Organized Information Transmission

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Abstract

In reality, the organizational structure of information — describing *how* information is transmitted to its recipients — is as important as its content. In this paper, we introduce families of (indirect) information structures, namely meeting schemes and delegated hierarchies, that capture the horizontal and vertical dimensions of real-world transmission. We characterize the outcomes that they implement in general (finite) games and show that they are optimal in binary-action environments with strategic complementarities. Our application to classical regime-change games illustrates the variety of optimal meeting schemes and delegated hierarchies as a function of the objective. (*JEL codes*: C72, D82, D83.)

Keywords: Incomplete information, information hierarchy, delegated transmission, meeting scheme, Bayes correlated equilibrium, information design.

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1 Introduction

Information is often transmitted in horizontal or vertical ways. Horizontal transmission refers to informing a group of listeners symmetrically and simultaneously. Academic seminars are an example of this, where a researcher speaks in front of an assembly of peers. Board meetings are another example in which management presents performance indicators to directors. Vertical transmission, instead, refers to information passed down sequentially, and perhaps partially, from one individual to another. In organizations, information is regularly communicated by directors to managers, then to supervisors, and finally to lower-level employees. In marketing, viral strategies rely on consumers "spreading the word" and forwarding information to friends and family.

There are many reasons why such protocols are ubiquitous in practice, even purely logistical reasons underlying the cost of information transmission. For one, giving information to many people at once, instead of giving the same information to each of them individually, saves on physical communication costs. In this vein, most researchers may prefer to present their work to an assembly of peers rather than give the same presentation to each of them in private (even abstracting from the benefits of collective engagement in seminars). Delegating information transmission to the receivers themselves can be another economical strategy. Downstream communication in organizations is an illustration, where an executive may inform a supervisor and instruct him how to inform other subordinates rather than personally bring that information to those subordinates.

In this paper, we introduce families of *indirect* information structures[1] that capture the horizontal and vertical dimensions of real-world transmission. In incomplete information games, information (about the state of the world and others' information about it) affects equilibrium behavior and, thus, the resulting outcomes. We characterize the outcomes that these families of information structures implement in general finite games, and show that they are optimal in binary-action games with strategic complementarities.[2] These games cover important economic applications, such as global games of regime-change, team effort problems and purchase decisions with network effects.

We capture horizontal transmission through information communicated publicly but to a restricted audience. This family of information structures we call *single-meeting schemes*.

[1] An information structure specifies a distribution over message profiles for each realization of the state, thereby formalizing the protocol by which players get informed.

[2] There is a high and a low action and the low action carries a zero payoff.

In organizations, for example, this is akin to the board calling a meeting during which a public announcement will be made to all people the board has chosen to invite. These information structures are intuitively appealing, because only one meeting ever takes place, during which information is communicated publicly to the participants or, equivalently, one email is ever sent to selected recipients.

We capture vertical transmission through *delegated hierarchies*, where transmission is handed to the players themselves and information flows down the hierarchy, in a sequential Cheap Talk fashion, after communicating directly with the highest ranked player only. Players are totally ordered according to how informed they are, so that they are able to inform the next player in the hierarchy, and they also have the strategic incentives to pass down information truthfully. In organizations, delegated transmission is used for obvious reasons, as executives cannot realistically deliver all the different pieces of information to different subordinates on a regular basis. In marketing, high-profile individuals are often the target of information about a product or an event, so that in turn they can inform their social media followers, who can in turn inform their followers, and so on.

In comparison, direct information structures, which invoke the Revelation Principle (Myerson (1991)) and make incentive-compatible action recommendations, do not constrain information to be commonly observed by some players or to be transmittable from one player to another in an incentive compatible manner. This can make them very difficult to implement in reality. Indeed, in many environments,[3] optimal direct information structures are not single-meeting schemes or delegated hierarchies, as the action recommendations they involve are often private information to each player, which others are uncertain about. To implement these privacy requirements, there is hardly any other way but to communicate with every single player individually. In large organizations or markets, this describes an unrealistic picture of information transmission.[4]

In incomplete information games, the strategic outcomes can be described by distributions over action profiles and states. Fixing the payoffs and the prior beliefs, an outcome distribution is (weakly) implemented by an information structure if it is the result of pure

[3]These include monotone environments, where players play an incomplete information game with strategic complementarities and the sender wants to foster large actions. For example, consider a seller of a good with positive network effects, who wants to maximize total sales by revealing information about its quality to a population of heterogeneous consumers.

[4]Concerns about direct mechanisms have been raised previously by Van Zandt (2007): "The Revelation Principle in mechanism design is both a blessing and a curse [...] It is a curse because direct mechanisms provide such an unrealistic picture of decision-making in organizations."

Bayes Nash equilibrium (BNE) behavior under that information structure. In which environments do single-meeting schemes and delegated hierarchies implement an optimal outcome distribution? What outcome distributions can they implement in general?

Our optimality results (Theorems 1 and 3) show that single-meeting schemes and delegated hierarchies are optimal in binary-action games with strategic complementarities. From Bergemann and Morris (2016), an outcome distribution is implementable if and only if it is a Bayes correlated equilibrium (BCE). We look at environments for which an optimal BCE[5] can be implemented by one of our information structures. Intuition suggests that if players' actions are strategic complements, then some degree of shared information should help coordination. Yet, we know that purely public information is often strictly suboptimal in such contexts, for example when the objective is to maximize the total probability of the high action among heterogenous players. Theorem 1 sheds light on this matter, as single-meeting schemes provide the optimal kind of shared information. This holds with great generality provided each player finds it more beneficial to choose the high action if others do so as well. Theorem 3 obtains under a stronger form of complementarities: if each player's utility in the high action is not only increasing but also supermodular in the others' actions, then a delegated hierarchy is optimal. This result implies that an informational line network (where players are totally ordered by informedness) implements an optimal BCE, even though players' interdependencies could be described by *any* network of supermodular interactions.

Our implementation results (Theorems 2 and 4) characterize the outcome distributions that can be implemented in pure Bayes Nash equilibrium by our families of information structures.[6] The theorems make a direct connection between the organizational constraints on information and the resulting constraints on strategic outcomes. These results are for general finite environments, and while the restriction to pure strategies entails a loss of generality, the analysis benefits from the additional structure and remains rich enough to be interesting. When thinking about how a group of players self-organize (or are organized by a third party) to receive information, horizontal and vertical transmission offer natural options, as modeled by our information structures. By shaping the kind of incomplete information that can emerge within the group, this also shapes the

[5] That is, maximizing the expected value of an objective function.

[6] While these characterizations are in terms of distributions over actions and states, direct implementation à la Bergemann and Morris (2016) and Taneva (2019) — by using action recommendations as messages — will generally not succeed in implementing the desired outcome distributions by a single-meeting scheme or a delegated hierarchy.

strategic outcomes that can emerge. Our results characterize these strategic outcomes by means of linear systems — with stronger conditions than the BCE constraints. A natural application of the characterizing inequalities is to information design, where they appear as constraints in linear programming.

Finally, we illustrate our optimality results in a classical global game of regime change (Sakovics and Steiner (2012)). We show how the optimal single-meeting scheme and delegated hierarchy change with the objective function. In a different application, we illustrate that optimization may require players to be treated equally, because they should receive the same information, yet delegation may require them to be ordered distinctively.

Related Literature. Various definitions of correlated equilibrium in games with incomplete information have been proposed (Forges (1993, 2006), Bergemann and Morris (2016)), depending on what variables the mediator or the correlating device can condition on to correlate players' actions (e.g., the state, private types). Our characterizations propose new forms of correlated equilibrium, where the ability to correlate behavior is not limited by the conditioning variables, but by the organizational structure of information. In doing so, we bring an organizational perspective to the formulation of incomplete information and study the resulting strategic implications.

This paper contributes to the information design literature, surveyed in Bergemann and Morris (2019) and Kamenica (2019), by importing organizational considerations into the designer's problem. In particular, our optimality results contribute to the recent interest in binary-action supermodular games. Arieli and Babichenko (2019), Candogan and Drakopoulos (2020) and Candogan (2020) study the optimal design of *direct* information structures in binary-action supermodular games (with binary states or in linear networks). Our motivation is different and highlights the role of *indirect* information structures. Oyama and Takahashi (2020) focus on equilibrium robustness in binary-action supermodular games and Morris, Oyama, and Takahashi (2020) on implementation in the smallest equilibrium through information design. Under adversarial equilibrium selection, Inostroza and Pavan (2020) derive an optimal public information structure, while Li, Song, and Zhao (2019) and Mathevet, Perego, and Taneva (2020) have shown the importance of indirect private information structures.

In the context of a multi-player email game, Morris (2002) introduces the concept of locally public communication where subgroups of players meet sequentially and share information that becomes common knowledge within each meeting. The size of the meetings is shown to play a pivotal role in making coordination possible. Two recent papers examine how information is structured or shared among many players. Brooks, Frankel, and Kamenica (2020) define an information hierarchy as a partially ordered set of agents, where the order describes who is better informed about the state in the Blackwell sense. They characterize the information hierarchies that are compatible with the strong "informedness" order, in which higher ranked players know all the information of less informed ones. Our definition of delegated hierarchy completely ranks the players under the strong informedness order and adds incentive compatibility to information transmission. Galperti and Perego (2020) study the impact of information spillovers on the outcomes of incomplete information games. Their notion of an information system assumes that messages are automatically shared between linked players on a network.

Hierarchical transmission relates to strategic communication. Ambrus, Azevedo, and Kamada (2013) and Laclau, Renou, and Venel (2020) study rich cheap talk intermediation networks between a sender and a single receiver, where the intermediators do not interact with each other beyond the transmission of messages. Our players both receive and send information, and strategically choose an action; their ordering and interdependence shape our characterization — for the special case of a line network. Hagenbach and Koessler (2010) and Galeotti, Ghiglino, and Squintani (2013) examine strategic (and simultaneous) pre-play communication and characterize the communication networks that emerge in equilibrium under quadratic payoffs. Our delegated hierarchies represent an equilibrium communication network in pre-play communication over multiple rounds, when only the highest-ranked player starts off with private information.

Finally, our work is also related to information transmission within organizations (Radner (1993), Van Zandt (1999), Rantakari (2008), Alonso, Dessein, and Matouschek (2008), Hori (2006), Dessein (2002), Crémer, Garicano, and Prat (2007) among others). In our framework, however, the principal is not trying to elicit information from better informed players, but instead to disseminate it effectively throughout the organization.

2 The Framework

2.1 Payoffs

A set of players $\mathscr{I} = \{1, \ldots, n\}$ interact in an environment where the state of the world, ω, is drawn from a finite set $\Omega \subseteq \mathbb{R}$ according to prior $\mu \in \Delta(\Omega)$. Players simultaneously choose one of two actions, $a_i \in A_i = \{0,1\}$ for all $i \in \mathscr{I}$, and payoffs are given by $u_i : A \times \Omega \to \mathbb{R}$ for each $i \in \mathscr{I}$ where $A = \times_i A_i$. Assume that $i \in \mathscr{I}$ gets utility 0 if $a_i = 0$. This framework captures well-known economic applications.

Network Interactions. Consider players making a binary choice while on a network:

$$u_i(a_i, a_{-i}; \omega) = \gamma_i(\omega) a_i + \sum_{j \neq i} \gamma_{ij}(\omega) a_i a_j,^{7}$$

where γ_i measures i's intrinsic preference for the high action and γ_{ij} measures the interdependence between i's and j's actions. In this incomplete information version of Ballester, Calvo-Armengol, and Zenou (2006), the state may affect the intrinsic preferences for action 1, but also the interdependencies in the network. This covers fixed networks where $\gamma_{ij}(\omega) = \gamma_{ij}$ for all $\omega \in \Omega$, but also random networks where the coefficients $\{\gamma_{ij}(\cdot)\}$ are uncertain.

This formulation subsumes the investment game of Carlsson and van Damme (1993), as well as "beauty contest" descriptions of social phenomena such as location choice of city versus suburb, or entry into versus exit from the labor force (see Brock and Durlauf (2001) and Glaeser and Scheinkman (1999)).

Regime-Change Models. These models are coordination games with binary actions in which a status quo is abandoned when a sufficiently large number of players choose action 1. Examples abound in the global games literature (Morris and Shin (2003)), such as currency attacks (Morris and Shin (1998)), bank runs (Goldstein and Pauzner (2005)), and many others. A finite set of investors make binary investment decisions into a common project of uncertain quality $\omega \in \Omega$. Let $\kappa_i > 0$ be i's contribution to success, $c_i > 0$ his investment cost, and $b_i > c_i$ his benefit from a successful project. If i invests, his payoff is:

$$u_i(a_{-i}; \omega) = \begin{cases} b_i - c_i & \text{if} \quad \kappa_i + \sum_{j \neq i} \kappa_j a_j > 1 - \omega \\ -c_i & \text{otherwise.} \end{cases}$$

2.2 Outcome Distributions

2.1.1 Definitions. An outcome distribution is any $p \in \Delta(A \times \Omega)$. Given prior $\mu \in \Delta(\Omega)$ and payoff functions $\{u_i\}$, Bayes correlated equilibria (BCE, Bergemann and Morris (2016))

[7] Other terms could be added, such as $\sum_{j \neq i, k \neq i, j} \gamma_{ijk}(\omega) a_i a_j a_k$ (for example, $\gamma_{ijk}(\omega) \geq 0$) to capture i's preference for coordination between j and k.

are outcome distributions p such that $p(A \times \{\omega\}) = \mu(\omega)$ for all $\omega \in \Omega$, and for all $i \in \mathscr{I}$ and $a_i \in A_i$,

$$\sum_{\omega \in \Omega} \sum_{a_{-i} \in A_{-i}} p(a,\omega)\big(u_i(a;\omega) - u_i(a_i', a_{-i};\omega)\big) \geq 0 \quad \forall a_i' \in A_i. \tag{1}$$

This equilibrium concept describes state-dependent correlated behavior that respects the prior μ and satisfies the obedience constraints given by (1). Let $BCE(\mu)$ denote the set of BCE given μ.

An information structure is a pair (S,P), where $S = \prod_i S_i$ is a finite message space and $P = \{P(\cdot|\omega)\}_{\omega \in \Omega}$ is a family of conditional distributions over S. In any state ω, message profile $s = (s_i)_i$ is drawn with probability $P(s|\omega)$ and i observes s_i. Given $\mathscr{I}' \subseteq \mathscr{I}$, we use $P((s_i)_{i \in \mathscr{I}'}) = \sum_\omega \sum_{j \notin \mathscr{I}'} \sum_{s_j} P(s|\omega)\mu(\omega)$ as a shorthand notation for the unconditional probability of $(s_i)_{i \in \mathscr{I}'}$. Without loss, assume $P(s_i) > 0$ for each $s_i \in S_i$. Given μ and information structure (S,P), let

$$\mu_i\left(s_{-i}', \omega | s_i\right) = \frac{\mu(\omega)P(s_{-i}', s_i|\omega)}{P(s_i)} \quad \text{and} \quad \mu_i\left(s_{-i}', \omega | s_i, s_j\right) = \frac{\mu(\omega)P(s_{-i}', s_i|\omega)}{P(s_i, s_j)}\, \mathbb{1}_{\{s_j = s_j'\}}$$

denote the conditional probabilities that the state is ω and others receive s_{-i}' given s_i and (s_i, s_j), and let

$$\mathscr{E}(S,P) = \Big\{ a^* = (a_i^*) : a_i^* : S_i \to A_i \text{ and } a_i^*(s_i) \in \operatorname*{argmax}_{a_i \in A_i} \sum_{\omega, s_{-i}} u_i(a_i, a_{-i}^*(s_{-i});\omega)\mu_i(s_{-i}, \omega | s_i) \quad \forall i \in \mathscr{I}, s_i \in S_i \Big\} \tag{2}$$

be the set of pure Bayes Nash equilibria (BNE).

A distribution $p \in \Delta(A \times \Omega)$ is (weakly) **implemented** by an information structure (S,P) if there is $a^* \in \mathscr{E}(S,P)$ such that $p(a,\omega) = \sum_{s \in S} \mu(\omega)P(\{s : a^*(s) = a\}|\omega)$ for all $a \in A$ and $\omega \in \Omega$. From Bergemann and Morris (2016, Theorem 1), p is implemented by some information structure if and only if $p \in BCE(\mu)$.[8]

2.1.2 Stochastic Orders. Our optimality results focus on activity and welfare enhancement. We define partial orders on $\Delta(A \times \Omega)$ that capture both of these. Let $v : A \times \Omega \to \mathbb{R}$ be weakly increasing if $a' \geq a$ implies $v(a';\omega) \geq v(a;\omega)$ for all $\omega \in \Omega$.

Definition 1. *Distribution p' dominates distribution p, denoted $p' \succeq_d p$, if* $\mathbb{E}_{p'}[v] \geq \mathbb{E}_p[v]$

[8]The proof makes use of pure BNE only.

for all weakly increasing v and $\mathbb{E}_{p'}[u_i] \geq \mathbb{E}_p[u_i]$ for all $i \in \mathscr{I}$.

A distribution dominates another one if the former first-order stochastically dominates the latter and also weakly improves every player's expected utility. Shaked and Shanthikumar (2007) provides a familiar characterization of stochastic dominance. Let $\hat{A} \subseteq A$ be an upper set if $a \in \hat{A}$ and $a' \geq a$ imply $a' \in \hat{A}$.[9] Then, $\mathbb{E}_{p'}[v] \geq \mathbb{E}_p[v]$ for all weakly increasing v if and only if $p'\left(\hat{A} \times \{\omega\}\right) \geq p\left(\hat{A} \times \{\omega\}\right)$ for all upper sets $\hat{A}$ and $\omega \in \Omega$.

Let

$$\underline{\omega} = \min\left\{\omega \in \Omega : \quad \omega' \geq \omega \Rightarrow u_i(1,\ldots,1;\omega') \geq 0 \ \ \forall i \in \mathscr{I}\right\} \tag{3}$$

and define $\Psi = A \times \left\{\omega < \underline{\omega}\right\}$.

Definition 2. *Distribution p' supermodular-dominates distribution p, denoted $p' \succeq_{sd} p$, if $\mathbb{E}_{p'}[v] \geq \mathbb{E}_p[v]$ for all weakly increasing v that are supermodular on Ψ and $\mathbb{E}_{p'}[u_i] \geq \mathbb{E}_p[u_i]$ for all $i \in \mathscr{I}$.*[10]

Supermodular dominance does not guarantee a larger expected value for all weakly increasing functions, but only for those that *also* value coordination between actions and the state in the sense of supermodularity.

2.3 Organized Information

Given an information structure (S,P) and $s \in S$ such that $P(s) > 0$, player i is **(weakly) more informed** than j at s, denoted $i \succeq^s_{\text{Inf}} j$, if $\mu_i(\omega, s'_{-i}|s_i, s_j) = \mu_i(\omega, s'_{-i}|s_i)$ for all $\omega \in \Omega$ and $s'_{-i} \in S_{-i}$. This is the strongest definition of 'being more informed,' as i knows everything that j knows, including j's message. We say that i and j are **equally informed** at s, denoted $i =^s_{\text{Inf}} j$, if $i \succeq^s_{\text{Inf}} j$ and $j \succeq^s_{\text{Inf}} i$.

An information structure (S,P) allows **horizontal transmission** to i and j at s if $i =^s_{\text{Inf}} j$. Since both players know each other's message, they both know that they both know it and so on, so that this fact is common knowledge among them; it is as if (s_i, s_j) were communicated simultaneously and overtly to both i and j. The ultimate example of horizontal transmission is public information. An information structure (S,P) allows **vertical transmission** from i to j at s if $i \succeq^s_{\text{Inf}} j$ and i satisfies communication incentives (see Section 4.1 for details).

[9] For example, $\{(0,1),(1,0),(1,1)\}$ is an upper set of $\{0,1\} \times \{0,1\}$.

[10] As standard, given a lattice $(Y,\geq)$ and a sublattice $(X,\geq)$, $f: Y \to \mathbb{R}$ is supermodular on X if for all $x', x'' \in X$, $f(x' \vee x'') + f(x' \wedge x'') \geq f(x') + f(x'')$.

The set of organized information structures provides the backdrop for the analysis

$$\mathscr{O} = \Big\{ \big((S,P), (I_h^s, I_v^s)_s\big) : \quad \text{for all } s \in S \text{ s.t. } P(s) > 0,\ I_h^s, I_v^s \subseteq \mathscr{I} \text{ and} \\ \big[i =_{\text{Inf}}^s j\ \ \forall i,j \in I_h^s\big] \text{ and } \big[i \succeq_{\text{Inf}}^s j \text{ or } j \succeq_{\text{Inf}}^s i\ \ \forall i,j \in I_v^s\big]\Big\}.$$

Information structures in this set organize the players into groups, I_h^s and I_v^s, at each message profile s, based on interactive knowledge that allows horizontal or vertical transmission. All information structures belong to $\mathscr{O}$, but many require $I_h^s = I_v^s = \emptyset$ for all s, and hence do not allow horizontal or vertical transmission between any i and j at any s. Some may allow it only between a few players and at a few messages. Yet others may allow horizontal transmission at some messages and vertical transmission at other messages, so that $I_h^s = \mathscr{I}$ for $s \in S'$ and $I_v^s = \mathscr{I}$ (plus communication incentives) for $s \in S \setminus S'$. The information structures in this paper are special within $\mathscr{O}$, because they capture either horizontal or vertical transmission across all messages in their support.

3 Single-Meeting Schemes

Meetings are a ubiquitous form of information dissemination in society.[11] When there are costs associated with the transmission of private messages, due to the necessity of creating and using separate communication channels, meetings are a cost-effective way of communicating content, simultaneously and overtly (i.e., on one and the same channel) to subsets of players. They embody horizontal transmission to those present at the meeting, while allowing for informational asymmetries, because those who are absent are less informed. While meetings can take many forms and serve many purposes, in this section we introduce a family of information structures that require a single communication channel, that is, only one meeting ever needs to be organized to communicate the content of any message realization. We first provide optimality results and then characterize the set of outcome distributions implementable by these information structures.

[11] For example, American businesses hold millions of meetings a day, a billion meetings a year, and the average employee spends hours in meetings every week. See Rogelberg, Scott, and Kello (2007) for numbers.

3.1 Definition

In communication contexts, a meeting is an assembly of individuals gathered for the purpose of receiving information. In principle, the content of a meeting becomes common knowledge among the participants, and the only information asymmetries between players come from attending or not attending a meeting. Our definition formalizes these stylized points by designating one message, $\tilde{s}_i$, to represent i's non-participation in any meeting, and by building common knowledge between the participants in a meeting.

Definition 3. *An information structure* (S,P) *is a single-meeting scheme if there exist a collection* $\{M(s) \subseteq \mathscr{I} : s \in S \text{ s.t. } P(s) > 0\}$ *and at most one* $\tilde{s}_i \in S_i$ *for each* $i \in \mathscr{I}$ *such that* $i \notin M(s)$ *implies* $s_i = \tilde{s}_i$ *and* $i \in M(s)$ *implies* $i \succeq^s_{Inf} j$ *for all* $j \in \mathscr{I}$.

For each message profile s, the subset of players $M(s)$ is invited to a meeting at which s is announced. Although many different meetings may be possible ex-ante, as described by the collection in the definition, at most one is ever realized in a single-meeting scheme, which corresponds to the realized s. When i and j are in $M(s)$, $i =^s_{\text{Inf}} j$. In addition, participation in a meeting perfectly reveals, to those invited, the subset of non-participants $\mathscr{I} \setminus M(s)$ and, therefore, their respective beliefs. Note that a player can have different beliefs depending on the meeting he participates in, while there is only one way of not participating in any meeting and, hence, from a player's perspective, only one belief associated with receiving message $\tilde{s}_i$. This does not mean $\mu_i(\cdot|\tilde{s}_i) = \mu_j(\cdot|\tilde{s}_j)$ for distinct i and j, as non-participation may carry different information for different players.

Single-meeting schemes capture the practical appeal of real-life meetings. They can be thought of as physical meetings or as emails, where one message is sent to a subgroup of players listed in the "To:" field. Their practicality stems from sending one email instead of having to send that same email individually to each recipient, or to use the blind carbon copy ("Bcc:") field to satisfy privacy requirements. (In this context, the message $\tilde{s}_i$ represents not receiving an email, while receiving an email is perfectly informative about who else has received the email and who has not).

3.2 Optimality

In this section, we show that in binary-action environments with strategic complementarities single-meeting schemes promote welfare and activity enhancement.

Assumption 1. (Weak Complementarities) For all $i \in \mathscr{I}$ and each $\omega \in \Omega$, $u_i(1, a_{-i}; \omega)$ is weakly increasing in a_{-i}.

Theorem 1. *Under Assumption 1, if $p \in BCE(\mu)$, then there exists $p' \succeq_d p$ such that p' can be implemented by a single-meeting scheme.*

Given $\mathcal{V} \subseteq \{v : A \times \Omega \to \mathbb{R}\}$, cone($\mathcal{V}$) is the convex cone of $\mathcal{V}$. Let $\mathcal{V}^{\mathrm{M}} = \{v : A \times \Omega \to \mathbb{R} : a' \geq a \Rightarrow v(a'; \omega) \geq v(a; \omega)\ \forall \omega \in \Omega\}$ be the family of action-wise weakly increasing functions.

Corollary 1. *If $\{u_i\}$ satisfy Assumption 1 and $v \in cone\left(\mathcal{V}^M \cup \{u_i\}\right)$, then there exists $p^* \in \underset{p \in BCE(\mu)}{\operatorname{argmax}} \mathbb{E}_p[v]$ that can be implemented by a single-meeting scheme.*

Dominance of the outcome distribution in the theorem translates into optimality for a large family of objective functions in the corollary. These functions include all action-wise weakly increasing functions, weighted welfare functions, and their positive linear combinations. The latter capture, for instance, the desire to maximize the probability that a subset of players play action 1 while simultaneously maximizing the welfare of the rest of the players.

It is worth pointing out that the theorem and its corollary hold regardless of the complementarities between each player's action and the state. For example, if two players have opposite relationships with the state, a higher state may incentivize player 1 to choose the high action, but deter player 2. Hence, all else equal, a high action by player 2 is interpreted as bad news about the state by player 1, who may then choose the low action in response, even if actions are strategic complements. In these situations, Theorem 1 implies that single-meeting schemes can induce some players to choose the higher action without depressing the beliefs about the state of the others, by not inviting them to the same meetings.

These results readily apply to multidimensional states, i.e., $\Omega \subseteq \mathbb{R}^m$, where information bears on multiple aspects of a decision problem. In organizations, for example, a manager may have information about more than one project or about multiple attributes of a project.

We conclude this section with an illustrative example.

Example 1. (An optimal single-meeting scheme). Consider a team effort problem with $\mathscr{I} = \{1, 2, 3\}$, $\Omega = \{0, 1\}$, $\mu(\omega = 0) = 4/5$ and $A_i = \{0, 1\}$. Let $u_1(a; \omega) = a_1(2\omega - 1)$, $u_2(a; \omega) = a_2(2\omega - 2 + a_1)$ and

$$u_3(a; \omega) = a_3 \times \begin{cases} -2 & \text{if } a_1 = a_2 = \omega = 0 \\ a_1 + a_2 & \text{otherwise.} \end{cases}$$

Player 1 wants to exert effort only if the state is high; player 2 only if the state is high and player 1 does so as well; and player 3's utility from effort is given by 1 and 2's total efforts, except in the low state where exerting effort alone is detrimental. Clearly, all $u_i(1, a_{-i}; \omega)$ are weakly increasing in a_{-i} for each ω so that Assumption 1 holds.

Suppose the objective is to maximize Prob$(a_3 = 1)$ so that $v(a) := a_3$. Then, the following $p^* \in BCE(\mu)$ uniquely maximizes $\mathbb{E}_p[v]$:

$p^*(\cdot;0)$	0,0	1,0	0,1	1,1
0	3/20	0	7/20	1/10
1	0	0	1/5	0

$p^*(\cdot;1)$	0,0	1,0	0,1	1,1
0	0	0	0	0
1	0	0	0	1/5

The direct information structure $(A, \{p^*(\cdot|\omega)\})$, which implements p^* by a standard argument, is *not* a single-meeting scheme. At recommendation profile $(1,0,1)$, no player is more informed than the others (player 1 does not know that $a_2 = 0$; player 2 does not know that $a_1 = 1$ or that $a_3 = 1$, and player 3 does not know that $a_1 = 1$ or that $a_2 = 0$). Thus, this profile is played when no one is in a meeting. Similarly, at recommendation profile $(0,0,1)$, no player is more informed than the others, implying that it too should be played when no one is in a meeting, a contradiction. In practice, the players' interactive knowledge at $(1,0,1)$ and $(0,0,1)$ requires each of them to receive his recommendation privately.

In line with Corollary 1 (since $v \in \mathcal{V}^{\mathrm{M}}$), p^* is implemented by the following single-meeting scheme: let $S_i = \{s_i^1, s_i^2, s_i^3, \tilde{s}_i\}$ for $i = 1,2$, $S_3 = \{s_3^1, \tilde{s}_3\}$, $P(\tilde{s}_1, \tilde{s}_2, \tilde{s}_3|1) = 1$ and

$$P\left(s_1^1, s_2^1, s_3^1 \middle| 0\right) = 3/16 \qquad P\left(\tilde{s}_1, s_2^3, \tilde{s}_3 \middle| 0\right) = 1/4$$
$$P\left(s_1^2, s_2^2, \tilde{s}_3 \middle| 0\right) = 7/16 \qquad P\left(s_1^3, \tilde{s}_2, \tilde{s}_3 \middle| 0\right) = 1/8.$$

In this single-meeting scheme, where $M(\tilde{s}) = \emptyset$, $M(s_1^1, s_2^1, s_3^1) = \{1,2,3\}$, $M(\tilde{s}_1, s_2^3, \tilde{s}_3) = \{2\}$, $M(s_1^2, s_2^2, \tilde{s}_3) = \{1,2\}$, and $M(s_1^3, \tilde{s}_2, \tilde{s}_3) = \{1\}$, an invitation to a meeting incentivizes action 0 as a response and the non-invitation $\tilde{s}_i$ incentivizes action 1.

3.3 Characterization

Our next result characterizes the outcome distributions that can be implemented by a single-meeting scheme.

Theorem 2. *A distribution $p \in BCE(\mu)$ can be implemented by a single-meeting scheme, if and only if, for all $i \in \mathscr{I}$, there is $\tilde{a}_i \in A_i$ such that for all $a_i \in A_i \setminus \{\tilde{a}_i\}$*

$$\sum_{\omega \in \Omega} p(a_i, a_{-i}, \omega)\big(u_i(a_i, a_{-i}; \omega) - u_i(a_i', a_{-i}; \omega)\big) \geq 0 \tag{4}$$

for all $a_i' \in A_i$ and $a_{-i} \in A_{-i}$.

For any μ, denote by $SMS(\mu)$ the set of BCE distributions that satisfy these necessary and sufficient conditions.

The theorem makes a clear connection between the organizational constraint on information, namely that it comes from a single-meeting scheme, and the resulting constraints on the strategic outcomes: Each player i has one action $\tilde{a}_i$ that satisfies the BCE obedience constraint (1), while every other action that i plays should be a best response to any a_{-i} against which it is played with positive probability. In comparison, the BCE constraints are summations over all a_{-i} of the constraints specified separately for each a_{-i} in (4). Thus, the above incentive constraints are stronger than the obedience constraints of a BCE, but weaker than (pure strategy) equilibrium play under public information, where all constraints are of the form (4) without any exception.

The characterization provides a system of linear inequalities for each $\tilde{a} \in A$, the solutions of which form a convex polytope $\mathscr{C}(\tilde{a}, \mu)$ that represents a class of single-meeting schemes. For each distribution in $\mathscr{C}(\tilde{a}, \mu)$, a realization $a \in A$ can be interpreted as a meeting amongst $\{i \in \mathscr{I} : a_i \neq \tilde{a}_i\}$, because by (4) each of these players would want to play his action even if he knew the action profile of all other players. As a union of convex sets, $SMS(\mu) = \cup_{\tilde{a} \in A} \mathscr{C}(\tilde{a}, \mu)$ need not be convex (see Figure 1 (b) below). That is, the randomization between two single-meeting schemes need not be a single-meeting scheme. This makes sense intuitively, as the randomization is effectively a public signal whose realization has to be disclosed to all players, which in turn requires an additional meeting.

Theorem 2 holds for all finite action sets and utility functions. Computing the set $SMS(\mu)$ involves solving $\prod_i |A_i|$ linear systems, one for each class of single-meeting schemes. In Section 6.1 and in Online Appendix C.2, we discuss how organizing more than a single meeting expands the set of implementable outcomes.

3.3.1 Structure. We next emphasize three properties of the structure of the set of single-meeting schemes, illustrated in Battle of the Sexes where complete information simplifies the graphical representations. The properties, however, apply more generally to incomplete information games.

	0	1
0	3, 2	0, 0
1	0, 0	2, 3

Table 1: Battle of the Sexes

We first define important sets of outcome distributions. Given $\mu \in \Delta(\Omega)$, let

$$\text{NE}(\mu) = \Bigg\{ p \in \Delta(A \times \Omega) : \quad \exists\, a^* \in A \text{ s.t. } p(a^*, \cdot) = \mu \text{ and } \sum_{\omega \in \Omega} \mu(\omega) u_i(a^*; \omega) \geq \sum_{\omega \in \Omega} \mu(\omega) u_i(a_i, a^*_{-i}; \omega) \quad \forall i \in \mathscr{I}, a_i \in A_i \Bigg\}$$

be the set of pure strategy Nash outcomes in the ex-ante normal form game in which it is common knowledge that all players have belief μ. Let

$$\text{Public}(\mu) = \bigcup \left\{ \sum_{\hat{\mu}} \alpha(\hat{\mu}) \text{Co}(\text{NE}(\hat{\mu})) : \quad \alpha \in \Delta(\Delta(\Omega)) \text{ s.t. } \sum_{\hat{\mu}} \alpha(\hat{\mu}) \hat{\mu} = \mu \right\}$$

be the set of pure strategy public information outcomes. In complete information games, NE is just the set of pure Nash equilibria and Public is its convex hull.

Figure 1 (a) depicts $BCE(\mu)$, which is simply the set of correlated equilibria in Battle of the Sexes. Part (b) of the figure depicts $SMS(\mu)$, which consists of four classes of single-meeting schemes. Letting $a = p(0,0)$, $b = p(0,1)$, $c = p(1,0)$ and $d = p(1,1)$, the four classes are:

$$\begin{aligned} \mathscr{C}((0,1),\mu) &= \{p : \ a, b, d \geq 0 \text{ and } c = 0\} \\ \mathscr{C}((1,0),\mu) &= \{p : \ a, c, d \geq 0 \text{ and } b = 0\} \\ \mathscr{C}((0,0),\mu) = \mathscr{C}((1,1),\mu) &= \{p : \ b = c = 0\}. \end{aligned}$$

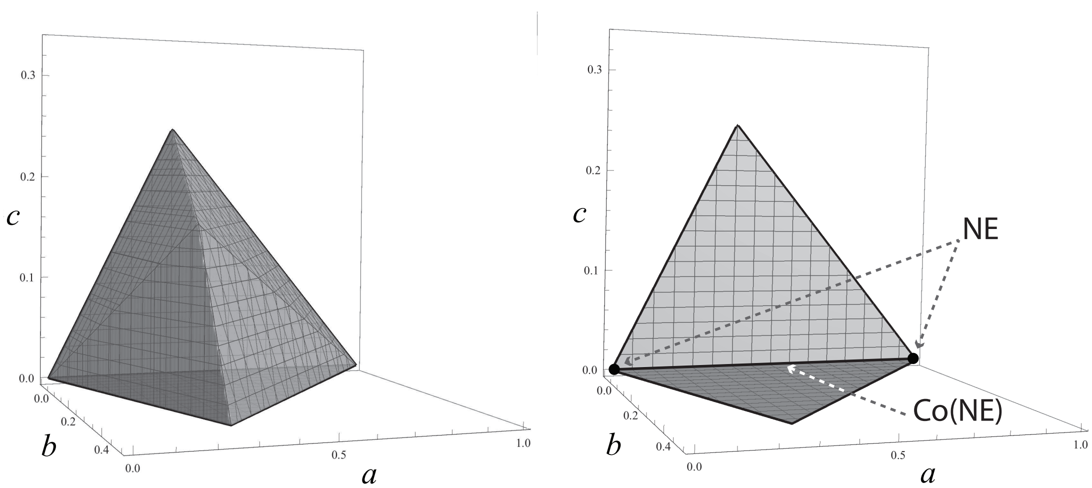
In $\mathscr{C}((0,1),\mu)$, the row player plays 1 when invited to a meeting and 0 when not. The opposite is true for the other player. This class corresponds to the bottom triangle in Figure 1 (b). $\mathscr{C}((1,0),\mu)$ is the mirror image of $\mathscr{C}((0,1),\mu)$, given by the top triangle. In $\mathscr{C}((0,0),\mu)$ and $\mathscr{C}((1,1),\mu)$, the players are always together in a meeting and coordinate their actions.

By comparing the two panels of Figure 1, we can see that $SMS(\mu)$ consists of faces of $BCE(\mu)$. This is a general property in coordination games with strict Nash equilibria.

Claim 1. *(Face structure) Suppose there exist distinct $a^*, a^{**} \in A$ which are strict Nash equilibria of $(\mathscr{I}, \{A_i, u_i(\cdot, \omega)\})$ for all $\omega \in \Omega$. Then, $SMS(\mu)$ is a union of faces of $BCE(\mu)$.*

Furthermore, note that Public(μ) lies at the intersection of the two triangles, $\mathscr{C}((0,0),\mu)$ and $\mathscr{C}((1,1),\mu)$. This property generalizes to all games: not only does public information produce outcomes in $SMS(\mu)$, as it is a special kind of single-meeting schemes, but its outcomes lie in the intersection of all classes of single-meeting schemes.

Claim 2. *(Public intersection)* Public(μ) $\subseteq \bigcap_{\tilde{a} \in A} \mathscr{C}(\tilde{a}, \mu) \subseteq SMS(\mu)$.



(a) Correlated Equilibria (b) SMS

Figure 1: Correlated Equilibria and SMS

Our last claim uses the fact that each class $\mathscr{C}(\tilde{a}, \mu)$ is structured around a profile $\tilde{a}$ that a collection of meetings is meant to incentivize. In light of this, $\max\{p(\tilde{a}, \omega) : p \in \mathscr{C}(\tilde{a}, \mu)\}$ is associated with the cheapest way of incentivizing $\tilde{a}$ at ω.

Claim 3. *(Extreme Points)* $\max\{p(\tilde{a}, \omega) : p \in SMS(\mu))\} = \max\{p(\tilde{a}, \omega) : p \in \mathscr{C}(\tilde{a}, \mu)\}$.

In Battle of the Sexes, $\mathscr{A} = \{(0,0),(1,1)\}$ supports the incentive compatibility of $\tilde{a}_1 = 1$ and $\tilde{a}_2 = 0$ in $\mathscr{C}((1,0),\mu)$. The cheapest way of incentivizing $\tilde{a}$ is by setting $a = d = 3/8$, which enables $\max_{p \in \mathscr{C}((1,0),\mu)} c = 1/4$ and corresponds to extreme point $(3/8, 0, 1/4, 3/8)$.

3.3.1 Existence. Well-known families of Bayesian games have a pure BNE for all information structures, in particular all single-meeting schemes. If the ex-post game $(\mathscr{I}, \{A_i,$

$u_i(\cdot,\omega)\}$) is supermodular for all $\omega \in \Omega$ (Milgrom and Roberts (1990)),[12] then the ex-ante Bayesian game is also supermodular for all priors and information structures. The same is true for potential games (Monderer and Shapley (1996)): if the ex-post game admits a potential $\phi_\omega : A \to \mathbb{R}$ for all $\omega \in \Omega$, then the ex-ante Bayesian game is also a potential game for all priors and information structures (Heumen et al. (1996)). In both families of games, existence of a pure equilibrium is guaranteed for all single-meeting schemes.

The nonemptiness of $SMS(\mu)$ is a weaker form of existence than existence of a pure BNE for *every* single-meeting scheme. As illustrated in Online Appendix C.1, Public(μ) can be empty and yet $SMS(\mu)$ be nonempty. This speaks to the ability of certain forms of organized information, in this case single-meeting schemes, to stabilize pure-strategy behavior in games where other forms, in this case public information, may not.

4 Delegated Hierarchies

Vertical transmission, whereby information flows from one individual to another, is also frequently observed in reality. Delegated hierarchies are a mode of vertical transmission in which information is delivered directly to a single player, de facto the most informed one, and subsequently gets (partially) transmitted from player to player down the hierarchy in an incentive compatible way. Therefore, communication happens on one channel at a time along a sequence of one-to-one transmissions. Delegated hierarchies are cost-effective when the communication costs are convex in the size of the audience, that is, when the cost of communicating directly with n individuals (privately or publicly) is larger than n times the cost of communicating with one person.[13] In general, delegated hierarchies can also implement outcomes that are not implementable by single-meeting schemes, in which case they offer an alternative implementation beyond cost considerations. As in Section 3, we first provide optimality results and then characterize the outcome distributions that are implementable by these information structures.

4.1 Definition

First, in order for players to be able to vertically transfer information to one another, they must be ordered with respect to how informed they are. This suggests the notion of

[12] Assuming the partial orders on the action sets are the same across ω.

[13] In this case, the sender would prefer to delegate transmission and compensate each player for the cost of transmission, rather than transmit the messages herself.

an information hierarchy. Second, in order for players to be willing to transfer information to one another, an information hierarchy must satisfy certain incentive constraints.

Definition 4. *An information hierarchy* $((S,P),\succ)$ *is an information structure* (S,P) *and a total order* $\succ$ *on* $\mathscr{I}$ *such that* $i \succ j$ *implies* $i \succeq^{s}_{\text{Inf}} j$ *for all* s *such that* $P(s) > 0$.

Whether a single-meeting scheme is also an information hierarchy can be determined in a straightforward way by comparing the invitees across different meetings, as formalized in the next result.

Proposition 1. *A single-meeting scheme* (S,P) *is an information hierarchy, if and only if, there is a total order* $\succ$ *on* $\mathscr{I}$ *such that, for all* s *such that* $P(s) > 0$, $i \succ j$ *and* $j \in M(s)$ *imply* $i \in M(s)$.

In Example 1, no total order can make the optimal single-meeting scheme an information hierarchy, as two distinct meetings have a different and unique invitee, hence no $\succ$ can rank them by informedness in the same way across all messages.

In an information hierarchy $((S,P),\succ)$, although player i such that $i \succ j$ is able to deliver s_j to j, i may have an incentive to report something else. Therefore, truthful information transmission down the hierarchy needs to be incentive compatible: not only must all players have an incentive to play their equilibrium action, but also to pass down the relevant information, for other players to do the same.

Given an information hierarchy $((S,P),\succ)$, a player $i \in \mathscr{I}$ and $s \in S$, let $s_{\prec i} = \left(s_j : j \prec i\right)$ and $s_{\succ i} = \left(s_j : j \succ i\right)$ and let the corresponding sets of profiles be denoted $S_{\prec i}$ and $S_{\succ i}$, respectively. Similarly, we use notation $a_{\prec i} \in A_{\prec i}$ and $a_{\succ i} \in A_{\succ i}$ to denote the action profiles of i's predecessors and i's successors.

Definition 5. *A distribution* $p \in \Delta(A \times \Omega)$ *can be implemented by a delegated hierarchy if there exist an information hierarchy* $((S,P),\succ)$ *and an equilibrium* $a^* \in \mathscr{E}(S,P)$ *such that*

$$p(a,\omega) = \sum_{s \in S} \mu(\omega) P(\{s : a^*(s) = a\}|\omega) \quad \forall a \in A, \omega \in \Omega$$

and for all $i \in \mathscr{I}$, $s_i \in S_i$ *and* $s_{\prec i} \in S_{\prec i}$ *such that* $P(s_i, s_{\prec i}) > 0$,

$$\mathbb{E}\left[u_i\left(a^*_i(s_i), a^*_{\prec i}(s_{\prec i}), a^*_{\succ i}(s_{\succ i}); \omega\right) \middle| s_i\right] \geq \mathbb{E}\left[u_i\left(a'_i, a^*_{\prec i}(s'_{\prec i}), a^*_{\succ i}(s_{\succ i}); \omega\right) \middle| s_i\right] \tag{5}$$

for all $a'_i \in A_i$ *and* $s'_{\prec i}$ *such that* $P(s'_{\prec i}) > 0$.

Delegated hierarchies allow for direct communication with only the most informed player, $i^* = \max_{\succ} \mathscr{I}$, who gets informed according to (S,P). The definition describes a truthful equilibrium of a sequential cheap talk game in which, starting with i^*, each player is both a receiver choosing an action strategically and an (strategic) informed sender to his immediate $\succ$-predecessor. Each player can deviate both from truthful information transmission and from his equilibrium action. Condition (5) requires these deviations to not be profitable in the desired equilibrium a^*. In this delegated process, each player must be more informed in a strong sense than all of his $\succ$-predecessors, as he is their only source of information. Hence, this process must build on an information hierarchy. Finally, since $((S,P),\succ)$ is common knowledge among the players and chosen with commitment, no one will believe a message $s'_{\prec i}$ passed down by i that would have zero probability under P.[14]

The definition also builds in robustness to communication as a by-product. In the standard information design framework, players are assumed to receive all their information from (S,P), and strategic communication between the players is assumed away. However, once players are in possession of their messages, they could potentially want to share some of their information with each other. If players have this ability, then by (5), they will be happy with the actions played by less informed players and, therefore, will not have an incentive to induce them to change their actions by disclosing different information to them. At the same time, no player can reveal anything to more informed players that the latter do not already know and, therefore, cannot impact their actions. Overall, no communication would be an equilibrium in this extended game and a^* would still be played, thus being robust to inter-player communication.

4.2 Optimality

4.2.1 Lack of delegation under weak complementarities. To motivate our next assumption, we first demonstrate the limitations of Assumption 1 in the perspective of delegation. We go back to Example 1, where the objective $v(a) = a_3$ is both weakly increasing in a and supermodular on $A \times \Omega$. However, note that

$$u_3(1,1,1;0) + u_3(1,0,0;0) = 2 - 2 = 0 < u_3(1,1,0;0) + u_3(1,0,1;0) = 1 + 1 = 2$$

[14]For the special case of a line network where information is seeded at i^*, the above definition can be related to the framework of Galperti and Perego (2020), where information transmission between linked players is full and mechanical. Here, in contrast, information transmission is partial and accounts for players' incentives.

so that $u_3(a_3, a_{-3}; \omega)$ is not supermodular in a (which violates Assumption 2 below).

In Example 1, the unique p^* cannot be implemented by a delegated hierarchy. If player 1 were the most informed in the hierarchy, then he would not be willing to play 1 at $a = (1,0,1)$, as that would reveal to him that $\omega = 0$ and he would prefer playing 0. If player 2 were the most informed, he would not be willing to play 1 at (0,1,1). Finally, if player 3 were the most informed in the hierarchy, then he would always have an incentive to recommend action 1 to the others and switch to playing 1 himself. Thus, no total ordering can satisfy (5).

4.2.2 Results. Recall that $\Psi = A \times \{\omega < \underline{\omega}\}$ where $\underline{\omega}$ is defined in (3).

Assumption 2. *(Supermodularity) For all $i \in \mathscr{I}$,* (a) $u_i(1, a_{-i}; \omega)$ *is weakly increasing in a_{-i} for each $\omega \geq \underline{\omega}$ and* (b) u_i *is supermodular on* Ψ.

This assumption implies Assumption 1 and adds the requirement that a_i and ω are complements and that $u_i(1, a_{-i}; \omega)$ is supermodular in a_{-i}. That is, every player is incentivized to choose action 1 by larger states and regards the other players' actions as complementary to each other in his own payoff. For example, for $\mathscr{I} = \{1,2,3\}$, $u_1(a;\omega) = a_1(\omega + a_2 a_3)$ is supermodular in a_{-1}, which implies that, at each ω and a_1, player 1 would prefer a coin toss between $a_{-1} = (1,1)$ and $a'_{-1} = (0,0)$ to a coin toss between $(0,1)$ and $(1,0)$. Note also that the choice of Ψ is not arbitrary: there are important applications, such as global games of regime change, in which players' payoffs fail to be supermodular on $A \times \Omega$ and yet are supermodular on Ψ.

Theorem 3. *Under Assumption 2, if $p \in BCE(\mu)$, then there exists $p' \succeq_{sd} p$ such that p' can be implemented by a delegated hierarchy.*

Let $\mathcal{V}^{\mathrm{SM}} = \{v : A \times \Omega \to \mathbb{R} : v \text{ is supermodular on } \Psi\} \bigcap \mathcal{V}^{\mathrm{M}}$ be the family of weakly increasing supermodular functions.

Corollary 2. *If $\{u_i\}$ satisfy Assumption 2 and $v \in cone\left(\mathcal{V}^{SM} \cup \{u_i\}\right)$, then there is $p^* \in \operatorname*{argmax}_{p \in BCE(\mu)} \mathbb{E}_p[v]$ that can be implemented by a delegated hierarchy.*

In supermodular environments, delegated hierarchies are a welfare and activity enhancing mode of transmission. While the results are agnostic about the exact optimal delegation order, Section 5.1 illustrates how the order may change with the objective v in a regime-change game. Moreover, note that the implementing information structure in the theorem and its corollary is also a single-meeting scheme.

The proof of Theorem 3 establishes that for any BCE p (i) there is a distribution $\hat{p} \succeq_{sd} p$ that can be implemented by a direct information structure satisfying an inclusion property (Lemma 2); (ii) the direct information structure obtained in (i) can be turned into an indirect structure which is an information hierarchy (perhaps many) implementing $p' \succeq_{sd} \hat{p}$ (Lemma 3); and (iii) at least one of the information hierarchies in (ii) allows delegated transmission (Lemma 4). Part (i) of the proof generalizes Lemma 5 of Arieli and Babichenko (2019) to many states, state-dependent v such as weighted welfare, and to a weaker supermodularity condition which, for instance, captures regime-change games.

Part (ii) is the information-enhancement phase which, due to complementarities, increases overall activity and all expected equilibrium payoffs. This phase transforms a baseline information structure into an information hierarchy by augmenting some action recommendations. In doing so, the physical content of a player's message is enriched with the information of his predecessors. This contrasts with Theorem 1, where the transformation into a single-meeting scheme leaves the physical nature of the message unchanged, but changes players' interactive knowledge by sending it publicly to its receivers. Step (ii) is interesting in its own right and its implications are especially striking in the network interactions from Section 2: although the environment might display no hierarchical ordering of players (that is, their dependencies on the state and each other might be unordered or only partially ordered), optimal design orders them totally by informedness.

Even if several total orders $\succ$ on $\mathscr{I}$ are consistent with $\succeq_{\text{Inf}}$, not all of them might meet the incentive requirements of delegation, as shown in Section 5.3. Part (iii) ties the existence of a total ordering that enables delegated transmission to the non-existence of coalitional deviations of subgroups of equally informed players. Unless there is room for welfare and activity enhancement, there cannot be room for such coalitional deviations. This, in turn, implies that (5) must hold along *some* total order $\succ$ of the players that is consistent with the informedness order $\succeq_{\text{Inf}}$ obtained in (ii).

4.3 Characterization

The next result characterizes the outcome distributions that can be implemented by a delegated hierarchy.

Theorem 4. *A distribution $p \in \Delta(A \times \Omega)$ can be implemented by a delegated hierarchy, if and only if, $p(A \times \{\omega\}) = \mu(\omega)$ for all $\omega \in \Omega$ and there exists a total order $\succ$ on $\mathscr{I}$ such that for all $a_i \in A_i$, $a_{\prec i} \in A_{\prec i}$ and $i \in \mathscr{I}$,*

$$\sum_{\omega \in \Omega} \sum_{a_{\succ i}} p(a_i, a_{\prec i}, a_{\succ i}, \omega) \big(u_i(a_i, a_{\prec i}, a_{\succ i}; \omega) - u_i(a'_i, a'_{\prec i}, a_{\succ i}; \omega) \big) \geq 0 \tag{6}$$

for all $a'_i \in A_i$ and $a'_{\prec i}$ such that $p(a'_{\prec i}) > 0$.

For any μ, denote by $DH(\mu)$ the set of outcome distributions that satisfy the above necessary and sufficient conditions.

This theorem also makes a clear connection between the organizational constraint on information, namely that it comes from a delegated hierarchy, and the resulting constraints on the strategic outcomes: Condition (6) requires that if player i knew the actions of his predecessors $a_{\prec i}$, he would not want to deviate from his recommended action or transmit any other recommendation $a'_{\prec i}$ that has positive probability under the outcome distribution. These incentive constraints are stronger than the BCE obedience constraints, which sum (6) over all $a_{\prec i} = a'_{\prec i}$.

A simple comparison between Theorems 2 and 4 reveals that some outcome distributions which cannot be implemented by a single-meeting scheme can be implemented by a delegated hierarchy, and vice versa. In other words, none of the sets $SMS(\mu)$ and $DH(\mu)$ is, in general, a subset of the other.

In Battle of the Sexes of Section 3.3.1, the two pure Nash equilibrium distributions, p and p' defined as $p(1,1) = 1$ and $p'(0,0) = 1$, are the only outcome distributions that can be implemented by a delegated hierarchy. Although each player prefers a different equilibrium, both p and p' can be implemented under any total order. As no other action profile is recommended with positive probability, the higher ranked player in the hierarchy cannot credibly recommend to his predecessor the action played in the other equilibrium. For this reason, (strict) public randomizations between the two pure Nash equilibria are not in the set DH, as the higher ranked player would have the incentive to always recommend his favorite equilibrium to the other player.

Theorem 4 provides a system of linear inequalities for each total order $\succ$ on $\mathscr{I}$, the solutions of which form a class of delegated hierarchies, denoted by $\mathscr{C}(\succ, \mu)$, with slight abuse of notation. Each class consists of the distributions built on the same hierarchical ordering of players, so that $DH(\mu) = \cup_{\succ} \mathscr{C}(\succ, \mu)$. Each class also contains the pure Nash outcomes, as demonstrated next.

Claim 4. $\text{NE}(\mu) \subseteq \bigcap_{\succ} \mathscr{C}(\succ, \mu) \subseteq DH(\mu)$.

We end this section with an example of a game with a dominance region, similar to other classic games (Rubinstein (1989), Carlsson and van Damme (1993) and Morris and Shin (2003)).

Example 2. Consider the following incomplete information game with two states $\Omega = \{\omega_\ell, \omega_h\}$ and uniform prior μ, where Battle of the Sexes is played in the low state and action 1 is strictly dominant in the high state:

ω_ℓ	$a_2 = 0$	$a_2 = 1$
$a_1 = 0$	3,2	0,0
$a_1 = 1$	0,0	2,3

ω_h	$a_2 = 0$	$a_2 = 1$
$a_1 = 0$	0,0	0,2
$a_1 = 1$	2,0	2,2

Observe that $\text{NE}(\mu) = \{p_1, p_2\}$ such that $p_1(1,1,\cdot) = \mu$ and $p_2(0,0,\cdot) = \mu$. In p_1, both players play action 1 with probability 1 and, in p_2, both play 0 with probability 1.

In this game, $\mathscr{C}(1 \succ 2, \mu)$ consists of all distributions

$p(\cdot,\cdot,\omega_\ell)$	$a_2 = 0$	$a_2 = 1$
$a_1 = 0$	x	0
$a_1 = 1$	0	$1/2 - x$

$p(\cdot,\cdot,\omega_h)$	$a_2 = 0$	$a_2 = 1$
$a_1 = 0$	y	0
$a_1 = 1$	z	$1/2 - y - z$

such that $x, y, z \geq 0$, and either $x = y + z = 1/2$ or $1/2 \geq x$, $1/2 \geq y + z$, and $x \geq \max\{y + z, 2y, 2(y+z) - 1/2\}$. The class $\mathscr{C}(2 \succ 1, \mu)$ consists of p_1 and all distributions

$p(\cdot,\cdot,\omega_\ell)$	$a_2 = 0$	$a_2 = 1$
$a_1 = 0$	x	$1/2 - x$
$a_1 = 1$	0	0

$p(\cdot,\cdot,\omega_h)$	$a_2 = 0$	$a_2 = 1$
$a_1 = 0$	y	$1/2 - y$
$a_1 = 1$	0	0

such that $2/5 \geq x \geq 1/2$, $0 \geq y \geq 1/2$ and $x \geq y$. By definition, $DH(\mu) = \mathscr{C}(1 \succ 2, \mu) \cup \mathscr{C}(2 \succ 1, \mu)$, while $\text{NE}(\mu) = \mathscr{C}(1 \succ 2, \mu) \cap \mathscr{C}(2 \succ 1, \mu)$.

These characterizations delineate what outcomes can emerge if information is received via delegated hierarchies. Hence, they can be used for information design purposes. For instance, if $v(a) = a_1 - a_2$, the optimal value $v^* = 1/2$ is achieved by $x = 1/2$, $y = 0$ and $z = 1/2$ in $\mathscr{C}(1 \succ 2, \mu)$. Although this scenario falls outside the scope of Theorem 3, because v is not monotone in a, $v^* = 1/2$ is the global maximum that a BCE can achieve.

5 Applications

5.1 Regime Change: Part I

Global games of regime change illustrate nicely the variety of optimal single-meeting schemes and delegated hierarchies as a function of the objective.

Consider a situation, be it effort choice or investment decision, modeled à la Sakovics and Steiner (2012) (see Section 2), where i gets utility 0 if $a_i = 0$, and if $a_i = 1$

$$u_i(a_{-i};\omega) = \begin{cases} b_i - c_i & \text{if} \quad \kappa_i + \sum_{j \neq i} \kappa_j a_j > 1 - \omega \\ -c_i & \text{otherwise.} \end{cases}$$

A designer wants to maximize the expected value of an objective function $v : A \times \Omega \to \mathbb{R}$ by optimally choosing an information structure, so that

$$\begin{aligned} &\max_{(S,P)} \mathbb{E}v(S,P) \\ &\text{where } \mathbb{E}v(S,P) = \max_{a^* \in \mathscr{E}(S,P)} \sum_{\omega \in \Omega} \sum_{s \in S} v(a^*(s);\omega) P(s|\omega)\mu(\omega) \end{aligned} \tag{7}$$

describes the design problem. In case there are multiple equilibria, (7) assumes favorable selection, as a^* is the equilibrium that yields the largest expected value of the objective.

Recall that $\underline{\omega} = \min\{\omega \in \Omega : \sum_i \kappa_i > 1 - \omega\}$ is the lowest state at which success is possible.

Example 3. Suppose the designer wants to maximize the probability that the project succeeds:

$$v(a;\omega) = \begin{cases} 1 & \text{if } \sum_{i \in \mathscr{I}} \kappa_i a_i \geq 1 - \omega \\ 0 & \text{otherwise.} \end{cases}$$

Note v is supermodular on Ψ (since $v = 0$ on Ψ) and weakly increasing in a. By Theorem 3 and Corollary 2, a delegated hierarchy $((S^*, P^*), \succ)$, where (S^*, P^*) is also a single-meeting scheme, solves (7).

It is trivial to see $\mathbb{E}v$ is maximized by telling all players the truth about the state: let $S^* = \Omega$ and $P^*(s|\omega) = 1$ iff $s = \omega$. From the point of view of horizontal transmission, this means inviting all players to one meeting and revealing the state truthfully. From the point of view of vertical transmission, $((S^*,P^*),\succ)$ is an optimal delegated hierarchy for any total order $\succ$ on $\mathscr{I}$, because any player receiving $\omega \geq \underline{\omega}$ will play 1 and be happy to forward that information to his successor, whereas receiving $\omega < \underline{\omega}$ means that the project will fail and so the player will play 0 and also be happy to forward his information.

Example 4. Monotone objectives are natural in this context, such as maximizing the total probability of the high action or insisting on the participation of a key player i^*

$$v(a,\omega) = \sum_{i \in \mathscr{I}} a_i \quad \text{or} \quad v(a,\omega) = a_{i^*} \sum_{i \in \mathscr{I}} a_i.$$

The reasoning below applies to all v that satisfy Assumption 2.

Fixing $\hat{\omega} < \underline{\omega}$ and $\beta \in (0,1]$, suppose that the designer sends message "$\omega \geq \hat{\omega}$" to i with probability 1 whenever $\omega > \hat{\omega}$, with probability β at $\omega = \hat{\omega}$, and with probability 0 otherwise. Then,

$$\mu_i\left(\{\omega \geq \underline{\omega}\}|\omega \geq \hat{\omega}\right) = \frac{\sum_{\omega:\omega \geq \underline{\omega}} \mu(\omega)}{\sum_{\omega:\omega > \hat{\omega}} \mu(\omega) + \mu(\hat{\omega})\beta}$$

is i's belief that the project succeeds given the message "$\omega \geq \hat{\omega}$". Because i invests if and only if

$$\mu_i\left(\{\omega \geq \underline{\omega}\}|\omega \geq \hat{\omega}\right) b_i - c_i \geq 0, \tag{8}$$

choose ω_i^* and β_i^* such that (8) holds with equality (recall $b_i > c_i$). That is,

$$\mu_i\left(\{\omega \geq \underline{\omega}\}|\omega \geq \omega_i^*\right) = \frac{c_i}{b_i}. \tag{9}$$

For simplicity, let us assume $\frac{c_i}{b_i} \neq \frac{c_j}{b_j}$ whenever $i \neq j$. The optimal delegated hierarchy $((S^*,P^*),\succ)$ is characterized by

1. $S_i^* = \{\min \Omega, \ldots, \omega_i^*, \{\omega \geq \omega_i^*\}\}$ for all $i \in \mathscr{I}$
2. The designer sends each i the message $s_i = \{\omega \geq \omega_i^*\}$ as often as possible subject to plausibility constraint. In particular, $P^*(\{\omega \geq \omega_i^*\}|\omega_i^*) = \beta_i^*$ for all i and

$$\begin{aligned} \omega > \omega_i^* \quad &\Rightarrow \quad s_i = \{\omega \geq \omega_i^*\} \\ \omega < \omega_i^* \quad &\Rightarrow \quad s_i = \omega \end{aligned}$$

   so that i's belief given $s_i = \{\omega \geq \omega_i^*\}$ satisfies (9).
3. $i \succ j$ if and only if $\frac{c_i}{b_i} > \frac{c_j}{b_j}$.

From the point of view of horizontal transmission, (S^*, P^*) can be interpreted as a single-meeting scheme which, at each ω, invites all i such that $\omega < \omega_i^*$ and reveals ω truthfully to them.

From the point of view of vertical transmission, players who have a lower cost-to-benefit ratio from the project are given less information (both about the state and about the other players' messages); play the high action more often; and occupy a lower position in the delegated hierarchy. Indeed, $\frac{c_i}{b_i} > \frac{c_j}{b_j}$ implies that, upon observing $s_i = \{\omega \geq \omega_i^*\}$, player i knows that $s_j = \{\omega \geq \omega_j^*\}$, since $\omega_i^* > \omega_j^*$, and upon observing $s_i = \omega$ player i knows if $\omega \geq \omega_j^*$ and $s_j = \{\omega \geq \omega_j^*\}$ or, alternatively, $\omega < \omega_j^*$ and $s_j = \omega$. Hence, i is more informed than j. The incentive constraints for truthful transmission are also satisfied: upon observing $s_i = \{\omega \geq \omega_i^*\}$, player i chooses $a_i^* = 1$ and therefore would like all players below him in the hierarchy to also play 1, which is achieved by truthfully transmitting $s_{i-1} = \{\omega \geq \omega_{i-1}^*\}$; on the other hand, upon observing $s_i = \omega$, player i chooses $a_i^* = 0$ and, hence, truthful transmission to his immediate predecessor is incentive compatible: $s_{i-1} = \{\omega \geq \omega_{i-1}^*\}$ if $\omega \geq \omega_{i-1}^*$ or $s_{i-1} = \omega$ otherwise.

Example 5. A (weighted) welfare-maximizing / utilitarian designer

$$v = \sum_{i \in \mathscr{I}} \lambda_i u_i \quad \text{(for any } \lambda_i \geq 0\text{)}$$

is an example of an objective that may not be weakly increasing, because a player may have a strictly negative utility from playing the high action. By revealing the true state to the players, $S^* = \Omega$ and $P^*(s|\omega) = 1$ iff $s = \omega$, the designer ensures that each player chooses $a_i = 1$ if and only if $\omega \geq \underline{\omega}$, which maximizes the expected utility of each player. For any total order $\succ$ on $\mathscr{I}$, $((S^*, P^*), \succ)$ is an optimal delegated hierarchy, and (S^*, P^*) is also a single-meeting scheme which organizes one truth-revealing meeting with all players.

Example 6. There is a rich class of objectives that represent designers who care (additively and separably) about the well-being of some players and about others playing the high action. For example, assume $n \geq 3$ and let

$$v(a;\omega) = u_1(a;\omega) + u_2(a;\omega) + \sum_{i \neq 1,2} a_i.$$

This describes benevolence toward 1 and 2 together with a desire to induce the rest of the population to adopt the high action. Under the optimal information structure (S^*, P^*), 1 and 2 should receive full information while others obey the same hierarchy as in Example

2. That is,

$$1 \succ 2 \succ i \quad (\text{or } 2 \succ 1 \succ i) \quad \forall i \in \mathscr{I} \setminus \{1,2\}$$

and for all $\{i,j\} \subseteq \mathscr{I} \setminus \{1,2\}$, $i \succ j$ if and only if $\frac{c_i}{b_i} > \frac{c_j}{b_j}$. From the point of view of horizontal transmission, (S^*, P^*) invites 1 and 2 at each $\omega \in \Omega$ as well as any i such that $\omega < \omega_i^*$, and the state is announced truthfully at the meeting. From the point of view of vertical transmission, 1 and 2 are fully informed about the state. When $\omega \geq \underline{\omega}$, 1 and 2 play action 1 and forward $s_i = \{\omega \geq \omega_i^*\}$ to the next player, who then forwards $s_j = \{\omega \geq \omega_j^*\}$ to the next and so on. When $\omega < \underline{\omega}$, 1 and 2 play action 0 and forward $s_i = \{\omega \geq \omega_i^*\}$ as often as possible as described in Example 4, forwarding $s_i = \omega$ the rest of the time, with each subsequent player down the hierarchy doing the same with regards to his predecessor.

5.2 Regime Change: Part II

In the previous section, all players received a positive payoff as long as the project was successful. Therefore, the only relevant question a player could ask himself was whether $\omega \geq \underline{\omega}$, since all other players played action 1 if that is the case, and if $\omega < \underline{\omega}$, their actions did not matter because the project failed anyway. Hence strategic uncertainty was minimal in the optimal information structures.[15]

However, we can enrich the strategic uncertainty and obtain an even greater variety of optimal single-meeting schemes and delegated hierarchies by allowing the benefits to depend on the state and the set of investors. That is, let $b_i : \Omega \times A_{-i} \to \mathbb{R}^+$ and allow $b_i(\omega, a_{-i}) < c_i$ for some (ω, a_{-i}) such that $\sum_i \kappa_i a_i > 1 - \omega$. In this setup, a project can be successful and yet not beneficial to a player, who might prefer action 0 even if he knew the project would succeed.

To satisfy Assumption 2, suppose $b_i(\omega' \vee \omega'', a'_{-i} \vee a''_{-i}) \geq b_i(\omega', a'_{-i}) + b_i(\omega'', a''_{-i})$ for all (ω', a'_{-i}) and (ω'', a''_{-i}). Let us revisit Examples 3 and 5 where full information was uniquely optimal and any ordering allowed delegation. The optimal information structure now depends on the κ_i's. Let $\underline{\omega}_i = \max\{\omega \in \Omega : b_i(\omega, \mathbf{1}) - c_i < 0\}$ and let $i^\dagger = \text{argmax}_{i \in \mathscr{I}} \underline{\omega}_i$ be the most state-sensitive player. If $\sum_{i \in \mathscr{I}} \kappa_i > 1 - \omega$ but $\sum_{i \neq i^\dagger} \kappa_i < 1 - \omega$ for all ω, then $i^\dagger$ is critical to success. In optimum, it may be worth giving partial information to $i^\dagger$ to incentivize him to invest at $\omega < \underline{\omega}_i$ so that the project succeeds even when $\omega < \underline{\omega}_i$ if this benefits some $i \neq i^\dagger$. In general, it may be worth downgrading the critical investors within the delegated hier-

[15] In particular, the κ_i's played no role at all, unlike in Sakovics and Steiner (2012).

archy, to keep them investing, while upgrading those whose welfare carries more weight and who need full information to make the right decision.

5.3 Delegation Ordering

In some environments, optimization requires players to be treated equally, because they should receive the same information, yet delegation requires them to be treated differently, because they should be arranged hierarchically in one particular total order. The next example illustrates this point.

Example 7. Consider a linear network with $\mathscr{I} = \{1,2,3\}$, $\Omega = \{-1/2, 1/2\}$, $\mu(\omega = 1/2) = \mu$, $u_1(a;\omega) = a_1\omega$, $u_2(a;\omega) = a_2(\omega + a_3)$, and $u_3(a;\omega) = a_3(\omega + a_1)$.

Given a utilitarian objective $v = \lambda_1 u_1 + \lambda_2(u_2 + u_3)$ with $\lambda_1 > 2\lambda_2 > 0$, the optimal BCE distribution p^* is $p^*(1,1,1,1/2) = \mu$ and $p^*(0,0,0,-1/2) = 1-\mu$. The information structure that implements p^* gives all players full information about the state, so that all players are equally informed: $i \succeq_{\text{Inf}} j$ and $j \succeq_{\text{Inf}} i$ for all i and j.

There are six possible total orders $\succ$ on $\mathscr{I}$ compatible with $\succeq_{\text{Inf}}$, one for each permutation of the players. Nevertheless, only one of those, namely $1 \succ 3 \succ 2$, allows delegated transmission. Indeed, any total order with $3 \succ 1$ fails the incentive for truthful transmission: upon receiving message 0, player 3 would not want to forward 0 to player 1 but instead prefer to play 1 and tell player 1 to play 1, even though he knows that the state is -1/2. For an analogous reason, any total order with $2 \succ 3$ fails to promote truthful transmission. Player 2 would want to misreport to player 3 and simultaneously deviate in action given message 0.

6 Discussion

The concepts presented in this paper can be extended in various ways. Here we discuss some of them, but leave their detailed exploration to future research.

6.1 Multiple Meetings

Organizing more than one parallel meetings allows for a greater diversity of incentives, which is especially useful beyond binary actions. In Definition 3, there is *only one* way of keeping a player imperfectly informed about others' information, which is to not invite him to any meeting. With many simultaneous meetings, there are many ways of keeping players imperfectly informed, as participation to a meeting does not give perfect information about what is said in another.

In Online Appendix C.2, we extend Definition 3 to m-meeting scheme, where m denotes the maximal number of simultaneous meetings that can happen with positive probability. Given an objective function v and n players having k actions and utility functions $\{u_i\}$:

(i) what is the minimal m necessary to implement $p^* \in \operatorname{argmax}_{p \in BCE(\mu)} \mathbb{E}_p[v]$?

(ii) given any $m < n$, what is the maximal $\mathbb{E}_p[v]$ that can be achieved subject to p being implementable by an m-meeting scheme?

In Online Appendix C.2, we present an example with $n = 4$ and $k = 3$ and argue that under monotone supermodular payoffs, $\max_{p \in BCE(\mu)} \mathbb{E}_p[v]$ can be achieved by a 3-meeting scheme. In the context of multiple parallel meetings, the average number of simultaneous meetings rather than the maximal number[16] could be used as an alternative indicator of organizational complexity, leading to a different analysis of (i) and (ii).

6.2 Random Delegated Hierarchies

In organizations, although it is simpler to have a fixed hierarchy, it is not unreasonable to assume that an executive would be able to choose the order of managers and supervisors, that is, the hierarchy, as function of the message she wants to transmit. In Online Appendix C.3, we generalize Definition 5 to random delegated hierarchies, by allowing the order of delegated transmission to change with the message profile, and characterize in Proposition 2 the distributions that can be implemented by such information structures. The extra flexibility allows some distributions, which cannot be implemented by a delegated hierarchy, to be implemented by a *random* delegated hierarchy.

A Appendix: Proofs

A.1 Theorem 1

Proof. Suppose that $p \in BCE(\mu)$. Let $p(a|\omega) := p(a,\omega)/\mu(\omega)$ be the conditional probability of a given ω. For all i, define $S_i = \{1\} \bigcup A$ and $s_i : A \to S_i$ such that

[16]This is the maximal number of meetings that can happen with positive probability.

$$s_i(a) = \begin{cases} a_i & \text{if} \quad a_i = 1 \\ a & \text{if} \quad a_i = 0. \end{cases}$$

Consider information structure $(S, \hat{P})$ such that $S = \prod_i S_i$ and $\hat{P}\big((s_i(a))_i|\omega\big) = p(a|\omega)$ for all a and ω. Let a^* be the strategy profile such that $a_i^*(s_i(a)) = a_i$ for all $a \in A$ and $i \in \mathscr{I}$. Given $(S, \hat{P})$, i's interim payoff when observing $s_i = a_i = 1$ and choosing $a_i^*(s_i) = 1$ while others follow a_{-i}^* is (proportional to):

$$\sum_{\omega \in \Omega} \sum_{a_{-i} \in A_{-i}} \mu(\omega) \hat{P}(1, s_{-i}(a)|\omega) u_i(1, a_{-i}^*(s_{-i}(a)); \omega) = \sum_{\omega \in \Omega} \sum_{a_{-i} \in A_{-i}} p(1, a_{-i}; \omega) u_i(1, a_{-i}; \omega) \geq 0 \quad (10)$$

by obedience of p. Thus, for all i, playing 1 is optimal conditional on $s_i = 1$.

Consider player i who observes message $s_i = a$ and notice that $\hat{P}(a, s_{-i}|\omega) = 1$ if $s_{-i} = s_{-i}(a)$ and 0 otherwise for all $\omega \in \Omega$. Hence, $(S, \hat{P})$ is a single-meeting scheme with $\tilde{s}_i = 1$ for all $i \in \mathscr{I}$. Moreover, player i's interim payoff given $s_i = a$ when choosing $a_i' = 1$ and others follow a_{-i}^* is (proportional to):

$$\sum_{\omega \in \Omega} \mu(\omega) \hat{P}(a, s_{-i}(a)|\omega) u_i(1, a_{-i}^*(s_{-i}(a)); \omega) = \sum_{\omega \in \Omega} p(0, a_{-i}; \omega) u_i(1, a_{-i}; \omega). \quad (11)$$

If $\sum_{\omega \in \Omega} p(0, a_{-i}; \omega) u_i(1, a_{-i}; \omega) < 0$ for all $i \in \mathscr{I}$ and $a_{-i} \in A_{-i}$, then $a_i^*(s_i) = 0$ is uniquely optimal given $s_i = a$ for any $a_{-i} \in A_{-i}$. Thus, a^* is an equilibrium such that

$$p'(a, \omega) = \sum_{s \in S} \mu(\omega) \hat{P}(\{s : a^*(s) = a\}|\omega) = p(a, \omega) \quad \forall a \in A, \omega \in \Omega$$

which ensures that $\mathbb{E}_{p'}[v] = \mathbb{E}_p[v]$ and $\mathbb{E}_{p'}[u_i] = \mathbb{E}_p[u_i]$ for all $i \in \mathscr{I}$. Hence, p' is implemented by a single-meeting scheme and $p' \succeq_d p$.

Alternatively, suppose $\sum_\omega p(0, a_{-i}; \omega) u_i(1, a_{-i}; \omega) \geq 0$ for some $i \in \mathscr{I}$ and $a_{-i} \in A_{-i}$. Then $a_i^{**}(s_i) = 1$ is an optimal action at $s_i = a$. Denote by $J \subseteq \mathscr{I}$ the subset of players such that for each $j \in J$ there is $A'_{-j} \subseteq A_{-j}$ such that $\sum_\omega p(0, a_{-j}, \omega) u_j(1, a_{-j}; \omega) \geq 0$ if and only if $j \in J$ and $a_{-j} \in A'_{-j}$. For every $a \in A$, let $J'(a) = \{j \in J : a_j = 0 \text{ and } a_{-j} \in A'_{-j}\}$, which must be nonempty for some a. Denote by $\delta(a)$ the action profile such that $\delta_j = 1$ if and only if $a_j = 1$ or $j \in J'(a)$. Then, any $i \in \mathscr{I}$ receiving $s_i = 1$ and choosing $a_i^*(s_i) = 1$ experiences a change in expected utility (proportional to):

$$\sum_{\omega \in \Omega} \sum_{a_{-i} : J'(1, a_{-i}) \neq \emptyset} \mu(\omega) \hat{P}(1, s_{-i}(a)|\omega) \big(u_i(1, \delta_{-i}(a); \omega) - u_i(1, a_{-i}; \omega) \big) \geq 0, \quad (12)$$

relative to (10), where the inequality follows by Assumption 1. Therefore, the incentive to play $a_i^*(s_i) = 1$ upon observing $s_i = 1$ is only strengthened if some player $j \neq i$ decides to deviate from 0 to 1 upon observing $s_j = a$. Let

$$p'(a,\omega) = \sum_{s\in S} \mu(\omega)\hat{P}(\{s : a^{**}(s) = a\}|\omega) \quad \forall a \in A, \omega \in \Omega$$

where $a_i^{**}(1) = 1$, $a_i^{**}(a) = 1$ if $i \in J'(a)$, and $a_i^{**}(a) = 0$ otherwise. Since $a^{**}(\cdot) \geq a^*(\cdot)$, it holds that $p'(\hat{A} \times \{\omega\}) \geq p(\hat{A} \times \{\omega\})$ for all upper sets $\hat{A}$ and $\omega \in \Omega$. Hence, $(S, \hat{P})$ implements p' such that $\mathbb{E}_{p'}[v] \geq \mathbb{E}_p[v]$ for all weakly increasing v.

Finally, we turn to the welfare implications. For any $a_i \in A_i$ and $\hat{p}$, let $\mathbb{E}_{\hat{p}} u_i(a_i) = \sum_{\omega, a_{-i}} \hat{p}(a_i, a_{-i}, \omega) u_i(a_i, a_{-i}; \omega)$. Consider first i playing 1 under p':

$$\begin{aligned}
\mathbb{E}_{p'} u_i(1) &= \sum_{\omega\in\Omega} \sum_{a_{-i}\in A_{-i}} \mu(\omega)\hat{P}(1, s_{-i}(a)|\omega) u_i(1, a_{-i}^{**}(s_{-i}(a)); \omega) \\
&\quad + \sum_{\omega\in\Omega} \sum_{a: i\in J'(a)} \mu(\omega)\hat{P}(a, s_{-i}(a)|\omega) u_i(1, a_{-i}^{**}(s_{-i}(a)); \omega) \\
&\geq \sum_{\omega\in\Omega} \sum_{a_{-i}\in A_{-i}} \mu(\omega)\hat{P}(1, s_{-i}(a)|\omega) u_i(1, a_{-i}^{*}(s_{-i}(a)); \omega) = \mathbb{E}_p u_i(1),
\end{aligned}$$

by Assumption 1 (since $a_{-i}^{**}(\cdot) \geq a_{-i}^*(\cdot)$) and by definition of J'. Moreover, $\mathbb{E}_{p'} u_i(0) = \mathbb{E}_p u_i(0) = 0$ (which is the only other relevant comparison because $a_i^{**}(s_i) = 0$ implies $a_i^*(s_i) = 0$). Therefore, for all $i \in \mathscr{I}$ it holds that $\mathbb{E}_{p'}[u_i] \geq \mathbb{E}_p[u_i]$. Hence, p' is implemented by a single-meeting scheme and $p' \succeq_d p$. ■

A.2 Theorem 2

Proof. (**Necessity**). Suppose $p \in BCE(\mu)$ can be implemented by a single-meeting scheme (S, P). Then, there is $a^* \in \mathscr{E}(S, P)$ such that $p(a, \omega) = \sum_{s\in S} \mu(\omega) P(\{s : a^*(s) = a\}|\omega)$ for all $a \in A$ and $\omega \in \Omega$. By definition of a single-meeting scheme, for all i and all s such that $P(s) > 0$ and $s_i \in S_i \setminus \{\tilde{s}_i\}$, $i \in M(s)$ and hence $\mu_i(s_{-i}|s_i) = 1$, implying

$$P(s|\omega)\mu(\omega) = \mu_i(\omega|s_i)P(s_i). \tag{13}$$

Let $\tilde{a}_i = a_i^*(\tilde{s}_i)$. Then, for all $a_i \in A_i \setminus \{\tilde{a}_i\}$ and all s such that $P(s) > 0$ and $a_i^*(s_i) = a_i$ (thus, $s_i \neq \tilde{s}_i$), we have

$$\sum_{\omega\in\Omega} \mu_i(\omega|s_i)\big(u_i(a_i, a_{-i}^*(s_{-i}); \omega) - u_i(a_i', a_{-i}^*(s_{-i}); \omega)\big) \geq 0, \tag{14}$$

for all $a_i' \in A_i$ in virtue of equilibrium. By (13) and (14), we obtain

$$\sum_{\omega\in\Omega}\mu(\omega)P(s|\omega)\left(u_i(a_i,a^*_{-i}(s_{-i});\omega)-u_i(a'_i,a^*_{-i}(s_{-i});\omega)\right)\geq 0,$$

for all $s_i \in S_i$ such that $a^*_i(s_i)=a_i$ and all $s_{-i}\in S_{-i}$. Now, given arbitrary $a_{-i}\in A_{-i}$ and $a_i \in A_i \setminus \{\tilde{a}_i\}$, let

$$S(a_i,a_{-i})=\left\{s\in S:\quad a^*_i(s_i)=a_i \text{ and } a^*_{-i}(s_{-i})=a_{-i}\right\}.$$

For all $a_{-i}\in A_{-i}$ and $a_i\in A_i\setminus\{\tilde{a}_i\}$,

$$\begin{aligned}
&\sum_{s\in S(a_i,a_{-i})}\sum_{\omega\in\Omega}\mu(\omega)P(s|\omega)\left(u_i(a_i,a^*_{-i}(s_{-i});\omega)-u_i(a'_i,a^*_{-i}(s_{-i});\omega)\right)\\
=&\sum_{\omega\in\Omega}\mu(\omega)P\left(\{s:a^*_i(s_i)=a_i,a^*_{-i}(s_{-i})=a_{-i}\}|\omega\right)\left(u_i(a_i,a_{-i};\omega)-u_i(a'_i,a_{-i};\omega)\right)\\
=&\sum_{\omega\in\Omega}p(a_i,a_{-i},\omega)\left(u_i(a_i,a_{-i};\omega)-u_i(a'_i,a_{-i};\omega)\right)\geq 0.
\end{aligned}$$

(**Sufficiency**). Suppose now that $p\in BCE(\mu)$ and for all $i\in\mathscr{I}$, there is $\tilde{a}_i\in A_i$ such that for all $a_i\in A_i\setminus\{\tilde{a}_i\}$, $\sum_\omega p(a_i,a_{-i},\omega)\left(u_i(a_i,a_{-i};\omega)-u_i(a'_i,a_{-i};\omega)\right)\geq 0$ for all $a'_i\in A_i$ and $a_{-i}\in A_{-i}$. Then, define (S,P) as follows:

1. for each $i\in\mathscr{I}$, let $S_i=\{\tilde{a}_i\}\bigcup A$ and $s_i:A\to S_i$ be $s_i(a)=\begin{cases}a_i & \text{if} \quad a_i=\tilde{a}_i\\ a & \text{if} \quad a_i\neq\tilde{a}_i\end{cases}$, and
2. let $S=\prod_i S_i$ and $P\left((s_i(a))_i|\omega\right)=p(a|\omega)$ for all $a\in A$ and $\omega\in\Omega$.

Consider strategy profile a^* such that $a^*_i(s_i(a))=a_i$ for all $a\in A$ and $i\in\mathscr{I}$. Since $p\in BCE(\mu)$, for all $i\in\mathscr{I}$ and $a_i\in A_i$,

$$\begin{aligned}
&\sum_{\omega\in\Omega}\sum_{a_{-i}\in A_{-i}}p(a_i,a_{-i},\omega)\left(u_i(a_i,a_{-i},\omega)-u_i(a'_i,a_{-i},\omega)\right)\\
=&\sum_{\omega\in\Omega}\sum_{a_{-i}\in A_{-i}}\mu(\omega)P\left((s_i(a))_i|\omega\right)\left(u_i(a^*_i(s_i(a)),a^*_{-i}(s_{-i}(a)),\omega)-u_i(a'_i,a^*_{-i}(s_{-i}(a)),\omega)\right)\geq 0,
\end{aligned}$$

which shows that a^* is a BNE. Clearly, given that $a^*_i(s_i(a))=a_i$ for all $a\in A$, $\sum_{s\in S}\mu(\omega)P(\{s: a^*(s)=a\}|\omega)=\mu(\omega)p(a|\omega)=p(a,\omega)$ for all $a\in A$ and $\omega\in\Omega$, so that p is implemented by (S,P). Finally, for each $i\in\mathscr{I}$, let $\tilde{s}_i=\tilde{a}_i$ and note that $\mu_i(s_{-i}(a)|s_i)=1$ for all $s_i\neq\tilde{s}_i$. Therefore, (S,P) is a single-meeting scheme. ■

A.3 Claims 1 and 2

Proof of Claim 1. Invoking Theorem 2, pick $a_i\in A_i\setminus\{\tilde{a}_i\}$ and without loss assume $a_i=a^*_i$. (If $\tilde{a}_i=a^*_i$, choose $a_i=a^{**}_i$). Since a^{**} is a strict equilibrium for all $\omega\in\Omega$, it must be that $u_i(a_i^*, a_{-i}^{**}; \omega) - u_i(a_i^{**}, a_{-i}^{**}; \omega) < 0$ for all $\omega \in \Omega$. Thus, the only way of satisfying (4) for $a_i = a_i^*$ and for all $a_{-i} \in A_{-i}$ is by setting $p^{\text{sms}}(a_i^*, a_{-i}^{**}, \omega) = 0$ for all $\omega \in \Omega$.

To show p^{sms} lies on a face, we need to show that there exists $p \in \text{BCE}(\mu)$ such that, for all $\omega \in \Omega$, $p^{\text{sms}}(a, \omega) > 0$ implies $p(a, \omega) > 0$, and, for some $\omega' \in \Omega$, $p(a_i^*, a_{-i}^{**}, \omega') > 0$. By assumption, both a^* and a^{**} are strict Nash equilibria at every state. Therefore, there exists $\hat{p} \in \text{BCE}(\mu)$ and $\omega' \in \Omega$ such that $\{(a^{**}, \omega'), (a^*, \omega'), (a_i^*, a_{-i}^{**}, \omega')\} \subseteq \operatorname{supp} \hat{p}$ where $\hat{p}(a_i^*, a_{-i}^{**}, \omega')$ is small enough. Next, define $p = (1-\alpha)p^{\text{sms}} + \alpha\hat{p}$ for any $\alpha \in (0,1)$. Given that $p^{\text{sms}}, \hat{p} \in \text{BCE}(\mu)$ and $\text{BCE}(\mu)$ is convex, $p \in \text{BCE}(\mu)$. Moreover, by construction, $p(a, \omega) > 0$ whenever $p^{\text{sms}}(a, \omega) > 0$ and $p(a_i^*, a_{-i}^{**}, \omega') > 0$. ■

Proof of Claim 2. Take $\alpha \in \Delta(\Delta(\Omega))$ such that $\sum_{\hat{\mu}} \alpha(\hat{\mu})\hat{\mu} = \mu$, and for every $\hat{\mu} \in \operatorname{supp} \alpha$, choose $p_{\hat{\mu}} \in \text{Co}(\text{NE}(\hat{\mu}))$. By definition of NE, for all $\hat{\mu} \in \operatorname{supp} \alpha$, $a \in \operatorname{supp} p_{\hat{\mu}}$ and $i \in \mathscr{I}$,

$$\sum_{\omega \in \Omega} \hat{\mu}(\omega) u_i(a_i, a_{-i}; \omega) \geq \sum_{\omega \in \Omega} \hat{\mu}(\omega) u_i(a_i', a_{-i}; \omega)$$

for all $a_i' \in A_i$. Multiplying both sides by $p_{\hat{\mu}}(a, \omega)/\hat{\mu}(\omega)$ gives $\sum_{\omega \in \Omega} p_{\hat{\mu}}(a, \omega) u_i(a_i, a_{-i}; \omega) \geq \sum_{\omega \in \Omega} p_{\hat{\mu}}(a, \omega) u_i(a_i', a_{-i}; \omega)$ for all $i \in \mathscr{I}$, $a_i, a_i' \in A_i$ and $a_{-i} \in A_{-i}$. Multiplying both sides by $\alpha(\hat{\mu})$ and summing across all $\hat{\mu}$ gives

$$\sum_{\omega \in \Omega} \sum_{\hat{\mu}} \alpha(\hat{\mu}) p_{\hat{\mu}}(a, \omega) u_i(a_i, a_{-i}; \omega) \geq \sum_{\omega \in \Omega} \sum_{\hat{\mu}} \alpha(\hat{\mu}) p_{\hat{\mu}}(a, \omega) u_i(a_i', a_{-i}; \omega) \tag{15}$$

for all $i \in \mathscr{I}$, $a_i, a_i' \in A_i$ and $a_{-i} \in A_{-i}$. By convexity of $\Delta(A \times \Omega)$, $p := \sum_{\hat{\mu}} \alpha(\hat{\mu}) p_{\hat{\mu}} \in \Delta(A \times \Omega)$; moreover,

$$\sum_{a \in A} p(a, \omega) = \sum_{\hat{\mu}} \alpha(\hat{\mu}) \sum_{a \in A} p_{\hat{\mu}}(a, \omega) = \sum_{\hat{\mu}} \alpha(\hat{\mu})\hat{\mu} = \mu. \tag{16}$$

Using the definition of p and summing (15) across all $a_{-i} \in A_{-i}$ implies $p \in BCE(\mu)$. This, together with (15), gives $p \in \mathscr{C}(\tilde{a}, \mu)$ for all $\tilde{a} \in A$ (because (15) ensures that (4) holds for all $a_i \in A_i$). This proves that $\sum_{\hat{\mu}} \alpha(\hat{\mu}) \text{Co}(\text{NE}(\hat{\mu})) \subseteq \mathscr{C}(\tilde{a}, \mu)$ for all $\tilde{a} \in A$. ■

A.4 Theorem 3

Given an information structure (A, P), let $(P \times \mu)(a, \omega) = P(a|\omega)\mu(\omega)$ for all $a \in A$ and $\omega \in \Omega$. For any (A, P) and $f : A \times \Omega \to \mathbb{R}$, let

$$\mathbb{E} f(A, P) = \sum_{\omega \in \Omega} \sum_{a \in A} f(a, \omega) P(a|\omega) \mu(\omega).$$

Lemma 1. *Sppose (A,P) is incentive compatible and implements $p \in BCE(\mu)$. If $(A,\hat{P})$ is such that $\hat{P} \times \mu \succeq_{sd} p$, then there is an incentive compatible (A,P^*) that implements $p^* \succeq_{sd} p$.*

Proof. Since $\mathbb{E}u_i(A,\hat{P}) \geq \mathbb{E}u_i(A,P) \geq 0$ for all $i \in \mathscr{I}$ and playing action 0 gives zero payoff, this implies that $a_i = 1$ is incentive compatible under $(A,\hat{P})$. If

$$\sum_{a_{-i}} \sum_{\omega} \hat{P}(0,a_{-i}|\omega)\mu(\omega)u_i(1,a_{-i};\omega) \leq 0 \tag{17}$$

then $(A,\hat{P})$ is incentive compatible and $p^* = \hat{P} \times \mu \succeq_{sd} p$. If (17) is violated for some $i \in \mathscr{I}$, then i plays 1 upon receiving $a_i = 0$ under $(A,\hat{P})$. This reinforces j's incentives to play 1 by Assumption 2 for all $j \neq i$ (and weakens j's incentives to play 0) and it also weakly increases everyone's total expected utility. After all such deviations have occurred, we obtain an incentive compatible (A,P^*) that implements $p^* := P^* \times \mu \succeq_{sd} p$. ■

For any $a \in A$, let $I(a) = \{i \in \mathscr{I} : a_i = 1\}$.

Lemma 2. *For any $p \in BCE(\mu)$, there exists $p^* \succeq_{sd} p$ which can be implemented by an incentive compatible information structure (A,P^*) such that:*

1. *For all ω, if $P^*(a|\omega) > 0$ and $P^*(a'|\omega) > 0$, then $I(a') \subseteq I(a)$ or $I(a) \subseteq I(a')$.*
2. *For all $\omega' < \omega''$, if $P^*(a'|\omega') > 0$ and $P^*(a''|\omega'') > 0$, then $I(a') \subseteq I(a'')$.*

Proof. (**Part 1**). Suppose that for some ω', a' and a'', (A,P) is such that $P(a'|\omega') > 0$ and $P(a''|\omega') > 0$, yet $I(a'') \subsetneq I(a')$ and $I(a') \subsetneq I(a'')$. Assume without loss that $P(a''|\omega') > P(a'|\omega')$.

Case 1: If $\omega' \geq \underline{\omega}$, then define $(A,\hat{P})$ as

$$\hat{P}(a|\omega) = \begin{cases} 0 & \text{if} \quad a = a' \text{ and } \omega = \omega' \\ P(a|\omega) + P(a'|\omega') & \text{if} \quad a = (1,\dots,1) \text{ and } \omega = \omega' \\ P(a|\omega) & \text{otherwise.} \end{cases}$$

If $i \in I(a')$, then $\mathbb{E}u_i(A,\hat{P}) - \mathbb{E}u_i(A,P) = \mu(\omega')P(a'|\omega')(u_i(1,\dots,1;\omega') - u_i(a';\omega')) \geq 0$, by Assumption 2(a). If $i \notin I(a')$, then

$$\mathbb{E}u_i(A,\hat{P}) - \mathbb{E}u_i(A,P) = \mu(\omega')P(a'|\omega')u_i(1,\dots,1;\omega') \geq 0,$$

which follows from $\omega' \geq \underline{\omega}$.

Case 2: If $\omega' < \underline{\omega}$, then define $(A,\hat{P})$ as

$$\hat{P}(a|\omega) = \begin{cases} 0 & \text{if} \quad a = a' \text{ and } \omega = \omega' \\ P(a|\omega) - P(a'|\omega') & \text{if} \quad a = a'' \text{ and } \omega = \omega' \\ P(a|\omega) + P(a'|\omega') & \text{if} \quad a = a' \vee a'' \text{ and } \omega = \omega' \\ P(a|\omega) + P(a'|\omega') & \text{if} \quad a = a' \wedge a'' \text{ and } \omega = \omega' \\ P(a|\omega) & \text{otherwise.} \end{cases}$$

If $i \in I(a')$ but $i \notin I(a'')$, then

$$\mathbb{E}u_i(A,\hat{P}) - \mathbb{E}u_i(A,P) = \mu(\omega')P(a'|\omega')(u_i(a' \vee a'';\omega') - u_i(a';\omega')) \geq 0,$$

by Assumption 2(b). If $i \in I(a'')$ but $i \notin I(a')$, then

$$\mathbb{E}u_i(A,\hat{P}) - \mathbb{E}u_i(A,P) = \mu(\omega')P(a'|\omega')(u_i(a' \vee a'';\omega') - u_i(a'';\omega')) \geq 0,$$

which follows by Assumption 2(b). If $i \in I(a') \cap I(a'')$, then

$$\begin{aligned}\mathbb{E}u_i(A,\hat{P}) - \mathbb{E}u_i(A,P) = \mu(\omega')P(a'|\omega')\Big(&\big[u_i(a' \vee a'';\omega') - u_i(a';\omega')\big] \\ &- \big[u_i(a'';\omega') - u_i(a' \wedge a'';\omega')\big]\Big) \geq 0,\end{aligned}$$

which follows by Assumption 2(b). If $i \notin I(a') \cup I(a'')$, then $\mathbb{E}u_i(A,\hat{P}) = \mathbb{E}u_i(A,P)$.

Furthermore, by Assumption 2, for all $v \in \mathcal{V}^{\text{SM}} \cup \{u_i\}$, we have, in Case 1

$$\mathbb{E}v(A,\hat{P}) - \mathbb{E}v(A,P) = \mu(\omega')P(a'|\omega')\big[v(1,\ldots,1;\omega') - v(a';\omega')\big] \geq 0,$$

and in Case 2,

$$\begin{aligned}\mathbb{E}v(A,\hat{P}) - \mathbb{E}v(A,P) = \mu(\omega')P(a'|\omega')\Big(&\big[v(a' \vee a'';\omega') - v(a';\omega')\big] \\ &- \big[v(a'';\omega') - v(a' \wedge a'';\omega')\big]\Big) \geq 0.\end{aligned}$$

Clearly, the same inequalities would hold for all positive linear combinations of functions in $\mathcal{V}^{\text{SM}} \cup \{u_i\}$, hence for all $v \in \text{cone}(\mathcal{V}^{\text{SM}} \cup \{u_i\})$. Hence, $\hat{P} \times \mu \succeq_{sd} p$. By Lemma 1, there is an incentive compatible (A,P^*) that implements $p^* \succeq_{sd} p$. Moreover, (A,P^*) preserves the inclusion property of $(A,\hat{P})$, because if $\sum_{a_{-i}} \sum_{\omega} \hat{P}(0,a_{-i}|\omega)\mu(\omega)u_i(1,a_{-i};\omega) > 0$, then $a_i = 1$ for all a such that $P^*(a) > 0$.

By repeating this procedure, for all a, a' and ω' such that $P(a|\omega') > 0$, $P(a'|\omega') > 0$, $I(a') \subsetneq I(a)$ and $I(a) \subsetneq I(a')$, we eventually obtain an incentive compatible (A,P^*) such that for any ω, $P^*(a|\omega) > 0$ and $P^*(a'|\omega) > 0$ imply $I(a') \subseteq I(a)$ or $I(a) \subseteq I(a')$, and (A, P^*) induces $p^* \succeq_{sd} p$.

(**Part 2**). Now suppose that for some $\omega' < \omega''$, (A, P) is such that $P(a'|\omega') > 0$, $P(a''|\omega'') > 0$ and yet $I(a') \subsetneq I(a'')$.

Case 1. If $\omega'' \geq \underline{\omega}$, then define $(A, \hat{P})$ as

$$\hat{P}(a|\omega) = \begin{cases} 0 & \text{if} \quad a = a'' \text{ and } \omega = \omega'' \\ P(a|\omega) + P(a''|\omega'') & \text{if} \quad a = (1, \dots, 1) \text{ and } \omega = \omega'' \\ P(a|\omega) & \text{otherwise.} \end{cases}$$

If $i \in I(a'')$, then

$$\mathbb{E}u_i(A, \hat{P}) - \mathbb{E}u_i(A, P) = P(a''|\omega'')\mu(\omega'')\big[u_i(1, \dots, 1; \omega'') - u_i(a''; \omega'')\big] \geq 0,$$

by Assumption 2(a). If $i \notin I(a'')$, then, because $\omega'' \geq \underline{\omega}$,

$$\mathbb{E}u_i(A, \hat{P}) - \mathbb{E}u_i(A, P) = P(a''|\omega'')\mu(\omega'')u_i(1, \dots, 1; \omega'') \geq 0.$$

Case 2. Suppose $\omega'' < \underline{\omega}$.

Case 2.1. If $P(a'|\omega')\mu(\omega') \geq P(a''|\omega'')\mu(\omega'')$, then define $(A, \hat{P})$ as

$$\hat{P}(a|\omega) = \begin{cases} 0 & \text{if} \quad a = a'' \text{ and } \omega = \omega'' \\ P(a|\omega) - \frac{\mu(\omega'')}{\mu(\omega')} P(a''|\omega'') & \text{if} \quad a = a' \text{ and } \omega = \omega' \\ P(a|\omega) + P(a''|\omega'') & \text{if} \quad a = a' \vee a'' \text{ and } \omega = \omega'' \\ P(a|\omega) + \frac{\mu(\omega'')}{\mu(\omega')} P(a''|\omega'') & \text{if} \quad a = a' \wedge a'' \text{ and } \omega = \omega' \\ P(a|\omega) & \text{otherwise.} \end{cases}$$

If $i \in I(a')$ and $i \notin I(a'')$, then

$$\mathbb{E}u_i(A, \hat{P}) - \mathbb{E}u_i(A, P) = P(a''|\omega'')\mu(\omega'')\big[u_i(a' \vee a''; \omega'') - u_i(a'; \omega')\big] \geq 0,$$

which follows by Assumption 2(b). If $i \in I(a'')$ and $i \notin I(a')$, then

$$\mathbb{E}u_i(A, \hat{P}) - \mathbb{E}u_i(A, P) = P(a''|\omega'')\mu(\omega'')\big[u_i(a' \vee a''; \omega'') - u_i(a''; \omega'')\big] \geq 0,$$

which follows by Assumption 2(b). If $i \in I(a') \cap I(a'')$, then

$$\mathbb{E}u_i(A, \hat{P}) - \mathbb{E}u_i(A, P) = P(a''|\omega'')\mu(\omega'')\big[u_i(a' \vee a''; \omega'') - u_i(a''; \omega'') + u_i(a' \wedge a''; \omega') - u_i(a'; \omega')\big] \geq 0,$$

by Assumption 2(b). If $i \notin I(a') \cup I(a'')$, then $\mathbb{E}u_i(A,\hat{P}) = \mathbb{E}u_i(A,P)$.

Case 2.2. If instead $P(a'|\omega')\mu(\omega') < P(a''|\omega'')\mu(\omega'')$, then define $(A,\hat{P})$ as

$$\hat{P}(a|\omega) = \begin{cases} 0 & \text{if} \quad a = a' \text{ and } \omega = \omega' \\ P(a|\omega) - \frac{\mu(\omega')}{\mu(\omega'')}P(a'|\omega') & \text{if} \quad a = a'' \text{ and } \omega = \omega'' \\ P(a|\omega) + \frac{\mu(\omega')}{\mu(\omega'')}P(a'|\omega') & \text{if} \quad a = a' \vee a'' \text{ and } \omega = \omega'' \\ P(a|\omega) + P(a'|\omega') & \text{if} \quad a = a' \wedge a'' \text{ and } \omega = \omega' \\ P(a|\omega) & \text{otherwise.} \end{cases}$$

If $i \in I(a')$ and $i \notin I(a'')$, then

$$\mathbb{E}u_i(A,\hat{P}) - \mathbb{E}u_i(A,P) = P(a'|\omega')\mu(\omega')\big[u_i(a' \vee a'';\omega'') - u_i(a';\omega')\big] \geq 0,$$

which follows by Assumption 2(b). If $i \in I(a'')$ and $i \notin I(a')$, then

$$\mathbb{E}u_i(A,\hat{P}) - \mathbb{E}u_i(A,P) = P(a'|\omega')\mu(\omega')\big[u_i(a' \vee a'';\omega'') - u_i(a'';\omega'')\big] \geq 0,$$

also by Assumption 2(b). Finally, if $i \in I(a') \cap I(a'')$, then

$$\begin{aligned} \mathbb{E}u_i(A,\hat{P}) - \mathbb{E}u_i(A,P) = P(a'|\omega')\mu(\omega')\big[&u_i(a' \vee a'';\omega'') - u_i(a'';\omega'') \\ &+ u_i(a' \wedge a'';\omega') - u_i(a';\omega')\big] \geq 0 \end{aligned}$$

by Assumption 2(b). If $i \notin I(a') \cup I(a'')$, then $\mathbb{E}u_i(A,\hat{P}) = \mathbb{E}u_i(A,P)$.

Furthermore, by Assumption 2, for all $v \in \mathcal{V}^{\mathrm{SM}} \cup \{u_i\}$, we have, in Case 2.1,

$$\begin{aligned} \mathbb{E}v(A,P^*) - \mathbb{E}v(A,P) = \mu(\omega'')P(a''|\omega'')\Big(&v(a' \vee a'';\omega'') - v(a'';\omega'') \\ &+ v(a' \wedge a'';\omega') - v(a';\omega')\Big) \geq 0 \end{aligned}$$

and in Case 2.2,

$$\begin{aligned} \mathbb{E}v(A,P^*) - \mathbb{E}v(A,P) = \mu(\omega')P(a'|\omega')\Big(&v(a' \vee a'';\omega'') - v(a'';\omega'') \\ &+ v(a' \wedge a'';\omega') - v(a';\omega')\Big) \geq 0. \end{aligned}$$

Clearly, the same inequalities would hold for all positive linear combinations of functions in $\mathcal{V}^{\mathrm{SM}} \cup \{u_i\}$, hence for all $v \in \mathrm{cone}(\mathcal{V}^{\mathrm{SM}} \cup \{u_i\})$. Hence, $\hat{P} \times \mu \succeq_{sd} p$. By Lemma 1, there is an incentive compatible (A,P^*) that implements $p^* \succeq_{sd} p$. Moreover, (A,P^*) preserves the inclusion property of $(A,\hat{P})$, because if $\sum_{a_{-i}} \sum_{\omega} \hat{P}(0,a_{-i}|\omega)\mu(\omega)u_i(1,a_{-i};\omega) > 0$, then $a_i = 1$

for all a such that $P^*(a) > 0$.

By repeating this procedure, for all $\omega' < \omega''$ such that $P(a'|\omega') > 0$, $P(a''|\omega'') > 0$ and $I(a') \subsetneq I(a'')$, we eventually obtain an incentive compatible (A, P^*) such that for any $\omega' < \omega''$, $P^*(a'|\omega') > 0$ and $P^*(a''|\omega'') > 0$ imply $I(a') \subseteq I(a'')$, and (A, P^*) induces $p^* \succeq_{sd} p$. ■

Lemma 3. *Let (A, P^*) be the incentive compatible information structure implementing p^* from Lemma 2. For all $i \in \mathscr{I}$, define $S_i = \{1\} \bigcup A$ and $s_i : A \to S_i$ such that $s_i(a) = a_i$ if $a_i = 1$ and $s_i(a) = a$ if $a_i = 0$. Let $S = \prod_i S_i$ and P' be such that $P'\big((s_i(a))_i|\omega\big) = P^*(a|\omega)$ for all $a \in A$ and $\omega \in \Omega$. Then, there exists a total order $\succ$ such that $((S, P'), \succ)$ is an information hierarchy that implements $p' \succeq_d p^*$. Further, (S, P') is a single-meeting scheme.*

Proof. Consider (S, P') as defined in the lemma. First, we show that there exists a total order $\succ$ such that $((S, P'), \succ)$ is an information hierarchy. Let $\mathbb{S} = \bigcup_{a,\omega: P^*(a|\omega)>0} I(a)$ be the collection of sets of players playing 1 in the action profiles occurring with positive probability in (A, P^*). If $|\mathbb{S}| = 1$, then (A, P^*) conveys no additional information to the players beyond their prior belief; thus, for any total order $\succ$, $((A, P^*), \succ)$ is an information hierarchy. In what follows, suppose $|\mathbb{S}| \geq 2$. By Lemma 2, $(\mathbb{S}, \subseteq)$ is a totally ordered set, the elements of which can be denoted $\{I_k\}_{k=1}^K$ such that $I_{k'} \subseteq I_{k''}$ iff $k'' \geq k'$. Now, define $G_k = I_k \backslash I_{k-1}$ for all k (assuming $I_0 = \emptyset$). Define $\succ$ such that $i \succ j$ iff $[i \in G_{k''}, j \in G_{k'}$ and $k'' > k']$ or $[\{i, j\} \subseteq G_k,\ i > j]$. By the inclusion property of (A, P^*) and by construction of (S, P'), $s_i(a) = 1$ if and only if $a_j = 1$ for all j such that $i \succ j$, which implies that $\mu_i(\{s_j = 1\}|1) = 1$ for all j such that $i \succ j$. Additionally, it holds that $\mu_i(\{s_{-i} = s_{-i}(a)\}|a) = 1$. Therefore,

$$\mu_i(\omega, s'_{-i}|s_j) = \mu_i(\omega, s'_{-i}|s_i, s_j) \quad \forall \omega \in \Omega, s'_{-i} \in S_{-i} \tag{18}$$

for all s_j whenever $i \succ j$. Equivalently, $i \succeq_{\text{Inf}} j$ for all $i \succ j$, making $((S, P'), \succ)$ an information hierarchy. Further, since $\mu_i(\{s_{-i} = s_{-i}(a)\}|a) = 1$ (i.e., $i \in M(s)$ iff $s_i = a$), (S, P') is a single-meeting scheme where $\tilde{s}_i = 1$ for all $i \in \mathscr{I}$. The proof of Theorem 1 already shows that such a transformation (here, of (A, P^*)) implements $p' \succeq_d p^*$. ■

Lemma 4. *The distribution p' from Lemma 3 can be implemented by a delegated hierarchy.*

Proof. In the proof of Lemma 3, we obtained an information hierarchy $((S, P'), \succ)$ (implementing p') from an incentive compatible direct information structure (A, P^*) with groups $\{G_k\}_{k=1}^K$. Suppose that for some G_k and $\hat{a}$ such that $I(\hat{a}) \subseteq \bigcup_{\ell<k} G_\ell$ and $P'(s(\hat{a})) = P^*(\hat{a}) > 0$, there exists $G'_k \subseteq G_k$ such that $\mathbb{E}\left[u_j(\tilde{a}; \omega)|s_j = \hat{a}\right] \geq 0$ for all $j \in G'_k$, where $\tilde{a}$ is defined by

$I(\tilde{a}) = G'_k \bigcup I(\hat{a})$. That is, there exists a subgroup G'_k, for which each $j \in G'_k$ finds the subgroup's joint deviation to action 1 weakly profitable upon receiving $s_j = \hat{a}$. Then, it must be that $u_j(\tilde{a};\tilde{\omega}) \geq 0$ for all $j \in G'_k$, where $\tilde{\omega} = \max\{\omega \in \Omega : P'(s(\hat{a})|\omega) > 0\}$.

Now define direct information structure $(A, \hat{P}_1)$ as:

$$\hat{P}_1(a|\omega) = \begin{cases} 0 & \text{if} \quad a = \hat{a} \text{ and } \omega = \tilde{\omega} \\ P^*(a|\omega) + P^*(\hat{a}|\tilde{\omega}) & \text{if} \quad a = \tilde{a} \text{ and } \omega = \tilde{\omega} \\ P^*(a|\omega) & \text{otherwise.} \end{cases}$$

Notice that $\mathbb{E}u_i(A, \hat{P}_1) \geq \mathbb{E}u_i(A, P^*)$ for all $i \in \mathcal{I}$ and $\mathbb{E}v(A, \hat{P}_1) \geq \mathbb{E}v(A, P^*)$ for all $v \in \mathcal{V}^{\mathrm{M}}$. Therefore, by Lemma 1, there exists an incentive compatible (A, P_1^*) which implements $p_1^* \succeq_d p'$. By applying Lemmas 2 and 3 to (A, P_1^*), we obtain an information hierarchy $((S, P'_1), \succ_1)$ that implements $p'_1 \succeq_{sd} p_1^* \succeq_{sd} p'$.

Performing this procedure recursively, we eventually obtain a direct information structure (A, P_M^*) with groups $\{G_k^*\}$ and an information hierarchy $((S, P'_M), \succ_M)$, such that no subgroup of any G_k^* would like to deviate jointly to action 1 upon receiving any message a. Hence, for all G_k^* and for $\hat{a}$ such that $I(\hat{a}) = \bigcup_{\ell < k} G_\ell^*$ and $P'_M(s(\hat{a})) > 0$, there is (at least one) player $j \in G_k^*$ such that $\mathbb{E}\left[u_j(\tilde{a};\omega)|s_j = \hat{a}\right] < 0$, where $\tilde{a}$ is defined by $I(\tilde{a}) = G_k^* \bigcup I(\hat{a})$. Let that player (any one of them, if many) be indexed as the largest player in group G_k^* and label him $(k, |G_k^*|)$. Next, there exists (at least one) player $j \in G_k^* \setminus (k, |G_k^*|)$ such that $\mathbb{E}\left[u_j(\bar{a};\omega)\middle|s_j = \hat{a}\right] < 0$ where $\bar{a}$ is defined by $I(\bar{a}) = (G_k^* \setminus (k, |G_k^*|)) \bigcup I(\hat{a})$. Let that player (any one of them, if many) be labeled $(k, |G_k^*| - 1)$. Proceeding in this manner, we obtain a total order within each group G_k^*, where $(k, 1)$ is the smallest indexed member of that group. Hence, we obtain a total ordering of players $\succ$, such that $i \succ j$ if and only if i is labeled (k, ℓ) and j is labeled (k', ℓ') such that $k > k'$ or $k = k'$ and $\ell > \ell'$.

Given $\succ$, we show that $((S, P'_M), \succ)$ is a delegated hierarchy. Pick any s and $i \in \mathcal{I}$ labeled (k, ℓ). First consider the case in which $a_i^*(s_i) = 0$, which by Lemma 2 implies that $a_j^*(s_j) = 0$ for all $j \succ i$ and $j \prec i$ that are labeled $(k, \cdot)$. If i deviates in transmission only, his expected payoff is still 0, which is not strictly profitable. Since i cannot influence $a_j^* = 0$ for all $j \succ i$, his most profitable deviation is to play $a_i = 1$ *and* induce $a'_j = 1$ for all $j \prec i$ by transmission. Denote by $\tilde{a}$ the resulting action profile: $\tilde{a}_j = 1$ iff $j = i$ or $j \prec i$. The most optimistic message about the state that i can receive given $a_i^*(s_i) = 0$ is $s_i = \hat{a}$ where $\hat{a}_j = 1$ iff j is labeled $(k', \cdot)$ and $k' < k$. By definition of (S, P'_M) and construction of $\succ$, $\mathbb{E}[u_i(\tilde{a};\omega)|s_i = \hat{a}] < 0$, and thus i prefers following $a_i^*(s_i) = 0$ and transmitting messages truthfully. Since the most profitable deviation is not profitable under the most optimistic message about the state, $\mathbb{E}[u_i(\tilde{a};\omega)|s_i = a] < 0$ for all $a \in A$ such that $a_i = 0$, so that the deviation is never profitable.

Second, consider the case in which $a_i^*(s_i) = 1$, which by Lemma 2 implies that $a_j^*(s_j) = 1$ for all $j \prec i$. If i plays his equilibrium action and forwards $a_j^* = 1$ to all $j \prec i$, he gets a weakly positive expected utility, because $a_i^*(s_i) = 1$ is optimal. Thus, it cannot be strictly profitable to deviate to action 0. The only deviation that can possibly be profitable is, therefore, to deviate in transmission only and induce $(a'_j)_{j\prec i} < (1,\dots,1)$. However, this cannot possibly result in a strictly higher expected utility given Assumption 2. Hence, truthful transmission is optimal. ■

Lemmas 2-4 prove Theorem 3. ■

A.5 Proposition 1

Proof. Suppose that (S,P) is a single-meeting scheme and that there exists a total order $\succ$ such that $((S,P),\succ)$ is an information hierarchy. Then, for any players i and j such that $i \succ j$, it must be that $i \in M(s)$ for all $s \in S$ such that $j \in M(s)$, for otherwise $i \succeq^s_{\text{Inf}} j$ (and hence $i \succ j$) would be violated. Suppose now that all players in a single-meeting scheme (S,P) can be totally ordered according to some $\succ$ such that $i \in M(s)$ whenever $j \in M(s)$ and $i \succ j$. Then, for all $s \in S$ such that $P(s) > 0$, $i \in M(s)$ and $i \succ j$ imply $i \succeq^s_{\text{Inf}} j$, and $i \notin M(s)$ and $i \succ j$ imply that i knows $j \notin M(s)$. In the latter, i knows that $s_j = \tilde{s}_j$, by definition of a single-meeting scheme, and thus $i \succeq^s_{\text{Inf}} j$ once again. This ensures that there is a total order $\succ$ such that $((S,P),\succ)$ is an information hierarchy. ■

A.6 Theorem 4

Proof. (**Necessity**). Suppose $p \in \Delta(A \times \Omega)$ can be implemented by a delegated hierarchy $((S,P),\succ)$. Then, there exists an equilibrium $a^* \in \mathscr{E}(S,P)$ such that

$$p(a,\omega) = \sum_{s\in S} \mu(\omega)P(\{s : a^*(s) = a\}|\omega) \quad \forall a \in A, \omega \in \Omega \tag{19}$$

and, by condition (5), for all $i \in \mathscr{I}$, $s_i \in S_i$ and $s_{\prec i} \in S_{\prec i}$ such that $P(s_i, s_{\prec i}) > 0$,

$$\sum_{\omega\in\Omega}\sum_{s_{\succ i}} u_i\left(a_i^*(s_i), a^*_{\prec i}(s_{\prec i}), a^*_{\succ i}(s_{\succ i});\omega\right)\mu_i(\omega, s_{\succ i}|s_i) \geq$$
$$\sum_{\omega\in\Omega}\sum_{s_{\succ i}} u_i\left(a'_i, a^*_{\prec i}(s'_{\prec i}), a^*_{\succ i}(s_{\succ i});\omega\right)\mu_i(\omega, s_{\succ i}|s_i)$$

for all $a'_i \in A_i$ and $s'_{<i} \in S_{<i}$ such that $P(s'_{<i}) > 0$. Equivalently, for each $s_i \in S_i$ and $s_{<i} \in S_{<i}$ such that $P(s_i, s_{<i}) > 0$, we have

$$\sum_{\omega\in\Omega}\sum_{a_{>i}}\sum_{s_{>i}:a^*_{>i}(s_{>i})=a_{>i}} u_i\left(a^*_i(s_i), a^*_{<i}(s_{<i}), a_{>i};\omega\right)\mu_i(\omega, s_{>i}|s_i) \geq \sum_{\omega\in\Omega}\sum_{a_{>i}}\sum_{s_{>i}:a^*_{>i}(s_{>i})=a_{>i}} u_i\left(a'_i, a^*_{<i}(s'_{<i}), a_{>i};\omega\right)\mu_i(\omega, s_{>i}|s_i) \quad (20)$$

for all $a'_i \in A_i$ and $s'_{<i} \in S_{<i}$ such that $P(s'_{<i}) > 0$. Since $((S,P),\succ)$ is an information hierarchy, for each $s_i \in S_i$ there is at most one $s_{<i} \in S_{<i}$ such that $P(s_i, s_{<i}) > 0$, which implies $\mu_i(s_{<i}|s_i) = 1$. Hence, multiplying each side of (20) by $P(s_i)$ and summing over all $s_i : a^*_i(s_i) = a_i$ yields the following inequalities for each $s_{<i} \in S_{<i}$:

$$\sum_{\omega\in\Omega}\sum_{s_i:a^*_i(s_i)=a_i}\sum_{a_{>i}}\sum_{s_{>i}:a^*_{>i}(s_{>i})=a_{>i}} P(s_i, s_{<i}, s_{>i}|\omega)\mu(\omega)u_i\left(a^*_i(s_i), a^*_{<i}(s_{<i}), a_{>i};\omega\right) \geq \sum_{\omega\in\Omega}\sum_{s_i:a^*_i(s_i)=a_i}\sum_{a_{>i}}\sum_{s_{>i}:a^*_{>i}(s_{>i})=a_{>i}} P(s_i, s_{<i}, s_{>i}|\omega)\mu(\omega)u_i\left(a'_i, a^*_{<i}(s'_{<i}), a_{>i};\omega\right) \quad (21)$$

for all $a'_i \in A_i$ and $s'_{<i} \in S_{<i}$ such that $P(s'_{<i}) > 0$. Summing each side of (21) over all $s_{<i} : a^*_i(s_{<i}) = a_{<i}$, and using (19), we get for all $i \in \mathscr{I}$, $a_i \in A_i$ and $a_{<i} \in A_{<i}$ such that $p(a_i, a_{<i}) > 0$,

$$\sum_{\omega\in\Omega}\sum_{a_{>i}} p(a_i, a_{<i}, a_{>i}, \omega)\left(u_i(a_i, a_{<i}, a_{>i};\omega) - u_i(a'_i, a'_{<i}, a_{>i};\omega)\right) \geq 0 \quad (22)$$

for all $a'_i \in A_i$ and $a'_{<i}$ such that $p(a'_{<i}) > 0$.

(**Sufficiency**). Suppose now that there exists a total order $\succ$ on $\mathscr{I}$ such that (6) holds. Then, define (S,P) as follows:

1. for each i, define $S_i = A_i \bigcup A_{<i}$ and $s_i : A \to S_i$ such that $s_i(a) = (a_i, a_{<i})$.

2. let $S = \prod_i S_i$ and P be such that $P\big((s_i(a))_i|\omega\big) = p(a|\omega)$ for all $a \in A$ and $\omega \in \Omega$.

We first prove that (S,P) is a delegated hierarchy. By construction, $s_i = (a_i, s_j)$ for all $i \in \mathscr{I}$, $j = \max\{j' : i \succ j'\}$ and s such that $P(s) > 0$. Therefore, i's message contains the messages of all of $j \prec i$. Hence, $i \succ j$ implies $i \succeq_{\text{Inf}} j$ and so (S,P) is an information hierarchy.

Furthermore, (6) implies that for all $a_i \in A_i$ and $a_{<i} \in A_{<i}$ such that $p(a_i, a_{<i}) > 0$,

$$\sum_{\omega \in \Omega} \sum_{a_{>i}} p(\omega, a_{>i} | a_i, a_{<i}) \left(u_i(a_i, a_{<i}, a_{>i}; \omega) - u_i(a'_i, a'_{<i}, a_{>i}; \omega) \right) \geq 0 \tag{23}$$

for all $a'_i \in A_i$ and $a'_{<i}$ such that $p(a'_{<i}) > 0$. Given strategy profile a^* defined as $a^*_i(s_i(a)) = a_i$ for all $a \in A$ and $i \in \mathscr{I}$, (23) implies that for all $s_i \in S_i$ and $s_{<i} \in S_{<i}$ such that $P(s_i, s_{<i}) > 0$

$$\sum_{\omega \in \Omega} \sum_{s_{>i}} \mu_i(\omega, s_{>i} | s_i) \left(u_i(a^*_i(s_i), a^*_{<i}(s_{<i}), a^*_{>i}(s_{>i}); \omega) - u_i(a'_i, a'_{<i}, a^*_{>i}(s_{>i}); \omega) \right) \geq 0$$

for all $a'_i \in A_i$ and $a'_{<i}$ such that $p(a'_{<i}) > 0$. Hence, for all $i \in \mathscr{I}$, $s_i \in S_i$ and $s_{<i} \in S_{<i}$ such that $P(s_i, s_{<i}) > 0$,

$$\begin{aligned} & \mathbb{E}\left[u_i\left(a^*(s); \omega\right) \middle| s_i\right] - \mathbb{E}\left[u_i\left(a'_i, a^*_{<i}(s'_{<i}), a^*_{>i}(s_{>i}); \omega\right) \middle| s_i\right] \\ = & \sum_{\omega \in \Omega} \sum_{s_{>i}} \mu_i(\omega, s_{>i} | s_i) \Big(u_i(a^*_i(s_i), a^*_{<i}(s_{<i}), a^*_{>i}(s_{>i}); \omega) \\ & \qquad - u_i(a'_i, a^*_{<i}(s'_{<i}), a^*_{>i}(s_{>i}); \omega) \Big) \geq 0, \end{aligned}$$

for all $a'_i \in A_i$ and $s'_{<i} \in S_{<i}$ such that $P(s'_{<i}) > 0$. This establishes the delegation property and also that a^* is a BNE. Clearly, by definition of a^*,

$$\sum_{s \in S} \mu(\omega) P(\{s : a^*(s) = a\} | \omega) = \mu(\omega) p(a | \omega) = p(a, \omega)$$

for all $a \in A$ and $\omega \in \Omega$, so that p is implemented by (S, P). ■

B Claim 4

Proof. By definition, if $p \in \mathrm{NE}(\mu)$, there exists $a^* \in A$ such that $p(a^*, \omega) = \mu(\omega)$ for all $\omega \in \Omega$ and

$$\sum_{\omega \in \Omega} \mu(\omega)(u_i(a^*; \omega) - u_i(a_i, a^*_{-i}; \omega)) \geq 0 \tag{24}$$

for all $i \in \mathscr{I}$ and $a_i \in A_i$. Take any total order $\succ$ on $\mathscr{I}$. Since $p(a^*, \cdot) = \mu$, (24) is equivalent to

$$\sum_{\omega \in \Omega} p(a^*_i, a^*_{<i}, a^*_{>i}, \omega) \left(u_i(a^*_i, a^*_{<i}, a^*_{>i}; \omega) - u_i(a'_i, a^*_{<i}, a^*_{>i}; \omega) \right) \geq 0 \tag{25}$$

for all $a'_i \in A_i$. Since $a^*_{<i}$ is the only action profile $a_{<i} \in A_{<i}$ such that $p(a_{<i}) > 0$, (25) holds for all $a_{<i} \in A_{<i}$ such that $p(a_{<i}) > 0$. Since $a^*_{>i}$ is the only action profile $a_{>i} \in A_{>i}$ such that $p(a_{>i}) > 0$, summing up (25) across all $a_{>i}$ maintains the inequality. ■

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C Online Appendix to Organized Information Transmission

In this online appendix, we first show by example that the set of single-meeting scheme (pure strategy) outcomes can be nonempty even though there is no (pure strategy) public-information BNE. Then, we define the notion of multiple-meeting scheme, provide a simple example of it, define the notion of a random delegated hierarchy, where the order of delegation can depend on the message realization, and characterize the corresponding implementable distributions.

C.1 Single-meeting Scheme Despite No Public Outcome

Consider an incomplete information game with $\omega = \{1,2\}$ and a uniform prior $\mu(1) = \mu(2) = 1/2$. The state-dependent payoffs are given in Table 2. Notice that for each $\omega \in \{1,2\}$, the corresponding (complete information) game has no pure NE. In fact, the set $\mathrm{NE}(\tilde{\mu})$ is empty for all $\tilde{\mu} \in \Delta(\{1,2\})$. Yet, the distribution in Table 3 (defined for each ω) is in $SMS(\mu)$ for all μ with $\tilde{a} = (1,0,1)$.

$a_3 = 0$	$a_2 = 0$	$a_2 = 1$
$a_1 = 0$	$0,0,0$	$0,\omega,0$
$a_1 = 1$	$-\omega,0,0$	$\omega,-\omega,0$

$a_3 = 1$	$a_2 = 0$	$a_2 = 1$
$a_1 = 0$	$0,0,-\omega$	$0,-3\omega,\omega$
$a_1 = 1$	$-\omega,0,3\omega$	$2\omega,\omega,-\omega$

Table 2: Joint Payoffs

$p(\cdot,\cdot,0;\omega)$	$a_2 = 0$	$a_2 = 1$
$a_1 = 0$	1/8	0
$a_1 = 1$	0	0

$p(\cdot,\cdot,1;\omega)$	$a_2 = 0$	$a_2 = 1$
$a_1 = 0$	1/8	0
$a_1 = 1$	1/8	1/8

Table 3: Implementable Distribution

In the distribution of Table 3, the incentive constraints for player 1 are:

$$\frac{1}{8}u_1(0,0,0;1) + \frac{1}{8}u_1(0,0,0;2) = 0 \geq \frac{1}{8}u_1(1,0,0;1) + \frac{1}{8}u_1(1,0,0;2) = -\frac{3}{8} \tag{26}$$

$$\frac{1}{8}u_1(0,0,1;1)+\frac{1}{8}u_1(0,0,1;2)=0\geq\frac{1}{8}u_1(1,0,1;1)+\frac{1}{8}u_1(1,0,1;2)=-\frac{3}{8} \tag{27}$$

and

$$\frac{1}{8}u_1(1,0,1;1)+\frac{1}{8}u_1(1,1,1;1)+\frac{1}{8}u_1(1,0,1;2)+\frac{1}{8}u_1(1,1,1;2)=\frac{3}{8}\geq$$
$$\frac{1}{8}u_1(0,0,1;1)+\frac{1}{8}u_1(0,1,1;1)+\frac{1}{8}u_1(0,0,1;2)+\frac{1}{8}u_1(0,1,1;2)=0. \tag{28}$$

It is easy to check that the corresponding constraints for players 2 and 3 are satisfied as well. It also holds that $\sum_a p(a;0)=\mu(0)=1/2$. Thus, the given outcome distribution can be implemented by a single-meeting scheme despite the absence of a pure NE for any μ.

C.2 m-Meeting Schemes

Organizing multiple meetings in parallel offers extra flexibility.

Definition 6. *An m-meeting scheme is an information structure (S,P) such that, for all s with $P(s)>0$, $\mathscr{I}$ is partitioned in at most $m+1$ groups $\{G_1(s),\dots,G_{m+1}(s)\}$ such that $i=^s_{\text{Inf}}j$ for all $i,j\in G_k(s)$ and $k\leq m$, and $s_i=s'_i$ whenever $i\in G_{m+1}(s)\cap G_{m+1}(s')$.*

An m-meeting scheme organizes at most m simultaneous meetings, $\{G_1(s),\dots,G_m(s)\}$, each of which makes its content common knowledge among the participants. The players who are not invited to any meeting at message profile s are contained in $G_{m+1}(s)$. There is an important distinction between the uncertainty from participation in a meeting (about what is said in other meetings) and the uncertainty from participation in no meeting. The latter must be the same across all messages, because if i is not present in any of $\{G_1(s),\dots,G_m(s)\}$ or $\{G_1(s'),\dots,G_m(s')\}$, then i must have the same belief given s_i as he does given s'_i.

Recall $\mathscr{I}=\{1,\dots,n\}$, let $A_i=\{0,\dots,k\}$ and fix the payoffs $\{u_i\}$. Given $(n,k)\in\mathbb{N}^2$ and v:

(i) what is the minimal m such that an outcome distribution $p\in\Delta(A\times\Omega)$ can be implemented by an m-meeting scheme?

(ii) Given $m'<n$, what is the set of outcome distributions p that can be implemented by an m'-meeting scheme?

(iii) Let $m(s)+1$ be the cardinality of the partition of $\mathscr{I}$ at s in Definition 6 ($m(s)$ is number of meetings plus 1 for the non-invited players). The maximal number of meetings that can occur with positive probability given an information structure (S,P) is

$$\bar{m}(S,P) = \max_{s\in S:P(s)>0} m(s)$$

which is the measure of organizational complexity used in (i) and (ii). The average number of meetings is another possible measure of complexity

$$\mathbb{E}m(S,P) = \sum_{s\in S} m(s)P(s|\omega)\mu(\omega)$$

which would lead to different answers in (i) and (ii).

Both questions (i) and (ii) are combinatorial problems. Of course, an n-meeting scheme imposes no restriction at all, because any message profile can be transmitted through individual meetings with each player. Therefore, the answer to (i) is at most n, making it especially interesting to identify problems for which the answer is strictly less than n. The reality of information transmission may impose an upper bound $m' < n$ on the maximal or the average number of meetings that can possibly be organized. Hence, (i) is a constrained optimization exercise and (ii) a characterization exercise.

As an example, assume $n = 4$; $k = 2$; u_i exhibits increasing differences in (a_i, a_{-i}) for each ω and in (a_i, ω) for each a_{-i}; and $v : A \to \mathbb{R}$ is strictly increasing, for example $v = \sum_i a_i$. In an optimal BCE, it follows from state monotonicity that each i must have two cutoff beliefs: one marking the transition from action 1 to 2, and the other from action 0 to 1. De facto, p^* induces at most 9 action profiles and it is always possible to implement it by a 3-meeting scheme. Here is an illustration (each $\{\cdot\}$ represents a meeting):

Meetings					$(a_1$	a_2	a_3	$a_4)$
$\{s_1\}$	$\emptyset$	$\{s_3\}$	$\emptyset$	$\Longleftrightarrow$	2	2	2	2
$\emptyset$	$\emptyset$	$\{s_3\}$	$\emptyset$	$\Longleftrightarrow$	1	2	2	2
$\emptyset$	$\{s_2\}$	$\{s_3\}$	$\emptyset$	$\Longleftrightarrow$	1	1	2	2
$\emptyset$	$\{s_2\}$	$\emptyset$	$\emptyset$	$\Longleftrightarrow$	1	1	1	2
$\emptyset$	$\{s_2\}$	$\emptyset$	$\{s_4\}$	$\Longleftrightarrow$	1	1	1	1
$\{s'_1\}$	$\{s_2\}$	$\emptyset$	$\{s_4\}$	$\Longleftrightarrow$	0	1	1	1
$\{\hat{s}_{12}$	$\hat{s}_{12}\}$	$\emptyset$	$\{s_4\}$	$\Longleftrightarrow$	0	0	1	1
$\{\hat{s}_{12}$	$\hat{s}_{12}\}$	$\{s'_3\}$	$\{s_4\}$	$\Longleftrightarrow$	0	0	0	1
$\{\hat{s}_{12}$	$\hat{s}_{12}\}$	$\{\hat{s}_{34}$	$\hat{s}_{34}\}$	$\Longleftrightarrow$	0	0	0	0

C.3 Random Delegated Hierarchies

We generalize Definition 5 and Theorem 4 by allowing the order of delegation to depend on the message profile.

Definition 7. *A distribution $p \in \Delta(A \times \Omega)$ can be implemented by a random delegated hierarchy if there is an information structure (S,P) and an equilibrium $a^* \in \mathscr{E}(S,P)$ such that*

$$p(a,\omega) = \sum_{s \in S} \mu(\omega) P(\{s : a^*(s) = a\}|\omega) \quad \forall a \in A,\, \omega \in \Omega$$

and if, for every s such that $P(s) > 0$, there exists a total order $\succ_s$ on $\mathscr{I}$ such that for all $i \in \mathscr{I}$,

$$\mu_i(s'_{-i},\omega|s_i, s_{\prec_s i}) = \mu_i(s'_{-i},\omega|s_i) \quad \forall s'_{-i} \in S_{-i},\, \omega \in \Omega \tag{29}$$

and

$$\mathbb{E}\Big[u_i\Big(a_i^*(s_i), a^*_{\prec_s i}(s_{\prec_s i}), a^*_{\succ_s i}(s_{\succ_s i});\omega\Big)\Big|s_i\Big] \geq \mathbb{E}\Big[u_i\Big(a'_i, a^*_{\prec_{s'} i}(s'_{\prec_{s'} i}), a^*_{\succ_s i}(s_{\succ_s i});\omega\Big)\Big|s_i\Big] \tag{30}$$

for all $a'_i \in A_i$ and $s' \in S$ such that $P(s') > 0$ and $\{j : j \prec_{s'} i\} = \{j : j \prec_s i\}$.

The definition assigns to each message profile realization s a total order $\succ_s$ such that two conditions hold. First, each s corresponds to a "local information hierarchy" in which every i knows the messages of his $\succ_s$-predecessors in the information structure (S,P). This condition, formalized in (29), is satisfied by all single-meeting schemes with a total order that ranks the players invited to a meeting at s arbitrarily amongst each other but above the non-invited players at s, who are also ranked arbitrarily amongst each other. Single-meeting schemes, however, may not satisfy the second condition in the definition, formalized in (30) and explained next.

In the expectations in (30), taken over $(s_{\succ_s i},\omega)$, player i can deviate to any a'_i and also misreport to all players $\succ_s$-below him by switching to any positive probability message profile $s'_{\prec_{s'} i} = (s'_j : j \prec_{s'} i)$ such that the set of i's predecessors at s' is the same as at s.[17] One subtlety in (30) is that upon observing s_i, i learns his rank in the total order $\prec_s$, because he can infer it from $\{j : j \prec_s i\}$, whose messages he is asked to forward and can manipulate. Hence, unlike in Halac, Lipnowski, and Rappoport (2020) and Morris, Oyama,

[17]We abstract from deviations to message profiles where $\{j : j \prec_s i\} \subseteq \{j : j \prec_{s'} i\}$ as those may be detected as misreports at some point in the hierarchy.

and Takahashi (2020), there is no own-rank uncertainty in our definition, as the only rank uncertainty pertains to players higher-up in the hierarchy.

In addition, we assume that for each message profile s, player i knows the identity of his immediate successor, $i^+ = \min\{j : j \succ_s i\}$, from whom he receives his message. This information is used in (30) to compute the expectation over all $s_{\succ i}$ such that i^+ is i's immediate successor. We could alternatively assume that i does not know the identity of i^+, in which case the expectation in (30) would be over all possible $s_{\succ i}$, rather than just those that have i^+ as i's immediate predecessor.

The next proposition generalizes the characterization in Theorem 4 by letting the total order depend on the action profile.

Proposition 2. *A distribution $p \in \Delta(A \times \Omega)$ can be implemented by a random delegated hierarchy, if and only if, for each $a \in A$ there exists a total order $\succ_a$ on $\mathscr{I}$ such that for all $i \in \mathscr{I}$,*

$$\sum_{\omega \in \Omega} \sum_{a_{\succ_a i}} p(a_i, a_{\prec_a i}, a_{\succ_a i}, \omega)\big(u_i(a_i, a_{\prec_a i}, a_{\succ_a i}; \omega) - u_i(a'_i, a'_{\prec_{a'} i}, a_{\succ_a i}; \omega)\big) \geq 0 \qquad (31)$$

for all $a'_i \in A_i$ and $a' \in A$ such that $p(a') > 0$ and $\{j : j \prec_{a'} i\} = \{j : j \prec_a i\}$.

For any μ, denote by $RDH(\mu)$ the set of outcome distributions that satisfy the above necessary and sufficient conditions.

What distributions are in $RDH(\mu)$ but *not* in $DH(\mu)$? Before giving a partial answer, let us see if p^* from Section 4.2.1 can be implemented by a random delegated hierarchy. Start with $a = (0,1,1)$. As discussed in Section 4.2.1, players 2 and 3 cannot be first in the ordering and so the possible orderings are $1 \succ 2 \succ 3$ or $1 \succ 3 \succ 2$. Whichever one is chosen should also apply to $a' = (1,1,1)$, for otherwise player 2 would infer from his rank that the state is 0 at a and hence refuse to play 1. This, however, creates a problem at $a'' = (1,0,1)$, where 2 has to be first and the possible orderings are $2 \succ 1 \succ 3$ or $2 \succ 3 \succ 1$. In either case, player 1 will learn that $\omega = 0$ at a'' because, unlike at a', he knows he is not first in rank. Hence, 1 will refuse to play 1 at a''. We conclude that p^* is *not* implementable by a random delegated hierarchy either.

From Claim 4 in Section 4.3 and the discussion that precedes it, we know that strict randomizations between profiles in NE(μ) are in general not included in $DH(\mu)$. Yet randomizations between strong Nash equilibria are included in $RDH(\mu)$. Let

$$\text{SNE}(\mu) = \Big\{ p \in \Delta(A \times \Omega): \quad \exists\, a^* \in A \text{ s.t. } p(a^*, \cdot) = \mu \quad \text{and}$$

$$\text{for all } J \subseteq \mathscr{I} \text{ and } a_J \in \times_{j \in J} A_j \text{ there exists } i \in J \text{ s.t.}$$

$$\sum_{\omega \in \Omega} \mu(\omega) u_i(a^*; \omega) \geq \sum_{\omega \in \Omega} \mu(\omega) u_i(a_J, a^*_{-J}; \omega) \Big\}$$

be the set of pure strategy *strong*-Nash outcomes in the ex-ante normal form game in which it is common knowledge that all players share belief μ. Let

$$\text{SPublic}(\mu) = \bigcup \left\{ \sum_{\hat{\mu}} \alpha(\hat{\mu}) \text{Co}(\text{SNE}(\hat{\mu})): \quad \alpha \in \Delta(\Delta(\Omega)) \text{ s.t. } \sum_{\hat{\mu}} \alpha(\hat{\mu}) \hat{\mu} = \mu \right\}$$

Claim 5. $\text{SPublic}(\mu) \subseteq RDH(\mu)$.