Subsidy policies and vertical integration in times of crisis: Can two virtues produce an evil?

Giuranno, Michele G. and Scrimitore, Marcella and Stamatopoulos, Giorgos

Università del Salento, Università del Salento, University of Crete

28 November 2020

Subsidy policies and vertical integration in times of crisis: Can two virtues produce an evil?

Michele G. Giuranno[*]

Marcella Scrimitore[†]

Giorgos Stamatopoulos[‡]

November 28, 2020

Abstract

Vertical integration in an environment without foreclosure, or more generally without any mechanisms that restrict competition among firms, and subsidization of firms' production are two separate mechanisms that raise consumer welfare, and both have been proposed as antidotes to certain aspects of the current economic crisis caused by COVID-19. In this paper we show that the interplay of the two can, surprisingly, be harmful for consumers. We consider a two-layer imperfectly competitive industry where each downstream firm purchases an input from its exclusive upstream supplier, in the presence of a welfare-maximizing government. We allow one (or more than one) of the downstream firms to integrate with its upstream counterpart and we identify two opposite resulting effects: on the one hand, integration alleviates the double marginalization problem and raises industry output and on the other, it alters the government's optimal subsidy policy in a way that reduces output. It turns out that the latter effect dominates the former and thus integration leads to lower market output and consumer surplus. This holds irrespective of the mode of downstream market competition (quantities or prices) or the nature of commodities (homogeneous or differentiated). It also holds when the fiscal policy of the government is subject to social costs. Our conclusions are in particular relevant to the current pandemic period which spurs heavy subsidization of firms and reformulation of firms' vertical relations.

Keywords: vertical industry; integration; subsidy policy; consumer surplus

JEL Classification: L13, L42, H21

[*]University of Salento, Dipartimento di Scienze dell'Economia, Lecce, 73100 Italy. Tel.: +39 0832 298773; fax: +39 0832 298757; E-mail: michele.giuranno@unisalento.it

[†]University of Salento, Dipartimento di Scienze dell'Economia, Lecce, 73100 Italy. Tel.: +39 0832 29 8772; fax: +39 0832 298757; E-mail: marcella.scrimitore@unisalento.it

[‡]University of Crete, Department of Economics, 74100 Rethymno, Crete, Greece. Tel.:+30 28310 77427; Email: gstamato@uoc.gr

1 Introduction

Vertical integration that fails to increase market power by eliminating competitors or raising entry barriers is unlikely to have adverse consequences for consumers.

This quote by Riordan (2008) seems to be well accepted in industrial economics. By eliminating double marginalization, and absent any factors that reduce market competition, the integration between firms located at different stages of the supply chain has a unequivocal positive effect for consumers in the form of higher output at lower prices. In this paper we provide a natural framework where the above does not hold, namely we show that vertical integration can, surprisingly, *hurt* consumers in a context free of mechanisms that reduce competition in the market, such as foreclosure, secret contracts, etc. Even more surprisingly, this happens when vertical integration interacts with another mechanism that promotes in general consumer welfare, the granting of government subsidies to firms.

Our motivation for analyzing the interplay between vertical integration and firm subsidization, and moreover our focus on consumer welfare, is not only theoretical, but also empirical as these factors are particularly relevant to the current period of the world-wide economic downturn. Indeed, the ongoing Covid-19 pandemic has caused market disruption which surges market strategies on behalf of firms that may eventually harm consumers.[1] This urges the regulatory authorities to promote and enhance appropriate competition policies.[2] Moreover, in response to the economic fallout, governments around the world have adopted measures such as tax reliefs and cash grants to support businesses. Then, a potential concern for regulators is to avoid any conflicts between the policies that aim to mitigate the impact of the crisis on firms, and the objectives of competition policy regarding consumer welfare. This paper contributes by tracing such a conflict. It examines the *interplay* between vertical integration, an activity which is more likely to occur in the current economic environment due to the underlying input shortage (and which is thus likely to be welcomed by competition authorities), and subsidization of firms by the government (which has already occured in various countries). The paper shows that the competitive effects of vertical integration are completely corroded when they interact with the effects of subsidization, leading eventually to a deterioration of consumer welfare.

Essentially, vertical integration in a foreclosure-free environment and subsidization of firms are substitute mechanisms: each one separately enchances efficiency but, as we show in the paper, when one of the two is promoted, the other is displaced. Hence a crowding-out effect is in order. We uncover this effect in a standard environment. In particular, we consider a two-layer oligopolistic industry consisting of a downstream and an upstream market. Each downstream firm deals with its upstream supplier for the provision of an input. The downstream firm pays the supplier a mark up over the marginal cost of the input (if there is no integration) or receives the input at marginal cost (if there is integration).

[1]See, among others, some recent relevant reports published by the OECD, in https://www.oecd.org/competition/competition-policy-responses-to-covid-19.htm. We note further that changes in the vertical relations of firms (in the direction of more intense vertical integration) as antidotes to the crisis the have been proposed by various bodies, like the OECD (OECD 2020) and others.

[2]See, again the aforementioned OECD reports.

Input prices are publicly observed. Non-integrated and integrated downstream firms compete then in the market by producing either homogeneous or differentiated goods, under either quantity or price competition. The industry is subject to a standard government policy: the government taxes/subsidizes the final (downstream) product by choosing the tax/subsidy rate that maximizes the total welfare generated by the two-layer industry.

Given the above context, we examine the competitive effects of vertical integration. We show that an integration of a downstream firm with its upstream supplier (or even multiple integrations by many pairs of such firms) leads, surprisingly, to a reduction in total final output and consumer surplus. The explanation of this result is as follows. First, for any given market structure, namely any given number of integrated and non-integrated firms, the optimal policy of the government consists of a negative tax, namely subsidies are paid to the downstream firms. Let now a non-integrated firm merge with its upstream supplier. This action creates two opposite effects. First, it reduces the distortions in the market, as the double marginalization problem (i.e., the imposition of a price markup by both the downstream firm and its upstream supplier) is alleviated; hence, final output tends to increase. Secondly, integration induces the government to reduce the optimal subsidy (as fewer distortions are now present in the market); hence final output tends to be reduced. It turns out that, if at least two firms compete downstreams, the second effect dominates the first and hence vertical integration eventually hurts consumers.[3] Therefore vertical integration crowds out, as we said, the optimal subsidy to a harmful degree.

The negative effect of the interplay between the two mechanisms survives against many of the fine details of the two-layer industry. In particular, it holds true irrespective of the nature of the downstream market competition, namely Cournot or Bertrand competition, or the nature of the products, namely substitutes or complements. We examine also this interplay when fiscal policy has a social cost that the government takes into account when it makes decisions on taxes/subsidies. The cost, which may reflect monetary or non-monetary costs that influence social welfare, affects the magnitude of the optimal subsidy, but it does not affect the consumer surplus detrimental result provided that it is not too high.

The current paper adds to the literature on the competitive effects of vertical integration. As noted before, the general view is that in the absence of barriers to competition, vertical integration benefits consumers. These barriers include market foreclosure and the raising of the cost of competitors (Salop and Scheffman, 1983; Hart and Tirole, 1990), secret contracts (Nocke and Rey, 2018), integration-driven collusion (Chen and Riordan, 2007; Nocke and White, 2007, Normann, 2009), etc. As our paper does not deal with such factors we won't review the relevant literature. We instead refer the reader to Rey and Tirole (2007) and Riordan (2008) for a description of how these (and other) mechanisms work. Regarding markets without foreclosure, a potential exception to the above general rule, apart from our tax/subsidy framework, is provided by a multiproduct market. Salinger (1991) points out that the merging of a multiproduct monopolist with one of his suppliers reduces the price of the good for which double marginalization is eliminated but also raises the prices of the other product(s) of the monopolist, resulting into an ambiguous net effect for consumers.

The paper is also related to the literature on taxation in two-layer industries. Colangelo and Galmarini (2001) analyzed taxation in vertically related industries, where the downstream firms purchase an intermediate good from upstreams. The paper examined the

[3] Interestingly, if one firm only exists downstreams, the two effects exactly offset one another.

relative efficiency of ad valorem taxation for the cases where the intermediate goods are taxed or not. Asker (2008) analyzed subsidization and taxation in an industry where a downstream monopolist procures inputs from upstream suppliers via a first-price auction. Peitz and Reisinger (2014) analyzed excise and ad valorem taxation in a vertical industry and derived a number of results on issues such as upstream vs. downstream taxation, tax overshifting, taxation under entry, etc.

In addition to the above strands of literature, the current work is linked indirectly to the work of Dinda and Mukherjee (2014) which examines how taxation in an one-layer oligopolistic market may distort the positive impact of more intense competition. Dinda and Mukherjee analyzed a market without any vertical relationships where cost-efficient and inefficient firms are subject to a tax policy. The paper showed that an increase in the number of cost-inefficient firms reduces consumer surplus. The result is driven by the effect this increase has on the policy of the government.

Irrespective of the current crisis, which may call for subsidies and re-formulation of vertical relations in the majority of the production sectors of an economy, the main features of our paper, namely the multi-layer structure, subsidization and vertical integration, fit particularly well with certain important sectors in the economy. Prominent among those are the food and agricultural sectors. These sectors have traditionally been the recipients of production subsidies in either of their layers, and have also witnessed quite intense vertical integration activities (typical examples where such activities take place are the dairy and meat industries).[4] Another important sector that fits our framework is the energy sector. Vertical integration and subsidization coexist, to a lower or larger degree, in the fossil fuel industry, the energy storage industry, and so on (of course, companies in the energy sector use heavily strategies -like spot or futures contracts- that are not analyzed in the current paper). The health industry is another significant sector which fits well with our analysis. This sector is often subsidized; further, vertical integration can be intense also.[5]

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 derives the results under the basic framework. Section 4 extends the analysis to various directions and the last section concludes.

2 Model

We consider a two-layer industry consisting of a downstream and an upstream market. Each downstream firm is dealing with an upstream firm within an exclusive relationship. The upstream firm provides an input which is used in the production of the downstream firm. Inputs are homogeneous across firms and they are produced at zero cost. Moreover a unit of input is transformed into a unit of a production in the downstream market. If a pair of firms in the two markets, i.e., an upstream firm and the associated downstream firm, are non-integrated the latter pays the former a mark up over the (zero) marginal cost of the input. If the two firms are integrated, the downstream firm uses the input for free.

[4] See the report of Feb 11, 2020, by Ernst and Young in https://www.ey.com/consumer-products-retail/how-vertical-integration-is-impacting-food-and-agribusiness.

[5] As Baker et al. (2014) report, vertical integration between hospitals and physicians in the US health market has more than doubled in the past decade.

There are n firms in the downstream market, m of which are non-integrated and $n-m$ are integrated. Denote by i a generic non-integrated firm and by j a generic integrated firm. The production cost for i if it produces q_i units of product and pays w_i per unit of input is $C_i(q_i, w_i) = w_i q_i + q_i^2/2$, whereas the production cost of j is $C_j(q_j) = q_j^2/2$, where q_j is j's production. Namely the cost of production includes a quadratic term, which is identical to all firms.[6] This cost specification allows us to capture the welfare effects of taxation/subsidization under vertical integration, thus avoiding the irrelevance under constant marginal costs.[7]

There is also a government which taxes/subsidizes the final product. In particular each firm in the downstream market is taxed/subsidized by t per unit of production. The government chooses the value of t by maximizing the total welfare generated by the vertical structure.

The interaction among downstream firms, upstream firms and the government has a standard structure:

- at the first stage the government decides upon the optimal tax rate;
- at the second stage the (non-integrated) upstream firms choose input prices;
- at the third stage the downstream firms choose quantities or prices[8]

The choices at each stage become commonly known ex post (this includes, in particular, input prices).

In what follows we solve for the sub-game perfect equilibrium outcome of the above interaction. We begin with a downstream market where the firms produce homogeneous goods (section 3). Then we move on to product differentiation and other extensions of the basic framework (section 4). We note in advance that we won't endogenize the numbers m and $n-m$. The goal of the paper is to analyze the impact of variations in these numbers, given the optimal tax policy.

3 Main results

3.1 Market equilibrium

Consider the downstream market. Let firms $1, 2, \ldots, m$ be the non-integrated firms and $m+1, m+2, \ldots, n$ the integrated ones. In this section we assume that the downstream firms compete in a Cournot fashion and produce homogeneous goods. The inverse demand function in the market is given by $p = a - Q$, where p is the price of the final good and Q is total final output.

Non-integrated firm i chooses q_i to maximize $\pi_i = (p - w_i - t)\, q_i - q_i^2/2$. At the same stage integrated firm j chooses q_j to maximize $\pi_j = (p - t)\, q_j - q_j^2/2$. The quantity choices, which we denote by q_i^i and q_j^*, are presented in Lemma A0 in the Appendix.

[6] Baake et al. 2002, among others, also use a downstream convex cost structure.

[7] Under constant marginal costs of production, taxation/subsidization brings on the first best allocation regardless of whether firms are integrated or not.

[8] Most of the paper deals with downstream quantity competition except from section 4.3 which deals with price competition.

We next move to the second stage. Denote by u_i the supplier of non-integrated firm i, for $i = 1, 2, \ldots, m$, and let $\pi_{u_i} = w_i q_i^*$ be the supplier's objective function. Supplier u_i chooses w_i by maximizing π_{u_i}. It is easy to see that input prices are strategic complements (the best-reply function of supplier u_i is increasing in the input price choices of the other suppliers).

Given symmetry, in equilibrium all suppliers chose the same price, which is given by $w_i^* = 2(a-t)/(3+2n-m)$, $i = 1, 2, \ldots, m$. Notice that as the number of non-integrated firms goes up, the equilibrium input price goes up: this is due to the above said property of strategic complementarity.

Using the equilibrium value of input prices and Lemma A0, we derive the following quantities of non-integrated and integrated firms

$$q_i^* = \frac{(1+n)(a-t)}{(2+n)(3+2n-m)}, \quad q_j^* = \frac{(3+2n)(a-t)}{(2+n)(3+2n-m)}$$

Consider finally the first stage, where the government maximizes social welfare. Denoting consumer surplus and social welfare by CS and W respectively, we have

$$CS = (mq_i^* + (n-m)q_j^*)^2/2 \tag{1}$$

$$W = CS + m\pi_{u_i}^* + m\pi_i^* + (n-m)\pi_j^* + t(mq_i^* + (n-m)q_j^*) \tag{2}$$

Namely, social welfare consists of downstream consumer surplus and profits, upstream profits and government revenue. The value of the tax rate that maximizes welfare is denoted by $t(m)$ and is given in Lemma A1 in Appendix A1. There we show that $t(m) < 0$, so firms are subsidized.

Using the optimal tax rate, Lemma A1 also derives the induced equilibrium input price and quantities of non-integrated and integrated firms, denoted by $w_i(m)$, $q_i(m)$ and $q_j(m)$ respectively. Total downstream equilibrium output is denoted by $Q(m)$, with $Q(m) = mq_i(m) + (n-m)q_j(m)$. The corresponding consumer surplus is denoted by $CS(m)$.

3.2 Impact of vertical integration

We will take as benchmark the case where all downstream firms are non-integrated and we consider a single firm integrating with its upstream supplier,[9] given (9). We examine how this change in market structure, namely the change from $m = n$ to $m = n-1$ non-integrated firms, affects the welfare of consumers.

Proposition 1 *Assume the government implements the optimal subsidy rate $t(m)$. Then $CS(n-1) < CS(n)$ for $n > 1$; and $CS(n-1) = CS(n)$ for $n = 1$, namely vertical integration reduces consumer surplus if at least two firms exist in the downstream market and it leaves consumer surplus unchanged if one firm exists in the downstream market.*

Proof It follows from $Q(n-1) < Q(n)$ for $n > 1$ and $Q(n-1) = Q(n)$ for $n = 1$, which are shown in Appendix A1. ■

[9] The case of multiple integrations is studied in the next section.

Vertical integration has primarily two opposite effects in our model. On the one hand, integration causes an output expansion by the integrated firm as it receives the input at marginal cost. This raises total downstream output and improves consumer surplus. This is the positive effect of integration, which alleviates the double marginalization problem, namely that prices are set above marginal cost in the two layers of the industry.

On the other hand, the higher efficiency achieved on the downstream market induces the government to reduce the optimal subsidy (in absolute terms) given to every downstream firm, namely $0 > t(n-1) > t(n)$. This is the negative effect of vertical integration. For the particular case of a monopoly ($n = 1$), the above two effects exactly cancel out each other and consumer welfare is unchanged. However, the presence of at least two firms in the downstream market, i.e. $n > 1$, weakens the positive effect of integration since there is at least one non-integrated firm still paying an above-marginal cost input price. Hence that effect becomes less important relative to the negative effect of subsidy reduction, leading the latter to prevail and to the reduction of consumer surplus.

The above are in sharp contrast with the impact of integration on consumers when the government does not intervene in the market. In such a case, irrespective of the number of downstream firms, vertical integration alleviates the double marginalization problem without affecting the rules of any government policy. So integration has a univocal positive effect on consumers, as we state in the following Proposition. For clarity all equilibrium variables in the no-taxation case will include superscript "N".

Proposition 2 *Assume the government does not interfere in the market. Then* $CS^N(n-1) > CS^N(n)$ *for all* n*, namely vertical integration raises consumer surplus.*

Proof It suffices to show that $Q^N(n-1) > Q^N(n)$, which can be easily seen to hold. ■

A question that arises is whether a vertical integration of the form analyzed above will take place. Namely is there an incentive for integration? To answer positively we need to show that the profit of the integrated entity, i.e., the entity comprised of (i, u_i), surpasses the sum of the profits of i and u_i when the two are non-integrated. Remark 1 below addresses this issue (for completeness we examine both cases of optimal taxation and of government's abstinence from the market).

Remark 1 *The following hold:*

(i) *If the government implements the optimal subsidy rate* $t(m)$ *then* $\pi_j(n-1) \geq \pi_i(n) + \pi_{u_i}(n)$ *for* $n \geq 4$.

(ii) *If the government does not interfere in the market then* $\pi_j^N(n-1) \geq \pi_i^N(n) + \pi_{u_i}^N(n)$ *for all* n.

Proof Appears in Appendix A1. ■

Integration under the optimal government policy takes place if at least four downstream firms are in the market. To see why, we first note that as the number of firms increases, the impact of subsidy on the profit of a firm becomes less significant. We further observe that by integrating, a downstream firm gets the input at its marginal cost and saves the overcharge by its upstream supplier, but also receives a lower per unit subsidy (recall that integration induces the government to lower the subsidy rate). Hence if the number of firms is sufficiently large, the impact of the (lower) subsidy weighs less, which titles the decision in favor of integration.[10]

Impact of vertical integration and market structure

An important question that arises is how the impact of vertical integration on consumer surplus behaves as a function of the market structure, and in particular the number of downstream firms. Recall that, under the optimal subsidization scheme, vertical integration between a downstream firm and its supplier affects the consumers negatively for all $n \geq 2$ and does not affect them for $n = 1$. To see how the loss in consumer surplus behaves as the number of downstream firms changes, Figure 1 displays the ratio $|CS(n-1) - CS(n)|/CS(n)$ as a function of n.

Observe that the loss is minimum at $n = 1$ (this value of n marks the origin of the horizontal axis): this is known from Proposition 1. We further notice in Figure 1 that the percentage loss is maximum at $n = 2$: this happens as the externality effect starts taking place (recall the explanation developed after Proposition 1). Finally, the percentage loss starts declining monotonically for $n > 2$ (but remains positive for all finite n): as n increases, the market approaches progressively a perfectly competitive market; in such case the optimal tax policy produces the first best market outcome irrespective of the number of integrated and non-integrated firms; in other words, as n increases, the change in the number of integrated firms plays progressively less of a role.

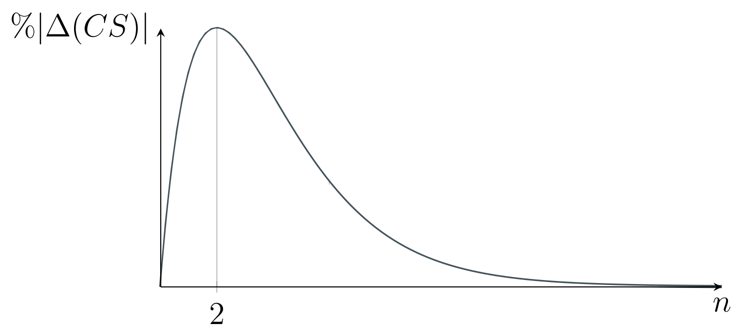

Figure 1: *Change in CS and n*

Apart from the corner monopoly case, vertical integration by a pair of upstream and downsteam firms hurts, percentage wise, the consumers more when there are few non-integrated competitors of the integrated firm. So the competition authorities should be more worried of vertical integration in markets with a small number of firms.[11]

[10] We refer the reader to Caves and Bradburd (1988), Lieberman (1991), and others, for *empirical* analyses of the factors that lead to vertical integration.

[11] This is in contrast with a finding in Loertscher and Reisinger (2014), where it is shown that vertical integration in an industry with partial foreclosure is more harmful for consumers when the integrated entity has many non-integrated competitors.

4 Extensions

In this section we analyze extensions of the basic framework. Since most parts of the section deal with cases needing at least two firms, for example when we have multiple vertical integrations and product differentiation, we set $n \geq 2$ throughout without referring to it again.

4.1 Multiple integrations

The previous results focused on the case where one pair of downstream and upstream firms integrate. In this subsection we show that vertical integration has negative effects on consumers when more than one downstream firms integrate with their suppliers, given the optimal policy of the government.

To see this, let's simply look on total downstream output. Under the optimal government policy when m firms are non-integrated, total output is

$$Q(m) = \frac{a(-m(2+n)+n(3+2n))^2}{\Omega_1},$$

where Ω_1 is defined in the Appendix (Lemma A1).

By simple calculations, $Q(m) < Q(n)$ iff $m^2(2+n)^2 + n(1+n)(3+2n)^2 - m(8+22n+17n^2+4n^3) > 0$. It is easy to show that this inequality holds for all $m \leq n$. Hence total output, and consumer welfare, falls when $n-m$ downstream firms integrate with their respective suppliers, compared to the case where no firms are integrated.

On the other hand, in the absence of any government policy, the total downstream output when m downstream firms integrate with their respective upstream suppliers is given by

$$Q^N(m) = \frac{a(-m(2+n)+n(3+2n))}{(3-m+2n)(2+n)}$$

It is straightforward to show that $Q^N(m) > Q^N(n) \Leftrightarrow 3-m+2n > 0$, which holds.

We summarize as follows.

Remark 2 *Vertical integration of $n-m$ downstream firms with their respective upstream suppliers reduces consumer surplus when the government implements the optimal subsidy rate $t(m)$; and it raises consumer surplus if the government does not interfere in the market.*

The intuition of the above findings follows the lines of the intuition behind Propositions 1 and 2. Vertical integration by multiple pairs of downstream and upstream firms alleviates the double marginalization problem but also reduces the optimal subsidy (when the government is present in the market). The latter effect again dominates the former and hence industry output and consumer surplus fall.

4.2 Differentiated Cournot competition

In this section we assume that downstream firms produce differentiated goods and compete in quantities. We will assume the following downstream inverse demand for firm k, where k is either integrated or not,

$$p_k = a - q_k - \delta Q_{-k}, \tag{3}$$

where p_k is the price of firm k's good, Q_{-k} is the sum of quantities of all downstream firms excluding k, and $\delta \in (0, 1)$ is the degree of product differentiation in the downstream market.

Consider the third stage of the interaction. Non-integrated firm i maximizes the function $\pi_i = (p_i - w_i - t)\, q_i - q_i^2/2$ with respect to q_i; and integrated firm j maximizes the function $\pi_j = (p_j - t)\, q_i - q_j^2/2$ with respect to q_j. The quantity choices appear in Lemma A2 in Appendix A2.

Consider next the second stage. The upstream supplier of non-integrated firm i maximizes the function $\pi_{ui} = w_i q_i$ with respect to w_i, $i = 1, 2, \ldots, m$. We easily obtain $w_i^* = (3 - \delta)(a - t)/\Phi_0$, where $\Phi_0 = 3(2 - \delta) + \delta(2n - m)$. Using the input price, we get

$$q_i^* = \frac{(3 - \delta(2 - n))(a - t)}{(3 - (1 - n)\delta)\Phi_0}, \quad q_j^* = \frac{(6 + \delta(2n - 3))(a - t)}{(3 - (1 - n)\delta)\Phi_0}$$

We finally move to the first stage. The government chooses the optimal tax rate by maximizing social welfare function (2) but with consumer surplus given now by

$$CS = \frac{(1 - \delta)\left(m\left(q_i^*\right)^2 + (n - m)\left(q_j^*\right)^2\right) + \delta\left(mq_i^* + (n - m)\, q_j^*\right)^2}{2} \tag{4}$$

The tax rate that maximizes social welfare is denoted by $t(\delta, m)$ and it appears in Lemma A3 (in Appendix A2). This Lemma also shows that $t(\delta, m) < 0$. Hence the firms in the downstream market are subsidized (irrespective of the degree of differentiation among their products). Using the optimal tax rate, Lemma A4 (in Appendix A2) computes the resulting equilibrium input price and quantities, $w_i(\delta, m)$, $q_i(\delta, m)$ and $q_j(\delta, m)$.

Let $CS(\delta, m)$ denote the corresponding consumer surplus. As in the previous section, we assume that initially all firms are non-integrated and we consider the case where a single integration takes place. We obtain the following result with regards to consumer surplus.

Proposition 3 *Assume the government implements the optimal subsidy rate* $t(\delta, m)$*. Then* $CS(\delta, n - 1) < CS(\delta, n)$*, namely vertical integration reduces consumer surplus.*

Proof Appears in Appendix A2. ■

The logic behind this result is similar to the logic of Proposition 1 (recall the intuition developed after that Proposition, taking into account the case $n \geq 2$).

The next result extends Proposition 2 to include product differentiation.

Proposition 4 *Assume the government does not interfere in the market. Then* $CS^N(\delta, n - 1) > CS^N(\delta, n)$*, namely vertical integration raises consumer surplus.*

Proof Appears in Appendix A2. ■

Again, the logic behind the last result is similar to the logic used to explain Proposition 2.

4.3 Differentiated Bertrand competition

In this section we assume that the downstream firms produce differentiated commodities and compete in prices. To analyze this case, we invert the inverse demand function in (3) and derive the demand function of downstream firm k, where k is either integrated or not,

$$q_k = \frac{a(1-\delta) - (1+(n-2)\delta)p_k + \delta P_{-k}}{1-\delta^2}, \tag{5}$$

where p_k is the price set by firm k and P_{-k} is the sum of the prices set by all downstream firms but k.

We note in advance that, because of their length, all the details about the choices of the firms and of the government, and the corresponding formulas and proofs of results, are not presented here but they are available from the authors upon request.

Noticing that the social welfare is as in (2) and that the consumer surplus is as in (4), the optimal tax rate $t(\delta, m)$ is of negative sign. Given this we again trace the impact of a sole integration between a downstream firm and its upstream counterpart, using as status quo the case where no firms are integrated.

We have the following result.

Proposition 5 *Let δ be sufficiently low and assume the government implements the optimal subsidy rate $t(\delta, m)$. Then $CS(\delta, n-1) - CS(\delta, n) < 0$, namely vertical integration reduces consumer surplus.*

Observe that Proposition 5 does not characterize the impact of integration as it restricts attention to a certain range of values of δ. To comprehend the role of this parameter we must note the following. First recall that under Bertrand downstream competition, the choices (prices) of the downstream firms are strategic complements. As we will explain, strategic complementarity may enhance the positive effect of vertical integration relatively to the negative effect.

When one of the downstream firms integrates vertically with its supplier, it receives the input at marginal cost and hence becomes more aggressive in the market, i.e., it reduces the price of its product. Strategic complementarity implies that the other downstream firms will also reduce their prices. Hence the positive impact of vertical integration, namely the alleviation of the double marginalization problem, is "fed" by the behavior of both the integrated and the non-integrated firms.[12] However, the lower the value of δ, the lower the strategic interdependence among the firms and hence the lower the contribution of the non-integrated firms towards the positive effect of integration. Therefore for low values of δ, the positive effect of integration is surpassed by the negative effect, namely the reduction

[12]This is in contrast with the case of strategic substitutability where the output expansion of the integrated firm induces an output contraction of the rival firms, enhancing the underproduction problem of double marginalization.

of the optimal subsidy, and post-integration downstream output and consumer surplus fall, as in the other frameworks we have examined in the paper.[13]

Finally the impact of integration absent any government policy is described in the following Proposition.

Proposition 6 *Assume the government does not interfere in the market. Then* $CS^N(\delta, n-1) - CS^N(\delta, n) > 0$*, namely vertical integration raises consumer surplus.*

4.4 Social cost of subsidies

Our analysis has assumed so far that subsidies can be financed through non-distortionary taxation, i.e., the cost of one unit of subsidy is simply one. The analysis is now extended to include a social cost of subsidies, which allows for divergences between the social valuation of consumer surplus and profits vs. the government's tax income/subsidy expenses (see for example Neary and Leahy, 2004; Liu et al., 2015 for the use and justification of this approach in strategic market models).

We assume product homegeneity and Cournot competition in the downstream market. Given the optimal quantity and input price choices derived in Section 3.1, we will search for the optimal government's policy at the first stage of the interaction. Assuming that the cost of one unit of subsidy is given by $\lambda > 0$, in the first stage of the interaction the government maximizes the social welfare function

$$W = CS + m\pi^*_{u_i} + m\pi^*_i + (n-m)\,\pi^*_j + \lambda t\left(mq^*_i + (n-m)\,q^*_j\right) \tag{6}$$

where CS is given by (1) and $\lambda t\left(mq^*_i + (n-m)\,q^*_j\right)$ measures the tax/subsidy payment weighted by the social cost of public funds (by setting $\lambda = 1$, we recover the case addressed in Section 3).

Notice that the outcomes of the second and third stages of the interaction are as in subsection 3.1, so we may omit writing them again. The optimal tax rate is denoted by $t(\lambda, m)$ and it is presented in Appendix A3. The sign of the tax rate depends on λ. If λ is below a threshold value, the cost of subsidization is relatively low and the government finds it optimal to subsidize the firms; otherwise, namely if λ is above that threshold, the cost of subsidies is high and the government taxes the firms.

To examine the impact of vertical integration, we again consider the status quo case where all firms are non-integrated and analyze the effect on consumer surplus by one firm getting integrated with its upstream counterpart. To be consistent with our benchmark case of section 3, in what follows we restrict attention to the values of λ consistent with a negative $t(\lambda, m)$ at both $m = n$ and $m = n-1$. For this we need to set $\lambda < \widehat{\lambda}(n)$, where the latter is presented in Appendix A3.

We denote the consumer surpluses before and after integration by $CS(\lambda, n)$ and $CS(\lambda, n-1)$ respectively. We have the following result.

[13] We must note that we don't claim that when the differentiation parameter is high then the consumer surplus detrimental result does not hold. As we said, if δ is high then the positive effect of integration is enhanced; still we do not have a full analytic comparison of the positive and negative effects for such values of δ.

Proposition 7 *Assume the government implements the optimal rate* $t(\lambda, m)$*. There exist* $\lambda_1(n)$ *and* $\lambda_2(n)$ *such that* $CS(\lambda, n-1) < CS(\lambda, n)$ *for* $\lambda \in (\lambda_1(n), \lambda_2(n))$.[14]

Proof Appears in Appendix A3. ■

By Proposition 7, a lower and an upper bound to the values of λ are needed for the consumer surplus detrimental result to hold. We note that the lower bound $\lambda_1(n)$ simply guarantees the positivity of market variables and it is not related to the effects of vertical integration on consumer surplus. Hence Proposition 7 essentially depends on the condition $\lambda < \lambda_2(n)$. The latter points out that the consumer surplus detrimental result is robust to costly subsidization provided that the cost of subsidization is not high.

To explain the above result, we first note that the two opposite effects of vertical integration on consumer surplus, i.e., the negative effect through the subsidy reduction and the positive effect through the alleviation of the double marginalization problem, are still in order. It can be shown that the subsidy reduction is higher, the lower the value of λ. Namely, the inequality $\partial\big(t(\lambda, n-1) - t(\lambda, n)\big)/\partial\lambda > 0$ holds.

Hence the social cost of subsidies, expressed by λ, works as a deflator of the negative impact that vertical integration has on the optimal subsidy rate. Hence, when λ is low, the negative impact of vertical integration is relatively high, thus dominating the positive effect; the opposite occurs for high values of λ.[15]

We note that when the cost of subsidization is high, the consumer detrimental result is reversed: namely vertical integration benefits consumers. This yields some implications of interest for the policy makers. In a subsidized two-layer industry, the policy makers should be worried about the consequences on consumer welfare of a pro-competitive firms' strategy, like vertical integration, if the social cost of subsidization is low. Conversely, a high cost of subsidization allows for the full ripping of the benefits of such a strategy. In this case, the authorities regulating the industry should be passive.

5 Conclusions

Motivated by the current pandemic and the resulting economic downturn, which spurs changes in vertical market structures and induces intense subsidization of firms by the government, in this paper we examined the competitive effects of the interplay between vertical integration and optimal subsidy policies. We showed that the interaction of the two mechanisms produces anti-competitive effects, even though each one alone acts pro-competitively. From a theoretical view point, our results contribute to the literature of vertical markets and regulation, and they may provide a guide for competition authorities when examining cases of integration in subsidized two-layer industries.

Our analysis has utilized a simple framework (linear downstream demand functions, linear input pricing, symmetric firms, etc). Relaxing some of these assumptions, and also introducing other types of government policies, such as ad valorem taxation, will allow -potentially- for further applicability of our results.

[14] The expressions for $\lambda_1(n)$ and $\lambda_2(n)$ are given in Appendix A3.

[15] Recall that we restrict attention to the values of λ that allow for a negative tax rate, i.e, $\lambda < \widehat{\lambda}(n)$. The relation between $\widehat{\lambda}(n)$ and $\lambda_2(n)$ is presented in the proof of Proposition 7.

References

1. Asker, J. (2008) Subsidizing (and taxing) business procurement, Journal of Public Economics 92, 1629-1643.

2. Baake, P., U., Kamecke, H.T. Normann (2002) Vertical integration and market foreclosure with convex downstream costs, Journal of Economics 75, 125-135.

3. Baker L.C., M., Bundorf K., Kessler D.P (2014) Vertical integration: Hospital ownership of physician practices is associated with higher prices and spending, Health Affairs 33, 756-763.

4. Caves R., M. Bradburd (1988) The empirical determinants of vertical integration, Journal of Economic Behavior and Organization 9, 265-279.

5. Chen Y., M. Riordan (2007) Vertical integration, exclusive dealing, and expost cartelization, The Rand Journal of Economics 38, 1-21.

6. Colangelo G., U. Galmarini (2001) Ad valorem taxation on intermediate goods in oligopoly, International Tax and Public Finance 8, 53-73.

7. Dinda S., A. Mukherjee (2014) A note on the adverse effect of competition on consumers, Journal of Public Economic Theory 16, 157-163.

8. Hart O., J. Tirole (1990) Vertical integration and market foreclosure, Brookings Papers on Economic Activity-Microeconomics, 205-286.

9. Lieberman M. (1991) Determinants of vertical integration: An empirical test, Journal of Industrial Economics 39, 451-466.

10. Liu, C.-C., Mukherjee, A., L.F.S. Wang (2015) Horizontal merger under strategic tax policy. Economics Letters,136, 184-186.

11. Loertscher S., M. Reisinger (2014) Market structure and the competitive effects of vertical integration, The Rand Journal of Economics 45, 471-494.

12. Neary, J.P., D. Leahy (2004) Revenue-constrained strategic trade and industrial policy, Economics Letters 82, 409-414.

13. Nocke V., P. Rey (2018) Exclusive dealing and vertical integration in interlocking relationships, Journal of Economic Theory 177, 183-221.

14. Nocke V., L. White (2007) Do vertical mergers facilitate upstream collusion? American Economic Review 97, 1321-1339.

15. Normann H.T. (2009) Vertical integration, raising rivals' costs and upstream collusion, European Economic Review 53, 461-480.

16. OECD (2020) COVID-19 and global value chains: Policy options to build more resilient production networks, Report.

17. Peitz M., M. Reisinger (2014) Indirect taxation in vertical oligopoly, Journal of Industrial Economics 62, 709-755.

18. Rey P., J. Tirole (2007) A primer in foreclosure, Handbook of Industrial Organization, M. Armstrong and R. Porter (eds.), vol. 3, 2145-2220.

19. Riordan M. (2008) Competitive Effects of Vertical Integration, in Handbook of Antitrust Economics, P. Buccirossi (ed.), MIT Press.

20. Salinger M. (1991) Vertical mergers in multi-product industries and Edgeworth's paradox of taxation, Journal of Industrial Economics 39, 546-566.

21. Salop S., D. Scheffman (1983) Raising rivals' costs, American Economic Review 73, 267-271.

Appendix A1

Lemma A0 *Quantity choices in the third stage of the interaction are given by*

$$q_i^* = \frac{2\,(a-t) - w_i\,(1+n) + \sum\limits_{k=1,k\neq i}^{m} w_k}{2\,(2+n)}, \quad i = 1, 2, \ldots, m \tag{7}$$

$$q_j^* = \frac{2\,(a-t) + \sum\limits_{k=1}^{m} w_k}{2\,(2+n)}, \quad j = m+1, m+2, \ldots, n \tag{8}$$

Proof By straightforward computations. ■

Lemma A1

(i) *The formula of* W *in (2) is given by*

$$W = \frac{a^2 g_1 + 2atg_2 - t^2 g_3}{2(2+n)^2(3+2n-m)^2}$$

(ii) *The optimal tax is given by*

$$t(m) = \frac{a(m(2+n)^2 - n(3+2n)^2)}{\Omega_1} < 0, \tag{9}$$

where $\Omega_1 = m^2(2+n)^2 + n(1+n)(3+2n)^2 - m(8+22n+17n^2+4n^3)$.

(iii) *The equilibrium values of input price and quantities are given by*

$$w_i(m) = \frac{2a(2+n)(-2m+3n-mn+2n^2)}{\Omega_2}, \quad i = 1, 2, \ldots, m$$

$$q_i(m) = \frac{a(1+n)\big(n(3+2n) - m(2+n)\big)}{\Omega_1}, \quad i = 1, 2, \ldots, m$$

$$q_j(m) = \frac{a(3+2n)\big(n(3+2n) - m(2+n)\big)}{\Omega_1}, \quad j = m+1, m+2, \ldots, n$$

where $\Omega_2 = -8m + 4m^2 + n(9 - 22m + 4m^2) + n^2(21 - 17m + m^2) + n^3(16 - 4m) + 4n^4$.

(iv) *Optimal welfare is* $W(m) = \frac{a^2\left(m(2+n)-n(3+2n)\right)^2}{2\left(m^2(2+n)^2+n(1+n)(3+2n)^2-m(8+22n+17n^2+4n^3)\right)}$

where $g_1 = m^2(2+n)^2 + n(3+n)(3+2n)^2 - m(16+30n+19n^2+4n^3)$, $g_2 = m(2+n)^2 - n(3+2n)^2$, $g_3 = m^2(2+n)^2 + n(1+n)(3+2n)^2 - m(8+22n+17n^2+4n^3)$.

Proof (i) By straightforward computations.

(ii) The numerator of $t(m)$ is negative as $m \leq n$. The denominator is a decreasing function of m. Hence, it takes the minimum value at $m = n$. It is easy to see that this minimum is positive; thus the denominator is positive and $t(m) < 0$.

(iii) By straightforward computations.

(iv) By straightforward computations. ■

Proof of Proposition 1 We will show that $Q(n-1) < Q(n)$. By straightforward computations, (9) gives us $t(n-1) = \frac{-af_1(n)}{f_2(n)}$ and $t(n) = \frac{-a(5+3n)}{1+n^2}$, where $f_1(n) = 4+9n+9n^2+3n^3$ and $f_2(n) = 12+19n+13n^2+5n^3+n^4$. Then we get

$$Q(n-1) = \frac{a(2+2n+n^2)^2}{f_2(n)}, \quad Q(n) = \frac{an}{1+n}$$

Given the above it is easy to verify that $Q(n-1) < Q(n)$ for $n > 1$ and $Q(n-1) = Q(n)$ for $n = 1$. ■

Proof of Remark 1

(i) We first note that, given optimal policy (9),

$$\pi_i(m) = \frac{3a^2(1+n)^2\big(m(2+n) - n(3+2n)\big)^2}{2\Omega_1^2}$$

$$\pi_j(m) = \frac{3a^2(3+2n)^2\big(m(2+n) - n(3+2n)\big)^2}{2\Omega_1^2}$$

$$\pi_{u_i}(m) = \frac{2a^2(1+n)(2+n)\big(m(2+n) - n(3+2n)\big)^2}{\Omega_1^2}$$

Plugging in the appropriate m each time gives $\pi_j(n-1) = \frac{3a^2(3+2n)^2(2+2n+n^2)^2}{2(12+19n+13n^2+5n^3+n^4)}$ and $\pi_i(n) + \pi_{u_i}(n) = \frac{a^2(11+7n)}{2(1+n)^3}$. By straightforward calculations, $\pi_j(n-1) \geq \pi_i(n) + \pi_{u_i}(n) \Leftrightarrow -738 - 2301n - 3169n^2 - 2480n^3 - 1136n^4 - 240n^5 + 30n^6 + 29n^7 + 5n^8 \geq 0$, which holds if $n \geq 4$.

(ii) Using straightforward computations, $\pi_j^N(n-1) = \frac{3a^2(3+2n^2)}{2(8+6n+n^2)^2}$ and $\pi_i^N(n) + \pi_{u_i}^N(n) = \frac{a^2(11+18n+7n^2)}{2(6+5n+n^2)^2}$. It can be readily verified that the desired inequality holds for all n. ■

Appendix A2: Differentiated Cournot competition

Lemma A2 *Quantity choices at the third stage of the interaction are given by*

$$q_i^* = \frac{(3-\delta)(a-t) - w_i(3-\delta(2-n)) + \delta \sum\limits_{k=1,k\neq i}^{m} w_k}{(3-\delta)(3-(1-n)\delta)}, \quad i = 1,2,\ldots,m$$

$$q_j^* = \frac{(3-\delta)(a-t) + \delta \sum\limits_{i=1}^{m} w_i}{(3-\delta)(3-(1-n)\delta)}, \quad j = m+1, m+2, \ldots, n$$

Proof By straightforward computations. ■

Lemma A3 *The optimal tax rate is given by* $t(\delta,m) = -\frac{a\Phi_1}{\Phi_2} < 0$, *where* $\Phi_1 = 4\delta^2 n^3 + (24\delta - 12\delta^2 + 3\delta^3 m)n^2 + (36 - 36\delta + 9\delta^2 - 7\delta^2 m + 3\delta^3 m)n - 2(3-\delta)\delta m$; *and* $\Phi_2 = 4\delta^3 n^4 + 4\delta^2(8 - 4\delta - \delta m)n^3 + \Phi_{22}n^2 + \Phi_{21}n + \Phi_{20}$, *with* $\Phi_{22} = \delta(84 - 84\delta + 21\delta^2 - 30\delta m + 13\delta^2 m + \delta^2 m)$, $\Phi_{21} = 72 - 108\delta + 54\delta^2 - 9\delta^3 - 72\delta m + 64\delta^2 m - 14\delta^3 m + 6\delta^2 m^2 - 2\delta^3 m^2$, $\Phi_{20} = m(-3+\delta)(18 - 19\delta + 5\delta^2 - 3\delta m + \delta^2 m)$.

Proof By straightforward calculations we derive the formula of $t(\delta,m)$. We can show that Φ_1 is increasing in m. Further for $m = 0$, Φ_1 is positive; hence it is positive for all m. Likewise, we can show that Φ_2 is positive too. Hence the tax rate is negative. ■

Lemma A4 *The equilibrium values of input price and quantities are given by*

$$w_i(\delta,m) = \frac{a\Phi_3(3-\delta)(3+\delta(n-1))}{\Phi_2}, \quad i = 1,2,\ldots,m$$

$$q_i(\delta,m) = \frac{a\Phi_3(3+\delta(n-2))}{\Phi_2}, \quad i = 1,2,\ldots,m$$

$$q_j(\delta,m) = \frac{a\Phi_3(6+\delta(2n-3))}{\Phi_2}, \quad j = m+1, m+2, \ldots, n$$

where $\Phi_3 = n\delta(2n-3) + 6n - (3+\delta(n-1))m$; $\Phi_4 = 4\delta^3 n^4 + 4\delta^2(7 - 4\delta - \delta m)n^3 + \Phi_{42}n^2 + \Phi_{41}n + \Phi_{40}$, $\Phi_{42} = \delta(60 - 72\delta + 21\delta^2 - 27\delta m + 13\delta^2 m + \delta^2 m^2$, $\Phi_{41} = 36 - 72\delta + 45\delta^2 - 9\delta^3 - 54\delta m + 56\delta^2 m - 14\delta^3 m + 6\delta^2 m^2 - 2\delta^3 m^2$, $\Phi_{40} = (-3+\delta)m(9 - 14\delta + 5\delta^2 - 3\delta m + \delta^2 m)$.

Proof By straightforward computations. ■

Proof of Proposition 3 By straightforward computations we have

$$CS(\delta, n-1) - CS(\delta, n) = -\frac{a^2\Phi_5(n-1)(3+\delta(n-1))^2}{2(2+(n-1)\delta)^2\Phi_6^2},$$

where

$$\Phi_5 = \Phi_{51}n^5 + \Phi_{52}n^4 + \Phi_{53}n^3 + \Phi_{54}n^2 + \Phi_{55}n + \Phi_{56}$$

$$\Phi_6 = \Phi_{61}n^4 + \Phi_{62}n^3 + \Phi_{63}n^2 + \Phi_{64}n + \Phi_{65}$$

and $\Phi_{51} = \delta^4(3-\delta), \Phi_{52} = 2\delta^3(13-10\delta+2\delta^2), \Phi_{53} = \delta^2(79-77\delta+26\delta^2-3\delta^3),\ \Phi_{54} = \delta(96-42\delta^2-6\delta^4-46\delta+33\delta^3),\ \Phi_{55} = 36-81\delta^4-316\delta^2+243\delta^3+10\delta^5+132\delta,\ \Phi_{56} = (3-\delta)(9+4\delta^2-11\delta)(2-\delta)^2;\ \Phi_{61} = \delta^3, \Phi_{62} = \delta^2(8-3\delta), \Phi_{63} = \delta(21-8\delta), \Phi_{64} = 18-26\delta^2+6\delta^3+21\delta,\ \Phi_{65} = 2(3-\delta)(9-2\delta(4-\delta))$.

Notice that all terms Φ_{5r} and Φ_{6r} are positive for $\delta \in (0,1)$ proves the result. ■

Proof of Proposition 4 The equilibrium market variables under no subsidies are derived by setting $t=0$ in Lemma A2. Then calculating consumer surplus $CS^N(\delta,m)$, and the difference $CS^N(\delta,n-1) - CS^N(\delta,n)$, gives us

$$CS^N(\delta,n-1) - CS^N(\delta,n) = \frac{a^2\Phi_7}{2(3+(n-1)\delta)^2(6+(n-2)\delta)^2(6+(n-3)\delta)^2},$$

where

$$\Phi_7 = \Phi_{71}n^4 + \Phi_{72}n^3 + \Phi_{73}n^2 + \Phi_{74}n + \Phi_{75}$$

and $\Phi_{71} = \delta^4(13-5\delta), \Phi_{72} = \delta^3(2-\delta)(87-35\delta), \Phi_{73} = 3\delta^2(267-387\delta)+\delta^4(556-88\delta), \Phi_{74} = -3\delta^2(1011-761\delta)-\delta^4(760-94\delta)+1512\delta, \Phi_{75} = 9(2-\delta)^2(27-42\delta+23\delta^2-4\delta^3)$.

Noticing that all terms Φ_{7r} are positive for $\delta \in (0,1)$ proves the result. ■

Appendix A3

Proof of Proposition 7 After calculating the optimal values $w_i(\lambda,m)$, $q_i(\lambda,m)$ and $q_j(\lambda,m)$ and at the optimal subsidy $t(\lambda,m)$ (the derivation of the formulas is skipped as it is straightforward), we compute the difference $CS(\lambda,n-1) - CS(\lambda,n) =$

$$\frac{a^2\lambda^2(2+n)(4(2+n)(n^2+2n+2)(n^3+5n^2+6n+3)\lambda+M_1)(4(3+2n)(n^2+2n+2)\lambda+M_2)}{2(2(n+3)(2+n)\lambda-n^2-8n-11)^2 M_3^2}$$

,

where $M_1 = -44-2n^6-184n^3-24n^5-93n^4-140n-213n^2$, $M_2 = -9n^3-30n^2-39n-22$ and $M_3 = (2(n+4)(2+n)(n^2+2n+2)\lambda-20-31n^2-37n-11n^3-n^4)$.

We then get $CS(\lambda,n-1) - CS(\lambda,n) < 0$ when $\lambda \in \left(\overline{\lambda}(n), \overline{\overline{\lambda}}(n)\right)$, with

$$\overline{\lambda}(n) = \frac{1}{4}\frac{2n^6+24n^5+93n^4+184n^3+213n^2+140n+44}{n^6+9n^5+32n^4+61n^3+68n^2+42n+12}$$

and $\overline{\overline{\lambda}}(n) = \frac{9n^3+22+30n^2+39n}{4(2n^3+7n^2+10n+6)}$.

There is $\lambda_1(n) = \frac{n^2+8n+11}{n^2+9n+12} < 1$, such that for $\lambda > \lambda_1(n)$, downstream and upstream quantities prices are positive. We note that $\lambda_1(n) > \overline{\lambda}(n)$, so we may focus our attention on $\lambda > \lambda_1(n)$. Consistently with the benchmark model in which $t(m)$ is negative, we focus on the values of λ ensuring the negativity of $t(\lambda,m)$ at both $m=n$ and $m=n-1$. For this we need the condition $\lambda < \widehat{\lambda}(n)$, where $\widehat{\lambda}(n) = \frac{n^2+8n+11}{n^2+5n+6}$ and $\lambda_1(n) < 1 < \widehat{\lambda}(n)$. So the feasible range of λ essentially is $(\lambda_1(n), \widehat{\lambda}(n))$.

There are two cases:

- Case I: if $\overline{\overline{\lambda}}(n) > \widehat{\lambda}(n)$, which holds when $n > 22$, vertical integration always hurts consumers.

- Case II: if $\overline{\overline{\lambda}}(n) \leq \widehat{\lambda}(n)$, which holds when $n \leq 22$, vertical integration hurts consumers for $\lambda \in \left(\lambda_1(n), \overline{\overline{\lambda}}(n)\right)$ and benefits consumers for $\lambda \in \left(\overline{\overline{\lambda}}(n), \widehat{\lambda}(n)\right)$.

The proof is completed by setting $\lambda_2(n) = \min\left\{\overline{\overline{\lambda}}(n), \widehat{\lambda}(n)\right\}$. ■