Competitive CSR in a strategic managerial delegation game with a multiproduct corporation

Garcia, Arturo and Leal, Mariel and Lee, Sang-Ho

Universidad de las Americas Puebla, Department of Business Administration, Mexico, Tecnologico de Monterrey, School of Engineering and Sciences, Mexico, Chonnam National University, Department of Economics, South Korea

November 2020

Competitive CSR in a strategic managerial delegation game with a multiproduct corporation

Arturo Garcia[a], Mariel Leal[b], Sang-Ho Lee[c]

[a] *Universidad de las Américas Puebla, Department of Business Administration, Mexico*
[b] *Tecnologico de Monterrey, School of Engineering and Sciences, Mexico*
[c] *Chonnam National University, Department of Economics, South Korea*

Abstract

We study the firm's strategic choice of corporate social responsibility (CSR) in a managerial delegation framework where a multiproduct corporation competes against a single plant firm. We examine simultaneous-move versus sequential-move in output choices when CSR decisions are simultaneous. We show that both firms adopt CSR in a simultaneous-move game, whereas only the follower firm adopts CSR (but not the leader firm) in sequential-move games. We also consider an endogenous timing game in output choices between the two firms and show that a simultaneous-move is an equilibrium when the products are substitutes or weak complements, while a single plant firm's leadership is an equilibrium when the products are sufficiently strong complements. Our findings can explain the widely observed phenomenon, in the real world, of different industries in which firms' CSR activities are more or less (even non-CSR or negative CSR) commonly widespread. It also partially helps us understand CSR's strategic motives and its relations with the firm's profits.

Keyword: corporate social responsibility; managerial delegation; multiproduct corporation; endogenous timing game

JEL Classification: L13; M14; D21

1. Introduction

The increasing emergence of adopting Corporate Social Responsibility (CSR) activities by the oligopolistic firms has been observed worldwide. As it has gained a higher profile on the political, economic, and business agendas in recent years, CSR has received increased attention from academics.[1] It is generally understood in the constitutive approach that CSR is an element of corporate governance as a social norm, based on the social contract. That is, firms have to meet the altruistic needs, and hence corporate governance is at least in part about managerial compliance with legal requirements surrounding CSR. It is then desirable for

Email addresses: arturo.garciam@udlap.mx (Arturo Garcia), mariellealc@tec.mx (Mariel Leal), sangho@jnu.ac.kr (Corresponding author) (Sang-Ho Lee)

[1] In particular, the academic debate on the expansive CSR wave has been raised more frequently in the fields of management, business ethics, law, and economics. For example, Wood (1991) examined corporate law while Carroll (1991, 1999) emphasized corporate governance. For recent research in the literature of economics, see Schreck (2011), Kitzmueller and Shimshack (2012), Crifo and Forget (2015), Hirose et al. (2017), and works cited therein.

firms to create fiduciary relationships with all their stakeholders beyond the shareholders and to satisfy the interests of all stakeholders.

In this stakeholder theory, the stakeholders exercise public pressure concerning the firm's objectives, whereas private owners make the firm's strategic choices such as price, output capacity, product location, advertising, and R&D. In particular, when the stakeholders' participation in governance applies to market decisions, CSR can be interpreted as part of consumer surplus (or consumer-friendliness) and thus, when a firm adopts CSR, the level of social concern is taken as exogenously given, for instance, by the customary toughness of the stakeholders. Accordingly, researchers have increasingly investigated various aspects of altruistic CSR in which the CSR is regarded as consumer-friendly behaviors and its level is an exogenously given variable that was a normative goal established in the social contract.[2]

On the other hand, Baron (2001) made a pioneering contribution to strategic CSR, in which CSR can be used to avoid political pressure from activists who might attempt to organize a boycott. Then, CSR can be an instrument of the firm's strategic variables to maximize their profits. For example, Reinhardt and Stavins (2010) and Manasakis et al. (2014, 2018) showed that CSR might be a business strategy that reflects management's incentive contracts if customers are willing to pay for ethical goods, which may be profitable. Most global firms focus a fair amount of attention on image signaling concerns and thus provide incentives for employee engagement in community service, boosting their public relations with local communities and attracting motivated employees in the local country.

Given that the firms compete for the same customers in the same industry, it can be reasonably assumed that the level of social engagement under the stakeholder's request is identical for all firms in a similar industry. However, it is widely observed that firms adopting CSR may present different levels of engagement in CSR activities; moreover, the simultaneous presence of CSR and non-CSR firms within the same industry occurs. This might be mainly because profits with CSR would be reduced in the case of oligopoly competition where the firms' strategic interaction exists.[3]

In the context of oligopolistic competition, recent works have formulated a delegation framework relevant to the strategic choice of the level of CSR from strategic motivations for adopting CSR behaviors. Plenty of works assumed an asymmetric market organization where CSR adoption can be endogenously chosen, but only some firms adopt CSR among all firms. For example, Leal et al. (2018) and Garcia et al. (2019) showed that the unilateral adoption of CSR might increase the profits of the CSR-initiated firm in the homogeneous product market. Dong and Wang (2019), Bárcena-Ruiz et al. (2020), and **?** also considered a market structure with multiproduct firms and showed that profit maximization could motivate a firm to

[2]For recent works on the normative approach of CSR, see Liu et al. (2015), Lambertini and Tampieri (2015), Fanti and Buccella (2017), Leal et al. (2018, 2019), and Xu and Lee (2019).

[3]Several works showed that CSR firms might earn higher or lower profits under CSR activities, but a higher level of CSR might not be beneficial to society. For more discussions in the different context, see Goering (2012), Kopel and Brand (2012), Brand and Grothe (2013, 2015), Fanti and Buccella (2017, 2019), Kim et al. (2019), and Planer-Friedrich and Sahm (2020) among others.

integrate with other firms and engage in CSR. They conclude that the effect of adopting unilateral CSR is to accelerate product market competition in the vein of the delegation game with managers.

Recent works also extend to a competitive managerial delegation game in which competitive firms choose the strategic level of CSR endogenously. For example, Bian et al. (2016), Hino and Zennyo (2017), and Fanti and Buccella (2017, 2019) examined an endogenous choice of adopting CSR. However, they confined into the symmetric market configuration of oligopoly competition and revealed that all firms adopt a positive CSR level in the market equilibrium. Hirose et al. (2020, 2017) and Lee and Park (2019) investigated the strategic environmental corporate social responsibility (ECSR) of polluting firms and showed that firms will adopt ECSR simultaneously or sequentially to mitigate price competition and earn higher profits. In their analysis, however, the economic role of some critical parameters such as product differentiation, economies of scale, multiproduct firms are limited.[4] As a result, the firms' endogenous heterogeneity regarding the strategic choice of CSR cannot be fully explained. To our knowledge, mainly, studies on the asymmetric equilibrium of strategic CSR in an endogenous choice game with a multiproduct corporation are scarce.

In this paper, we examine the firm's strategic choice of CSR in a competitive managerial delegation framework in which a multiproduct company competes vis-a-vis a single plant one. This asymmetric model formulation follows the work by Dong and Wang (2019) and Garcia et al. (2020a,b) in which product substitutability or complementarity plays a vital role in determining CSR's strategic level. We then examine and compare simultaneous output choices and sequential output choices when CSR decisions are simultaneous. We further consider an endogenous timing game in output choices between the two firms and figure out how the game's timing will affect the firm's strategic choice of asymmetric CSR.

We show that both firms adopt CSR when output decisions are simultaneous, whereas only one firm adopts CSR in sequential moves. It implies that CSR's adoption might bring about a more intensive product market competition, but the relations between the differentiated products and the inter-firm interaction between the multiproduct corporation and its rival with single plant are crucial to determine CSR's strategic level.

First, in a simultaneous move where both firms adopt CSR, an increasing product substitutability leads the multiproduct corporation to choose a higher level of CSR, which is stronger with substitute than with complement products. As a response, single plant firm decreases its CSR level as the substitutability increases. Since the higher weight attached to the consumer's surplus induces an aggressive market behavior of the corporation via output expansion, the rival firm decreases its output production. We also show that the multiproduct corporation might choose negative level of CSR, so called Corporate Social IrResponsibility (CSiR),[5] when the products are sufficiently strong complements. As a result, multiproduct corporation gets

[4]Kumar (1992) and Eckel and Neary (2010) point out that one characteristic of current economies is the presence of multiproduct firms and economies of scale. In the literature of CSR, the heterogeneity of objectives among multiproducts firms has also emerged as an important research topic in recent literature of CSR. See Garcia et al. (2020a,b), Kim et al. (2019), Dong and Wang (2019) and Bárcena-Ruiz et al. (2020) among others.

[5]CSiR is understood as being reactive as opposed to proactive in addressing corporate issues in relation to CSR that often

higher profits than single plant firm but both firm's profits decrease with substitutability.

Second, in a sequential-move game where only one firm adopts CSR, it adopts non-zero CSR when its rival firm takes a leadership. That is, the leading firm does not adopt CSR under its output leadership position because it already has the first-mover advantage, and thus it can strategically commit and choose its output. This implies that the first-mover advantage in the stage of output choice induces no-CSR, and thus the profit-oriented CSR is not effective in the strategic managerial delegation game when the firm has a first-mover advantage in the sequential-move game. This finding is similar to the work by Hino and Zennyo (2017), who examined a homogeneous product duopoly market and showed that a leader does not adopt CSR while a follower adopts a positive CSR when they regard CSR as a concern about social welfare, instead of consumer surplus.[6] However, if both firms consider consumer surplus as CSR in their model, they choose a positive CSR level, but the leader always chooses a higher level of CSR than that of a follower. This implies that the leader produces output more aggressively than that in a simultaneous-move game with homogeneous products between symmetric firms. However, our analysis with a multiproduct firm creates contrasting findings that the leader has no profit-motivated CSR activities in a sequential-move game irrespective of product substitutability.

Third, we show that both firms might decrease their outputs in both simultaneous and sequential games, compared with no delegation case, even with a higher level of CSR. This contrasts with the unilateral case where only one firm adopts CSR, wherein the increase of CSR level always implies an aggressive output production of the firm. However, in a competitive bilateral case where both firms choose the level of CSR competitively, due to the strategic substitutes effect, it will not always increase its output production at equilibrium where the rival's CSR is endogenously taken into consideration.

Finally, we examine an endogenous timing game and show that a simultaneous move is an equilibrium when the products are substitutes or weak complements, whereas a sequential move by the single plant firm's leadership is an equilibrium when the products are sufficiently strong complements. Hence, both firms adopt CSR unless the products are sufficiently strong complements. This implies that modeling with a unilateral approach where only one firm adopts CSR in a simultaneous-move game is problematic when the products are substitutes. We also show that in the equilibrium with a sequential move in an endogenous timing game, the multiproduct corporation will be a follower but might choose a negative CSR level, i.e., CSiR. This also implies that in a unilateral model where only one firm adopts CSR, the strategic relations between CSR and CSiR should be incorporated to understand the strategic motivation of CSR.

Our findings can explain the widely observed phenomenon, in the real world, of different industries in

occurs when the things are perceived as having "gone wrong." For recent conceptional discussions in management literature, see Jones et al. (2009) and Riera and Iborra (2017).

[6]Their findings are closely related to a mixed market configuration in which the welfare-maximizing public firm compete with a profit-maximizing private firm. Then, the private firm chooses a leader position to achieve the highest profits while the public firm chooses a follower position to achieve the highest welfare in an endogenous choice of market role with homogeneous products. See, for example, Ino and Matsumura (2010), Matsumura and Ogawa (2010), and Chen et al. (2019).

which firms' CSR activities are more or less (even non-CSR or negative CSR) commonly widespread. It also partially helps us understand CSR's strategic motives and its relations with the firm's profits.

The remainder of the paper is organized as follows. Section 2 presents the basic duopoly model with multiproduct firms deciding the level of CSR. Section 3 describes the fixed timing game analysis, and Section 4 discusses an endogenous timing game. Finally, Section 5 concludes the paper.

2. Model

We consider two differentiated products markets with a CSR-corporation, A, which has two plants and produces goods in market 1 and 2, denoted by $A1$ and $A2$, respectively, and a single plant consumer-friendly (CF) firm that produces only product 1, denoted by $B1$.

On the demand side, there is a continuum of consumers of the same type. Following Singh and Vives (1982), the representative consumer has a utility function $U(q_1, q_2)$, which is quadratic, strictly concave and symmetric in q_1 and q_2: $U(q_1, q_2) = (q_1 + q_2) - ((q_1^2 + 2\gamma q_1 q_2 + q_2^2))/2$, where $\gamma \in [-1, 1]$, $q_1 = q_{A1} + q_{B1}$ and $q_2 = q_{A2}$ are the quantity of good 1 and 2, respectively, and q_{ki} is the output produced by firm or plant ki, $k = A, B$; $i = 1, 2$. The consumer maximizes $U(q_1, q_2) - p_1 q_1 - p_2 q_2$, where p_i is the price of good i. Then, the inverse demand functions are linear and given by:

$$p_1 = 1 - (q_{A1} + q_{B1}) - \gamma q_{A2}, \quad p_2 = 1 - q_{A2} - \gamma(q_{A1} + q_{B1}); \tag{1}$$

where parameter γ $(-1 \leq \gamma \leq 1)$ measures the degree of product differentiation. The products are regarded as complements if $\gamma < 0$, substitutes if $\gamma > 0$ and independent if $\gamma = 0$.

On the supply side, we assume that firms have identical technologies with decreasing return to scale represented by the following quadratic cost function: $C(q_{ki}) = \frac{q_{ki}^2}{2}$, $k = A, B$; $i = 1, 2$. The profit function of plants of firms are given by:

$$\pi_{Ai} = p_i q_{Ai} - \frac{q_{Ai}^2}{2}, \ \ i = 1, 2 \tag{2}$$

$$\pi_{B1} = p_1 q_{B1} - \frac{q_{B1}^2}{2} \tag{3}$$

and the profit of the multiplant CSR-corporation is:

$$\pi_A = \pi_{A1} + \pi_{A2} \tag{4}$$

We consider a competitive managerial delegation model, in which the owners and managers are separated in both the CSR-corporation and the CF firm. To maximize the joint profits (profits), the owner of the CSR-corporation (CF firm) specifies an incentive contract with the manager. The CSR-corporation's manager is assumed to maximize the joint profits of its two plants plus a fraction (θ_A) of consumer surplus (CS) in production, whereas the CF firm manager maximizes the profits of its plant plus a fraction (θ_B) of consumer surplus (CS) in production. Thus, the objective function of the manager of CSR-corporation (CF firm) is

given by, respectively:

$$V_A = \pi_A + \theta_A CS \tag{5}$$

$$V_B = \pi_{B1} + \theta_B CS \tag{6}$$

where $CS = ((q_{A1} + q_{B1})^2 + 2\gamma(q_{A1} + q_{B1})(q_{A2}) + q_{A2}^2)/2$. Note that parameter $\theta_k \in [-1, 1]$ $k = A, B$. represents the extent to which firm k adopts either CSR ($\theta_k > 0$) or CSiR ($\theta_k < 0$) activities specified in the incentive contract.

Our goal is to study the firms' strategic choice of CSR in a managerial delegation framework in which the firms have the profit-increasing CSR motives in a multiproduct duopoly model. We also examine an endogenous timing game in output choices between the two asymmetric firms and figure out how the timing of the game and the degree of product differentiation will affect the firm's strategic level of CSR.

The setting is a multi-stage game. In the first stage, both firms simultaneously and independently set up the managerial incentive scheme, θ_k, to maximize their profits. In the second stage, firms compete with outputs: the manager of the CSR-corporation chooses the outputs given θ_A prescribed in the first stage to maximize his objective function, whereas the manager of CF firm chooses its output, given θ_B, to maximize its objective function. In this stage of output choice, we have either simultaneous or sequential choice. We will first examine the fixed timing game and then analyze an endogenous timing game.

3. Fixed timing game

3.1. Simultaneous choice in an output stage

In the last stage, we solve equilibrium output decisions of the game. The manager of the CSR-corporation A chooses the outputs q_{A1} and q_{A2} that maximise eqn. (5). CF firm $B1$ chooses the output q_{B1} that maximises eqn. (6), respectively.

According the first-order conditions, the best response functions of the firms are derived as

$$\begin{aligned} q_{A1} &= \frac{1 + \gamma q_{A2}\,(-2 + \theta_A) + q_{B1}\,(-1 + \theta_A)}{3 - \theta_A} \\ q_{A2} &= \frac{1 + \gamma q_{A1}\,(-2 + \theta_A) + \gamma q_{B1}\,(-1 + \theta_A)}{3 - \theta_A} \\ q_{B1} &= \frac{1 + q_{A1}\,(-1 + \theta_B) + \gamma q_{A2}\,(-1 + \theta_B)}{3 - \theta_B} \end{aligned} \tag{7}$$

From (7), it is easy to see that output of plant $A1$ decreases with that of $B1$, whereas the output produced by plant $A2$ decreases (increases) with that of $B1$ if $\gamma > (<)0$. Similarly, the output of $B1$ decreases with that of plant $A1$, whereas it decreases (increases) with that of plant $A2$ if $\gamma > (<)0$. Solving them, we obtain the following Lemma.[7]

[7] N_i is provided in Appendix A

Lemma 1. *Under a simultaneous output competition, we obtain the following equilibrium outputs and profits:*

$$q_{A1} = \frac{6-5\gamma+\gamma^2+\left(-1+\gamma^2\right)\theta_A^2+\left(-3+\gamma+\gamma^2\right)\theta_B+\theta_A\left(1+2\gamma-2\gamma^2-\left(-1+\gamma^2\right)\theta_B\right)}{\Delta_1}, \quad q_{A2} = \frac{8-6\gamma+(-2+3\gamma)\theta_A+(-2+\gamma)\theta_B}{\Delta_1}$$

$$q_{B1} = \frac{(3-2\gamma+(-1+\gamma)\theta_A)(2+\gamma-(1+\gamma)\theta_A+(1+\gamma)\theta_B)}{\Delta_1}$$

$$\pi_A = \frac{N_1}{2\Delta_1^2}, \qquad \pi_{B1} = \frac{N_2}{2\Delta_1^2}$$

where $\Delta_1 \equiv 24 - 11\gamma^2 - 2\left(-1+\gamma^2\right)\theta_A^2 + 3\left(-2+\gamma^2\right)\theta_B - 2\theta_A\left(7-5\gamma^2+\left(-1+\gamma^2\right)\theta_B\right) > 0$.

By Lemma 1, other things being equal, it can be easily checked that q_{A1} increases with θ_A whereas q_{B1} decreases with θ_A, and that q_{B1} increases with θ_B whereas q_{A1} decreases with θ_B. If the level of CSR is given, since the higher weight attached to the consumer's surplus induces an aggressive market behavior of the firm via output expansion, the rival firm decreases its output production. A firm's higher CSR leads to its increasing output but decreasing the rival firm's output in the competitive market 1. Note that total output in market 1 will be increased as θ_A increases; however, the effect of CSR on the monopolistic market 2 depends on the degree of product differentiation. In particular, q_{A2} decreases with θ_A when products are sufficiently strong complements, whereas it increases (decreases) with θ_B when products are complements (substitutes).

In the first stage of the game, firms' owners simultaneously choose θ_A and θ_B to maximize their respective profits which are given by Lemma 1. The equilibrium strategies at the first stage of the game are found as a solution to the following system of two equations: $\frac{\partial \pi_A}{\partial \theta_A} = 0$ and $\frac{\partial \pi_{B1}}{\partial \theta_B} = 0$. However, it is impossible to derive explicit equilibrium values because of mathematical complexity. In the below, in order to have an explicit comparison, we will now use a numerical analysis to show equilibrium outcomes. The results of the calculations for various levels of γ are given in Table 1. In particular, Figure 1 shows that the competitive levels of strategic CSR depends on the degree of product differentiation, which have opposite direction between the two firms.

γ	-1	$-\frac{9}{10}$	$-\frac{4}{5}$	$-\frac{7}{10}$	$-\frac{3}{5}$	$-\frac{1}{2}$	$-\frac{2}{5}$	$-\frac{3}{10}$	$-\frac{1}{5}$	$-\frac{1}{10}$	0	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{7}{10}$	$\frac{4}{5}$	$\frac{9}{10}$	1
θ_A^{Sm}	-0.0876	-0.0414	0.00197	0.0246	0.0403	0.0532	0.0648	0.0755	0.0856	0.0953	0.104	0.113	0.122	0.13	0.138	0.145	0.152	0.158	0.164	0.169	0.172
θ_B^{Sm}	0.725	0.48	0.362	0.297	0.256	0.227	0.206	0.19	0.177	0.167	0.159	0.152	0.147	0.143	0.14	0.137	0.136	0.137	0.139	0.143	0.15
q_{A1}^{Sm}	0.903	0.729	0.614	0.528	0.463	0.411	0.369	0.334	0.305	0.281	0.26	0.243	0.227	0.214	0.203	0.193	0.185	0.179	0.174	0.171	0.17
q_{B1}^{Sm}	0.463	0.413	0.378	0.353	0.334	0.32	0.307	0.297	0.289	0.281	0.275	0.269	0.264	0.26	0.257	0.254	0.251	0.249	0.248	0.248	0.249
q_1^{Sm}	1.37	1.14	0.992	0.881	0.797	0.73	0.676	0.631	0.594	0.562	0.535	0.512	0.492	0.474	0.46	0.447	0.436	0.428	0.422	0.419	0.42
q_{A2}^{Sm}	1.1	0.896	0.761	0.663	0.587	0.526	0.477	0.436	0.401	0.371	0.345	0.322	0.302	0.283	0.266	0.249	0.234	0.219	0.204	0.188	0.17
p_1	0.732	0.665	0.618	0.582	0.555	0.533	0.515	0.5	0.486	0.475	0.465	0.456	0.448	0.441	0.434	0.428	0.423	0.419	0.415	0.412	0.41
p_2	1.27	1.13	1.03	0.954	0.891	0.839	0.793	0.753	0.717	0.685	0.655	0.627	0.6	0.575	0.551	0.527	0.504	0.481	0.458	0.435	0.41
π_{A1}^{Sm}	0.253	0.219	0.191	0.168	0.15	0.135	0.122	0.111	0.102	0.094	0.0871	0.0812	0.076	0.0715	0.0675	0.0641	0.0612	0.0589	0.057	0.0558	0.0553
π_{B1}^{Sm}	0.232	0.189	0.162	0.143	0.13	0.119	0.111	0.104	0.0987	0.094	0.09	0.0865	0.0835	0.0808	0.0785	0.0765	0.0748	0.0733	0.0722	0.0714	0.0711
π_{A2}^{Sm}	0.79	0.612	0.496	0.413	0.351	0.303	0.265	0.233	0.207	0.185	0.166	0.15	0.135	0.123	0.111	0.1	0.0906	0.0815	0.0727	0.0642	0.0553
π_A^{Sm}	1.04	0.831	0.686	0.581	0.501	0.438	0.387	0.344	0.309	0.279	0.254	0.231	0.211	0.194	0.178	0.165	0.152	0.14	0.13	0.12	0.111

Table 1: Equilibrium outcomes in a simultaneous choice

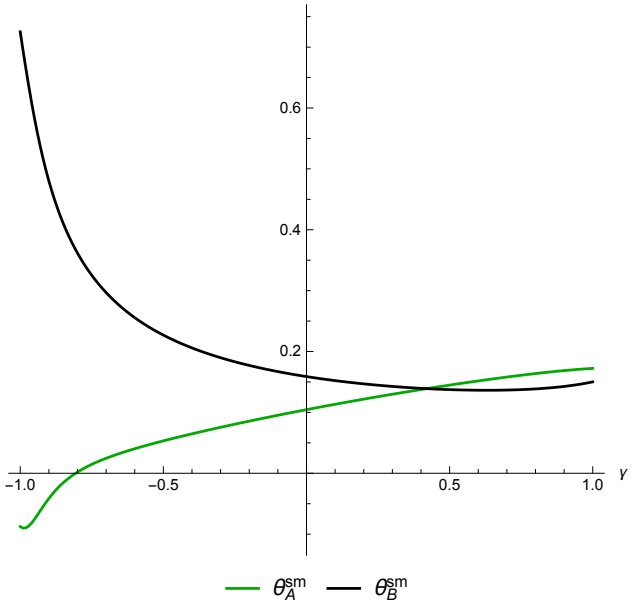


Figure 1: Strategic level of CSR in a simultaneous choice

Proposition 1. *Under a simultaneous output competition in a managerial delegation game, we have that*

(a) θ_A^{sm} *decreases when* $\gamma \leq \gamma_0 \approx -0.987$*, otherwise, it increases.* θ_B^{sm} *is a convex function of* γ*, which first decrease when the products are complements and then increases in the vicinity of* $\gamma = \frac{3}{5}$*. Also, let* $\gamma_1 \in \left(\frac{2}{5}, \frac{1}{2}\right)$ *satisfy* $\theta_A^{sm} = \theta_B^{sm}$*, then* $\theta_A^{sm} \gtreqless \theta_B^{sm}$ *if* $\gamma \gtreqless \gamma_1$*, whereas* θ_A^{sm} *becomes positive in the vicinity of* $\gamma = -\frac{4}{5}$ *and* θ_B^{sm} *is always positive.*

(b) q_{Ai}^{sm}, $i = 1, 2.$*, decreases monotonically with* γ *whereas* q_{B1}^{sm} *is a convex function of* γ*, which first decreases and then increases in the vicinity of* $\gamma = \frac{9}{10}$*. Also,* $q_{A1}^{sm} \leq q_{A2}^{sm}$*, and let* $\gamma_2 \in \left(-\frac{1}{5}, -\frac{1}{10}\right)$ *satisfy* $q_{A1}^{sm} = q_{B1}^{sm}$*, then* $q_{A1}^{sm} \lesseqgtr q_{B1}^{sm}$ *if* $\gamma \gtreqless \gamma_2$*.*

(c) $\pi_A^{sm} > \pi_{B1}^{sm} > 0$ *and each firm's profit decreases with* γ*.*

Proposition 1 (a) states that the strategic levels of CSR between the firms in a simultaneous output choice game are inversely proportional to each other. That is, CSR is a strategic substitute between the firms in the managerial delegation. In particular, increasing product substitutability leads the multiproduct corporation to choose a higher level of CSR, which is stronger with substitute than with complement products. As a response, firm B decreases its CSR level, whereas firm A increases its level as the substitutability increases.

Two extreme cases are noteworthy. First, when the products are sufficiently substitutes, $\theta_A^{sm} > \theta_B^{sm}$, i.e., firm A will choose a higher level of CSR than that of firm B. Note that even though the higher weight is attached to the consumer's surplus, it does not necessarily induce an aggressive market behavior of the firm via output expansion, as shown in Proposition 1 (b), i.e., $q_{A1}^{sm} < q_{B1}^{sm}$ if $\gamma \geq \gamma_2$.

Second, when the products are sufficiently complements, firm A will choose a negative CSR level, whereas firm B chooses a high positive level of CSR. Due to the strong effects of complementarity between the two products, firm A produces a large amount of product 2, but both firms produce a small amount of product 1.

Further, as shown in Proposition 1 (b), both firms monotonically decrease their outputs with $\gamma < \frac{9}{10}$, even with a positive level of θ_k^{sm}. This contrasts with the unilateral case where only one firm adopts CSR, wherein the increase of the level of CSR implies an aggressive output production of the firm. However, in a bilateral case where the firms choose the level of CSR competitively, it will not always increase its output production at equilibrium because the reaction of its rival's CSR is endogenously taken into its strategic consideration. As a result, firm A gets higher profits than firm B, but the substitutability reduces both firms' profits.

Remark 1: In Appendix B.1, we compare with no delegation case where both firms are pure profit-seekers, by setting $\theta_i = 0$ in Lemma 1, and obtain that market i's output is increased under both firms' strategic CSR. Thus, the managerial delegation with the strategic choice of CSR in a simultaneous movement has a competitive effect on the market. Note that when the products are sufficiently strong complements, the multiproduct corporation A adopts negative CSR, which makes that plant $A1$ reduces its production, but still increases market output, compared to the no delegation case. Further, regarding profits it can be shown that when the products are sufficiently strong complements, firm A's joint-profits and firm B's profits increase under both firms' strategic CSR. Otherwise, their profits are reduced regardless of product differentiation. In that case, no adoption of CSR is better off for both firms. Therefore, a prisoner's dilemma exists between the two firms, especially when the products are substitutes.

3.2. Sequential choice with CSR-corporation leadership in an output stage

We address the Stackelberg situation, in which the CSR-corporation A, plays the leading position whereas the CF firm responds. Using backward induction, the CF firm's manager chooses q_{B1} that maximizes eqn. (6), that is, according to $\frac{\partial V_B}{\partial q_{B1}} = 0$, we have:

$$q_{B1} = \frac{1 + q_{A1}\left(-1 + \theta_B\right) + \gamma q_{A2}\left(-1 + \theta_B\right)}{3 - \theta_B}$$

By inserting $q_{B1}(q_{A1}, q_{A2})$ into the CSR-corporation manager's objective function and solving $\frac{\partial V_A}{\partial q_{A1}} = 0$ and $\frac{\partial V_A}{\partial q_{A2}} = 0$ simultaneously, we get the following Lemma:

Lemma 2. *Under a sequential output competition with CSR-corporation leadership, we obtain the following equilibrium outputs and profits:*

$$q_{A1} = \frac{2\left(-1+\gamma^2\right)\theta_A^2+(3-\theta_B)\left(6-4\gamma-\left(3-2\gamma^2\right)\theta_B\right)+\theta_A\left(2(2-\gamma)\gamma+5\left(1-\gamma^2\right)\theta_B+\left(-1+\gamma^2\right)\theta_B^2\right)}{\Delta_2}, \quad q_{A2} = \frac{(3-\theta_B)(7-5\gamma-\theta_B)-(4-6\gamma)\theta_A}{\Delta_2}$$

$$q_{B1} = \frac{15-3\gamma-5\gamma^2+2\left(1-\gamma^2\right)\theta_A^2+\left(6+4\gamma-5\gamma^2\right)\theta_B-\left(3+\gamma-2\gamma^2\right)\theta_B^2+\theta_A\left(-11+7\gamma^2+2\left(-1+\gamma^2\right)\theta_B+\left(1-\gamma^2\right)\theta_B^2\right)}{\Delta_2}$$

$$\pi_A = \frac{N_3}{2\Delta_2^2}, \qquad \pi_{B1} = \frac{N_4}{2\Delta_2^2}$$

where $\Delta_2 \equiv 4\left(1-\gamma^2\right)\theta_A^2 + (3-\theta_B)\left(21 - 10\gamma^2 + \left(-3+2\gamma^2\right)\theta_B\right) + \theta_A\left(-33 + 25\gamma^2 + 10\left(1-\gamma^2\right)\theta_B + \left(-1+\gamma^2\right)\theta_B^2\right) > 0$.

By Lemma 2, other things being equal, it can be easily checked that q_{A1} increases with θ_A whereas q_{B1} decreases with θ_A. In addition, q_{A2} decreases with θ_A if products are sufficiently strong complements,

otherwise q_{A2} increases with θ_A, as expected. However, the effect of θ_B on the equilibrium output depends on the degree of product differentiation. In particular, q_{B1} increases with θ_B whereas q_{A1} decreases with θ_B unless both θ_A and θ_B are high enough. Finally, q_{A2} increases with θ_B: (i) for small θ_A when $\gamma < 0$ or (ii) when $\gamma > 0$ and both θ_A and θ_B are high enough. Otherwise, q_{A2} decreases with θ_B.

In the first stage of the game, firms' owners simultaneously choose θ_A and θ_B to maximize their respective profits which are given by Lemma 2. The equilibrium strategies at the first stage of the game are found as a solution to the following system of two equations: $\frac{\partial \pi_A}{\partial \theta_A} = 0$ and $\frac{\partial \pi_{B1}}{\partial \theta_B} = 0$.

We will use a numerical analysis to show equilibrium outcomes. The results of the calculations for various levels of γ are given in Table 2. In particular, Figure 2 shows that the leader firm, CSR-corporation, chooses non-CSR whereas the CF firm's CSR depends on the degree of product differentiation.

γ	-1	$-\frac{9}{10}$	$-\frac{4}{5}$	$-\frac{7}{10}$	$-\frac{3}{5}$	$-\frac{1}{2}$	$-\frac{2}{5}$	$-\frac{3}{10}$	$-\frac{1}{5}$	$-\frac{1}{10}$	0	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{7}{10}$	$\frac{4}{5}$	$\frac{9}{10}$	1
θ_A^{AL}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
θ_B^{AL}	0.341	0.427	0.4	0.366	0.337	0.314	0.296	0.281	0.269	0.26	0.253	0.247	0.243	0.24	0.238	0.238	0.24	0.244	0.25	0.261	0.278
q_{A1}^{AL}	0.906	0.732	0.612	0.525	0.459	0.407	0.366	0.332	0.304	0.28	0.259	0.241	0.226	0.212	0.2	0.19	0.181	0.173	0.167	0.163	0.161
q_{B1}^{AL}	0.423	0.406	0.384	0.364	0.348	0.335	0.323	0.314	0.306	0.299	0.294	0.289	0.285	0.281	0.279	0.277	0.276	0.275	0.276	0.278	0.282
q_1^{AL}	1.33	1.14	0.996	0.889	0.807	0.742	0.689	0.646	0.609	0.579	0.553	0.53	0.511	0.494	0.479	0.467	0.457	0.449	0.443	0.441	0.443
q_{A2}^{AL}	1.09	0.899	0.762	0.66	0.581	0.519	0.468	0.426	0.39	0.36	0.333	0.31	0.289	0.271	0.254	0.238	0.223	0.209	0.194	0.179	0.161
p_1	0.765	0.672	0.614	0.573	0.542	0.517	0.498	0.482	0.469	0.457	0.447	0.439	0.432	0.425	0.419	0.414	0.409	0.405	0.402	0.399	0.397
p_2	1.23	1.12	1.03	0.963	0.903	0.852	0.808	0.768	0.732	0.698	0.667	0.637	0.608	0.581	0.554	0.528	0.503	0.477	0.452	0.425	0.397
π_{A1}^{AL}	0.283	0.224	0.188	0.163	0.143	0.128	0.115	0.105	0.0962	0.0888	0.0824	0.0768	0.072	0.0677	0.0639	0.0606	0.0578	0.0553	0.0532	0.0516	0.0508
π_{B1}^{AL}	0.234	0.19	0.162	0.142	0.128	0.117	0.109	0.102	0.0966	0.0921	0.0883	0.0851	0.0823	0.08	0.078	0.0763	0.0749	0.0737	0.0728	0.0722	0.0721
π_{A2}^{AL}	0.753	0.607	0.498	0.418	0.356	0.308	0.268	0.236	0.209	0.186	0.167	0.149	0.134	0.121	0.109	0.0975	0.0873	0.0778	0.0688	0.0599	0.0508
π_A^{AL}	1.04	0.831	0.686	0.58	0.499	0.435	0.384	0.341	0.306	0.275	0.249	0.226	0.206	0.188	0.172	0.158	0.145	0.133	0.122	0.112	0.102

Table 2: Equilibrium outcomes in a sequential choice with CSR-corporation leadership

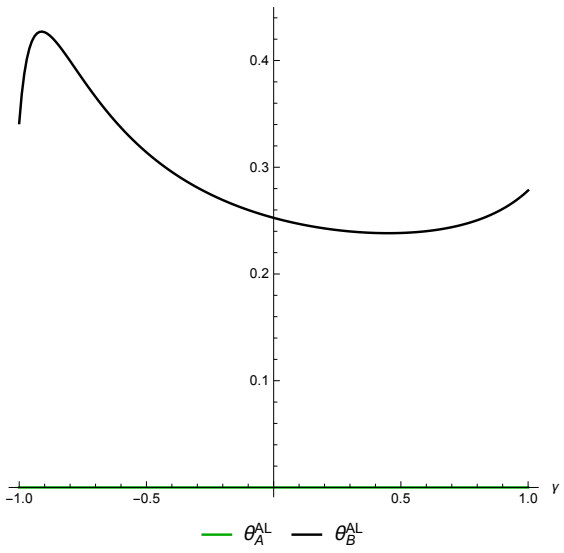


Figure 2: Strategic level of CSR in a sequential choice with CSR-corporation leadership

Proposition 2. *Under a Stackelberg output competition with multiproduct corporation leadership in a managerial delegation game, we have that*

(a) $\theta_A^{AL} = 0$ *and* $\theta_B^{AL} > 0$. *Also,* θ_B^{AL} *is non-monotone in* γ *when the products are complements, i.e.,* θ_B^{AL} *first increases and then decreases in the vicinity of* $\gamma = -\frac{9}{10}$. *When the products are substitutes, however,* θ_B^{AL} *is a convex function of* γ, *which first decreases and then increases in the vicinity of* $\gamma = \frac{2}{5}$.

(b) q_{Ai}^{AL}, $i = 1, 2.$, *decreases monotonically with* γ *whereas* q_{B1}^{AL} *is a convex function of* γ, *which first decreases and then increases in the vicinity of* $\gamma = \frac{7}{10}$. *Also,* $q_{A1}^{AL} \leq q_{A2}^{AL}$, *and let* $\gamma_3 \in \left(-\frac{3}{10}, -\frac{1}{5}\right)$ *satisfy* $q_{A1}^{AL} = q_{B1}^{AL}$, *then* $q_{A1}^{AL} \lessgtr q_{B1}^{AL}$ *if* $\gamma \gtrless \gamma_3$.

(c) $\pi_A^{AL} > \pi_{B1}^{AL} > 0$ *and each firm's profit decreases with* γ.

Proposition 2 (a) states that when the multiproduct corporation, firm A, is a leader in the output choice stage, it will not adopt CSR, whereas the follower, firm B, chooses positive CSR in a sequential output choice game. This is because the leader firm can always commit its optimal output in advance before the follower chooses, and thus the first-mover advantage in the stage of output choice induces no-CSR when the owner maximizes its profit only. The profit-oriented CSR is not effective in the managerial delegation game when the firm has a first-mover advantage in the sequential-move game.[8]

It also shows that the follower firm B's strategic level of CSR is non-monotone and depends on the degree of product differentiation. In particular, firm B increases CSR level either when the products are sufficiently complements or substitutes, whereas it decreases otherwise. When the products are sufficiently substitutes, firm B increases CSR level, which induces a larger output than that of the rival firm, as shown in Proposition 2 (b). However, when the products are sufficiently complements, even though firm B increases its level of CSR with the substitutability, the strong effects of complementarity between the two products induce both firms to produce a small amount of product 1, in which $q_{A1}^{AL} > q_{B1}^{AL}$. This contrasts with the unilateral case where only one firm adopts exogenously given CSR wherein the increase of the level of CSR implies an aggressive output production of the firm. However, Proposition 2 (b) shows that when the unilateral case is endogenously chosen in a sequential output choice game, the adoption of higher CSR does not always support an aggressive production.

Finally, both firms monotonically decrease their outputs with $\gamma < \frac{7}{10}$ and both firms' profits decrease with substitutability.

Remark 2: In Appendix B.2, we compare with no delegation case where both firms are pure profit-seekers, by setting $\theta_i = 0$ in Lemma 2, and obtain that market 1's output increases, whereas market 2's output increases (decreases) when products are complements (substitutes) under firm A's leadership where only the

[8] As related work, Hirose et al. (2020) investigated Stackelberg's price competition where the firms chose ESCR and showed that only the follower adopts ECSR, but it increases the leaders' profits under the leader's first-mover advantage. Further, Lee and Park (2019) considered the sequential ECSR choice game under the simultaneous price competition and showed that the follower adopts lower ECSR that increases both firms' profits.

firm B adopts strategic CSR. Then, the firm A will reduce output in a competitive market with the firm B, which will lead to the increasing (decreasing) profits of the firm B (plant $A1$), compared to the no delegation case. However, firm A's joint-profits increase when products are sufficiently strong complements; otherwise, they will shrink. Therefore, firm B can utilize CSR's strategic degree to increase its profit in the competitive CSR choice game.

3.3. Sequential choice with CF firm leadership in an output stage

We finally address another situation, in which the CF firm B, acts as the leader whereas the CSR-corporation responds. Similarly, solving $\frac{\partial V_A}{\partial q_{A1}} = 0$ and $\frac{\partial V_A}{\partial q_{A2}} = 0$ simultaneously, we have:

$$q_{A1}(q_{B1}) = \frac{3-2\gamma+(-1+\gamma)\theta_A+q_{B1}(-1+\theta_A)\left(3-2\gamma^2+\left(-1+\gamma^2\right)\theta_A\right)}{9-4\gamma^2+(-6+4\gamma^2)\theta_A-(-1+\gamma^2)\theta_A^2}, \quad q_{A2}(q_{B1}) = \frac{3-2\gamma+\gamma q_{B1}(-1+\theta_A)+(-1+\gamma)\theta_A}{9-4\gamma^2+(-6+4\gamma^2)\theta_A-(-1+\gamma^2)\theta_A^2}$$

By inserting both $q_{A1}(q_{B1})$ and $q_{A2}(q_{B1})$ into the CF firm manager's objective function and using $\frac{\partial V_B}{\partial q_{B1}} = 0$, we get the following Lemma:

Lemma 3. *Under a sequential output competition with CF firm leadership, we obtain the following equilibrium outputs and profits:*

$$\begin{aligned}
q_{A1} &= \frac{N_5}{\Delta_3}, \qquad q_{A2} = \frac{N_6}{\Delta_3}, \\
q_{B1} &= \frac{(3-2\gamma+(-1+\gamma)\theta_A)^2\left(-\left(5+8\gamma+3\gamma^2\right)\theta_A+(1+\gamma)^2\theta_A^2+(2+\gamma)(3+2\gamma+(1+\gamma)\theta_B)\right)}{\Delta_3} \\
\pi_A &= \frac{N_7}{2\Delta_3^2}, \qquad \pi_{B1} = \frac{N_8}{2\Delta_3^2}
\end{aligned}$$

where $\Delta_3 \equiv 189 - 174\gamma^2 + 40\gamma^4 - 4\left(4 - 7\gamma^2 + 3\gamma^4\right)\theta_A^3 + \left(1-\gamma^2\right)^2\theta_A^4 + \left(-36 + 35\gamma^2 - 8\gamma^4\right)\theta_B + \theta_A^2\big(90 - 132\gamma^2 + 46\gamma^4 + \left(-4 + 7\gamma^2 - 3\gamma^4\right)\theta_B\big) + 2\theta_A\left(-4\left(27 - 32\gamma^2 + 9\gamma^4\right) + \left(12 - 17\gamma^2 + 5\gamma^4\right)\theta_B\right) > 0$.

By Lemma 3, other things being equal, it can be easily checked that q_{A1} increases with θ_A and q_{B1} decreases with θ_A unless both θ_A and θ_B are high enough. Further, as usual, it also can be checked that q_{B1} increases with θ_B whereas q_{A1} decreases with θ_B. Regarding the effect of CSR on the monopolistic market 2, we have that q_{A2} decreases with θ_A if products are sufficiently strong complements, whereas it increases (decreases) with θ_B when products are complements (substitutes), as expected.

In the first stage of the game, firms' owners simultaneously choose θ_A and θ_B to maximize their respective profits which are given by Lemma 3. The equilibrium strategies at the first stage of the game are found as a solution to the following system of two equations: $\frac{\partial \pi_A}{\partial \theta_A} = 0$ and $\frac{\partial \pi_{B1}}{\partial \theta_B} = 0$.

We will use a numerical analysis to show equilibrium outcomes. The results of the calculations for various levels of γ are given in Table 3. In particular, Figure 3 shows that the leader firm, CF firm chooses non-CSR whereas the CSR-corporation's CSR depends on the degree of product differentiation.

γ	-1	$-\frac{9}{10}$	$-\frac{4}{5}$	$-\frac{7}{10}$	$-\frac{3}{5}$	$-\frac{1}{2}$	$-\frac{2}{5}$	$-\frac{3}{10}$	$-\frac{1}{5}$	$-\frac{1}{10}$	0	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{7}{10}$	$\frac{4}{5}$	$\frac{9}{10}$	1
θ_A^{BL}	-0.8	-0.169	0.00608	0.0644	0.0937	0.113	0.129	0.143	0.156	0.168	0.179	0.19	0.2	0.21	0.22	0.23	0.24	0.249	0.259	0.27	0.281
θ_B^{BL}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
q_{A1}^{BL}	0.857	0.71	0.614	0.537	0.474	0.424	0.383	0.349	0.321	0.297	0.277	0.259	0.244	0.231	0.22	0.21	0.202	0.196	0.191	0.188	0.188
q_{B1}^{BL}	0.524	0.424	0.378	0.35	0.33	0.314	0.301	0.291	0.282	0.274	0.267	0.261	0.256	0.251	0.247	0.243	0.24	0.238	0.235	0.234	0.234
q_1^{BL}	1.38	1.13	0.992	0.887	0.804	0.738	0.685	0.64	0.603	0.571	0.544	0.52	0.5	0.482	0.467	0.454	0.442	0.433	0.426	0.422	0.421
q_{A2}^{BL}	1.14	0.893	0.762	0.666	0.592	0.533	0.485	0.444	0.41	0.38	0.354	0.332	0.311	0.293	0.276	0.26	0.245	0.231	0.217	0.203	0.188
p_1	0.762	0.671	0.617	0.58	0.551	0.528	0.509	0.493	0.479	0.467	0.456	0.447	0.438	0.43	0.423	0.416	0.41	0.405	0.4	0.395	0.391
p_2	1.24	1.13	1.03	0.954	0.89	0.836	0.789	0.748	0.711	0.677	0.646	0.616	0.589	0.563	0.538	0.513	0.489	0.466	0.442	0.417	0.391
π_{A1}^{BL}	0.286	0.224	0.191	0.167	0.149	0.134	0.122	0.111	0.102	0.0946	0.088	0.0822	0.0771	0.0727	0.0688	0.0655	0.0626	0.0601	0.0582	0.0567	0.0558
π_{B1}^{BL}	0.262	0.194	0.162	0.142	0.127	0.117	0.108	0.101	0.0953	0.0904	0.0862	0.0825	0.0793	0.0765	0.074	0.0717	0.0697	0.068	0.0665	0.0652	0.0641
π_{A2}^{BL}	0.762	0.607	0.496	0.414	0.352	0.304	0.265	0.234	0.207	0.185	0.166	0.149	0.135	0.122	0.11	0.0996	0.0899	0.0809	0.0723	0.0641	0.0558
π_A^{BL}	1.05	0.832	0.686	0.581	0.501	0.438	0.387	0.345	0.31	0.28	0.254	0.232	0.212	0.195	0.179	0.165	0.152	0.141	0.13	0.121	0.112

Table 3: Equilibrium outcomes in a sequential choice with CF firm leadership

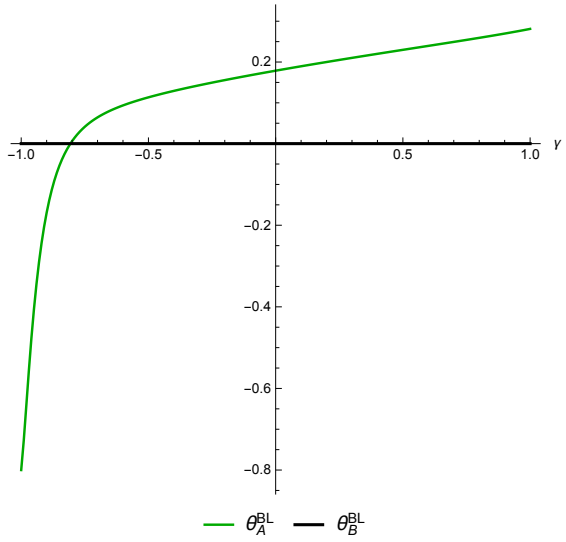


Figure 3: Strategic level of CSR in a sequential choice with CF firm leadership

Proposition 3. *Under a Stackelberg output competition with single plant firm leadership in a managerial delegation game, we have that*

(a) $\theta_B^{BL} = 0$ *and* θ_A^{BL} *becomes positive in the vicinity of* $\gamma = -\frac{4}{5}$. *Also,* θ_A^{BL} *increases with* γ.

(b) q_{ki}^{BL}, $k = A, B$; $i = 1, 2$, *decreases monotonically with* γ. *Also,* $q_{A1}^{BL} \leq q_{A2}^{BL}$, *and let* $\gamma_4 \in \left(0, \frac{1}{10}\right)$ *satisfy* $q_{A1}^{BL} = q_{B1}^{BL}$, *then* $q_{A1}^{BL} \lesseqgtr q_{B1}^{BL}$ *if* $\gamma \lesseqgtr \gamma_4$

(c) $\pi_A^{BL} > \pi_{B1}^{BL} > 0$ *and each firm's profit decreases with* γ.

Proposition 3 (a) states that when the single plant firm B is a leader in the output choice stage, it will not adopt CSR whereas the follower, multiproduct corporation A, chooses non-zero (even negative) CSR in a sequential output choice game. The reason is the same in the case with firm A's leadership, where the leader firm can commit its optimal output in advance before the follower chooses. It also shows that the

follower firm A's strategic level of CSR is monotone and increases with the degree of product differentiation. In particular, firm A will choose a positive level of CSR unless the products are sufficiently complements. Otherwise, firm A will choose a negative level of CSR. Thus, when the products are sufficiently complements, firm A adopts a negative level of CSR, i.e., $\theta_B^{BL} = 0 > \theta_A^{BL}$ and thus its output is higher than that of firm B, but they can raise market prices, as shown in Proposition 3 (b). However, when the products are not sufficiently complements, firm A adopts a positive level of CSR but its output is not always larger than that of firm B, as shown in Proposition 3 (b), i.e., $q_{A1}^{BL} < q_{B1}^{BL}$ when $\gamma \geq \gamma_4$. This also contrasts with the unilateral case where only one firm adopts exogenously given CSR wherein the increase of the level of CSR implies an aggressive output production of the firm. Finally, not only firms' outputs but both firms' profits monotonically decrease with γ.

Remark 3: In Appendix B.3, we compare with no delegation case where both firms are pure profit-seekers, by setting $\theta_i = 0$ in Lemma 3, and obtain that market i's output increases under firm B's leadership where only the firm A adopts strategic CSR. Note that when the products of firm A are sufficiently strong complements, plant $A1$ shrinks its outputs, and firm B increases its output, compared to the no delegation case, which yields that firm B profits increase. Otherwise, firm B decreases its production and its profits as well. However, as a follower, firm A's joint-profits always increases irrespective of product differentiation. Therefore, firm A can utilize the strategic degree of CSR to increase its profit in the competitive CSR choice game.

4. Endogenous timing game

We examine the modified format of endogenous timing game where both firms choose its timing to move between "early" ($t = 1$) and "late" ($t = 2$) in determining output decisions.[9] If both firms choose the same period, the equilibrium is a simultaneous-move game. Otherwise, the equilibrium is a sequential-move game. Table 4 provides the payoff matrix of the observable delay game.

Firm A/B	$t_B = 1$	$t_B = 2$
$t_A = 1$	$(\pi_A^{sm}, \pi_{B1}^{sm})$	$(\pi_A^{AL}, \pi_{B1}^{AL})$
$t_A = 2$	$(\pi_A^{BL}, \pi_{B1}^{BL})$	$(\pi_A^{sm}, \pi_{B1}^{sm})$

Table 4: Payoff Matrix of the Observable Delay Game in a mixed market

In order to find the equilibrium of the endogenous timing output game, we will compare the firms' profits under each scenario, which give us the following results:

Lemma 4. *Regarding multiproduct corporation profits for any* $\gamma \in [-1, 1]$, $\pi_A^{BL} > \pi_A^{sm} > \pi_A^{AL}$.

[9] Regarding model descriptions on the endogenous timing game, see Hamilton and Slutsky (1990) and Garcia et al. (2019).

Lemma 5. *Regarding single plant firm profits, (i)* $\pi_{B1}^{BL} > \pi_{B1}^{AL} > \pi_{B1}^{sm}$ *if* $\gamma < -\frac{4}{5}$*; (ii)* $\pi_{B1}^{BL} < \pi_{B1}^{AL} < \pi_{B1}^{sm}$ *if* $\gamma \in [-\frac{4}{5}, \frac{3}{5})$*; and (iii)* $\pi_{B1}^{BL} < \pi_{B1}^{sm} < \pi_{B1}^{AL}$ *if* $\gamma \in [\frac{3}{5}, 1]$.

Using these lemmas, we have the following result:

Proposition 4. *The equilibrium of an endogenous timing game is as follows:*

(a) *If* $\gamma \in [-1, -\frac{4}{5})$*, the only equilibrium is the sequential-move outcome,* $(t_A, t_B)=(2, 1)$*, in which the single plant firm B firm acts as the leader. Furthermore,* $\theta_A^{BL} < 0$ *and* $\theta_B^{BL} = 0$.

(b) *If* $\gamma \in [-\frac{4}{5}, 1]$*, the only equilibrium of the game is the simultaneous move,* $(t_A, t_B)=(2, 2)$*. Furthermore, the equilibrium CSR is positive.*

Proposition 4 states that a simultaneous move is an equilibrium when the products are substitutes or weak complements, whereas a sequential move by the single plant firm's leadership is an equilibrium when the products are sufficiently strong complements. In a simultaneous move equilibrium, both firms choose positive CSR, but their profits are reduced, compared with no-CSR case by both firms. However, in a sequential move equilibrium, the leading firm B does not adopt CSR, whereas the multiproduct firm chooses negative CSR, i.e., CSiR. In that case, both firms can increase profits, compared with a no-CSR case. Therefore, the degree of product differentiation plays an important role in determining the equilibrium of the endogenous timing game and the strategic level of CSR or CSiR.

5. Concluding Remarks

In this study, we investigated firm's strategic choice of CSR in a competitive managerial delegation framework where a multiproduct corporation competes against a single plant firm. We showed that both firms adopt CSR in a simultaneous move in output choices, whereas only the follower firm adopts CSR, but the leader firm does not adopt CSR in a sequential move. It implies that the adoption of CSR brings about a more intensive product market competition, but the relations between the product differentiation and the inter-firm interaction in the multiproduct corporation are crucial to determining the strategic level of CSR. We also showed that when the multiproduct corporation adopts CSR, it might choose a negative level of CSR when the products are sufficiently strong complements. In both simultaneous and sequential games in which the firms choose the level of CSR competitively, however, it will not always increase its output production at equilibrium where the rival's CSR is endogenously taken into consideration.

We further examined an endogenous timing game and showed that CSR's strategic level critically depends on the degree of product differentiation. We showed that a simultaneous move is an equilibrium when the products are substitutes or weak complements, whereas a sequential move by a single plant firm's leadership is an equilibrium when the products are sufficiently strong complements. Hence, both firms adopt CSR unless the products are sufficiently strong complements. Further, in the equilibrium with sequential move in an endogenous timing game, the multiproduct corporation chooses a negative level of CSR.

Our work can help shed light on the rationale for the emergence of various equilibria in different industries, regarding firms' strategic moves in CSR behaviors, and may be of great interest to the current policy debate on firms' CSR. Also, detailed information about the degree of product differentiation and the firm's endogenous choice of the movement should be taken into account if the government intends to evaluate the welfare effect of firm's consumer-friendly activities.

Finally, our analysis has limitations in the duopoly modeling with linear demand and symmetric quadratic cost functions. Further strategic avenues of the firms such as product design and R&D investments should include different modeling of managerial delegation game and the investigation of real-world evidence. These are challenging future research.

References

Baron, D., 2001. Private politics, corporate social responsibility, and integrated strategy. Journal of Economics and Management Strategy 10, 7–45.

Bian, J., Li, K., Guo, X., 2016. A strategic analysis of incorporating csr into managerial incentive design. Transportation Research Part E: Logistics and Transportation Review 86, 83–93.

Brand, B., Grothe, M., 2013. A note on 'corporate social responsibility and marketing channel coordination'. Research in Economics 67, 324–327.

Brand, B., Grothe, M., 2015. Social responsibility in a bilateral monopoly. Journal of Economics 115, 275–289.

Bárcena-Ruiz, J., Dong, Q., Wang, L., 2020. Foreign-owned firms and partial privatisation of state holding corporations. Japanese Economic Review doi:10.1111/jere.12223.

Carroll, A., 1991. The pyramid of corporate social responsibility: Toward the moral management of organizational stakeholders. Business Horizons 34, 39–48.

Carroll, A., 1999. Corporate social responsibility: Evolution of a definitional construct. Business and Society 38, 268–295.

Chen, J., Lee, S.H., Muminov, T., 2019. Time-inconsistent output subsidy/tax policies in free-entry mixed markets. Hitotsubashi Journal of Economics 60, 61–77.

Crifo, P., Forget, V., 2015. The economics of corporate social responsibility: A firm-level perspective survey. Journal of Economic Surveys 29, 112–130.

Dong, Q., Wang, L., 2019. Mixed oligopoly with state holding corporations and consumer-friendly firm. International Review of Economics and Finance 62, 121–130.

Eckel, C., Neary, J., 2010. Multi-product firms and flexible manufacturing in the global economy. Review of Economic Studies 77, 188–217.

Fanti, L., Buccella, D., 2017. Corporate social responsibility, profits and welfare with managerial firms. International Review of Economics 64, 341–356.

Fanti, L., Buccella, D., 2019. Managerial delegation games and corporate social responsibility. Managerial and Decision Economics 40, 610–622.

Garcia, A., Leal, M., Lee, S.H., 2019. Endogenous timing with a socially responsible firm. Korean Economic Review 35, 345–370.

Garcia, A., Leal, M., Lee, S.H., 2020a. Cooperation with a multiproduct corporation in a strategic managerial delegation. Managerial and Decision Economics 41, 3–9.

Garcia, A., Leal, M., Lee, S.H., 2020b. Welfare-improving cooperation with a consumer-friendly multiproduct corperation. Managerial and Decision Economics 41, 1144–1155.

Goering, G.E., 2012. Corporate social responsibility and marketing channel coordination. Research in Economics 66, 142 – 148.

Hamilton, J.H., Slutsky, S.M., 1990. Endogenous timing in duopoly games: Stackelberg or Cournot equilibria. Games and Economic Behavior 2, 29 – 46.

Hino, Y., Zennyo, Y., 2017. Corporate social responsibility and strategic relationships. International Review of Economics 64, 231–244.

Hirose, K., Lee, S.H., Matsumura, T., 2017. Environmental corporate social responsibility: A note on the first-mover advantage under price competition. Economics Bulletin 37, 214–221.

Hirose, K., Lee, S.H., Matsumura, T., 2020. Noncooperative and cooperative environmental corporate social responsibility. ournal of Institutional and Theoretical Economics 176, 549–571.

Ino, H., Matsumura, T., 2010. What role should public enterprises play in free-entry markets? Journal of Economics 101, 213–230.

Jones, B., Bowd, R., Tench, R., 2009. Corporate irresponsibility and corporate social responsibility: Competing realities. Social Responsibility Journal 5, 300–310.

Kim, S.L., Lee, S.H., Matsumura, T., 2019. Corporate social responsibility and privatization policy in a mixed oligopoly. Journal of Economics 128, 67–89.

Kitzmueller, M., Shimshack, J., 2012. Economic perspectives on corporate social responsibility. Journal of Economic Literature 50, 51–84.

Kopel, M., Brand, B., 2012. Socially responsible firms and endogenous choice of strategic incentives. Economic Modelling 29, 982 – 989.

Kumar, A., 1992. The state holding company. issues and options. World Bank Discussion Papers WDP 187 .

Lambertini, L., Tampieri, A., 2015. Incentives, performance and desirability of socially responsible firms in a Cournot oligopoly. Economic Modelling 50, 40 – 48.

Leal, M., Garcia, A., Lee, S.H., 2019. Excess burden of taxation and environmental policy mix with a consumer-friendly firm. Japanese Economic Review 70, 517–536.

Leal, M., García, A., Lee, S.H., 2018. The timing of environmental tax policy with a consumer-friendly firm. Hitotsubashi Journal of Economics 59, 25–43.

Lee, S.H., Park, C.H., 2019. Eco-firms and the sequential adoption of environmental corporate social responsibility in the managerial delegation. B.E. Journal of Theoretical Economics 19.

Liu, C.C., Wang, L., Lee, S.H., 2015. Strategic environmental corporate social responsibility in a differentiated duopoly market. Economics Letters 129, 108–111.

Manasakis, C., Mitrokostas, E., Petrakis, E., 2014. Strategic corporate social responsibility activities and corporate governance in imperfectly competitive markets. Managerial and Decision Economics 35, 460–473.

Manasakis, C., Mitrokostas, E., Petrakis, E., 2018. Strategic corporate social responsibility by a multinational firm. Review of International Economics 26, 709–720.

Matsumura, T., Ogawa, A., 2010. On the robustness of private leadership in mixed duopoly. Australian Economic Papers 49, 149–160.

Planer-Friedrich, L., Sahm, M., 2020. Strategic corporate social responsibility, imperfect competition, and market concentration. Journal of Economics 129, 79–101.

Reinhardt, F., Stavins, R., 2010. Corporate social responsibility, business strategy, and the environment. Oxford Review of Economic Policy 26, 164–181.

Riera, M., Iborra, M., 2017. Corporate social irresponsibility: review and conceptual boundaries. European Journal of Management and Business Economics 26, 146–162.

Schreck, P., 2011. Reviewing the business case for corporate social responsibility: New evidence and analysis. Journal of Business Ethics 103, 167–188.

Wood, D.J., 1991. Corporate social performance revisited. The Academy of Management Review 16, 691–718.

Xu, L., Lee, S.H., 2019. Tariffs and privatization policy in a bilateral trade with corporate social responsibility. Economic Modelling 80, 339–351.

Appendix A. Values of N_i

$$N_1 := 300-276\gamma-85\gamma^2+122\gamma^3-21\gamma^4-5\left(-1+\gamma^2\right)^2\theta_A^4-2\left(102-51\gamma-60\gamma^2+22\gamma^3+7\gamma^4\right)\theta_B+\left(39-6\gamma-32\gamma^2+2\gamma^3+7\gamma^4\right)\theta_B^2+2\left(-1+\gamma^2\right)\theta_A^3$$
$$\left(3\left(-3-4\gamma+4\gamma^2\right)+\left(-1+\gamma^2\right)\theta_B\right)+\theta_A^2\left(67-158\gamma-13\gamma^2+134\gamma^3-48\gamma^4+\left(-40+26\gamma+48\gamma^2-26\gamma^3-8\gamma^4\right)\theta_B+3\left(-1+\gamma^2\right)^2\theta_B^2\right)-2\theta_A$$
$$\left(166-169\gamma-83\gamma^2+119\gamma^3-25\gamma^4+\left(-89+52\gamma+82\gamma^2-42\gamma^3-7\gamma^4\right)\theta_B+\left(13-3\gamma-17\gamma^2+3\gamma^3+4\gamma^4\right)\theta_B^2\right)$$

$$N_2 := (3-2\gamma+(-1+\gamma)\theta_A)(2+\gamma-(1+\gamma)\theta_A+(1+\gamma)\theta_B)\left(-3\left(-6+\gamma+2\gamma^2\right)+\left(3-3\gamma^2\right)\theta_A^2-\left(15+\gamma-8\gamma^2\right)\theta_B+\theta_A\left(-15+9\gamma^2-5\left(-1+\gamma^2\right)\theta_B\right)\right)$$

$$N_3 := -20\left(1-\gamma^2\right)^2\theta_A^4+8\left(1-\gamma^2\right)\theta_A^3\left(7+12\gamma-10\gamma^2+4\left(1-\gamma^2\right)\theta_B+\left(-1+\gamma^2\right)\theta_B^2\right)+(3-\theta_B)\left(21-10\gamma^2+\left(-3+2\gamma^2\right)\theta_B\right)\left(33-30\gamma+5\gamma^2-2\left(11-5\gamma\right.\right.$$
$$\left.\left.-2\gamma^2\right)\theta_B+2\left(2-\gamma^2\right)\theta_B^2\right)+\theta_A^2\left(-4\left(-120+189\gamma+74\gamma^2-165\gamma^3+40\gamma^4\right)-4\left(119-60\gamma-151\gamma^2+60\gamma^3+32\gamma^4\right)\theta_B+\left(129-12\gamma-218\gamma^2+12\gamma^3+89\gamma^4\right)\theta_B^2\right.$$
$$\left.-14\left(-1+\gamma^2\right)^2\theta_B^3+\left(-1+\gamma^2\right)^2\theta_B^4\right)-2\theta_A\left(-33+30\gamma-5\gamma^2-2\left(-11+5\gamma+2\gamma^2\right)\theta_B+2\left(-2+\gamma^2\right)\theta_B^2\right)\left(-33+25\gamma^2-10\left(-1+\gamma^2\right)\theta_B+\left(-1+\gamma^2\right)\theta_B^2\right)$$

$$N_4 := \left(-45+9\gamma+15\gamma^2+6\left(-1+\gamma^2\right)\theta_A^2+\left(48-25\gamma^2\right)\theta_B+\left(-9-\gamma+6\gamma^2\right)\theta_B^2+\theta_A\left(33-21\gamma^2+16\left(-1+\gamma^2\right)\theta_B+3\left(1-\gamma^2\right)\theta_B^2\right)\right)\left(-15+3\gamma+5\gamma^2\right.$$
$$\left.+2\left(-1+\gamma^2\right)\theta_A^2+\left(-6-4\gamma+5\gamma^2\right)\theta_B+\left(3+\gamma-2\gamma^2\right)\theta_B^2+\theta_A\left(11-7\gamma^2+2\left(1-\gamma^2\right)\theta_B-\left(1-\gamma^2\right)\theta_B^2\right)\right)$$

$$N_5 := 45-39\gamma-12\gamma^2+18\gamma^3-4\gamma^4+\left(8+\gamma-14\gamma^2-\gamma^3+6\gamma^4\right)\theta_A^3-\left(-1+\gamma^2\right)^2\theta_A^4+\left(-18+7\gamma+14\gamma^2-4\gamma^3-2\gamma^4\right)\theta_B-\theta_A^2\left(16+11\gamma-29\gamma^2-9\gamma^3+13\gamma^4\right.$$
$$\left.+(-1+\gamma)^2\left(2+3\gamma+\gamma^2\right)\theta_B\right)+\theta_A\left(-12+37\gamma-11\gamma^2-23\gamma^3+12\gamma^4+\left(12-4\gamma-15\gamma^2+4\gamma^3+3\gamma^4\right)\theta_B\right)$$

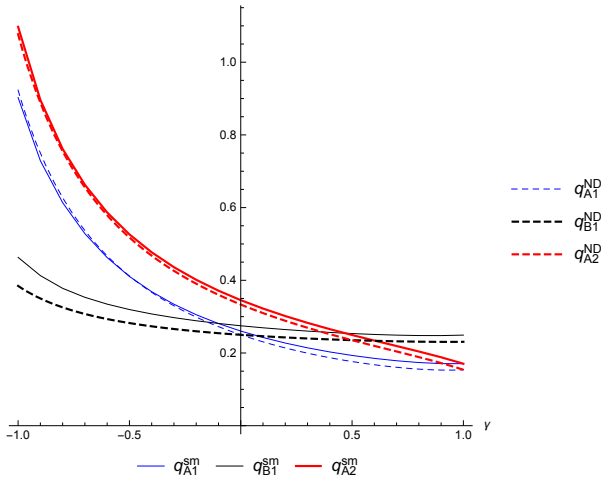
$$N_6 := 63-48\gamma-29\gamma^2+22\gamma^3+\left(13-18\gamma-11\gamma^2+14\gamma^3\right)\theta_A^2+\left(-1+2\gamma+\gamma^2-2\gamma^3\right)\theta_A^3-3\left(4-2\gamma-2\gamma^2+\gamma^3\right)\theta_B+\theta_A\left(-51+52\gamma+33\gamma^2-31\gamma^3+2\left(2-\gamma\right.\right.$$
$$\left.\left.-2\gamma^2+\gamma^3\right)\theta_B\right)$$

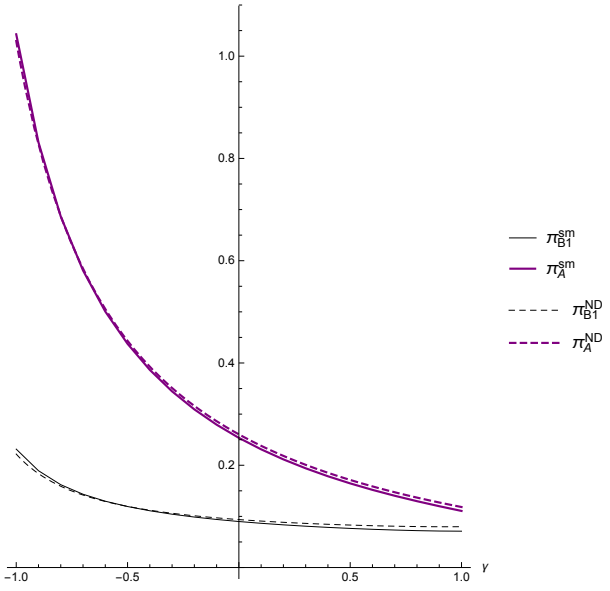
$$N_7 := (3-2\gamma)^2\left(1998+738\gamma-2259\gamma^2-720\gamma^3+783\gamma^4+176\gamma^5-76\gamma^6\right)+2\left(-1+\gamma^2\right)^3\left(-28-4\gamma+21\gamma^2\right)\theta_A^7-3\left(-1+\gamma^2\right)^4\theta_A^8-2\left(4698-2565\gamma-7119\gamma^2\right.$$
$$\left.+3651\gamma^3+3766\gamma^4-1737\gamma^5-770\gamma^6+276\gamma^7+40\gamma^8\right)\theta_B+3\left(468-108\gamma-819\gamma^2+172\gamma^3+516\gamma^4-92\gamma^5-135\gamma^6+16\gamma^7+12\gamma^8\right)\theta_B^2+\left(1-\gamma^2\right)^2\theta_A^6\left(-403\right.$$
$$\left.-178\gamma+635\gamma^2+138\gamma^3-254\gamma^4+2\gamma\left(2-\gamma-2\gamma^2+\gamma^3\right)\theta_B\right)-2\left(-1+\gamma^2\right)\theta_A^5\left(613+822\gamma-1644\gamma^2-1272\gamma^3+1477\gamma^4+482\gamma^5-438\gamma^6+\left(18-39\gamma-26\gamma^2\right.\right.$$
$$\left.\left.+70\gamma^3+2\gamma^4-31\gamma^5+6\gamma^6\right)\theta_B\right)+\theta_A^4\left(45-8218\gamma+3663\gamma^2+19070\gamma^3-9170\gamma^4-14450\gamma^5+7359\gamma^6+3566\gamma^7-1905\gamma^8+2\left(-290+333\gamma+757\gamma^2-863\gamma^3\right.\right.$$
$$\left.\left.-637\gamma^4+735\gamma^5+159\gamma^6-205\gamma^7+11\gamma^8\right)\theta_B+\left(1-\gamma^2\right)^3\left(12-4\gamma-5\gamma^2\right)\theta_B^2\right)-2\theta_A^3\left(5598-12084\gamma-8213\gamma^2+25311\gamma^3-31\gamma^4-17206\gamma^5+4060\gamma^6\right.$$
$$\left.+3791\gamma^7-1350\gamma^8+\left(-1812+1526\gamma+4380\gamma^2-3590\gamma^3-3447\gamma^4+2736\gamma^5+884\gamma^6-672\gamma^7-5\gamma^8\right)\theta_B+\left(-1+\gamma^2\right)^2\left(88-32\gamma-86\gamma^2+22\gamma^3+15\gamma^4\right)\theta_B^2\right)$$
$$-2\theta_A\left(19683-20466\gamma-27765\gamma^2+33255\gamma^3+9908\gamma^4-17549\gamma^5+810\gamma^6+3028\gamma^7-648\gamma^8+\left(-8154+4995\gamma+15000\gamma^2-8837\gamma^3-9059\gamma^4+4998\gamma^5\right.\right.$$
$$\left.\left.+1953\gamma^6-916\gamma^7-76\gamma^8\right)\theta_B+2\left(468-144\gamma-1019\gamma^2+291\gamma^3+747\gamma^4-183\gamma^5-216\gamma^6+36\gamma^7+20\gamma^8\right)\theta_B^2\right)+\theta_A^2\left(32427-42174\gamma-50961\gamma^2+78680\gamma^3\right.$$
$$\left.+17442\gamma^4-47470\gamma^5+4938\gamma^6+9284\gamma^7-2448\gamma^8-2\left(5508-3852\gamma-11805\gamma^2+8000\gamma^3+8214\gamma^4-5313\gamma^5-1931\gamma^6+1129\gamma^7+50\gamma^8\right)\theta_B+\left(888-312\gamma\right.\right.$$
$$\left.\left.-2294\gamma^2+744\gamma^3+2019\gamma^4-564\gamma^5-684\gamma^6+132\gamma^7+71\gamma^8\right)\theta_B^2\right)$$

$$N_8 := (3-2\gamma+(-1+\gamma)\theta_A)^2\left(-\left(5+8\gamma+3\gamma^2\right)\theta_A+(1+\gamma)^2\theta_A^2+(2+\gamma)(3+2\gamma+(1+\gamma)\theta_B)\right)\left(\left(-15+26\gamma^2-11\gamma^4\right)\theta_A^3+\left(1-\gamma^2\right)^2\theta_A^4-(2+\gamma)\left(-63+42\gamma\right.\right.$$
$$\left.\left.+30\gamma^2-20\gamma^3+\left(45-23\gamma-32\gamma^2+18\gamma^3\right)\theta_B\right)+\theta_A^2\left(77-\gamma-109\gamma^2+\gamma^3+36\gamma^4+\left(-10+\gamma+17\gamma^2-\gamma^3-7\gamma^4\right)\theta_B\right)+\theta_A\left(-165+10\gamma+181\gamma^2-8\gamma^3-46\gamma^4\right.\right.$$
$$\left.\left.+\left(60-6\gamma-82\gamma^2+6\gamma^3+22\gamma^4\right)\theta_B\right)\right)$$

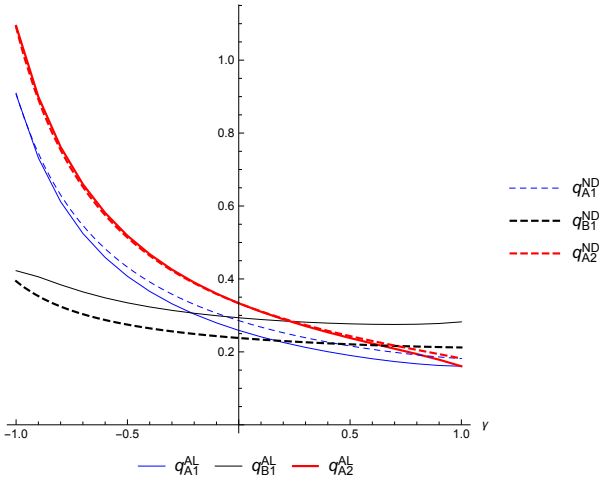
Appendix B. Comparisons of no delegation case vs competitive CSR case

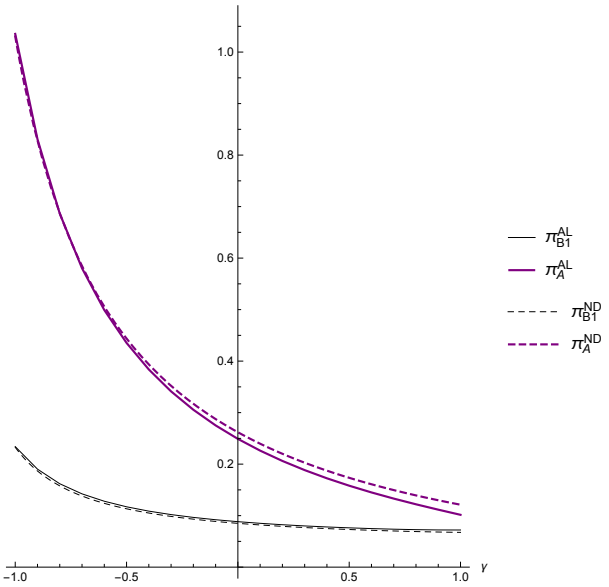
Appendix B.1. Simultaneous choice in output stage



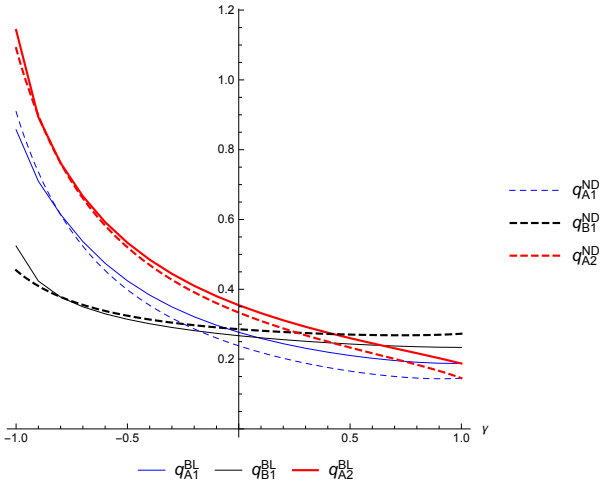


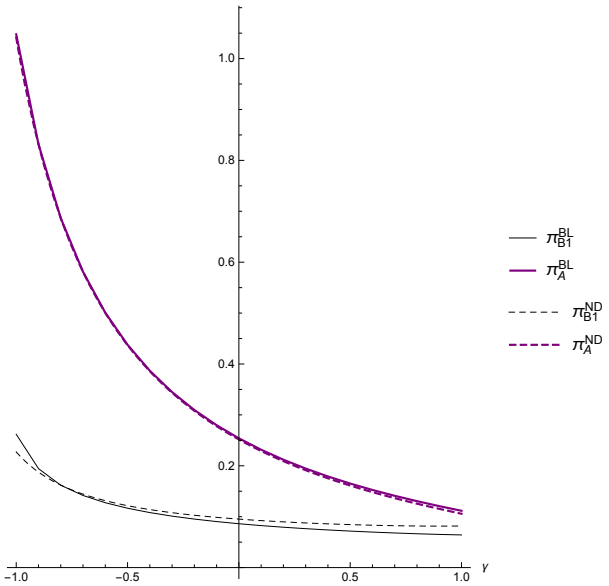
Appendix B.2. Sequential choice with multiproduct corporation leadership in output stage





Appendix B.3. Sequential choice with single plant firm leadership in output stage




1.0
0.8
0.6
0.4
0.2
−1.0
−0.5
0.5
1.0
γ
π_{B1}^{BL}
π_{A}^{BL}
π_{B1}^{ND}
π_{A}^{ND}