Debt and demand regimes in canonical growth models: a comparison of neo-Kaleckian and Supermultiplier models

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Abstract

The paper addresses the features of stock-flow consistent (SFC) canonical versions of neo-Kaleckian (NK) and Supermultiplier (SM) models that introduce either the accumulation of debt of households or firms. The aim of this comparison is twofold: (i) to analyze the implications of a debt accumulation process in each of these models; (ii) to evaluate the extent to which these debt accumulation processes differ due to each model's specific closure. For that purpose, we investigate the relation of debt ratios, demand, and growth, focusing on the conditions for the paradox of debt to arise in each model; and the stability conditions of firms' and household debt ratios. Results suggest that not only the paradox of debt in the firms' sector is not a necessary result of SM models, but the Minskyan debt regime is the sole economically viable one in the long-run. For the NK model, as extensively explored in the literature, firms' leverage ratio can be either pro-cyclical or anti-cyclical. About the demand regimes, in this canonical version of the SM model, firms' debt has no direct effect on demand; as for the NK model, demand can be either debt-led or debt-burdened. As for the household sector, the paradox of debt is a feature of the canonical Supermultiplier model in the long run, yet there may be episodes of rising debt-to-income ratios and financial crises as precipitated by policy decisions. Besides, only the debt-led demand regime is compatible with a stable household debt ratio. In turn, in the neo-Kaleckian model, results are ambiguous in what concerns the two-way relation between demand and household debt.

Keywords: Paradox of debt, neo-Kaleckian model, Supermultiplier model, autonomous expenditures, SFC

JEL classification codes: E11, E12, O41.

1 Introduction

In Supermultiplier models, non-capacity creating autonomous expenditures lead growth and private business investment endogenously adjusts to the deviation of capacity utilization from a desired rate in the long-run (Freitas and Serrano, 2015; Allain, 2015; Lavoie,

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2016). The closure of the model has been targeted with some criticism as it allegedly reflects the model's inability to address financial phenomena such as financial crises and episodes of rising debt-to-income ratios (Nikiforos, 2018).

Underlying these critical remarks there seems to be a misconception of what autonomous expenditures are. Autonomous expenditures in the Supermultiplier literature are those expenditures that are not directly connected to the circular flow of income (Serrano, 1995) or to the income accruing from firms' production decisions (Freitas and Serrano, 2015). Nowhere in this definition there is a hint that points to the belief that they are to be necessarily considered as *exogenous* or that they are not affected by other economic variables. Autonomous expenditures have determinants that can be explicitly addressed in a theoretical model.

That said, in canonical models, it is a common practice to assume as a matter of simplification that some variables are not explained within the model. The Supermultiplier model in that respect is no different from the canonical neo-Kaleckian model that assumes autonomous investment is explained by animal spirits, which in turn are not explained by the model. But few in the non-mainstream field of economics would say that in reality aggregate demand or income has no role to play in the entrepreneurs' drive to invest. In the canonical Supermultiplier model, autonomous expenditures are exogenous as a means of simplifying the analysis of a growth scenario.

Besides that, saying that a kind of expenditure is autonomous in theory should not be mixed up with saying that there are no political and policy constraints for the working of an expenditure as such. Especially considering the high level of abstraction in which canonical models are situated.

In what concerns the real-financial linkages in this approach, there have been some initial efforts to address household debt accumulation (Pariboni, 2016; Fagundes, 2017; Mandarino et al., 2020) and to make financial determinants of autonomous expenditures endogenous (Brochier and Macedo e Silva, 2019) employing the stock-flow consistent (SFC) methodology. Those papers that focus on household debt already take into account the negative effect that debt service payments or debt amortization may have on an economic system in which household credit-based consumption leads growth. Household debt accumulation becomes unstable if the negative rate of amortization of loans exceeds the growth rate of autonomous expenditures (Fagundes, 2017; Pariboni, 2016). This condition can be interpreted as saying that the rate of expansion of the autonomous expenditures (and thus of capital accumulation and income in the long run) must be higher than the rate at which households roll over their debt principal.

Brochier and Macedo e Silva (2019) also show that by making non-capacity creating autonomous expenditures endogenous to the supermultipler model, in their case consumption out of household wealth, changes in the propensities to spend and in income distribution may have permanent effects on the long run growth rate of the economy, not only on the average growth rate.

Yet as has been the case for other non-mainstream growth models in their early development years, the Supermultiplier approach has a long way to go in exploring the role finance can play in the model. In turn, the neo-Kaleckian literature has been dealing with financial issues for at least 30 years. Since both approaches are in different stages of development, it only seems fair to compare the implications of a debt accumulation process in variants of canonical models under these approaches.

Building on this, the paper addresses the features of stock-flow consistent (SFC) canonical versions of neo-Kaleckian and Supermultiplier models that introduce either the accumulation of debt of households or firms. The aim of this comparison is twofold: (i) to analyze the implications of a debt accumulation process in these models; (ii) to evaluate the extent to which these debt accumulation processes differ due to each model's specific closure. For that purpose, we investigate the relation of debt ratios, demand, and growth, focusing on the conditions for the paradox of debt to arise in each model; and the stability conditions of firms' and household debt ratios.

Besides this introduction, the paper is organized as follows. Section 2 presents a benchmark model with the institutional and financial assets structure employed to compare Supermultiplier and neo-Kaleckian models. This section also defines the behavioral equations of consumption and investment (and capital accumulation) that are specific to the closure of each model. In section 3, we discuss the process of firms' debt accumulation in each model, addressing (i) the effects debt might have on the capacity utilization rate and growth; (ii) the conditions for a Minsky or Steindl debt regime to happen in firms' sector; and (iii) the partial stability conditions of firms' debt-to-capital ratio. Following this, in section 4 we deal with household debt accumulation adopting the same procedure of section 3. At last, in section 5, we present our conclusions based on the comparison made throughout the paper.

2 Framework of canonical Supermultiplier and neo-Kaleckian SFC models

In order to compare the results of canonical neo-Kaleckian and Supermultiplier models that take explicitly into account the process of debt accumulation, we adopt the same institutional and asset structure for both of them. We build a benchmark SFC model with both household and firms' debt, however, to keep matters as simple as possible, we analyze each sector debt accumulation process separately. We do that by adopting two specifications: for analyzing firms' debt, we assume that household debt and the related parameters are zero (specification 1); for analyzing household debt, we assume firms' debt and the related parameters are zero and that firms issue equities to finance the part of investment that is not covered by the available internal funds (specification 2). When presenting the equations, and where it applies, we point to the differences between these specifications. In the sections where we deal with firms (section 3) and household (section 4) debt accumulation, we summarize the features of specifications 1 and 2 respectively.

In what concerns neo-Kaleckian models, there have been several efforts to extend its canonical version to deal with firms' debt (Lavoie, 1995; Dutt, 1995; Hein, 2006; Taylor, 2004; Ryoo, 2013) or with household debt (Dutt, 2006; Kim, 2012; Setterfield et al., 2016). The latter have emerged mainly in the context of the 2007-08 *subprime* crisis. There are also more elaborate versions of the neo-Kaleckian model that combine both household and firms' debt, such as Isaac and Kim (2013) and van Treeck (2009).

For addressing household debt in the neo-Kaleckian model, we adopt a variant of the models presented by Kim (2012), Hein (2012), and Setterfield et al. (2016) in which household debt is as a function of workers' desire to emulate capitalists' consumption. For dealing with firms' debt we adopt a variant of the models in Lavoie (1995), Dutt (1995) and Taylor (2004) that adds the debt service as an argument of the investment function. If we were to address both households' and firms' debt accumulation at the same time, the model would become a variant of Isaac and Kim (2013)'s model.

As for the Supermultiplier model, we adopt a variant of the models in Fagundes (2017) and Pariboni (2016) that extend the canonical Supermultiplier model (Serrano, 1995; Freitas and Serrano, 2015; Lavoie, 2016) to address the process of household debt accumulation. To deal with firms' debt, we simply add firms' debt into the canonical model in a similar vein to what is usually done in neo-Kaleckian models: firms' take on loans to finance the part of investment demand that is not covered by retained earnings.[1]

In the next paragraphs, we present the models' basic accounting framework and their behavioral assumptions. As for the latter, unless said otherwise they will be the same for both models. We assume a pure credit closed economy with no government sector.

Table 1 presents the balance sheet of the three institutional sectors: households,

[1]For Supermultiplier models that take firms' debt explicitly into account see Brochier and Macedo e Silva (2019); Mandarino et al. (2020).

firms, and banks. We further suppose that both households and firms take on debt for consumption and investment purposes, respectively. For the sake of simplicity, we assume that banks do not profit and that interests paid on households deposits and households' and firms' loans are the same, and that banks set the interest rate exogenously.

Table 1: Balance sheet matrix

Assets	Households	Firms	Banks	$\sum$
1. Deposits	$+M$		$-M$	0
2. Loans⋆	$-L_h$	$-L_f$	$+L$	0
3. Fixed capital		$+K$		$+K$
4. Equities⋆	$+p_eE$	$-p_eE$		0
5. Net worth	V_h	V_f	0	$+K$

⋆ In specification 1: $L_h = 0$, $peE = 0$ and $L_f \neq 0$; in specification 2: $L_h \neq 0$, $peE \neq 0$ and $L_f = 0$.

Table 2 shows the transactions between the institutional sectors in its first part and the flow of funds in the second part. At this point we can describe the transactions of each sector and the behavioral assumptions.

Households earn wage and financial income (interests on deposits and distributed profits of firms) (1). Income distribution is exogenous to the model (2).[2] Behavioral assumptions in what concerns household consumption will be different in neo-Kaleckian (NK) and Supermultiplier (SM) models.

$$Y_h = W + FD + iM \tag{1}$$

$$W = (1 - \pi)Y \tag{2}$$

In the Supermultiplier model, if households take on loans ($L_h > 0$) (specification 2), autonomous consumption (Z) can be interpreted as workers' consumption financed by credit, otherwise ($L_h = 0$) (specification 1) autonomous consumption could be interpreted as capitalists' autonomous consumption based on social habits (see Fagundes (2017); Dutt (2019)). Since we are dealing with a closed economy with no government, autonomous consumption that grows at an exogenously given rate (g_z) will be the non-capacity creating autonomous expenditure that leads growth in the long run (3). For specification 2, households take on new loans to finance autonomous consumption (4) and banks grant all loans demanded by households for consumption purposes. Besides autonomous con-

[2] This is the case with both neo-Kaleckian and Sraffian income distribution theories. They generally assume that there is no regular or systematic relationship through which the growth of output affects income distribution.

Table 2: Transactions and Flow of Funds matrix

	Household	Firms Current	Firms Capital	Banks	$\sum$
1. Consumption	$-C$	$+C$			0
2. Investment		$+I$	$-I$		0
3. Wages	$+W$	$-W$			0
4. Firms' Profit	$+FD$	$-F$	$+FU$		0
5. Deposits interest	$+iM$			$-iM$	0
6. Loans interest*	$-iL_h$	$-iL_f$		$+iL$	0
7. Subtotal	S_h	0	S_f	0	0
8. Change in Deposits	$-\dot{M}$			$+\dot{M}$	0
9. Change in Loans*	$+\dot{L}_h$		$+\dot{L}_f$	$-\dot{L}$	0
10. Change in Equities*	$-p_e\dot{E}$		$+p_e\dot{E}$		0
11. $\sum$	0	0	0	0	0

$\star$ In specification 1: $iL_h; \dot{L}_h = 0$, $pe\dot{E} = 0$ and $iL_f; \dot{L}_f \neq 0$; in specification 2: $iL_h; \dot{L}_h \neq 0$, $pe\dot{E} \neq 0$ and $iL_f; \dot{L}_f = 0$.

sumption, poorer households consume a fraction (α_1) of their wage income after interest payments on loans and richer households consume a fraction (α_2) of their financial income (5). For specification 1, since we understand the autonomous consumption component as richer households' consumption, it seems redundant to include also an induced component for consumption out of financial income. Therefore we consider the consumption function takes the form of equation 6.

$$Z = Z_0 e^{g_z} \tag{3}$$

$$\dot{L}_h = Z \qquad \text{(spec.2)} \tag{4}$$

$$C = \alpha_1(W - iL_h) + Z + \alpha_2(FD + iM) \qquad \text{(spec.2)} \tag{5}$$

$$C = \alpha_1 W + Z \qquad \text{(spec.1)} \tag{6}$$

As for the neo-Kaleckian model, for specification 2, poorer households take on loans to emulate richer households' consumption (Kim, 2012; Ryoo and Kim, 2014; Setterfield et al., 2016) as represented by equation 7. The parameter η represents poorer households' willingness to emulate richer households' consumption and at the same time banks' willingness to grant credit to these households (Dutt, 2006; Lavoie, 2014). Poorer households also consume a fraction of their wage income (α_1) after debt service payments and richer households consume a fraction (α_2) of their financial income (8). Following Fagundes (2017) and differently from most of the models that deal with emulation effects, we do not assume that households consume all of their wages, since in the aggregate this is not a necessary condition for indebtedness to take place in the household sector.[3] For specification 1, poorer households' consumption does not depend on richer households consumption: the emulation parameter (or banks' willingness to lend to households) is zero ($\eta = 0$) in equation 8, so that the consumption function is given by equation 9. As should be clear by now, we abstract from household debt amortization in the NK and SM model.

$$\dot{L}_h = \eta[\alpha_2(FD + iM)] \qquad \text{(spec.2)} \tag{7}$$

$$C = \alpha_1(W - iL_h) + \eta[\alpha_2(FD + iM)] + \alpha_2(FD + iM) \qquad \text{(spec.2)} \tag{8}$$

$$C = \alpha_1 W + \alpha_2(FD + iM) \qquad \text{(spec.1)} \tag{9}$$

For both models, changes in households' net wealth will be given by households' savings (since there are no capital gains in these models), which is also equivalent to the changes in deposits and held equities (assets) less changes in loans (liabilities) for specification 2; and to changes in deposits for specification 1 (10). Deposits are an increasing function of new loans and households' savings and a decreasing function of equities acquisition for specification 2. For specification 1, deposits are an increasing function of households' savings (11).

$$\begin{cases} \dot{V}_h = S_h = \dot{M} + p_e\dot{E} - \dot{L}_h, & \text{if } L_h > 0 \qquad \text{(spec.2)} \\ \dot{V}_h = S_h = \dot{M}, & \text{if } L_h = 0 \qquad \text{(spec.1)} \end{cases} \tag{10}$$

$$\begin{cases} \dot{M} = S_h + \dot{L}_h - p_e\dot{E}, & \text{if } L_h > 0 \qquad \text{(spec.2)} \\ \dot{M} = S_h, & \text{if } L_h = 0 \qquad \text{(spec.1)} \end{cases} \tag{11}$$

As for the **firms**' sector, in both models they finance investment (I) through retained earnings (FU) and any additional demand for funds is covered by banks loans

[3]There may coexist workers that consume all their income and incur into debt to finance the consumption pattern they desire with workers that save part of their income and do not take on loans. Depending on the proportion of each group in total workers, the marginal propensity to consume may be lower or higher than one (Fagundes, 2017).

(specification 1) or equity issuance (specification 2) (12). As was the case for households, firms are not credit constrained. Firms retain a fraction of their profit (s_f) (specification 2) discounting the payment of interests on loans (specification 1) (13) and distribute the rest of profits to households (14).

$$\begin{cases} \dot{L_f} = I - FU, & \text{if } L_f > 0 \quad \text{(spec.1)} \\ p_e\dot{E} = I - FU, & \text{if } L_f = 0 \quad \text{(spec.2)} \end{cases} \tag{12}$$

$$\begin{cases} FU = s_f(\pi Y - iL_f), & \text{if } L_f > 0 \quad \text{(spec.1)} \\ FU = s_f \pi Y, & \text{if } L_f = 0 \quad \text{(spec.2)} \end{cases} \tag{13}$$

$$\begin{cases} FD = (1 - s_f)(\pi Y - iL_f), & \text{if } L_f > 0 \quad \text{(spec.1)} \\ FD = (1 - s_f)\pi Y, & \text{if } L_f = 0 \quad \text{(spec.2)} \end{cases} \tag{14}$$

In the Supermultiplier approach, firms' investment behavior is based on the flexible accelerator principle. Aggregate investment of firms is induced by income (15) and the marginal propensity to invest (h) will endogenously react to the discrepancies between the actual (u) and the normal capacity utilization rate (u_n) according to a sensitivity parameter γ (16) (Freitas and Serrano, 2015; Lavoie, 2016; Allain, 2015). Investment decisions are not directly affected by firms' indebtedness.

$$I = hY \tag{15}$$

$$\dot{h} = h\gamma(u - u_n) \tag{16}$$

$$g_k = g_I = \frac{hu}{v} \tag{17}$$

There are several variants of a neo-Kaleckian investment function. We adopt a variant of the functions presented in Lavoie (1995), Dutt (1995) and Taylor (2004) that takes into account the negative effect debt service may have on investment demand. Thus firms base their investment decisions on "animal spirits" or the expected trend growth rate of sales that is represented by the exogenous parameter β_0, on the capacity utilization rate (β_1) and, negatively, on the leverage ratio (l_f) depending on the weight debt service has on investment demand (β_2) (18).

$$I = K(\beta_0 + \beta_1 u - \beta_2 i l_f) \tag{18}$$

$$g_k = g_I = \beta_0 + \beta_1 u - \beta_2 i l_f \tag{19}$$

In both models, the actual capacity utilization rate will be given by the ratio of output to full-capacity output (20) and full-capacity output is determined by a ratio of the capital stock to the given capital-output ratio (21). Besides that, since we abstract from capital depreciation for simplicity, in both cases the capital accumulation rate will be given by the investment growth rate (17 and 19). In the SM model, the investment function and the capital accumulation rate are the same for both specifications; regarding the NK model, the difference of specification 2 in relation to specification 1 is that the third term on the RHS of equation 19 will amount to zero ($\beta_2; l_f = 0$).

$$u = \frac{Y}{Y_{fc}} \tag{20}$$

$$Y_{fc} = \frac{K}{v} \tag{21}$$

Also for both models real output is the sum of households' consumption and business investment (22).

$$Y = C + I \tag{22}$$

At last, since banks grant all loans demanded by households and firms and do not profit, it follows from the accounting framework that the total amount of the deposits will match firms' or households loans (depending on the adopted specification).

3 Growth, demand and firms' debt dynamics

In this section we compare the features of firms' debt dynamics in NK and SM models. Specifically we analyze: (i) firms' debt ratio impact on capacity utilization (and growth in the case of the NK model) to check whether demand is debt-led or debt-burdened in the terminology of Taylor (2004); (ii) the impact of demand and growth on firms' debt ratio to establish the conditions for a Steindlian or a Minskyian regime to happen in firms' sector (Lavoie, 2014); and finally (iii) firms' debt dynamic stability conditions in each model.

For this purpose, we adopt specification 1 of the models, in which: (i) households do not take on loans ($L_h; \eta = 0$); (ii) firms do not issue equities ($peE = 0$) and take on loans to finance investment that is not covered by internal funds ($L_f > 0$); (iii) firms' loans have a directly negative impact on investment demand in the NK model ($\beta_2 > 0$).

That said we can obtain the short run capacity utilization rate and the equilibrium growth rate (in the NK model) and compare the effects debt may have on the level of activity in each model.

3.1 Firms' leverage ratio impact on demand and growth

Substituting equations 15 and 6 and equations 18 and 9 into equation 22, respectively for the SM and for the NK model, and normalizing it by the capital stock, we get the short-term equilibrium capacity utilization rates (u) (equations 23 and 24) (see table 3). In equation 23, z denotes the autonomous consumption normalized by the capital stock. From these equations and assumptions, it is clear that the terminology of a debt-led or debt-burdened demand regime does not apply to this version of the Supermultiplier model.

As for the neo-Kaleckian model, assuming the Keynesian stability condition is satisfied (and therefore the denominator of equation 24 is positive), if we take the derivative of u with respect to firms' debt to capital ratio (l_f),[4] we notice it has an ambiguous effect on the capacity utilization rate as in Taylor (2004) and Lavoie (2014). Through distributed profits, a higher loans to capital ratio reduces retained earnings and has a net positive effect on households' consumption. On the other hand, firms' leverage ratio has a negative impact on investment. The economy will be debt-led (debt-burdened) if the positive (negative) effect on consumption (investment) more than compensates the negative (positive) effect on investment (consumption)(see table 3). It is also important to notice that the ambiguous effect of debt on demand arises from the investment function specification that firms' leverage ratio has a direct impact on investment demand. Otherwise, firms' debt ratio would only have a positive effect on consumption. This is what would happen also in the Supermultiplier model if we were to include the induced component of richer households' consumption into the current specification of the model (demand would be debt-led).

These rather unrealistic results – that firms' debt could have an unambiguously positive effect on the capacity utilization rate just by taking debt out of the investment function – are related to a common simplifying assumption of the first (and many) SFC models. Namely that banks do not profit and as such it follows from the accounting framework that deposits will equal loans. If deposits are not equivalent to loans, it is easy to see that interest payment on these assets would have separate effects on demand. In that case, firms' leverage ratio would have an unambiguously negative effect on the level of

[4]For the full set of derivatives from which the conditions presented in tables 3 to 8 were obtained see Appendix A.

activity through the reduction of distributed profits that affects households' consumption out of financial income.

For the NK model, if we substitute equation 24 into equation 19, we get the capital accumulation growth rate (25). Where x is the inverse of the multiplier. Taking the derivative of g^* with respect to l_f, we notice that the condition for growth to be debt-led is more restrictive than the condition for demand to be debt-led, depending also on the multiplier. Cet.par., the higher the multiplier (the lower x) the more likely growth will debt-led and vice-versa (table 3).

3.2 Demand and growth impacts on firms' leverage ratio

So far we have talked about how firms' debt ratio might affect the level of activity. But it is well known that an expansion in firms' investment expenditures financed by loans increases both firms' profits and their debt burden.

In the aggregate, this process may lead to a lower leverage ratio, when higher investment expenditures allow for an improvement in firms' ability to finance investment with retained earnings (Pedrosa, 2019). In that case, the leverage ratio is anti-cyclical and constitutes what is called a Steindl debt regime, or the paradox of debt (Steindl, 1952).[5] In periods of economic activity contraction, this means that firms attempts to de-leverage by shrinking their investment plans may lead to a higher debt ratio in the end as profits may fall to a larger extent in comparison to firms liabilities.

When a faster pace of investment leads to a higher leverage ratio in the aggregate as agents are more willing to assume riskier positions, the leverage ratio is said to be pro-cyclical, constituting a Minsky debt regime (Taylor, 2004; Lavoie, 2014).

That said, we can also investigate the conditions under which an increase in demand and a faster pace of accumulation will lead to either a reduction (paradox of debt, Steindl regime) or an increase (Minsky regime) in firms' debt ratio.

We start by finding the differential equation that describes the evolution of firms' debt to capital ratio through time. For the SM model, we substitute equations 13 and 15 into 12, normalize it by the capital stock and then substitute 17 into the resulting equation. After some mathematical manipulation, we get the differential equation for firms' debt ratio in the SM model (26). Taking the derivative of $\dot{l}_f$ with respect to u, we get the conditions for demand to have a positive or negative effect on the evolution of

[5] As highlighted by Ryoo (2013), the paradox of debt is usually attributed to Steindl, but his analysis of this issue builds directly on Kalecki's work on the matter.

Table 3: Firms' debt effect on demand and growth

		Capacity utilization rate	Debt demand regimes
Specification 1: $L_h = 0$; $\eta = 0$; $p_e E = 0$; $L_f > 0$; $\beta_2 > 0$	SM	$$u = \frac{vz}{1 - \alpha_1(1-\pi) - h} \quad (23)$$	–
	NK	$$u = \frac{v[\beta_0 + (\alpha_2 s_f - \beta_2) i l_f]}{1 - \alpha_1(1-\pi) - \alpha_2(1-s_f)\pi - \beta_1 v} \quad (24)$$	Debt-led: $\alpha_2 s_f - \beta_2 > 0$ Debt-burd: $\alpha_2 s_f - \beta_2 < 0$
		Growth rate	**Debt growth regimes**
	SM	$g^* = g_z$	–
	NK	$$g^* = g_I = \beta_0 + \beta_1 \left(\frac{v[\beta_0 + (\alpha_2 s_f - \beta_2) i l_f^*]}{x} \right) - \beta_2 i l_f^* \quad (25)$$	Debt-led: $\alpha_2 s_f - \beta_2 \left(1 + \frac{x}{\beta_1 v}\right) > 0$ Debt-burd: $\alpha_2 s_f - \beta_2 \left(1 + \frac{x}{\beta_1 v}\right) < 0$

Legend: $x = 1 - \alpha_1(1-\pi) - \alpha_2(1-s_f)\pi - \beta_1 v$

Note: All conditions assume Keynesian stability holds.

firms' leverage ratio (see table 4).

$$\dot{l}_f = (h - s_f\pi)\frac{u}{v} + \left(s_f i - \frac{hu}{v}\right) l_f \tag{26}$$

From the conditions presented in table 4, one can notice that, in the short or medium run, the likelihood of a Minskyian regime for firms' sector to happen increases the lower the leverage ratio and the lower the retained share of profits in relation to firms' propensity to invest. The opposite holds for a Steindlian regime: the paradox of debt in firms' sector is likely to emerge the higher firms' leverage ratio and the higher the retained profit share in comparison to firms' propensity to invest. Therefore, a scenario in which a higher level of activity leads to an increase in "financial fragility" as represented by a higher leverage ratio in firms' sector according to the main view of Minsky's Financial Instability Hypothesis (Minsky, 1982) can also happen in the SM model.

In the case of the SM model, we can also easily calculate the impact the growth rate of autonomous expenditures will have on the steady growth firms' debt to capital ratio and, therefore, define Minskyian and Steindlian regimes in terms of growth. In the long run, all stock and stock-flow ratios converge to their steady growth values ($\dot{l}_f = 0$), that is, all variables grow at the same rate, g_z, and firms have adjusted their investment behavior so that the capacity utilization converges to the normal utilization rate ($\dot{h} = 0$). Given these steady growth conditions, the long run firms' debt to capital ratio (l_f^*) will be given by equation 27. Taking the derivative of l_f^* with respect to g_z, we notice that for a Minskyian debt regime to happen the normal profit rate ($r_n = \pi u_n / v$) has to be higher than the interest rate (the normal profit rate net of interests must be greater than zero) and for a Steindlian regime to happen, the interest rate paid on loans and deposits has to exceed the normal profit rate (table 4).

The condition for a Minskyian debt regime to emerge in the long run of the SM model is also an economic feasibility condition, as one would expect firms to invest in the long run only when the profit rate exceeds the interest rate. Therefore, one can say that a Steindlian debt regime is not feasible in the SM model in the long run since it would mean firms would not invest and, therefore, there would not be a positive capital accumulation.

$$l_f^* = \frac{g_z - s_f\pi\dfrac{u_n}{v}}{g_z - s_f i} \tag{27}$$

For the NK model, firms' debt ratio differential equation can be written as in (28). Since the conditions for Minsky or Steindl regimes in terms of demand to emerge have been explored elsewhere for the NK model, we simply present the results for our variant of the NK model in table 4.[6] Solving equation 28 for l_f, considering both the feedback effects of l_f on u and g, would leads to a very long second degree polynomial equation. To keep the discussion within limits, we will not pursue this avenue. We follow once more Taylor (2004) and Lavoie (2014). We assume that, for a steady debt ratio ($\dot{l}_f = 0$) and for a debt ratio lower than one, a Minsky debt regime may arise in the long run if the effect of the capacity utilization rate on the capital accumulation rate is strong enough to compensate for the direct negative effect of the capacity utilization rate on the debt ratio (as an increase in retained profits reduces the need for external borrowing), making the total derivative of the debt ratio with respect to the capacity utilization in the surroundings of the equilibrium greater than zero ($[\partial \dot{l}_f / \partial u^*]_{\dot{l}_f=0} > 0$). This implies the conditions for the debt regimes to happen in the long run of NK model are virtually the same ones that apply to the short run.

$$\dot{l}_f = g_I(1 - l_f) - s_f \left(\frac{\pi u}{v} - i l_f \right) \tag{28}$$

In the next section, we discuss the dynamic stability conditions for firms' debt ratio in both models.

3.3 Dynamic stability conditions

To analyze the dynamic stability of firms' debt ratio in the SM model, it will be helpful to define an alternative equation for the capacity utilization that explicitly takes the growth rate of autonomous expenditures into account. Assuming that we have partial equilibrium (a constant z ratio), namely, that the capital accumulation rate is given by autonomous expenditures growth rate in equation 17 and solving for u^*, we get an alternative definition for the capacity utilization rate (29).

$$u^* = \frac{v g_z}{h} \tag{29}$$

Substituting (29) into (26), we get the following dynamic equation for firms' debt ratio:

$$\dot{l}_f = g_z \left(1 - \frac{s_f \pi}{h} \right) + (s_f i - g_z) l_f \tag{26A}$$

From table 5, we notice that for firms' debt ratio to stabilize it is a necessary and sufficient condition that the growth rate exceeds the interest rate multiplied by the retention rate (since firms do not save all of their net profits). A similar partial stability condition

[6] For more on firms' debt dynamics in canonical NK models see Dutt (1995),Lavoie (1995), Hein (2006), and Taylor (2004) among others.

Table 4: Steindl and Minsky debt regimes in NK and SM models

	Steindl regime (demand)	Minsky regime (demand)
SM	$\frac{\partial \dot{l}_f}{\partial u} < 0$, if $\frac{s_f \pi}{h} + l_f - 1 > 0$	$\frac{\partial \dot{l}_f}{\partial u} > 0$, if $\frac{s_f \pi}{h} + l_f - 1 < 0$
NK	$\frac{\partial \dot{l}_f}{\partial u} < 0$, if $\frac{s_f \pi}{\beta_1 v} + l_f - 1 > 0$	$\frac{\partial \dot{l}_f}{\partial u} > 0$, if $\frac{s_f \pi}{\beta_1 v} + l_f - 1 < 0$
	Steindl regime (growth)	Minsky regime (growth)
SM*	Not economically viable in the long run	$\frac{\partial l_f^*}{\partial g_z} > 0$, if $\frac{\pi u_n}{v} - i > 0$
NK$^\diamond$	Same conditions of the short run apply	Same conditions of the short run apply

Notes: All conditions assume Keynesian stability holds.
$^\diamond$ In the NK model, for a pro-cyclical debt ratio to happen in the long run, $\left[\partial \dot{l}_f / \partial g^*\right]_{\dot{l}_f=0} > 0$. For $l_f < 1$, this happens when $(\partial g^* / \partial u)$ exceeds the partial derivative $(\partial \dot{l}_f / \partial u)$ to the extent that makes the total derivative $\left[\partial \dot{l}_f / \partial u^*\right]_{\dot{l}_f=0} > 0$.

applies to the NK model (from the derivative of equation 28 ($\dot{l}_f$) with respect to l_f), however, in this case the growth rate is a function of a set of parameters as shown in equation 25. While in the SM model a sustainable debt accumulation process depends on the growth rate of autonomous consumption that is exogenous; in the NK model, overall stability of the debt accumulation process depends on how debt itself affects the capacity utilization rate and the growth rate – directly through debt service commitments and indirectly through the capacity utilization rate.

Table 5: Firms' debt ratio dynamic stability conditions

SM model	NK model (partial)
$\frac{\partial \dot{l}_f}{\partial l_f} < 0$	$\frac{\partial \dot{l}_f}{\partial l_f} < 0$
$g_z - s_f i > 0$	$g^* - s_f i > 0$

So far we have not talked about the household sector, however, some comments are in order. From the accounting framework[7] one can observe that households' net wealth

[7]From net worth row in table 1, for specification 1, we have that: $V_h + V_f = V_h + K - L_f = K \rightarrow$

is equivalent to firms' loans. That means whenever a Minskyian debt regime is observed in the firms' sector, households' wealth ratio increases as a result of a stronger level of activity and vice-versa. Therefore, even if we cannot talk about financial fragility in the households' sector, we can say its relative net wealth is decreasing when firms experience a Steindl debt regime for both the SM and the NK model.

Even if both models seem to be able to account for a rising firms' debt ratio during the transition from one steady growth state to the other and, consequently, for a rising financial fragility in its main interpretation (Minsky, 1982, 1986; Taylor and O'Connell, 1985), it is important to acknowledge that there have been some relevant work that highlight the possibility that financial fragility at the micro level does not necessarily add up to a higher aggregate debt to capital ratio (Lavoie and Seccareccia, 2001). In addition to this, more recently some researchers have also questioned the relevance of aggregate debt ratios as indicators of rising financial fragility in firms' sector (Pedrosa, 2019).

4 Growth, demand and households' debt dynamics

In this section, we compare the household debt dynamics in canonical versions of NK and SM models. With that purpose in mind, we adopt a second specification of the benchmark model. To focus on how household debt affects demand and the other way around, we assume that firms take no external funding for investment that is not covered by retained earnings ($L_f = 0$; $\beta_2 = 0$). They otherwise issue equities for that purpose ($p_e E > 0$). We also assume there are no capital gains. This specification allows us to concentrate on household debt and demand interactions without making additional arbitrary assumptions regarding households and firms' savings (since banks do not profit).[8]

Introducing household debt into the analysis ($L_h > 0$) of the SM model implies that induced consumption out of financial income is no longer redundant since the autonomous consumption component can be interpreted as consumption out of new loans taken by poorer households. Therefore the consumption function adopted here will be the one represented by equation 5. As for the NK model, introducing household debt means poorer households emulate richer households consumption ($\eta > 0$) which is given by consumption out of financial income as in equation 8.

After clarifying the adopted specification, we can move on to the comparison of how household debt might affect demand and growth in both models.

$V_h = L_f \rightarrow \frac{V_h}{K} = \frac{L_f}{K} \rightarrow v_h = l_f$.

[8]Equities as a buffer of firms' investment demand financial needs are also found in other NK models that deal with household debt, such as in Kim (2012) and Lavoie (2014).

4.1 Households' leverage ratio impact on demand and growth

Substituting equations 15 and 5 and equations 18 and 8 into equation 22, respectively for the SM and for the NK model, and normalizing it by the capital stock, we get the short-term equilibrium capacity utilization rates (u_1)(equations 30 and 32). Where l_h denotes households' debt ratio in both equations. If we further assume a constant z ratio for the SM model and a constant emulation consumption to capital ratio in the NK model, we obtain equations 31 and 33 (u_2) that allow us to address both the impacts of new loans and debt services on demand. Otherwise, using equations 30 and 32, from the derivative of u with respect to l_h we get just the net impact of interest payment on loans over the capacity utilization rate (see table 6). For analyzing the derivatives of u_2 with respect to l_f in the SM and the NK model, we assume the Keynesian stability condition holds.

In the SM model, taking the derivative of u_2 (equation 31) with respect to l_h, we notice that household debt ratio will have a positive impact on the capacity utilization rate in the short and medium run if the growth rate of autonomous expenditures exceeds the difference between the propensity to consume out of net wage income and the one out of financial income times the interest rate. The intuition behind this condition is that the positive effect of autonomous consumption on income must exceed the net negative impact of debt servicing on households' consumption and, thus, on income. The net effect of interest payments on loans over induced consumption is represented here by the difference between the propensities to consume out of net wage income and out of financial income, once interest payments on loans have a positive effect on richer households consumption and a negative effect on poorer households consumption. Since we are assuming the propensity to consume out of wages is larger than the one out of financial income the net effect of interest payments on loans over income and the level of activity is always negative.

One can notice that the condition for demand to be debt-led is the same that must hold for a positive rate of capacity utilization (and level of output) to exist. Therefore, only the debt-led regime makes sense economically for the SM model (table 6).

In the NK model, from the derivative of u_1 (equation 32) with respect to l_h, we notice that the net debt service will also have a negative impact on demand. This effect will be higher the greater the propensity to consume out of net wage income in relation to the propensity to consume out of financial income and the smaller the emulation parameter for a given interest rate.

Also taking the derivative of u_2 (equation 33) with respect to l_h sheds some light into the mechanisms through which household debt affects demand in the NK model.

Since consumption out of credit is induced by income and only business investment is (partially) autonomous, the condition for demand to be debt-led says that the effect of investment over income has to be greater than the net (negative) impact of interest payments on loans over households' consumption. The likelihood of an increase in households' debt ratio having a positive effect on demand is higher the greater the autonomous investment component and the greater the weight of induced investment on the multiplier (excluding the propensity to consume arising from emulation) for a given net debt service (table 6).

Regarding the impact of household debt on growth in the NK model, since the conditions are virtually the same as those for the impact of household debt on demand we do not repeat them. Since in the SM model the growth rate of autonomous expenditures is exogenous – as a matter of *simplification* – the terminology of a debt-led (burdened) growth regime does not apply.

It is worth noticing that while in the SM only the debt-led regime makes sense economically since both autonomous demand injections and demand leakages are related to household consumption; in the NK model, the debt-burdened regime is also feasible economically as the autonomous investment component may compensate for a net negative effect of household debt on demand.

In the next section, we investigate how demand and growth may impact household debt in both models.

4.2 Demand and growth impacts on households' leverage ratio

At this point, we turn our attention to the conditions under which a higher aggregate demand and/or a faster pace of accumulation leads to a higher or lower household debt ratio. It is important to say that we employ the same terminology we did when analyzing firms debt regimes (specification 1): when a faster pace of economic activity leads to a higher household debt ratio, debt is said to be pro-cyclical, constituting a Minsky regime; when it leads to a lower household debt ratio, debt is said to be anti-cyclical, constituting a Steindl regime. Yet we are aware that we cannot say the Steindl regime, in this case, is the same as the paradox of debt, since the latter refers to the relation of each sector expenditure behavior with its own leverage ratio. Therefore we only refer to the paradox of debt in the household sector when analyzing the effects of the parameters related to household expenditures on the household debt ratio.

We start by finding the differential equation that shows how household debt ratio evolves through time. In the SM model, we normalize equation 4 by the capital stock

Table 6: Households' debt effect on demand and growth

		Capacity utilization rate	Debt demand regimes
Specification 2: $L_h > 0$; $\eta > 0$; $p_e E > 0$; $L_f = 0$; $\beta_2 = 0$	SM	$u_1 = \dfrac{v[z + (\alpha_2 - \alpha_1)il_h]}{1 - \alpha_1(1-\pi) - \alpha_2(1-s_f)\pi - h}$ (30)	
		$u_2 = \dfrac{v[l_h g_z + (\alpha_2 - \alpha_1)il_h]}{1 - \alpha_1(1-\pi) - \alpha_2(1-s_f)\pi - h}$ (31)	Debt-led: $g_z - (\alpha_1 - \alpha_2)i > 0$ Debt-burd: $g_z - (\alpha_1 - \alpha_2)i < 0$
	NK	$u_1 = \dfrac{v\{\beta_0 + [(\eta+1)\alpha_2 - \alpha_1]il_h\}}{1 - \alpha_1(1-\pi) - (\eta+1)\alpha_2(1-s_f)\pi - \beta_1 v}$ (32)	
		$u_2 = \dfrac{v\{\beta_0 + [\beta_0 + (\alpha_2 - \alpha_1)i]l_h\}}{1 - \alpha_1(1-\pi) - \alpha_2(1-s_f)\pi - \beta_1 v(1+l_h)}$ (33)	Debt-led: $\beta_0 - \dfrac{(\alpha_1 - \alpha_2)i}{1 + \dfrac{\beta_1 v}{x}} > 0$ Debt-burd: $\beta_0 - \dfrac{(\alpha_1 - \alpha_2)i}{1 + \dfrac{\beta_1 v}{x}} < 0$
		Growth rate	**Debt growth regimes**
	SM	$g^* = g_z$	–
	NK	$g^* = \beta_0 + \beta_1 \left\{ \dfrac{v\{\beta_0 + [\beta_0 + (\alpha_2 - \alpha_1)i]l_h^*\}}{1 - \alpha_1(1-\pi) - \alpha_2(1-s_f)\pi - \beta_1 v(1+l_h^*)} \right\}$ (34)	Same conditions for debt demand regimes apply

Legend: $x = 1 - \alpha_1(1-\pi) - \alpha_2(1-s_f)\pi - \beta_1 v$
Note: All conditions assume Keynesian stability holds.

and then substitute equation 17 into the resulting equation. This leads to the differential equation in 35.

$$\dot{l}_h = z - \frac{hu}{v} l_h \tag{35}$$

For analyzing the effects demand and growth may have on households leverage ratio it is useful to obtain an equation for the autonomous consumption ratio. We do that by solving equation 22 for Z and normalizing the result by the capital stock (equation 36).

$$z = \frac{u}{v}(s - h) + (\alpha_1 - \alpha_2) i l_h \tag{36}$$

Where $s = 1 - \alpha_1(1 - \pi) - \alpha_2(1 - s_f)\pi$ represents the marginal propensity to save.

$$\dot{l}_h = \frac{u}{v}(s - h) + (\alpha_1 - \alpha_2) i l_h - \frac{hu}{v} l_h \tag{35A}$$

Substituting equation 36 into equation 35 (equation 35A) and taking the derivative of $\dot{l}_h$ with respect to u, we notice the likelihood of a higher level of activity being associated with a lower household debt ratio is higher the lower the marginal propensity to save, the higher the household debt ratio and the firms' propensity to invest. On the other hand, it is more likely that an increase in the capacity utilization rate will be associated with a higher household debt to capital ratio the higher the marginal propensity to save in relation to the propensity to invest and the lower the household debt to capital ratio (table 7). This condition shows that in the SM model there can happen surges in household debt ratio during expansions in the level of economic activity.

In comparison with the conditions for firms' debt regimes in specification 1, we notice that, cet.par., while a higher marginal propensity to invest reduces (increases) the likelihood of a Steindl (Minsky) debt regime in firms' sector for specification 1; it increases (reduces) the likelihood of a Steindl (Minsky) debt regime in households' sector for specification 2. As for the debt ratio, cet.par., a higher (lower) debt ratio increases (decreases) the likelihood of a Steindl (Minsky) debt regime in the household sector in specification 2, as was the case for firms' sector in specification 1.

In the steady growth path where all variables grow at the same rate g_z ($\dot{l}_h = 0$) and firms have adjusted their investment plans ($\dot{h} = 0$ and $u = u_n$), households' debt ratio will be given by equation 37 and the autonomous consumption ratio by equation

38.[9]

$$l_h^* = \frac{z^*}{g_z} \tag{37}$$

$$z^* = \frac{su_n}{v} - g_z + (\alpha_1 - \alpha_2)il_h^* \tag{38}$$

In the long run, from the derivative of l_h^* (equation 37) with respect to g_z we know that for an expansion in the pace of households expenditures to reduce their leverage ratio – that is, for the paradox of debt to emerge – it is a necessary and sufficient condition having a positive autonomous consumption to capital ratio (table 7).

Yet since we have an equilibrium autonomous consumption ratio, we can further substitute equation 38 into equation 37 and solve it for l_h in order to get the ultimate steady growth solution for household debt ratio (equation 37A).

$$l_h^* = \frac{\frac{su_n}{v} - g_z}{g_z - (\alpha_1 - \alpha_2)i} \tag{37A}$$

We know that the denominator of equation 37A must be positive if we are dealing with a debt-led demand regime, which is the only case of interest in this canonical SM model. Taking that into consideration, a positive steady growth household debt ratio also requires its numerator to be positive. That happens when the growth rate of autonomous demand is lower than the maximum demand-led growth rate of capacity (su_n/v). One can observe that the combination of these two conditions shows the range of values in which the growth rate is demand-led and demand is debt-led, the latter also implies a condition for the model to make sense economically.

$$(\alpha_1 - \alpha_2)i < g_z < \frac{su_n}{v} \tag{39}$$

Also taking the derivative of equation 37A with respect to g_z, we observe that the condition for the paradox of debt to emerge in the long run (that is, a positive autonomous consumption ratio) requires a maximum demand-led growth rate of capacity that is greater than the net debt service effect ($su_n/v - (\alpha_1 - \alpha_2)i > 0$) (table 7).

For the NK model, we do the same procedure: we normalize equation 7 by the capital stock (equation 40) and then substitute equation 19 into the resulting equation,

[9]To obtain equation 38, besides supposing $u = u_n$, we substitute the assumption that $g_k = g_z$ into equation 17 and solve it for h. We then substitute the steady growth propensity to invest $h^* = \frac{vg_z}{u_n}$ into equation 36.

which leads to equation 41. From the derivative of $\dot{l}_h$ (equation 41) with respect to u we arrive at the conditions required for the debt regimes to emerge.

From table 7, we notice that a Steindl (Minsky) debt regime in the household sector is more likely to happen the lower (the higher) the emulation of household consumption out of dividends in relation to the induced part of investment and the higher (the lower) the household debt ratio. We notice that similarly to what happens in the SM model, in the NK model, cet. par., a higher induced investment ratio contributes to bringing about an anti-cyclical household leverage ratio in the household sector (specification 2) and a pro-cyclical firms' leverage ratio (specification 1).

$$\dot{l}_h = \eta\alpha_2(1-s_f)\pi\frac{u}{v} + \eta\alpha_2 i l_h - g_k l_h \tag{40}$$

$$\dot{l}_h = \left[\frac{\eta\alpha_2(1-s_f)\pi}{v} - \beta_1 l_h\right] u + (\eta\alpha_2 i - \beta_0) l_h \tag{41}$$

As was the case for the analysis of firms' debt ratio in the NK model, in the case of household debt, solving equation 41 for $\dot{l}_h = 0$ also leads to a second-degree polynomial equation. Since the stability conditions of these two possible equilibrium solutions for household debt ratio have been explored elsewhere, as in Setterfield et al. (2016) and Kim (2012), we do not go deeper into this matter. Instead we employ the same procedure used in specification 1 to analyze firms' debt regimes in the NK model in the long run.

We assume that, for a steady household debt ratio ($\dot{l}_h = 0$) and lower than one, a Minsky regime in the household sector arises when the direct positive effect of the capacity utilization on the debt ratio (since a higher emulation of consumption out of distributed profits increases households borrowing needs) more than compensates the effect of capacity utilization on the capital accumulation rate, making the total derivative of household debt ratio with respect to the capacity utilization in the surroundings of the equilibrium greater than zero ($[\partial\dot{l}_h/\partial u^*]_{\dot{l}_h=0} > 0$). Once again the conditions for the debt regimes to happen in the long run of the NK model are the same ones that hold in the short run.

To account for the final effect of the emulation parameter on household debt ratio in the long run we have to consider both its direct effect on household debt – that we know to be positive[10]– and its indirect effect on the capacity utilization rate (and, consequently, on the capital accumulation rate) and, in turn, the effect of this variable on household debt ratio. We also know that whenever the emulation parameter has a positive effect on

[10] As shown for similar variants in Kim (2012); Lavoie (2014)

the capacity utilization rate it will also have a positive effect on the capital accumulation rate as it is defined in the NK model.

Taking the derivative of the capacity utilization rate (equation 32) with respect to the emulation parameter, we notice that it has positive effect on the level of activity (and growth) when both the capacity utilization rate and household debt assume positive values (assuming also that Keynesian stability holds). We will focus only on this case since it represents a zone of the model where the variables have economic meaning.[11]

Therefore, for the case in hand where the emulation parameter has a positive effect on the capacity utilization rate, two scenarios are possible in what concerns the household debt ratio: (i) when both the emulation parameter and the capacity utilization rate have a positive effect on household debt ratio, there is no paradox of debt in the long run; (ii) when the emulation parameter has a direct positive effect on household debt ratio, but capacity utilization has a negative effect on household debt ratio, the result is ambiguous. It will depend on the magnitudes of the derivatives. The paradox of debt would emerge only when the capacity utilization rate effect on household debt (through its impact on the accumulation rate that negatively affects the debt ratio) exceeds the one of emulation on this same ratio (table 7).

We notice that both in the SM and the NK model episodes of rising household debt ratio may follow a demand stimulus as proxied by an increase in the capacity utilization rate in the short run. For both models, the intuition is again similar: cet. par., the lower the household debt and the lower the marginal propensity to invest (or the induced component of investment in the case of the NK model) the more likely household debt will be pro-cyclical.

In the long run, while in the SM model there is always the paradox of debt, as any increase in the pace of consumption out of credit (autonomous consumption) leads to a lower household debt ratio; in the NK model, the partial direct effect of an increase in the wish to emulate richer households consumption, also financed by credit, is always positive. Yet the total effect of the emulation parameter on household debt ratio also depends on how it affects the capacity utilization rate – and the growth rate – of the economy and on how the latter affects household debt. Assuming the emulation parameter has a positive effect on the capacity utilization rate, there will be no paradox of debt if the capacity utilization rate also positively impacts the household debt ratio. Otherwise, if the capacity utilization rate has a negative effect on household debt ratio (through its impact on the growth rate), the paradox of debt will emerge when the magnitude of this negative effect exceeds the positive effect of emulation on household debt (table 7).

[11]For the derivatives see Appendix A.

Table 7: Household debt regimes in NK and SM models

	Steindl debt regime (demand)	Minsky debt regime (demand)
SM	$\frac{\partial \dot{l}_h}{\partial u} < 0$, if $\frac{s}{h} - 1 - l_h < 0$	$\frac{\partial \dot{l}_h}{\partial u} > 0$, if $\frac{s}{h} - 1 - l_h > 0$
NK	$\frac{\partial \dot{l}_h}{\partial u} < 0$, if $\frac{\eta \alpha_2 (1 - s_f) \pi}{\beta_1 v} - l_h < 0$	$\frac{\partial \dot{l}_h}{\partial u} > 0$, if $\frac{\eta \alpha_2 (1 - s_f) \pi}{\beta_1 v} - l_h > 0$
	Paradox of debt	No paradox
SM*	$\frac{\partial l_h^*}{\partial g_z} < 0$, if $z^* > 0$ or $\frac{s u_n}{v} - (\alpha_1 - \alpha_2) i > 0$	Not economically meaningful
NK$^\diamond$	$\left[\frac{\partial \dot{l}_h}{\partial \eta}\right]_{\dot{l}_h = 0} < 0$, if $\left(\frac{\partial \dot{l}_h}{\partial \eta} > 0\right) \cap \left(\frac{\partial \dot{l}_h}{\partial u^*} < 0\right) \cap \left(\frac{\partial \dot{l}_h}{\partial u^*} > \frac{\partial \dot{l}_h}{\partial \eta}\right)$	$\left[\frac{\partial \dot{l}_h}{\partial \eta}\right]_{\dot{l}_h = 0} > 0$, if $\left(\frac{\partial \dot{l}_h}{\partial \eta} > 0\right) \cap \left(\frac{\partial \dot{l}_h}{\partial u^*} > 0\right)$ $\left(\frac{\partial \dot{l}_h}{\partial \eta} > 0\right) \cap \left(\frac{\partial \dot{l}_h}{\partial u^*} < 0\right) \cap \left(\frac{\partial \dot{l}_h}{\partial u^*} < \frac{\partial \dot{l}_h}{\partial \eta}\right)$

Note: All conditions assume Keynesian stability holds.
$^\diamond$ In the NK model, the paradox of debt conditions assume $[\partial u^* / \partial \eta] > 0$.

We should mention that there is a main difference in the way the condition for the paradox of debt to exist can be formulated in each model relating to their specific closure: in the SM model, the condition for the paradox of debt to exist is formulated in terms of growth, since the growth rate of autonomous consumption financed by credit is exogenous. The same does not hold for the NK model, in which the growth rate is endogenous and led by investment. Therefore, in the NK model, the condition for the paradox to emerge must be put in terms of the exogenous emulation parameter that proxies an increase in household wish to borrow for consumption purposes.

Besides that, in the NK model, cet.par., an increase in the emulation parameter also leads, for positive values of the capacity utilization rate, to a higher emulation consumption (consumption out of credit) to capital ratio.[12] In the SM model, the equivalent consumption out of credit ratio is z. From equation 38, we know that, for a given growth rate, an increase in the autonomous consumption ratio due to an increase in the propen-

[12]We obtain this ratio by normalizing the second term on the RHS of equation 8 by the capital stock.

sity to save (caused by a redistribution of income towards profits for instance), will lead to a higher household debt ratio. Therefore, other things being equal, in both models, a higher consumption out of credit ratio is associated with a higher household debt ratio.

Even though an increase in the growth rate of autonomous consumption always leads to a reduction in the steady growth household debt ratio in the SM model, since it will trigger firms' reaction to the observed increase in capacity utilization that follows the higher consumption pattern and, thus, will lead to an acceleration in the pace of capital accumulation that makes income grow at a faster pace than debt; it is not true that the model does not allow for an increase in the liabilities-to-income ratio in the household sector at all circumstances. We have seen that a higher autonomous consumption ratio increases the household debt ratio for a given growth rate. Besides that, taking the derivative of equation 37A with respect to the interest rate as shown in 42, we notice that an increase in the interest rate – as long as households have a larger propensity to consume out of wages in relation to the one out of financial income and as long as growth is demand-led – will increase household debt ratio. So even if higher interest rates do not affect the pace of debt accumulation (dictated by g_z) they affect negatively the disposable wage income and, as a result, household consumption. In turn, this will temporarily slow down the pace of capital accumulation, leading to a higher household debt to capital ratio.

$$\frac{\partial l_h^*}{\partial i} = \frac{(\alpha_1 - \alpha_2)\left(\dfrac{su_n}{v} - g_z\right)}{[g_z - (\alpha_1 - \alpha_2)i]^2} \tag{42}$$

That is also to say that even if the model does not allow in its canonical version for an increase in household debt ratio as the economy grows at a faster pace, it allows for a rough representation of a financial crisis process as triggered, for instance, by a credit constraint imposed by banks or a reduction in households' willingness to borrow more from banks. In the model, this could be represented by a lower rate of growth of autonomous expenditures. The process that follows shows that since this reduction in the pace of autonomous expenditures has a negative effect on income and capital accumulation, households' debt to income ratio increases as a result of a change in households and/or banks' behavior.

4.3 Dynamic stability conditions

Analysing the household debt ratio dynamics in the SM model (table 8), we realize that for the ratio to be stable autonomous expenditures (and the capital stock in the short run) must grow at a faster pace than the negative effect of net debt service on household consumption.[13] At this point, a careful reader would have noticed that the stability condition for the household debt ratio is the same condition required for demand to be debt-led. That said, a debt-burdened demand regime would fall into an unstable zone of the model. If we think about the economic reasoning, it makes sense that in an economy where household consumption financed by credit leads growth if debt services (or debt amortization, which we are not considering for simplification) put a very strong drag on demand its trajectory will be unstable.

As for the NK model, household debt ratio stability requires the condition in column two, first row, of table 8 to be verified.[14] We can see that for household debt ratio to be stable autonomous investment must grow at a faster pace than the one given by these parameters. If we attribute values to these parameters in the stability condition, we observe that the growth rate of autonomous investment required for the stability condition to hold is positive but not very high. Therefore the condition can be easily satisfied.

For the stability in the neighbourhood of the two feasible equilibrium values for household debt, we rely on the analysis of the NK model presented by Setterfield et al. (2016) since household debt dynamics of our variant is very similar to theirs. Therefore, if we are dealing with a debt-burdened growth regime ($\partial g^*/\partial l_h < 0$) a high value of household debt ratio may be destabilizing, increasing the likelihood of an unstable debt dynamics ($\partial \dot{l}_h/\partial l_h > 0$) for high values of household debt ratio, so the smaller of the two solutions ($l^*_{h_1}$) will be the stable steady state solution. Otherwise, when the growth regime is debt-led ($\partial g^*/\partial l_h > 0$), a high value for household debt ratio may be stabilizing, increasing the likelihood of a stable debt dynamics ($\partial \dot{l}_h/\partial l_h < 0$) for high values of household debt. In that case, the larger of the two solutions is associated with a stable steady growth state ($l^*_{h_2}$).

5 Final remarks

In the previous sections, we carried out a comparative analysis of firms' and households' debt accumulation process in canonical versions of both the SM and the NK model. In this comparison, we addressed both the effects of debt on short run capacity utilization and on the growth rate (for the NK model) and the effects of short run capacity utilization and growth on debt accumulation. We also analyzed the partial stability conditions of debt ratios of households and firms sector. From this comparative exercise, we have reached

[13] To get this condition we take the derivative of equation 35A with respect to l_h.

[14] We obtain this condition by substituting equation 32 into equation 41 and taking its derivative with respect to l_h. For this derivative see Appendix A.

Table 8: Households' debt ratio dynamic stability conditions

SM model	NK model
$\dfrac{\partial \dot{l}_h}{\partial l_h} < 0$, if $g_k - (\alpha_1 - \alpha_2)i > 0$	$\dfrac{\partial \dot{l}_h}{\partial l_h} < 0$, if $\beta_0 - \dfrac{[\eta\alpha_2(1-s_f)\pi - 2\beta_1 v l_h][(\eta+1)\alpha_2 - \alpha_1]i + \eta\alpha_2 i\phi}{\phi + \beta_1 v} > 0$
$\left[\dfrac{\partial \dot{l}_h}{\partial l_h}\right]_{l_h^*} < 0$, if $g_z - (\alpha_1 - \alpha_2)i > 0$	$\left[\dfrac{\partial \dot{l}_h}{\partial l_h}\right]_{l_{h_1}^*} < 0$, if $\dfrac{\partial g^*}{\partial l_h} < 0$ $\left[\dfrac{\partial \dot{l}_h}{\partial l_h}\right]_{l_{h_2^*}} < 0$, if $\dfrac{\partial g^*}{\partial l_h} > 0$

Legend: $\phi = 1 - \alpha_1(1-\pi) - (\eta+1)\alpha_2(1-s_f)\pi - \beta_1 v$
Note: All conditions assume Keynesian stability holds.

some conclusions in what regards the process of debt accumulation in these models.

Starting by firms' debt (specification 1), we have seen that both in the SM and the NK model, firms can face either a Steidlian or a Minskyian debt regime. So in the SM model as in the NK one, an increase in financial fragility as represented by a higher debt ratio in the firms' sector is feasible. And, in fact, in the long run of the SM model, only the Minskyan debt regime is economically feasible, since the condition for a Steindlian debt regime to hold in firms' sector requires a profit rate lower than the interest rate, scenario in which there would be no positive capital accumulation.

That said, the intuition for the emergence of a debt regime in firms' sector is very similar in both models. For instance, a Minsky debt regime is more likely to emerge, cet. par., the lower the leverage ratio and the lower the retained share of profits in relation to the propensity to invest (or the induced part of investment in the NK model).

The difference between the debt regimes in these models is that while in the SM model household behavior has a more patent effect on firms' debt regime through the effect of autonomous expenditures on firms' propensity to invest; in the NK model it relies mostly on firms own behavior.

In the SM model, firms' debt ratio will have no effect neither on the short run capacity utilization rate nor on the growth rate of the economy[15] while in the NK model it does affect these variables. Besides that, in the SM model firms' leverage ratio stability depends directly on the autonomous consumption growth rate since it dictates the pace

[15] It goes without saying that for specification 2, household debt ratio will also have no impact on the growth rate in the long run.

of capital accumulation. In the NK model, firms' leverage ratio stability depends on the investment growth rate which is a function of the model's parameters. These results can be said to depend on the specific closure of each model.

In what concerns household debt accumulation (specification 2), in the canonical version of the SM model either a pro-cyclical or anti-cyclical household debt regime in relation to demand (to the capacity utilization rate) is feasible in the short and medium run. This also holds for the NK model. In both models, household indebtedness depends on firms behavior through the propensity to invest (or the induced part of investment in the NK model) with the opposite sign of its effect on firms' indebtedness in specification 1.

Regarding the paradox of debt in the household sector, in the SM model, there is always the paradox of debt in the long run provided a positive autonomous consumption ratio exists. In turn in the NK model, it is likely there will be no paradox of debt in the long run, yet a paradox of debt is possible when an increase in poorer households' wish to emulate richer households consumption positively affects the growth rate and the latter negatively affects the leverage ratio to a larger extent than the emulation parameter itself.

From these results, we notice that more than depending on each model closure, the specific closures define how the paradox of debt can be postulated in the household sector: in the SM model, debt-financed household expenditure behavior is a function of the autonomous consumption growth rate; while in the NK model, it is a function of the emulation parameter that affects household expenditures directly and indirectly through the capacity utilization (and growth) rate.

For both SFC versions of the canonical models, it is fair to say that household expenditure behavior is dictated by an exogenous variable. Therefore in the NK model, as in the SM one, an increase in household debt ratio will *not* be followed by an *endogenous* adjustment of expenditure decisions by households even if they are in a financially fragile situation. That said, it is clear this qualification does not apply exclusively to the SM model as seems to be claimed by Nikiforos (2018). Whether within the SFC approach or not, models of this nature rely, by definition, on the assumption that part of investment or consumption demand is exogenous.

Also for both models, whether demand is debt-led or debt-burdened depends on the relation between autonomous demand injections (autonomous consumption for the SM model and autonomous investment for the NK model) and autonomous leakages (net debt service payments that put a drag on induced consumption). What changes is that, for a given interest rate, in the SM model this relation depends basically on household consumption behavior, while in the NK model it depends also on firms investment behavior.

As for household leverage ratio stability, while in the SM model it is only stable when demand is debt-led; in the NK model, a stable household debt ratio is compatible either with a debt-led or debt-burdened demand regime. While household debt stability depends on a minimum autonomous consumption growth rate in the SM model; in the NK model, it depends on a minimum autonomous investment growth rate.

In a nutshell, in the SM model: (i) increases in autonomous consumption financed by debt certainly lead to a higher accumulation of household debt. Yet when it triggers firms' reaction through a higher marginal propensity to invest, the pace of accumulation will also increase. If the pace of capital accumulation temporarily exceeds the one of autonomous expenditures, household debt ratio will decrease in the long run; (ii) there will be episodes of rising household leverage ratio following increases in interest rates, in the profit share, and also following a decrease in the growth rate of autonomous expenditures. Even if interest rates and income distribution (profit share) have no permanent effect on the growth rate in the canonical version of the model, a positive shock to these variables will lead to a permanently higher household debt ratio. That said, even if there is no permanent increase in the leverage ratio following an increase in the autonomous expenditure growth rate, an episode of financial crisis as triggered by policy decisions can still be represented in the SM model.

As a more general assessment of these models, none of them in their canonical versions seem appropriate to deal with an endogenous process of rising financial fragility. As Lavoie points out: "(...) another line of defence of the financial instability hypothesis has been that steady-state models cannot claim to be faithful to Minsky's views, since a crucial feature of these is that 'stability is destabilizing' (...)"(Lavoie, 2014, p.446). But then this is not a matter of choosing the appropriate model closure but rather choosing the appropriate method of analysis to the issue of interest.

Moreover, for firms' sector, a higher or increasing aggregate leverage ratio may not say as much as we would like about financial fragility as an indicator of firms' ability to come up with the funds needed to cover for their cash commitments (Pedrosa, 2019). As for the household sector, we also know that the distribution of liabilities across different income groups and income inequality also matter for financial fragility (Cardaci, 2018). That might indicate that purely macroeconomic models are not the right place to deal in detail with these issues.

Given these caveats, one can still represent the outcomes of a very stylized financial crisis in the sense employed by Toporowski (2005), according to whom a financial crisis and/or the turning point of a boom depends on an explicit policy decision, in a simplified macroeconomic model provided the appropriate financial crisis channels one is interested into are in place. We, therefore, claim that the SM model can be modified to tackle changes in household demand for credit and banks lending rules and to address how these could affect the growth rate of autonomous expenditures by the introduction of the appropriate behavioral equations as has been done in NK models.

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Appendix A

A.1 Derivatives for specification 1

Supermultiplier model

$$\frac{\partial u}{\partial l_f} = 0$$

$$\frac{\partial g_z}{\partial l_f} = 0$$

$$\frac{\partial \dot{l}_f}{\partial u} = \frac{h(1-l_f) - s_f\pi}{v}$$

$$\frac{\partial \dot{l}_f}{\partial u} \gtreqless 0 \rightarrow \frac{s_f\pi}{h} + l_f - 1 \lesseqgtr 0$$

$$\frac{\partial l_f^*}{\partial g_z} = \frac{s_f\pi\frac{u_n}{v} - s_f i}{(g_z - s_f i)^2}$$

$$\frac{\partial l_f^*}{\partial g_z} \gtreqless 0 \rightarrow \pi\frac{u_n}{v} - i \gtreqless 0$$

$$\frac{\partial \dot{l}_f}{\partial l_f} = s_f i - g_z < 0 \rightarrow g_z - s_f i > 0$$

Neo-Kaleckian model

$$\frac{\partial u}{\partial l_f} = \frac{(\alpha_2 s_f - \beta_2)iv}{x}$$

Where $x = 1 - \alpha_1(1-\pi) - \alpha_2(1-s_f)\pi - \beta_1 v$

$$\frac{\partial u}{\partial l_f} \gtreqless 0 \rightarrow \alpha_2 s_f - \beta_2 \gtreqless 0$$

$$\frac{\partial g^*}{\partial l_f} = \frac{\beta_1 v(\alpha_2 s_f - \beta_2)i - \beta_2 i x}{x}$$

$$\frac{\partial g^*}{\partial l_f} \gtreqless 0 \rightarrow \alpha_2 s_f - \beta_2\left(1 + \frac{x}{\beta_1 v}\right) \gtreqless 0$$

$$\frac{\partial \dot{l}_f}{\partial u} = \beta_1(1-l_f) - s_f\frac{\pi}{v}$$

$$\frac{\partial \dot{l}_f}{\partial u} \gtreqless 0 \rightarrow \frac{s_f\pi}{\beta_1 v} + l_f - 1 \lesseqgtr 0$$

$$\frac{\partial \dot{l}_f}{\partial l_f} = s_f i - g_I < 0 \rightarrow g_I - s_f i > 0$$

A.2 Derivatives for specification 2

Supermultiplier model

$$\frac{\partial u_2}{\partial l_h} = \frac{v[g_z + (\alpha_2 - \alpha_1)i]}{1 - \alpha_1(1-\pi) - \alpha_2(1-s_f)\pi - h}$$

$$\frac{\partial u_2}{\partial l_h} \gtreqless 0 \rightarrow g_z - (\alpha_1 - \alpha_2)i \gtreqless 0$$

$$\frac{\partial g_z}{\partial l_h} = 0$$

$$\frac{\partial \dot{l}_h}{\partial u} = \frac{s - h(1 + l_h)}{v}$$

$$\frac{\partial \dot{l}_h}{\partial u} \lesseqgtr 0 \rightarrow \frac{s}{h} - 1 - l_h \gtreqless 0$$

$$\frac{\partial l_h^*}{\partial g_z} = -\frac{z^*}{g_z^2}$$

$$\frac{\partial l_h^*}{\partial g_z} \lesseqgtr 0 \rightarrow z^* \gtreqless 0$$

Or

$$\frac{\partial l_h^*}{\partial g_z} = \frac{(\alpha_1 - \alpha_2)i - \frac{su_n}{v}}{[g_z - (\alpha_1 - \alpha_2)i]^2}$$

$$\frac{\partial l_h^*}{\partial g_z} \lesseqgtr 0 \rightarrow \frac{su_n}{v} - (\alpha_1 - \alpha_2)i \gtreqless 0$$

$$\frac{\partial l_h^*}{\partial i} = \frac{(\alpha_1 - \alpha_2)\left(\frac{su_n}{v} - g_z\right)}{[g_z - (\alpha_1 - \alpha_2)i]^2}$$

$$\frac{\partial \dot{l}_h}{\partial l_h} = (\alpha_1 - \alpha_2)i - g_k < 0$$

$$\rightarrow g_k - (\alpha_1 - \alpha_2)i > 0$$

$$\left[\frac{\partial \dot{l}_h}{\partial l_h}\right]_{l_h^*} = (\alpha_1 - \alpha_2)i - g_z < 0$$

$$\rightarrow g_z - (\alpha_1 - \alpha_2)i > 0$$

Neo-Kaleckian model

$$\frac{\partial u_1}{\partial l_h} = \frac{v[(\eta+1)\alpha_2 - \alpha_1]i}{1 - \alpha_1(1-\pi) - (\eta+1)\alpha_2(1-s_f)\pi - \beta_1 v}$$

$$\frac{\partial u_1}{\partial l_h} \gtreqless 0 \rightarrow \eta + 1 - \frac{\alpha_1}{\alpha_2} \gtreqless 0$$

$$\frac{\partial u_1}{\partial \eta} = \frac{v\alpha_2 i l_h \phi}{\phi^2}$$
$$\frac{+\alpha_2(1-s_f)\pi v[\beta_0 + [(\eta+1)\alpha_2 - \alpha_1]il_h]}{\phi^2}$$

Where $\phi = 1 - \alpha_1(1-\pi) - (\eta+1)\alpha_2(1 - s_f)\pi - \beta_1 v$

$$\frac{\partial u_1}{\partial \eta} \gtreqless 0 \rightarrow u_1, l_h \gtreqless 0$$

$$\frac{\partial u_2}{\partial l_h} = \frac{v\varphi[\beta_0 + (\alpha_2 - \alpha_1)i]}{\varphi^2}$$
$$+\frac{\beta_1 v^2[\beta_0 + (\beta_0 + (\alpha_2 - \alpha_1)i)l_h]}{\varphi^2}$$

Where $\varphi = 1 - \alpha_1(1-\pi) - \alpha_2(1-s_f)\pi - \beta_1 v(1 + l_h)$

$$\frac{\partial u_2}{\partial l_h} \gtreqless 0 \rightarrow \beta_0 - \frac{(\alpha_1 - \alpha_2)i}{1 + \frac{\beta_1 v}{x}} \gtreqless 0$$

Where $x = 1 - \alpha_1(1-\pi) - \alpha_2(1-s_f)\pi - \beta_1 v$

$$\frac{\partial \dot{l}_h}{\partial u} = \frac{\eta\alpha_2(1-s_f)\pi}{v} - \beta_1 l_h$$

$$\frac{\partial \dot{l}_h}{\partial u} \gtreqless 0 \rightarrow \frac{\eta\alpha_2(1-s_f)\pi}{\beta_1 v} - l_h \gtreqless 0$$

Neo-Kaleckian model (cont.)

Substituting equation 32 into equation 40, we get the following equation for household debt dynamics:

$$\dot{l_h} = \frac{\{\eta\alpha_2(1-s_f)\pi - \beta_1 l_h v\}\{[(\eta+1)\alpha_2 - \alpha_1]il_h + \beta_0\}}{\phi} + (\eta\alpha_2 i - \beta_0)l_h$$

This is the equation we use to derive the stability condition of table 8 departing from the following derivative:

$$\frac{\partial\dot{l_h}}{\partial l_h} = \frac{\eta\alpha_2(1-s_f)\pi[(\eta+1)\alpha_2 - \alpha_1]i - 2l_h\beta_1 vi[(\eta+1)\alpha_2 - \alpha_1] - \beta_1 v\beta_0}{\phi} + \eta\alpha_2 i - \beta_0$$