest rates on CBDC.

Proposition 5. *Paying negative interest on CBDC makes the initial old generation worse off, and it also reduces the welfare of all other generations.*[29]

Proof. That the initial old are made worse off follows from the proof of Proposition

[28] Note that the assumption that the stock of CBDC is fixed is innocuous, and it is relaxed in the next section. One could alternatively assume the stock of money follows some quantity rule, which would not change the main results here.

[29] I have assumed here that the government acts as a "benevolent planner" with the objective of maximizing agent's utility in the steady state. With other objectives for the government, however, the results could change.

4, $m^* < \hat{m}$. That is, the payment of negative interest on CBDC reduces savings, and hence real money balances. Endowed with a fix stock of digital currency, the real purchasing power of CBDC falls, which reduces the welfare of the initial old. That all other generations are also made worse is proved as follows. Note that their utility if CBDC earns no interest is $\hat{W} = U(y - \hat{Q}) + V[(\phi x + 1 - \phi)\hat{Q}]$, which I have used $\hat{R} \equiv \phi x + (1 - \phi)$ and $\hat{\tau} = 0$. Finding $\hat{Q}$ would need to satisfy (13). From equation (8), it is also clear that their utility when CBDC earns negative interest is $W^* = U(y - Q^*) + V[(\phi x + 1 - \phi)Q^*]$. However, from equation (13), by definition $\hat{W} = U(y - \hat{Q}) + V[(\phi x + 1 - \phi)\hat{Q}] \geq W = U(y - Q) + V[(\phi x + 1 - \phi)Q], \forall Q \in [0, y]$. Moreover, the inequality is strict if $Q \neq \hat{Q}$, since V is a strictly concave function.

Alternatively, steady-state utility may be written as a function of ρ, and $W(\rho) = U(y - Q(\rho)) + V[(\phi x + 1 - \phi)Q(\rho)]$. One gets:

$$W'(\rho) = (\partial Q/\partial \rho)[-U' + (\phi x + 1 - \phi)V']. \tag{15}$$

Substituting the first-order condition (4) into (15) yields:

$$W'(\rho) = (\partial Q/\partial \rho)[-\rho(1 - \phi)V'], \tag{16}$$

which reveals that steady-state utility reaches a maximum at $\rho = 0$. **Q.E.D.**

The economic intuition behind Proposition 5 is straightforward. Each young agent faces a lower expected return on money holdings when the government pays negative interest on CBDC. Albeit they receive government subsidies when old, a substitution effect induces them to save less. Thus savings fall, so do real money balances, which in turn results in the initial old realizing a capital loss on their CBDC. They therefore are worse off from this scheme. All young generations are also injured by the policy, from a welfare perspective. Since the legal requirement on holding CBDC is binding at all times, in the aggregate, the social return to savings for young generation is $\hat{R} = \phi x + (1 - \phi)$. However, each member of any young generation perceives the private return on CBDC to be $R = \phi x + (1 - \phi)(1 + \rho) < \hat{R}$. From the perspective of any young generation, paying negative interest on CBDC makes the return on savings artificially low, resulting in a low level of capital investment and output, which has detrimental effects on the economy. The payment of negative interest on CBDC simply acts as a distortionary tax to savings, leading the public to demand inefficiently little public money. Note that this result is also reminiscent of the "Friedman rule" (see Friedman, 1969), which suggests to set the nominal interest rate at zero–with neither a tax nor a subsidy on government money. The above analysis implies that paying negative interest rates on CBDC does not seem to have clear welfare justifications.

4 A model with government debt

To sharpen our understanding of the mechanics of negative interest payments on CBDC, one crucial assumption made in the benchmark model is that there exists only one type of government money, i.e., central bank digital currency. The key of the analysis was a substitution effect of consumption/saving induced by a negative interest rate on government money. It is important to note that in the benchmark economy, both capital investment and CBDC are proportional to total savings, and there are no portfolio allocations among asset classes albeit agents hold two types of assets. That is, for a given level of savings, agents would always hold the minimum amount of CBDC required by the liquidity requirement and allocate the rest to capital investment, due to the fact that the real return on CBDC is dominated by that on capital.

In this section, this assumption is relaxed and I introduce government debt into the model. The motivations are twofold. First, in reality, there do exist other types of government money (or outside money), namely cash, bank reserves, and government bonds.[30] And crucially, these monetary assets (except cash) typically yield a positive return.[31] Would previous results still hold if we include other monetary assets? Second and more importantly, introducing another asset allows us to analyze a non-trivial *portfolio effect*. How would agents make portfolio adjustments when they perceive a lower return on CBDC? How would this change the macroeconomic impact of negative interest rates on CBDC? These questions are thoroughly examined in this section.

4.1 The model economy

Following Champ et al. (2011) and Andolfatto (2015), I now lay out a stylized monetary model with government debt. The physical setup is similar to that of the benchmark model, and thus I shall keep the presentation as simple as possible. Note, nevertheless, that the model differs from the benchmark environment in two ways. First, I adopt a standard production function where the rate of return on capital is endogenously determined. As such, agents can make portfolio adjustments when they perceive a change in the rate of interest on fiat money (see Freeman and Huffman, 1991). Second, I consider a consolidated monetary/fiscal authority, which I call "the government". The two authorities are interconnected by the government budget constraint (see Champ and Freeman, 1990; Andolfatto, 2015).

[30] Note that since I consider a consolidated government sector, government bonds can be viewed as money. In fact, both CBDC and government bonds are liabilities of the government. Moreover, as CBDC is a replacement of physical cash, I consider a cashless economy. But unlike the benchmark model, this economy does not need to be cashless *per se*. If the net interest rate on bonds is zero, government bonds can be viewed as cash. Introducing cash is straightforward but would not change my main results. Finally, banks reserves (also electronic central bank money) are generalized in our "reserves for all" economy, thus I omit the discussion of bank reserves.

[31] Since the focus of my analysis is on the macroeconomic impact of negative interest payments on government fiat money (i.e., CBDC), I restrict attention to public money and omit private money (e.g., bank deposits) altogether.

4.1.1 Preferences and technology

Time is discrete and denoted $t = 1, 2, ..., \infty$. The economy is populated by a sequence of two-period-lived overlapping generations. At each date $t \geq 1$, a unit mass of young agents enter the economy and a unit mass of old agents leave the economy. In the initial period, there also exists a unit mass of old agents, who live only for one period. The total population is fixed at $2N$ across time and is at every date t divided evenly between the young and the old.

Agents of every generation $t \geq 1$ are endowed with y units of output when young and nothing when old. I assume individuals value consumption only in the second period of life. For simplicity, I also assume that preferences are linear, thus, preferences are given by $U_t = c_{t+1}$, for $t \geq 0$. As a result, the young face a trivial consumption-saving decision: it will always be optimal for them to save all of their income, and the substitution effect of consumption/saving is shut down. This simplified consumption-saving problem allows me to focus on a portfolio effect, which I would like to emphasize later.[32]

Instead of using a linear capital production technology, I assume the young possess a standard investment technology, where k_t units of output invested at date t yield $f(k_t)$ units of output at date $t+1$. The investment function yields a rate of return $f'(k) > 0$ that diminishes with the scale of the capital investment, $f''(k) < 0$. Also assume that $f'(0) = \infty$, so that agents would always prefer to have some investment. Finally, I assume that capital depreciates fully after it is used in production.

[32] Incorporating a non-trivial consumption-saving problem is easy. But doing so would only complicate the analysis without adding anything to the main points I intend to make in this section.

4.1.2 Government policy

In what follows, I consider a consolidated monetary/fiscal authority, the government. There are two nominal monetary assets, central bank digital currency (CBDC) M_t and government bonds B_t, each issued by the government. Assume that both CBDC and government bonds are perpetual instruments that yield gross nominal rates of return equal to R_t^m and R_t^b, respectively. Note that since the focus of discussion is on paying negative interest on CBDC, $R_t^m \leq 1$; and by construction government bonds yield a positive return, $R_t^b > 1$. Assume that the interest and principal owed on maturing government debt $R_{t-1}^m M_{t-1} + R_{t-1}^b B_{t-1}$ must be financed by a combination of new debt $M_t + B_t$ and a lump-sum tax (or subsidy) T_t,[33] that is,

$$R_{t-1}^m M_{t-1} + R_{t-1}^b B_{t-1} = T_t + M_t + B_t. \tag{17}$$

Let D_t denote the nominal value of the government's total outstanding debt at date t, that is, $D_t = M_t + B_t$. In this modelling environment, I assume that the fiscal authority determines the path of D_t and T_t, and the monetary authority determines the path of interest rates R_t^m and R_t^b. Note that equation (17) explicitly shows how monetary and fiscal policy are interacted through the consolidated government budget constraint. Next, I turn to characterize fiscal and monetary policy.

Fiscal policy. I assume that the fiscal authority grows the nominal government debt (or government money) at a fixed rate μ, so that,

$$D_t = \mu D_{t-1}. \tag{18}$$

As before, the initial amount of government money (CBDC and government bonds) is injected to the initial old. I also assume that the fiscal authority passively adjusts the

[33] In this setup, central bank digital currency is *de facto* government debt.

lump-sum tax (or subsidy) T_t to satisfy the government budget constraint, that is, fiscal policy is Ricardian.[34] As it proves to be convenient to express variables in real terms, I let p_t denote the date t price level and define $\tau_t = T_t/p_t$, $d_t = D_t/p_t$, $m_t = M_t/p_t$, which represent real taxes (or subsidies), real holdings of government money, and real holdings of CBDC, respectively. Following (18), the government budget constraint is rewritten as follows:

$$\tau_t = \frac{R^m_{t-1} m_{t-1} + R^b_{t-1}(d_{t-1} - m_{t-1})}{\mu} - d_t, \tag{19}$$

where I have used the condition $b_t = B_t/P_t = d_t - m_t$, in which b_t denotes real holdings of government bonds. Again, I assume that the tax τ_t (or subsidy) falls entirely on the old.

Monetary policy. Since the focus of analysis is on the economic impact of negative interest on digital currency, I consider that the interest rate on CBDC is set exogenously to some level $R^m \leq 1$. The key exercise is to compare the case where CBDC earns no interest ($\hat{R}^m = 1$) with the case where CBDC earns negative interest ($R^{m,*} < 1$). In addition, I consider an interest peg policy: $R^b_t = R^b > 1$. The stability of equilibrium is checked later. Since digital currency and government bonds share identical risk and liquidity characteristics in this setup, to motive a demand for CBDC when it is dominated in rate of return ($R^m < R^b$) by bonds, again, a legal liquidity requirement (or a reserve requirement) is imposed: $m_t \geq \gamma k_t$.

4.1.3 Optimal behavior

The young enter the economy with y units of real income. Since consumption is not valued when young, all income is saved, with savings divided among the three available

[34] Note that T_t can also be interpreted as government surplus (or deficit) if one include government spending into the model.

assets: CBDC, government bonds, and capital. Thus,

$$m_t + b_t + k_t = y. \tag{20}$$

Given a portfolio choice, future consumption is given by:

$$c_{t+1} = f(k_t) + R^b(p_t/p_{t+1})b_t + R^m(p_t/p_{t+1})m_t - \tau_{t+1}. \tag{21}$$

To characterize agent's optimal behavior, substitute (20) into (21) and form the expression:

$$W_t = f(k_t) + R^b(p_t/p_{t+1})(y - m_t - k_t) + R^m(p_t/p_{t+1})m_t - \tau_{t+1} + \lambda_t(m_t - \gamma k_t),$$

where W_t denotes expected consumption (or utility) and $\lambda_t \geq 0$ is the Lagrange multiplier associated with the liquidity requirement. Maximizing W_t with respect to m_t and k_t yields the following conditions:

$$\lambda_t = (R^b - R^m)(p_t/p_{t+1}), \tag{22}$$

$$R^b(p_t/p_{t+1}) = f'(k_t) - \gamma\lambda_t. \tag{23}$$

Condition (22) shows that the liquidity requirement will bind tightly ($\lambda_t > 0$) since bonds strictly dominate CBDC in rate of return ($R^b > R^m$). In this case, $m_t = \gamma k_t$ and $b_t = y - (1+\gamma)k_t$. Condition (23) implicitly defines the demand function for capital investment. The left side of equation (23) represents the real return on government bonds, whereas the right side of equation (23) represents the difference between the rate of return on capital and the *cost* incurred by capital investing. Note that as the real return on capital is endogenously determined, agents may make portfolio adjustments in response to exogenous shocks. For instance, consider the case in which the liquidity constraint is slack ($\lambda_t = 0$), if the government suddenly increases R^b, agents would choose to hold more bonds and less capital until the rate of return on capital equals the real return on bonds.

In our experiment, the liquidity constraint is binding ($\lambda_t > 0$). The expected rate of return on capital would exceed the return on government bonds, $f'(k_t) > R^b(p_t/p_{t+1})$, and agents would prefer to expand their capital spending. Doing so, however, is costly in that agents would have to accumulate additional low-yield money (i.e., negative-return CBDC) simultaneously. Hence, the liquidity requirement imposed by the government serves as an effective *tax* on capital investment.

To show the demand for capital investment more transparently, combine conditions (22) and (23) to form:

$$f'(k_t) = [(1+\gamma)R^b - \gamma R^m]/\Pi_{t+1}, \tag{24}$$

where $\Pi_{t+1} \equiv p_{t+1}/p_t$ is defined as the gross inflation rate. Condition (24) is essentially the Fisher equation, which equates the real interest rate to the inflation-adjusted rate of return on government debt.

4.2 Equilibrium

In equilibrium, the market for CBDC and government debt must clear. The market clearing conditions are given by:

$$M_t = P_t N m_t, \tag{25}$$

$$B_t = P_t N b_t, \tag{26}$$

for all $t \geq 1$. Since $D_t = M_t + B_t$, we also have:

$$D_t / N = p_t d_t. \tag{27}$$

Note that equation (27) determines the price level in this economy. Because $D_t = \mu D_{t-1}$, the expected rate of inflation satisfies:

$$\Pi_{t+1} = p_{t+1}/p_t = (D_{t+1}/D_t)(d_t/d_{t+1}) = \mu(d_t/d_{t+1}). \tag{28}$$

Next, combine (28) and (24), together with $k_t = y - d_t$, to get:

$$f'(y - d_t) = [\frac{(1+\gamma)R^b - \gamma R^m}{\mu}](\frac{d_{t+1}}{d_t}). \tag{29}$$

Since the liquidity requirement binds, i.e., $m_t = \gamma k_t$, one gets:

$$m_t = \gamma(y - d_t). \tag{30}$$

Finally, from the government budget constraint, we have:

$$\tau_t = \frac{R^m_{t-1} m_{t-1} + R^b_{t-1}(d_{t-1} - m_{t-1})}{\mu} - d_t. \tag{31}$$

An equilibrium in this model consists of bounded sequences for d_t, m_t, and τ_t, given a policy vector (R^m, R^b, μ), that satisfy (29), (30), and (31), for $t \geq 1$. A stationary equilibrium is defined as an equilibrium that satisfies $(d_t, m_t, \tau_t) = (d, m, \tau)$, for all t. In a similar setup, Andolfatto (2015) shows that the equilibrium has a simple recursive structure. In this model, given initial values of CBDC and government bonds, equation (29) determines $\{d_t\}_{t=1}^{\infty}$. With d_t determined, equation (30) determines the sequence of m_t. With $\{d_t, m_t\}$ so determined, equation (31) then determines the lump-sum tax (or subsidy) τ_t that is used to balance the government budget.

Let us now characterize the equilibrium path of d_t. In addition, let us assume $0 < d^* < \infty$, this means in the steady state equilibrium the real purchasing power of government money never goes to zero, and $d^* \to \infty$ is also ruled out because it violates feasibility constraint: $d_t \leq y$ for all t. Define $A^{-1} \equiv [\frac{(1+\gamma)R^b - \gamma R^m}{\mu}] > 0$, and (29) is rewritten as:

$$d_{t+1} = Af'(y - d_t)d_t \equiv G(d_t). \tag{32}$$

Given the assumption that f is strictly concave, it is easy to verify that there exists a unique (non-zero) stationary equilibrium, satisfying $1 = Af'(y - d^*)$.[35] Then, one can use equations (30) and (31) to solve for m^* and τ^*. Note that under an interest rate peg policy, there exists a continuum of nonstationary equilibria indexed by the initial condition $d_0 \in (0, d^*)$ with the property that $d_t \to 0$.[36] Again, the focus of analysis is on the stationary equilibrium.

4.3 Negative interest rates on CBDC

As before, the key exercise is to compare the equilibria with and without negative interest payments on CBDC. Let us start by characterizing the steady state level of capital investment k^*, when the government pays negative interest on digital currency. From equation (29), we get:

[35] Technically, there also exists a degenerate equilibrium, $d = 0$, which is ruled out by our assumption.

[36] The stability properties of the equilibrium are familiar in OLG models with fiat money, where the return on money is pegged. This set of nonstationary equilibria are ruled out by our assumption. Note also that there are no equilibria if the initial condition satisfies $d_0 \in (d^*, \infty)$.

$$f'(k^*) = \frac{(1+\gamma)R^b - \gamma R^{m,*}}{\mu}. \tag{33}$$

Proposition 6. *Capital investment is lower in the policy regime in which the government pays a negative interest rate on CBDC, i.e.,* $k^* < \hat{k}$.

Given that $R^{m,*} < \hat{R}^m = 1$ and $f''(k) < 0 < f'(k)$, it is immediately clear from (33) that k is increasing in R^m, so that $k^* < \hat{k}$. Recall from conditions (22) and (23) that, intuitively, the payment of negative interest on CBDC increases the tax on capital spending. In response, agents cut their capital investment and reach the new equilibrium. This leads to the following corollary.

Corollary 1. *As* R^m *falls, the tax on capital spending increases, i.e.,* $\lambda^* > \hat{\lambda}$.

Following equation (33), it is interesting to note that a higher R^b or a lower μ can also induce agents to reduce their capital investment. This is because both policy actions would increase the real return on government bonds, and induce a portfolio substitution away from capital into government bonds.

Corollary 2. *Capital investment is increasing in* μ *and decreasing in* R^b.

Similar to (14), GDP in this economy can be defined as:

$$GDP_t = Ny + Nf(k_{t-1}). \tag{34}$$

Proposition 7. *Paying negative interest on CBDC causes a fall in real output in the steady state, i.e.,* $GDP^* < \widehat{GDP}$.

Following the result of Proposition 6, one gets $k^* < \hat{k}$. From (34), as N and y are exogenously set, real output in the steady state would depends solely on the level of capital stock. Since capital spending falls when the government pays a negative interest rate on CBDC, real output falls as well, $GDP^* < \widehat{GDP}$.

Notice that the results of Proposition 6 and Proposition 7 are consistent with those of the benchmark economy discussed before such that paying negative interest on CBDC reduces capital investment and real output. However, the underlying mechanism is completely different. In the benchmark model, earning negative interest on CBDC induces agents to save less and consume more, and the key mechanism is a substitution effect. As total savings fall, so do real money balances and capital investment. In the current model with government debt, however, a fall in capital spending (and the subsequent drop in real output) works through a *portfolio effect*, shown as follows. The payment of negative interest on CBDC cause an increase in the tax on capital spending, making capital become less attractive than government bonds. As the expected real return on bonds does not change (i.e., R^b/μ in the steady state), agents would make portfolio adjustments to hold more bonds and less capital. This in turn causes a fall in real output.

In addition, such a portfolio allocation decision has interesting implications on the monetary sector, which I now tend to discuss.

Proposition 8. *If* $R^{m,*} < \hat{R}^m$, $m^* < \hat{m}$ and $b^* > \hat{b}$.

Corollary 3. *If* $R^{m,*} < \hat{R}^m$, $d^* > \hat{d}$.

Proof. From equation (30) $m = \gamma(y - d)$, it is easy to show $\frac{\partial m}{\partial d} = -\gamma < 0$, so that real holdings of CBDC is decreasing in d in the steady state. Following the result of Proposition 6, $k^* < \hat{k}$, and given that $k^* + d^* = y = \hat{k} + \hat{d}$, it is obvious that $d^* > \hat{d}$, implying that agents increase their holdings of government money. Thus, $m^* < \hat{m}$. In addition, given $d^* > \hat{d}$ and $m^* < \hat{m}$, together with $d^* = m^* + b^*$ and $\hat{d} = \hat{m} + \hat{b}$, one gets $b^* > \hat{b}$. **Q.E.D.**

Paying negative interest on central bank digital currency increases the tax on capital spending, thus productive capital is less attractive as an asset group. There is then a "flight to quality" towards government bonds, and *total* government debt increases. However, despite the increase in the total demand for government money, agent's holdings of CBDC falls due to the drop in capital investment. Thus, such a policy has different effects on the holdings of government bonds and CBDC, which has important implications on the price level.

Proposition 9. *Paying negative interest on CBDC can cause a decline in the price level, i.e.* $p_t^* < \hat{p}_t$.

Proof. From equation (27), we have $D_t/N = p_t d_t$. In this economy, the path of D_t is determined by the fiscal authority through $D_t = \mu D_{t-1}$, which also implies the long-run inflation rate is anchored by μ. Since $R^{m,*} < \hat{R}^m$, $d^* > \hat{d}$, and that $p_t^* d^* = \hat{p}_t d_t$, we immediately have $p_t^* < \hat{p}_t$. **Q.E.D.**

Interestingly, the above result shows that negative interest payments on CBDC can cause a *decrease*, instead of an increase, in the price level, inducing a transitory deflation. This result is different from that of the benchmark model and may seem surprising. The economic reasoning is as follows. Although in both economies the holdings of CBDC fall, the introduction of government debt opens up the possibility of an increase in the demand for government money.[37] Due to a portfolio allocation decision, agents hold more bonds and less capital in their portfolio. The increase in the demand for bonds dominates the fall in CBDC holdings, thereby the demand for public money increases.[38] To clear the money market, the price level now needs to fall.

My analysis makes it clear that, ultimately, it is the total demand for government money that influences the price level in the economy. The result shows that driving the interest rate into negative territory does not necessarily cause inflation if agents can make portfolio adjustments and substitute for other types of government money.[39]

[37] Recall that both CBDC and government bonds serve as money in this economy.

[38] Following the result of Corollary 3, $d^* > \hat{d}$, the increase in the real demand for government money is unambiguous.

[39] Note that the welfare effects of such a policy is ambiguous and are ultimately determined by the marginal productivity of capital. To show this, write steady-state utility (or equivalently consumption) as a function of d, and $W(d) = c(d) = f(y-d) + \frac{R^b b}{\mu} + \frac{R^m m}{\mu} - \tau$. Given that $b = d - m$ and $\tau = \frac{R^m m + R^b(d-m)}{\mu} - d$, the above function can be simplified as: $W(d) = f(y-d) + d$. Note that

It is also worth noting that the model is in the same spirit of the *quantity theory of money*. With the quantity of government money being determined exogenously, one simply needs to track the demand for government money, in order to understand the changes in the price level. For example, when the demand for money is high, due to a portfolio substitution for bonds, the value of money (i.e., $1/p$) has to increase, inducing deflation.[40]

5 Conclusion

In an era where physical cash is becoming increasingly vestigial in legal transactions, and where technological advances have opened the door to the negative interest payment on electronic money, it is perhaps time to stop treating the zero lower bound (ZLB) as a natural constraint and start considering negative interest rate policy. The appearance of privately-issued monies, such as Bitcoin and Ethereum, has also triggered a wave of interest among major central banks in exploring sovereign digital currency. To overcome the ZLB, the most direct, and arguably the easiest, approach is to pay negative interest on central bank digital currency (CBDC). This paper provides, for the first time, a formal analysis to evaluate the macroeconomic impact of negative interest rates on CBDC through the lens of a neoclassical general equilibrium model with monetary aggregates. In the benchmark model, agents have access to two types of assets: CBDC and productive capital. The demand for digital currency is motivated by a liquidity requirement (or a reserve requirement). I show that paying negative interest on CBDC induces agents to save less and consume more via a substitution

$d = y - k$, we have $W(k) = f(k) + y - k$, and $W'(k) = f'(k) - 1$. Of course, the initial old benefit from paying negative interest on CBDC, because real money balances increase. Since I have assumed non-standard preferences to ease exposition, I omit the discussion of welfare implications in this section.

[40] In the benchmark economy, however, paying negative interest on CBDC causes a fall in savings, and thus a fall in the demand for money, which induces inflationary pressures.

effect. A drop in savings in turn causes a fall in capital investment, subsequent output, and real money balances. To clear the money market, the price level has to increase.

I then extend the model to include positive-yielding government bonds. This allows me to study a non-trivial portfolio effect: when the government pays a negative interest rate on CBDC, the tax on agents' capital spending increases, inducing a decrease in capital investment and an increase in government bonds in agents' portfolio. Such a policy causes a drop in investment and output. However, there is a transitory decline in the price level due to a "flight to quality" (i.e., government bonds). That is, although there is a fall in the holdings of CBDC, the real demand for bonds increases by more due to the portfolio effect. As a result, real holdings of government money actually increase, and the price level has to decline to clear the money market. Overall, my analysis shows that there exists a trade-off in implementing negative interest payments on CBDC: such a policy can boost consumption and may induce inflation, but at the cost of causing a fall in capital investment and output.

No doubt that both negative interest rate policy and CBDC constitute major shifts in macroeconomics thinking, with important implications for payment systems, financial stability, and the whole economy. This study focuses on the macroeconomic impact of negative rates on CBDC and provides a first step into this issue. Note that the results draw from a stylized yet rigorous model that abstracts from several important aspects in real-world settings, including the absence of uncertainty and an explicit banking sector. However, it is not immediately clear how extending the model to account for these considerations would change the essential points made in this paper. Future research should devote to answering this question. Note also that negative interest rates on CBDC are certainly no panacea for all of an economy's ills, and they can have potential side-effects as summarized above. They do not substitute for other macroeconomic policies, such as fiscal policy and macroprudential policy. Understanding the interactions between negative rates and other economic policies is also interesting.

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