Asymmetric Guessing Games

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20 October 2020

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Abstract

This paper theoretically and experimentally investigates the behavior of asymmetric players in guessing games. The asymmetry is created by introducing $k > 1$ replicas of one of the players. Two-player and restricted N-player cases are examined in detail. Based on the model parameters, the equilibrium is either unique in which all players choose zero or mixed in which the weak player ($k = 1$) imitates the strong player ($k > 1$). A series of experiments involving two and three-player repeated guessing games with unique equilibrium is conducted. We find that equilibrium behavior is observed less frequently and overall choices are farther from the equilibrium in two-player asymmetric games in contrast to symmetric games, but this is not the case in three-player games. Convergence towards equilibrium exists in all cases but asymmetry slows down the speed of convergence to the equilibrium in two, but not in three-player games. Furthermore, the strong players have a slight earning advantage over the weak players, and asymmetry increases discrepancy in choices (defined as the squared distance of choices from the winning number) in both games.

Keywords: Guessing game, asymmetry, convergence, game theory, experimental economics
JEL Classification: C72, C92

1 Introduction

Any investment situation that can be considered as a complex game with its potentially high number of players and strategies, requires both deep reasoning and strategic thinking due to

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the mutual determination of the resulting outcomes. This insight regarding the behavior of investors especially in financial markets can be captured by the "guessing" or "p-beauty contest" games (inspired by Keynes, 1936). In the standard N-player guessing game ($N \geq 3$), players simultaneously choose a number from a closed interval, generally $[0, 100]$, and the player whose number is closest to a given fraction (p) of the average of all of the chosen numbers is the winner. The game is dominance solvable and iterated elimination of weakly dominated strategies leads to the unique Nash equilibrium in which all players choose zero. However, experimental findings are not found to be aligned with this theoretical prediction: in general, participants apply the iterated dominance process up to three rounds and choose dominated strategies, especially in the first periods of play, although convergence is achieved as the game is played out repeatedly.[1] Grosskopf and Nagel (2008) introduce an even simpler version of this game with $N = 2$ in which iterative reasoning is unnecessary since choosing zero is the weakly dominant strategy (it is isomorphic to the game: "whoever chooses the smaller number wins").[2] In two-player games, the same pattern of findings persists except in the cases that involve professional participants, who tend to adopt the equilibrium strategy relatively more often.

In the literature, players in guessing games are typically treated symmetrically in terms of their influence on the formation of the target number. However, in very few markets are players actually identical in regards to their market power (e.g., budget). For example, in stock markets, "stronger" investors have the financial power to influence prices more than "weaker" investors. Due to these asymmetries, players may hold different beliefs (e.g., overconfidence) and behave differently. Moreover, their influence on the determination of the realized prices is potentially different.

In this paper, we introduce asymmetric players that better reflect real-life situations where market players are not identical. We model this asymmetry by introducing replicas of the players that essentially render some players relatively more powerful in influencing the target number. In two-player games, this comes down to multiplying the chosen numbers of strong and weak players by $k_s \geq k_w \geq 1$, respectively. Integer coefficients k_s and k_w can be considered as the strengths of the players. The target number is some fraction of the "weighted" average of the two choices. This implies that the strong player has more power to influence the target number in comparison

[1] The first experiments on guessing game were conducted by Nagel (1995). After this paper, a fruitful literature has emerged to study iterative reasoning, bounded rationality, and learning. See Duffy and Nagel (1997), Ho et al. (1998), Nagel (1995), and Stahl (1996) for early applications of iterated best reply and learning models. For more recent reviews, see Akin and Urhan (2011), Camerer (2003), Crawford et al. (2013), Nagel (2008) and Nagel et al. (2017). Most recently, Mauersberger and Nagel (2018) use the beauty contest game as a generative framework that embeds many different games and present an extensive review.

[2] See Burchardi and Penczynski (2014), Costa-Gomes and Crawford (2006), Chou et al. (2009), Fragiadakis et al. (2013) and Nagel et al. (2017) for other examples of two-player guessing games. Grosskopf and Nagel (2009) use feedback structure as the treatment variable in repeated two-player guessing games and find that learning is weakened with less information and bounded rationality argument is supported.

to the weak player.

There are few papers in the literature involving asymmetry, but they do not focus on the relative influence of players on the target. To characterize which decision rules players employ, Costa-Gomes and Crawford (2006) create asymmetry in one-shot two-player continuous payoff guessing games through having different p values in the target calculations and support (the set of alternatives that can be chosen). Similarly, Güth et al. (2002) introduce heterogeneity with different p values in guessing games with continuous payoff and interior equilibrium and find that heterogeneity slows down convergence and reduces earnings.[3] The focus and structure of the asymmetry in the aforementioned papers are different than in ours. In our model, there is only one target and the power of players to influence the target is different, whereas different players have different targets in the models mentioned above.[4]

Another type of asymmetry introduced within groups is based on experience and sophistication. Slonim (2005) examines the effect of experience and finds that experienced players who have a better grasp of how the game is actually played, tend to shift their choices away from the equilibrium as inexperienced players are added to the population, and they are observed to earn more. With a similar design, Liu (2016) systematically varies the proportion of players who know how the game should be played by informing them of the game theoretic solution and finds that choices tend to decrease as the proportion of informed players increases, but exclusively when this proportion is large. Agranov et al. (2012) also change the composition of players by varying the proportion of random-choosing computers and graduate students and find that as the proportion of graduate students increases, players tend to systematically lower their choices. In these papers, there is informational asymmetry among players, but they are symmetric in terms of their ability to influence the target. This type of asymmetry influences the reasoning process of players regarding what the target can be according to the composition of the population. By contrast, in our design, heterogeneity is driven solely by the differing strengths of players to influence the target without any informational asymmetry.

Kopányi et al. (2019) investigate how price dynamics in a learning to forecast (LtF) asset pricing experiment are influenced by financial advisors who attract more investors by forecasting more accurately and are able to influence market prices asymmetrically. They motivate their model with the observations that the effects of market participants on market prices may partially depend on past successes. Successful financial advisors attract more money and therefore they

[3] Kovac et al. (2008) replicate Güth et al. (2002) and find conversely that heterogeneous players guess closer and converge faster to the equilibrium.

[4] Note that with the continuous payoff structure used by these two papers, both players choosing zero is still the unique equilibrium. However, zero is the only rationalizable strategy obtained through an iterated elimination of *strictly dominated* strategies. Moreover, we can also talk about (Pareto) efficiency of choices and zero is also the unique Pareto optimal equilibrium. Nagel et al. (2017) examine both two and $N > 2$ player games with fixed and continuous payoff and find that behavior is not affected by the payoff structure, is boundedly rational and can be described by the level k model.

have a greater impact on market prices. They show that the asymmetry driven by the competitive market forces may reduce price volatility and mispricing unless the competition is not fierce. This paper is closely related to ours in terms of both the types of markets it models and the asymmetric impact of the players on the outcome. Our model can also be motivated by the fact that in financial markets, there are investment advisors who manage different account sizes and their impact on the markets differ accordingly. Kopányi et al. (2019) use experimental LtF asset markets in which the impact of players and prices are endogenously determined. We use guessing games to model these markets where the impact of each player is taken as fixed.

The theoretical solution of our model depends on the relative strength of the players and the value of p. We first characterize the equilibrium for two-player case and then extend our theoretical analysis to N-players. Our theoretical analyses show that for any given k_w and k_s values, if p is small enough ($p < \frac{k_w+k_s}{2k_s}$), the equilibrium is the same as with the standard guessing game. On the other hand, if p is high enough ($p > \frac{k_w+k_s}{2k_s}$), the payoff structure of the game changes and there is at least one mixed strategy (no pure strategy) Nash Equilibrium. In this case, the weak player is disadvantaged in the sense that her winning strategy is imitating the strong player. She can only win if she chooses a smaller (but not too small) number than the number of the strong player. Moreover, for any given k_w and k_s values, there is a high enough threshold p^* such that for all $p > p^*$, the strong player always wins the game unless there is a tie (there is a unique completely mixed uniform strategy Nash equilibrium). However, as relative strength k ($\frac{k_s}{k_w}$) increases, the range in which the strong player wins gets larger only up to a threshold value of k^* after which payoffs do not change. For general N-player cases where each player i has a potentially different $k_i \geq 1$, we characterize a sufficient condition for choosing zero for all players to be the unique pure strategy Nash equilibrium. This condition implies that if the strongest player is strong enough with a sufficiently small p, or if the asymmetry among players is not high, then choosing zero for all players is the unique Nash Equilibrium.

We then report the results of an experiment we conducted that adopts commonly used design features in the literature, excluding the asymmetric influence of players on the target. We form two and three-player groups (In many studies, $N \geq 3$. At one extreme, Bosch-Rosa and Meissner (2020) examine one-player guessing games and in the other, Bosch-Domenech et al. (2002) conduct a newspaper experiment with $N = 7900$). To keep the game as simple as possible, we conduct the experiment by using $p = 1/2$ such that in all games, there is a unique equilibrium in which all players choose zero as in the standard game (Duffy and Nagel (1997) also used only $p = 1/2$. Costa-Gomes and Crawford (2006), Güth et al. (2002) and Nagel (1995) use p as a treatment variable including this value but most frequently, $p = 2/3$ is observed to be used in the literature). In our design with two players, the control is the standard two-player guessing game where $k_s = k_w = 1$. In the treatment, the players are asymmetric, $k_w = 1$ and $k_s = 9$. In three-player games, we have one strong and two weak players. In the control, we have $k_{w1} = k_{w2} = k_s = 1$, whereas we set the k values as $k_{w1} = k_{w2} = 1$ and $k_s = 8$ in the treatment.[5] We use (weighted) mean as the order statistics in the calculation of the target (The treatment variable in Duffy and Nagel (1997) is the order statistics -mean, median or maximum of the chosen numbers). We use the common bounded and fixed support for guesses $[0, 100]$ (In Costa-Gomes and Crawford (2006), the support is a treatment variable. Benhabib et al. (2019) have an unbounded guessing interval). We have a tournament structure and there is only one winner unless there is a tie (See Costa-Gomes and Crawford (2006), Güth et al. (2002), Kocher and Sutter (2006) and Nagel et al. (2017) for continuous payoff structure where each player is paid according to their distance to the target).

Regarding the formation of the groups, participants are randomly matched, and they have no knowledge regarding their partners. In the asymmetric games, the roles are randomly assigned. Participants play the game individually against each other (see Kocher and Sutter (2005), Kocher et al. (2006) and Sutter (2005) for guessing games played by teams) and there is no communication (See Baethge (2016), Burchardi and Penczynski (2014) and Penczynski (2016) for the effect of communication). Participants play the game for ten rounds in a fixed pair setting that allows us to observe convergence and behavioral dynamics (There are a few papers that involve repeated play in the two-player guessing games. Burchardi and Penczynski (2014) has three rounds but did not report the results. Fragiadakis et al. (2013) have two phases and ask subjects to recall their choices or play against their own previous choices from memory in phase II. Ours is similar to Grosskopf and Nagel (2009)). Finally, at the end of each round, we give full feedback to all players in the group including all the chosen numbers in the group, the calculated target, and the winner (See Grosskopf and Nagel (2009), Kocher et al. (2014), Sbriglia (2008), and Weber (2003) for the effect of feedback).

Our main research questions are: *i*) How do the first period behavior and overall choices differ between the symmetric and the asymmetric cases? *ii*) Do the choices in each of the symmetric and asymmetric cases converge to the equilibrium over time? *iii*) If there is convergence, are there any differences in terms of the speed of convergence between the symmetric and asymmetric cases? Our results imply that non-equilibrium behavior is more common and overall choices are farther from the equilibrium in two-player asymmetric games than in symmetric games. In all cases, there is convergence towards equilibrium. Introducing asymmetry slows down the convergence to the equilibrium in two, but not in three-player games. Finally, strong players earn more than weak players and asymmetry increases discrepancy in choices (defined as the squared distance of choices from the winning number) in both games.

The rest of the paper is organized as follows. Section 2 introduces the model and characterization of the equilibrium. Section 3 describes the experimental design in detail. Section 4 presents

[5]Together with $p = 1/2$, we choose these k values to make the target easy to calculate. The denominator in the target is 20 in all asymmetric games. See the instructions for details.

the experimental results. Finally, section 5 concludes with a discussion of the results.

2 The Model and the Equilibrium Analysis

Before introducing the asymmetric case, we briefly explain the standard guessing game. In the standard N-player guessing game ($N \geq 3$), players simultaneously choose an integer number from a closed interval $[0, z]$ where $z \in \mathbb{Z}^+$ (generally $z = 100$). The player whose number is the closest to the target number (T) wins the game. The target number is calculated as follows:

$$T = \left(\frac{1}{N} \sum_{i=1}^{N} g_i \right) p$$

where g_i is the player i's guess, $0 < p < 1$, and p is common knowledge. When $N = 2$, the target simply becomes

$$T = \left(\frac{g_1 + g_2}{2} \right) p.$$

We have a tournament structure where the winner of the game receives a pre-determined fixed prize, and the other players receive nothing. If there is a tie, the prize is equally divided among the winners. The standard N-player guessing game is dominance-solvable under rationality and common knowledge of rationality assumptions. It is straightforward to see that all choosing zero is the only rationalizable strategy combination that survives the infinitely repeated simultaneous elimination of weakly dominated strategies. Hence, given $0 < p < 1$, all players choosing zero is the unique pure strategy Nash equilibrium of the game. In two-player guessing games with tournament payoff, iterative reasoning is unnecessary since choosing zero is the weakly dominant strategy (the lower number always wins).

2.1 Two-player Asymmetric Guessing Games

We now introduce the two-player guessing game with asymmetric players. This is achieved by introducing replicas of the players (the one who has more replicas is called the strong player and she has more power to determine the target number). To do this, guesses of the players are multiplied by k_s and k_w (s stands for strong, w stands for weak) in the target number calculation. The parameters $k_s \geq k_w \geq 1$ are positive integers[6] and represent the power level of the strong and weak players. We define a parameter called relative strength $k = \frac{k_s}{k_w}$ to simplify the exposition throughout the paper (the standard two-player guessing game is represented by $k_s = k_w \geq 1$ or $k = 1$). The target is calculated as follows:

$$T = \left(\frac{g_w + k g_s}{k + 1} \right) p$$

[6] $k \geq 1$ values can be assumed to be real numbers and the results still hold.

where $0 < p < 1$ and g_w, g_s are the guesses of weak and strong players, respectively. Parameters p and k are common knowledge. As in the standard game, players simultaneously choose an integer from the closed interval $[0, 100]$.[7] The player with the closest guess to the target number wins the game, and in the event of a tie, the prize is equally divided between the players. Mathematically,

$$
\begin{aligned}
|g_w - T| - |g_s - T| < 0 &\Rightarrow \text{ the weak player wins.}\\
|g_w - T| - |g_s - T| > 0 &\Rightarrow \text{ the strong player wins.}\\
|g_w - T| - |g_s - T| = 0 &\Rightarrow \text{ there is a tie.}
\end{aligned}
$$

In contrast to the standard guessing game, the equilibrium in the asymmetric case is not unique for all $p < 1$. The following lemmas characterize the equilibrium in detail. All proofs are in the Appendix A1.

Lemma 1 *Let $k > 1$. If $p < \frac{k+1}{2k} < 1$, then the player whose number is smaller wins the game. This implies that $(g_s, g_w) = (0, 0)$ is the unique Nash Equilibrium in weakly dominant strategies.*

Lemma 1 states that although we have an asymmetry with $k > 1$, two-player asymmetric guessing games are indistinguishable from the standard two-player guessing games if p is small enough ($p < \frac{k+1}{2k} < 1$), and the player with a smaller number wins the game.

Lemma 2 *Let $k > 1$. If $\frac{k+1}{2k} < p < 1$, then*

i) the weak player can win only by imitating the strong player, $ag_s < g_w < g_s$ where $a = \frac{2pk-k-1}{k+1-2p} < 1$;

ii) there is at least one mixed strategy Nash Equilibrium that involves the weak player imitating the strong player.

Lemma $2 - i$ is about the payoff structure of the game in case which $\frac{k+1}{2k} < p < 1$, and implies that choosing a smaller number guarantees a win for the strong player, but she may also win by choosing a larger number than the weak player. However, the weak player cannot win by choosing a larger number and choosing a smaller number is not enough for her to win. She must also choose a number that is greater than some proportion of the number of the strong player. That is, the weak player can win only if she imitates the strong player. This lemma also implies that as either p or relative strength k or both increase, the weak player must follow the strong player more closely to be able to win. The reason is as follows: since $\frac{da}{dk} > 0$ and $\frac{da}{dp} > 0$, an increase in k or p implies a higher a value. This further implies that ag_s gets closer to g_s (for the

[7] Lopez (2001) states that when calculating the target number, the experimenter must use decimal approximation. For this reason, he calls the game as a"beauty contest decimal game". He proves that the beauty contest decimal game is equivalent to the beauty contest integer game. Thus, he concludes that any experimental guessing game is equivalent to its integer restricted version.

extreme case of this, see proposition 5). This shrinks the range of guesses that enables the weak player to win. Thus, the weak player must follow the strong player more closely to win.

Lemma $2-ii$ characterizes the equilibrium in this case, and states that if p is large enough, then the equilibrium is no longer pure. It is essentially mixed and may not be unique. More importantly, imitation emerges as an equilibrium behavior.[8] The strong player randomizes over her action space, but the weak player randomizes over her actions such that the maximum number she assigns positive probability is always less than or equal to the maximum number the strong player assigns positive probability. Furthermore, the equilibrium does not involve the weak player always choosing small numbers. Thus, in any equilibrium, the weak player randomizes to imitate the strong player. By calculating mixed equilibria for some two-player games with a narrower action space such as $[0, 10]$, we provide examples of this phenomena. The strong player randomizes over small and some large numbers, but the weak player only randomizes over numbers that are less than the largest number the strong player assigns positive probability (see Appendix A2 for these examples).

Lemma 3 *Let* $k > 1$. *If* $p = \frac{k+1}{2k}$,

i) and $g_w = 0$, *there will be a tie regardless of the value of* g_s;

ii) and $g_w \neq 0$, *the player whose number is smaller wins the game;*

iii) in any equilibrium, $g_w = 0$. *Pure strategy Nash equilibria are* $(g_s, g_w) = (0, 0)$ *and* $(g_s, g_w) = (1, 0)$. *There are infinitely many mixed strategy Nash equilibria including randomization between* $g_{s1} = 0$ *and* $2 \leq g_{s2} \leq z$ *equally and any randomization between* $g_{s1} = 0$ *and* $g_{s2} = 1$.

Lemma $3-i$ and $3-ii$ are about the payoff structure of the game in case which $p = \frac{k+1}{2k}$, and state that there will be a tie for any choice of the strong player if the weak player chooses zero, but that the player who chooses a smaller number wins if the weak player does not choose zero.

Lemma $3-iii$ characterizes the equilibrium in this case, and indicates that there are only two pure strategy and infinitely many mixed strategy Nash equilibria with randomization by the

[8] The imitation behavior in our case is completely different from the well-known strategic non-equilibrium behavior of the players in standard N-player guessing games who are rational but believe that others are boundedly rational. Initially, since actions of the subjects are strategic complements (See Hanaki et al. (2019) for the strategic environment effect in guessing games), experienced/sophisticated players try to imitate the less experienced/sophisticated players and guess higher numbers than zero to win the game, which amplifies deviations from the equilibrium. As the game is played repeatedly, guesses converge to the unique equilibrium. It is also different from the adaptive behavior of less sophisticated players who learn to play equilibrium by imitating the winners. Here, imitation emerges as an equilibrium phenomena. Our usage of the term imitation may also suggest the pooling equilibrium in signaling games where one (low) type imitates the other (high) type. In our case, there is neither asymmetric information nor signaling. The weak player needs to follow the strong player to win, and in equilibrium she does not have to follow the same strategy as the strong player.

strong player that always include $g_{s1} = 0$.[9] Moreover, the weak player always chooses zero and there will be a tie in all equilibria.[10] For some example games and their solutions related to lemma 1, 2, and 3, see Appendix A2.[11]

The payoff matrix is influenced by the values of p and k. Figure 1, proposition 4 and proposition 5 investigate this influence. Since the strategy space for each player is the set of integers in $[0, 100]$, we have a 101×101 finite game that has a payoff matrix of $101^2 = 10201$ cells. Thus, given p and k, it is easy to figure out the number of cells (in percentage) at which the strong player wins. Figure 1 shows this winning percentage of the strong player as a function of p and k. For example, if $k = 7$ and $p = 0.74$, then the winning percentage is % 67.91. This means that with these parameter values, the strong player is the winner in approximately % 68 of all cells in the payoff matrix. Figure 1a and 1b are identical except that they are captured from different angles.

Notice that there are mainly two regions in the Figure 1: the flat region and the sloped region. The two regions are separated by a white curve which corresponds to the points satisfying $p = \frac{k+1}{2k}$. The flat region below the white curve corresponds to the points satisfying $p < \frac{k+1}{2k}$, and the steeper region above the white curve corresponds to the points satisfying $p > \frac{k+1}{2k}$.

Remember that if $p < \frac{k+1}{2k}$, the player with the smaller number wins the game (See Lemma 1). Then, once $p < \frac{k+1}{2k}$ is satisfied, the structure of the payoff matrix does not change. This is the reason why the surface below the white curve is flat. In this flat region, the winning percentage (about 49%) of the strong player does not change in accordance with changes in p and k.

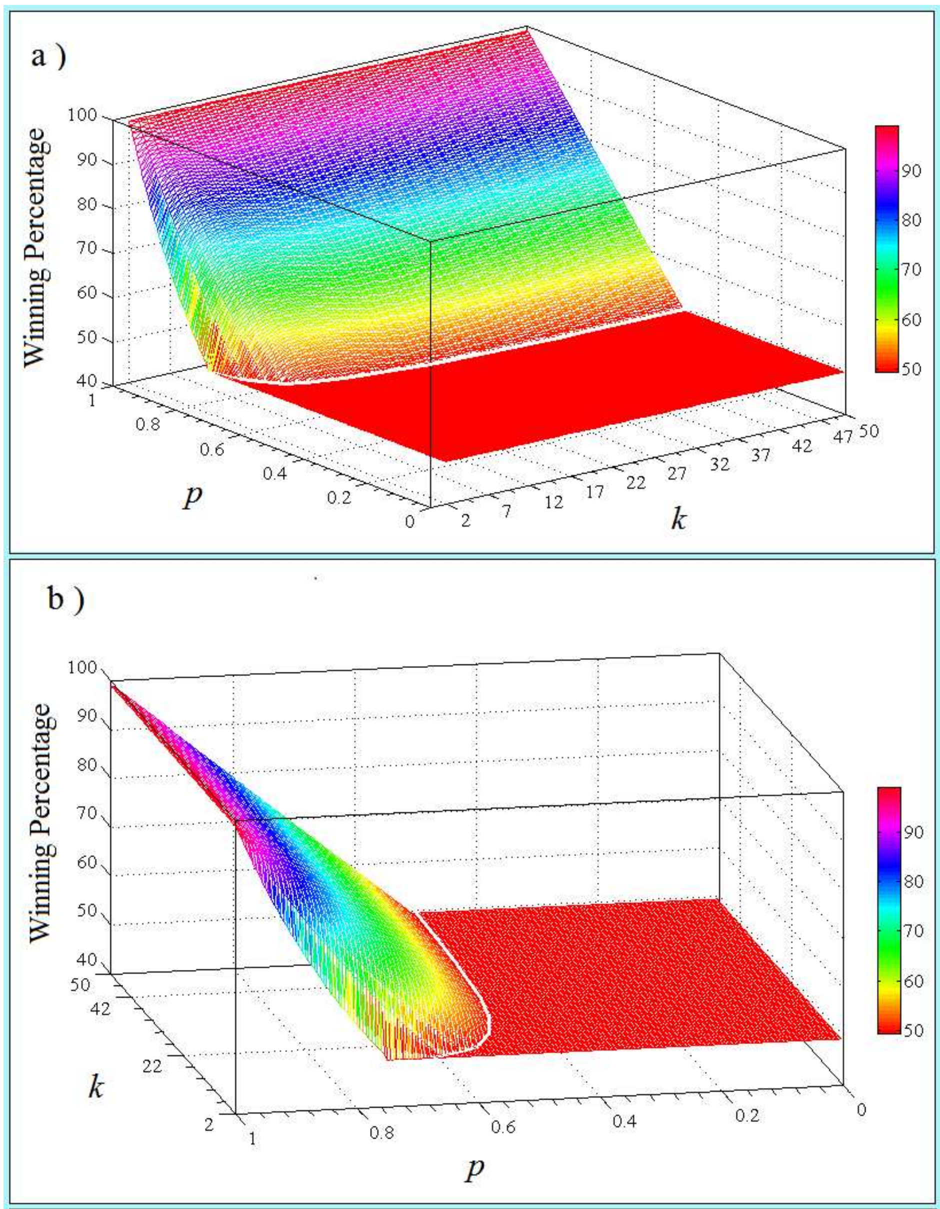
However, if $p > \frac{k+1}{2k}$, we know that for the weak player to win, g_w should satisfy $ag_s < g_w < g_s$ where $a = \frac{2pk-k-1}{k+1-2p}$ (See Lemma 2). Since the parameter a is a function of p and k, the structure of the payoff matrix changes in accordance with the changes in p and k. Therefore, we have a non-flat region above the white curve.

One observation about the steeper region of the Figure 1 is that for any fixed $p > \frac{k+1}{2k}$ and

[9] There is evidence that in games with strict strategic complementarities, mixed Nash equilibria are unstable (Echeniquea and Edlin (2004); Heinemann et al. (2009)). Since ours is also a game of strategic complementarity and there are mixed strategy equilibria (if p is not small enough), it can be expected that these equilibria would also be unstable.

[10] If players are allowed to choose real numbers instead of integers, (this does not matter practically as behavior in experiments does not change, but has theoretical importance) we conjecture the following about the equilibria: 1. The mixed strategy equilibrium involving imitation behavior in Lemma 2 still holds; 2. In Lemma 3, $(g_s, g_w) = (0, 0)$ is the unique pure strategy Nash equilibrium. The pure strategy $(g_s, g_w) = (1, 0)$ and the set of mixed strategies including $g_w = 0$ and any randomization between $g_{s1} = 0$ and $g_{s2} = 1$ are no longer equilibria. Furthermore, the set of mixed strategy Nash equilibria includes $g_w = 0$ and equal randomization between $g_{s1} = 0$ and $0 < g_{s2} \leq z$.

[11] If the payoff scheme is continuous (for example, with payoff functions $\pi_i = 100 - (g_i - p\frac{g_w + k.g_s}{k+1})^2$ where $i = w, s$, as in Nagel et al. (2017)), then out of equilibrium, it is optimal to play $g_w = \frac{p.k}{k+1-p} g_s$ and $g_s = \frac{k}{k+1-k.p} g_w$. Zero is the only rationalizable strategy obtained by iterated elimination of *strictly dominated* strategies for all $1 > p > 0$ and $k \geq 1$. Moreover, since $\frac{p.k}{k+1-p} > \frac{k}{k+1-k.p}$ for all $k > 1$, in asymmetric games, iteration steps converge to zero faster for the strong player.



Note: 0 < ***p*** < 1. ***k*** is an integer and ***k*** > 1. ***k*** values above 50 are not included since there is no significant change in the figure's shape for ***k*** > 50.

Figure 1: Winning Percentage of the Strong Player as a Function of p and k

lower values of k, as k increases, the surface firstly becomes steeper and then its slope becomes constant in the direction of k axis. This implies that for any fixed $p > \frac{k+1}{2k}$, increasing the power level causes the winning percentage of the strong player to rise, but only to an extent. The following proposition formalizes this observation.

Proposition 4 *Consider a guessing game with $p > \frac{k+1}{2k}$. There exists $k^* > 1$ such that for all $k < k^*$, as k increases, the range in which the strong player wins gets larger and for no $k > k^*$, the game structure changes.*

Proposition 4 implies that in asymmetric guessing games, there is a limit to the strength of the strong player in the sense that for any given p, there is a certain threshold level of strength (k) after which becoming stronger does not bring any more advantage.

Another observation is that for any fixed $k > 1$ with $p > \frac{k+1}{2k}$, as p increases, the winning percentage of the strong player rises gradually and reaches its maximum ($\frac{100(z-1)}{z}\%$) that can be formalized as follows:

Proposition 5 *Consider a guessing game with $p > \frac{k+1}{2k}$. For any $k > 1$, there exists $p^* < 1$ such that for all $p > p^*$ the strong player wins the game if $g_w \neq g_s$. This payoff structure implies that there is a unique completely mixed uniform strategy Nash equilibrium.*

In the standard guessing game, the game turns into a pure coordination game when $p = 1$ although it is still a constant sum game. Proposition 5 generalizes this result into asymmetric games. For any given k value, if p is close enough to one, then the asymmetric game turns into a pure coordination game where the strong player wins whenever players choose a different number. This is the extreme case of lemma 2 that implies that the weak player should imitate the strong player more and more closely to win the game as p increases. This proposition states that if p is very high, then the weak player cannot win and can get a draw only if she imitates the strong player perfectly ($g_w = g_s$).

Figure 2 summarizes how the equilibrium changes depending on the value of k and p. In the region below the curve (solid line), there is a unique pure strategy Nash Equilibrium which is choosing zero for both players. In this region, whoever chooses the smaller number wins (Lemma 1). On the curve that converges to 0.5 as k goes to infinity, there are infinitely many mixed strategy Nash Equilibria where $g_w = 0$ (Lemma 3). Above the curve, there is at least one mixed strategy Nash Equilibrium and there is imitation behavior such that the weak player can only win if she imitates the strong player (Lemma 2). Finally, above the dashed curve that converges to $p = 1 - \frac{1}{2z}$ (in the current game, $z = 100$), the game turns into a coordination game and there is a unique mixed strategy Nash Equilibrium that involves uniform randomization on full support and the strong player always wins unless there is a tie (Proposition 5).

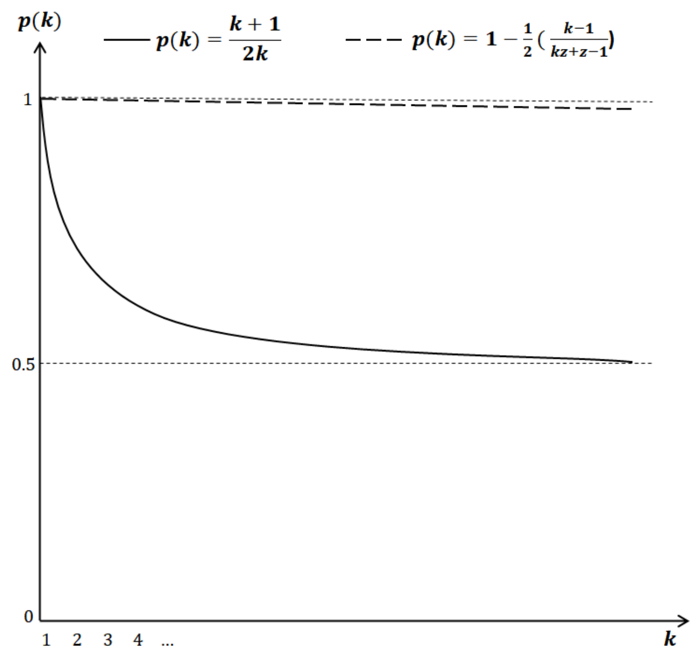


Figure 2: Equilibrium Characterization Based on the Value of k and p.

2.2 N-player Asymmetric Guessing Games

We now introduce the asymmetric N-player case. Players simultaneously choose a number from a closed interval, e.g. $[0, 100]$. Let $\{g_1, g_2, g_3, ..., g_N\}$ be the numbers chosen by the players. Asymmetry among the players comes from the fact that each player has a potentially different k value, namely $k_i \geq 1$, $i = 1, 2, .., N$; implying that player i has k_i replicas. This means that the guess of player i will be multiplied by k_i. The target number is given by:

$$T = \frac{\sum_{i=1}^{N} k_i.g_i}{\sum_{i=1}^{N} k_i}.p$$

The player whose number is the closest to the target number wins the game and the others receive nothing. If there is a tie among the players, they share the prize. The following lemma characterizes the sufficient condition for $(0, 0, ..., 0)$ strategy profile to be the unique pure strategy Nash equilibrium in this game.

Lemma 6 *Let there be $N \geq 2$ asymmetric players playing the asymmetric guessing game described above. $(0, 0, ..., 0)$ strategy profile is the unique pure strategy Nash equilibrium achieved by* *the iterated elimination of weakly dominated strategies if*

$$p < min\{\frac{\sum_{i=1}^{N} k_i}{2k^*}, 1\}, \ \textit{where } k^* = max\{k_1, k_2, k_3, ..., k_N\}$$

This result implies that if the strongest player is not stronger than the sum of all of the other players ($1 \leqslant \frac{\sum_{i=1}^{N} k_i}{2k^*}$), then strategy profile $(0, 0, ..., 0)$ is the unique Nash equilibrium of this game since the above condition is satisfied by definition ($0 < p < 1$). But even in the highly asymmetric case (the strongest player is stronger than the sum of all other players), $(0, 0, ..., 0)$ is still the unique Nash equilibrium if p is small enough. Lemma 1 is a special case of this lemma where $N = 2$, $k_1 = 1$ and $k_2 = k$. The classical symmetric N-player guessing game is also a special case of this lemma where $k_i = 1$ for all i.

3 Experimental Design and Procedures

The experiment was conducted at the experimental laboratory of a university in one of the big cities of Turkey. We collected data from 313 subjects who were mostly undergraduate students. A session lasted approximately 45-50 minutes. Computers in the experimental laboratory were isolated so that the subjects could not see other screens. Subjects were also not allowed to communicate during the sessions. The experimental code was programmed with z-Tree (Fischbacher, 2007).

Both two-player and three-player games were played. In addition to control groups (symmetric games) in which standard (two-player and three-player) guessing games were played, there were treatment groups in which players have different types (asymmetric games). In the two-player asymmetric games, players in each pair had different k values: $k_w = 1$ and $k_s = 9$. In the three-player asymmetric games, two of the players had $k_{w1} = k_{w2} = 1$ and the other had $k_s = 8$. Table 1 summarizes the design of the experiment. For brevity, we use the following abbreviations: S2 for two-player symmetric games; AS2 for two-player asymmetric games, S3 for three-player symmetric games, and AS3 for three-player asymmetric games.

Table 1: Experimental Design Overview

2-Player		3-Player	
Symmetric (S2)	Asymmetric (AS2)	Symmetric (S3)	Asymmetric (AS3)
N = 68	N = 98	N = 33	N = 114
$k_s=k_w=1$	$k_s=9$, $k_w=1$	$k_s=k_{w1}=k_{w2}=1$	$k_s=8$, $k_{w1}=k_{w2}=1$

In all the games, the strategy space for each player was the set of integers in $[0, 100]$. We set $p = 1/2$ in all sessions so that the unique pure strategy Nash equilibrium for both two and three-player asymmetric games is choosing zero for everyone (See Lemma 6).

The experiment was announced via e-mail and participants were registered to one of the experimental sessions online. We screened subjects so that any subject could participate in only one session. When subjects arrived at the laboratory, they were placed randomly to separate computer stations (In cases where more participants than needed showed up, the participant arriving last was dismissed with a 10 Turkish Lira (TL) show-up fee). Then, all subjects were given an instructions sheet in which the general rules and the rules of the game were written (See Appendix B1 for the instructions). Instructions were read aloud, and questions related to the instructions were answered. Then, subjects were given a multiple-choice quiz in a computer environment. The quiz consisted of five questions related to the calculation of the target number and the payment scheme. After the quiz, any remaining questions of the subjects were answered. Then, the software randomly formed two and three-player groups that did not change throughout a given session and randomly assigned player types to the subjects. Except for the controls, in each session, there was one weak player and one strong player in two-player games. In three-player games, there were two weak players and one strong player. Before starting the first period, subjects were informed about their types on the computer screen for 15 seconds. Instructions were framed neutrally and terms such as "weak" or "strong" were avoided. Instead, participants were given the following information on the screen "You are Player A (B) and your number will be multiplied by 1 (9)" or "You are Player A (B or C) and your number will be multiplied by 1 (1 or 8)" depending on the subject's type in the two and three-player games, respectively. They were also informed that their types and groups were assigned randomly by the software, and other player(s) in the group will not change throughout the whole session.

After informing players about their types, they played the game for 10 periods. During each period, the formula of the target number and the type of the player were displayed on the screen. Subjects were given 45 seconds for each period to decide on and submit their numbers. At the end of each period, full feedback was provided for 15 seconds. That is, after all subjects submitted their numbers, they were provided feedback about their own number, the number(s) the other player(s) chose in the group, the calculated target number and the winner. At the end of the last period, they were asked to fill out a short survey including some demographic questions. They were also provided a space to describe their strategies in their own words. After subjects completed the survey, the software randomly chose 3 of the 10 periods and calculated the earnings in those periods. For each chosen period, the winner in each group earned 10 TL and 15 TL and the other player(s) earned nothing in two and three-player games, respectively. In case of a tie, winning subjects split the fixed amount among themselves. This payment scheme information was also explained in detail in the instructions. Each subject was paid 10 TL show up fee plus what they earned from the selected 3 periods. On average, participants earned 30 TL in total (approximately \$11 at the time). At the end of each session, subjects received their payments individually.

4 Experimental Results

Since there is strong experimental evidence that initial behavior is quite different than the equilibrium predictions, we first examine the behavior in the first period. Then, we will examine the data in the later periods.

4.1 The First Period Behavior

We now investigate whether the chosen numbers in the first period differ across treatments. We first compare proportion of zero choices, and then compare all choices in the first period.

4.1.1 Comparison of Proportions of Zero Choices in the First Period

There is no particular reason to not expect rational players to play equilibrium in their first encounter with games. However, there is substantial experimental evidence showing systematic deviations from equilibrium in initial responses (Costa-Gomes and Crawford (2006)). This is also the case in two-player guessing games that do not require iterated reasoning (Grosskopf and Nagel (2008, 2009)). We first examine the initial equilibrium behavior in our experiment.

Result 1: The equilibrium choices in the first round are significantly higher in two-player symmetric games than in asymmetric games, but there is no difference in three-player games.

Table 2: Proportion of Subjects Who Choose Zero in the First Round

Treatment\Players	All Players	Weak Players	Strong Players
	2-Players		
Symmetric (S2)	46% (31/68)	NA	NA
Asymmetric (AS2)	14% (14/98)	16% (8/49)	12% (6/49)
	3-Players		
Symmetric (S3)	33% (11/33)	NA	NA
Asymmetric (AS3)	32% (36/114)	29% (22/76)	37% (14/38)

Table 2 presents the proportion of the subjects who choose zero in the first period for both two and three-player games. In two-player games, 27% percent (45/166) of all subjects chose zero. Almost half of the subjects in S2 played the equilibrium strategy (46%) whereas only 14% of two-player asymmetric game players chose zero. We compared proportions of zero choices by using the two-sided Fisher's Exact Test. Apparently, proportions of zero choices in symmetric and asymmetric games are significantly different (46% vs. 14%, $p < 0.001$).[12] When we further analyze asymmetric games by taking into account the roles of the players, we find that the behavior of the weak players and their strong counterparts are virtually the same (16% vs. 12%, $p = 0.77$).

For three-player games, when we compare the proportions of zero choices in the first period of S3 and AS3, we find no difference (33% vs. 32%, $p = 0.5$). Furthermore, we could not reject the hypothesis that the behavior of players with different roles are the same (29% vs. 37%, $p = 0.4$).

Finally, we compare the observed behavior in two-player and three-player games. The frequency of equilibrium play in S2 as compared to S3 is relatively higher but not significantly so (46% vs. 33%, $p = 0.17$). However, when we compare AS2 with AS3, we find significant differences for both all players and strong players (14% vs. 32%, $p = 0.002$; 16% vs. 29%, $p = 0.13$; 12% vs. 37%, $p = 0.01$).

Our baseline treatment (S2) is very similar to the two-player guessing game in Nagel et al. (2008) who use $p = 2/3$. They observed that only 9.85% of students chose zero in the first period while we observed 45.6%. In the standard two-player game, iterative reasoning is unnecessary since zero is the weakly dominant strategy. For this reason, the two-player guessing game is much simpler than its N-player versions. We further simplify the game by choosing $p = 1/2$ instead of $p = 2/3$. Thus, one possible reason for this high proportion is that the game in our baseline treatment is the simplest in the literature.[13]

To sum up, regarding the first period equilibrium choices, there is a significant difference between symmetric and asymmetric two-player games but introducing asymmetry seems to have essentially no effect in three-player games.

4.1.2 Comparison of All Choices in the First Period

In addition to the initial equilibrium behavior, examining the distribution of choices and how close players get to the equilibrium in the first period is also informative.

Result 2: The distributions of first round choices in the two-player symmetric and asymmetric games are significantly different from each other, but there is no significant difference in three-player games.

[12] We set the level of significance as $\alpha = 0.05$ and consider all p values greater than 0.05 as insignificant.

[13] Another reason may be the backgrounds of the subjects. 58% of all subjects are from engineering and 67% of people who choose zero are from engineering. These figures are 65% and 71% in the S2 treatment and they may have recognized the game form better than others (as in Chou et al. (2009) comparing Caltech vs. community college subjects). For the subjects who choose less than or equal to five as in Chou et al. (2009), these figures are 58% and 63% for all subjects and 65% and 70% for S2. However, none of these proportions are significantly different (two-sided Fisher's Exact Test).

Table 3: Means and Medians of the First Round Choices

Treatment\Players	All Players	Weak Players	Strong Players
	2-Players (Mean / Median)		
Symmetric (S2)	18 / 5		
Asymmetric (AS2)	26 / 13	25 / 17	27 / 10
	3-Players (Mean / Median)		
Symmetric (S3)	19 / 13		
Asymmetric (AS3)	25 / 10	25 / 14	24 / 10

Table 3 presents means and medians of the first period choices. When all data is considered, S2 has the lowest mean and median values and AS2 has the highest values.

We used two sample Kolmogorov-Smirnov (KS) test to identify whether chosen numbers in the first period of symmetric and asymmetric games are drawn from the same distribution. Figure 3 shows the cumulative frequencies of chosen numbers in S2, AS2, S3 and AS3 in the first period. It is clear from Figure 3 that the cumulative distribution function of first period choices in S2 lies above the cumulative distribution function of first period choices in AS2. Thus, we reject the null hypothesis that these two samples come from the same distribution (two-sided KS: $p < 0.001$). However, we cannot reject this null hypothesis for S2 - S3 and S3 - AS3 (two-sided KS: $p = 0.84$ and $p = 0.78$, respectively). Moreover, there is a significant difference between the choices of players in AS2 and AS3 in the first period ($p = 0.042$).

Figure 4 demonstrates the cumulative frequency of the first period choices of weak and strong players in AS2 and AS3 treatments. We find no significant differences between the choices of weak players and the choices of strong players neither within AS2 nor within AS3 (two-sided KS test; AS2 weak vs. AS2 strong, $p = 0.62$; AS3 weak vs. AS3 strong, $p = 0.99$). Moreover, while there are no significant differences between weak players in AS2 and AS3, choices of strong players seem to come from different distributions (two-sided KS test; AS2 weak vs. AS3 weak, $p = 0.66$; AS2 strong vs. AS3 strong, $p = 0.047$).[14]

In brief, choice behavior in the first period in both two and three-player games support our result in the previous subsection that introducing asymmetry causes the first period choices to differ in two-player games, but not in three-player games.

[14] We also test whether the choices in the treatments are drawn from the same distribution with the Kruskal-Wallis test. For all the treatments (S2, AS2, S3, and AS3), we reject this hypothesis ($p = 0.048$).

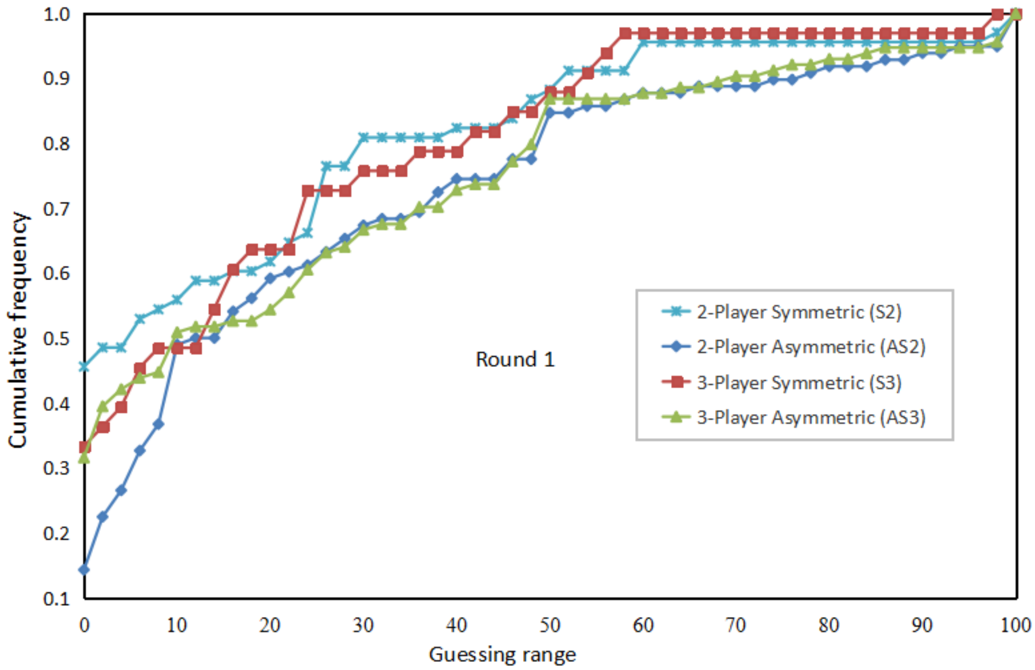


Figure 3: Cumulative Frequency of All Chosen Numbers in the First Round

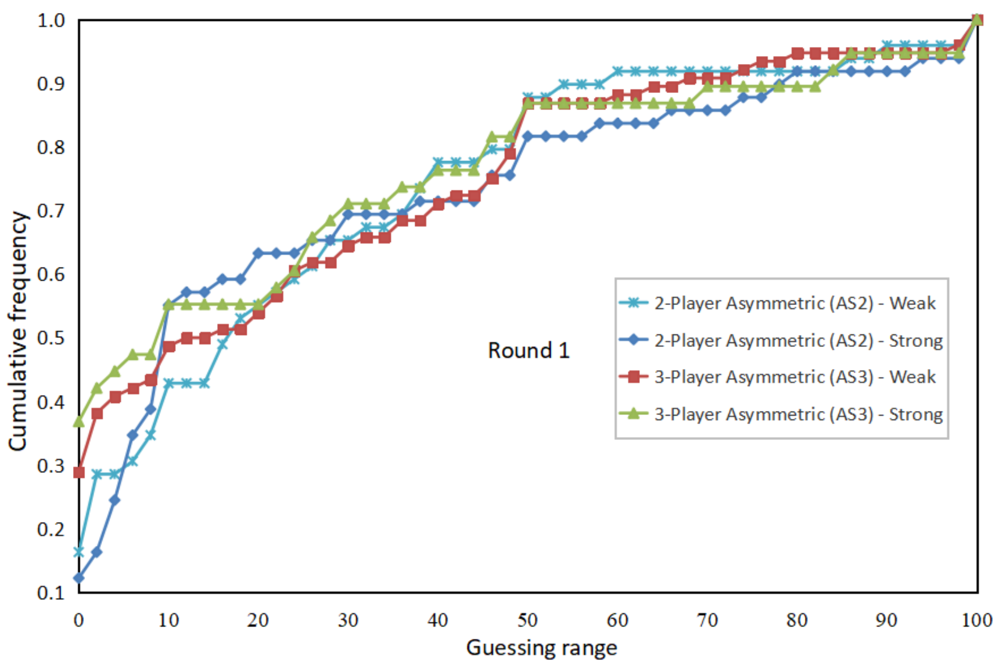


Figure 4: Cumulative Frequency of Weak and Strong players in the First Round

4.2 The Behavior in Later Periods

Table 4 reports the mean and median choices over all subjects for each game over time (for asymmetric games, choices are indicated for both all players and for weak and strong players separately). Note that in two-player games, the median choices for all subgroups monotonically decrease (not strictly) and become at most 1 after period 4 and 0 after period 6 (mean choices also decrease with some exceptions, but since there are few subjects who choose high numbers even in the last periods, median is a more reliable measure to summarize behavioral tendencies). Moreover, the median choices seem to be higher in two-player asymmetric games especially in the first 4-5 periods. Figure 5 clearly shows the difference in median choices over time between two-player symmetric and asymmetric games.

Result 3: The respective significant and insignificant distributional differences between symmetric and asymmetric games in the two and three-player cases, observed in the first round choices, continue to hold for later periods.

Table 4: Means and Medians of Chosen Numbers at Each Period

2-PLAYER	Mean Choice (Period 1 -10)									
	1	2	3	4	5	6	7	8	9	10
Symmetric (68)	18	10	7	4	5	1	1	1	0	3
Asymmetric (98)	26	22	15	12	10	9	8	7	4	6
Strong (49)	27	22	12	12	8	12	10	8	5	6
Weak (49)	25	22	17	11	11	7	5	6	3	6

2-PLAYER	Median Choice (Period 1 -10)									
	1	2	3	4	5	6	7	8	9	10
Symmetric (68)	5	0	0	0	0	0	0	0	0	0
Asymmetric (98)	13	10	4	2	1	1	0	0	0	0
Strong (49)	10	10	4	2	1	1	0	0	0	0
Weak (49)	17	13	5	2	1	0	0	0	0	0

3-PLAYER	Mean Choice (Period 1 -10)									
	1	2	3	4	5	6	7	8	9	10
Symmetric (33)	19	13	13	13	9	5	2	5	2	2
Asymmetric (114)	25	17	16	12	7	10	8	7	4	5
Strong (38)	24	17	14	10	7	13	11	8	3	8
Weak (76)	25	17	16	13	7	8	7	6	4	4

3-PLAYER	Median Choice (Period 1 -10)									
	1	2	3	4	5	6	7	8	9	10
Symmetric (33)	13	6	5	3	1	1	0	0	0	0
Asymmetric (114)	10	10	5	4	2	2	1	1	1	0
Strong (38)	10	10	4	3	2	1	0	1	1	0
Weak (76)	14	10	6	5	2	2	2	1	0	1

For three-player games, the median choices exhibit similar patterns, but subgroups contain irregularities. Median choices of weak and strong players decrease but not monotonically and become zero at the last period for strong players, and do not become zero for the weak players. Moreover, as can be seen from Figure 6, the median choices do not seem to differ between the symmetric and asymmetric games in any of the periods which support our findings about the first period behavior and they can be generalized into later periods.

To check whether the aforementioned observations are statistically valid, we conducted period-wise comparisons of subgroups by using the Kolmogorov–Smirnov test. Table 5 shows p-values for all those comparisons. For two-player games, choices in the symmetric and the asymmetric case are significantly different for all periods except the last period (Table 5, row 1). There is no difference between the weak and strong player (Table 5, row 2). For three-player games, neither choices in symmetric and asymmetric games nor choices of different types in asymmetric games are found to be stochastically different from one another (Table 5, row 3 and 4).

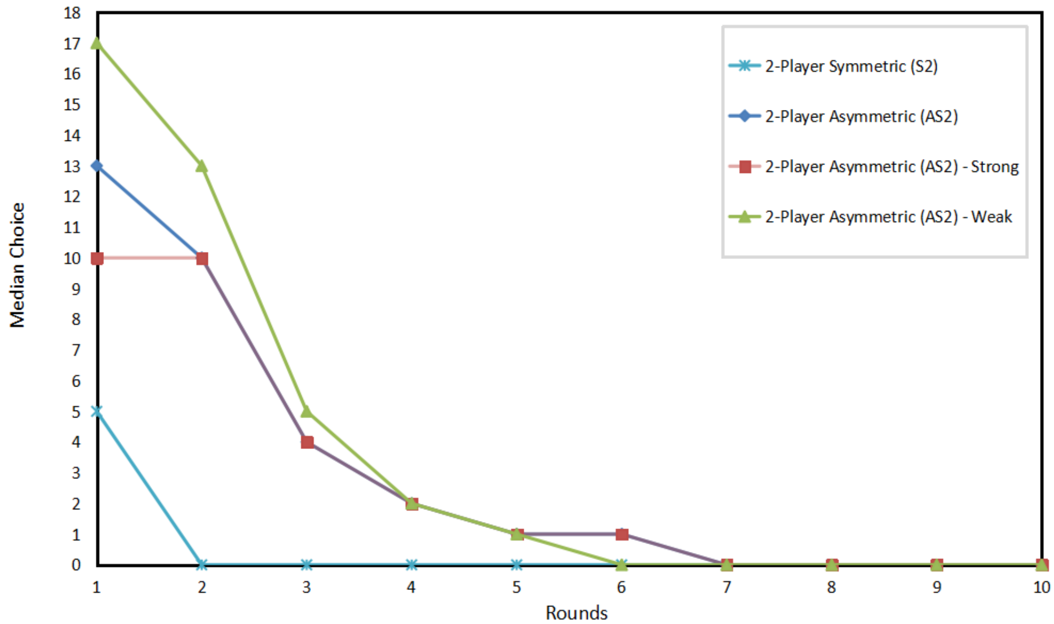


Figure 5: Median Choices for Two-player Games

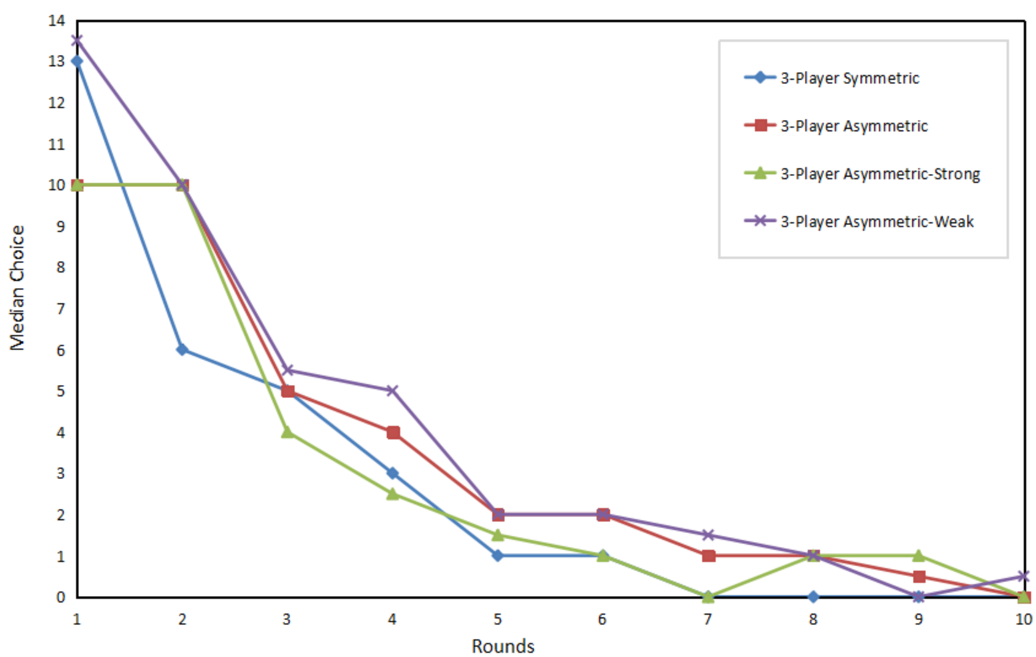


Figure 6: Median Choices for Three-player Games

Table 5: Results of two-sample Kolmogorov –Smirnov tests of the equality of distributions

Compared Samples		Period 1	2	3	4	5	6	7	8	9	10
S2 (68)	**AS2 (98)**	0.00**	0.00**	0.00**	0.00**	0.001**	0.00**	0.018*	0.012*	0.012*	0.068
Weak (49)	**Strong (49)**	0.623	0.994	0.196	0.939	0.994	0.939	0.994	0.939	0.803	0.994
S3 (33)	**AS3 (114)**	0.782	0.803	0.754	0.169	0.963	0.274	0.274	0.308	0.207	0.278
Weak (76)	**Strong (38)**	0.995	0.995	0.210	0.912	1.000	0.912	0.468	1.000	0.995	0.704
S2 (68)	**S3 (33)**	0.845	0.228	0.024*	0.002**	0.026*	0.039*	0.017*	0.091	0.143	0.597
AS2 (98)	**AS3 (114)**	0.042*	0.171	0.921	0.733	0.608	0.869	0.132	0.282	0.062	0.070

** significant at 1% significance level, * significant at 5% significance level.

When we make cross comparisons of choices between two-player and three-player games, we find that the choices are significantly lower in two-player symmetric games in the mid-periods relative to the choices in three-player symmetric games. This observation together with the fact that the choices tend to decrease (Table 4) implies that choices are closer to the equilibrium in two-player games than the choices in three-player games in the mid periods (Table 5, row 5). Finally, there is essentially no difference, except in the first period, between choices in asymmetric two-player games and three-player games (Table 5, last row).

These results show that introducing asymmetry has a lasting effect on the choices especially in two-player games but not a significant effect in three-player games. Moreover, all median choices tend to converge in time to the equilibrium choice, which will be examined in the next section in more detail.

4.2.1 Existence and Extent of Convergence

As mentioned in the previous section, it is evident that all the median choices and almost all the mean choices monotonically decrease over time and converge to zero, although not fully (Table 4, Figure 5 and 6). This monotonically increasing trend in the proportion of zero choices can be seen clearly in Table 6.

Result 4: Choices unravel to the equilibrium in all games due to both lower choices and the increasing percentage of zero choices over time.

We compared the proportions of zero choices pair by pair by using the "Fisher's Exact Test" for the first period in section 4.1.1. We now extend this comparison to all periods. These results are presented in Table 7. In two-player games, the frequency of the equilibrium play is significantly higher in symmetric games than in asymmetric games (Table 7, row 1).

In three-player games, we found no difference in equilibrium play frequencies between symmetric and asymmetric games in the first period. Table 6 shows that this trend continues for the first four periods but convergence seems to slow down in asymmetric cases, especially after period four (speed of convergence will be examined in the next section in more detail). Only in the last two periods are equilibrium choices significantly more frequent in three-player symmetric games than in asymmetric games (Table 7, row 2).

When we make cross comparisons between two and three-player games, we find that in symmetric games, there is convergence but the frequencies of equilibrium play in three-player games are significantly lower than in two-player games after the first two periods (Table 7, row 3). In asymmetric games, equilibrium play frequencies are also significantly lower in three-player games in comparison to two-player games, especially in the last four periods (Table 7, last row). These findings suggest that the extent of convergence in three-player games is lower than in two-player games for both symmetric and asymmetric cases. If we focus on the last period, we observe that there is a significant difference in proportions between two and three-player symmetric and asymmetric games (S2 vs. AS2, S3 vs. AS3, and AS2 vs. AS3).

Table 6: Percentages of Zero Choices Overtime

2-PLAYER	Periods									
	1	2	3	4	5	6	7	8	9	10
Symmetric (68)	46	51	62	71	74	76	82	85	88	85
Asymmetric (98)	14	17	26	32	44	48	59	61	67	71
Strong (49)	12	16	27	37	43	43	57	61	61	67
Weak (49)	16	18	24	27	45	53	61	61	73	76

3-PLAYER	Periods									
	1	2	3	4	5	6	7	8	9	10
Symmetric (33)	33	39	39	33	45	48	52	61	70	73
Asymmetric (114)	32	29	29	32	39	40	44	48	50	54
Strong (38)	37	29	42	34	39	47	53	47	45	63
Weak (76)	29	29	22	30	38	37	39	49	53	50

Table 7: Results of Fisher's Exact tests of the equality of proportions of zero choices

Compared Samples		Period 1	2	3	4	5	6	7	8	9	10
S2 (68)	**AS2 (98)**	0.000**	0.000**	0.000**	0.000**	0.000**	0.000**	0.001**	0.001**	0.001**	0.027*
S3 (33)	**AS3 (114)**	0.503	0.177	0.177	0.503	0.305	0.262	0.281	0.146	0.035*	0.045*
S2 (68)	**S3 (33)**	0.169	0.177	0.028*	0.000**	0.006**	0.005**	0.002**	0.007**	0.025*	0.108
AS2 (98)	**AS3 (114)**	0.002**	0.034*	0.343	0.555	0.261	0.165	0.018*	0.04*	0.008**	0.008**

** significant at 1% significance level, * significant at 5% significance level.

Thus, we conclude that subjects in all games revise their choices towards the equilibrium but introducing asymmetry and the number of players seem to affect the extent of convergence negatively, measured as the proportion of equilibrium choices.

4.2.2 The Speed of Convergence

A clear convergence to the equilibrium is observed in choices, as shown in the previous section. How fast this unraveling process occurs is also of importance.

Result 5: Introducing asymmetry slows down the convergence to the equilibrium in two (S2 vs. AS2), but not in three-player games (S3 vs. AS3). Convergence is somewhat faster in symmetric two-player games than in symmetric three-player games (S2 vs. S3), but this is not valid in asymmetric games (AS2 vs AS3).

Table 8 presents the medians of chosen numbers at each period for all treatments and the rates of decrease in medians (defined by Nagel (1995)). The rate of decrease is calculated by using the following formula:

$$w_{1-t}^{median} = \frac{median_{period=1} - median_{period=t}}{median_{period=1}}$$

where w_{1-t}^{median} denotes the rate of decrease from period 1 to period t. The above definition implies that the larger the w_{1-t}^{median} value is, the faster the convergence tends to be. Specifically, if there is full convergence from period 1 to period N, w_{1-t}^{median} value is one and if the median choice does not change from period 1 to period t, w_{1-t}^{median} value is zero.

In two-player games, there is immediate convergence in symmetric cases. That is, in the second period, median choice becomes zero. In all two-player games, convergence happens in period 7. In three-player games, zero median is observed in period 7 in symmetric games, and in the very last period in asymmetric games. We calculated the rates of decrease for $t = 2, 3, 4, 5$ and the average of these rates, since the median is already zero in period 5 for most treatments. It seems that rates in symmetric games are higher than in asymmetric games.

Table 8: Medians of Chosen Numbers at Each Period and Rates of Decreases in Medians

2-PLAYER	**Median Choice (Period 1 -10)**										**Rate of Decrease**				**Average**
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**1-2**	**1-3**	**1-4**	**1-5**	
Symmetric (68)	5	0	0	0	0	0	0	0	0	0	1.00	1.00	1.00	1.00	1.00
Asymmetric (98)	13	10	4	2	1	1	0	0	0	0	0.23	0.69	0.85	0.92	0.67
Strong (49)	10	10	4	2	1	1	0	0	0	0	0.00	0.60	0.80	0.90	0.58
Weak (49)	17	13	5	2	1	0	0	0	0	0	0.24	0.71	0.88	0.94	0.69
3-PLAYER	**Median Choice (Period 1 -10)**										**Rate of Decrease**				**Average**
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**1-2**	**1-3**	**1-4**	**1-5**	
Symmetric (33)	13	6	5	3	1	1	0	0	0	0	0.54	0.62	0.77	0.92	0.71
Asymmetric (114)	10	10	5	4	2	2	1	1	0.5	0	0.00	0.50	0.60	0.80	0.48
Strong (38)	10	10	4	2.5	1.5	1	0	1	1	0	0.00	0.60	0.75	0.85	0.55
Weak (76)	13.5	10	5.5	5	2	2	1.5	1	0	0.5	0.26	0.59	0.63	0.85	0.58

Since we have individual data, we are able to make a more detailed analysis based on individual choices over periods. We define the rate of decrease in the "choice" for each player as follows:

$$w_{1-t}^{choice} = \frac{choice_{period=1} - choice_{period=t}}{max\{choice_{period=1}, choice_{period=t}\}}$$

where w_{1-t}^{choice} denotes the rate of decrease of choices from period 1 to period N. Notice that the formula of the rate of decrease in "choice" is different from the formula of the rate of decrease in median. We now divide the difference by the maximum of the choice in period 1 and the choice in period N instead of the choice in period 1. By doing this, we prevent the rate of decrease of some subjects to be undefined (i.e. the subjects who choose zero but increase their choice later) and the rate of decrease to take extreme negative values (i.e. the subjects who choose a small number but increase their choice later). All w_{1-t}^{choice} values are between -1 and +1. We observe that most of the subjects who choose zero in the first period also continue to choose zero later. The rates of decrease are not defined for those subjects. But since these subjects chose the equilibrium choice directly, we assign rates of decrease of these subjects as +1. We calculated w_{1-t}^{choice} values for each subject for all periods, and to see whether there are differences in speed of convergence, we compared these values across treatments. Table 9 shows the p values of two-sample Kolmogorov-Smirnov tests of the equality of distributions.

Table 9: Comparions of Rates of Decreases (Two-sample Kolmogorov-Smirnov test)

		Rates of Decrease								
Compared Samples		**1-2**	**1-3**	**1-4**	**1-5**	**1-6**	**1-7**	**1-8**	**1-9**	**1-10**
S2 (68)	**AS2 (98)**	0.000**	0.000**	0.000**	0.000**	0.000**	0.002**	0.001**	0.002**	0.102
S3 (33)	**AS3 (114)**	0.247	0.593	0.681	0.754	0.350	0.119	0.725	0.207	0.278
S2 (68)	**S3 (33)**	0.295	0.053	0.002**	0.004**	0.011*	0.011*	0.028*	0.099	0.263
AS2 (98)	**AS3 (114)**	0.083	0.650	0.277	0.128	0.328	0.132	0.282	0.041*	0.070

** significant at 1% significance level, * significant at 5% significance level.

Results show that the distributions of rates of decrease are significantly different between two-player symmetric and asymmetric games. The former has a higher speed of convergence (Table 9, row 1). In three-player games, there is no difference in speed of convergence between symmetric and asymmetric cases (Table 9, row 2).

Cross comparison between two and three-player symmetric games shows that the speed of convergence in the former is generally higher than in the latter, but the differences are significant only in the mid periods (Table 9, row 3).[15] There is no difference in the speed of convergence between asymmetric two-player games and three-player games (Table 9, last row).

When we compare the rates of decrease in median from period 1 to 5 and from period 1 to 10 with a rank order test, the rates are significantly higher in symmetric games than in asymmetric games for two-player games (higher but not significantly so for three-player games).[16]

Furthermore, in order to test whether having a weak or a strong opponent influences the speed of convergence, we compared the behavior of weak players in asymmetric cases and all the

[15]This is expected due to the equilibrium structure. In two-player games, there is a weakly dominant strategy, which does not hold in three-player games.

[16]w_{1-5}^{median} and w_{1-10}^{median} in two-player games $p = 0.001$ and $p = 0.029$; in three-player games $p = 0.36$ and $p = 0.06$.

players in symmetric cases (weak players in AS2 vs. all players in S2). In two-player games, the KS tests (also Wilcoxon rank-sum tests) show that the speed of convergence for the players in symmetric cases is significantly higher than the weak players in asymmetric cases until period eight ($p < 0.01$). The same comparison implies no distributional differences in three-player cases (weak players in AS3 vs. all players in S3). We also compared the behavior of strong players in two and three-player asymmetric games (strong players in AS2 vs. strong players in AS3) to see whether having one or two weak opponents influences the speed of convergence but found no significant difference in their behavior. As a robustness check, we ran the same tests by excluding the players who always chose zero in all periods (we assigned rates of decrease of these subjects as +1). The results still mostly hold, although we lost significance in a few cases.

4.3 Earnings and Discrepancy in Choices

In this section, we report two more measures to detect the possible effects of introducing asymmetry in guessing games. We first compare the earnings of strong and weak players. Then, we define discrepancy in choices and compare symmetric/asymmetric games and strong/weak players in asymmetric games in terms of discrepancy in choices.[17]

4.3.1 Earnings

In this paper, we examine discrete (tournament) payment version of guessing games in which the player with the closest number to the target wins and gets the full prize while others get zero (in case of a tie, winners share the total prize). This discreteness makes a player a winner, regardless of how much closer she is to the target than the other player(s). Although previous analyses about the number choices of different types is very informative, this discreteness in determining the winner makes analyses of the average earnings of different types[18] important.

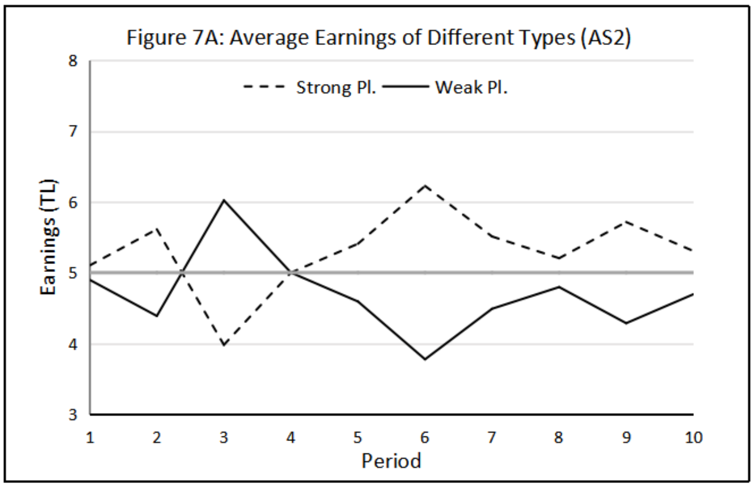
Result 6: The strong players have an earning advantage over the weak players in both two and three-player games, more prominently in the latter.

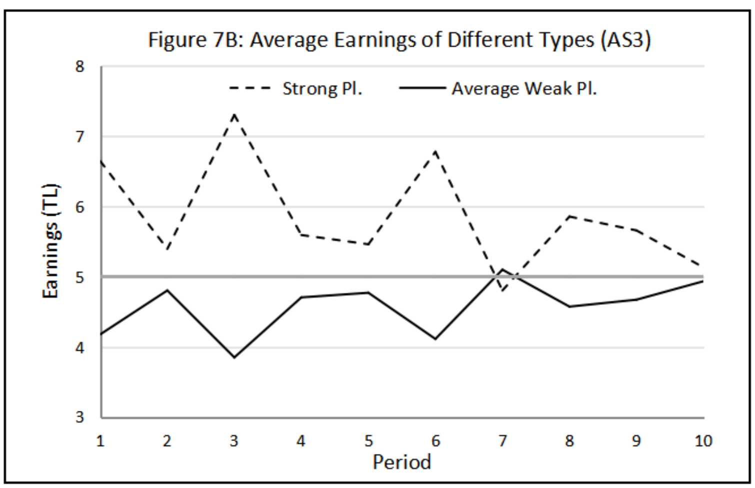
Figure 7A and 7B, respectively, show the average earnings of strong and weak types in asymmetric two-player and asymmetric three-player games over periods 1-10. The winner earned 10 TL and 15 TL in each period in two and three-player games, respectively. In case of a tie, the winners share these amounts which makes the average earnings 5 TL. If all players were to play the equilibrium strategy (or choose the same number), the earnings for all would be on the gray line at 5 TL in each figure. However, we observe that the average earnings deviate from this

[17] These two measures would be very closely related if we used a continuous payment scheme since choices directly affect the level of earnings in continuous payment games where all players are paid depending on their distance to the target.

[18] We can also compare winning percentages of different types, but this complicates aggregation in three-player game because there are different combinations of ties.

equilibrium earning in both two and three-player games. In two-player games (Figure 7A), strong players earn strictly more than weak players in all except two periods but these differences are only observed to be significant in periods 6 and 9. In three-player games (Figure 7B), strong players earn more in all but one period and the earnings converge to 5 TL over time. In this case, these differences are only significant in periods 1, 3, and 6. When we compare the overall earnings for the whole game, we find that for both games, strong players earn significantly more than weak players (5.31 vs 4.69 TL, $p = 0.016$ in AS2 and 5.98 vs. 4.51 TL, $p < 0.001$ in AS3).





These findings suggest that strong players in asymmetric games overall seem to have an earning advantage over weak players especially in three-player games, but this advantage is not observed to be consistent across periods and is significant only in 2-3 periods.[19]

[19]Wilcoxon rank-sum test is used, and significance level is set to be 0.05. When we repeat the same analysis with the KS test, the difference loses its significance for two-player games, but it is still significant for three-player games.

4.3.2 Discrepancy in Choices

We define discrepancy in choices (shortly, discrepancy) as the squared distance of the chosen numbers to the winning number. It is calculated in each period for every player and then the average is found for different games and player types. Since all choices must be equal in equilibrium, this measure should always be zero. It can also be considered as a metric for performance. It shows how losing players choose their numbers differently in comparison to their winning rivals and how unsuccessful they are in anticipating the behavior of other player(s). Higher values of this measure imply that losers tend to lose with a higher margin and are more unsuccessful.

Result 7: Introducing asymmetry increases discrepancy in choices in both two and three-player games, more prominently in the former.

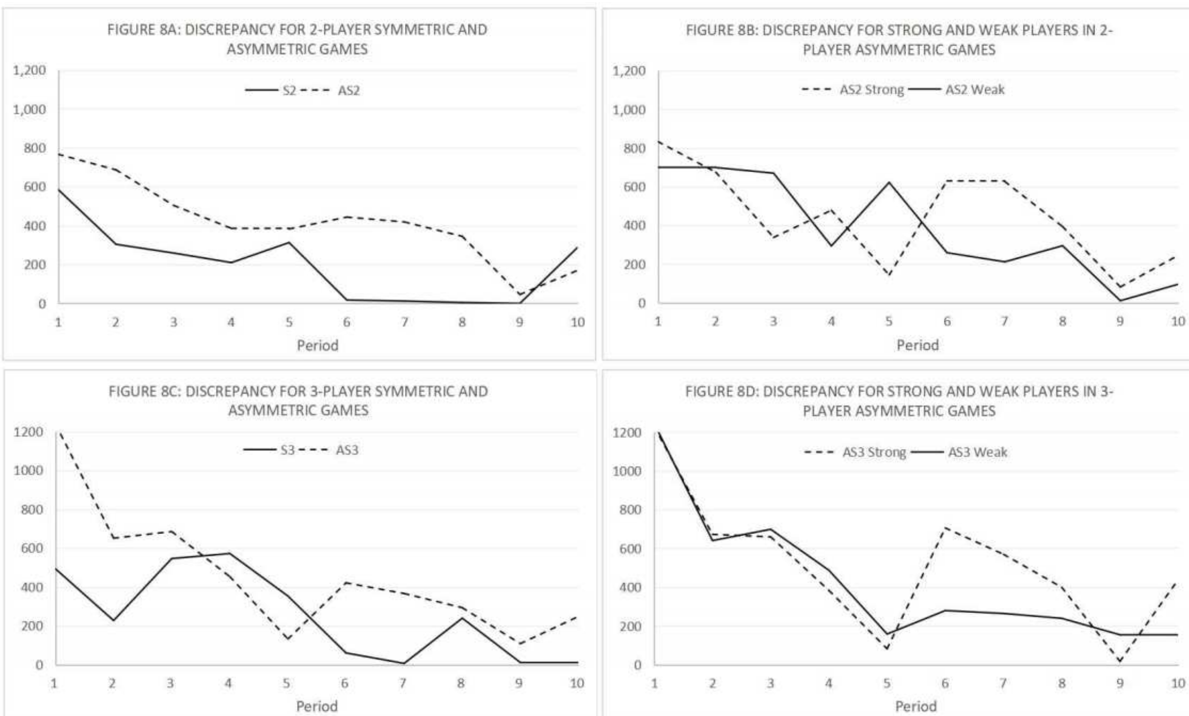


Figure 8 shows the comparison of symmetric and asymmetric games and weak and strong players in terms of average discrepancy. Firstly, discrepancy is reduced over time as all the lines have negative slopes.[20] Secondly, discrepancy is higher in asymmetric games in comparison to symmetric ones, especially for two-player games (First column, Figure 8A and 8C) since the

[20]Interestingly, in almost all cases, the value increases in the last period. This can also be seen from Table 4. Some players, at least, might have chosen higher numbers in period 10 due to boredom and fatigue and they may have wanted to do something different by experimenting with some extreme values (Portfolio effect and/or wealth effect might have played a role as well). In period 10, the number of people who choose more than 50 are 2 (out of 68), 4 (out of 98), 0 (out of 33) and 4 (out of 114) in S2, AS2, S3 and AS3, respectively.

dashed curve is above the solid curve with a few exceptions. Finally, we see that discrepancy for the weak players is less than the strong players in later periods (Second column, Figure 8B and 8D).

Thus, these findings suggest that introducing asymmetry increases discrepancy and strong players seem to be more susceptible to this asymmetry (e.g., strong losers lose with a higher margin than weak losers).[21] When we consider the combined evidence regarding earnings and discrepancy, we can say that size, to some extent, does matter more prominently in two-player games. One reason for this seems to be the asymmetry driven perceptions of weak and strong players about their relative ability to win the game that can be inferred from the debriefing part at the end of the experiment (See Appendix B2). This might be considered as a self-fulfilling prophecy of players. Since some, if not most, strong (weak) players think that they are (dis)advantageous and that they are more (less) likely to win, they play according to these beliefs. As a result, the earnings of strong players are somewhat higher than weak players but (due to this overconfidence, some strong players are more careless in choosing their numbers and) this leads to strong players losing with a higher margin when they do, and more discrepancy in asymmetric games.

5 Discussion and Conclusion

In this paper, we have examined the behavior in guessing games with asymmetric players. This asymmetry is introduced by creating replicas of the players that influence their relative strength in determining the target number. We characterized the equilibria of this class of guessing games and demonstrated the cases in which these games are observed to be indistinguishable from standard guessing games that have a unique pure strategy Nash equilibrium. In all other cases, the equilibrium is mixed, may not be unique, and involves the weak player imitating the strong player.

In our experimental design with two and three-player repeated guessing games with full feedback, we chose the parameters such that all versions of the game have the same equilibrium prediction that all players choose zero. Hence, the observed differences in behavior can be attributed to the perceived effect of created asymmetry.[22] We find that in the first period, equilibrium be-

[21] In the pooled data, there is a clear difference in discrepancy between symmetric and asymmetric games (first column in Figure 8, t-test, $p < 0.01$). Moreover, if we look at the pooled data of the ones who lose in asymmetric games, we see that the average difference between the choices of losers and winners is significantly different for weak and strong losers in AS3 (15.91 vs. 23.87, t-test, $p < 0.001$). We also see that the average choice for weak and strong losers is significantly different in AS3 (19.04 vs. 27.7, t-test, $p < 0.001$).

[22] One can argue that since the situation is more complicated in the asymmetric treatments, the difference in behavior can be caused by this increased complexity. Firstly, to keep the game as simple as possible, the p value ($p = 1/2$) and k values ($k = 9$ and $k = 8$ in two and three-player games, respectively) are intentionally chosen to make the target fairly easy to calculate. Second, it is difficult to say whether the observed change in the behavior is

havior is observed significantly more frequently, and overall choices are closer to the equilibrium in two-player symmetric games in contrast to two-player asymmetric games. But behavior in three-player symmetric and asymmetric games is similar with respect to first period choices. We also find support for these observations in the later periods. We conclude that although equilibrium is the same in all games, introducing asymmetry moves players away from the equilibrium in two-player games.

There is clear convergence to equilibrium in all games but there are some differences across treatments. Equilibrium frequencies (the extent of convergence) in asymmetric games and three-player games are significantly less than symmetric and two-player games, respectively. Convergence speed measures (rate of decrease in median and choices) are significantly higher in symmetric games than in asymmetric games for two-player games (higher but not significantly so for three-player games).

When we examine the earnings of different types in asymmetric games, we find that the strong players seem to have a slight earning advantage over the weak players. When we finally compare the treatments in terms of discrepancy of choices (defined as the squared distance of choices from the winning number), we find that asymmetry increases discrepancy in both two and three-player games, and strong players are influenced more negatively from the asymmetry in comparison to the weak players.

Two-player guessing games are partly studied to address the challenge of distinguishing between two sources of non-equilibrium behavior, self-bounded rationality and believing others are boundedly rational. Grosskopf and Nagel (2009) deal with this problem and conclude that the former dominates the latter. Our design does not allow us to make this distinction, but based on explanations of subjects at the end of the experiment, we can say that both sources are in play. We observe that there are many subjects who mention that they start choosing smaller numbers after seeing their rival's small numbers (learning to best respond/bounded rationality). We also observe that there are a considerable number of subjects who do not choose zero at the beginning in order to not "awaken" their opponent (the belief that others are boundedly rational). Thus, our already high proportion of equilibrium behavior in the first period may be an underestimate of the actual ratio of rational players (This is reinforced by the fact that there are some subjects who mention choosing the lowest number "1", they were under the belief that one -not zero- was the smallest number that can be chosen). One twist that would help isolate these two sources of non-equilibrium behavior is to make the matching process random in each round. To the best of our knowledge, there is no study employing random matching in the context of repeated guessing games. This may eliminate the second consideration by way of limiting learning opportunities about the opponent.

due to the new structure or increased complexity of the game because we do not have a good measure of complexity in the literature. Nevertheless, some extensions can be made in future studies to address this issue such as having a within subject design or to correlate behavior with some measure of cognitive capacity.

As an extension related to the role assignments, entitlement effects may be investigated. The right to be a specific player can be allocated to the player who performs better in an unrelated task (e.g., general knowledge questions as in Hoffman et al. (1994)). This perceived entitlement may generate false impressions such as overconfidence and influence behavior in guessing games.[23]

In our design, we restrict the parameter values, (especially $p = 1/2$) such that the equilibrium is easy to calculate, and we think that this was effective in obtaining relatively quick convergence in both two and three-player games. But even in this simple case, introducing asymmetry in the relative strength of players leads to asymmetry in the convergence to equilibrium and its speed (in two-player games). We think that this result is important because it shows how fragile the rational reasoning process is (e.g., adding a small twist without changing the equilibrium distorts rational reasoning by triggering other considerations). In future studies, higher p and appropriate k values without changing the equilibrium might be used to observe whether the results still hold. We think that it is also worthwhile to investigate $N = 3$ case further and $N > 3$ cases. Moreover, Proposition 4 also has a testable implication of whether increasing the k value without changing the payoff structure affects behavior.[24]

Finally, one goal of guessing game experiments like experimental asset markets and expectation feedback experiments is to better understand behavior in financial markets. In the latter two models, non-convergence to the fundamental value in the form of mispricing is an established finding (Heemeijer et al., 2009; Hommes et al., 2005, 2008; Kirchler, 2009; Noussair et al., 2001; Smith et al., 1988). However, in guessing games literature, especially the fast convergence result is surprising because financial markets are volatile (Sonnemans and Tuinstra (2010) argue that these models correspond to different markets, e.g., speculative vs. dividend yield markets). With a simple modeling twist, we observe a significant difference between symmetric and asymmetric cases in two-player games in terms of convergence and its speed (although not in three-player games). Hence, we think that our framework has the potential to capture this phenomenon regarding non-convergence and instability by means of different parametrization that allows both pure and mixed strategy Nash Equilibria[25] (for example, $p = 2/3$ and $k \geq 4$ result in mixed equilibria. Even with a much smaller strategy space such as $[0, 10]$, one would easily get cases where there

[23] We run an extra two-player treatment with an auction stage where the roles are assigned based on bids that participants submit in a pre-game second price auction. This addresses the question of whether players are willing to pay for any role and bidding for roles affects behavior. Since in this paper we are directly interested in the effects of asymmetry on behavior, we do not discuss the results of this treatment.

[24] In another pilot session that we did not report here, we run the two-player game with $k = 2$ and get virtually the same results as the symmetric game. However, we think that it is worthwhile to run treatments with higher k values to see its effects on behavior.

[25] As an extension, our static model where the strengths of players are fixed can be made dynamic as in Kopányi et al. (2019). The strengths of players that are given at the beginning can be adjusted positively depending on their performance throughout the game in a continuous payoff setting (by fixing the equilibrium of the game). This extra competitiveness may have an effect on the dynamics and speed of convergence.

are many mixed equilibria. See Appendix A2 for some examples). Our findings regarding the mixed strategy equilibrium involving imitation is related to this point as well, because imitative behavior that is generally considered to be a decision making heuristic in complex environments emerges as an equilibrium phenomena. Imitation is indeed used as an adaptive strategy by players, especially when they do not recognize the game form. It is also observed in guessing games where players learn to play the equilibrium strategy, not through a self-initiated rationality process, but through the imitation of the winning players. Our model implies that imitation does not always have to emerge as a heuristic but may also emerge as a part of equilibrium behavior. Since imitation is also a crucial concept in financial markets, we believe that this is an endeavor that is worth further investigation because it has the potential to better represent the observed empirical regularities.

References

[1] Agranov, M., Potamites, E., Schotter, A., and Tergiman, C. (2012). Beliefs and endogenous cognitive levels: An experimental study. *Games and Economic Behavior*, 75 (2), 449-463.

[2] Akin, Z., and Urhan, U. B (2011). A Game Theoretic Approach to Model Financial Markets: Guessing Games. *ISE Review*, 12(47), 43-60.

[3] Baethge, C. (2016). Performance in the beauty contest: How strategic discussion enhances team reasoning. Passauer Diskussionspapiere - Betriebswirtschaftliche Reihe, No. B-17-16, Universität Passau.

[4] Benhabib, J., Duffy J., and Nagel R. (2019). (De)-Anchoring Beliefs in Beauty Contest Games. Working Paper.

[5] Bosch-Domenech A., GarcÃ-a-Montalvo, J., Nagel, R. and Antoni, S. A. (2002). One, Two, (Three), Infinity: Newspaper and Lab Beauty-Contest Experiments. *American Economic Revie*w, Vol. 92, No 5, 1687-1701.

[6] Bosch-Rosa, C., and Meissner, T. (2020). The one player guessing game: a diagnosis on the relationship between equilibrium play, beliefs, and best responses. *Experimental Economics*, 1-19.

[7] Burchardi, K. B., and Penczynski, S. P. (2014). Out of your mind: Eliciting individual reasoning in one shot games. *Games and Economic Behavio*r, 84, 39-57.

[8] Camerer, Colin F. (2003). *Behavioral Game Theory*. Princeton University Press.

[9] Chou, E., McConnell, M., Nagel, R., and Plott, Charles R. (2009). The control of game form recognition in experiments: Understanding dominant strategy failures in a simple two person guessing game. *Experimental Economics*, 12, 159-179.

[10] Costa-Gomes, M.A. and Crawford, V.P. (2006). Cognition and behavior in two-player guessing games: An experimental study. *American Economic Review*, 96, 1737-1768.

[11] Crawford, V.P., Costa-Gomes, M.A., and Iriberri, N. (2013). Structural models of non-equilibrium strategic thinking: Theory, evidence, and applications. *Journal of Economic Literature*, 51(1), 5-62.

[12] Duffy, J. and Nagel, R. (1997). On the robustness of behavior in experimental "beauty contest" games. *The Economic Journal*, Vol. 107, 1684-1700.

[13] Echenique, F., and Edlin, A. S. (2004). Mixed equilibria are unstable in games of strategic complements. *Journal of Economic Theory*, 118(1), 61-79.

[14] Fischbacher, Urs, (2007). z-Tree: Zurich toolbox for ready-made economic experiments. *Experimental Economics* 10(2), 171-178.

[15] Fragiadakis, D. E., Knoepfle, D. T., and Niederle, M. (2013). Identifying predictable players: Relating behavioral types and subjects with deterministic rules. mimeo.

[16] Grosskopf, B. and Nagel, R. (2008). The two-player beauty contest. *Games and Economic Behavior*, 62 (1), 93-99.

[17] Grosskopf, B. and Nagel, R. (2009). Unraveling in two-player Guessing Games. Working Paper.

[18] Güth, W., Kocher, M. and Sutter, M. (2002). Experimental beauty contests with homogeneous and heterogeneous players and with interior and boundary equilibria. *Economics Letters*, Vol. 74, 219-228.

[19] Hanaki, N., Koriyama, Y., Sutan, A., and Willinger, M. (2019). The strategic environment effect in beauty contest games. *Games and Economic Behavior*, 113, 587-610.

[20] Heemeijer, P., Hommes, C., Sonnemans, J., and Tuinstra, J. (2009). Price stability and volatility in markets with positive and negative expectations feedback: An experimental investigation. *Journal of Economic Dynamics and Control*, 33(5), 1052-1072.

[21] Heinemann, F., Nagel, R., and Ockenfels, P. (2009). Measuring strategic uncertainty in coordination games. *The Review of Economic Studies*, 76(1), 181-221.

[22] Ho, T., Camerer, C., and Keith, W. (1998). Iterated dominance and iterated best response in experimental p-beauty-contests. *American Economic Review*, Vol. 88, 947-969.

[23] Hoffman, E., McCabe, K., Shachat, K., and Smith, V. (1994). Preferences, property rights, and anonymity in bargaining games. *Games and Economic behavior*, 7(3), 346-380.

[24] Hommes, C., Sonnemans, J., Tuinstra, J., and Van de Velden, H. (2005). Coordination of expectations in asset pricing experiments. *Review of Financial Studies*, 18, 955- 980.

[25] Hommes, C., Sonnemans, J., Tuinstra, J., and Van de Velden, H. (2008). Expectations and bubbles in asset pricing experiments. *Journal of Economic Behavior and Organization*, 67(1), 116-133.

[26] Keynes, John Maynard (1936). *The General Theory of Employment, Interest and Money*. London: MacMillan.

[27] Kocher, M., Straub, S., and Sutter, M. (2006). Individual or team decision-making - Causes and consequences of self-selection. *Games and Economic Behavior*, Vol. 56, 259-270.

[28] Kocher, M. and Sutter, M (2005). The decision maker matters: individual versus group behavior in experimental beauty-contest games. *The Economic Journal*, 115 (500), 200–223.

[29] Kocher, M., Sutter, M., and Wakolbinger, F. (2014). Social learning in beauty-contest games. *Southern Economic Journal*, 80(1), 586-613.

[30] Kopányi, D., Rabanal, J. P., Rud, O. A., and Tuinstra, J. (2019). Can competition between forecasters stabilize asset prices in learning to forecast experiments?. *Journal of Economic Dynamics and Control*, 109, 103770.

[31] Kovac, E., Ortmann, A., and Vojtek, M. (2008). Comparing guessing games with homogeneous and heterogeneous players: Experimental results and a CH explanation. *Economics Bulletin*, 3(9), 1-9.

[32] Kirchler, M. (2009). Under-reaction to fundamental information and asymmetry in mispricing between bullish and bearish markets. An experimental study. *Journal of Economic Dynamics and Control*, 33(2), 491-506.

[33] Liu, T. (2016). Heterogeneity in Guessing Games: An Experiment. Working Paper.

[34] Lopez, R. (2001). On p-beauty contest integer games. UPF Economics and Business Working Paper No:608.

[35] Mauersberger, F. and Nagel, R. (2018). Levels of reasoning in Keynesian Beauty Contests: a generative framework. In *Handbook of computational economics* (Vol. 4, 541-634). Elsevier.

[36] McKelvey, Richard D., McLennan, Andrew M., and Turocy, Theodore L. (2016). Gambit: Software tools for game theory, Version 16.0.1. http://www.gambit-project.org.

[37] Nagel, R. (1995). Unraveling in guessing games: an experimental study. *American Economic Review*, Vol. 85, 1313-1326.

[38] Nagel, R. (2008). Experimental beauty contest games: levels of reasoning and convergence to equilibrium. *Handbook of experimental economics results*, 1, 391-410.

[39] Nagel, R., Bühren, C., and Frank, B. (2017). Inspired and inspiring: Hervé Moulin and the discovery of the beauty-contest game. *Mathematical Social Sciences*, 90, 191–207.

[40] Noussair, C., Robin, S., and Ruffieux, B. (2001). Price bubbles in laboratory asset markets with constant fundamental values. *Experimental Economics*, 4(1), 87-105.

[41] Penczynski, S. P. (2016). Persuasion: An experimental study of team decision making. *Journal of Economic Psychology*, 56, 244-261.

[42] Sbriglia, Patrizia. (2008). Revealing the depth of reasoning in p-beauty contest games. *Experimental Economics*, Vol. 11, No.2, 107-121.

[43] Slonim, R. L. (2005). Competing Against Experienced and Inexperienced Players. *Experimental Economics*, Vol. 8, 55-75.

[44] Smith, V. L., Suchanek, G. L., and Williams, A. W. (1988). Bubbles, crashes, and endogenous expectations in experimental spot asset markets. *Econometrica*, 1119-1151.

[45] Sonnemans, J. and Tuinstra, J. (2010). Positive expectations feedback experiments and number guessing games as models of financial markets. *Journal of Economic Psychology*, 31(6), 964-984.

[46] Stahl, Dale O. (1996). Boundedly rational rule learning in a guessing game. *Games and Economic Behavior*, Vol. 16, 303-330.

[47] Sutter, M. (2005). Are four heads better than two? An experimental beauty-contest game with teams of different size. *Economics Letters*, Vol. 88 (1), 41-46.

[48] Weber, Roberto A. (2003). Learning with no feedback in a competitive guessing game. *Games and Economic Behavior*, Vol. 44, 131-144.

Appendix A1 - Proofs

Proof of Lemma 1. Assume $p < \frac{k+1}{2k} < 1$ for any $k > 1$. Suppose $g_w < g_s$. Remember that the target number is the weighted average of the two numbers multiplied by p ($T = (\frac{g_w+kg_s}{k+1})p$). Therefore, it can never be greater than or equal to g_s. Then, we have two possible cases: 1) $T \leq g_w < g_s$ and 2) $g_w \leq T < g_s$. If $T \leq g_w < g_s$, clearly the weak player wins. If $g_w \leq T < g_s$, the weak player is the winner again, since

$$|g_w - T| - |g_s - T| = 2T - g_w - g_s = \underbrace{(\frac{2p}{k+1} - 1)}_{<0} g_w + \underbrace{(\frac{2pk}{k+1} - 1)}_{<0} g_s < 0.$$

Now suppose that $g_s < g_w$. If $T \leqslant g_s < g_w$, clearly the strong player wins. If $g_s \leqslant T < g_w$, the strong player is the winner again, since

$$|g_s - T| - |g_w - T| = 2T - g_s - g_w = \underbrace{(\frac{2pk}{k+1} - 1)}_{<0} g_s + \underbrace{(\frac{2p}{k+1} - 1)}_{<0} g_w < 0.$$

Thus, this implies that if $p < \frac{k+1}{2k}$, the player who chooses a smaller number wins the game. This further implies that choosing zero is the weakly dominant strategy for each player because choosing zero guarantees a win or at least a draw. Thus, (0,0) is the unique Nash Equilibrium in weakly dominant strategies. ■

Proof of Lemma 2. Assume $1 > p > \frac{k+1}{2k}$.

i) Suppose $g_w > g_s$. Since $T = (\frac{g_w+kg_s}{k+1})p$, the weighted average is already closer to g_s than g_w. It is easy to see that if the weighted average is multiplied by $p < 1$, the resulting number, (T), will be even more closer to g_s than g_w. Thus, for all $p < 1$ and $k > 1$, playing $g_w < g_s$ is necessary for the weak player to win.

Now, suppose $g_w < g_s$. Then, we have two possible cases: 1) $T \leq g_w < g_s$ and 2) $g_w \leq T < g_s$. If $T \leq g_w < g_s$, clearly the weak player wins. If $g_w \leq T < g_s$, the required condition for the weak player to win is:

$$\begin{aligned}
|g_w - T| - |g_s - T| < 0 \quad &\Rightarrow \quad 2T - g_w - g_s < 0 \\
&\Rightarrow \quad (\frac{2p}{k+1} - 1)g_w + (\frac{2pk}{k+1} - 1)g_s < 0 \\
&\Rightarrow \quad (\frac{2pk-k-1}{k+1-2p})g_s < g_w \\
&\Rightarrow \quad ag_s < g_w \qquad\qquad (0 < a = \frac{2pk-k-1}{k+1-2p} < 1)
\end{aligned}$$

Hence, for the weak player to win she should choose a number such that $ag_s < g_w < g_s$. In all other cases except $g_w = g_s$ that leads to a tie, strong player wins.[26]

ii) Suppose for a contradiction that (g_s, g_w) is a pure strategy Nash Equilibrium (PSNE).

a) If a winner exists at (g_s, g_w), the other player always has an incentive to deviate because she always has the chance to share the prize by choosing her opponent's strategy at (g_s, g_w). Hence, if one of the players is the winner at (g_s, g_w), this point cannot be PSNE.

b) If there is a tie at (g_s, g_w) with $g_w \neq 0$, the strong player has an incentive to deviate because she can win by choosing a $g_s < g_w$. Hence, such a point cannot be a PSNE.

c) In the case where $(g_s, g_w) = (0, 0)$, the strong player has an incentive to deviate because, given $p > \frac{k+1}{2k}$, choosing any $g_s > 0$ guarantees winning. Thus, (0,0) cannot be a PSNE, either.

Hence, the game has no PSNE. Since the game is finite, we have at least one mixed strategy Nash Equilibrium.

Now we show that the weak player imitates the strong player in equilibrium. Suppose that the strong player randomizes between g_{si} where $i = 1, ..., l_s$ and $l_s \leq z$. Without loss of generality, we order pure strategies of the strong player such that $0 \leq g_{s1} < g_{s2} < ... < g_{sl_s}$. By part *i*, we know that the weak player cannot win by choosing a pure strategy $g_w > max\{g_{s1}, g_{s2}, ..., g_{sl_s}\}$.[27] In other words, all pure strategies that satisfy $g_w > max\{g_{s1}, g_{s2}, ..., g_{sl_s}\}$ are weakly dominated. Now suppose that the weak player randomizes between g_{wi} where $i = 1, ..., l_w$ and $l_w \leq z$ and $max\{g_{w1}, g_{w2}, ..., g_{wl_w}\} > max\{g_{s1}, g_{s2}, ..., g_{sl_s}\}$. In this case, the weak player can always increase her expected payoff by reducing the numbers over which she randomizes such that $max\{g_{w1}, g_{w2}, ..., g_{wl_w}\} \leq max\{g_{s1}, g_{s2}, ..., g_{sl_s}\}$. The reason is that for no $g_w > max\{g_{s1}, g_{s2}, ..., g_{sl}\}$ that is assigned positive probability does the weak player have a chance of winning the game. Alternatively, she has a chance to win or attain a draw by playing smaller than or equal to the highest value of the mixed strategy of the strong player. Thus, no $g_w > max\{g_{s1}, g_{s2}, ..., g_{sl}\}$ can be a part of a mixed strategy for the weak player in equilibrium. In other words, any mixed strategy that assigns positive probability to a $g_w > max\{g_{s1}, g_{s2}, ..., g_{sl_s}\}$ is weakly dominated (See Appendix A2 for simple examples). This implies that in any equilibrium, the weak player should randomize between her strategies g_{wi} where $i = 1, ..., l_w$ and $l_w < z$ such that $max\{g_{w1}, g_{w2}, ..., g_{wl_w}\} \leq max\{g_{s1}, g_{s2}, ..., g_{sl_s}\}$.

Furthermore, given any p and k such that $1 > p > \frac{k+1}{2k}$, if a g_{si} satisfies $a.g_{si} > g_{si} - 1$ where $a = \frac{2pk-k-1}{k+1-2p}$, then for all pure strategies less than or equal to g_{si}, the strong player wins unless there is a tie. This implies that the strong player randomizes over $0, 1, ..., g_{si}$ equally, and possibly

[26] From the first case mentioned above, a stronger imitation condition arises such that $bg_s < g_w < g_s$ where $b = \frac{pk}{k+1-p} < 1$ and $b > a$. Since this is a more restrictive condition and we have tournament payoff structure, we continue using $ag_s < g_w < g_s$ to represent the imitation behavior of the weak player.

[27] We assume $1 > p^* > p > \frac{k+1}{2k}$ such that the weak player wins for some strategy pairs. For the cases where $1 > p \geq p^* > \frac{k+1}{2k}$, the weak player can never win, and may attain a draw only if she perfectly imitates the strong player (See proposition 5 for the definition of p^* and other details).

some other pure strategies in equilibrium. As a best response to this mixed strategy, the weak player must randomize over $0, 1, ..., g_{wj} = g_{si}$ equally, and also some other pure strategies such that $max\{g_{w1}, g_{w2}, ..., g_{wl_w}\} \leq max\{g_{s1}, g_{s2}, ..., g_{sl_s}\}$ based on the above argument. Note that the weak player does not randomize only over $0, 1, ..., g_{wj} = g_{si}$ in equilibrium, because if she does then the strong player has an incentive to deviate (e.g., the strong player can win by choosing $g_s = z$).

We showed that the weak player never plays greater than the highest value of the mixed strategy of the strong player and that the equilibrium does not involve the weak player always choosing small numbers. These two arguments together prove that the weak player imitates the strong player in equilibrium. ■

Proof of Lemma 3. Suppose $k > 1$ and $p = \frac{k+1}{2k}$.

i) Assume $g_w = 0$. Then, $T = \frac{g_s}{2}$. Thus, $|g_w - T| - |g_s - T| = 0$ for any g_s.

ii) If $g_w \neq 0$ and $g_s < g_w$, the strong player will be the winner because for all $p < 1$ and $k > 1$, the necessary condition for the weak player to win is $g_w < g_s$ by Lemma 2. If $g_w \neq 0$ and $g_w < g_s$, the weak player will be the winner, since $|g_w - T| - |g_s - T| = 2T - g_w - g_s = (\frac{1}{k} - 1)g_w < 0$.

iii) Firstly, we show that in any equilibrium, $g_w = 0$. Suppose a pure strategy by the weak player where $g_w \neq 0$. Then, the strong player can always win the game by choosing $0 \leq g_s < g_w$. Now suppose that the weak player randomizes between g_{wi} where $i = 1, ..., l$ and $l < z$. Again, the strong player can always win or guarantee a draw by choosing $0 \leq g_s \leq min\{g_{w1}, g_{w2}, ..., g_{wl}\}$. Thus, there cannot be an equilibrium that includes a pure or mixed strategy by the weak player with $g_w \neq 0$ and in any equilibrium, $g_w = 0$.

Secondly, it is easy to show that $(g_s, g_w) = (0, 0)$ and $(g_s, g_w) = (1, 0)$ are pure strategy Nash Equilibria because neither player has an incentive to deviate, given the other player's strategy (the strong player is actually indifferent between choosing zero and one and in both cases, there is a tie). Moreover, by Lemma $2 - ii$ a and b, there cannot be any other pure strategy Nash equilibria.

Finally, we show that there are infinitely many mixed strategy Nash Equilibria including randomization between $g_{s1} = 0$ and $2 \leq g_{s2} \leq z$ equally and any randomization between $g_{s1} = 0$ and $g_{s2} = 1$. Note that if $g_w = 0$, any pure strategy g_s (part *i*) or any mixed strategy of the strong player will result in a tie. However, $g_w = 0$ is the best response of the weak player only for the mentioned mixed strategies. The second part is trivial: any (infinitely many) randomization between $g_{s1} = 0$ and $g_{s2} = 1$ will provide the same expected payoff since all payoffs are the same in both pure strategy Nash Equilibria ($(g_s, g_w) = (0, 0)$ and $(g_s, g_w) = (1, 0)$). We now show the first part that $g_w = 0$ and randomization between $g_{s1} = 0$ and $2 \leq g_{s2} \leq z$ equally are the best responses to each other. Given the randomization between $g_{s1} = 0$ and $2 \leq g_{s2} \leq z$ equally, any $0 \leq g_w < g_{s2}$ will give the same expected payoff (no $g_w \geq g_{s2}$ or no randomization by the weak player can provide a higher expected payoff) to the weak player. So, $g_w = 0$ is a best response to the equal randomization between $g_{s1} = 0$ and $2 \leq g_{s2} \leq z$. Given $g_w = 0$, randomizing equally between $g_{s1} = 0$ and $2 \leq g_{s2} \leq z$ will provide the exact same expected payoff to the strong player as any other pure or mixed strategy. So this set of mixed strategies are best responses to $g_w = 0$. Moreover, for any other mixed strategy of the strong player that does not include zero, the weak player can strictly increase her expected payoff by deviating from $g_w = 0$ and choosing less than or equal to the minimum of actions the strong player assigns positive probability. More precisely, suppose that the strong player randomizes between $g_{si} \neq 0$ where $i = 1, ..., l$ and $l < z$. However, $g_w = 0$ cannot be a best response to this mixed strategy because the weak player can strictly increase her expected payoff by choosing any $1 \leq g_w \leq min\{g_{s1}, g_{s2}, ..., g_{sl}\}$ (she even wins for sure if $1 \leq g_w < min\{g_{s1}, g_{s2}, ..., g_{sl}\} \neq 1$). ■

Proof of Proposition 4. Suppose $1 > p > \frac{k+1}{2k}$. Remember that $a = \frac{2pk-k-1}{k+1-2p}$. Then, for all $g_s \in \{0, 1, .., z\}$, $\lim_{k\to\infty} ag_s = \lim_{k\to\infty} \frac{2pk-k-1}{k+1-2p} g_s = (2p-1)g_s$. Thus, for each $g_s \in \{0, 1, .., z\}$, there exists $k_i^* > 1$, $i \in \{0, 1, .., z\}$, such that

$$\frac{2pk_i^* - k_i^* - 1}{k_i^* + 1 - 2p}\ g_s = a_i^* g_s = \lceil (2p-1)g_s - 1 \rceil.$$

Since $\frac{da}{dk} > 0$, for all $k > k_i^*$, $ag_s \in (a_i^* g_s,\ (2p-1)g_s)$, which means that for each g_s, there exists $k_i^* > 1$ such that for all $k > k_i^*$ the range in which the weak player wins does not change. Now, if we set $k^* = \max\{k_0^*, k_1^*, .., k_z^*\}$, then for all $g_s \in \{0, 1, .., z\}$, there exists $k^* > 1$ such that for all $k > k^*$ the game structure does not change. Since $\frac{da}{dk} > 0$, it is clear that for $k < k^*$, as k increases the range in which the strong player wins gets larger. ■

Proof of Proposition 5. Suppose $k > 1$, $p > \frac{k+1}{2k}$ and $g_w \neq g_s$. Remember that if $p > \frac{k+1}{2k}$, for the weak player to win, she must choose her action (g_w) such that $ag_s < g_w < g_s$ where $a = \frac{2pk-k-1}{k+1-2p}$. Since $g_w, g_s \in \{0, 1, .., z\}$, if we have $ag_s > g_s - 1$ for all g_s, the weak player can never win the game. To prove the result, it is sufficient to find $p^* < 1$ such that for all $p > p^*$, $ag_s > g_s - 1$ for all g_s. Moreover, note that finding p^* satisfying $az = z - 1$ is sufficient, since

$$\begin{aligned} ag_s = g_s - 1 \quad &\Rightarrow \quad ag_s - 1 = g_s - 2 \\ &\Rightarrow \quad ag_s - a > g_s - 2 \text{ (since } 0 < a < 1) \\ &\Rightarrow \quad a(g_s - 1) > g_s - 2. \end{aligned}$$

Then,

$$\begin{aligned} az = z - 1 \quad &\Rightarrow \quad \left(\frac{2p^*k - k - 1}{k + 1 - 2p^*}\right)z = z - 1 \\ &\Rightarrow \quad p^* = 1 - \frac{1}{2}\left(\frac{k-1}{kz + z - 1}\right) < 1. \end{aligned}$$

Since $\frac{da}{dp} > 0$, given any $k > 1$, for all $p > p^* = 1 - \frac{1}{2}(\frac{k-1}{kz+z-1})$, $ag_s > g_s - 1$ is satisfied for all g_s. ■

Proof of Lemma 6. Assume $p < min\{\frac{\sum_{i=1}^{N} k_i}{2k^*}, 1\}$ where $k^* = max\{k_1, k_2, k_3, ..., k_N\}$ for any given set of k values, $\{k_i\}|_{i=1}^{N}$, $k_i \geq 1$ for all i. There are two cases: Case 1: $p < \frac{\sum_{i=1}^{N} k_i}{2k^*} \leq 1$ and case 2: $p < 1 < \frac{\sum_{i=1}^{N} k_i}{2k^*}$. Without loss of generality, we can order the players such that $1 \leq k_1 \leq k_2 \leq ... \leq k_{N-1} \leq k_N$, implying that $k^* = k_N = max\{k_1, k_2, ..., k_{N-1}, k_N\}$.

Case 1: Suppose $p < \frac{\sum_{i=1}^{N} k_i}{2k^*} \leq 1$. Note that this case implies a sufficiently high asymmetry. Specifically, the strongest player is assumed to be stronger than or equal to the sum of the strengths of all of the other players ($2k^* \geq \sum_{i=1}^{N} k_i$ or $k^* \geq \sum_{i=1}^{N-1} k_i$). To to show that the strategy profile $(0, 0, ..., 0)$ is the unique Nash equilibrium, we need to show that the strongest player cannot win the game by choosing the largest number among the players. In other words, for any given distribution of guesses of the other $N-1$ players, the strongest player does not have an incentive to guess a larger number than the maximum of the rest of the players. Moreover, it is trivial to show that if the strongest player does not have an incentive to guess the largest number, no other player does. If this is the case, then all players have an incentive to guess smaller numbers to win the game. Then, by iterated elimination of weakly dominated strategies, the strategy profile $(0, 0, ..., 0)$ is obtained as the unique pure strategy Nash equilibrium. Now, we show that the strongest player cannot win the game by choosing the largest number among the players.

Let $g^* = max\{g_1, g_2, g_3, ..., g_N\}$ be the maximum of the guesses of N players. To prove our claim, we suppose that $g^* = g_N$ and show that the strongest player N cannot win by guessing g^*. Let $\hat{g} = max\{g_1, g_2, g_3, ..., g_{N-1}\}$ and $\hat{k}$ be the coefficient of the player who guesses $\hat{g}$. Given that $T = \frac{\sum_{i=1}^{N} k_i.g_i}{\sum_{i=1}^{N} k_i}.p$ and $p < \frac{\sum_{i=1}^{N} k_i}{2k^*} \leq 1$, we can write

$$T < \frac{\sum_{i=1}^{N} k_i.g_i}{\sum_{i=1}^{N} k_i} * \frac{\sum_{i=1}^{N} k_i}{2k^*} \Longrightarrow 2T < \frac{\sum_{i=1}^{N} k_i.g_i}{k^*} = \frac{\sum_{i=1}^{N-1} k_i.g_i}{k^*} + g^*.$$

If $T \leq \hat{g} < g^*$, then the strongest player cannot win anyways. If $\hat{g} < T < g^*$, then $g^* - T > T - \hat{g} \Longrightarrow \hat{g} + g^* > 2T$ is sufficient to show that the strongest player N cannot win by guessing g^*. Note that if we can show $\hat{g} + g^* \geq \frac{\sum_{i=1}^{N-1} k_i.g_i}{k^*} + g^*$, this automatically implies $\hat{g} + g^* > 2T$.

$$\hat{g} + g^* \geq \frac{\sum_{i=1}^{N-1} k_i.g_i}{k^*} + g^* \Longrightarrow k^*\hat{g} \geq \sum_{i=1}^{N-1} k_i.g_i$$

The condition we supposed at the beginning, $p < \frac{\sum_{i=1}^{N} k_i}{2k^*} \leq 1$, implies $k^* \geq \sum_{i=1}^{N-1} k_i$ or $k^*\hat{g} \geq \sum_{i=1}^{N-1} k_i.\hat{g}$. But since $\hat{g} = max\{g_1, g_2, g_3, ..., g_{N-1}\}$, $k^*\hat{g} \geq \sum_{i=1}^{N-1} k_i.\hat{g} \geq \sum_{i=1}^{N-1} k_i.g_i$ (The second inequality is not strict to include the extreme case where $\hat{g} = g_1 = g_2 = g_3 = ... = g_{N-1}$). Thus, the above condition is satisfied.

Case 2: Suppose $p < 1 < \frac{\sum_{i=1}^{N} k_i}{2k^*}$. This implies $k^* < \sum_{i=1}^{N-1} k_i$. First, suppose the extreme case where $\hat{g} = g_1 = g_2 = g_3 = ... = g_{N-1}$. Then, $T = \frac{\hat{g}.\sum_{i=1}^{N-1} k_i + g^*.k^*}{\sum_{i=1}^{N} k_i}.p$. But since $k^* < \sum_{i=1}^{N-1} k_i$, the weighted average $\frac{\hat{g}.\sum_{i=1}^{N-1} k_i + g^*.k^*}{\sum_{i=1}^{N} k_i}$ will be closer to $\hat{g}$ than g^*even without multiplying it with $p < 1$. But since $\hat{g} = max\{g_1, g_2, g_3, ..., g_{N-1}\}$, the weighted average $\frac{\sum_{i=1}^{N-1} k_i.g_i + g^*.k^*}{\sum_{i=1}^{N} k_i}$ will be even smaller and will be farther away from g^*. Thus $g^* - T > T - \hat{g}$ is satisfied, which proves that the strongest player N cannot win by guessing g^*.

Thus, $p < min\{\frac{\sum_{i=1}^{N} k_i}{2k^*}, 1\}$ is a sufficient condition for all players choosing zero to be the unique pure strategy Nash equilibrium of this game. ■

Appendix A2 - Some Example Games and Their Solutions

Tie | Weak player wins | Strong player wins

Strong Player \ Weak Player	0	1	2	3	4	5
0	1 , 1	2 , 0	2 , 0	2 , 0	2 , 0	2 , 0
1	0 , 2	1 , 1	2 , 0	2 , 0	2 , 0	2 , 0
2	0 , 2	0 , 2	1 , 1	2 , 0	2 , 0	2 , 0
3	0 , 2	0 , 2	0 , 2	1 , 1	2 , 0	2 , 0
4	0 , 2	0 , 2	0 , 2	0 , 2	1 , 1	2 , 0
5	0 , 2	0 , 2	0 , 2	0 , 2	0 , 2	1 , 1

k = 9	p = 0.5	((k+1)/2k) = 0.56

Unique solution:		0	1	2	3	4	5
Strong Player	*Prob.:*	1	0	0	0	0	0
Weak Player	*Prob.:*	1	0	0	0	0	0

This case is an example for Lemma 1 (Standard two-player guessing game).

Strong Player \ Weak Player	0	1	2	3	4	5
0	1 , 1	2 , 0	2 , 0	2 , 0	2 , 0	2 , 0
1	2 , 0	1 , 1	2 , 0	2 , 0	2 , 0	2 , 0
2	2 , 0	0 , 2	1 , 1	2 , 0	2 , 0	2 , 0
3	2 , 0	0 , 2	0 , 2	1 , 1	2 , 0	2 , 0
4	2 , 0	0 , 2	0 , 2	0 , 2	1 , 1	2 , 0
5	2 , 0	2 , 0	0 , 2	0 , 2	0 , 2	1 , 1

k = 9	p = 0.67	((k+1)/2k) = 0.56	a = 0.23

Solution 1:		0	1	2	3	4	5
Strong Player	*Prob.:*	0.4	0.4	0	0	0	0.2
Weak Player	*Prob.:*	0.4	0.4	0.2	0	0	0
Solution 2:		0	1	2	3	4	5
Strong Player	*Prob.:*	0.4	0.4	0	0	0	0.2
Weak Player	*Prob.:*	0.4	0.4	0	0.2	0	0
Solution 3:		0	1	2	3	4	5
Strong Player	*Prob.:*	0.4	0.4	0	0	0	0.2
Weak Player	*Prob.:*	0.4	0.4	0	0	0.2	0

Strong Player \ Weak Player	0	1	2	3	4	5	6	7
0	1 , 1	2 , 0	2 , 0	2 , 0	2 , 0	2 , 0	2 , 0	2 , 0
1	2 , 0	1 , 1	2 , 0	2 , 0	2 , 0	2 , 0	2 , 0	2 , 0
2	2 , 0	2 , 0	1 , 1	2 , 0	2 , 0	2 , 0	2 , 0	2 , 0
3	2 , 0	2 , 0	0 , 2	1 , 1	2 , 0	2 , 0	2 , 0	2 , 0
4	2 , 0	2 , 0	2 , 0	0 , 2	1 , 1	2 , 0	2 , 0	2 , 0
5	2 , 0	2 , 0	2 , 0	0 , 2	0 , 2	1 , 1	2 , 0	2 , 0
6	2 , 0	2 , 0	2 , 0	2 , 0	0 , 2	0 , 2	1 , 1	2 , 0
7	2 , 0	2 , 0	2 , 0	2 , 0	0 , 2	0 , 2	0 , 2	1 , 1

k = 20	p = 0.8	((k+1)/2k) = 0.525	a = 0.567

Solution 1:		0	1	2	3	4	5	6	7
Strong Player	*Prob.:*	4/15	4/15	4/15	0	2/15	0	0	1/15
Weak Player	*Prob.:*	4/15	4/15	4/15	1/15	2/15	0	0	0
Solution 2:		0	1	2	3	4	5	6	7
Strong Player	*Prob.:*	4/15	4/15	4/15	0	2/15	0	1/15	0
Weak Player	*Prob.:*	4/15	4/15	4/15	1/15	2/15	0	0	0

Strong Player \ Weak Player	0	1	2	3	4	5	6	7	8	9	10
0	1 , 1	2 , 0	2 , 0	2 , 0	2 , 0	2 , 0	2 , 0	2 , 0	2 , 0	2 , 0	2 , 0
1	2 , 0	1 , 1	2 , 0	2 , 0	2 , 0	2 , 0	2 , 0	2 , 0	2 , 0	2 , 0	2 , 0
2	2 , 0	0 , 2	1 , 1	2 , 0	2 , 0	2 , 0	2 , 0	2 , 0	2 , 0	2 , 0	2 , 0
3	2 , 0	0 , 2	0 , 2	1 , 1	2 , 0	2 , 0	2 , 0	2 , 0	2 , 0	2 , 0	2 , 0
4	2 , 0	0 , 2	0 , 2	0 , 2	1 , 1	2 , 0	2 , 0	2 , 0	2 , 0	2 , 0	2 , 0
5	2 , 0	2 , 0	0 , 2	0 , 2	0 , 2	1 , 1	2 , 0	2 , 0	2 , 0	2 , 0	2 , 0
6	2 , 0	2 , 0	0 , 2	0 , 2	0 , 2	0 , 2	1 , 1	2 , 0	2 , 0	2 , 0	2 , 0
7	2 , 0	2 , 0	0 , 2	0 , 2	0 , 2	0 , 2	0 , 2	1 , 1	2 , 0	2 , 0	2 , 0
8	2 , 0	2 , 0	0 , 2	0 , 2	0 , 2	0 , 2	0 , 2	0 , 2	1 , 1	2 , 0	2 , 0
9	2 , 0	2 , 0	2 , 0	0 , 2	0 , 2	0 , 2	0 , 2	0 , 2	0 , 2	1 , 1	2 , 0
10	2 , 0	2 , 0	2 , 0	0 , 2	0 , 2	0 , 2	0 , 2	0 , 2	0 , 2	0 , 2	1 , 1

k = 9	p = 0.67	((k+1)/2k) = 0.56	a = 0.23

Solution:		0	1	2	3	4	5	6	7	8	9	10
Strong Player	*Prob.:*	0.4	0.4	0	0	0	0.2	0	0	0	0	0
Weak Player	*Prob.:*	0.4	0.4	0	0.2	0	0	0	0	0	0	0

These three cases are examples for Lemma 2.

Strong Player \ Weak Player	0	1	2	3	4	5
0	1 , 1	2 , 0	2 , 0	2 , 0	2 , 0	2 , 0
1	1 , 1	1 , 1	2 , 0	2 , 0	2 , 0	2 , 0
2	1 , 1	0 , 2	1 , 1	2 , 0	2 , 0	2 , 0
3	1 , 1	0 , 2	0 , 2	1 , 1	2 , 0	2 , 0
4	1 , 1	0 , 2	0 , 2	0 , 2	1 , 1	2 , 0
5	1 , 1	0 , 2	0 , 2	0 , 2	0 , 2	1 , 1

k = 9	p = 0.56	((k+1)/2k) = 0.56

This case is an example for Lemma 3.

In any equilibrium, the weak player chooses zero. Pure strategy Nash Equilibria are (0,0) and (1,0). There are infinitely many mixed strategy Nash Equilibria including the strong player randomizing equally between zero and another action from her action space and any randomization between zero and one.

*Solutions are found by using Gambit (McKelvey et al. (2016)).

Appendix B1 - Instructions

(Translated from Turkish) Thank you for coming. You are participating in an experiment on decision making. You can earn substantial amounts of money in this experiment. Understanding the instructions correctly will help you in making better decisions to increase your earnings. You will also take a quiz about the instructions before proceeding to the actual experiment. Please follow the instructions carefully. From now on you are not allowed to talk with any other participant in the experiment. Whenever you have a question, raise your hand and wait for the experimenter.

The experiment will take approximately one hour. At the end of the experiment, you will be paid in cash whatever amount you have earned and an additional 10 Turkish Lira (TL) as a show-up fee. All the decisions you make during the experiment will be kept anonymous. They will be used for research purposes only and will not be shared with anyone.

Groups and Roles: At the beginning, we will randomly assign you into a 2-person (3-person) group. Each group member will be randomly assigned to either "Role A" or "Role B" (or "Role C") and these assignments will stay constant over all 10 rounds of the experiment, but none of you will know with whom you are paired. You will play the game for 10 rounds.

Game: In each round, each participant must choose an integer number between 0 and 100 (including these boundaries). The winner of each group will be the person who selects the number that is closest to 1/2 of the mean of the numbers chosen in that group. In other words, there will be a target number and the participant who selects a closer number to this target will win the game in that round. The target number is calculated as follows:

$$H = \frac{(g_A + g_B)}{2} * \frac{1}{2} \text{ for S2 where } g_i \text{ is the number chosen by Player } i.$$

$$(H = \frac{(g_A + 9.g_B)}{10} * \frac{1}{2},\ H = \frac{(g_A + g_B + g_C)}{3} * \frac{1}{2}, \text{ and } H = \frac{(g_A + g_B + 8.g_C)}{10} * \frac{1}{2} \text{ for AS2, S3, and AS3, respectively.})$$

After each round, we will inform you about the number you have chosen, the number chosen by the other member(s) of the group, the target number, and the winning player.

Payment: The winner of each group receives 10 (15) TL for 3 randomly chosen rounds out of 10 rounds. In the case of a tie, this is divided between the winners (i.e., each winner gets 5 TL). The one(s) who loses receives nothing (zero).

In the decision screen, you will see the current round of the game and the remaining time to enter your number in that round. If the time has expired and you have not made your decision, a warning will be displayed in the upper right corner. The allotted time (45 seconds) is sufficient for you to choose your number. The game will not proceed to the next round unless the players in all groups enter their numbers and press the "OK" button. Please manage your time accordingly. Please do not enter a number outside of the determined range, from 0 to 100 (including 0 and 100).

In each decision screen, you will see the target number (H) formula and your role, that will not change during the game. At the end of 10 rounds, you will be asked to answer a few short questions and complete a questionnaire. The last screen will show how much you will be paid in total.

To summarize, the experiment will proceed as follows:

• First, you will be asked to answer the multiple choice quiz regarding the rules of the game you will play and how you will be paid. The correct answer will appear on the screen after each question.

• You will see your randomly selected player role on the screen, then the first round of the game will start.

• In each round, after you choose your number and click the "OK" button, you will be given feedback on that round. This feedback will remain on the screen for 15 seconds and the next round will start.

• After 10 rounds are completed, you will be asked a few questions and asked to complete a questionnaire.

• Finally, the 3 rounds that have been randomly selected from 10, and your earnings in these rounds will be shown on screen. By adding the participation fee to your earnings, your total earnings will be determined. When you see this screen, remain seated. We will call and pay everyone individually. After the payment, the experiment will end.

Any questions?

Appendix B2 - Examples of Subject Responses

Reports of some weak players are as follows:

"Although I had a lower chance, I tried to predict the other player's choices from his/her previous choices,"

"I think I was disadvantaged, the other player was often the one who determined the target,"

"I was disadvantaged so I tried to mimic the other player,"

"I thought that my number was ineffective since it was multiplied by 1,"

"I was disadvantaged because my number was multiplied by 1,"

"Since player C (strong player) was decisive, I focused on her choices,"

"I wish I was player C (strong player) because (s)he had the advantage,"

"I especially focused on player C's choices,"

"I tried to choose lower numbers to eliminate the advantage of player C,"

"Player C had the advantage by all means,"

"I was not able to affect the target since I was player A (weak player),"

"I chose based on Player C's choices,"

"The outcome was mostly up to player C,"

"Player C was obviously favored,"

"Since I was disadvantaged I followed player C's strategies."

Reports of some strong players are as follows:

"I thought that whatever I chose, the target would be closer to me,"

"I thought that I could win since I was player B (strong player),"

"I had the highest impact on the target,"

"Since I was player B (strong player), I thought if I chose a big number, target would be closer to me"

"I had the advantage so I chose zero to get target closer to the smaller number,"

"Since I was player B (strong player), I thought I could get closer to the target by choosing a number as big as possible,"

"I had the advantage, so I chose small numbers,"

"I thought that the target would be closer to me since I was player B,"

"I thought that I had the advantage, and I could win by choosing big numbers"

"Since my number was multiplied by 8, I had the advantage,"

"I governed the game for the first couple periods since I was player C,"

"I tried to use my advantage but I failed,"

"I was player C and I thought I could govern the game,"

"Since my coefficient was higher, I thought I had the advantage,"

"Player B (strong player) had the advantage by all means,"

"I had the coefficient advantage."