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# Using PDE-model and system dynamics model for describing multi-operation production line

O. Pihnastyi, D. Yemelianova, D. Lysytsia

*The two classes models for describing production flow lines is analyzed. The models use of these classes for the design of production lines control highly efficient systems, the technological route of which consists of a large number of technological operations, is analyzed. The division of the technological route into a large number of operations is caused by the modern production lines development trend. The production line equipment performance synchronization is provided by an accumulating buffer. A formalized description of the production line was used as a foundation for constructing equations for each models class. The features of the use of each models class in the description of production systems, and the conditions for their application are shown. The form of the system dynamics model and the PDE-model equations is substantiated. The assumption about a deterministic rate of processing of parts and the absence of a time delay and feedback between the parameters of technological operations was made, when deriving the equations. The use of generalized technological operations in the system dynamics model as a way to reduce the number of model equations is discussed. Two limiting transitions from the PDE-model equations to the system dynamics equations are demonstrated. It is shown that the system dynamics equations are a special case of the PDE-model equations, the production line within the technological operation parameters aggregation result. The method for constructing level the system dynamics model equations is substantiated. For production lines with a different number of operations, a solution to the problem of the processing of details along a production line is presented. The solutions comparative analysis of using the system dynamics and the PDE-model equations is obtained.*

*Key words: production line; technological route, system dynamics, PDE-model of production*

## 1. Introduction

A production line is the production organizing progressive method. The details processing along the production line occurs in accordance with the given product manufacture technological route [1]. The technological route of modern production lines consists of a large number ( $10^2$ - $10^3$ ) of technological operations. The global trend of a constant increase in the operations number and production lines throughput determines the high relevance of the further development of models of production lines used to design effective production control systems. In [2], a production line of 100 machines, in [3] of 26 machines, in [4] of 300 machines, in [5] of 250 operations and 100 machines, in [6] of 18 units equipment, in [7] of 10 and 50 machines is considered. The papers analysis demonstrates that a multi-operation production line is a complex dynamic distributed system. The increased requirement to improve the modeling production systems accuracy was directly related to the need for a detailed description of the distributed system parameters state, which limited the use of many common models for describing production lines. Among the limited models range that are currently successfully used to describe production lines with distributed along the technological route parameters, two most common classes of models should be distinguished: the system dynamics model [8] and the PDE-model of the production line [9]. Therefore, the improvement of the method for constructing level equations and the tempo equations of the system dynamics using the PDE-model equations is

actual. The improvement makes it possible to use, instead of the phenomenological approach, which for the system dynamics is characteristic, the statistical approach, which is the foundation of the PDE model, which makes it possible to take into account the exact model concepts of the parts interaction with equipment and among themselves as a technological processing result.

## **2. Literature review and problem statement**

In [8], the principles of the industrial system model constructing using the system dynamics equations are given, the structure of the model and six interconnected networks of production activity are determined. The material flow network takes into account the details processing along a technological route. The equations system of levels and tempos is a tool for describing distributed production systems, technological routes of which consist of a large number of technological operations. The delay parameter presence is associated with the methods of dividing the technological route into generalized technological operations [10]. Methods of diagrams of random cycles [11] and identification of resources [12] do not take into account the structure of stocks and flows of systems. The use of general structures of the subject area [13] and component strategy [14] did not solve this problem either. One of the laborious ways to solve the problem is based on the concept of an interaction matrix that links resource flows [15]. The use of the PDE-model equations can be an option to overcome the corresponding difficulties for production lines with a large number of technological operations [16]. The use of partial differential equations was made it possible to significantly expand the design capabilities of control systems for production lines [17]. PDE-models, as well as models of system dynamics, are used to describe a multi-operation production line that operates under steady-state and transient mode [18–20].

## **3. The aim and objectives of the study**

The aim of the study is to compare the use of equations of the system dynamics model and the PDE model to describe production lines and identify the relationship between the models.

To achieve the aim, the following tasks were set:

- to show that the system dynamics equations are the limiting case of the PDE-model equations for the technological route with a large number of operations;
- to demonstrate the results of solving the details processing problem along the technological route using the system dynamics model and the PDE model equations.

## **4. Materials and methods of production lines research**

### **4. 1. The production line formalized description**

A production line at the company with the flow production method is a set of in-line or parallel equipment  $a_m$  in accordance with the technological route of manufacturing a product. Each  $m$  technological operation is a complete part of the technological process [1], is performed on the  $m$  technological equipment. For the production line synchronized operation in order to prevent downtime of the equipment, the  $m$  technological operation has the  $b_m$  accumulator for storing details awaiting processing. The storage device size is determined by the permissible deviation between

the actual and standard performance during the production process [21]. A similar structure is typical for many industries, among which the automotive industry should be distinguished. The analysis of the structure of the production serial-line, lines with converging, branching, re-entrant technological routes is presented in [6]. To describe the production lines, distributed parameters are used that characterize the capacity of the buffer  $b_m$  and the processing equipment performance  $a_m$ . The use of these parameters is a common approach when constructing models of production lines [2, 6, 7, 9]. These parameters are used in the system dynamics equations and the PDE-model equations.

The structure of the production serial-line is used for comparing the two classes of models [6] (Fig. 1). Details arrive at the first technological operation with equipment  $a_1$ , buffer  $b_1$ , are processed in sequentially located one after another technological operations. The change in the number of details in the buffer  $b_m$  will be characterized by the parameter  $V_m(t)$ , the time change in the productivity of the equipment  $a_m$  by the parameter  $[\chi]_{1m}(t)$ .

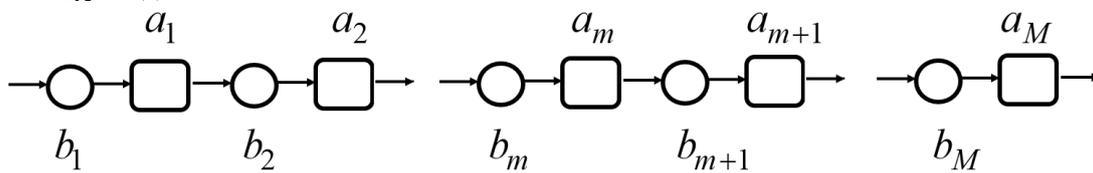


Fig. 1. Production serial-line

The serial-line structure is widely used in industry in the form of separate multi-operational sections of the technological route, it allows to simplify the process of formalized description of the production line and the interpretation of the models qualitative analysis results. Complex branched structures of production lines are represented as a combination of separate serial-lines for main and slave products [10]. This structure of the production line is widely used in industry in the form of separate multi-operational sections of the technological route, it allows to simplify the process of formalized description of the production line and the interpretation of the results of qualitative analysis of models. Complex branched structures of production lines are represented as a combination of separate serial-lines for main and slave products [10].

#### 4. 2. Features of the system dynamics model equations application

The system of the production line equations (Fig. 1), which determines the number of details  $V_m(t)$  in the buffer  $b_m$  with a capacity of  $V_{maxm}$  for the  $m$  technological operation with the processing tempo of details  $[\chi]_{1m}(t)$  on the equipment  $a_m$  at time  $t$  is presented as an equation:

$$\frac{dV_m(t)}{dt} = [\chi]_{1m-1}(t) - [\chi]_{1m}(t), \quad m=1..M, \quad (1)$$

$$[\chi]_{1m}(t) = [\chi]_{1vm}(t), \quad (2)$$

with the number of details in the buffer  $b_m$  at time  $t=0$  equal to  $V_m(0)=V_{m0}$ . The level equation (1) is supplemented by the tempo equation (2) [8]. The productivity  $[\chi]_{1m}(t)$  of the equipment  $a_m$  depends on the conditions and mode of processing, it is

characterized by the average value of the tempo  $[\chi]_{\psi m}(t)$  and the standard deviation. When describing the production line, we assume that equality (2) is fulfilled, there are no time delays and feedbacks between the model parameters. The tempo of incoming of details at the input of the production line is denoted by  $[\chi]_{10}(t)$ , the output flow of details from the production line is  $[\chi]_{1M}(t)$ . The solution of the system of equations (1), (2) with the given initial conditions determines the distribution of details by technological operations at the time, has the form:

$$V_m(t) = V_m(0) + \int_0^t ([\chi]_{1m-1}(\tau) - [\chi]_{1m}(\tau)) d\tau, \quad m=1 \dots M. \quad (3)$$

For a constant value of the operation processing  $[\chi]_{1m}(t)=[\chi]_{1m}=\text{const}$  at the  $m$  operation, solution (4) has the form

$$V_m(t) = V_m(0) + ([\chi]_{1m-1} - [\chi]_{1m})t, \quad m=1 \dots M. \quad (4)$$

The solution of equations (3), (4) determines the state of the parameters of the production line in the absence of time delays and feedbacks. To reduce the number of equations for  $M \gg 1 (10^2 - 10^3)$ , several technological operations are combined into one generalized operation [10].

### 4. 3. Features of the PDE-model equations application

When building a PDE-model, the flow of details along the production route is considered as a continuous flow. We assume that the position of the detail in the technological route at the moment of time  $t$  satisfies the trajectory of movement  $S=S(t)$ , which is set in accordance with the route maps for the product manufacture. For the conveyor line of mining and processing companies, the coordinate  $S$  determines the path traveled by the material [22], for models of semiconductor lines, the coordinate  $S$  corresponds to the degree of completeness of the product manufacturing  $S \in [0, 1]$  [23]. The choice of the  $S$  coordinate in the form of the cost of resources spent on the manufacture of the product [16] makes it possible to build efficient dynamic distributed models of the inventory control. Let us consider the construction of PDE-model equations for a production line with the structure shown in Fig. 1. Let's introduce the concept of the density of details by the  $m$  operation along the technological route

$$[\chi_0](t, S_m) = \frac{V_m(t)}{\Delta S_m}, \quad V_m(t) = \Omega(t, S_m) - \Omega(t, S_{m-1}). \quad (5)$$

The value  $\Omega(t, S_m)$  specifies the number of details that are in the state of technological processing,  $S \in [0, S_m]$  (not processed at the  $m$  technological operation). The value  $V_m(t)$  is the number of details in the interval  $\Delta S_m = \Delta S_m - \Delta S_{m-1}$  between the  $(m-1)$  and  $m$  operation in the technological space  $(S, \mu)$ . Equation (5) defines the relationship between parameters in the system dynamics model and the PDE model. The production line model (Fig. 1) is considered in a one-moment approximation [9, 16, 23]:

$$\frac{\partial [\chi]_0(t, S)}{\partial t} + \frac{\partial [\chi]_1(t, S)}{\partial S} = 0, \quad (6)$$

$$[\chi]_1(t, S) = [\chi]_{1\psi}(t, S), \quad (7)$$

with the distribution of details at the initial moment of time

$$[\chi]_0(0, S) = [\chi]_{0S}(S). \quad (8)$$

As for the system dynamics model, it is assumed that the tempo  $[\chi]_1(t, S)$  is equal to the average value  $[\chi]_{1\psi}(t, S)$ , there are no feedbacks between the parameters of the production line. The amount of transport delay is determined by equation (6), which is the law of conservation of the number of details as a result of their processing along the technological route.

Equation (6) can be represented as

$$[\chi]_0(t, S) = [\chi]_0(0, S) - \int_0^t \frac{\partial[\chi]_1(\tau, S)}{\partial S} d\tau = [\chi]_0(0, S) - \int_0^t \frac{\partial[\chi]_{1\psi}(\tau, S)}{\partial S} d\tau. \quad (9)$$

For a constant value of the tempo of operation of the equipment  $[\chi]_{1\psi}(t, S) = [\chi]_{1\psi S}(S)$  at the  $m$  operation, solution (5) has the form

$$[\chi]_0(t, S) = [\chi]_0(0, S) - \frac{\partial[\chi]_{1\psi}(S)}{\partial S} t. \quad (10)$$

The backlog amount at the  $m$  operation  $S \in [S_{m-1}, S_m]$  is determined by the expression

$$V_m(t) = \int_{S_{m-1}}^{S_m} [\chi]_0(t, S) dS = [\chi]_0(t, S_m) \Delta S_m + 0(\Delta S_m). \quad (11)$$

The solution of equations (6), (7) determines the state of the production flow line parameters.

## 5. Interconnection of system dynamics and PDE-model equations

System dynamics model equations (1), (2) and PDE-model equations (6), (7) are examples of discrete and continuous description of a production line [24, 25]. With a discrete description, to reduce the number of equations of levels and tempos, technological operations are combined [11] into a generalized operation. Let us consider the construction of equations of system dynamics for a production line with the structure shown in Fig. 1. Let us integrate equation (6) within the  $m$  operation

$$\int_{S_{m-1}}^{S_m} \frac{\partial[\chi]_0(t, S)}{\partial t} dS + \int_{S_{m-1}}^{S_m} \frac{\partial[\chi]_1(t, S)}{\partial S} dS = 0, \quad \Delta S_m = S_m - S_{m-1}. \quad (12)$$

Taking into account the constraint equation (11), the integration of the first and second terms leads to the following form

$$\int_{S_{m-1}}^{S_m} \frac{\partial[\chi]_0(t, S)}{\partial t} dS \approx \frac{d}{dt} \int_{S_{m-1}}^{S_{m-1} + \Delta S_m} [\chi]_0(t, S) dS = \frac{dV_m(t)}{dt}, \quad (13)$$

$$\int_{S_{m-1}}^{S_m} \frac{\partial[\chi]_1(t, S)}{\partial S} dS \approx [\chi]_1(t, S_m) - [\chi]_1(t, S_{m-1}) = [\chi]_{1m}(t) - [\chi]_{1m-1}(t). \quad (14)$$

Substituting expressions (13) and (14) into equation (12), we obtain an equation for changing the value of the backlog of the  $m$  operation

$$\frac{dV_m(t)}{dt} = [\chi]_{1m-1}(t) - [\chi]_{1m}(t), \quad (15)$$

which is the level equation of the system dynamics model (1). Integration of the one-moment PDE-model equation (6) within the  $m$  operation  $S \in [S_{m-1}, S_m]$  allows the transition from PDE-model equations to system dynamics equations. The tempo equation can be obtained from the two-moment approximation [26]. This method makes it possible to use not a phenomenological approach, but a statistical approach when constructing the tempo equations. At the same time, when constructing equations, exact model ideas about the statistical mechanism of interaction of details with each other and with equipment as a result of technological processing are taken into account. This allows us to conclude that the equations of system dynamics are a special case of the equations of the PDE model. The PDE model equations are the foundation for constructing level equations and tempo equations in the system dynamics model.

Let us carry out the passage to the limit, demonstrating the interconnection of the models, using the finite difference method. Let us represent equation (6) in the form of a finite difference equation

$$\frac{[\chi]_0(t_{i+1}, S_m) - [\chi]_0(t_i, S_m)}{\Delta t} + \frac{[\chi]_{1\psi}(t, S_m) - [\chi]_{1\psi}(t, S_{m-1})}{\Delta S_m} + O(\Delta S) = 0,$$

where  $O(\Delta S)$  is the error of replacing equation (6) with an equation in finite differences,  $\Delta S = \max\{\Delta S_m\}$ . Then the expression for the density of backlogs within the  $m$  technological operation takes the form

$$[\chi]_0(t_{i+1}, S_m) = [\chi]_0(t_i, S_m) - \left( [\chi]_{1\psi m+1}(t_i) - [\chi]_{1\psi m}(t_i) \right) \frac{\Delta t}{\Delta S_m} - O(\Delta S) \frac{\Delta t}{\Delta S_m}. \quad (16)$$

Taking into account the limiting expression (11), we write

$$V_m(t_{i+1}) = V_m(t_i) + \left( [\chi]_{1\psi m}(t_i) - [\chi]_{1\psi m+1}(t_i) \right) \Delta t - O(\Delta S) \Delta t. \quad (17)$$

Using the recurrence relation (17) for a constant tempo of operation of the equipment, we obtain the equation of the levels of system dynamics [8] in the form

$$V_m(t_{i+1}) = V_m(0) + \left( [\chi]_{1\psi m} - [\chi]_{1\psi m+1} \right) t_i - O(\Delta S) t_i, \quad m=1 \dots M, \quad (18)$$

determining the state of interoperational backlogs  $V_m(t)$  for the  $m$  operation at time  $t_{i+1}$ .

## 6. The problem solving of the serial-line functioning

Consider the problem of the functioning of a production line which consists of  $M$  technological operations with the initial distribution of parts over interoperational backlogs

$$V_m(0) = V_S(S_m),$$

$$V_S(S) = A \left( 1 + 0.5 \sin(2\pi S / S_d) \right), \quad S_m = S_d \cdot m / M, \quad A=10^4, \quad (19)$$

and a constant tempo of processing parts at the  $m$  operation

$$[\chi]_{1\psi m} = [\chi]_{1\psi S}(S_m), \quad [\chi]_{1\psi S}(S) = MB \left( 1 + 0.5 \cos(2\pi S / S_d) \right), \quad B=2. \quad (20)$$

The division of the technological route into an additional number of technological

operations leads to a decrease in the execution time of a separate operation  $\Delta\tau_{\psi m}$ , and, accordingly, to an inversely proportional increase in the value of the operation tempo  $[\chi]_{1\psi m} = \Delta\tau_{\psi m}^{-1}$ . When each operation of a technological process, which consists of  $M$  technological operations is divided into two operations with equal execution time

$$\Delta\tau_{\psi m 1} + \Delta\tau_{\psi m 2} = \Delta\tau_{\psi m},$$

$$\Delta\tau_{\psi m 1} = \Delta\tau_{\psi m 2} = 0.5\Delta\tau_{\psi m} = 0.5[\chi]_{\psi m}^{-1},$$

a new technological route, consisting of  $2M$  operations, has rate values for a separate technological operation  $[\chi]_{\psi m} = 2\Delta\tau_{\psi m}^{-1}$ . This determined the presence of the factor  $M$  in conditions (19), (20). To construct a solution to problem (5), (16), we introduce dimensionless parameters [27, 28]:

$$\begin{aligned} \xi &= S / S_0, & \xi_m &= S_m / S_0, & \Delta\xi_m &= (S_m - S_{m-1}) / S_0, & \tau &= t / T_0, \\ n_m(\tau) &= V_m(t) / V_0, & n_s(S) &= V_s(S) / V_0, & \theta_{1m} &= \theta_{1s}(\xi_m), & \theta_{1s}(\xi) &= [\chi]_{1\psi s}(S) / [\chi]_{1\psi 0}, \\ \theta_0(\tau, \xi) &= [\chi]_0(t, S) \frac{S_0}{[\chi]_{1\psi 0} T_0}, & \theta_{0s}(\xi) &= [\chi]_0(0, S) \frac{S_0}{[\chi]_{1\psi 0} T_0}. \end{aligned} \quad (21)$$

As the characteristic parameters of the production line model, we use the length of the technological route in the state space  $S_0$ , the average number of details in the buffer  $V_0$ , and the average processing tempo of details for  $M$  operations  $[\chi]_{1\psi 0}$ . The scale of the parameters was chosen for the convenience of performing calculations [27, 29]. Due to the arbitrariness of the choice, we define  $T_0 = V_0 / [\chi]_{1\psi 0}$ ,  $S_0 = S_d$ , which allows us to interpret the value of  $T_0$  as the time during which the average interoperational backlog  $V_0$  will be processed with the average rate  $[\chi]_{1\psi 0}$ . Solution (4), corresponding to the system dynamics model with initial conditions (19), (20) taking into account dimensionless parameters (21), can be written in the following form:

$$n_m(\tau) = 1 + 0.5 \sin(2\pi\xi_m) + 0.5M (\cos(2\pi\xi_{m-1}) - \cos(2\pi\xi_m))\tau, \quad (22)$$

$$n_s(\xi) = 1 + 0.5 \sin(2\pi\xi), \quad \theta_{1s}(\xi) = M(1 + 0.5 \cos(2\pi\xi)), \quad (23)$$

where  $A = [\chi]_{1\psi 0} = B$ ,  $n_m(0) = n_s(\xi_m)$ ,  $\theta_{1m} = \theta_{1s}(\xi_m)$ . Using the formula for the difference of cosines and taking into account that for  $M \gg 1$   $(\xi_{m-1} + \xi_m) \approx 2\xi_m$ ,  $(\xi_{m-1} - \xi_m) \approx \xi_m = M^{-1}$  it follows

$$n_m(\tau) = n_m(0) + \sin(2\pi\xi_m)\pi\tau = 1 + \sin(2\pi\xi_m)(0.5 + \pi\tau). \quad (24)$$

We represent in dimensionless form the solution obtained using the PDE-model equations

$$\theta_0(\tau, \xi) = \theta_0(0, \xi) - \frac{\partial\theta_{1s}(\xi)}{\partial\xi}\tau = \theta_0(0, \xi) + \pi M \sin(2\pi\xi)\tau. \quad (25)$$

Taking into account the dimensionless notation (21), we write

$$n_m(\tau) = \theta_0(\tau, \xi_m) / M, \quad \theta_0(0, \xi_m) = Mn_m(0) = Mn_s(\xi_m), \quad (26)$$

From where

$$\theta_0(\tau, \xi) = M(1 + \sin(2\pi\xi)(0.5 + \pi\tau)), \quad \theta_0(0, \xi) = M(1 + 0.5 \sin(2\pi\xi)). \quad (27)$$

The transition from the dimensionless density of the distribution of interoperative backlogs  $\theta_0(\tau, \xi)$  to the distribution of backlogs  $n_m(\tau)$  is carried out in accordance with

(27)

$$n_m(\tau) = \theta_0(\tau, \xi_m) / M = 1 + \sin(2\pi\xi_m)(0.5 + \pi\tau). \quad (28)$$

Solution (28) coincides with solution (24). Within the specified time, the modeling of changes in the state of interoperational backlogs for a production line with a different number of operations  $M=\{500,100,20,10,5\}$  is shown in Fig. 2. In Fig. 3 the relative error of replacing the equations of the system dynamics model with the equation of the PDE model ( $\tau=0.1$ ) is shown

$$\Delta n_m = 0.5M(\cos(2\pi\xi_{m-1}) - \cos(2\pi\xi_m))\tau - \sin(2\pi\xi_m)\pi\tau. \quad (29)$$

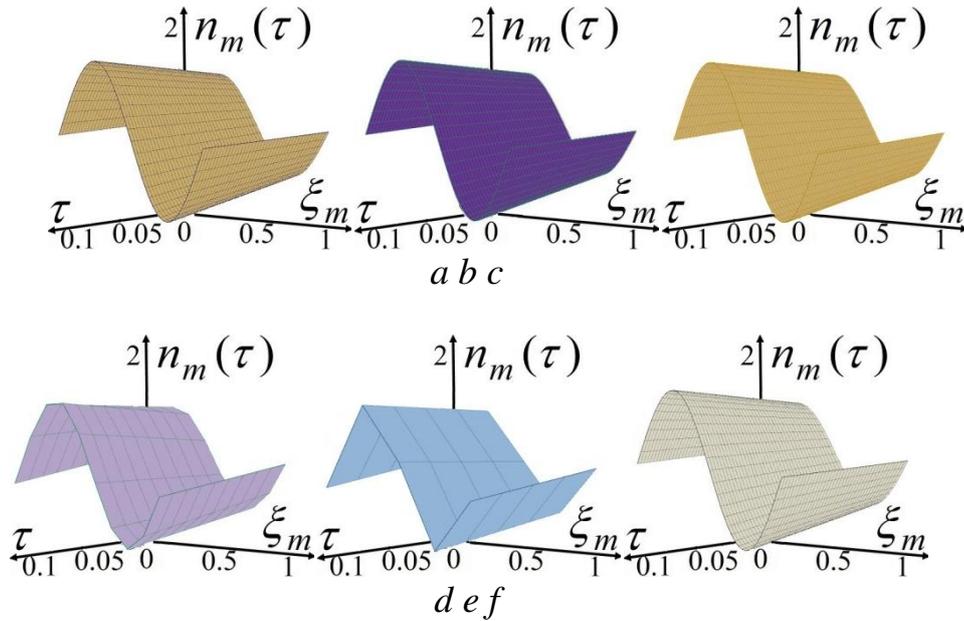


Fig. 2. Modeling the state of interoperational backlogs: *a* –  $M=500$ ; *b* –  $M=100$ ; *c* –  $M=20$ ; *d* –  $M=10$ ; *e* –  $M=5$ ; *f* – PDE

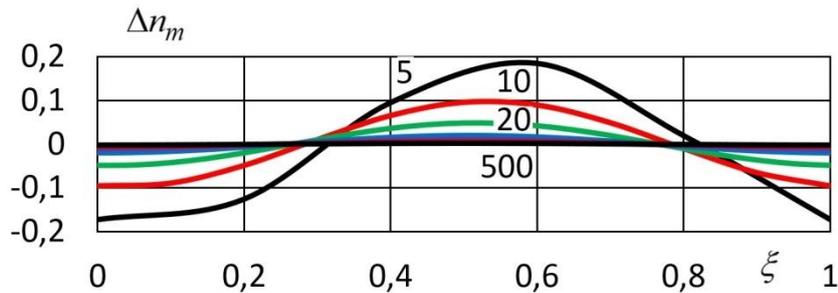


Fig. 3. Relative deviation of inter-operational backlogs  $\Delta n_m$ ,  $M=\{500, 100, 20, 10, 5\}$

The difference between the solutions using the system dynamics model equations and the PDE-model equations is observed for small values of  $M=\{20,10,5\}$ . With a large number of operations, the use of the equations of system dynamics and equations of the PDE-model leads to the same solution, which is a consequence of the use of the limiting relation  $\Delta\xi_m=M^{-1}\ll 1$ .

## 7. Discussion of the research results of describing production lines methods

The paper substantiates and presents a criterion for the limit transition from the

system dynamics equations to the PDE-model equations, which allows us to consider the system dynamics equations as the limit case of the PDE-model equations when describing production multi-operation lines.

A strong limitation of the system dynamics model as a general apparatus for the macroscopic description of a production system is that the system dynamics equations contain phenomenological coefficients that determine the relationship between the amount of interoperational backlogs and the flows of parts entering and leaving the technological operation. The presence of coefficients is due to the use of a phenomenological approach to construct the system dynamics equations. The PDE equations are based on another principle based on the fact that the production line is considered as a system consisting of a large number of details that are in different stages of production (work in progress). Despite the fact that the detail processing along the technological route obeys strictly specified laws of the technological process of a product manufacturing [1], the presence of such a number of details leads to the emergence of strict dependencies between the production line macro-parameters - the backlog amount and the tempo of details processing in a technological operation. These dependencies take into account the structure of the technological process and can be determined for stationary and transient modes of production. Thus, the substantiation of the limit passage allows using the PDE-model as a tool for phenomenological coefficients constructing for transient modes of production lines operation, both in the absence and in the presence of feedbacks. The proposed tool provides effective methods for solving the problem of accounting for the structure of reserves and flows [13–15] when constructing system dynamics equations.

The results of the functioning problem solving of a serial-line [6] with the number of technological operations  $M=\{500, 100, 20, 10, 5$  are shown in Fig. 2. The characteristic time of the process dynamic characteristics analysis (28) is determined by the inequality  $\tau_d \sim (2\pi)^{-1}$ . Criterion  $M^{-1}$  is the basis for choosing a model for describing a production line with a sequential order of performing technological operations (Fig. 1), determines the deviation between the results of calculating the interoperational backlogs of a production line (Fig. 2), performed using the system dynamics equations (22) and PDE-models equations (28). The deviation is shown in Fig. 3, in accordance with expression (29) has the estimate  $\Delta n_m \sim M^{-1}$ . For  $M^{-1} > 0.01$ , it is recommended to use the system dynamics model equations to describe the production line. Otherwise, it is preferable to use the PDE model equations. The criterion value choice is determined by various factors that determine the production line modeling features, among which the time complexity of the model implementation algorithm is of no small importance.

The transition criterion  $M^{-1}$  defines the constraint for replacing the system dynamics equations with the PDE model equations. It should be noted that this limitation applies to production lines with a non-linear dependence of the technological equipment productivity on the number of technological operations at a given moment in time. For synchronized or quasi-synchronized production lines, due to the form of equations (6), (9), this limitation is absent. The transition is possible for an arbitrary number of technological operations. The limitation also does not apply to the case of production lines with a linear dependence of the technological equipment productivity

on the number of technological operations. The remark is quite important due to the fact that the standard mode of operation of the production line is a synchronized or quasi-synchronized mode [10].

The further research prospect is the development of methods for constructing criteria for the similarity of production systems, taking into account both the number of technological operations and the operating mode of the production line, in order to justify the choice of a production line model.

## 8. Conclusions

1. The features and conditions of application of the system dynamics model and the PDE-model equations for the production lines description are considered. It is shown that with an increase in the number of technological operations, the duration of the production line parameters calculating using the system dynamics model equations can exceed the allowable time required to ensure control influences. For the considered serial-line model with a time-independent intensity of processing a product for a technological operation and the absence of feedbacks, the processor time costs linearly depend on the number of system dynamics equations. The presence of feedbacks caused by control actions leads to a nonlinear dependence of the computing resources costs.

2. The limiting transition from PDE-models equations to system dynamics equations for  $M \gg 1$  is presented, showing the models interconnection, allows us to assert that the system dynamics equations are the limiting case of the PDE-model equations. The conditions for choosing a model for describing a production line with a sequential order of performing technological operations are determined by the value of the criterion  $M^{-1}$ , which characterizes the number of production operations in the technological route. A comparative analysis of the production line operation problem solution is carried out, which shows that with a large number of technological operations, the system dynamics equations can be replaced by the PDE-model equations.

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