Institutions-Augmented Solow Model
And Club Convergence

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August 2008
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Abstract

Growth economists still face challenges and limitations to incorporate institutions into the standard growth framework. This article develops a simple augmented Solow growth model that accounts for the interactions between institutions and factor-productivity and examine the impacts of the quality of institutions on levels and growth rates of output. The institutions augmented growth model shows that differences in the quality of institutions preclude convergence and determine both the level and the growth rate of output per worker. The model also shows that poor institutions induce poverty traps. Furthermore, the income gap between rich and poor countries will increase if poor countries’ institutions do not improve relative to their rich counterpart.

JEL Classification: I32, O17, O43
Keywords: Solow Model, Institutions, Club Convergence, Poverty Traps

August 27, 2008

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1. INTRODUCTION

Recent work in the growth literature has placed institutions as one of the engines of long-run economic growth (Chong and Calderón, 2000; Acemoglu et al. 2001; Rodrik et al., 2004; Tebaldi and Elmslie, 2008). However, growth economists still face enormous challenges and limitations in terms of modeling institutions into the standard theoretical framework of economic growth. According to Sala-i-Martin (2002), “[w]e are still in the early stages when it comes to incorporating institutions into our growth theories” (p. 18). Important theoretical contributions in terms of modeling institutions within the realm of long-run economic growth include Huang and Xu (1999), Fedderke (2001), Gradstein (2002 and 2004), and Tebaldi and Elmslie (2008).

This article develops a simple institutions augmented Solow model that accounts for the impacts of the quality of institutions on levels and growth rates of output. In particular, we modify the production function and the capital accumulation equation found in the traditional Solow model allowing for interactions between institutions and factor-productivity. Despite the simplicity of the model, it theorizes a formal link for specifying an empirical model for studying the impacts of institutions on economic performance. The workable theoretical institutions-augmented Solow model also allows analyzing the role of poor/good institutions in creating club convergence and/or poverty traps.

2. THE MODEL

The model economy is a modified version of the Solow (1956) model. Goods are produced using a constant return to scale (CRS) technology in a market characterized by perfect competition. Institutions are assumed to play a central role in determining factors’ productivity and technology adoption, so output (Y) is produced using the following production function:

\[ Y = f(A(T,t), K(T,t), L(T,t)) \]

where \( L \) denotes labor, \( A \geq 1 \) is an index denoting the level of state-of-art technology, \( K \) is capital, \( T \) is an index denoting the quality of institutions, and \( t \) is time.

We assume that the representative economy is small and has access to a pool of technology generated exogenously that grows at a constant rate of \( g \). In addition, the growth rate of the labor force and the labor force participation rate are constant over time, which implies that

\[ \frac{\dot{L}}{L} = n, \]

where \( n \) is the population growth rate. Moreover, \( T \) is assumed to be increasing with the quality of institutions that accounts for the enforcement of contracts and property rights, perceptions that the judiciary system is predictable and effective, transparency of the public
administration, control of corruption and pro-market regulations (e.g., no price controls). For simplicity, $T$ is treated as a constant and normalized to range between zero and one ($0 < T \leq 1$). Therefore, $T$ is equal to one for an economy with the best relative institutions.

Equation 1 poses a major question: how do institutions affect the adoption of available technologies and the productivity of physical capital?\(^1\) It can be argued that poor institutions prevent the use of available technologies (Tebaldi and Elmslie, 2008) and limit the efficiency gains from current innovation (Matthews, 1986). Therefore, good (bad) institutions increase (decrease) the efficacy of technology and augment both labor and capital productivity. With respect to capital, it has been shown that poor institutional arrangements (translated into corruption and poor enforcement of laws and contracts) decrease the returns to investments and reduce capital accumulation (Mauro, 1995; Brunetti, Kisunko and Weder, 1997; Lambsdorff, 1999; Wei, 2000). We consider these ideas by developing two alternative specifications.\(^2\) First, we ignore the impacts of institutions on technology adoption and focus the analysis on the influences of institution on physical capital productivity. Then we develop a more general model that accounts for the impacts of institutions on technology adoption and capital productivity. In both specifications we also examine the case of institutions-driven club convergence and/or poverty traps.

### 2.1 Baseline model

This section presents a heuristic way to account for the impacts of institution on physical capital productivity. In particular, we assume that the elasticity of output with respect to capital is affected by institutions. More precisely, better institutions augment capital productivity and, therefore, influence the contribution of capital to output. Formally:

$$ Y = K^{\alpha T} (AL)^{1-\alpha T} $$

(2)

where $0 < \alpha < 1$. Defining $y = \frac{Y}{AL}$ and $k = \frac{K}{AL}$ allows writing the production function as follows:

$$ y = k^{\alpha T} $$

(3)

Combining equation (3) with the capital accumulation equation produces:

$$ \dot{k} = sk^{\alpha T} - (\delta + n + g)k $$

(4)

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\(^1\) Another relevant question is: how do institutions affect technology adoption and human capital accumulation? While important, this is not the focus of our current paper and could be addressed in future research.

\(^2\) Although restrictive, this specification generates a workable model. Other general functional specifications have created difficulties in solving the model.
Equation 4 implies that the economy will converge to a balanced growth path where
\[ \frac{\dot{y}}{y} = \frac{k}{k} = 0. \]
This allows solving equation (4) for the steady state capital stock:
\[ k^* = \frac{s}{(\delta + n + g)} \left( \frac{1}{1-\alpha T} \right) \]  
(5)
where “*” denotes steady state values. Equation 5 implies that institutions impact positively the steady state stock of capital and, consequently, the steady state level of output per effective worker. To be precise, better institutions (larger T) increases the return to capital accumulation, which boosts investments and leads to a higher steady-state effective capital \((k^*)\) and effective output per worker\((y^*)\). However, the long-term growth rate of output per worker is still determined by the rate of technological progress. Defining \(\bar{y} = \frac{y}{L}\), using the fact that \(\frac{k^*}{k} = 0\), and log-differentiating equation 3 generates:
\[ g_{\bar{y}} = \frac{\dot{\bar{y}}}{\bar{y}} = g \]
(6)
Therefore, this simple model suggests that countries are richer or poorer because of the quality of their institutions. Equation 5 implies that wealthier countries should have better institutions than poorer countries. Equation 6 entails that there should be no effect of the quality of institutions in a country’s long run growth rate. Therefore, institutions have level effects but not growth effects. The lack of growth effects found in equation 6 above is inconsistent with the existing growth literature (see details in Tebaldi and Elmslie, 2008) and is further examined below.

This modified-Solow model also formalizes the idea that poor institutions might induce poverty traps and club convergence.\(^3\) Equation 4, simply depicted in Figure 1, implies that the quality of institutions generates different steady states. Consider two economies with identical \(\delta, n, g,\) savings rate \(s\), technology \(A\), and initial stock of capital \((k_0)\), but economy P is endowed with poor institutions \((T_p)\) relatively to economy R, so that \(T_R > T_P\). The model implies that the differences in the quality of institutions will produce different steady states indicated by \(k^*_P\) and \(k^*_R\). Country P will growth until reaching \(k^*_P\) and stuck at that point. On the other hand, country R, which has identical initial conditions, but is endowed with better institutions \((T_R)\),

\(^3\) The literature also shows that non-constant savings (Galor and Ryder, 1989), learning-by-doing and spillover effects (Barro, 1995) might generate poverty traps.
will grow steadily reaching a higher steady state $k^*_R$. The lower steady state $k^*_p$ can be interpreted as a poverty trap for a country that is endowed with poor institutions. Therefore, the model suggests that poor institutions might create poverty traps and the only way to escape it is through improvements in the quality of institutions. This result is consistent with North (1990), who questioned the ability of societies to eradicate an eventual inferior institutional framework that prevents poor countries to close the income gap with rich countries.

**Figure 1: Institutions and Club Convergence**

2.2 Extended Model

The literature suggests that institutions might create difficulties (e.g. labor market imperfections - restrictive labor contracts, or union’s bargaining power, and/or government regulation) to utilize available technologies (Tebaldi and Elmslie, 2008; Baldwin and Lin, 2002, Haucap and Wey, 2004). It has also been argued that better institutional arrangements enable economic agents “to cooperate with one another more efficiently” (Matthews, 1986: 908) which ultimately boost factors’ productivity. We account for these ideas by formally extending the baseline model. In particular, we re-specify the production function as follows:

$$ Y = A^{(T-1)}K^{\alpha T}(AL)^{1-\alpha T} $$  \(7\)

Equation 7 incorporates the impacts of institutions on output in a traditional Solow production function. Since $T$ is a normalized measure of institutional quality ranging from zero to one, an economy with the relative best institutions ($T=1$) would have a production function
identical to the one used in the standard Solow model. However, not all countries will have similar quality of institutions. Therefore, the Solow model is a particular case when institutions play no role in affecting the production process. Moreover, the term $A^{(\tau-1)}$ accounts for the external effect of institutions on technology adoption and total factor productivity. It implies that a country with poor institutions will be unable to fully benefit from the potential productivity gains generated by available technologies. The model is solved by defining $y = \frac{Y}{A^\tau L}$ and $k = \frac{K}{A^\tau L}$, which allows us to write the production in terms of per effective labor:

$$y = k^{\alpha \tau} \quad (8)$$

The per effective capital accumulation equation is given by:

$$\frac{\dot{k}}{k} = sk^{\alpha \tau - 1} - (\delta + n + Tg) \quad (9)$$

This model has a well-behaved steady state solution in which $\ddot{y} = \ddot{k} = 0$. Thus:

$$k^* = \left[\frac{s}{(\delta + n + Tg)}\right]^{\frac{1}{1-\alpha \tau}} \quad (10)$$

The extended model implies that institutions impact the long-run level and growth rate of output per worker. Defining $\ddot{y} = \frac{\ddot{y}}{y}$, using the fact that $\frac{\dot{k}}{k^*} = 0$, and log-differentiating equation 7 generates:

$$g\ddot{y} = \frac{\ddot{y}}{y} = Tg \quad (11)$$

Therefore, the model implies that the growth rate of the output per worker is not only determined by technological change, but also affected by the quality of institutions. An economy may have access to state-of-art technology, but its poor institutions may hinder the adoption of available technologies and diminish the productivity of factors of production, which impede economic growth. Institutions also affect the time path level of output per worker. Figure 2 depicts the case in which an economy is growing at the rate $T_1 g$ and subsequently, at time $t_k$, an exogenous shock improves the quality of institutions from $T_1$ to $T_2$ ($T_2 > T_1$). The improvement

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4 $\lim_{T \to 1} A^{(1-\alpha)\tau} K^{\alpha \tau} L^{1-\alpha \tau} = K^\alpha (AL)^{1-\alpha}$

5 Equation 6 also satisfies the Constant Return to Scale (CRS) assumption, that is, if $c$ is a nonnegative constant, then: $cY = A^{(1-\alpha)\tau} (cK)^{\alpha \tau} (cL)^{1-\alpha \tau} = c(A^{(1-\alpha)\tau} K^{\alpha \tau} L^{1-\alpha \tau})$.

6 It is worth noticing that our definition of “effective labor” accounts not only for the state-of-art technology but also for the quality of institutions.
in the quality of institutions causes a once-for-all change in the trajectory of the level of output per worker.

**Figure 2: Institutional Quality and Time Path of GDP per worker**

![Graph showing the time path of GDP per worker](image)

The influence of institutions on output per worker originates not only from its impacts on transitional and steady state technological efficiency, but also from its impacts on capital accumulation. Institutions affect the marginal product of capital and therefore impact investments and capital accumulation. In particular, given that the ratio \( \frac{y}{k} \) is constant around the steady state, deriving equation 8 with respect to \( k \) and evaluating its derivative around the steady state produces:

\[
MP_k = \frac{\partial y^*}{\partial k^*} = \alpha T k^{\alpha r - 1} = \alpha T \frac{y^*}{k^*} > 0 \tag{12}
\]

This implies that improvement in the quality of institutions has a proportional impact on the steady state marginal product of capital. In other words, good institutions increase the returns to investments, which ultimately boost capital accumulation, leading to high level of output per worker. This result is consistent with empirical studies that find that capital accumulation is adversely affected by poor institutions (Mauro, 1995; Brunetti, Kisunko and Weder, 1997; Wei, 2000).

The extended modified-Solow model also predicts that poor institutions induce club convergence. Consider a case in which shows two economies (R and P) have identical \( \delta, n, g, \) savings rate \( (s), \) technology \( (A), \) initial stock of capital \( (k_0), \) and institutions, which implies that
that income per worker in these economies are also equal \((\bar{y}_{0,R} = \bar{y}_{0,P})\). However, at time \(t_k\), economy R experiences an institutional shock that permanently improves the quality of its institutions, so that \(T_R > T_P\). Using the fact that the long-term trajectory of the output per worker is determined based on equation 11, we can easily derive the trajectory of the relative output per capita \((\frac{\bar{y}_P}{\bar{y}_R})\) of these two economies. Figure 3 shows that the differences in the quality of institutions will generate an income gap that increases over time. The increasing income gap can be interpreted as the institutions-induced club convergence and/or poverty trap.

Figure 3: Institutions-induced Club Convergence

\[ \frac{\bar{y}_P}{\bar{y}_R} = e^{(T_P - T_R)\theta} \]

3. CONCLUSION

In this paper we modify the traditional Solow production and capital accumulation equations and allows for interactions between institutions and factor-productivity. The institutions augmented Solow growth model shows that differences in the quality of institutions preclude convergence and determine both the level and growth rate of output per worker. The model also shows that poor institutions induce poverty traps and the income gap between rich and poor countries will increase if poor countries’ institutions do not improve relative to their rich counterparts.
**BIBLIOGRAPHY**


