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Bazhanov, Andrei

M.V. Lomonosov Moscow State University, Moscow School of Economics

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Andrei Bazhanov

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Abstract Hamilton (2016) shows that a general closed imperfect economy with extraction cost and any substitutability among inputs is sustainable along an exponentially decreasing path of extraction (EDP) under a generalized Hartwick rule with resource rent measured in SEEA-2012 (System of Environmental-Economic Accounting) units. Mathematically, the result is correct. The problem is that Hamilton offers this approach as "the correct policy rule for sustainability," although this saving rule may be inapplicable to real economies because the prescribed investment may exceed output. In particular, the Cobb-Douglas economy's output goes to zero along EDP even if there is no cost, no health damage from resource use, and all output is invested. The economy with infinite elasticity of substitution between the resource and capital *may* be sustainable along EDP depending on initial conditions. This result extends Hartwick (2003) disclaimer about substitutability among inputs to the generalized version of the rule. Moreover, the result shows that the assessment of sustainability and accounting prices for real economies depends, besides allocation mechanism, on specification of technology and initial conditions.

Keywords Natural nonrenewable resource · Imperfect economy · Resource policy · Sustainability accounting price

JEL classification Q32 · Q36

1 Introduction

Hamilton (2016) shows that a general imperfect economy with extraction cost and any substitutability among inputs is sustainable along an exponentially decreasing path of extraction (EDP) under a generalized Hartwick rule with resource rent measured in SEEA (2014) units.

Mathematically, the result is correct. The problem is that Hamilton offers this approach as "the correct policy rule for sustainability" although this combination

A. Bazhanov
M.V. Lomonosov Moscow State University, Moscow School of Economics

of extraction path and saving rule may be inapplicable to real economies because the prescribed investment is not linked to economy's *current* investment abilities. The required investment may exceed output, but Hamilton (2016) and Hamilton and Ruta (2017), who use the same saving rule, do not offer any practical feasibility conditions for this investment. Moreover, there are already practical-oriented papers that use Hamilton's recommendation.[1]

This comment provides two counterexamples. In the Cobb-Douglas case, output goes to zero along EDP even if there is no cost, no health damage from resource use like in Hamilton and Ruta (2017), and all output is invested. In a linear case (infinite elasticity of substitution between the resource and capital), the economy *may* be sustainable along EDP depending on initial conditions.

Section 2, for self-sufficiency, outlines the model and main result of Hamilton (2016), Section 3 provides counterexamples and an expression for accounting price of a natural resource that arises from a condition guaranteeing sustainability of production possibilities, and Section 4 concludes.

2 The model and main result of Hamilton (2016)

2.1 Model

To model a closed resource economy, Hamilton (2016, Sec.2) uses a neoclassical production function $F(K,R)$[2] with no capital depreciation and TFP equal to unity, where K is the stock of produced capital, R is the flow of resource extraction, and the following conditions hold:

$$F_K > 0,\ F_R > 0,\ F_{KK} < 0,\ F_{RR} < 0,\ F_{KR} > 0,\ F_{RK} > 0; \tag{1}$$
$$F(K,0) = F(0,R) = 0. \tag{2}$$

All variables are the functions of time, unless otherwise specified. Production of a homogeneous good is either consumed (C) or invested,

$$F(K,R) = C + \dot{K}. \tag{3}$$

Extraction of the resource decreases the resource stock S,

$$\dot{S} = -R. \tag{4}$$

2.2 Main result

In Sec. 2.1, Hamilton introduces cost of extraction, which, optimistically, is assumed zero here, and the value of the resource stock that equals the present value of total rents,

$$N = \int_t^\infty F_R(z)R(z) \cdot e^{-\int_t^z F_K(\tau)d\tau} dz \tag{5}$$

with $\dot{N} = F_K N - F_R R$. The unit value of the resource in the ground, by SEEA-2012, is $p \equiv N/S$ (average asset value per unit).

[1] For example, Mardones and del Rio (2019).

[2] Using a similar setup of Hamilton and Hartwick (2005, p. 618), labor is fixed.

The main result of Hamilton (2016) is that if this economy (i) measures genuine saving as $G = \dot{K} - pR$ and (ii) follow the extraction rule $R/S \equiv \phi = const < 1$, which implies $\dot{R}/R = -\phi = -R_0/S_0$ and $R = \tilde{R}(t) = R_0 e^{-\phi t}$, then the generalized Hartwick rule $\dot{C} = F_K G - \dot{G}$ holds.

A short and elegant proof deserves to be included in this paper for self-completeness. Namely, the derivative of (3) leads to $\ddot{K} = \dot{F} - \dot{C} = F_K \dot{K} + F_R \dot{R} - \dot{C}$ implying $\dot{G} = \ddot{K} - \phi \dot{N} = F_K \dot{K} + F_R \dot{R} - \dot{C} - \phi(F_K N - F_R R)$. Then

$$F_K G - \dot{G} = \dot{C} - F_R \dot{R} - \phi F_R R = \dot{C}. \tag{6}$$

It is easy to see that, indeed, as long as $G > 0$ and G is growing more slowly than the interest rate (F_K), consumption will be rising, and if this saving rule applies at each point in time, the economy is sustainable in a sense $\dot{C} \geq 0$ for any $t \geq 0$.

This result is derived for any substitutability σ_{KR} between the resource and capital, which raises concern because it is known[3] that, for example, the economies from CES (Constant Elasticity of Substitution) family with $\sigma_{KR} < 1$, satisfying (1) and (2), are doomed to collapse regardless of saving and extraction rules.

A possible problem is that feasibility conditions ($K > 0, S > 0, \forall t \geq 0$) do not include the condition $\dot{K} < F$ (which should follow from (3)), and it is not clear if the "benchmark investment" ϕN is linked to the *current* economy's investment ability.[4] That is, a prescribed $\dot{K}$ may exceed the economy's output F. In order to shed light on this issue, the following section analyzes output of specific economies along the path $\tilde{R}(t) = R_0 e^{-\phi t}$.

3 Counterexamples

Let, for simplicity, $G \equiv 0$ or $\dot{K} = \phi N$ which, by (6), guarantees constant consumption along $\tilde{R}$ for all $t \geq 0$ if applied at every moment in time. The definition of N given by (5) provides the following integro-differential equation for capital:

$$\dot{K} = \phi \int_t^{\infty} F_R[K(z), \tilde{R}(z)] \tilde{R}(z) \cdot e^{-\int_t^z F_K[K(\tau), \tilde{R}(\tau)] d\tau} dz. \tag{7}$$

Instead of solving this challenging equation directly, assume optimistically (again) that economy can survive without consumption and invest all output in produced capital. This approach should provide an upper bound for the output.

In a general case, the balance equation (3) implies that if production possibilities are not going to zero ($F \not\to 0$) and $w := \frac{\dot{K}}{F} \in (0,1)$ is such that $\dot{w} \leq \frac{\dot{F}}{F}(1-w)$ for any $t \geq 0$, then $\dot{C} \geq 0$ for any $t \geq 0$. In particular, if $w = const$, then $\dot{C} \geq 0$ is equivalent to $\dot{F} \geq 0$. As to sustainability of production, the inequality $\dot{F} = F_K \dot{K} + F_R \dot{R} \geq 0$ immediately implies the following

Lemma 1 *The output of a closed economy is sustainable ($\dot{F} \geq 0$ for all $t \geq 0$) along the path of extraction R_s, if and only if there exists such $\dot{K}_s \in (0, F)$ that the condition*

$$\dot{K}_s + p_s \dot{S}_s \geq 0 \tag{8}$$

[3] Dasgupta and Heal (1974), Solow (1974).

[4] For example, a benchmark investment in the standard Hartwick rule is $F_R R$, which, for $F = K^\alpha R^\beta (0 < \alpha < \beta < 1)$ is $F_R R = \beta F < F$, providing $\dot{C} = 0$ for $\dot{K} = F_R R$. Although, as Hartwick (2003) put it, "This result is local in time" i.e., it does not guarantee sustainability.

holds for all $t \geq 0$*, where* $\dot{S}_s = -R_s$*,*

$$p_s = \frac{F_R}{F_K}\frac{(-\dot{R}_s)}{R_s} \tag{9}$$

is a sustainability accounting price, and $-\dot{R}_s \frac{F_R}{F_K} = \dot{K}_s^{min}$ *is a "benchmark investment."*

The price p_s shows by how much capital must be increased to compensate for a unit of the extracted resource in a sense that the economy is still able to maintain constant output F along R_s. This definition follows from equality in (8) with $\dot{S}_s = -R_s = -1$ and can be generalized to any number of assets.

3.1 Cobb-Douglas case

Consider $\sigma_{KR} = 1$, that is $F = K^\alpha R^\beta, 0 < \alpha, \beta < 1$.[5] As mentioned above, CES functions with $\sigma_{KR} < 1$ do not reflect the research question because they are unsustainable regardless of extraction and saving rules, and functions with $\sigma_{KR} > 1$ do not satisfy (2). "So only the Cobb-Douglas remains" (Solow 1974). For this function, the following proposition holds.

Proposition 1 *For* $F = K^\alpha R^\beta$*, output* F *asymptotically goes to zero if* $\dot{K} \equiv F$ *and* $R(t) = \tilde{R} = R_0 e^{-\phi t}$.

Proof The saving rule $\dot{K} = K^\alpha R^\beta$ and $R = \tilde{R}$ imply $K^{-\alpha} dK = R_0^\beta e^{-\phi\beta t} dt$. Integration, using $\phi = R_0/S_0$, leads to

$$K^{1-\alpha} = K_0^{1-\alpha} + \frac{S_0(1-\alpha)}{\beta R_0^{1-\beta}}(1 - e^{-\phi\beta t})$$

or $K(t) = K_0[1 + K_1(1 - e^{-\phi\beta t})]^{\frac{1}{1-\alpha}}$, where $K_1 = \frac{S_0(1-\alpha)}{\beta R_0^{1-\beta} K_0^{1-\alpha}} > 0$. Note that K is bounded from above along $\tilde{R}$ by a constant $K_0(1 + K_1)^{\frac{1}{1-\alpha}}$.

With this K and $\tilde{R}$, F becomes $F = K_0^\alpha R_0^\beta e^{-\phi\beta t}[1 + K_1(1 - e^{-\phi\beta t})]^{\frac{\alpha}{1-\alpha}}$ or

$$F = F_0 \left\{ e^{-\frac{\phi\beta(1-\alpha)}{\alpha}t}(1 + K_1) - K_1 e^{\frac{-\phi\beta}{\alpha}t} \right\}^{\frac{\alpha}{1-\alpha}}, \tag{10}$$

implying $F \to 0$ with $t \to \infty$ since the bracket $\{\cdot\}$ in (10) goes to zero with $t \to \infty$ □

There is another approach to show that $\tilde{R}$ leads to an infeasible investment for a closed economy $F = K^\alpha R^\beta$ if the economy maintains forever a constant consumption $C_0 > 0$. Denote the path in (10) by $\tilde{F}$. Then the investment is

$$\dot{K} = F - C_0 \leq \tilde{F} - C_0.$$

Since $\tilde{F} \to 0$ as $t \to \infty$, there exists such $\bar{t}$ that $\tilde{F}(\bar{t}) - C_0 = 0$ and $\dot{K} < 0$ for any $t > \bar{t}$. That is, this economy, indeed, maintains constant consumption forever but only due to decapitalization ($\dot{K} < 0$) after a finite moment in time, which contradicts $\dot{K} = \phi N > 0$ and, eventually, the feasibility condition $K > 0$.[6]

[5] There is no need in $\alpha > \beta$ here because $\int_0^\infty \tilde{R} dt$ converges regardless of α and β.

[6] This conclusion is verified by numerical estimates for various $C_0 > 0$.

One more approach is to use Lemma 1 where p_s becomes $\tilde{p}_1 = \phi\frac{\beta K}{\alpha R}$, which goes to infinity as $t \to \infty$. It can be shown that inequality $\dot{K}_s^{min} > F$ (benchmark investment exceeds output) is equivalent to $K^{\alpha-1}R^{\beta} < \frac{\beta R_0}{\alpha S_0}$, where LHS goes to zero as $t \to \infty$, that is, this inequality holds for any sufficiently large t.

Recall that the paths of extraction derived from normative approaches, which lead to (locally) nondecreasing consumption, for example, maximin (Solow 1974, p. 37) or classical utilitarian (Dasgupta and Heal 1979, p. 305), belong to a family

$$\hat{R} = R(t) = R_0(1 + R_1 t)^{R_2}, \;\; R_1 > 0, R_2 < -1.$$

The problem with the path $R_0e^{-\phi t}$ is that it decreases slower than $\hat{R}$ in the short run and approaches zero faster than $\hat{R}$ in the long run. In terms of distribution properties, the path $R_0e^{-\phi t}$ has a thin tail, that is, it redistributes the resource from the future to the present compared to $\hat{R}$.

The following subsection considers an extremely optimistic case with $\sigma_{KR} = \infty$ just for curiosity: may be at least in this unrealistic economy, the path $\tilde{R}$ can guarantee sustainability for a reasonable investment rule.

3.2 Linear case

The case $\sigma_{KR} = \infty$, that is, $F = \alpha K + \beta R, 0 < \alpha, \beta < 1$, is too optimistic. As Dasgupta and Heal (1974) put it "the case [$\sigma_{KR} = \infty$,] ... is, of course, just silly." Assumption (2) does not hold (resource is not necessary), and inequalities $F_{KK} < 0$ and $F_{RR} < 0$ in assumption (1) hold as equalities. Nevertheless, it is illustrative to find out the behavior of F along $\tilde{R}$ as an "upper bound" for a real economy. Lemma 1 with specified F and $R_s = \tilde{R}$ immediately implies the following

Proposition 2 *Economy $F = \alpha K + \beta R$ with $\dot{K} = wF$, where $w = const \in (0,1)$, is sustainable ($\dot{C} \geq 0 \; \forall t \geq 0$) along the path $\tilde{R}(t) = R_0 e^{-\frac{R_0}{S_0}t}$ if and only if*

$$\dot{K} + \tilde{p}_{\infty}\dot{S} \geq 0 \;\; or \;\; \frac{R_0}{S_0} \leq \alpha w \left(1 + \frac{\alpha K}{\beta \tilde{R}}\right), \tag{11}$$

where $\tilde{p}_{\infty} = \frac{R_0\beta}{S_0\alpha}$ is a sustainability accounting price for $F = \alpha K + \beta R$ along $\tilde{R}$.

By (11), even the extreme case $w \equiv 1$ does not guarantee sustainability along $\tilde{R}$ if α and/or K are relatively small (low capital and/or capital efficiency), and/or β and R_0/S_0 are high (high resource dependence and overextraction). However, the period of possible unsustainability is finite since $K/\tilde{R}$ is monotonically increasing. Moreover, if (11) holds at $t = 0$, the economy is globally sustainable.

Note that the price $\tilde{p}_{\infty}$ in Proposition 2 is constant unlike $\tilde{p}_1$ in the Cobb-Douglas case. An intuitive difference of $\tilde{p}_{\infty}$ from $\tilde{p}_1$ is that $\tilde{p}_{\infty}$ does not depend on capital (capital does not use the resource) and depends only on the relative impact of the resource on output β/α and the intensity of extraction R_0/S_0.

4 Conclusion

This comment shows that the sustainability policies offered by Hamilton (2016) in a form of a generalized Hartwick rule may be inapplicable to real economies because the investment of resource rent measured in SEEA-2012 units may exceed current output. Investments that do not exceed output may lead to unsustainability or even collapse along the offered exponentially decreasing path of extraction (EDP) $R(t) = R_0 e^{-\phi t}$.

In particular, the Cobb-Douglas model collapses along EDP even if all output is invested, there is no cost of extraction, and no health damage from resource use (Hamilton and Ruta 2017). An unrealistically optimistic model, where produced capital and natural resource are perfect substitutes, *may* be sustainable along EDP depending on the initial conditions. These examples extend Hartwick (2003) disclaimer about substitutability among inputs to the generalized version of the rule.

This comment offers an expression for accounting price of a natural resource that arises from a condition guaranteeing sustainability of production possibilities. Estimation of this price needs specification of extraction path and production function, which supports the claim of Hamilton and Ruta (2017) that "accounting prices can only be measured with respect to the assumed allocation mechanism . . . And [this mechanism] needs to be fully specified." This claim should be even stronger because the assessment of sustainability may depend, besides allocation mechanism, on specification of technology and initial conditions.

References

Dasgupta, P., and G. Heal. 1974. The optimal depletion of exhaustible resources. *Rev. Econ. Stud.* 41: 3–28.

Dasgupta, Partha S, and Geoffrey M Heal. 1979. *Economic theory and exhaustible resources*. Cambridge University Press.

Hamilton, Kirk. 2016. Measuring sustainability in the UN system of environmental-economic accounting. *Environmental and Resource Economics* 64 (1): 25–36.

Hamilton, Kirk, and John M Hartwick. 2005. Investing exhaustible resource rents and the path of consumption. *Canadian Journal of Economics* 38 (2): 615–621.

Hamilton, Kirk, and Giovanni Ruta. 2017. Accounting price of an exhaustible resource: response and extensions. *Environmental and Resource Economics* 68 (3): 527–536.

Hartwick, J. M. 2003. "Net investment" and sustainability. *Nat. Resour. Model.* 16 (2): 145–160.

Mardones, Cristian, and Ricardo del Rio. 2019. Correction of Chilean GDP for natural capital depreciation and environmental degradation caused by copper mining. *Resources Policy* 60: 143–152.

SEEA. 2014. *System of environmental-economic accounting 2012: Central framework*. United Nations. Bureau of the Committee of Experts on Environmental-Economic Accounting: United Nations Publications.

Solow, R. M. 1974. Intergenerational equity and exhaustible resources. *Rev. Econ. Stud.* 41: 29–45.