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Marjit, Sugata and Yang, Lei

Indian Institute of Foreign Trade; Centre for Studies in Social
Sciences Calcutta; CESifo, Hong Kong Polytechnic University

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An Elementary Theorem on Gains from Virtual Trade

Sugata Marjit

Indian Institute of Foreign Trade

Kolkata, India

CTRPFP

Centre for Studies in Social Sciences Calcutta, India

CESifo

Munich, Germany

&

Lei Yang

Hong Kong Polytechnic University

November, 2020

Abstract

Virtual markets allow consumers to save time costs to purchase goods and services. Countries lose relative to the conventional welfare gain when they increase consumption of non-virtual goods under free trade. We include the classical gains from trade theorem as a special case. For two identical countries that have same endowment and technology, the income difference between them can generate trade when we consider the time cost of purchasing goods. The rich country exports the non-virtual good and imports the virtual good while the poor country exports the virtual good and imports the non-virtual good.

Key Words: Virtual Trade, Time Cost

JEL Classification: F10, O30

1. Introduction

Virtual markets, exchange of goods and services have become the order of the day. Radical and novel technologies made it possible and COVID-19 pandemic has increased its relevance thousand fold. With this backdrop we explore how the fundamental theorem of gains from trade and the pattern of trade functions when trade can take place on virtual platform. We have the following two findings in this paper. First, countries may lose relative to the conventional welfare gain under free trade when we consider the time cost of purchasing goods. As the consumption level of non-virtual good declines, the welfare converges to the level of classical gains from trade. Second, for two identical countries in terms of endowment and technology, the income difference between them can generate trade when we consider the time cost of purchasing goods. The rich country exports the non-virtual good and imports the virtual good while the poor country exports the virtual good and imports the non-virtual good.

In this paper, we consider the fact that consumers have to sacrifice time to purchase goods and accordingly consumption of non-virtual good leads to the leakage of leisure, which was neglected by previous literature. We fill this void by providing new insights on the gains from trade and establishing a link between differences in income and the pattern of trade when we take into considerations of the time cost of purchasing goods and its impact on the allocation of leisure.

Virtual market implies that consumers can place order for the goods without spending any time to physically visit the location where they would be made available otherwise, such as retail stores. Generally speaking, we can conceive of a set of goods which can be accessed via virtual platform thus saving the time cost for purchasing them and another that of non-virtual goods which have to be gathered physically by spending time cost. Therefore, consumption of non-virtual good leads to a leakage of leisure but consumption of virtual good

does not. Accordingly, the relative effective cost of consuming non-virtual good to virtual good is different from their relative price as determined in the market. Based on that, we find that countries may lose relative to the conventional welfare gain under free trade. Further, we reveal the link between the difference in income level and the pattern of trade. Rich countries cut back their consumption of non-virtual goods under trade but increase the consumption of virtual goods, thereby enjoying more leisure. Poor countries gain from trade because of the relative price difference, but lose because they have to spend time to consume. Therefore, we provide the new insight that the income difference among different countries can generate trade.

Theoretical literature on the link between benefits of trade and exchange on virtual platforms was pointed out first by Marjit (2007) which provides a Ricardian model where exactly identical countries located across non-overlapping time zones gain from trading intermediates on virtual platform leading to quicker delivery of the product. Time essentially becomes the source of comparative advantage. Following this work, Kikuchi and Marjit (2011) and Marjit and Mandal (2017) analyze how trade across time zones can lead to automatic increase in the growth rate. Nakanishi and Van Long (2020) discuss virtual mobility of labor and wage inequality in models with separated time zones and finally in a recent book Marjit, Mandal and Nakanishi (2020) discuss various aspects of trade due to separated time zones.

Our paper is also related to the empirical literature on internet and virtual markets not international trade. It shows that the internet can facilitate business transaction because it can lower transaction costs. Varian et al. (2004) shows that internet has greatly facilitated business transactions by making search and communication faster and cheaper. Freund and Weinhold (2002) provide evidence that internet is related to growth in service trade. Levin (2011) discusses the economic impacts of internet markets. Keller and Yeaple (2013) find that internet reduces spatial barriers to disembodied knowledge transfer. Ricci and Trionfetti (2012) find that access to internet has positive impacts on export performance using multi-country multi-

industry firm-level dataset. Kneller and Timmis (2016) provide evidence for a causal effect from the broadband use on trade in business service using U.K. data. Fernandes et al. (2019) show that the internet rollout boosted firm manufacturing exports, even before the rise of major e-commerce platforms in China based on Chinese firm-level data in 1999–2007. The positive impact of internet on exports comes from the fact that internet can affect both demand side by reducing trade costs and supply side of international trade allowing firms to source their inputs and organize production more efficiently.

Previous literature including Kikuchi and Marjit (2011), Marjit and Mandal (2017) have discussed issues related to quick delivery of intermediate goods. In contrast, in this paper we focus on the leisure or time cost related to procure final goods for consumption. Our paper also builds on the strand of literature that considers the impacts of transportation cost on the relative price in international trade (Jones, Marjit and Beladi, 1999).

Our paper is also related to the line of literature on the consumption time of a good ¹. Our paper is different from this line of the literature in that our paper focuses on the time cost of purchasing the goods while the aforementioned focuses on the time of physically consume the goods.

None of the aforementioned studies, however, has touched upon how the fundamental theorem of gains from trade functions when trade can take place on virtual platform. Our paper fills this gap in the literature by offering a new perspective on gains from trade where virtual and non-virtual goods coexist. In this set up we modify the classical gains from trade theorem applicable only to non-virtual goods. We find that countries lose (gain) relative to the conventional welfare gain under free trade if they export (import) non-virtual goods. As the consumption level of non-virtual good declines, the welfare converges to the level of classical

¹ See Becker (1965) and Roy (2005).

gains from trade. Our theorem includes the classical theorem as a special case. Furthermore, we find that greater volume of trade will compensate for consumption distortion due to the time cost.

The remainder of the paper is organized as follows. In the next section, we present a basic model of virtual goods and analyze the gains of trade and the pattern of trade when considering the time cost of purchasing goods. We offer concluding remarks in the section 3.

2. The Model

2.1 The Gains from Trade

Consider an economy endowed with a set of n goods $(X_1 \dots \dots \dots X_n)$. We partition the set into two groups. $(X_1 \dots \dots \dots X_m)$ ($n \geq m$) represents the set of virtual goods that can be traded online and no time cost is incurred for physically purchasing the goods. $(X_{m+1} \dots \dots \dots X_n)$ is the set of offline goods which need time cost to be added to the market price and for them consumers' price is different from producers' price. Thus X_i , $i = 1, \dots \dots m$ is defined as virtual goods and X_j , $j = m + 1, \dots \dots n$ is defined as non-virtual or offline goods. We postulate a very simple price equation for x_j which represents the consumption of the j^{th} good, where $(x_1, \dots \dots \dots x_n)$ is the set of consumption or demands. Suppose each worker is endowed with H hours in total, among which L is the leisure time he has. Let t_j denote the time cost of physically purchasing the goods for good j . Therefore the net leisure time after deducing the time cost of physically purchasing all non-virtual goods for are given

by $L = L - \sum_{j=m+1}^n t_j x_j$. Let w denote the wage rate. Let $u(\cdot)$ denote the utility function.

Therefore, we have the following maximization problem.

$$\text{Max} \quad u(x_1, \dots, x_m, x_{m+1}, \dots, x_n, L - \sum_{i>m} t_i x_j) \quad (1)$$

$$\text{s.t } w(H - L) = \sum_{i=1}^n P_i x_i$$

$$L = L - \sum_{j=m+1}^n t_j x_j \quad (2)$$

$$\frac{\partial u}{\partial x_i} = \lambda p_i \quad i = 1 \dots m$$

$$\frac{\partial u}{\partial x_j} = \lambda p_j + \frac{\partial u}{\partial L} t_j \quad j = m+1 \dots n$$

$$\frac{\partial u}{\partial L} = \lambda w$$

Therefore, we have

$$\frac{MU_{x_i}}{MU_L} = \frac{\frac{\partial u}{\partial x_i}}{\frac{\partial u}{\partial L}} = \frac{p_i}{w} \quad i = 1 \dots m \quad (3)$$

$$\frac{MU_{x_j}}{MU_L} = \frac{\frac{\partial u}{\partial x_j}}{\frac{\partial u}{\partial L}} = \frac{p_j}{w} + t_j \quad j = m+1 \dots n \quad (4)$$

Choose $W = 1$, then we have

$$\frac{MU_{x_i}}{MU_L} = p_i \quad i = 1 \dots m \quad (5)$$

$$\frac{MU_{x_j}}{MU_L} = p_j + t_j \quad j = m+1 \dots n \quad (6)$$

Hence, we have

$$\begin{aligned} du &= \sum_{i=1}^m \frac{\partial u}{\partial x_i} dx_i + \sum_{j=m+1}^n \frac{\partial u}{\partial x_j} dx_j + \frac{\partial u}{\partial L} dL \\ &= \sum_{i=1}^m \lambda P_i dx_i + \sum_{j=m+1}^n \lambda (P_i + t_j) dx_j + \lambda dL \end{aligned} \quad (7)$$

Define Ω as the welfare. Therefore, we have

$$d\Omega = \sum_{i=1}^m P_i dx_i + \sum_{j=m+1}^n P_j dx_j + dL \quad (8)$$

As $L = L - \sum_{j=m+1}^n t_j x_j$, we have

$$dL = dL - \sum_{j=m+1}^n t_j dx_j \quad (9)$$

Assume that $dt_j = 0$, we have

$$d\Omega = \sum_{i=1}^m P_i dx_i + \sum_{j=m+1}^n P_j dx_j + dL - \sum_{j=m+1}^n t_j dx_j \quad (10)$$

Let \bar{x}_i $i = 1 \dots n$ denote the output of good i .

From the budget constraint, we have

$$\sum_{i=1}^m P_i \bar{x}_i + \sum_{j=m+1}^n P_j \bar{x}_j = \sum_{i=1}^m P_i x_i + \sum_{j=m+1}^n P_j x_j - \text{Balance of Trade} \quad (11)$$

Total differentiation equation (9), we have

$$\sum_{i=1}^m P_i d\bar{x}_i + \sum_{j=m+1}^n P_j d\bar{x}_j + \sum_{i=1}^m \bar{x}_i dP_i + \sum_{j=m+1}^n \bar{x}_j dP_j = \sum_{i=1}^m x_i dP_i + \sum_{j=m+1}^n x_j dP_j + d\Omega - dL.$$

Total value of production can change either through changes in the prices when the levels of output held constant (i.e. at a given allocation of labour in each sector, hence with constant total L), or through changes in the levels of output with prices held constant. The change in the value of output at given prices is caused by increase in labour supply and through a decline in L . As we have used leisure as the numeraire, the incremental value of output in terms of leisure when leisure changes must match the fall in leisure. In other words with labour as the only factor of production, the incremental value of output at given prices when labour supply is changing must be equal to the extra wage bill. Thus the change in the labour value of

production at given prices is the same as $-dL$ (increase in labour supply) or in money price terms, which is $w(-dL)$.

Therefore, equating the first two terms of LHS with $-dL$ in the RHS of the above relationship we get

$$d\Omega = \sum_{i=1}^m dP_i(\bar{x}_i - x_i) + \sum_{j=m+1}^n dP_j(\bar{x}_j - x_j) - \sum_{j=m+1}^n t_j dx_j \quad (12)$$

The first two components of RHS in (12) denotes the standard gains from trade. From the right-hand side of equation (12), we can see that a fall in leisure is compensated by higher income.

but a fall in L by a rise in $\sum_{j=m+1}^n t_j x_j$ is not. Note that for $t_j = 0 \quad \forall j$, we get the classical result.

We thus have the following proposition.

Proposition 1: *Countries lose relative to the conventional welfare gain under trade when we consider the virtual trade if they increase the consumption of the non-virtual goods. As the consumption level of non-virtual good declines, the welfare converges to the level of classical gains from trade.*

Proof- See the discussion above.

QED

Proposition 1 implies that the reduction in time cost of purchasing the non-virtual goods (a decrease in t_j) can lead to the gains from trade even if the volume of consumption does not

change. In fact in our set up if there were only virtual goods, we would have the same expression for gains from trade as in the standard model.

2.2 Income and the Pattern of Trade

Consider country A and country B produce both a set of virtual goods x_i ($i = 1 \dots m$) and a set of non-virtual goods x_j $j = m + 1 \dots n$ with one factor of production labor in a Ricardian setting. Assume that both countries have the labor endowment L . Suppose country A has absolute advantage in all goods relative to country B such that there is no comparative advantage for them.

Suppose for each good i ($i = 1 \dots n$) the unit labor requirement in country A is a with $a < 1$ no matter the goods belongs to virtual good or non-virtual good. And the unit labor requirement for each good i ($i = 1 \dots n$) in country B is b with $b = 1$. Hence, we have

$\frac{p_j}{p_i} = \frac{1}{a}$ is the same for both countries in autarky. Also assume t_j is the same for all good

j in both home country and foreign country. Accordingly, we have $t_j = t_j^*$. Suppose that country A is richer than country B in the sense that the wage rate in country A is higher than that in country B. Let w and w^* denote the wage of country A and B respectively. Thus we have $w > w^*$.

From equation (3) and (4), the relative price between a non-virtual good and a virtual good $\forall i$ and $\forall j$ is given by

$$\frac{\frac{\partial u}{\partial x_j}}{\frac{\partial u}{\partial x_i}} = \frac{\frac{P_j}{w} + t_j}{\frac{P_i}{w}} = \frac{P_j}{P_i} + t_j \frac{w}{P_i} \quad (13)$$

Let $\frac{MU_{x_j}}{MU_{x_i}} = f\left(\frac{x_j}{x_i}\right)$, $f' < 0$. We have

$$f\left(\frac{x_j}{x_i}\right) = \frac{P_j}{P_i} + t_j \frac{w}{P_i} \quad (14)$$

Thus, we have

$$\frac{P_j}{P_i} = f\left(\frac{x_j}{x_i}\right) - t_j \frac{w}{P_i} \quad (15)$$

From the market equilibrium condition, we have

$$\frac{\bar{x}_j}{\bar{x}_i} = \frac{\bar{x}_j}{\bar{x}_i}.$$

Here \bar{x}_j and \bar{x}_i are constant. Similarly, in the foreign country we have

$$\frac{x_j^*}{x_i^*} = \frac{\bar{x}_j^*}{\bar{x}_i^*}.$$

As the labor endowment and unit labor requirement ratio of good j to good i is the same for both home country and foreign country, we have

$$\frac{\bar{x}_j}{\bar{x}_i} = \frac{\bar{x}_j^*}{\bar{x}_i^*}.$$

Therefore, we have

$$\frac{x_j}{x_i} = \frac{x_j^*}{x_i^*}$$

Define $P_i \equiv 1$ and $P = P_j$. Let P_A and P_A^* denote the relative price of good j to good i in the home country and that in the foreign country under autarky respectively. Given that

$\frac{x_j}{x_i} = \frac{x_j^*}{x_i^*}$. From equation (15), in both countries we have

$$P_A = f\left(\frac{x_j}{x_i}\right) - t_j \frac{w}{P_i}$$

$$P_A^* = f\left(\frac{x_j^*}{x_i^*}\right) - t_j^* \frac{w^*}{P_i^*}$$

As $t_j = t_j^*$ and $w > w^*$, we have $P_A < P_A^*$.

We illustrate the above results in Figure 1. Here the horizontal axis represents the relative quantity of the non-virtual good to the virtual good and the vertical axis represents the relative price of the non-virtual good to the virtual good. The relative supply of the non-virtual good to

the virtual good in the home country is the same as that in the foreign country ($\frac{\bar{x}_j}{\bar{x}_i} = \frac{\bar{x}_j^*}{\bar{x}_i^*}$).

Hence, the vertical line represents relative supply curve of the non-virtual good to the virtual good in both home country and foreign country. RD and RD^* denotes the relative demand curve of the non-virtual good to the virtual good of the home country and the foreign country respectively.

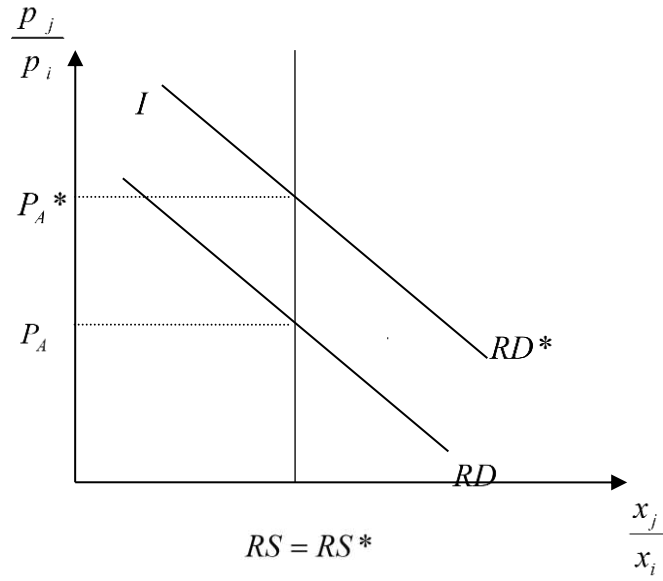


Figure 1. Equilibrium in Both Countries under Autarky

Let us use a two-good example to show the relationship between income and the pattern of trade. Consider country A and country B produce both good 1 (virtual good) and good 2 (non-virtual good) with labor as the only factor of production.

$$\begin{aligned} \text{Max} \quad & u(x_1, x_2, L - tx_2) \\ \text{s.t} \quad & \frac{w}{P_1}(H - L) = x_1 + \frac{P_2}{P_1}x_2 \end{aligned}$$

Define $P_1 \equiv 1$ and $P = P_2$. Thus we have

$$w(H - L) = x_1 + Px_2$$

$$\frac{\frac{\partial u}{\partial x_2}}{\frac{\partial u}{\partial x_1}} = \frac{\frac{P_2}{P_1} + t}{\frac{P_1}{w}} = P + tw$$

Let $\frac{MU_{x_2}}{MU_{x_1}} = f\left(\frac{x_2}{x_1}\right)$, $f' < 0$. We have

$$f\left(\frac{x_2}{x_1}\right) = P + tw$$

From the market equilibrium condition, we have

$$\frac{x_2}{x_1} = \frac{\bar{x}_2}{\bar{x}_1}$$

Here \bar{x}_1 and \bar{x}_2 are constant. Similarly, in the foreign country we have

$$\frac{x_2^*}{x_1^*} = \frac{\bar{x}_2^*}{\bar{x}_1^*}$$

As the labor endowment and unit labor requirement ratio of good 2 to good 1 is the same for both home country and foreign country, we have

$$\frac{\bar{x}_2}{\bar{x}_1} = \frac{\bar{x}_2^*}{\bar{x}_1^*}$$

Therefore, we have

$$\frac{x_2}{x_1} = \frac{x_2^*}{x_1^*}$$

Let P_A and P_A^* denote the relative price of good 2 to good 1 in the home country and that in

the foreign country respectively. Given that $\frac{x_2}{x_1} = \frac{x_2^*}{x_1^*}$. In both countries, we have

$$f\left(\frac{x_2}{x_1}\right) = P + tw$$

$$P_A = f\left(\frac{x_2}{x_1}\right) - tw$$

$$P_A^* = f\left(\frac{x_2^*}{x_1^*}\right) - tw^*$$

As $w > w^*$, we have $P_A < P_A^*$. Therefore the rich country exports good 2 (non-virtual good) and imports good 1 (virtual good).

This leads to the following proposition.

Proposition 2: *Income difference between different countries can generate trade when we consider both virtual good and non-virtual good. The pattern of trade is as follows. The rich country exports the non-virtual good and imports the virtual good while the poor country exports the virtual good and imports the non-virtual good.*

Proof: See the discussion above which shows the relevant differences in autarkic prices.

QED

Proposition 2 implies that trade rich countries cut back their consumption of non-virtual goods under trade but increase the consumption of virtual goods, thereby enjoying more leisure. Poor countries gain from trade because of the relative price difference, but lose because they have to spend time to consume. For $t_j = 0$ all are virtual goods and we are back with the classical approach. But income difference generating trade is a new result in tune with Linder's early observations (Acharyya, 2013). Interestingly Acharyya and Kar (2014) point out that Malthus had a conjecture regarding how availability of cheaper consumption goods through trade can lead to greater labour supply. But import of virtual goods gives even more of a gain as it reduces overall leisure costs of consumption.

3. Concluding Remarks

Virtual markets has attracted growing interest in recent years, and COVID-19 pandemic has increased its relevance. In this paper we try to explore how the fundamental theorem of gains from trade and the pattern of trade can act when trade can take place on virtual platform by developing a simple theoretical model. In our model we modify the classical gains from trade theorem applicable only to non-virtual goods by including both virtual goods - a set of goods which can be accessed via virtual platform thus saving the time cost for purchasing them and non-virtual goods which have to be gathered physically by spending time cost. We show that countries lose relative to the conventional welfare gain under trade when we consider the time cost of purchasing goods. As the consumption level of non-virtual good declines, the welfare converges to the level of classical gains from trade. Therefore, our theorem includes the classical theorem as a special case. For two identical countries with same endowment and technology, the income difference between them can generate trade when we consider the time cost of purchasing goods. The rich country exports the non-virtual good and imports the virtual good while the poor country exports the virtual good and imports the non-virtual good.

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