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Abstract

We propose a stochastic spanning approach to assess whether a traditional portfolio of stocks and bonds spans augmented portfolios including commodities, foreign exchange, and real estate. We empirically show that in all seven portfolio combinations, the augmented portfolio is not spanned by the traditional one. Our results are further confirmed by both parametric and non-parametric tests in an out-of-sample setting. Therefore, traditional investors can generally benefit in terms of higher Sharpe ratios from augmenting their portfolio with alternative asset classes. Additional analysis demonstrates that diversification benefits can be explained by the current state of the U.S. economy and stock markets.

Key words: Stochastic Dominance, Stochastic Spanning, Commodities, FX, Real Estate, Diversification.

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I. Introduction

Traditionally, U.S. investors focus mostly on stock and bond portfolios.[1] The optimal portion in either asset class and a sufficient number of individual items ensure portfolio diversification and reduce its idiosyncratic risk. Moreover, the possibility of diversifying internationally (Solnik, 1974; Carrieri, Errunza, and Hogan, 2007; Berger, Pukthuanthong, and Yang, 2011; Christoffersen, Errunza, Jacobs, and Langlois, 2012; Liu, 2016) as well as the financialization of alternative asset classes such as commodities and real estate (Tang and Xiong, 2012; Basak and Pavlova, 2016) allow investors to benefit from different return-risk characteristics, and especially from the low correlation of the alternative assets with U.S. stocks or bonds. Along these lines, adding alternative asset classes may thus increase the portfolio's expected return, decrease its risk, and hedge inflation (Adams, Füss, and Kaiser, 2008).

Early research suggests including commodities, real estate, or foreign currencies in the traditional portfolio universe (Friedman, 1971; Bodie and Rosansky, 1980; Eun and Resnick, 1988; Ankrim and Hensel, 1993) and often refers to the aspect of the low or even negative correlation (Gorton and Rouwenhorst, 2006). However, more recent literature is ambiguous regarding potential diversification benefits across asset classes. Daskalaki and Skiadopoulos (2011); Bessler and Wolff (2015); Cotter, Eyiah-Donkor, and Potì (2017) show, for example, that identified in-sample potential benefits do not hold in out-of-sample tests.

[1] Boubaker, Gounopoulos, Nguyen, and Paltalidis (2018) examine, for example, the asset allocations for 151 state pension funds in the United States from January 1998 to December 2013 and show that the shares of stocks and bonds in their portfolios exceed 80% and only less than 20% are allocated to cash, real estate, and alternative investments. This portfolio composition also remains intact over subperiods such as 1998-2001, 2001-2006, 2007-2008, and 2009-2013.

Gao and Nardari (2018)'s findings suggest that only forward-looking strategies allow gains from adding commodities to a stock-bond portfolio. One of the common explanations for the reduced potential of commodity diversification benefits is their increased financialization since 2004, which transformed the formerly negative/low correlations to positive moderate correlations (Adams and Glück, 2015; Bhardwaj, Gorton, and Rouwenhorst, 2015). More importantly, despite significant insights and lessons in terms of asset class diversification, conclusions of related past studies are conditional on the choice of asset pricing models as well as their assumptions about return distributions and investor's risk preferences.

In this study, we extend the existing literature on asset class diversification and employ the stochastic spanning approach to test whether commodities, currencies, and real estate should be included in stock-bond portfolios to improve the investment universe of a risk averse investor. The concept of stochastic spanning, introduced by Arvanitis, Hallam, Post, and Topaloglou (2019), can be perceived as a model-free alternative to mean-variance (MV) spanning (Huberman and Kandel, 1987) and is useful for portfolio decisions when investment constraints and distributional characteristics of asset class returns are relaxed. It goes beyond the standard stochastic dominance (SD) commonly used for pairwise performance comparison of two particular portfolios and allows, in our case, the examination of whether optimal portfolios augmented by a set of new asset classes outperform a portfolio constructed from bonds and stocks while considering all possible combinations of allocation weights. Many studies on diversification benefits either test only one additional asset class (mostly commodities, see among others, Satyanarayan and Varangis, 1996; Belousova and Dorfleitner, 2012; Bessler and Wolff, 2015) or use funds or indices instead of individual securities of an asset class (e.g., Daskalaki, Skiadopoulos, and Topaloglou, 2017; Gao and Nardari, 2018). Our study considers 14 individual commodity futures and 18 different currency pairs to represent the commodity and foreign exchange (FX) asset classes.

We also conduct both in-sample and out-of-sample tests to assess the diversification benefits of alternative asset classes. Using individual commodity futures, FX rates, and a U.S. real estate index, we show that U.S. investors benefit and can improve the risk-adjusted performance, when augmenting their traditional portfolios. In particular, we test seven cases of augmenting stocks and bonds portfolios with additional asset classes both individually and jointly. The in-sample analysis shows striking evidence that traditional portfolios are not able to span any of the seven expanded portfolios. Additionally, parametric and non-parametric tests confirm the in-sample results in an out-of-sample setting. In an attempt to investigate the factors driving the return differences between augmented and traditional portfolios, we document that although they vary across seven portfolio designs, the key drivers of diversification benefits from adding new asset classes are the Leading Index for the United States (LIUS) (all cases), MSCI world market returns (5 cases), implied volatility (VIX) in the US equity market (5 cases), 10-year Treasury bond returns (4 cases), Global Real Economic Activity (GREA, 3 cases), 3-month Treasury bill rate (3 cases), and Global Economic Policy Uncertainty (GEPU, 2 cases). When all regressors are included, the most important driving force is the return on the world stock market, followed by the LIUS, GREA, T-bill rate, and 10-year T-bond returns. As an illustration, the results with respect to the MSCI world market index and LUIS indicate that benefits of asset class diversification are reduced when stock markets are booming and the economy is expected to grow.

Overall, this study provides three contributions to the related literature. First, unlike previous works that mainly address the issue of asset class diversification from asset pricing perspectives, market co-movement, and MV framework as discussed in Arouri, Nguyen, and Pukthuanthong (2014), our proposed approach based on stochastic spanning constructs optimal portfolios (with and without alternative asset classes) in a non-parametric way and compare their performance. Indeed, whereas the majority of previous studies in the literature use standard MV criteria to construct optimal portfolios[2], its use is questionable for portfolio selection if investment returns are not normally distributed or if the utility functions are not quadratic. Most of the assets in our sample depict such non-normality with high volatility, skewness, and kurtosis. Moreover, it is commonly known that the MV criterion is consistent with expected utility for elliptical distributions such as the normal distribution (Chamberlain, 1983; Owen and Rabinovitch, 1983; Berk, 1997), but it has limited economic meaning when the probability distribution cannot be characterized completely by its location and scale.[3] Second, the in- and out-of-sample statistical finding that the optimal stock-bond portfolio is always spanned by the augmented portfolio with either commodity futures, currencies, or real estate (and any combination of them) shows that the augmented portfolio is a good option for risk-averse investors to help diversify their portfolio risks from a long-term perspective. This result is important as the diversification benefits across asset classes have not been statistically proved, particularly when the number of assets considered is high. Third, we propose the first attempt to identify the finance and macroeconomic factors that drive the superior performance of the augmented portfolios. In the related literature, studies such as Pukthuanthong, Roll, and Subrahmanyam (2019) have searched

[2]Exceptions are Kroencke, Schindler, and Schrimpf (2014); Daskalaki et al. (2017); Henriksen (2018); Henriksen, Pichler, Westgaard, and Frydenberg (2019).

[3]For example, the monthly returns of many stocks exhibit positive skewness. The phenomena of skewness preferences and loss aversion have attracted much attention from financial economists (Harvey and Siddique, 2000).

for the priced risk factors that determine portfolio returns in the cross-section, but little is known about factors underlying the asset class diversification benefits.

The remainder of the paper is organized as follows. Section II briefly reviews the literature on diversification benefits from investing in commodity, currency, and real estate markets. Section III presents the concept of stochastic spanning and introduces the corresponding test. Section IV discusses the empirical findings. Section V concludes the paper.

II. Literature Review on Diversification Benefits

Regarding the inclusion of additional asset classes in a traditional portfolio of stocks and bonds, the literature covers two main advantages: (i) diversification benefits, that is, an investor yields a higher premium for the same level of risk or can reduce her risk exposure without sacrificing any return; and (ii) inflation hedge capabilities, meaning that the appended asset class reduces the risk of unanticipated inflation due to its positive correlation with consumer prices. Adams et al. (2008) note that the inflation hedging capability also implies the dynamic behavior of commodity futures returns through business cycles. Thus, diversification benefits are expected to vary through time. In what follows, our review focuses on the diversification benefits of commodity, currency, and real estate asset classes.

Commodities

Commodities and especially commodity futures have long been seen as a perfect diversification tool because of their low correlation with stocks and bonds (Bodie and Rosansky, 1980; Anson, 1998). Along this line, Gorton and Rouwenhorst (2006) conclude that the diversification benefits from commodities are partially because of their hedging ability against inflation and the counter-cyclical returns compared with stocks and bonds (see also Bjornson and Carter, 1997; Scherer and He, 2008). The inflation-hedge potential is even used for tactical asset allocation (Jensen, Johnson, and Mercer, 2000, 2002).

Several studies have examined individual commodity futures. Galvani and Plourde (2010) show that energy commodities do not help to diversify energy stock portfolios. Geman and Kharoubi (2008) provide evidence that when the right "time-to-maturity" is chosen, WTI oil futures provide diversification benefits to stock-bond portfolios during bull and bear markets. Belousova and Dorfleitner (2012) find evidence to support the valuable diversification gains of several individual commodities, despite their diversification contributions being dissimilar. You and Daigler (2013) find diversification benefits for individual commodities for a traditional stock-bond portfolio within an optimal Markowitz universe. More recently, Bessler and Wolff (2015) analyze different commodity groups and especially find that commodity indices, metals, and energy commodities offer diversification benefits, whereas agricultural commodities do not. In the meanwhile, these authors note that out-of-sample Sharpe ratios are much smaller than in-sample ones. By contrast, limited diversification benefits of alternative asset classes are provided in, among others, Cheung and Miu (2010) and Huang and Zhong (2013). For example, Cheung and Miu (2010) show that diversification benefits of commodity futures are only present during bull markets. If the equity market is bearish, no diversification benefits from commodities are found for the United States and Canada.

When considering a global portfolio, Satyanarayan and Varangis (1996) conclude that commodities move the efficient frontier upwards and thus provide higher returns for a given risk level. Daskalaki and Skiadopoulos (2011)'s study investigates the potential of commodities for MV and non-MV investors in an in- and out-of-sample setting. Whereas they show that in-sample, non-MV investors can profit from adding commodities to their portfolios, the finding does not hold for the out-of-sample analysis.

Yan and Garcia (2017) and Platanakis, Sakkas, and Sutcliffe (2019) do not find that commodities improve Sharpe ratios in- or out-of-sample. Their findings are in contrast to Daskalaki et al. (2017)'s, which show that commodities can add value to investors' portfolios. In particular, using the SD efficiency approach, the authors show that the results are independent from investors' utility functions. However, both studies agree that using more sophisticated commodity products (e.g. momentum driven indices) results in even better performance.

These hitherto mixed results are challenged by You and Daigler (2010) and Gao and Nardari (2018), who show that exploiting the predictability of higher individual and co-moments advocates diversification benefits from commodities for stock-bond portfolios.

Foreign exchange

Regarding the diversification benefits of foreign currencies, the literature mostly focuses on hedging the currency risk (e.g., Solnik, 1974; Eun and Resnick, 1988; Glen and Jorion, 1993; de Roon, Nijman, and Werker, 2003; Campbell, Serfaty-de Medeiros, and Viceira, 2010). For instance, an early study by Solnik (1974) examines the diversification effect of adding international stocks to a U.S. stock portfolio. Typical investors are assumed to hedge the exchange risk of their international position with a forward exchange contract. Solnik (1974) points out that a portfolio unprotected from exchange risk implicitly speculates on local currencies, but both protected and unprotected portfolios result in lower portfolio risk than a pure domestic portfolio. Eun and Resnick (1988) confirm these findings in an out-of-sample setting for different allocation strategies. Glen and Jorion (1993) investigate the benefits of currency hedging in a similar setting, but also include bonds and consider possible short-selling restrictions. Their results indicate that international diversification and hedging of exchange risk improve the Sharpe ratios.

Campbell et al. (2010) examine whether an investor can manage the risk of a portfolio of domestic stocks or bonds while taking positions in foreign currency and show that long positions in the US dollar, the Euro, and the Swiss franc as well as short positions in the British pound, the Japanese Yen and the Canadian dollar are optimal currency exposures. In particular, equity risk can be hedged effectively with a long position in the US-Canadian exchange rate. Kroencke et al. (2014) document that global investors can obtain large FX diversification benefits for their stocks and bond portfolios by employing different FX style-based investment strategies. Moreover, the inclusion of a composite FX strategy was found to increase the out-of-sample Sharpe ratio by 64%. Using a different approach based on spanning tests, de Roon et al. (2003) show that dynamic exchange risk hedging improves an international diversified stock portfolio, both for MV and power utility investors. Cotter et al. (2017) find evidence that currency futures do not improve the performance of a portfolio of stocks, bonds, and T-bills, using a number of portfolio investment strategies.

Real estate

Early studies in the literature on real estate investments and their diversification benefits for stock and bond portfolios argue that real estate investment can be used as an inflation hedge (Fogler, 1984; Ibbotson and Siegel, 1984) or find mixed results regarding the risk-reduction potential of REITs (Kuhle, 1987). By contrast, Zerbst and Cambon (1984) and Lee (2005) advocate that real estate investments have diversification potential for traditional portfolios because of their negative or low correlation with stocks and bonds. More recent studies such as Hung, Lee, and Liu (2008a); Huang and Zhong (2013), and Lizieri (2013) investigate the time-varying nature of diversification with real estate investments and find that it is beneficial, but only in times when the equity market is not in distress. Interestingly, Georgiev, Gupta, and Kunkel (2003)'s study concludes that direct real estate investments offer diversification, but REITs do not.

Comparison of alternative assets

Some studies have investigated the diversification benefits of adding not only one new asset class, but several asset classes with respect to a traditional stock-bond portfolio. Irwin and Landa (1987) and Ankrim and Hensel (1993) compare the (dis-)advantages of commodity futures and real estate investments and argue that these asset classes offer similar benefits for traditional investors. Using spanning tests, Cotter et al. (2017) provide evidence that commodities as well as FX offer in-sample diversification benefits for traditional portfolios of stocks, bonds, and T-bills. This effect is, however, weakened in their second period (2000–2014) for commodities, which might be due to the increased financialization of commodity markets (Tang and Xiong, 2012; Adams and Glück, 2015), and does not hold in the out-of-sample testing. An alternative explanation is presented in (Huang and Zhong, 2013) where the authors show that REITs and commodities are not spanned by stocks and bonds before the financial crisis. It is worth noting that the aforementioned studies are challenged by Platanakis et al. (2019)'s study, which finds no potential benefit from adding commodities and real estate into stock-bond portfolios.

Looking at the literature of the past ten years, we find 12 studies that confirm in-sample diversification benefits across asset classes. However, 14 studies in the same period find mixed results or conclude that there are no benefits for investors from augmenting their portfolios. Turning to the out-of-sample evidence, only four studies confirm diversification benefits, but nine do not, or only partially with exceptions. A table with an overview is provided in the Online Appendix.

III. The Stochastic Dominance Approach

In contrast to the MV dominance criterion, which only accounts for the first and second moment of the asset's return distribution, SD is a model-free alternative that takes into account all moments and does not rely on the assumption of any particular distribution. SD compares random variables, such as asset returns, in the sense of stochastic orderings expressing the common preferences of rational decision-makers and is used in many applications in economics and finance (Scaillet and Topaloglou, 2010). Being non-parametric, second order stochastic dominance (SSD) ranks investments based on conditions that characterize decision-making under uncertainty regarding the class of utilities that exhibit non-satiation and risk aversion. SSD is additionally represented by sets of conditions in the form of lower partial moment inequalities between the distributions compared, which essentially represent the risk properties relevant to the aforementioned class of utilities. These conditions are defined by mild non-parametric restrictions on the distributions involved. The non-parametric nature of SSD makes it particularly appealing for asset classes and investment strategies that involve securities with asymmetric risk profiles, like commodities and FX.

SD is traditionally applied for comparing a pair of given prospects, for example, two income distributions or two medical treatments. Davidson and Duclos (2000); Barrett and Donald (2003), and Linton, Maasoumi, and Whang (2005), among others, develop statistical tests for such pairwise comparisons. However, investors are not necessarily limited to only two securities.

A more general, multivariate problem is that of testing whether a given prospect is stochastically efficient relative to all mixtures of a discrete set of alternatives (Bawa, Bodurtha Jr., Rao, and Suri, 1985; Shalit and Yitzhaki, 1994; Post, 2003; Kuosmanen, 2004; Roman, Darby-Dowman, and Mitra, 2006). This problem arises naturally in applications of portfolio theory and asset pricing theory, where the mixtures are portfolios of financial securities. Post and Versijp (2007); Scaillet and Topaloglou (2010); Linton, Post, and Whang (2014), and Post and Potì (2017) address this problem using various statistical methods. Their stochastic efficiency tests can be seen as model-free alternatives to test for MV efficiency, such as the Shanken (1985, 1986) test (without a riskless asset) and the Gibbons, Ross, and Shanken (1989) test (with a riskless asset).

In a similar manner, the concept of stochastic spanning, introduced by Arvanitis et al. (2019), can be perceived as a model-free alternative to MV spanning (Huberman and Kandel, 1987; Gibbons et al., 1989). Spanning is defined as the situation where assets added to a set of investment opportunities do not improve the situation of the investor. In this study, we test whether augmenting a stock-bond portfolio by commodities, FX, and/or real estate is spanned by the original stock-bond portfolio. In particular, if we (cannot) reject spanning, the additional asset does (not) improve the investment opportunity set of any risk-averse investor. To do so in an empirical manner, we employ the Arvanitis et al. (2019) stochastic spanning test that we briefly describe in the following paragraphs.

A. Preliminaries and Definitions

We work with a portfolio space defined as the set of positive convex combinations of N assets and represented by the $\{\boldsymbol{\lambda} \in \mathbb{R}_+^N : \boldsymbol{\lambda}'\mathbf{1}_N = 1\}$. The returns of the assets form the random vector $X := (x_1, \ldots, x_N)$. We work under the assumption that its support is bounded by $\mathcal{X}^N := [\underline{x}, \overline{x}]^N$, $-\infty < \underline{x} < \overline{x} < +\infty$, in accordance with realistic investment frameworks (see Arvanitis et al., 2019).

F denotes the continuous cumulative distribution function (CDF) of X and $F(y, \boldsymbol{\lambda}) := \int 1(X^{\mathrm{T}}\boldsymbol{\lambda} \leq y)dF(X)$ the marginal CDF for portfolio $\boldsymbol{\lambda}$. Consider the CDF integrals $L(x, \boldsymbol{\lambda}; F) := \int_{-\infty}^{x} F(y, \boldsymbol{\lambda})dy$. Because of the integration by parts formula for Lebesgue-Stieljes integrals $L(x, \boldsymbol{\lambda}; F)$ equals the first-order lower-partial moment (LPM), or expected shortfall $\int_{-\infty}^{x}(x - y)dF(y, \boldsymbol{\lambda})$, for each return threshold $x \in \mathcal{X}$ (see Bawa, 1975). Let $D(x, \boldsymbol{\lambda}, \boldsymbol{\kappa}; F) := L(x, \boldsymbol{\lambda}; F) - L(x, \boldsymbol{\kappa}; F)$, the LPM spread between portfolios $\boldsymbol{\lambda}$ and $\boldsymbol{\kappa}$. Then, $\boldsymbol{\lambda}$ stochastically dominates $\boldsymbol{\kappa}$ by SSD, or $\boldsymbol{\lambda} \succeq_F \boldsymbol{\kappa}$, iff $D(x, \boldsymbol{\lambda}, \boldsymbol{\kappa}; F) \leq 0$, $\forall x \in \mathcal{X}$. Using normalizations, and integral representations of convex functions on bounded intervals that are also continuous at endpoints, it is possible to show that $\boldsymbol{\lambda} \succeq_F \boldsymbol{\kappa}$ iff $\boldsymbol{\lambda}$ achieves a higher expected utility than $\boldsymbol{\kappa}$ for every increasing and concave utility function (see for example Proposition 2 of Arvanitis et al., 2019).

Here, we focus on the changes followed by augmenting a traditional stock-bond portfolio with one or more alternative asset classes. Thus, consider two subsets of the general portfolio space, $\mathrm{K} \subset \Lambda$, which are further assumed to be closed and simplicial, to facilitate among others the invocation of convex optimization properties. In our framework, K is constructed as the convex hull of the traditional assets, whereas Λ is also the convex hull of the aforementioned set of traditional assets augmented with a new asset class (or combination of asset classes).

B. Stochastic Spanning

The concept of stochastic spanning compares the two distinct portfolio sets via SSD. Specifically:

Definition 1. (Stochastic Spanning): K spans Λ by SSD iff for every portfolio $\boldsymbol{\lambda} \in \Lambda$ that includes additional asset classes, there exists a portfolio $\boldsymbol{\kappa} \in \mathrm{K}$ of the traditional assets that dominates it by SSD:$\forall \boldsymbol{\lambda} \in \Lambda, \exists \boldsymbol{\kappa} \in \mathrm{K} : \forall x \in \mathcal{X} : \ D(x, \boldsymbol{\kappa}, \boldsymbol{\lambda}; F) \leq 0$.

Using the continuity properties of $D(\cdot, \cdot, \cdot; F)$ and the compactness of the "parameter sets" Λ, K, $\mathcal{X}$ it is easy to characterize spanning by the following scalar-valued function of F:

(1) $$\eta(F) := \sup_{\Lambda} \inf_{K} \sup_{\mathcal{X}} D(x, \boldsymbol{\kappa}, \boldsymbol{\lambda}; F);$$

spanning occurs iff $\eta(F) = 0$, so long as some $\boldsymbol{\lambda} \in \Lambda$ that is not stochastically dominated by any portfolio $\boldsymbol{\kappa} \in K$ by SSD exists, that is, no spanning occurs iff $\eta(F) > 0$.

1. Hypothesis structure, test statistic and, critical values

In empirical applications, F is latent so $\eta(F)$ is unknown, while the analyst has access to a time series sample of realized returns $(X_t)_{t=1}^{T}$, $X_t \in \mathcal{X}$, t $= 1, ..., $ T, for the traditional assets. Given the previous statement, the hypothesis structure of a statistical test for spanning is

$$\mathbf{H_0} : \eta(F) = 0 \text{ vs. } \mathbf{H_1} : \eta(F) > 0.$$

The null hypothesis $\mathbf{H_0}$ is that the traditional set spans the augmented set with additional asset classes, whereas the alternative hypothesis $\mathbf{H_1}$ is that there are some portfolios augmented with additional asset classes that are not spanned by the traditional assets.

Under an assumption framework involving stationarity and mixing for the traditional asset return process, a function scaled by a $\sqrt{T}$ empirical analogue of $\eta(F)$ is used as a K-S type test statistic for the null hypothesis:

$$\eta_T := \sqrt{T} \sup_{\Lambda} \inf_{\mathrm{K}} \sup_{\mathcal{X}} D(x, \boldsymbol{\kappa}, \boldsymbol{\lambda}; F_T),$$

where $F_T(x) := T^{-1} \sum_{t=1}^{T} 1(X_t \leq x)$ denotes the function associated with the sample empirical CDF (ECDF).

The asymptotic decision rule is to reject $\mathbf{H}_0$ in favor of $\mathbf{H}_1$ iff $\eta_T > q(\eta_\infty, 1-\alpha)$ is the $(1-\alpha)$ quantile of the distribution of η_∞ for any significance level $\alpha \in]0, 1[$. Because the distribution of $q(\eta_\infty, 1-\alpha)$ depends on the underlying distribution, we use the subsampling procedure of Arvanitis et al. (2019) to approximate it by feasible decision rules. Specifically, given the choice of the subsampling rate $1 \leq b_T < T$, this generates the maximally overlapping subsamples $(X_s)_{s=t}^{t+b_T-1}$, $t = 1, \cdots, T - b_T + 1$, evaluates the test statistic on each subsample, thereby obtaining $\eta_{b_T;T,t}$ for $t = 1, \cdots, T - b_T + 1$, hence resulting in the evaluation of $q_{T,b_T}(1-\alpha)$, the $(1-\alpha)$ quantile of the empirical distribution of $\eta_{b_T;T,t}$ across the subsamples. Using the deduction above, the modified decision rule is to reject $\mathbf{H}_0$ in favor of $\mathbf{H}_1$ iff $\eta_T > q_{T,b_T}(1-\alpha)$. This results in an asymptotically exact and consistent test as long as the significance level α is appropriately chosen (in our empirical application it suffices that $\alpha < 0.25$ for $N = 4$) and the subsampling rate b_T diverges to infinity at a slower rate than T.

We also employ the proposed bias correction by Arvanitis et al. (2019) for the quantile estimates $q_{T,b_T}(1-\alpha)$ to mitigate their sensitivity to the choice of b_T in finite samples of realistic time series and cross sectional dimensions. They propose choosing $b_T = \lfloor T^c \rfloor$, with c ranging from 0.6 to 0.9, then estimating a regression of the estimated critical values and the subsample length ($\lfloor T^c \rfloor$) for several values of c in the aforementioned range, and finally using the estimated regression line evaluated at T to obtain the bias corrected critical value. They argue that this procedure does not affect the limit theory, and they provide evidence that this method is more efficient and powerful in small samples.

2. Computational strategy for spanning

The utility class interpretation of Arvanitis et al. (2019)'s Proposition 2 implies that η can also be represented in terms of expected utility as:

$$\eta(F) := \sup_{\boldsymbol{\lambda}\in\Lambda; u\in\mathcal{U}} \inf_{\boldsymbol{\kappa}\in\mathrm{K}} \mathbb{E}_F \left[u\left(X^{\mathrm{T}}\boldsymbol{\lambda}\right) - u\left(X^{\mathrm{T}}\boldsymbol{\kappa}\right)\right]; \tag{2}$$

$$\mathcal{U} := \left\{u \in \mathcal{C}^0 : u(y) = \int_{\underline{x}}^{\overline{x}} v(x) r(y;x) dx \; v \in \mathcal{V}\right\}; \tag{3}$$

$$\mathcal{V} := \left\{v : \mathcal{X} \to \mathbb{R}_+ : \int_{\mathcal{X}} v(x) = 1\right\} \tag{4}$$

$$r(y;x) := (y-x)1(y \leq x), \; (x,y) \in \mathcal{X}^2. \tag{5}$$

$\mathcal{U}$ is comprised of normalized, increasing, and concave utility functions that are constructed as convex mixtures of elementary Russell and Seo (1989) ramp functions $r(y;x)$, $x \in \mathcal{X}$. This implies that K spans Λ, iff for any $\boldsymbol{\lambda} \in \Lambda$ there exists some $\boldsymbol{\kappa} \in \mathrm{K}$, weakly preferred to the former by every utility in $\mathcal{U}$. Equivalently, spanning occurs iff no risk averter in $\mathcal{U}$ loses expected utility from the excision of Λ-K from Λ. This representation can be used for the numerical implementation of the associated testing procedure.

The test statistic can be obviously expressed as:

$$\eta_T := \sqrt{T} \sup_{u\in\mathcal{U}} \left(\sup_{\boldsymbol{\lambda}\in\Lambda} \mathbb{E}_{F_T}\left[u\left(X^{\mathrm{T}}\boldsymbol{\lambda}\right)\right] - \sup_{\boldsymbol{\kappa}\in\mathrm{K}} \mathbb{E}_{F_T}\left[u\left(X^{\mathrm{T}}\boldsymbol{\kappa}\right)\right]\right). \tag{6}$$

The computational complexity of evaluating η_T stems from the functional complexity of the set $\mathcal{U}$. However, because of the properties of the admissible utilities, Arvanitis et al. (2019) approximate every element of $\mathcal{U}$ with arbitrary prescribed accuracy using a finite set of increasing and concave piecewise-linear functions in the following way:

Let N_1, N_2 denote integers greater than or equal to 2. First $\mathcal{X}$ is partitioned into N_1 equally spaced values as $\underline{x} = z_1 < \cdots < z_{N_1} = \overline{x}$, where $z_n := \underline{x} + \frac{n-1}{N_1-1}(\overline{x} - \underline{x})$, $n = 1, \cdots, N_1$. Second, $[0,1]$ is partitioned as $0 < \frac{1}{N_2-1} < \cdots < \frac{N_2-2}{N_2-1} < 1$. Using these partitions, consider:

$$\underline{\eta_T} := \sqrt{T} \sup_{u \in \underline{\mathcal{U}}} \left(\sup_{\boldsymbol{\lambda} \in \Lambda} \mathbb{E}_{F_T} \left[u \left(X^{\mathrm{T}} \boldsymbol{\lambda} \right) \right] - \sup_{\boldsymbol{\kappa} \in \mathrm{K}} \mathbb{E}_{F_T} \left[u \left(X^{\mathrm{T}} \boldsymbol{\kappa} \right) \right] \right); \tag{7}$$

$$\underline{\mathcal{U}} := \left\{ u \in \mathcal{C}^0 : u(y) = \sum_{n=1}^{N_1} v_n r(y; z_n) \; v \in V \right\}; \tag{8}$$

$$V := \left\{ v \in \left\{ 0, \frac{1}{N_2 - 1}, \cdots, \frac{N_2 - 2}{N_2 - 1}, 1 \right\}^{N_1} : \sum_{n=1}^{N_1} v_n = 1 \right\}. \tag{9}$$

By construction, every $u \in \underline{\mathcal{U}}$ consists of at most N_2 linear line segments with endpoints at N_1 possible outcome levels. Furthermore $\underline{\mathcal{U}} \subset \mathcal{U}$, which is finite as it has $N_3 := \frac{1}{(N_1-1)!} \prod_{i=1}^{N_1-1} (N_2 + i - 1)$ elements and $\underline{\eta_T}$ approximates η_T from below as the partitioning scheme is refined ($N_1, N_2 \to \infty$). Then for every $u \in \underline{\mathcal{U}}$, the two embedded maximization problems in (7) can be solved using LP: consider

$$c_{0,n} := \sum_{m=n}^{N_1} \left(c_{1,m+1} - c_{1,m} \right) z_m; \tag{10}$$

$$c_{1,n} := \sum_{m=n}^{N_1} w_m; \tag{11}$$

$$\mathcal{N} := \{ n = 1, \cdots, N_1 : v_n > 0 \} \bigcup \{ N_1 \}. \tag{12}$$

Then for any given $u \in \underline{\mathcal{U}}$, $\sup_{\boldsymbol{\lambda} \in \Lambda} \mathbb{E}_{F_T} \left[u \left(X^{\mathrm{T}} \boldsymbol{\lambda} \right) \right]$ is the optimal value of the objective function of the following LP problem in canonical form:

$$\max T^{-1} \sum_{t=1}^{T} y_t \tag{13}$$

$$\text{s.t. } y_t - c_{1,n} X_t^{\mathrm{T}} \boldsymbol{\lambda} \leq c_{0,n},\ t = 1, \cdots, T; n \in \mathcal{N};$$

$$\sum_{i=1}^{M} \lambda_i = 1;$$

$$\lambda_i \geq 0,\ i = 1, \cdots, M;$$

$$y_t \text{ free},\ t = 1, \cdots, T.$$

The LP problem always has a feasible solution and has $\mathcal{O}(T+N)$ variables and constraints, making it manageable for typical data dimensions. The empirical application is based on the entire available history of monthly investment returns to a standard set of traditional assets ($N = 48$, $T = 228$), and uses $N_1 = 10$ and $N_2 = 5$. This gives $N_3 = \frac{1}{9!} \prod_{i=1}^{9} (4+i) = 715$ distinct utility functions and $2N_3 = 1,430$ small LP problems, which is perfectly manageable with modern-day computer hardware and solver software.[4]

IV. Empirical Investigation

A. Data

Our empirical assessment includes the following asset classes: U.S. Treasury notes, corporate bonds, commodities, stocks (S&P 500 Equity Sector Indexes), FX, and equity in

[4] The total run time of all computations for our application amounts to several working days on a standard desktop PC with a 2.93 GHz quad-core Intel i7 processor, 16GB of RAM and using MATLAB and GAMS with the Gurobi solver.

real estate markets. Except for the U.S. Treasury notes, all data is retrieved from Thomson Reuters DataStream. The U.S. Treasury note data are collected from Bloomberg.

Our dataset covers the period from 1 January 1990 to 31 December 2018. We collected the data for monthly and daily frequencies. Table 1 reports the summary statistics of the employed assets' returns over this period. In addition, we provide correlation plots for the total and the two sub-samples (1990-2000 and 2000-2018) in the Online Appendix.

We design the numerical experiments to use an increasing number of asset classes to test the hypothesis that the traditional (benchmark) asset class portfolio spans the augmented portfolios. We start with an additional asset class portfolio. Next, we use combinations of asset classes, and finally we include all asset classes. We test the following cases:

- Case 1: traditional vs augmented with commodities
- Case 2: traditional vs augmented with FX
- Case 3: traditional vs augmented with Real Estate index
- Case 4: traditional vs augmented with commodities and FX
- Case 5: traditional vs augmented with commodities and Real Estate
- Case 6: traditional vs augmented with FX and real estate
- Case 7: traditional vs augmented with commodities, FX and, real estate

We test whether the traditional asset class portfolios span the portfolios augmented with any of the additional asset classes in an in-sample analysis. We also compare the performance of the traditional asset classes with the augmented in out-of-sample (dynamic) tests, using a rolling window analysis.

[Table 1 about here]

B. In-sample Analysis

In this section, we test in-sample the null hypothesis that the traditional set of stocks and bonds spans the portfolios augmented with the additional asset classes. We get the subsampling distribution of the test statistic for subsample size $b_T \in [T^{0.6}, T^{0.7}, T^{0.8}, T^{0.9}]$. Using ordinary least squares regression on the empirical quantiles $q_{T,b_T}(1-\alpha)$ and for significance level $\alpha = 0.05$, we get the estimate q_T^{BC} for the critical value. We reject spanning if the test statistic $\eta_T^\star$ is higher than the regression estimate q_T^{BC}.

Table 2 reports the test statistics $\eta_T^\star$ as well as the regression estimates q_T^{BC} when we test for spanning. As can be seen in all cases, the optimal stock-bonds portfolio cannot span its optimal augmented counterparts. Thus, our in-sample spanning tests indicate that adding commodities, FX, and real estate to a stock-bonds portfolio results in increased performance and some risk averse investors could benefit from the augmentation.

[Table 2 about here]

C. Out-of-sample Analysis

This section examines whether the diversification benefits found in-sample also hold in an out-of-sample setting (Daskalaki and Skiadopoulos, 2011; Bessler and Wolff, 2015).

For each case outlined above, we optimize portfolios from two different investment universes: one that includes the stocks and bonds, and an augmented one with the additional asset classes. The out-of-sample analysis spans the period from 1 January 1990 to 31 December 2018 (348 months). We use the first 120 months as the first training set. Thus, our out-of-sample period is from January 2000 to December 2018 (228 months). After constructing the SSD optimal portfolios for the first month (January 2000), we roll the training window (120 months) one month ahead and solve the stochastic spanning models again to get the new optimal portfolios based on the new training set (February 1990 to January 2000). The procedure is then repeated each month, to derive realized returns for each optimal portfolio for each month, which are depicted in Figure 1 and analyzed in the following section.

[Figure 1 about here]

D. Out-of-sample Performance Assessment

In this section, we test whether our findings of improved augmented portfolio performance from the in-sample tests hold for the out-of-sample analysis. In the following, we compare the results using non-parametric and parametric tests.

1. Non-parametric tests

There are a number of pairwise SD tests presented in the literature; see, for example Barrett and Donald (2003); Davidson and Duclos (2000); Linton et al. (2005); Davidson (2009). Scaillet and Topaloglou (2010) develop SD efficiency tests, which could be used for pairwise comparisons as well. Here we prefer to use the Scaillet and Topaloglou (2010) SD test, mainly for two reasons. First, the test allows for correlated samples. Second, it allows for time-dependent data, and it does not assume i.i.d. returns. To our knowledge, there is no other SD test that explicitly accounts for such time-series effects.

The general hypotheses for testing the SD dominance of the optimal traditional τ over the optimal portfolio augmented with an additional asset class λ, can be written compactly as:

$$
\begin{aligned}
H_0 &: J(z, \boldsymbol{\tau}; F) \leq J(z, \boldsymbol{\lambda}; F) \quad \text{for all } z \in \mathbb{R} \\
H_1 &: J(z, \boldsymbol{\tau}; F) > J(z, \boldsymbol{\lambda}; F) \quad \text{for some } z \in \mathbb{R}.
\end{aligned}
$$

The empirical counterpart is simply obtained by integrating with respect to the empirical distribution $\hat{F}$ of F, which yields:

$$
\mathcal{J}(z, \boldsymbol{\lambda}; \hat{F}) = \frac{1}{T} \sum_{t=1}^{T} (z - \boldsymbol{\lambda}' \boldsymbol{Y}_t)_+ .
$$

We consider the weighted Kolmogorov-Smirnov type test statistic

$$
\hat{S} := \sqrt{T} \frac{1}{T} \sup_{z,} \left[J(z, \boldsymbol{\tau}; \hat{F}) - J(z, \boldsymbol{\lambda}; \hat{F}) \right],
$$

and a test based on the decision rule:

$$
\text{“ reject} \quad H_0 \quad \text{if} \quad \hat{S} > c\text{”},
$$

where c is some critical value (Scaillet and Topaloglou, 2010).

To make the result operational, we need to find an appropriate critical value c. Because the distribution of the test statistic depends on the underlying distribution, this is not an easy task, and we decide hereafter to rely on a block bootstrap method to simulate p-values.

Block bootstrap methods extend the nonparametric i.i.d. bootstrap to a time series context (see Barrett and Donald, 2003; Abadie, 2002, for use of the non-parametric i.i.d. bootstrap in SD tests). They are based on "blocking" arguments, in which data are divided into blocks and these blocks, rather than individual data, are re-sampled to mimic the time dependent structure of the original data. We focus on a block bootstrap method because we face moderate sample sizes in the empirical applications, and wish to exploit the full sample information.

[Table 3 about here]

The findings are reported in Table 3. We present the test statistics and the p-values for the non-parametric SD test. We can reject the null hypothesis of the traditional portfolio dominating the augmented counterpart in all 7 cases at 5%. Accordingly, the augmented portfolio dominates the traditional optimal portfolio in this non-parametric test. Moreover, we can confirm the results of the in-sample analysis. In that sense, we contradict earlier findings, for example Daskalaki and Skiadopoulos (2011) and Bessler and Wolff (2015), who could not show that the diversification benefits that they find in-sample hold in an out-of-sample setting.

2. Parametric tests

In addition to the non-parametric tests, we analyze our results using a set of well-known parametric performance measures, which are described in what follows. We also calculate the average portfolio return, the standard deviation of the returns, and the Sharpe ratio.

The downside Sharpe ratio S_P (Ziemba, 2005) is defined as:

$$S_P = \frac{\bar{R}_P - \bar{R}_f}{\sqrt{2}\sigma_{P-}}, \tag{14}$$

where $\bar{R}_P$ is the average period return of portfolio P, $\bar{R}_f$ is the average risk-free rate, and σ_{P-} is the downside risk measure

$$\sigma_{P\_} = \sqrt{\frac{\sum_{t=1}^{T} \left(\min\left[x_t, 0\right]\right)^2}{T-1}}, \tag{15}$$

for $t \in 1, \ldots, T$ in the out-of-sample period. This way, the variance only accounts for losses relative to zero.

The upside potential ratio (Sortino, Meer, Plantinga, and Forsey, 2003) is calculated as

$$\mathrm{UP} = \frac{\frac{1}{T}\sum_{t=1}^{K} \max[0, R_{P,t} - R_{f,t}]}{\sqrt{\frac{1}{T}\sum_{t=1}^{T} (\max[0, R_{f,t} - R_{P,t}])^2}}, \tag{16}$$

and puts average excess returns over the average losses relative to the risk-free rate.

Lastly, we follow DeMiguel, Garlappi, and Uppal (2009) and employ the portfolio turnover (PT) and the return loss to evaluate the average changes to the portfolio weights at each re-balancing moment and the associated costs. PT can be calculated as:

$$PT = \frac{1}{T} \sum_{t=1}^{T} \sum_{i=1}^{N} (|w_{P,i,t+1} - w_{P,i,t}|), \tag{17}$$

where N is the number of assets in the portfolio and $w_{P,i,t}$ is the weight of asset i at time t.

The return loss is defined as:

$$\text{Return Loss} = \frac{\mu_{Aug}}{\sigma_{Aug}} \times \sigma_{Tr} - \mu_{Tr}, \tag{18}$$

where μ_{Aug} and μ_{Tr} are the average portfolio returns net of transaction costs and σ_{Aug} and σ_{Tr} are the associated standard deviations. We define the net returns as $\frac{NW_{P,t+1}}{NW_{P,t}} - 1$, where $NW_{P,t}$ is the wealth net of transaction costs trc for portfolio P at time t:

$$NW_{P,t+1} = NW_{P,t} \left(1 + R_{P,t+1}\right) \left[1 - trc \times \sum_{i=1}^{N} \left(|w_{P,i,t+1} - w_{P,i,t}|\right)\right]. \tag{19}$$

Here, the trc are proportional transaction costs of 50 bps (following e.g. DeMiguel et al., 2009).

To analyze the economic value of augmenting the traditional portfolio, we use the test proposed by Simaan (1993) to calculate the opportunity costs between the two portfolios in each case. The opportunity cost measure θ is defined as the return trade-off of choosing the traditional portfolio over the augmented portfolio:

$$E[U(1 + R_{Tr} + \theta)] = E[U(1 + R_{Aug})]. \tag{20}$$

where R_{Aug} and R_{Tr} is the return of the augmented and traditional portfolios, respectively. For the utility functions, we use the exponential and power utility at the risk aversion parameters of 2, 4, and 6. Because the measure considers the whole distribution of the returns, it is especially suitable for evaluating non-normal cases.

[Table 4 about here]

Table 4 reports the results of the parametric performance measures (Panel A) and the opportunity costs (Panel B) described above. In addition, we report the average portfolio weights for each asset class in Panel C.

For each case, we find that the augmented portfolio is superior in terms of the Sharpe, downside Sharpe, and upside potential ratios. Hence, the parametric performance measures complement the results of the non-parametric dominance and in-sample spanning tests. Only regarding the portfolio turnover does the augmented portfolio have more changes on average than the traditional one. This might be due to the additional assets. However, the return loss measure indicates that the higher turnover, and therefore, higher cumulative transaction cost, does not translate to less performance. Quite the contrary is true. The return loss measure shows that, net of the transaction costs, the augmented portfolio has a higher Sharpe ratio than the traditional one in all cases, and thus, justifies the higher turnover.

The opportunity costs reported in Panel B show that an investor is better off by augmenting the traditional portfolio with commodities, FX, and/or real estate assets. The result holds for the exponential and power utility functions under three different risk-aversion parameters. It is worth noting, that the calculated opportunity costs consider higher order moments rather than just the first and second moment in the case of the Sharpe ratio.

Nevertheless, we find positive evidence for the case of diversification benefits from augmenting the stock-bond portfolio.

We observe that in most cases the augmented portfolios have more frequent rebalancing and incur higher cumulative transaction costs (higher portfolio turnover), which is consistent with Carroll, Conlon, Cotter, and Salvador (2017)'s findings. The overall performance of the augmented portfolios compared with the traditional portfolios justifies these fees. In addition, the models generate well-diversified portfolios where all assets are included in the optimal portfolios. We observe from Panel C of Tab. 4 that the optimal stock-bonds portfolio includes 90% in stocks and 10% in bonds. The optimal augmented portfolios include more than 40% of the additional asset class, whereas the percentage of bonds is limited.

To sum up the out-of-sample findings: the non-parametric SD test as well as the parametric performance measures indicate that the investment universe of the augmented portfolio dominates the traditional portfolio of only stocks and bonds, yielding diversification benefits and providing better investment opportunities.

E. Determinants of the Diversification Benefits

Another intriguing question arises as to what the potential factors are that drive the diversification benefits associated with the inclusion of new asset classes. In the following analysis, we attempt to answer this question by means of a time series analysis on the difference between the returns of the augmented portfolios and those of the traditional portfolios for each case. For the whole out-of-sample period of the previous analysis, we run an autoregressive model with additional contemporaneous explanatory variables. These variables include the level of the GREA activity (Kilian, 2009, 2019), LIUS, monthly logarithmic returns or changes of the MSCI World Market Index, as well as the 3-month US Treasury Bill, 10-year US Governmental Bond, the S&P500 VIX, the Global Economic Policy Uncertainty (Baker, Bloom, and Davis, 2016), and the TED spread at a monthly frequency.[5] The time series regression model is specified as follow:

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + X_t \beta + \varepsilon_t. \tag{21}$$

Here, y_t is the return differential between the augmented and traditional portfolios at time t, and X_t is the row vector of explanatory variables at time t. The coefficients α_0 and α_1 refer to the intercept and the auto-regressive parameter of the regression. The column vector β contains the respective coefficients for the explanatory variables.

[Table 5 to Table 11 about here]

After checking for possible high correlations among the explanatory variable[6] and stationarity issues, we run time series regressions for each case with individual explanatory variables and a full model including all variables. The results are given in Tables 5-11.

The regression results show that the returns of the MSCI World explain most cases. Except for the two cases where either commodities or FX are mixed with real estate to augment the traditional portfolio (cases 5 and 6), we find that the MSCI World has a statistically significant and negative effect on the diversification benefits in the full model

[5]We retrieve the data for GREA from https://sites.google.com/site/lkilian2019/research/data-sets, the Leading Index from https://fred.stlouisfed.org/series/USSLIND, and the Economic Policy Uncertainty from https://www.policyuncertainty.com/global_monthly.html. The rest of the data is obtained from the Thomson Reuters DataStream.

[6]The highest absolute correlations are found between the returns of the MSCI World and VIX (−0.6843), and between the returns of the MSCI World and the U.S. 10-year Government Bond (−0.3270).

and also individually. This finding suggests that an increase in the stock market decreases the diversification benefits and vice versa for a decline in the stock market. This behavior appears quite logical because it lowers the contribution from additional investments to the traditional case.

A similar explanation can be derived upon the negative coefficient of the LIUS for all seven cases. A positive LIUS level reduces the benefits from diversification. By contrast, when the economy is in distress and the LIUS is negative, the spread between the traditional and the augmented portfolio increases. The picture is, however, more complex for the second business cycle indicator. The GREA is positively associated with the diversification benefit for the portfolio augmented with only commodities, and commodities and real estate. A negative coefficient is found when investigating the diversification benefit from a portfolio augmented with only the FX asset class. We have to recall that the GREA is based on dry bulk cargo prices and may translate directly to commodities. Thus, on the one hand, a positive GREA level increases the benefits from investing in commodities. On the other hand, a positive GREA level also indicates the relative strength of economy outside the United States and pressures the benefits from investments in FX. In combined portfolios, these effects might offset one another.

In addition, the VIX is positively associated with the diversification benefits when commodities and FX are used to augment the traditional set (cases 1, 2, 4, and 7). High stock market uncertainty thus increases the return spread between the augmented and traditional portfolios. However, because of the high absolute correlation with MSCI World, we do not find a statistically significant effect in the full model. Interestingly, we find that the short-term 3-month T-bills are negatively associated with diversification benefits of portfolios involving real estate (cases 3, 5, and 6). Furthermore, the returns of the long-term 10-year government bond have a positive relation in most of the other cases (cases 1, 4, 5, 7). Hence, it appears that the rise in short-term interest rates lowers the diversification potential from real estate, whereas positive changes in long-term interest rates increase the benefit from commodities. Lastly, the TED spread (i.e., the perceived risk in international loan markets) does not show any relevance for explaining the diversification benefits of commodities, FX, or real estate.

In summary, the diversification benefits are highly affected by market conditions. Earlier evidence in Belousova and Dorfleitner (2012) and Bessler and Wolff (2015) shows that diversification benefits are time-varying and behave differently in different market environments. A different view is presented by Chan, Treepongkaruna, Brooks, and Gray (2011) who find in-sample evidence of diversification benefits only in "tranquil" periods.

V. Concluding Remarks

Whether diversification across asset classes brings significant benefits to investors and portfolio managers is an important issue in finance, particularly within the context of the successive crises and financial turmoil over the last thirty years and the resulting high economic policy uncertainty (Baker et al., 2016). Our literature reviews show that more than 50% of past studies find weak or no evidence of in-sample diversification benefits from augmenting portfolios of stocks and/or bonds with additional asset classes, whereas very few of them document out-of-sample benefits.

This study proposes a stochastic spanning approach to assess whether traditional portfolios of stocks and bonds span portfolios augmented with other asset classes, with both in-sample and out-of-sample analysis. Particularly, we employ individual commodity futures, various FX rates, and a real estate index. In our empirical application, we discuss seven portfolio designs where one, two, or three of the asset classes augment the traditional portfolio. Our results for the in-sample assessment show that all augmented portfolio combinations under consideration cannot be spanned by the traditional set of stocks and bonds. The out-of-sample performance, conducted in the second step with the help of several non-parametric and parametric tests, confirms our previous findings. We conclude that traditional investors can generally benefit in terms of higher returns and lower volatility by augmenting their portfolios' stocks and bonds with alternative asset classes. Finally, higher diversification benefits are found to be associated with high market uncertainty, bearish stock markets, and economic downturns.

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Table 1: Descriptive Statistics

Assets	Mean	Std. Dev.	Median	Min.	Max.	Skewn.	Ex. Kurt.
US Equity (S&P 500 Sector Indices)							
Health Care	1.0442	4.5534	1.2280	−16.9288	13.8124	−0.2595	0.8179
Consumer Discretionary	0.9798	5.2481	1.3667	−19.8210	19.6656	−0.1792	1.4761
Consumer Staples	0.9202	3.7901	1.0706	−12.1026	19.0259	−0.0477	2.0529
Industrials	0.9211	5.1551	1.2372	−20.7370	18.6080	−0.3552	2.0656
Information Technology	1.1768	7.1344	1.2038	−25.8973	24.2201	−0.1157	1.3631
Materials	0.7997	5.8611	0.9079	−21.4641	27.2166	0.1316	2.9019
Communication Systems	0.5979	5.3529	0.9100	−14.3041	31.6710	0.3067	3.0695
Utilities	0.7625	4.3327	1.0256	−16.1161	12.7984	−0.5522	1.1491
Financials	0.9191	6.6161	1.5405	−32.4672	29.9052	−0.5262	4.4511
Energy	0.8895	5.6429	1.1704	−18.3789	18.6642	−0.2626	1.0613
US Corporate Bond Indices							
Moody's Seasoned AAA	−0.1390	3.8947	−0.6092	−17.0543	13.3333	0.0019	2.2529
Moody's Seasoned BAA	−0.1268	3.3017	−0.3722	−9.6612	20.5845	0.8942	4.7876
Bloomberg Barclays Aggr.	−0.0033	1.0683	0.0080	−3.9888	3.8718	−0.2076	1.0968
US Government Bonds (Continuous Series, Bloomberg)							
5Y Treasury Note	0.2464	10.5214	−0.5641	−32.3685	50.5170	0.7745	3.3548
30Y Treasury Note	−0.1051	5.4186	−0.3849	−22.1640	34.6786	0.4701	6.7906
Commodity Futures (Front Month Continuous Series)							
Brent (ICE)	0.7854	9.3947	0.7021	−36.5572	37.2596	0.1371	1.8758
Live Cattle (CME)	0.2510	4.7344	0.2698	−21.4686	13.6152	−0.3643	1.8811
Feeder Cattle (CME)	0.2573	4.4005	0.3548	−21.6236	16.6540	−0.1980	2.0732
Lean Hogs (CME)	0.5504	9.7004	0.0714	−31.7311	40.9213	0.3382	1.5253
Corn (CBT)	0.4345	7.6909	0.4479	−22.8406	24.5565	−0.1681	0.7871
Soybeans (CBT)	0.4072	7.3881	0.3373	−29.1446	21.1497	−0.2417	1.2740
Wheat (CBT)	0.4024	8.2338	0.1390	−20.6079	38.7194	0.4638	1.4813
WTI (NYMEX)	0.6854	8.9922	0.5575	−34.0380	37.8788	0.0626	1.6433
Gold (CMX)	0.4152	4.5412	−0.0672	−17.7642	19.4118	0.2660	1.4752
Silver (CMX)	0.6012	8.1932	−0.0470	−26.1537	25.9808	0.1267	0.8189
Cotton (CSCE)	0.3791	8.2614	0.2127	−35.0440	31.8710	0.0496	1.8883
Coffee (CSCE)	0.6461	10.8249	−0.6365	−30.9904	58.6585	1.0331	3.0257
Cocoa (CSCE)	0.6080	8.5746	−0.0181	−23.1537	38.6544	0.4698	1.1026
Sugar (CSCE)	0.4181	9.4605	0.0000	−24.9775	33.8537	0.2637	0.4519
US Dollar Foreign Exchange Rates							
Canadian $	0.0595	2.2063	−0.0036	−8.4092	11.8695	0.4537	3.2946
Danish Krone	0.0274	2.9349	−0.1863	−9.2579	10.2819	0.3874	0.8176
Japanese Yen	−0.0185	3.0917	0.0417	−15.6390	10.2454	−0.2137	2.1243
Norwegian Krone	0.1131	3.1398	−0.0560	−7.9762	13.6386	0.4506	0.9405
South African Rand	0.5639	4.2139	0.3617	−10.7571	20.0266	0.6976	2.5504
Swedish Krona	0.1548	3.3479	−0.1970	−10.6280	15.1566	0.4347	1.3924
Swiss Franc	−0.0847	3.1609	−0.3088	−11.6181	15.0924	0.3362	2.0518
Australian $	0.0709	3.3110	0.1273	−10.0111	16.3271	0.4706	2.0901
New Zealand $	0.0112	3.4207	−0.2385	−12.6287	15.6984	0.5227	2.8076
UK £	0.0974	2.7887	−0.0190	−9.5270	15.1096	1.0376	4.3386
Indian Rupee	0.4357	2.3147	0.0573	−7.0477	14.1397	1.5514	7.4833
Sri Lankan Rupee	0.4384	1.2529	0.2129	−4.3333	9.2447	1.9199	11.7305
Chinese Yuan	0.2152	3.1520	−0.0036	−3.2184	49.8703	13.1841	192.9047
Hong Kong $	0.0001	0.1297	0.0000	−0.7693	0.4832	−0.8041	7.4952
Singapore $	−0.0887	1.6470	−0.1264	−6.0941	8.9911	0.7937	4.1196
Thai Bhat	0.1072	2.8642	−0.0396	−13.3333	31.3214	3.8828	43.6078
South Korean Won	0.2115	3.8812	−0.0739	−12.3246	42.6768	4.3824	44.5104
Taiwan $	0.0573	1.5068	0.0363	−5.5863	9.5604	0.7225	5.5098
Real Estate							
S&P Index US REIT	0.9795	6.1455	1.2847	−36.6389	46.3883	−0.1474	16.1129

Note: Descriptive statistics of the different assets for monthly arithmetic returns for the period 1 January 1990 to 31 December 2018. Mean, Median, Minimum, Maximum, and Standard Deviation are given in percentage.

Table 2: Stochastic Spanning Tests

Case	Test statistic $\eta_T^\star$	Regression estimates q_T^{BC}	Result
Case 1	0.0132	0.0011	Reject Spanning
Case 2	0.0384	0.0246	Reject Spanning
Case 3	0.0023	0.0014	Reject Spanning
Case 4	0.0476	0.0034	Reject Spanning
Case 5	0.0139	0.0098	Reject Spanning
Case 6	0.0416	0.0274	Reject Spanning
Case 7	0.0480	0.0312	Reject Spanning

Note: Stochastic Spanning tests of the traditional asset class with respect to the augmented set with additional asset classes. Entries report the test statistics $\eta_T^\star$ as well as the regression estimates q_T^{BC} for each case. We reject spanning if the test statistic $\eta_T^\star$ is higher than the regression estimate q_T^{BC}.

Table 3: Non-parametric Stochastic Dominance Tests

Case	Test statistic	p-value (%)
Case 1	0.0005	3.34%
Case 2	0.0013	2.47%
Case 3	0.0021	4.33%
Case 4	0.0014	4.89%
Case 5	0.0015	3.56%
Case 6	0.0020	2.84%
Case 7	0.0007	4.49%

Note: Entries report test statistics and p-values from the distribution of the 228 out-of-sample portfolio returns with the null hypothesis that the traditional optimal portfolio dominates the augmented portfolio with alternative asset classes. The null hypothesis is that the traditional optimal portfolio dominates the augmented portfolio with each asset class. We reject the null hypothesis if the p-value is lower than 5%.

Table 4: Out-of-sample Performance: Parametric Performance Measures.

	Case 1		Case 2		Case 3		Case 4		Case 5		Case 6		Case 7	
	Trad.	Augm.	Trad.	Augm.	Trad.	Augm.	Trad.	Augm.	Trad.	Augm.	Trad.	Augm.	Trad.	Augm.
Panel A: Performance Measures														
Mean	0.0052	0.0060	0.0014	0.0036	0.0031	0.0068	0.0034	0.0058	0.0044	0.0071	0.0027	0.0063	0.0036	0.0049
SD	0.0487	0.0421	0.0191	0.0165	0.0415	0.0474	0.0443	0.0223	0.0489	0.0487	0.0283	0.0287	0.0446	0.0203
Sharpe ratio	0.0796	0.1118	0.0022	0.1396	0.0423	0.1165	0.0463	0.2017	0.0626	0.1180	0.0495	0.1729	0.0500	0.1739
D. Sharpe Ratio	0.0787	0.1257	0.0021	0.1702	0.0410	0.1185	0.0449	0.3380	0.0606	0.1263	0.0494	0.2176	0.0487	0.2179
UP ratio	0.5529	0.6631	0.4979	0.7186	0.5120	0.5754	0.4604	0.9440	0.5284	0.6194	0.5128	0.7419	0.4706	0.7692
Portfolio Turnover	32.94%	38.45%	18.24%	18.45%	41.67%	41.47%	17.10%	21.63%	27.31%	27.49%	28.18%	26.67%	17.96%	27.83%
Return Loss		0.178%		0.283%		0.292%		0.819%		0.271%		0.347%		0.710%
Panel B: Opportunity Cost														
Exponential Utility														
ARA=2		0.231%		0.149%		0.316%		0.393%		0.273%		0.356%		0.287%
ARA=4		0.241%		0.231%		0.245%		0.563%		0.283%		0.359%		0.463%
ARA=6		0.252%		0.330%		0.151%		0.753%		0.298%		0.367%		0.657%
Power Utility														
RRA=2		0.231%		0.153%		0.312%		0.397%		0.274%		0.357%		0.290%
RRA=4		0.242%		0.241%		0.230%		0.574%		0.285%		0.361%		0.473%
RRA=6		0.253%		0.349%		0.115%		0.773%		0.300%		0.370%		0.677%
Panel C: Average allocation of the optimal portfolios														
Stocks	89.17%	54.85%	47.95%	34.74%	84.29%	40.27%	67.46%	27.83%	92.00%	43.54%	60.30%	31.55%	69.54%	26.01%
Bonds	10.83%	3.71%	52.05%	1.43%	15.71%	14.30%	32.54%	0.35%	8.00%	0.58%	39.70%	0.89%	30.46%	0.24%
Commodities		41.43%						19.69%		27.29%				14.84%
FX				68.83%				52.12%				47.06%		44.67%
Real Estate						45.43%				28.59%		20.50%		14.24%

Note: Entries in Panel A report the performance measures (Mean, Standard Deviation, Sharpe ratio, Downside Sharpe ratio, UP ratio, Portfolio Turnover, Returns Loss and Opportunity Cost) for the traditional as well as the augmented optimal portfolios for each case. The results for the opportunity cost are reported in Panel B for different degrees of absolute risk aversion (ARA=2,4,6) and different degrees of relative risk aversion (RRA=2,4,6). Panel C exhibits the average weight allocation of the optimal portfolios. The dataset spans the out-of-sample period from 1 January 2000 to 31 December 2018.

Table 5: Time series analysis on the return difference between augmented and traditional portfolio. Case 1: Commodities

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
Const.	0.0053	0.7887**	0.1410	0.0897	0.0774	0.0576	0.0737	0.0849	0.2664
	(0.2150)	(0.3504)	(0.1960)	(0.2073)	(0.2199)	(0.2149)	(0.2170)	(0.2180)	(0.3365)
AR(1)	−0.0568	−0.0560	−0.0548	−0.0349	−0.0367	−0.0500	−0.0454	−0.0413	−0.0839
	(0.0661)	(0.0661)	(0.0663)	(0.0662)	(0.0661)	(0.0663)	(0.0665)	(0.0663)	(0.0668)
GREA	0.0070**								0.0068**
	(0.0029)								(0.0027)
LIUS		−0.6453**							−0.1824
		(0.2543)							(0.2474)
MSCI			−0.3059***						−0.2713***
			(0.0438)						(0.0656)
VIX				0.0574***					0.0128
				(0.0108)					(0.0147)
3MTB					0.0007				−0.0001
					(0.0021)				(0.0020)
10YG						0.2263**			0.0001
						(0.1067)			(0.1027)
GEPU							0.0151		−0.0070
							(0.0121)		(0.0116)
TED								0.0081	0.0006
								(0.0081)	(0.0078)
Obs.	228	228	228	228	228	228	228	228	228
LogL	−602.03**	−601.71**	−582.76***	−591.44***	−604.80	−602.62**	−604.07	−604.35	−577.95***
BIC	1225.78	1225.13	1187.25	1204.61	1231.31	1226.95	1229.85	1230.43	1215.63

Note: Model 1 to 8 include only one explanatory variable at the time. Model 9 includes all explanatory variable. HAC standard errors in parentheses. Asterisks indicate statistically significant estimates at level of significance of 10% (*), 5% (**), and 1%(***). The log-likelihood is tested against the null model without explanatory variables based on a Log-likelihood ratio test.

Table 6: Time series analysis on the return difference between augmented and traditional portfolio. Case 2: FX

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
Const.	0.2600**	0.4945***	0.2483**	0.2245**	0.2197*	0.2215*	0.2221*	0.2216*	0.4761***
	(0.1128)	(0.1841)	(0.1069)	(0.1143)	(0.1153)	(0.1151)	(0.1150)	(0.1154)	(0.1810)
AR(1)	0.1296**	0.1318**	0.1596**	0.1647**	0.1472**	0.1463**	0.1459**	0.1503**	0.1569**
	(0.0661)	(0.0660)	(0.0655)	(0.0657)	(0.0659)	(0.0665)	(0.0675)	(0.0659)	(0.0666)
GREA	−0.0034**								−0.0036**
	(0.0015)								(0.0015)
LIUS		−0.2477*							−0.1571
		(0.1331)							(0.1313)
MSCI			−0.1285***						−0.1526***
			(0.0192)						(0.0276)
VIX				0.0166***					−0.0036
				(0.0046)					(0.0060)
3MTB					0.0004				−0.0002
					(0.0009)				(0.0008)
10YG						0.0109			−0.0779*
						(0.0467)			(0.0438)
GEPU							0.0009		−0.0063
							(0.0052)		(0.0048)
TED								−0.0024	−0.0029
								(0.0034)	(0.0032)
Obs.	228	228	228	228	228	228	228	228	228
LogL	−411.00**	−411.76*	−393.03***	−407.16***	−413.35	−413.42	−413.43	−413.21	−386.08***
BIC	843.73	845.24	807.77	836.04	848.42	848.56	848.58	848.13	831.89

Note: Model 1 to 8 include only one explanatory variable at the time. Model 9 includes all explanatory variable. HAC standard errors in parentheses. Asterisks indicate statistically significant estimates at level of significance of 10% (*), 5% (**), and 1%(***). The log-likelihood is tested against the null model without explanatory variables based on a Log-likelihood ratio test.

Table 7: Time series analysis on the return difference between augmented and traditional portfolio. Case 3: Real Estate

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
Const.	0.3439*	0.8130**	0.3706*	0.3763*	0.4349**	0.3589*	0.3579*	0.3759*	1.0378***
	(0.2056)	(0.3332)	(0.2057)	(0.2045)	(0.2076)	(0.2015)	(0.2016)	(0.2043)	(0.3564)
AR(1)	−0.1261*	−0.1303**	−0.1162*	−0.1222*	−0.0747	−0.1350**	−0.1195*	−0.1236*	−0.0787
	(0.0656)	(0.0656)	(0.0669)	(0.0664)	(0.0675)	(0.0658)	(0.0656)	(0.0656)	(0.0679)
GREA	0.0030								−0.0000
	(0.0028)								(0.0029)
LIUS		−0.3984*							−0.6191**
		(0.2420)							(0.2620)
MSCI			0.0310						0.1655**
			(0.0490)						(0.0692)
VIX				−0.0018					0.0147
				(0.0117)					(0.0154)
3MTB					−0.0081***				−0.0083***
					(0.0021)				(0.0021)
10YG						0.1654			0.2523**
						(0.1071)			(0.1082)
GEPU							0.0343***		0.0333***
							(0.0120)		(0.0122)
TED								−0.0020	−0.0051
								(0.0083)	(0.0082)
Obs.	228	228	228	228	228	228	228	228	228
LogL	−606.23	−605.46	−606.61	−606.79	−599.73***	−605.62	−602.83***	−606.78	−589.87***
BIC	1234.18	1232.64	1234.93	1235.31	1221.18	1232.97	1227.38	1235.27	1239.47

Note: Model 1 to 8 include only one explanatory variable at the time. Model 9 includes all explanatory variable. HAC standard errors in parentheses. Asterisks indicate statistically significant estimates at level of significance of 10% (*), 5% (**), and 1%(***). The log-likelihood is tested against the null model without explanatory variables based on a Log-likelihood ratio test.

Table 8: Time series analysis on the return difference between augmented and traditional portfolio. Case 4: Commodities and FX

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
Const.	0.2107	1.2261***	0.3628*	0.2571	0.2475	0.1997	0.2314	0.2477	0.4605
	(0.2963)	(0.4635)	(0.1908)	(0.2527)	(0.2943)	(0.2725)	(0.2831)	(0.2914)	(0.3360)
AR(1)	0.0645	0.0403	−0.0633	0.0262	0.0657	0.0138	0.0353	0.0595	−0.0737
	(0.0660)	(0.0661)	(0.0664)	(0.0674)	(0.0660)	(0.0681)	(0.0691)	(0.0664)	(0.0675)
GREA	0.0031								0.0034
	(0.0040)								(0.0027)
LIUS		−0.8957***							−0.0975
		(0.3360)							(0.2469)
MSCI			−0.6033***						−0.6411***
			(0.0432)						(0.0650)
VIX				0.0918***					−0.0078
				(0.0125)					(0.0145)
3MTB					−0.0005				−0.0024
					(0.0026)				(0.0020)
10YG						0.4242***			0.0379
						(0.1310)			(0.1028)
GEPU							0.0243		−0.0167
							(0.0151)		(0.0115)
TED								0.0088	−0.0017
								(0.0097)	(0.0077)
Obs.	228	228	228	228	228	228	228	228	228
LogL	−647.46	−644.33***	−578.46***	−622.79***	−647.74	−642.62***	−646.45	−647.35	−575.70***
BIC	1316.64	1310.37	1178.64	1267.30	1317.20	1306.96	1314.62	1316.41	1211.12

Note: Model 1 to 8 include only one explanatory variable at the time. Model 9 includes all explanatory variable. HAC standard errors in parentheses. Asterisks indicate statistically significant estimates at level of significance of 10% (*), 5% (**), and 1%(***). The log-likelihood is tested against the null model without explanatory variables based on a Log-likelihood ratio test.

Table 9: Time series analysis on the return difference between augmented and traditional portfolio. Case 5: Commodities and Real Estate

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
Const.	0.1618	1.0913***	0.2826	0.2717	0.2960	0.2279	0.2630	0.2701	0.8798***
	(0.1908)	(0.3167)	(0.1993)	(0.1998)	(0.1999)	(0.1944)	(0.2007)	(0.2007)	(0.3181)
AR(1)	−0.2154***	−0.2036***	−0.1794***	−0.1697***	−0.1764***	−0.1846***	−0.1749***	−0.1750***	−0.2413***
	(0.0646)	(0.0648)	(0.0651)	(0.0652)	(0.0651)	(0.0650)	(0.0651)	(0.0651)	(0.0649)
GREA	0.0097***								0.0072***
	(0.0026)								(0.0025)
LIUS		−0.7515***							−0.6441***
		(0.2302)							(0.2355)
MSCI			−0.0745						0.0696
			(0.0485)						(0.0692)
VIX				0.0248**					0.0232
				(0.0118)					(0.0159)
3MTB					−0.0038*				−0.0052**
					(0.0022)				(0.0021)
10YG						0.3747***			0.3956***
						(0.1064)			(0.1073)
GEPU							0.0099		−0.0018
							(0.0126)		(0.0125)
TED								0.0050	−0.0016
								(0.0085)	(0.0084)
Obs.	228	228	228	228	228	228	228	228	228
LogL	−606.49***	−607.99***	−611.94	−610.93**	−611.57*	−607.08***	−612.80	−612.93	−593.00***
BIC	1234.69	1237.71	1245.60	1243.59	1244.85	1235.88	1247.32	1247.58	1245.72

Note: Model 1 to 8 include only one explanatory variable at the time. Model 9 includes all explanatory variable. HAC standard errors in parentheses. Asterisks indicate statistically significant estimates at level of significance of 10% (*), 5% (**), and 1%(***). The log-likelihood is tested against the null model without explanatory variables based on a Log-likelihood ratio test.

Table 10: Time series analysis on the return difference between augmented and traditional portfolio. Case 6: FX and Real Estate

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
Cons.	0.3643**	0.9272***	0.3649**	0.3561**	0.3778**	0.3523**	0.3495**	0.3536**	1.0397***
	(0.1692)	(0.2644)	(0.1632)	(0.1670)	(0.1746)	(0.1664)	(0.1660)	(0.1673)	(0.2910)
AR(1)	0.0408	0.0157	0.0183	0.0387	0.0873	0.0341	0.0364	0.0422	0.0488
	(0.0662)	(0.0663)	(0.0685)	(0.0667)	(0.0695)	(0.0682)	(0.0664)	(0.0663)	(0.0717)
GREA	−0.0008								−0.0027
	(0.0023)								(0.0023)
LIUS		−0.5212***							−0.5780***
		(0.1917)							(0.2126)
MSCI			−0.0451						−0.0165
			(0.0354)						(0.0506)
VIX				0.0023					0.0008
				(0.0080)					(0.0110)
3MTB					−0.0032**				−0.0036**
					(0.0016)				(0.0016)
10YG						0.0330			0.0242
						(0.0782)			(0.0794)
GEPU							0.0115		0.0065
							(0.0085)		(0.0087)
TED								−0.0056	−0.0057
								(0.0057)	(0.0058)
Obs.	228	228	228	228	228	228	228	228	228
LogL	−525.41	−521.90***	−524.66	−525.42	−523.41**	−525.38	−524.55	−524.97	−517.52**
BIC	1072.53	1065.51	1071.04	1072.57	1068.53	1072.47	1070.82	1071.67	1094.76

Note: Model 1 to 8 include only one explanatory variable at the time. Model 9 includes all explanatory variable. HAC standard errors in parentheses. Asterisks indicate statistically significant estimates at level of significance of 10% (*), 5% (**), and 1%(***). The log-likelihood is tested against the null model without explanatory variables based on a Log-likelihood ratio test.

Table 11: Time series analysis on the return difference between augmented and traditional portfolio. Case 7: Commodities, FX, and Real Estate

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9
Const.	0.1541	0.8778**	0.2255	0.1400	0.1375	0.0828	0.1091	0.1306	0.4010
	(0.2498)	(0.3973)	(0.1677)	(0.2192)	(0.2492)	(0.2270)	(0.2350)	(0.2479)	(0.2971)
AR(1)	0.0237	0.0111	−0.1358**	−0.0216	0.0297	−0.0300	−0.0072	0.0261	−0.1437**
	(0.0662)	(0.0662)	(0.0663)	(0.0675)	(0.0663)	(0.0677)	(0.0675)	(0.0668)	(0.0667)
GREA	−0.0022								−0.0023
	(0.0034)								(0.0024)
LIUS		−0.6822**							−0.1382
		(0.2881)							(0.2191)
MSCI			−0.4856***						−0.4890***
			(0.0402)						(0.0609)
VIX				0.0690***					−0.0107
				(0.0114)					(0.0137)
3MTB					−0.0010				−0.0028
					(0.0023)				(0.0018)
10YG						0.4508***			0.1530
						(0.1129)			(0.0949)
GEPU							0.0394***		0.0035
							(0.0128)		(0.0108)
TED								0.0013	−0.0052
								(0.0086)	(0.0072)
Obs.	228	228	228	228	228	228	228	228	228
LogL	−618.25	−615.71**	−564.02***	−601.25***	−618.35	−610.78***	−613.79***	−618.44	−560.57***
BIC	1258.21	1253.14	1149.77	1224.21	1258.42	1243.27	1249.30	1258.61	1180.85

Note: Model 1 to 8 include only one explanatory variable at the time. Model 9 includes all explanatory variable. HAC standard errors in parentheses. Asterisks indicate statistically significant estimates at level of significance of 10% (*), 5% (**), and 1%(***). The log-likelihood is tested against the null model without explanatory variables based on a Log-likelihood ratio test.

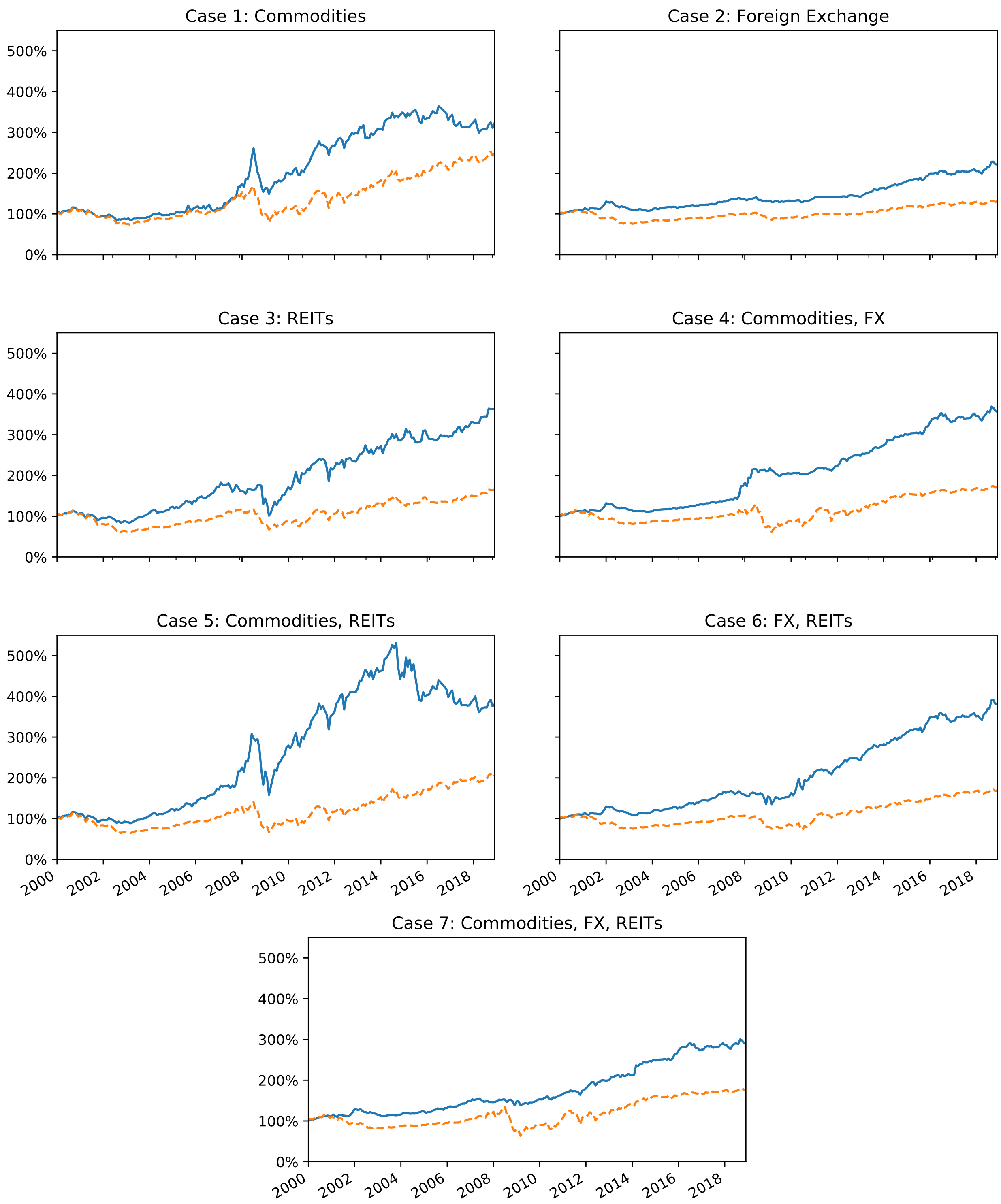

Figure 1: Cumulative performance of the traditional optimal portfolio (orange, stripped) as well as the optimal augmented portfolio (blue, solid) in each case for the out-of-sample period from 1 January 2000 to 31 December 2018.

Appendix A: Literature Review

Table 12: Literature overview

Author(s)	Asset Class			Benefits		Sample			Remarks
	C	R	F	InS	OoS	Data	Freq.	Period	
Friedman (1971)		✓		✓		I	A	1963-1968	Real Estate dominate mixed-asset portfolios
Robichek, Cohn, and Pringle (1972)	✓	✓		✓		X/I	A	1949–1969	Multimedia diversification may offer substantial improvement in portfolio performance
Solnik (1974)			✓	✓		I	W	1966-1971	Foreign Exchange decrease the risk in diversified international portfolios
McDonald and Solnick (1977)	✓			✓		Gold	M	1945–1976	Gold and gold equity may reduce the variability of stock-bond portfolios
Greer (1978)	✓			✓		X	S	1960–1974	Commodity futures increase risk-adjusted performance
Bodie and Rosansky (1980)	✓			✓		I	Q	1950–1976	Commodity futures reduce variance of S&P 500 portfolio
Burns and Epley (1982)		✓		✓		I	Q	1970–1979	Combination of REITs and stocks is superior to single investments
Sherman (1982)	✓			✓		Gold	M	1972–1981	Gold decreases volatility
Bodie (1983)	✓			✓		I	A	1953–1981	Commodities futures as inflation hedge for a stock-bond portfolio
Herbst (1983)	✓			(✓)		X	A	1800–1976	Gold offers no inflation hedge, but is valuable for diversification
Brueggeman, Chen, and Thibodeau (1984)		✓		✓		X	Q	1972–1983	Optimal portfolios are weighted heavily in Real Estate
Fogler (1984)		✓		(✓)		X	A	1915–1978	A well diversified portfolio has a real estate share of 15%-20%
Ibbotson and Siegel (1984)		✓		✓		X	A	1947–1982	Real Estate offers diversification and inflation hedge
Webb and Rubens (1986)		✓		✓		X	A	1967–1982	High amount of Real Estate in optimal portfolios
Elton, Gruber, and Rentzler (1987)	✓			✗		X	M	1979–1985	Commodity funds do not offer profitable addition to stock-bond portfolios
Kuhle (1987)		✓		✗		I	M	1980–1985	No performance benefits from adding REITs to common stock portfolio
Irwin and Landa (1987)	✓	✓		✗		X	A	1975–1985	Gold does not move efficient frontier
Webb and Rubens (1987)		✓		✓		X	A	1947–1984	Real estate occupies are major weight in optimal portfolios
Eun and Resnick (1988)			✓	✓	✓	I	W	1980–1985	Hedging strategies covering exchange and estimation risk outperform
Webb, Curcio, and Rubens (1988)		✓		✓		X	A	1947–1983	Purely financial diversification produces inefficient portfolios
Fortenbery and Hauser (1990)	✓			(✓)		X/I	M	1976–1985	Agricultural futures rarely increase returns, but may lower portfolio risk
Fischmar and Peters (1991)	✓			✓		X	M	1980–1988	Portfolios with a major share in managed futures dominates all others
Ankrim and Hensel (1993)	✓	✓		✓		X	M	1972–1990	Commodities offer inflation hedge capabilities similar to Real Estate
Glen and Jorion (1993)			✓	(✓)	(✓)	I	M	1974–1990	Only conditional hedging with currencies improves stock-bond portfolio performance

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Irwin, Krukemyer, and Zulauf (1993)	✓			✗		X	M	1979–1990	Commodity funds do not improve stock-bond portfolios
Lummer and Siegel (1993)	✓			✓		X	M/A	1970–1991	Inflation hedge for risk-averse and diversifier for other investors
Greer (1994)	✓			✓		X	A	1970–1993	Passively managed commodity index improves risk-return of a portfolio with stocks
Froot (1995)	✓	✓		(✓)		X/I	Q	1970–1993	Highy energy component (GSCI or Crude Oil) can reduce portfolio risk
Grauer and Hakansson (1995)		✓		(✓)		X	A	1955-1988	Only active management yields significant gains
Edwards and Park (1996)	✓			✓		I	M	1983–1992	Commodities increase Sharpe ratios of stock & bond portfolios
Kallberg, Liu, and Greig (1996)		✓		(✓)		I	Q	1982–1989	Incremental benefits diminish with larger property
Satyanarayan and Varangis (1996)	✓			✓		X	M	1985–1992	Commodities shift efficient frontier of global portfolios upwards
Miles and Mahoney (1997)		✓		✓		X	Q	1971-1996	Commercial Real Estate provides a hedge against inflation
Mull and Soenen (1997)		✓		(✓)		X	M	1985–1994	including REITs did not yield increases in risk-adjusted return over the whole period
Schneeweis and Spurgin (1997)	✓			✓		X	M	1987–1995	Commodity indices differ in return and risk characteristics
Ziobrowski and Ziobrowski (1997)		✓		✗		X	A	1970–1995	Real Estate can have a negative impact on mixed-asset portfolios
Anson (1998)	✓					X	Q	1985–1997	Commodity indices provide diversification due to futures returns
Halpern and Warsager (1998)	✓					X	M/A	1974–1996	Diversified portfolios benefit from commodities especially in inflationary periods
Kaplan and Lummer (1998)	✓			✓		X	M	1970–1997	Inflation hedge for risk-averse and diversifier for other investors
Abanomey and Mathur (1999)	✓			✓	✓	I	M	1970–1995	International portfolio
Anson (1999)	✓			✓		X	Q	1974–1997	Commodity futures offer great benefits to risk-averse investors
Chandrashekaran (1999)		✓		✓		X	M	1975–1996	Using ex-ante information improves the investment opportunity set
Gibson (1999)	✓	✓		✓		X	A	1972–1997	Robust return enhancement and volatility reduction
Goldstein and Nelling (1999)		✓		✗		X	M	1972–1998	Equity and Mortgage REITs have increase correlation with stocks in bull markets
Seiler, Webb, and Myer (1999)		✓		(✓)		–	–	–	Literature review finds recommendations between 0-67% of real estate in portfolios
Greer (2000)	✓			✓		X	A	1970–1999	Commodity index returns are negatively correlated with stocks and bonds
Jensen et al. (2000)	✓	✓		(✓)		X	M	1973–1997	Benefits vary with monetary policy
Georgiev (2001)	✓			✓		X	M	1990–2000	Adding a commodities results in enhanced risk-adjusted performance
Johnson and Jensen (2001)	✓	✓		(✓)		X	M	1974–1999	Benefits vary with monetary policy
Jensen et al. (2002)	✓			(✓)		X	M	1973–1999	Benefits vary with monetary policy
Georgiev et al. (2003)		✓		✗		X	Q	1990–2002	Only direct real estate investments may offer benefits; REITs do not
Hudson-Wilson, Fabozzi, and Gordon (2003)		✓		✓		X	Q	1982-2002	Based on correlation, real estate earns its place in a well-diversified portfolio
de Roon et al. (2003)			✓	(✓)	(✓)	I	M	1975–1998	Only very risk averse investors profit from static strategies; Dynamic strategies are beneficial
Chen, Ho, Lu, and Wu (2005)		✓		✓		X	M	1980-2002	REITs provide benefits starting 1985. Mortgage REITs do not
Hudson-Wilson, Gordon, Fabozzi, Anson, and Giliberto (2005)		✓		✓		X	Q	1982-2004	Based on correlation, real estate can play a significant role in a mixed-asset portfolios

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Lee (2005)		✓		✓		X	A	1952-2001	Adding Real Estate to a mixed-asset portfolio increases return due diversification
Erb and Harvey (2006)	✓			(✓)		X/I	M	1982–2004	Positive diversification returns only with portfolio re-balancing
Gorton and Rouwenhorst (2006)	✓			✓		X	M	1959–2004	Commodity futures diversify the cyclical variation in stock and bond returns
Idzorek (2007)	✓			✓		X	A	1970–2004	Including commodities in the opportunity set improved the risk-return characteristics
Scherer and He (2008)	✓			✓		X	M	1989–2006	Strong evidence of diversification benefits
Nijman and Swinkels (2008)	✓			(✓)		X	M	1970–2006	GSCI offers diversification to inflation-protected pension schemes (but not for nominal)
Geman and Kharoubi (2008)	✓			✓		WTI	D	1990–2006	Adding WTI Futures reduce (increase) volatility and kurtosis (return)
Hung, Onayev, and Tu (2008b)		✓		(✓)		X	M/Q	1988-2005	Only mortgage & hybrid REITs offer benefits, but only in bull markets
Büyüksahin, Haigh, and Robe (2009)	✓			(✓)		X	D/W/M	1991–2008	No benefits in times of stock market distress
Campbell et al. (2010)			✓	✓		I	M	1975–2005	Using long- and short-positions in FX reduces portfolio risk
Cheung and Miu (2010)	✓			(✓)		X	M	1970–2005	Diversification benefits only in bullish stock markets
Sa-Aadu, Shilling, and Tiwari (2010)	✓	✓		✓		X	M	1972–2008	Additional asset classes may serve as a hedge for traditional portfolios
You and Daigler (2010)	✓		✓	✓		X/I	W	1992–2006	Even a naive portfolio can reduce four-moment tail risk
Chan et al. (2011)	✓	✓		(✓)		X/I	M	1987–2008	Diversification only in tranquil regimes
Daskalaki and Skiadopoulos (2011)	✓			(✓)	✗	X/I	M	1989–2009	Diversification benefits are partially confirmed in-sample and rejected out-of-sample
Belousova and Dorfleitner (2012)	✓			✓		I	M	1995–2010	For European investors. Across commodities, strong variation diversification benefits
Pojarliev and Levich (2012)			✓	✓		X	M	1990–2010	FX markets may be beneficial if correctly managed
Graham, Kiviaho, and Nikkinen (2013)	✓			(✓)		X	W	1999–2009	Long-term co-movement implies less diversification benefits
Lizieri (2013)		✓		(✓)		X	M	1990–2011	Time-varying diversification potential, but less when it is most needed
Silvennoinen and Thorp (2013)	✓			✗		I	W	1990–2009	Volatility in stock markets is connected to correlation with commodities
Simon (2013)	✓			(✓)		X	W	1991–2011	Commodity-Equity correlation increased, but might still be low enough for diversification
You and Daigler (2013)	✓		✓	✓	✓	X/I	W	1994–2010	An augmented portfolio outperfoms the traditional one
Huang and Zhong (2013)	✓	✓		✓		X	D/M	1970–2010	Commodities and REIT are not spanned by traditional portfolios
Kroencke et al. (2014)			✓	✓	✓	I	M	1976–2011	Carry Trade, Value, and Momentum FX styles lead to large diversification benefits
Bessler and Wolff (2015)	✓			✓	✗	X	M	1983–2013	Only metals and energy add value. Agricultural & livestock does not.
Bhardwaj et al. (2015)	✓			(✓)		X	M	1959–2014	Commodity-Equity correlation increased compared to Gorton and Rouwenhorst (2006)
Kremer (2015)	✓			(✓)	(✓)	X	M	1991–2013	Only momentum strategy portfolios are not spanned by traditional portfolios
Lombardi and Ravazzolo (2016)	✓				✗	X	W	1980–2015	Commodities increase portfolios returns at the cost of higher volatility
Cotter et al. (2017)	✓		✓	(✓)	✗	X	M	1986–2014	Commodities and FX do not improve investment opportunity set
Daigler, Dupoyet, and You (2017)	✓		✓	✓	✓	I	D	1990–2012	MV portfolio incl. commodity futures has higher Sharpe ratios than equity benchmarks
Daskalaki et al. (2017)	✓			✓	✓	X	M	1990–2013	Effect is even stronger if commodity indices mimic trading strategies

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Yan and Garcia (2017)	✓			(✓)	(✓)	X/I	M	1991–2015	Only momentum strategy commodity indices enhance traditional portfolios
Gao and Nardari (2018)	✓				(✓)	X	M/Q	1976-2012	Only forward looking strategies and exploiting higher moments increases economic value
Henriksen (2018)	✓				(✓)	X/I	M	2001–2015	Only time-varying benefits from long/short commodity indices
Demiralay, Bayraci, and Gaye Gencer (2019)	✓			✓		I	W	1992–2014	Commodities offer conditional diversification benefits.
Henriksen et al. (2019)	✓			(✓)	(✓)	X	W	1995–2017	Only gold and managed commodity indices may outperform a traditional portfolio.
Platanakis et al. (2019)	✓	✓		✗		X	M	1997-2015	Adding alternative assets are harmful to U.S. investors.

Note: Literature summary on the diversification benefits and inflation hedge capabilities of commodities, real estate, and foreign exchange. Note that we abbreviate Commodities (C), Real Estate (R), Foreign Exchange (F), In-Sample (InS) and Out-of-Sample (OoS) analysis, Frequency (Freq.), daily (D), weekly (W), monthly (M), quarterly (Q), semi-annual (S), and annually (A) data, individual assets (I) and Indices (X). Moreover, ✓, (✓), and ✗ indicate whether diversification benefits and inflation hedge capabilities are confirmed, partially confirmed/mixed, or rejected in the particular study, respectively.

Appendix B: Data

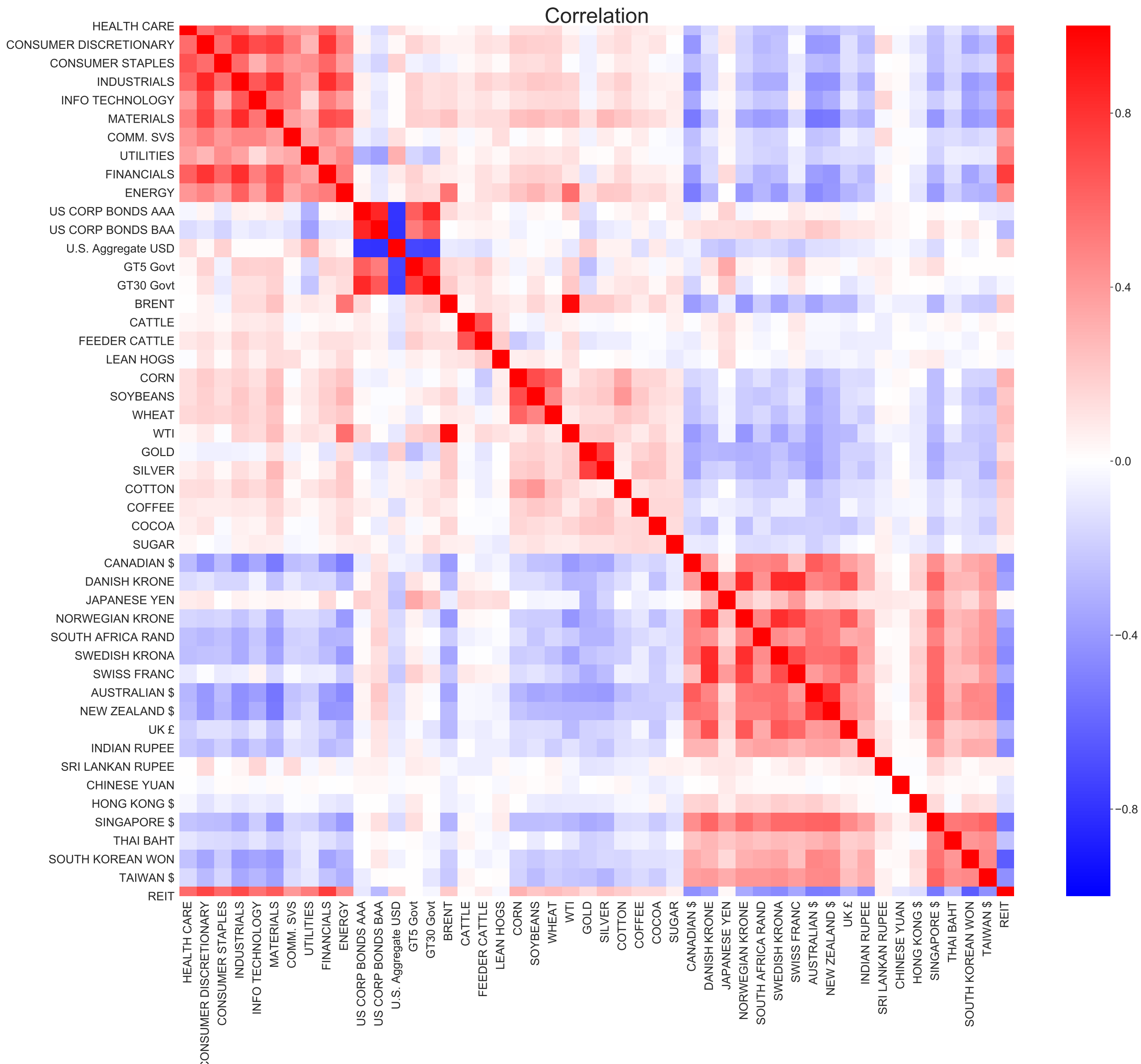

Figure 2: Correlation Heatmap of monthly returns 01-1990–12-2018 (Total Sample Period)

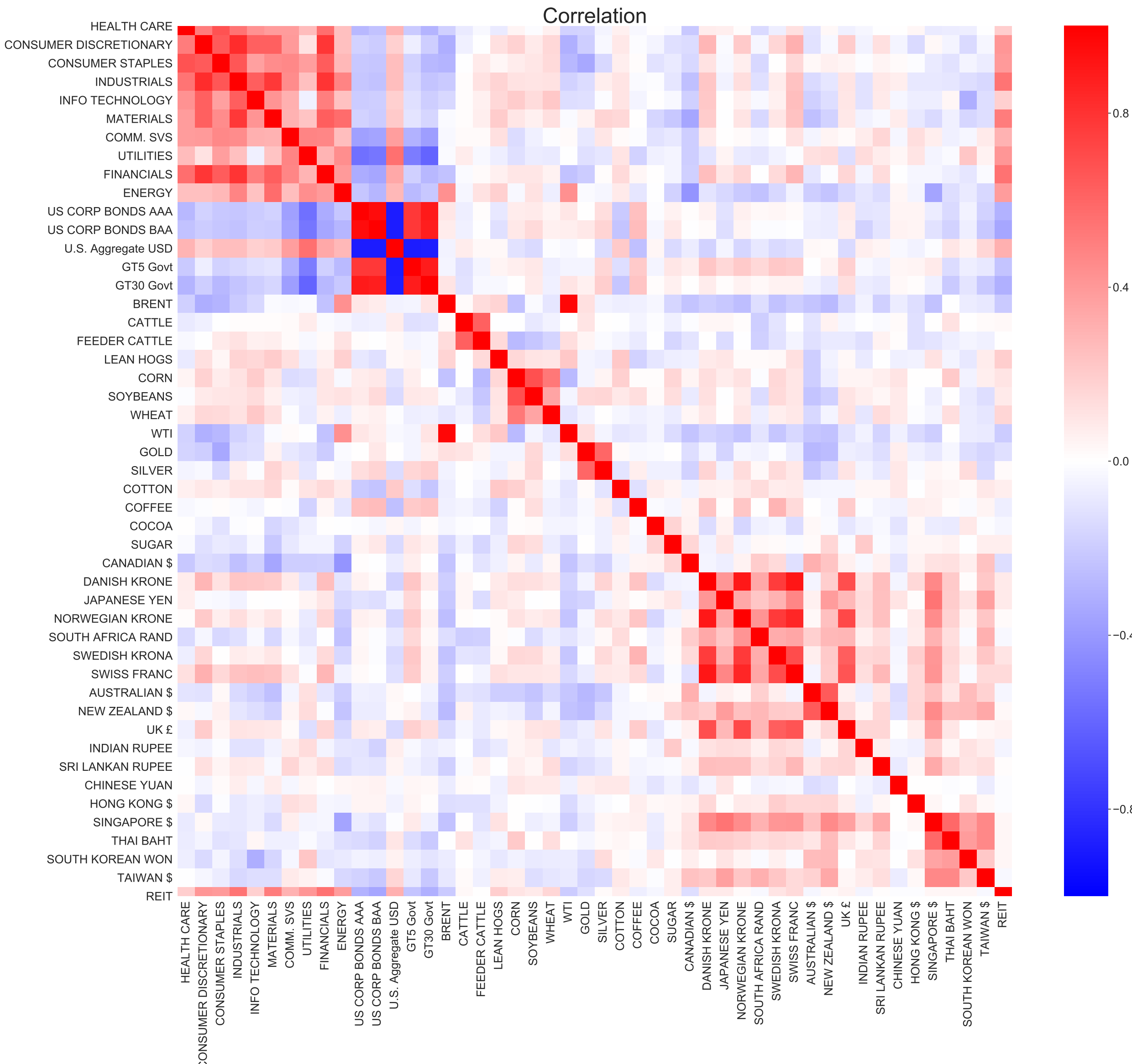

Figure 3: Correlation Heatmap of monthly returns 01-1990–12-1999 (In-Sample Period)

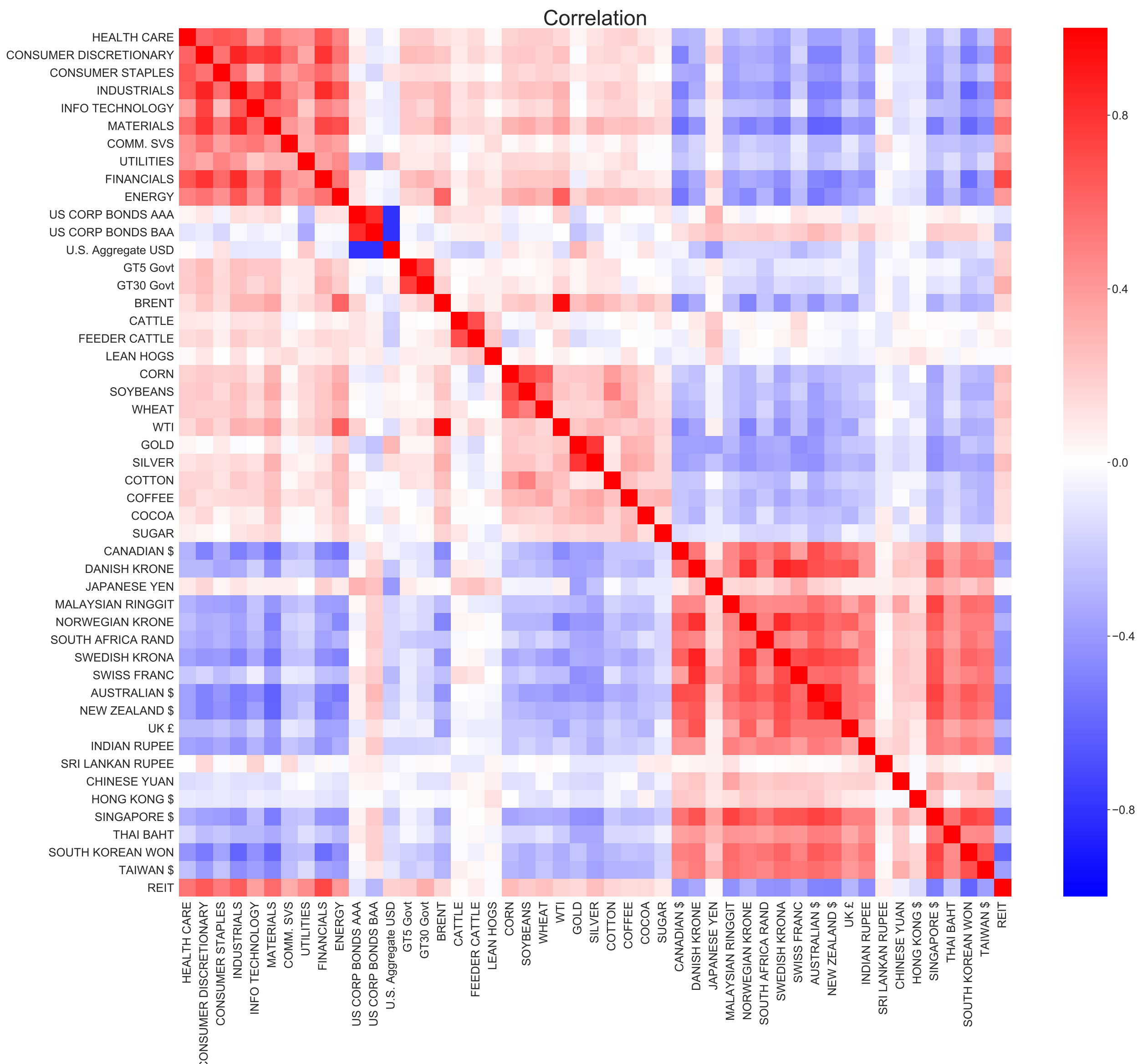

Figure 4: Correlation Heatmap of monthly returns 01-2000–12-2018 (Out-of-Sample Period)