Bull and Bear Markets During the COVID-19 Pandemic

Maheu, John M and McCurdy, Thomas H and Song, Yong

McMaster University, University of Toronto, University of Melbourne

November 2020

Bull and Bear Markets During the COVID-19 Pandemic*

John M. Maheu† Thomas H. McCurdy‡ Yong Song§

November 2020

Abstract

The COVID-19 pandemic has caused severe disruption to economic and financial activity worldwide. We assess what happened to the aggregate U.S. stock market during this period, including implications for both short and long-horizon investors. Using the model of Maheu, McCurdy and Song (2012), we provide smoothed estimates and out-of-sample forecasts associated with stock market dynamics during the pandemic. We identify bull and bear market regimes including their bull correction and bear rally components, demonstrate the model's performance in capturing periods of significant regime change, and provide forecasts that improve risk management and investment decisions. The paper concludes with out-of-sample forecasts of market states one year ahead.

Key Words: predictive density, long-horizon returns, Markov switching

*We thank Marco Salerno, Xiaofei Zhao and workshop participants at the Rotman School of Management, Deakin University, Shanghai University of Finance and Economics, Kobe University and the NBER-NSF conference for comments. Maheu thanks SSHRC for financial support.

†DeGroote School of Business, McMaster University, maheujm@mcmaster.ca

‡Rotman School of Management, University of Toronto, tmccurdy@rotman.utoronto.ca

§Department of Economics, University of Melbourne, yong.song@unimelb.edu.au

1 Introduction

This paper dates and forecasts bull and bear markets for the COVID-19 pandemic period based on aggregate equity return data from 1885-2020. Using the model of Maheu et al. (2012) applied to weekly data from 1885-2020, we document where the market was and what has happened to equity markets in 2020.

There are several reasons for using the restricted 4-state Markov-Switching (MS) model of Maheu et al. (2012). First, unlike *ex post* dating methods (Pagan & Sossounov 2003, Lunde & Timmermann 2004), it treats the market states as latent and provides probability estimates for future states and regimes. This probability framework provides a full specification of the data generating process. As such, it generates probability statements about the market dynamics which are essential for investment and risk management decisions.

Secondly, in contrast to simple specifications which focus on two states of the market, our model allows for four states including bull corrections and bear rallies. Conventional methods of partitioning regimes do not identify intra-regime dynamics that can be very important for forecasts and investment decisions. For example, if a rally (positive sub-trend) starts during a bear regime, what is the probability that it will continue and transition to a bull market regime as opposed to falling back to the bear market state? Will a bull market correction (negative sub-trend) continue to a bear regime or recover to stay in the bull market regime?

Thirdly, higher-order moments of the state-specific distributions also provide useful information – for example, risk assessments associated with different states. They combine to provide the aggregate mixture distribution that governs the market dynamics and the associated predictive mean and density forecasts that are key for decisions.

Finally, the parameters and states are identified with economic restrictions that are consistent with investors attitudes to market phases. The data supports these economically motivated restrictions compared to an unrestricted 4-state model.

We find that the market moved from a bull state at the start of 2020 to a bull correction and quickly to a bear market on February 26. This bear market dominated until June 3 when the market transitioned to a bear rally and has remained in this phase. This is in contrast to the dating methods used in the popular press ("New bull market in stocks could last three years and may produce another 30% in gains, veteran strategist says", Aug 31, 2020, https://www.marketwatch.com/). Those methods focus solely on price trends and ignore risk. Our model classifies the recent market into the bear rally due to the larger variance in returns. This elevated risk is not consistent with past bull market phases that display lower variability.

Traditional 2-regime models using *ex post* dating methods are unable to distinguish between bear rally and bull market states. Our approach does this, in part, due to probability estimates of risk differences across those states. We show the dramatic impact that the COVID-19 pandemic has had on the return distribution and risk measures. The model provides very accurate forecasts of turning points and these would have been available in real-time to investors.

Given the benefits of a full probability model of stock market phases, it is natural ask whether forecasts from this mixture-distribution model can improve investment and risk management decisions. To this end, we define a pseudo Sharpe ratio to characterize in-sample estimates of the state density parameters. We then extend this measure to an out-of-sample predictive Sharpe ratio, derived from the predictive density of returns which is sensitive to the forecasted market states. This measure can be useful in assessing the risk and return of entering the market.

Several market timing investments are explored. We show that simple timing rules directing when to exit and enter the market lead to improved investment decisions relative to a buy and hold strategy in 2020. These results are robust to different timing strategies and are a result of the precise turning points our model identifies and forecasts. Each of these market timing strategies are in real time and would have been available to an investor using the model for forecasts.

The paper concludes with long-horizon forecasts of state probabilities one year ahead. If the effects of COVID-19 were to disappear today, then our model predicts months, and not weeks, till the stock market returns to normal times.

Our paper is organized as follows. Sections 2 and 3 briefly review the structure and estimation of the Maheu et al. (2012) bull and bear market model; and Section 4 summarizes the data. Section 5 reports the results for the pre and post COVID-19 periods. Notably, Section 5.3 provides forecasts based on one-week-ahead predictive densities and out-of-sample forecasts of future states and the associated risk and return measures. Section 5.4 reports market timing strategies that exploit those forecasts. Long-horizon forecasts are discussed in Section 5.5. Section 6 provides robustness results, comparing our model to competing models from several perspectives. Section 7 concludes and an Appendix provides additional results and model comparison details.

2 Model

Define log-returns as r_t, $t = 1, \dots, T$ and $r_{1:t-1} = \{r_1, \dots, r_{t-1}\}$. Consider the following 4-state Markov-Switching (MS4) model from Maheu et al. (2012) for returns,

$$\begin{aligned} r_t | s_t &\sim N(\mu_{s_t}, \sigma^2_{s_t}) \\ p_{ij} &= p(s_t = j | s_{t-1} = i), \quad i = 1, ..., 4, \quad j = 1, ..., 4. \end{aligned}$$

in which $s_i, i = 1, ..., 4$, denote the latent states, parameterized as Normally distributed with mean μ_{s_i} and variance $\sigma^2_{s_i}$, and $p_{i,j}$ denote the state transition probabilities.

The following restrictions and labels are imposed for identification purposes,

$$\begin{aligned} \mu_1 &< 0 \text{ (bear state)}, \\ \mu_2 &> 0 \text{ (bear rally state)}, \\ \mu_3 &< 0 \text{ (bull correction state)}, \\ \mu_4 &> 0 \text{ (bull state)}. \end{aligned}$$

No restriction is imposed on $\{\sigma_1^2, \dots, \sigma_4^2\}$. Note that there are four states but we refer to two distinct regimes by B_t as

$$\begin{aligned} B_t = 1 \quad &\text{if} \quad s_t = 1 \text{ or } 2 \text{ (bear regime)}, \\ B_t = 2 \quad &\text{if} \quad s_t = 3 \text{ or } 4 \text{ (bull regime)}. \end{aligned}$$

The transition matrix takes the following form.

$$\boldsymbol{P} = \begin{pmatrix} p_{11} & p_{12} & 0 & p_{14} \\ p_{21} & p_{22} & 0 & p_{24} \\ p_{31} & 0 & p_{33} & p_{34} \\ p_{41} & 0 & p_{43} & p_{44} \end{pmatrix}$$

This specification implies that bear states and bear rally states cannot move to a bull correction and bull states and bull correction states cannot move to a bear rally state. This is done to avoid confounding these states that display common mean trends up or down in prices. For instance, it would be difficult to separate states 1 and 3 which both display negative average growth. The restricted transition matrix also means that the start of a new regime must be from a bear state or a bull state.

To further enforce economic restrictions on this model, the following long-run trends are imposed on the model. Solving for the stationary distribution associated with P we can compute the vector of unconditional state probabilities:

$$\boldsymbol{\pi} = (\boldsymbol{A}'\boldsymbol{A})^{-1}\boldsymbol{A}'\boldsymbol{e} \tag{2.1}$$

where $\boldsymbol{A}' = [\boldsymbol{P}' - \boldsymbol{I},\ \boldsymbol{\iota}]$ and $\boldsymbol{e}' = [0, 0, 0, 0, 1]$ and $\boldsymbol{\iota} = [1, 1, 1, 1]'$. The long-run restrictions are

$$E[r_t|\text{bear regime}, B_t = 1] = \frac{\pi_1}{\pi_1 + \pi_2}\mu_1 + \frac{\pi_2}{\pi_1 + \pi_2}\mu_2 < 0 \tag{2.2}$$

$$E[r_t|\text{bull regime}, B_t = 2] = \frac{\pi_3}{\pi_3 + \pi_4}\mu_3 + \frac{\pi_4}{\pi_3 + \pi_4}\mu_4 > 0. \tag{2.3}$$

3 Estimation

We perform posterior simulation with Gibbs sampling steps which reject any draws that violate the parameter restrictions, coupled with the simulation smoother of Chib (1996) to sample the latent state vector. Estimation follows exactly from Maheu et al. (2012) with the same priors employed in our analyses. For the MCMC output simulation, consistent posterior moments or predictive density quantities can be computed and are detailed in Maheu et al. (2012). We collect $30,000$ posterior draws for inference after dropping an initial $5,000$ draws for burn-in.

4 Data

Daily equity capital gains for 1885 - 1927 are from Schwert (1990). Equity data from 1928 - 2020 use the S&P500 index daily adjusted close reported by Yahoo Finance (ĜSPC symbol). Risk-free return data are from the U.S. Department of the Treasury. From these data, weekly continuously compounded returns, scaled by 100, are obtained for 1885 - 2020. Weekly returns are computed using Wednesday data and Thursday if Wednesday data is missing. The last observation is November 25, 2020. In the following we refer to the equity index returns as the S&P500 or the market. A matching weekly realized variance measure RV_t, is computed as the sum of intra-week daily squared returns. Summary statistics for the weekly data are reported in Table 1.

5 Results

Full sample parameter estimates for the four states are found in Table 2. Those estimates include posterior means and 0.95 probability density intervals for μ_i and σ_i associated with each of the four states. Below those estimates, we report a pseudo Sharpe ratio, μ_i/σ_i, for each state i for $i = 1, 2, 3, 4$. This measures the expected return adjusted for risk assuming a zero risk-free rate for each state. The parameter estimates are broadly similar to those in Maheu et al. (2012). The average return in the bear states, -0.94, is more negative than -0.11 in the bull correction phases of the bull regimes. Analogously, the upward trend for returns in bull states (0.52) is stronger than 0.23 in the bear rally phase of the bear regime. Combining the return estimates with state volatilities, the Sharpe ratios also make sense, ranked from highest to lowest in the bull, bear rally, bull correction and bear states respectively.

The posterior means of the state transition matrix $\boldsymbol{P}$ indicate persistence of bull and bear regimes in that states within the bear regime ($s_t = 1, 2$) and those in the bull regime ($s_t = 3, 4$) are likely to cluster together so that moves between regimes will be infrequent. Interestingly, a bull correction state is much more likely (0.097) to transition back to the bull state than to move to a bear state (0.013); whereas, a bear rally state is more likely (0.019) to transition to the bull state than it is to fall back to a bear state (0.013). Furthermore, regime changes are almost always transitions from a bear rally or from a bull correction. Directly jumping from a bull state to a bear state or vice versa is very rare.

Table 3 reports the unconditional probabilities associated with the four states from which one can compute the unconditional probabilities for the regimes. For example, the bull market regime has a long-run probability of 0.672. Using the formulae in equation 2.2, one can compute that the associated long-run mean of weekly returns in bull regimes as 0.186; whereas, that for bear regimes is -0.069.

5.1 Before COVID-19

Figure 1 displays the cumulative log-return and realized volatility (top), the probability of a bull regime $P(B_t = 2 \mid r_{1:T})$ (middle), and the probabilities of the 4 individual states (bottom) during 2019, the year before the COVID-19 pandemic. Very early in 2019, the market moved from a bear rally state into the bull state. Throughout 2019 the model decisively identifies a bull regime with fluctuations between the bull state and bull corrections.

5.2 After COVID-19

Figure 2 reports the same information as Figure 1 but for the year 2020 during which COVID-19 erupted. The year began with strong evidence of a bull market, although with the probability of a bull correction building. The week of January 29 began a sequence of large negative drops in the market leading to increasing evidence of a transition from a bull market to a bear market. By February 26, the market had transitioned decisively from the bull correction state to the bear market state. By April the probability of the bear market state declined until April 22 revealed a transition to a bear rally state. At the end of our estimation window (November 25, 2020), the probability of a bear rally state is still very high at 0.824. This is because, even with a visible upward trend in the index since April, the market volatility is still too high and variable to be consistent with a typical bull regime. This is evinced by the observation that the weekly realized volatility (red line in the top panel) is variable, with spikes (for example, during September and October 2020), and higher on average in comparison to its historical average value 1.94 from Table 1.

5.3 Forecasts

Although one can date stock market cycles after the fact with smoothed probability estimates, it is much more difficult to forecast changes out-of-sample. In this section, we report results for which the model has been estimated at each point t using all past data $1, \ldots, t$ to produce a forecast of the state one week ahead $t + 1$.

Figure 3 uses these model forecasts of the state probabilities one week ahead to generate out-of-sample regime forecasts. This figure compares the out-of-sample forecast of the market regime probabilities with the full-sample (smoothed) probability estimates. It shows a relatively accurate week-by-week classification of regimes that was available in real-time to an investor using the model to forecast stock market cycles.

One challenging period for the model forecasts was late August to early September. From July to September the index displayed a strong positive trend, and the model forecasts allocated a nontrivial probability to a bull regime. In hindsight, this episode is precisely identified as a bear regime using the smoothed estimates.

The breakdown of the regime forecasts into the constituent state forecasts one week ahead in seen in Figure 4. Given this additional disaggregation, there is more deviation between the state forecasts and smoothed estimates, but overall there is a strong correspondence between the two. Note again that, with the exception of the July to September period, the forecasts assign the highest probability to the bear rally state rather than a bull state.

The COVID-19 period has had an important impact on several features of the return distribution. For example, Figure 5 displays the one-week-ahead predictive density generated by the model. From the middle of March 2020 there was a dramatic impact that flattened the return distribution for the rest of the year, along with more subtle changes in the location. This implied a sharp increase in risk associated with holding the S&P500 portfolio.

Figure 6 illustrates the one-week-ahead predictive Sharpe ratios defined as $\frac{E(r_t|r_{1:t-1})}{\sqrt{\mathrm{Var}(r_t|r_{1`:t-1})}}$. The flattened density of returns in March is accompanied by a sudden drop of the predictive Sharpe ratio. This ratio becomes positive in June and continues to improve over the summer months until a decline in August. For the remainder of our sample, the predictive Sharpe ratio never does attain the values at the start of 2020 before COVID-19 struck.

To illustrate another way, these changes in risk can be clearly seen from Value-at-Risk levels estimated for our model and illustrated in Figure 7. The dashed Value-at-Risk levels obtained from an assumption that returns are normally distributed would significantly understate risk as compared to those levels implied by our model that incorporates a mixture of four state distributions.

5.4 Market Timing

In this section, we consider some simple market timing strategies that exploit the forecasts from our model. All of the investment strategies are based on the out-of-sample predictions of states and regimes; and the simple maxim to buy low and sell high. This can take several forms such as selling at the end of a bull market and buying at the end of a bear market. However, our MS4 model provides much more detailed and useful information. For example, the states that identify increasing prices are state 4 (bull state) and the riskier state 2 (bear rally). Holding the market during periods for which forecasts assign significant probability for these states could be fruitful.

In each case, the investor can buy the market and continue to hold the market; or sell and hold a risk-free asset. No short selling is allowed. Here are the market timing strategies we consider.

1. Strategy B: buy or continue to hold the market when
   $P(B_t = 2|r_{1:t-1}) > \tau_B$ and otherwise sell.

2. Strategy S: buy or continue to hold the market if
   $P(s_t = 2|r_{1:t-1}) > \tau_S$ or $P(s_t = 4|r_{1:t-1}) > \tau_S$ and otherwise sell.

The first strategy B only uses the aggregate regime information associated with the probability forecasts for B_t. The second strategy exploits the positive expected return in both the bear rally ($s_t = 2$) and bull states ($s_t = 4$). We focus on using one cutoff value τ_S for both of those states but this could be generalized and we present some evidence of this below.

Table 4 shows the investment results for 2020. All values are annualized. The annualized return is 13.1% with a Sharpe Ratio of 0.566 if the investor buys in the first week of 2020 and holds the position until the last week of our data sample (last Wednesday of November). This compares with a hypothetical buy and hold return of 6.46% if an investor held the index for our entire sample from 1885.

As reported in Table 4, using the market timing strategy B does not perform well in 2020 even with a range of alternative cutoff values, τ_B, for buying and selling. However, exploiting the additional information about states provided by the MS4 model (as in strategy S), yields positive results. For example, with $\tau_S = 0.5$, the market timing strategy generates a 22% annualized return with a Sharpe Ratio of 1.203. This investment strategy performs significantly better than the buy and hold strategy.

Figure 8 displays returns for strategy S as a function of different values of τ_S. For most values of $\tau_S \in (0.5, 0.9)$ a positive return is achieved with the best performance for values less than 0.65.

Figure 9 shows the returns from strategy S while relaxing the constraint of a common τ_S associated with both bear rally and bull states. This figure fixes τ_S at 0.5 for the bear rally state, that is, buying or continuing to hold the market if $P(s_t = 2|r_{1:t-1}) > 0.5$; while allowing τ_S to vary for the bull state, that is, buy if $P(s_t = 4|r_{1:t-1}) > \tau_S$ and sell otherwise. The results show that it is possible to achieve even larger gains by separating the thresholds in strategy S. This is further evidence that the added value associated with using information inherent in the state probabilities and predictive state densities for investment strategies is quite robust.

5.5 Long-horizon Predictions

We have focused on one-week-ahead predictions from the model and how they might be used for investment decisions. However, the model estimates have implications for the long-run behaviour of stock market returns. Being stationary, our MS4 model implies that any long-horizon predictions converge to the implied stationary distribution. Figure 10 and 11 report one-year-ahead probability forecasts for states and associated regimes looking forward from our most recent observations at the end of November 2020. What is notable is that the transition back to *normal* times in the form of the long-run values of the states is slow. Even if the effects of COVID-19 were to disappear today, our model predicts months, and not weeks, until the stock market returns to normal times.

6 Robustness Results

There are a number of alternative models and comparisons we have conducted. The details are collected in the Appendix. Here we highlight a few of these results. Table 5 reports log-predictive likelihood values for the 2020 data for several alternative models. Included is our proposed 4-state model (MS4), an unrestricted version of MS4, a MS4 model with Student-t innovations, a GARCH model and a MS2 model. None of these specifications improve on our proposed MS4 model. In the Appendix, we show how these alternative models date the turning points in 2020 and generally see a close correspondence with the MS4 model. One notable exception is a 2-state Markov switching model (MS2). The MS2 classifies the summer months as a bull state while our MS4 identifies this period as a bear market rally. This could be considered as a drawback of the 2-state model in that it does not allow for intra-regime dynamics.

7 Conclusion

This paper estimates and forecasts bull and bear markets during the COVID-19 pandemic. Using the model of Maheu et al. (2012), we document where the market was and what has happened to U.S. equity markets in 2020. We find that the market moved from a bull state at the start of 2020 to a bull correction and quickly to a bear market on February 26. This bear market dominated until April 22 when the market transitioned to a bear rally, remaining in this phase until the current date. We show the dramatic impact of the COVID-19 pandemic on the forecasts of the return distribution and how effective market timing strategies can exploit model forecasts. Long-horizon forecasts from the model predict months, and not weeks, until the stock market returns to normal times.

8 Appendix

8.1 MS4 versus GARCH

Figure 12 shows the smoothed standard deviations from the MS4 versus a GARCH(1,1) model. The MS4 model picks up the volatility surge quickly at the beginning of 2020 and does not have the long tapering-off period associated with the GARCH model. The implications for Value-at-Risk are illustrated in Figure 13. The quick adjustment of the Value-at-Risk estimates generated by the MS4 model match the high potential losses after March 2020. This adjustment is substantially delayed and less flexible using a GARCH(1,1) model. The latter is due to the single exponential decay parameter in the GARCH model which invokes higher persistence and less flexible adjustment to shocks.

8.2 MS4 versus MS2

Figure 14 shows the probability of a bull market regime inferred from our 4-state model versus a simple 2-state model for the period 2019-2020. There is one striking difference between these two models in the period from April to September 2020. While the MS4 model classifies this period as a bear regime (in a bear rally state), the MS2 model signals a bull regime. This difference is due to the 2-state model not having the structure to incorporate intra-regime dynamics. The September and October evidence from the 2-state model had the bull market probability plunge to the bottom with small humps, while the 4-state model always estimated that the bull regime had not been confirmed yet.

8.3 MS4-t

Figure 16 shows the bull regime probability (middle) and the probability of each individual state (bottom) associated with assuming a Student-t distribution for each state. There is no qualitative difference from our proposed MS4 model in Figure 2.

8.4 Predictive Likelihood Comparisons

Figure 17 illustrates differences in the cumulative predictive likelihood associated with alternative models for year 2020, using the GARCH(1,1) model as the benchmark value. The values at time t are log predictive Bayes factor as $\log\left[\frac{p(r_{2020 \text{ until } t}|r_{\text{before } 2020}, \text{M})}{p(r_{2020 \text{ until } t}|r_{\text{before } 2020}, \text{GARCH}(1,1))}\right]$, where M indicates various alternative models including the MS4. According to Kass & Raftery (1995), a value $\frac{p(r|M_0)}{p(r|M_1)}$ that is larger than 5 indicates strong data evidence to support model M_0. The 2020 data clearly favours the MS4 model against the GARCH(1,1). In addition, having a Student-t distribution for each state does not provide any additional value.

8.5 MS4 with unrestricted $\boldsymbol{P}$

Economic intuition suggests a zero restriction on p_{13}, p_{23}, p_{32} and p_{42}. Without that restriction, the 4-state model can still identify key dates of regime change in 2020 as shown in Figure 18. However, a closer look at Figure 19, showing the bull regime and 4-state probabilities from both MS4 and MS4 with unrestricted $\boldsymbol{P}$, reveals that our proposed MS4 model, which has a restricted transition probability structure, has less uncertainty between a bull market state and a bear rally state. From the figure, during February and July 2020 the MS4 with unrestricted $\boldsymbol{P}$ has higher bull state probability than the MS4, because the unrestricted transition matrix has no structure to prevent transition from the bull state to a bear rally state. As a result, at the aggregate level, (top panel of Figure 19), the MS4 has sharper identification of regimes than without the restriction on $\boldsymbol{P}$. Further evidence can be seen from Figure 17, which reveals that the out-of-sample performance of the unrestricted MS4 model is dominated by our proposed MS4 model.

Interestingly, the MS4 model can be interpreted as a two-level hierarchical hidden Markov model (Fine et al. (1998)), in which each level has two states with simple restrictions. The first restriction is on the vertical down probability to incorporate the zero restrictions on the transition matrix. The second restriction is the mean restriction for state-identification purposes. These restrictions add value to bull and bear regime identification, and proves the usefulness of a priori restrictions motivated by economic intuition.

References

Chib, S. (1996), 'Calculating posterior distributions and modal estimates in markov mixture models', *Journal of Econometrics* **75**, 79–97.

Fine, S., Singer, Y. & Tishby, N. (1998), 'The hierarchical hidden markov model: Analysis and applications', *Machine learning* **32**(1), 41–62.

Kass, R. E. & Raftery, A. E. (1995), 'Bayes factors', *Journal of the American Statistical Association* **90**(420), 773–795.

Lunde, A. & Timmermann, A. G. (2004), 'Duration dependence in stock prices: An analysis of bull and bear markets', *Journal of Business & Economic Statistics* **22**(3), 253–273.

Maheu, J. M., McCurdy, T. H. & Song, Y. (2012), 'Components of bull and bear markets: Bull corrections and bear rallies', *Journal of Business & Economic Statistics* **30**(3), 391–403.

Pagan, A. R. & Sossounov, K. A. (2003), 'A simple framework for analysing bull and bear markets', *Journal of Applied Econometrics* **18**(1), 23–46.

Schwert, G. W. (1990), 'Indexes of u.s. stock prices from 1802 to 1987', *Journal of Business* **63**(3), 399–426.

Table 1: Weekly Return Statistics

N	Mean	Mean($RV^{.5}$)	Skewness	Excess Kurtosis
7064	0.125	1.938	-0.565	8.007

Table 2: Posterior Estimates

	mean	95% DI
bear μ_1	-0.94	(-1.09, -0.79)
bear rally μ_2	0.23	(0.14, 0.32)
bull correction μ_3	-0.11	(-0.21, -0.02)
bull μ_4	0.52	(0.42, 0.64)
σ_1	5.60	(5.21, 6.03)
σ_2	2.44	(2.27, 2.61)
σ_3	1.85	(1.69, 2.04)
σ_4	1.09	(0.97, 1.21)
μ_1/σ_1	-0.17	(-0.20, -0.14)
μ_2/σ_2	0.10	(0.06, 0.13)
μ_3/σ_3	-0.06	(-0.12, -0.01)
μ_4/σ_4	0.49	(0.35, 0.65)

$$\text{Transition matrix } \boldsymbol{P} = \begin{pmatrix} 0.906 & 0.092 & 0 & 0.002 \\ 0.013 & 0.968 & 0 & 0.019 \\ 0.013 & 0 & 0.891 & 0.097 \\ 0.001 & 0 & 0.122 & 0.876 \end{pmatrix}$$

Table 3: Unconditional State Probabilities

	mean
bear π_1	0.084
bear rally π_2	0.245
bull correction π_3	0.356
bull π_4	0.316

Table 4: Investment Returns

	Return	Sharpe Ratio
Strategy B[a]: $\tau_B = 0.5$	-0.009	-0.048
Strategy S[b]: $\tau_S = 0.5$	0.220	1.203
Buy-and-hold	0.131	0.566

The returns are annualized.

[a] Buy if $P(B_t = 2 \mid r_{1:t-1}) > \tau_B$ and sell otherwise.

[b] Buy if $P(s_t = 2 \mid r_{1:t-1}) > \tau_S$ or $P(s_t = 4 \mid r_{1:t-1}) > \tau_S$. Sell otherwise.

.

Table 5: Log-Predictive Likelihood in 2020

MS4	MS4 Unrestricted	MS4t	GARCH	MS2
-127.6	-128.7	-127.9	-146.3	-132.8

Figure 1: Estimates for 2019

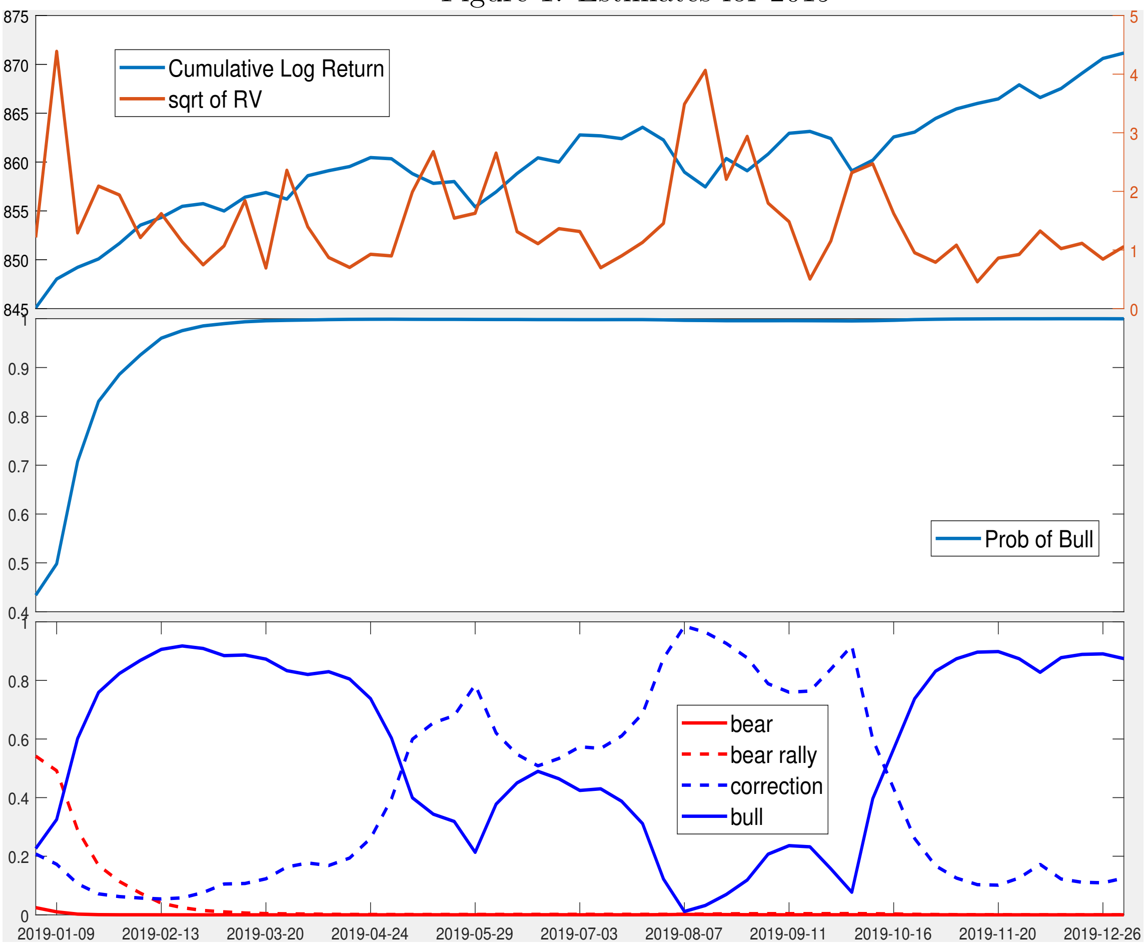
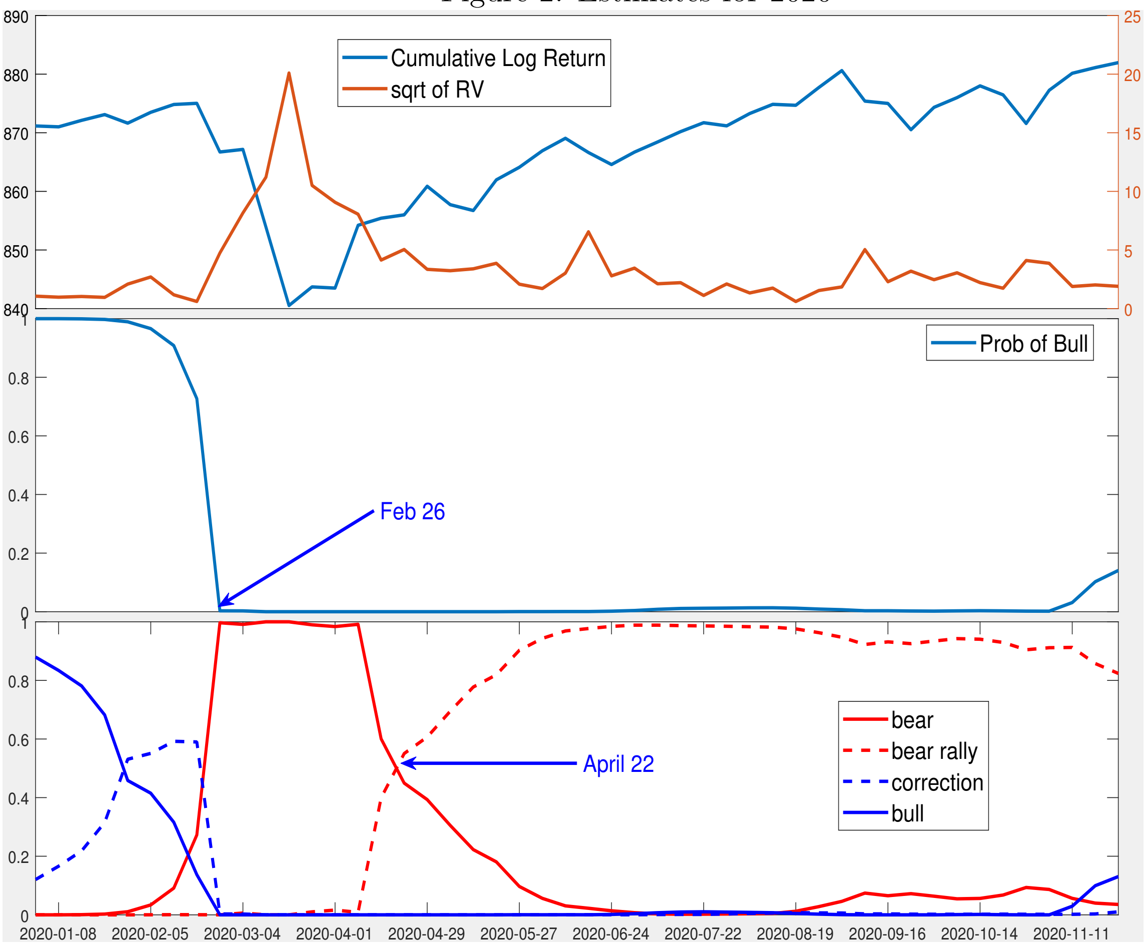


Figure 2: Estimates for 2020



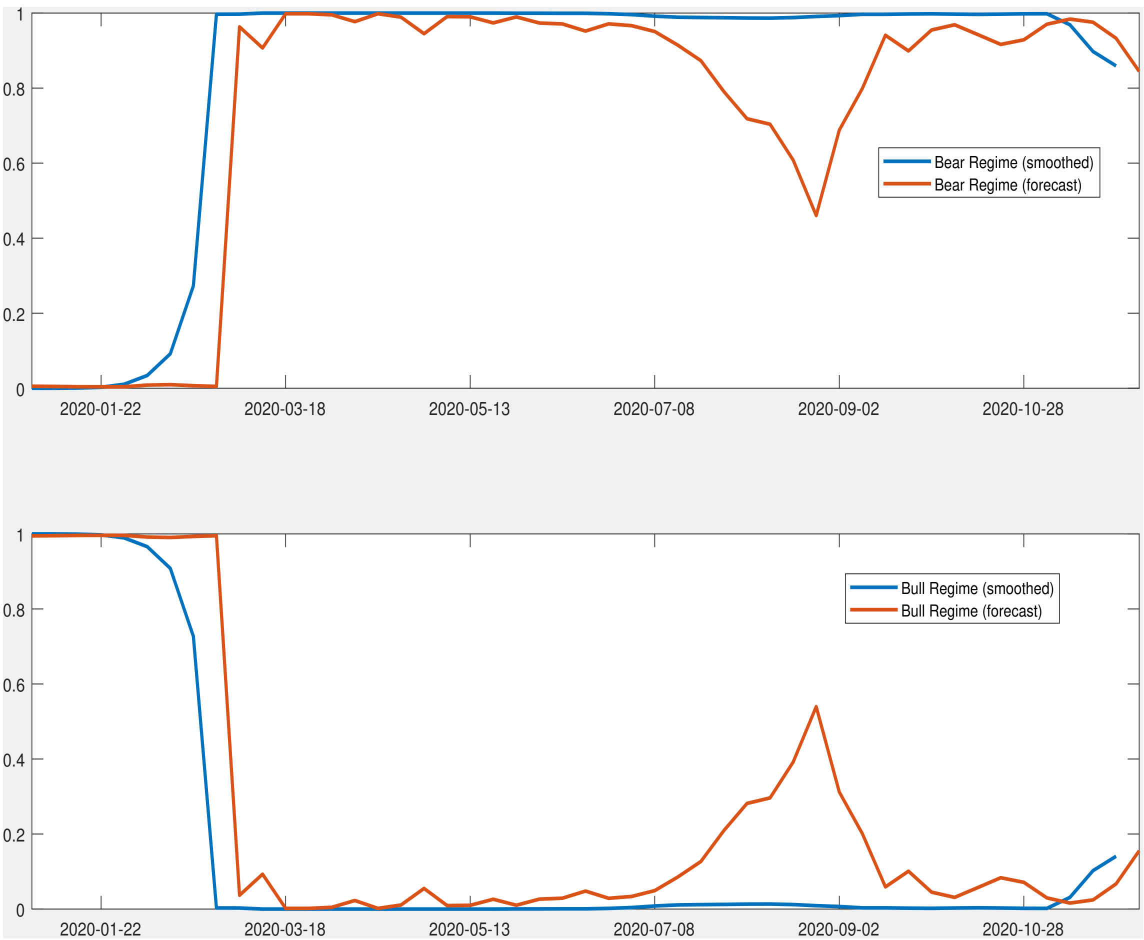


Figure 3: Out-of-sample: One-week-ahead Regime Probability Forecasts

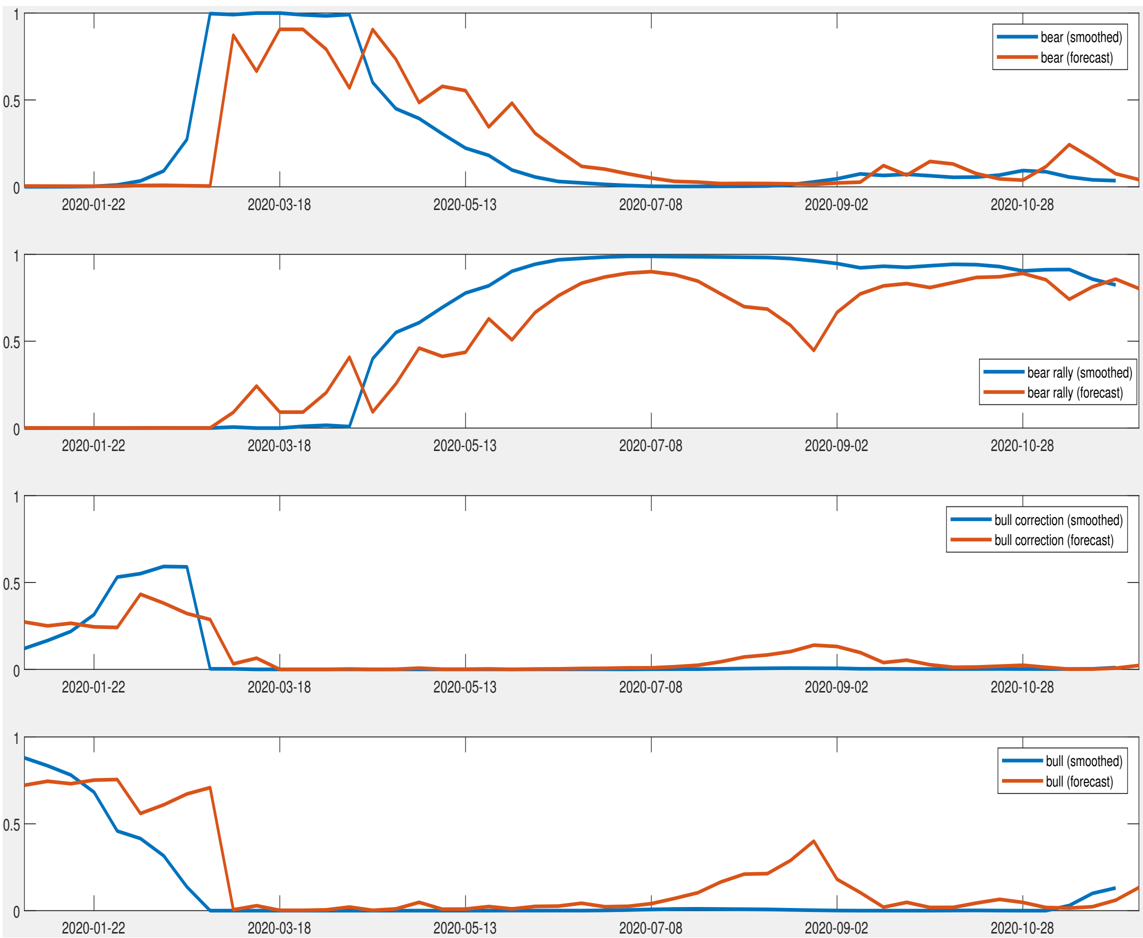


Figure 4: Out-of-sample: One-week-ahead State Probability Forecasts

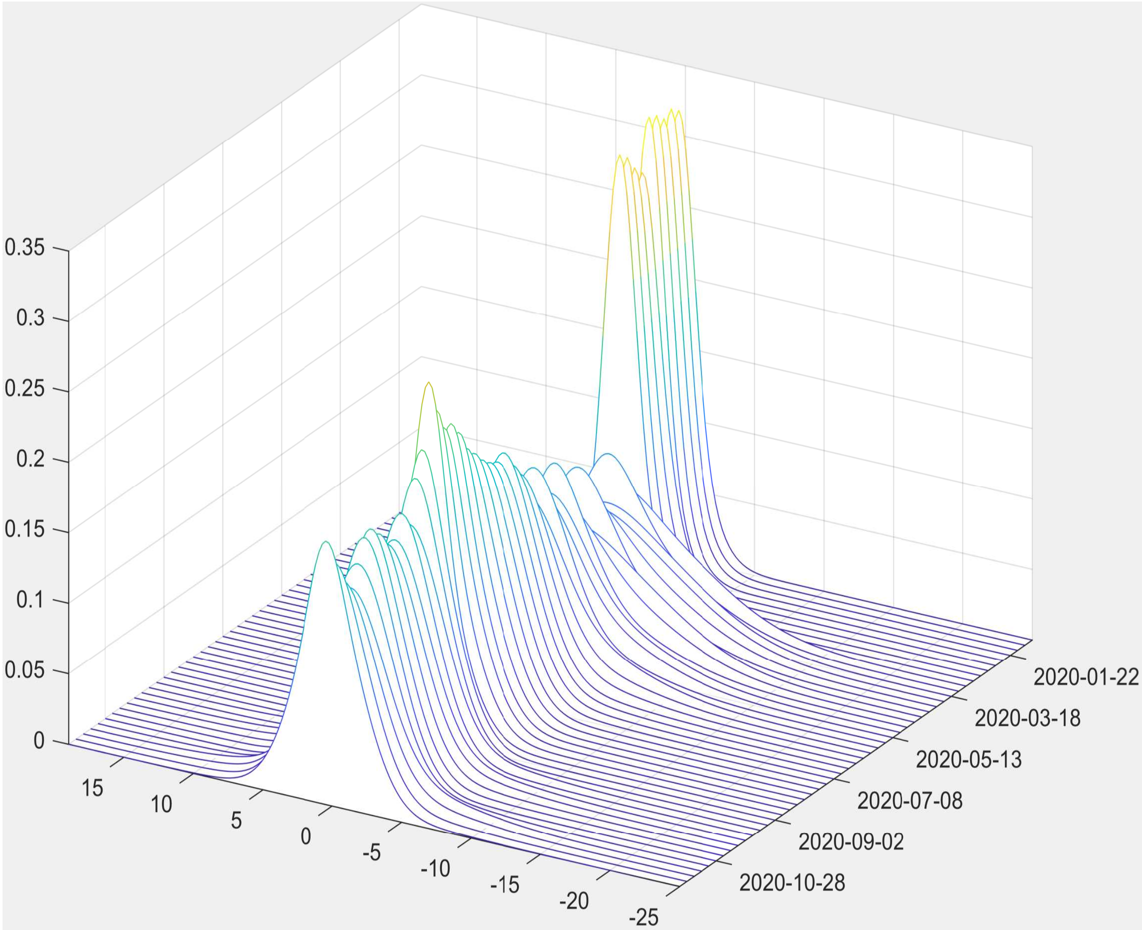
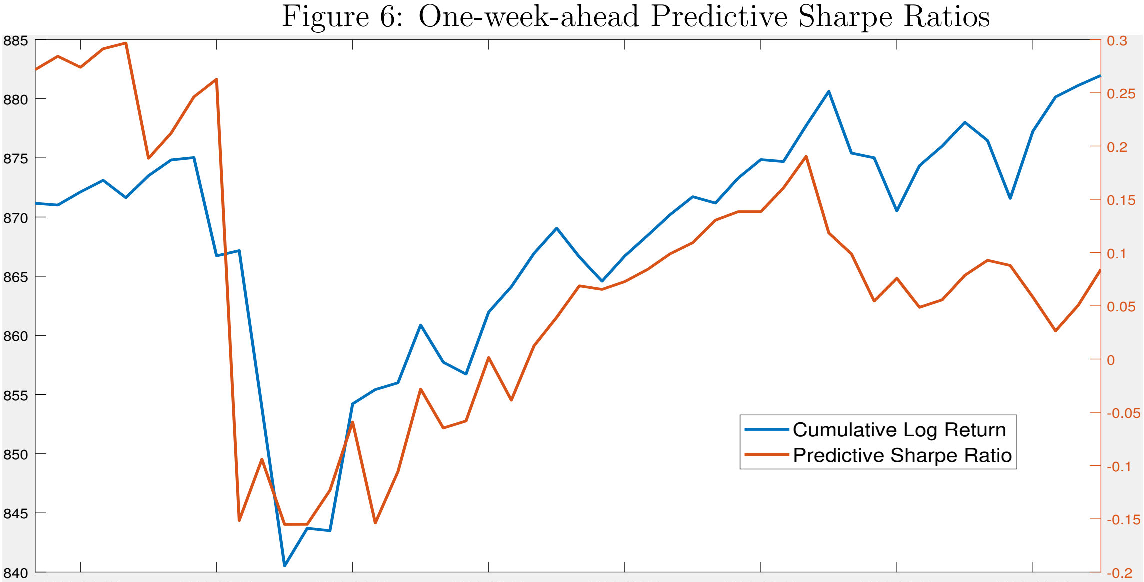


Figure 5: Out-of-sample: One-week-ahead Predictive Densities

The X-axis is a grid of possible return values. The Y-axix is the time. The Z-axis is the probability density function values.

Figure 6: One-week-ahead Predictive Sharpe Ratios



The Predictive Sharpe ratio is defined as the ratio of predictive mean and standard deviation, $\frac{E(r_t|r_{1:t-1})}{\sqrt{\text{Var}(r_t|r_{1:t-1})}}$.

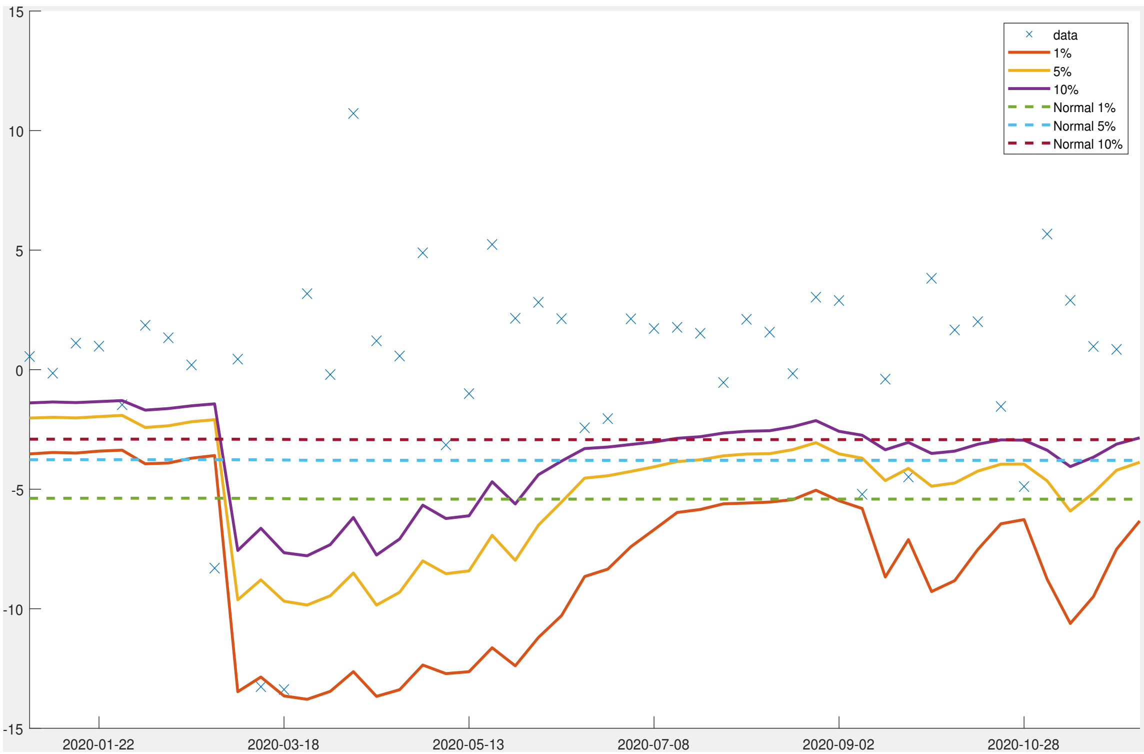
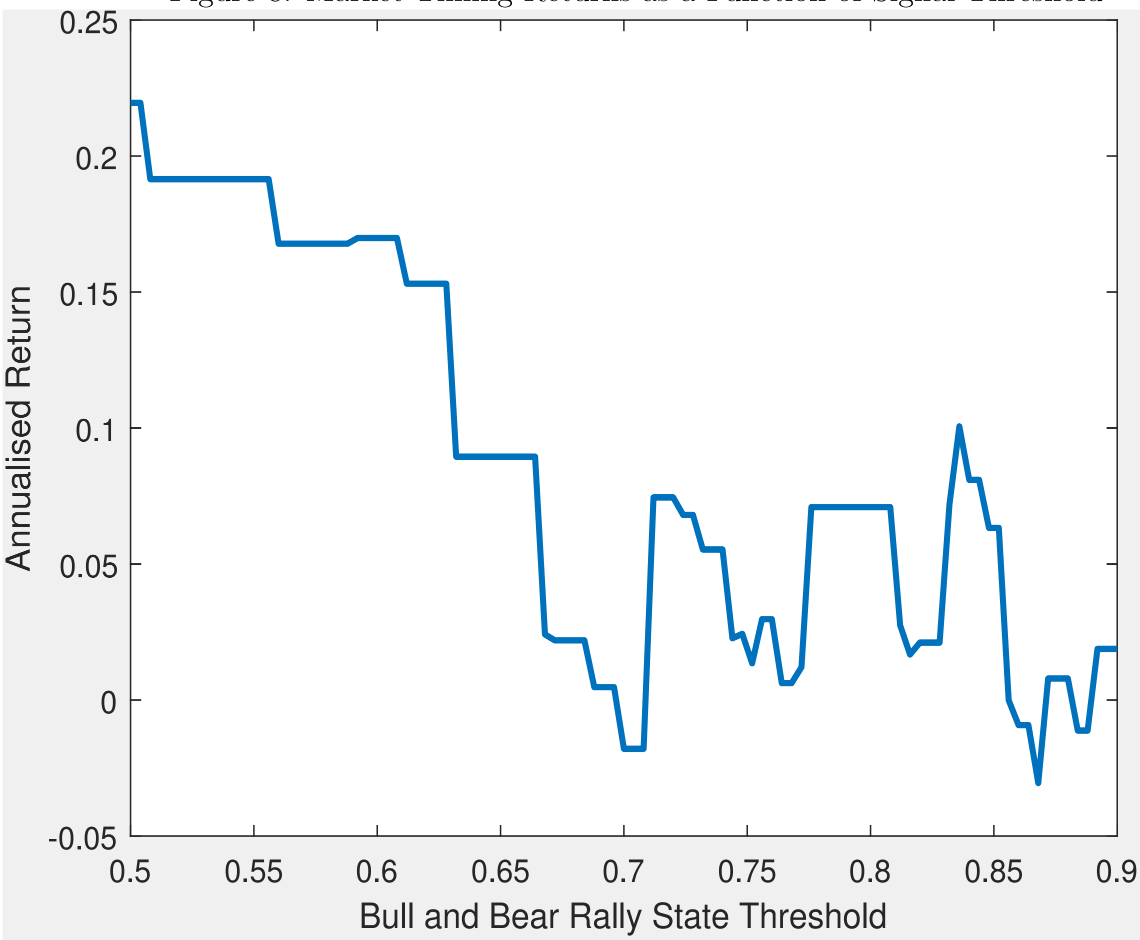


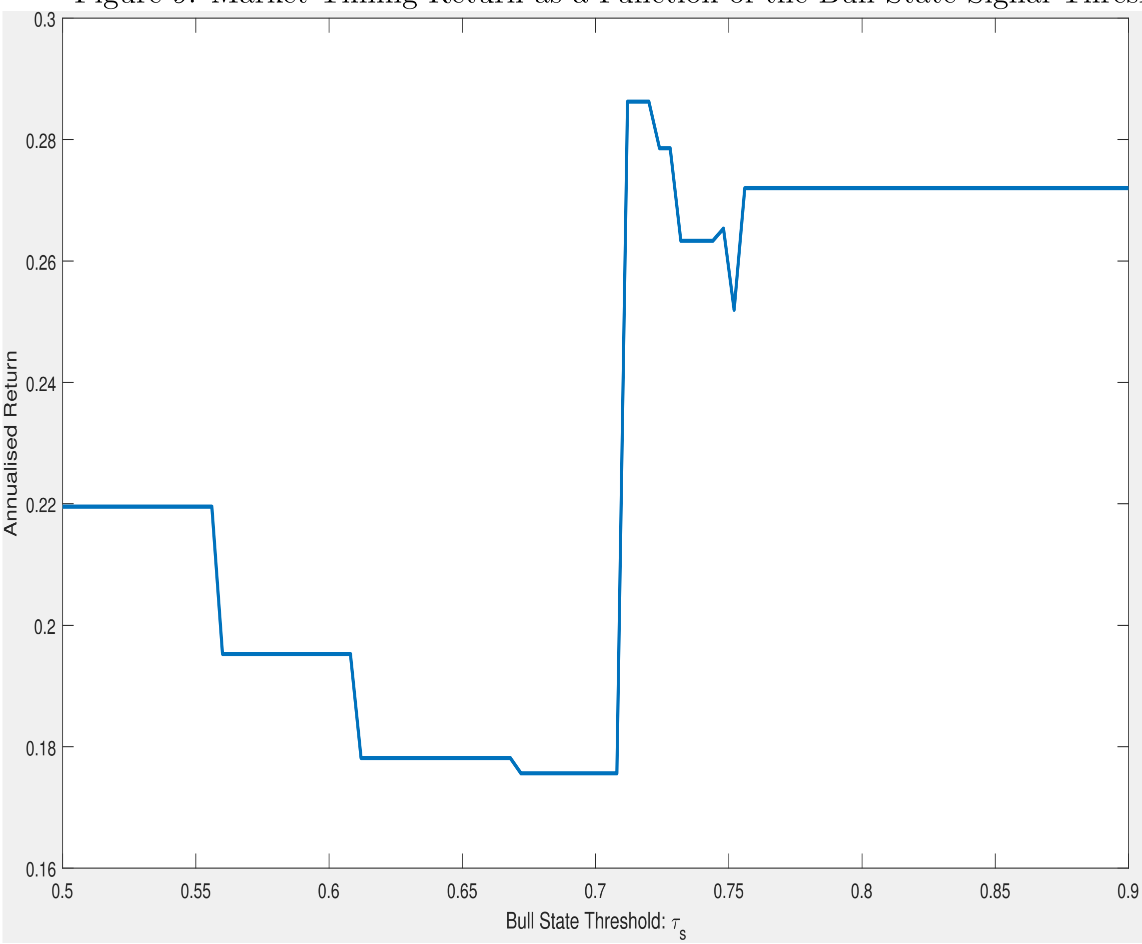
Figure 7: Out-of-sample: One-week-ahead Value-at-Risk Forecasts

Figure 8: Market Timing Returns as a Function of Signal Threshold



The blue line is the return in 2020 till the end of the sample period as a function of τ_S for the investment strategy S: buy or continue to hold the market if $P(s_t = 2|r_{1:t-1}) > \tau_S$ or $P(s_t = 4|r_{1:t-1}) > \tau_S$ and otherwise sell.

Figure 9: Market Timing Return as a Function of the Bull State Signal Threshold



The blue line is the return in 2020 till the end of the sample period as a function of τ_S for investment strategy S: buy or continue to hold the market if $P(s_t = 2|r_{1:t-1}) > 0.5$ or $P(s_t = 4|r_{1:t-1}) > \tau_S$ and otherwise sell.

Figure 10: States Estimates for 2020 and One-year Forecasts

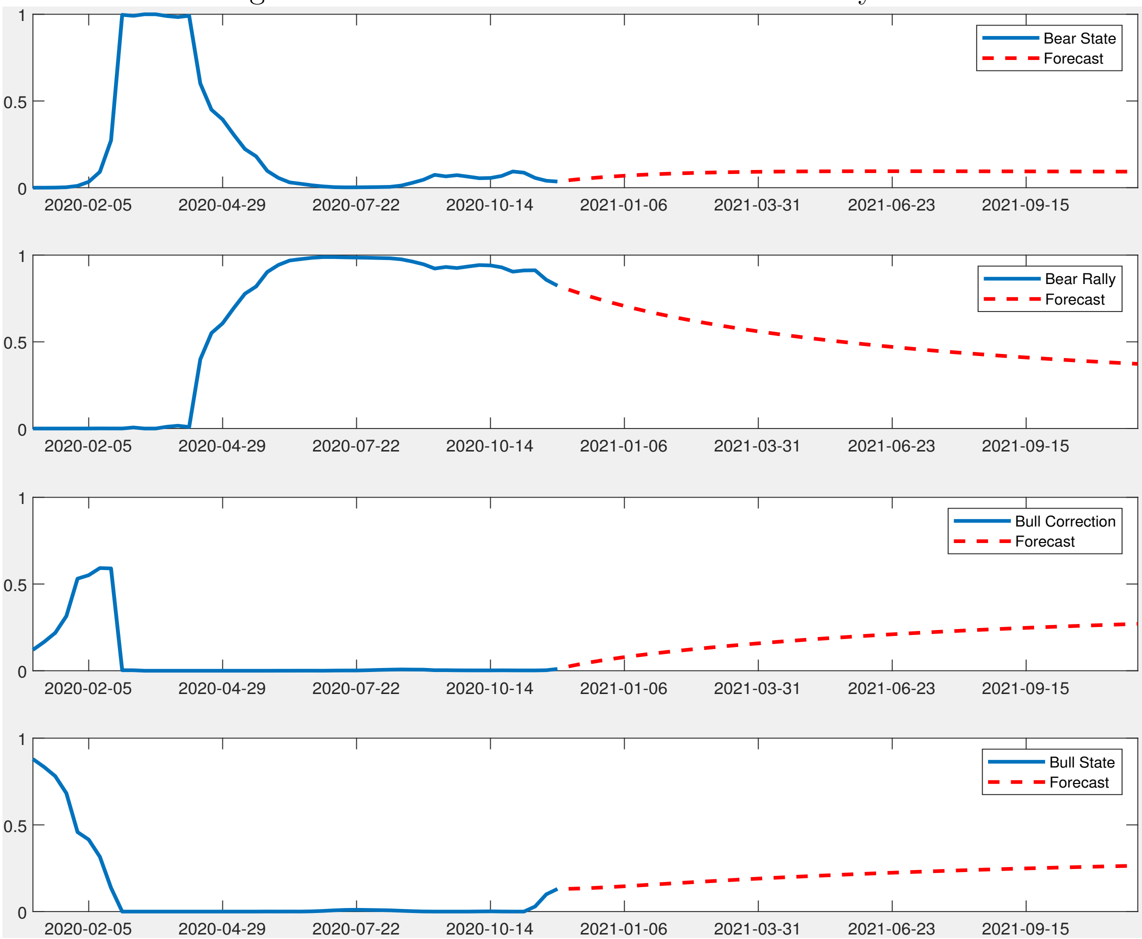
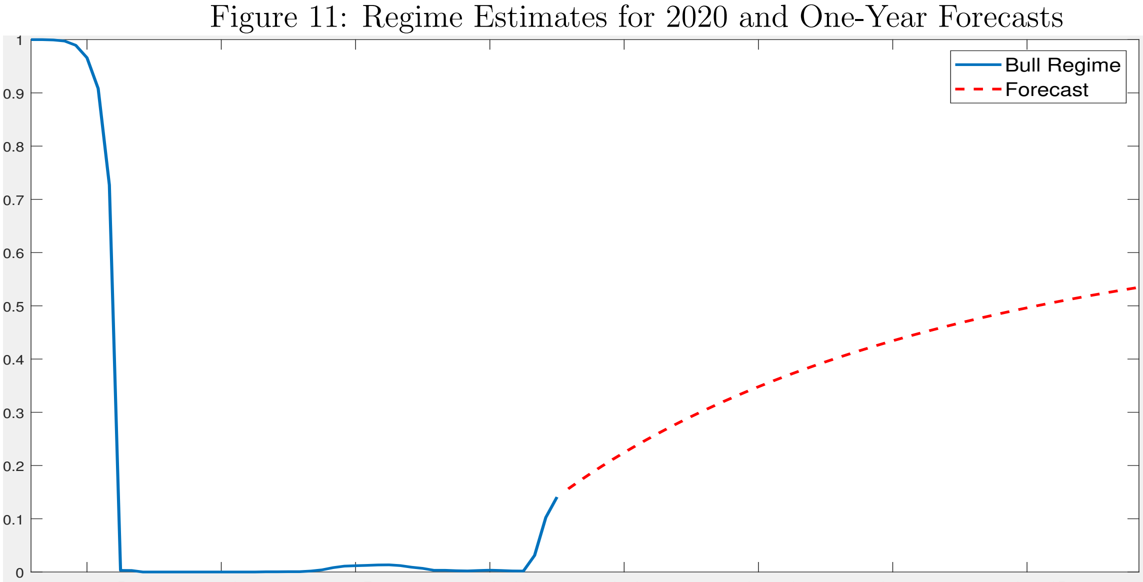


Figure 11: Regime Estimates for 2020 and One-Year Forecasts



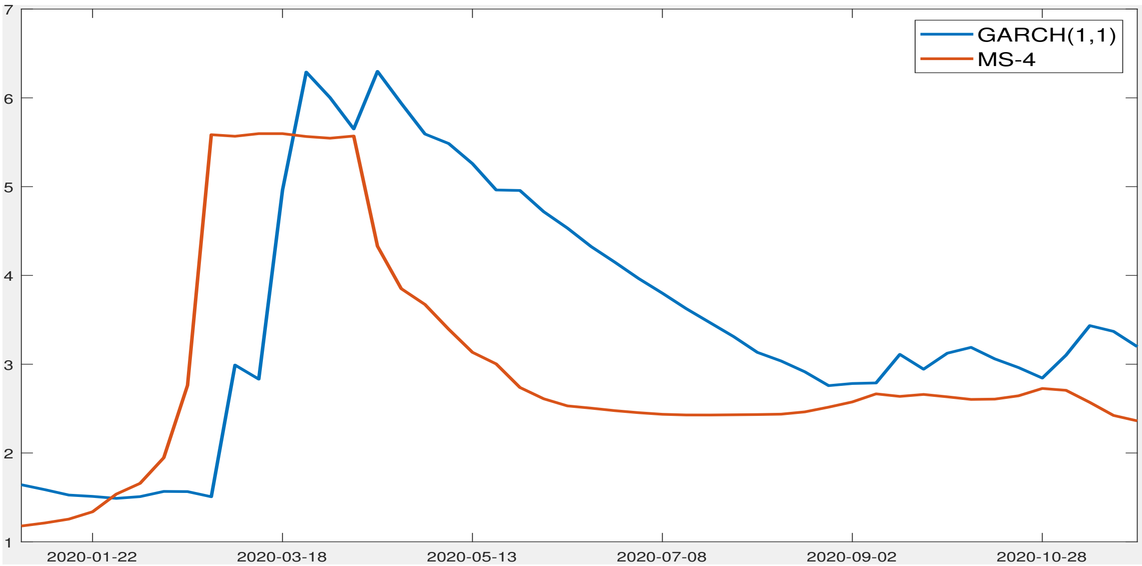


Figure 12: Standard Deviations from GARCH(1,1) and MS4 in 2020.

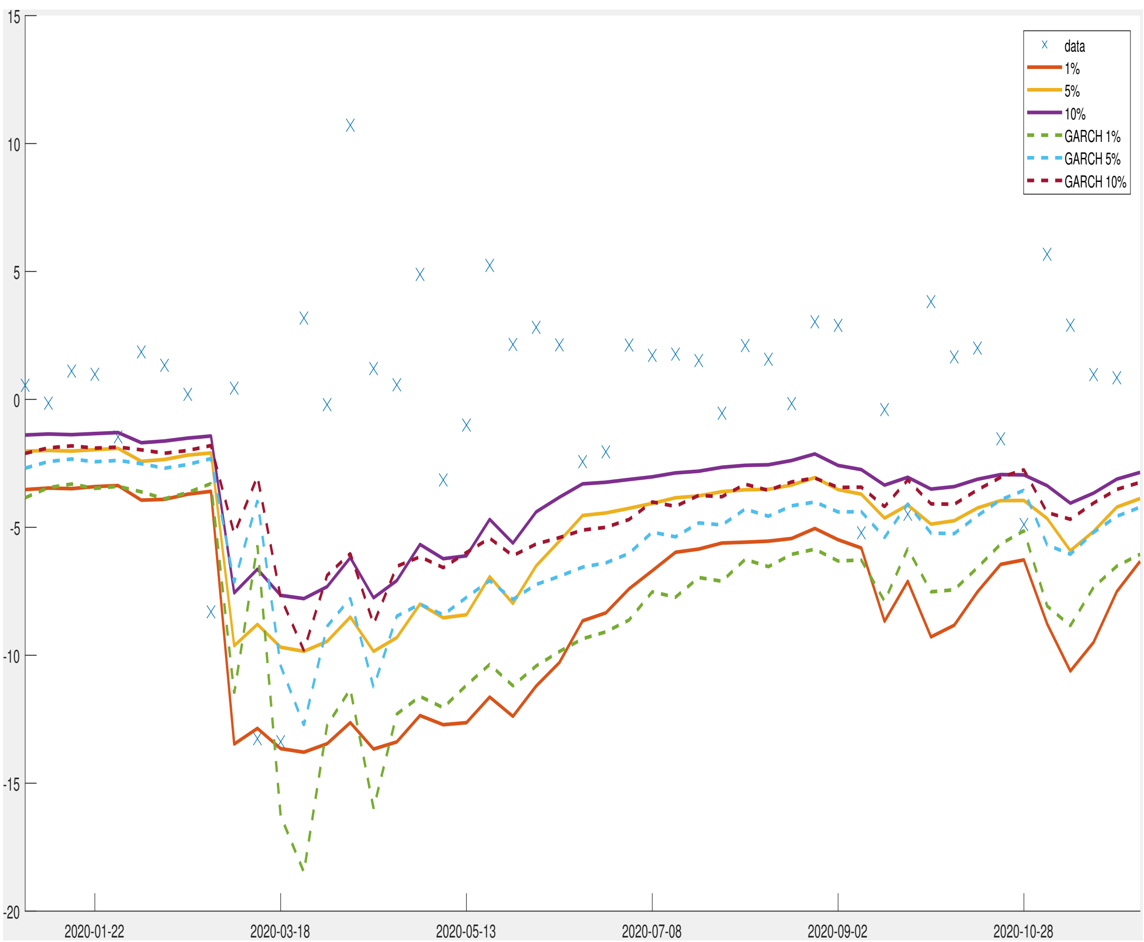


Figure 13: Out-of-sample: One-week-ahead Value-at-Risk Forecasts

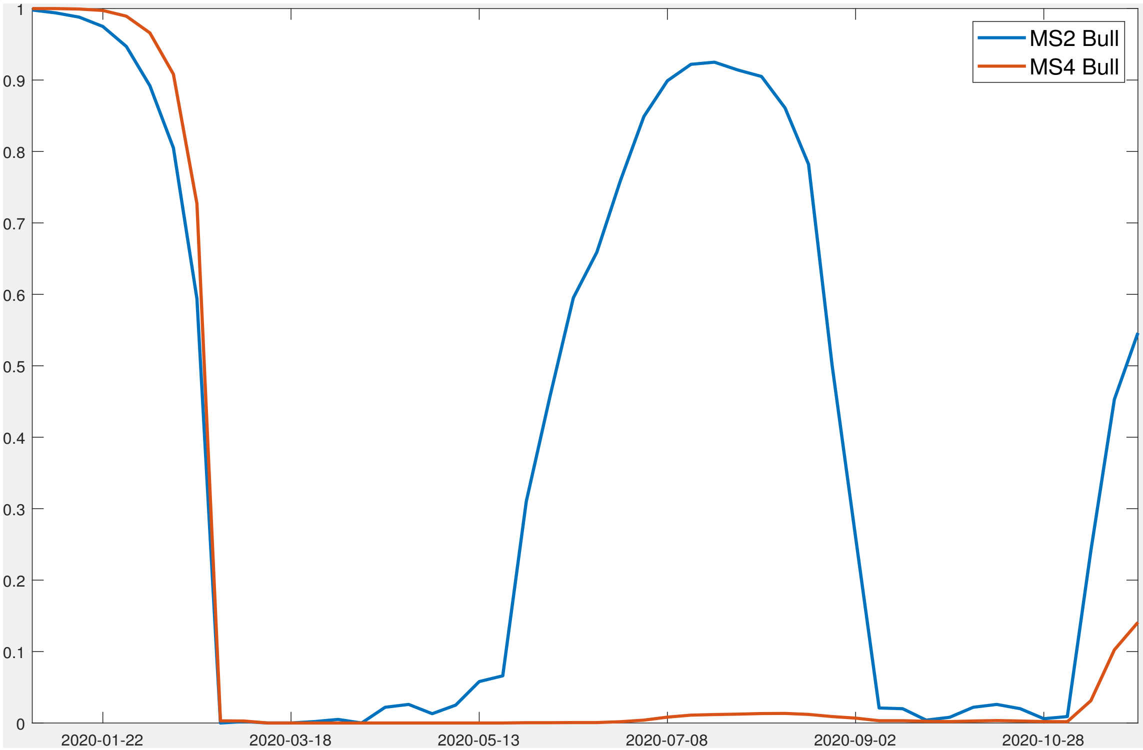


Figure 14: Bull Regime Probability: MS2 vs MS4 Model

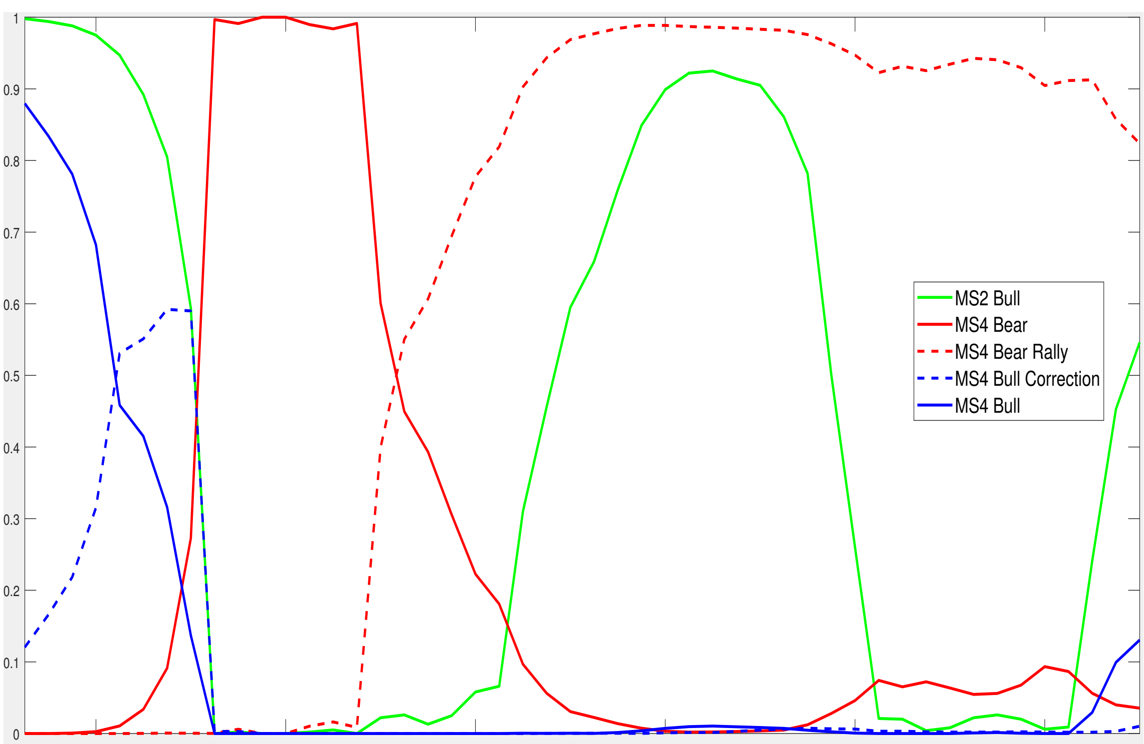


Figure 15: Bull Regime Probability: MS2 & State Probabilities: MS4

Figure 16: State Probability Estimates: MS4t

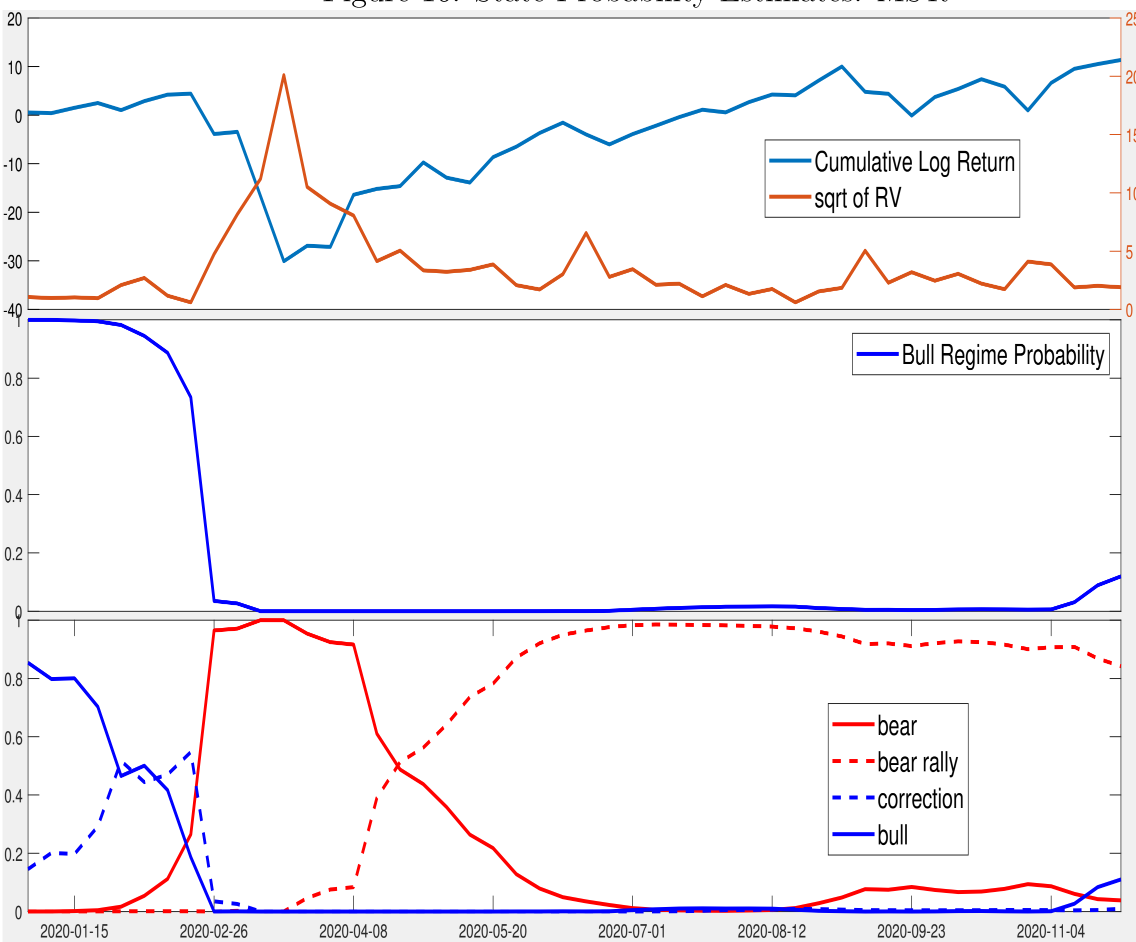


Figure 17: Cumulative Log Predictive Likelihoods

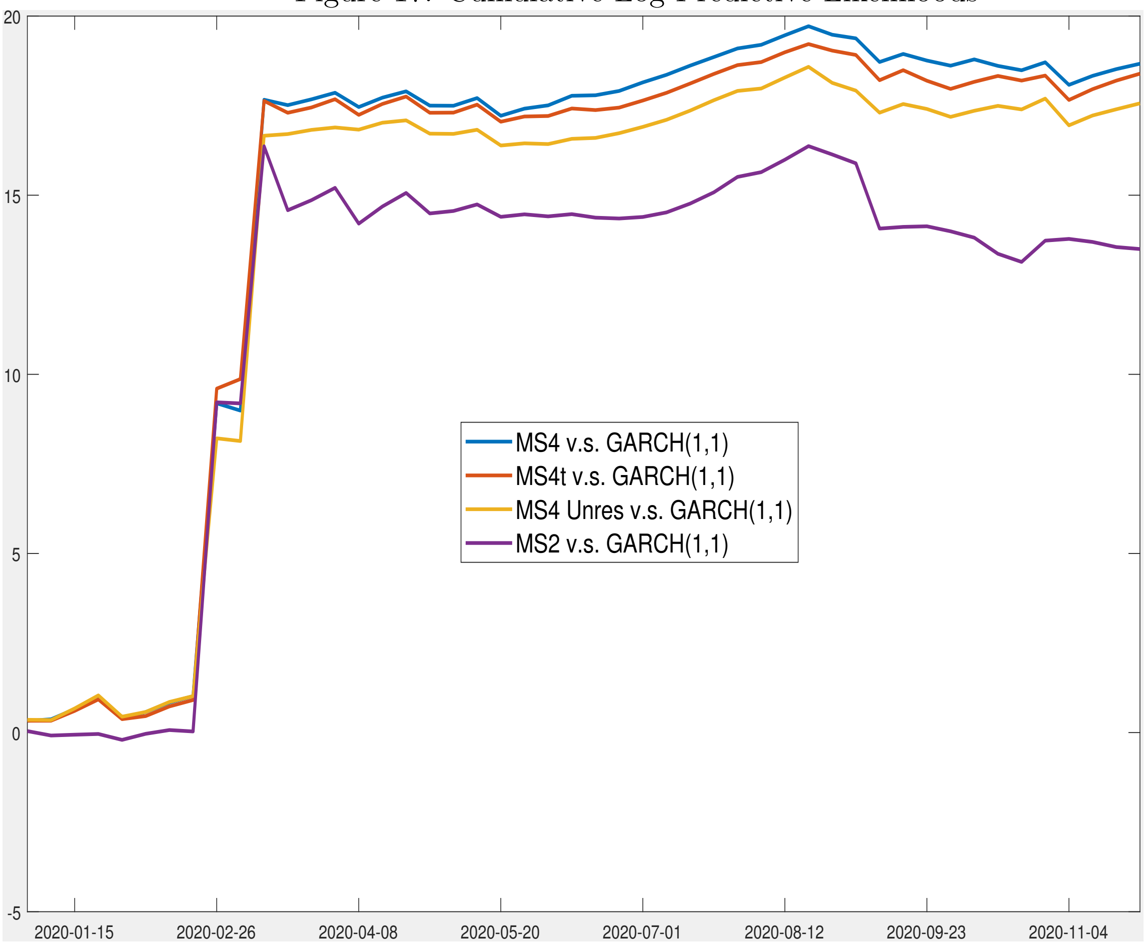


Figure 18: State Probability Estimates: MS4 with Unrestricted $\boldsymbol{P}$

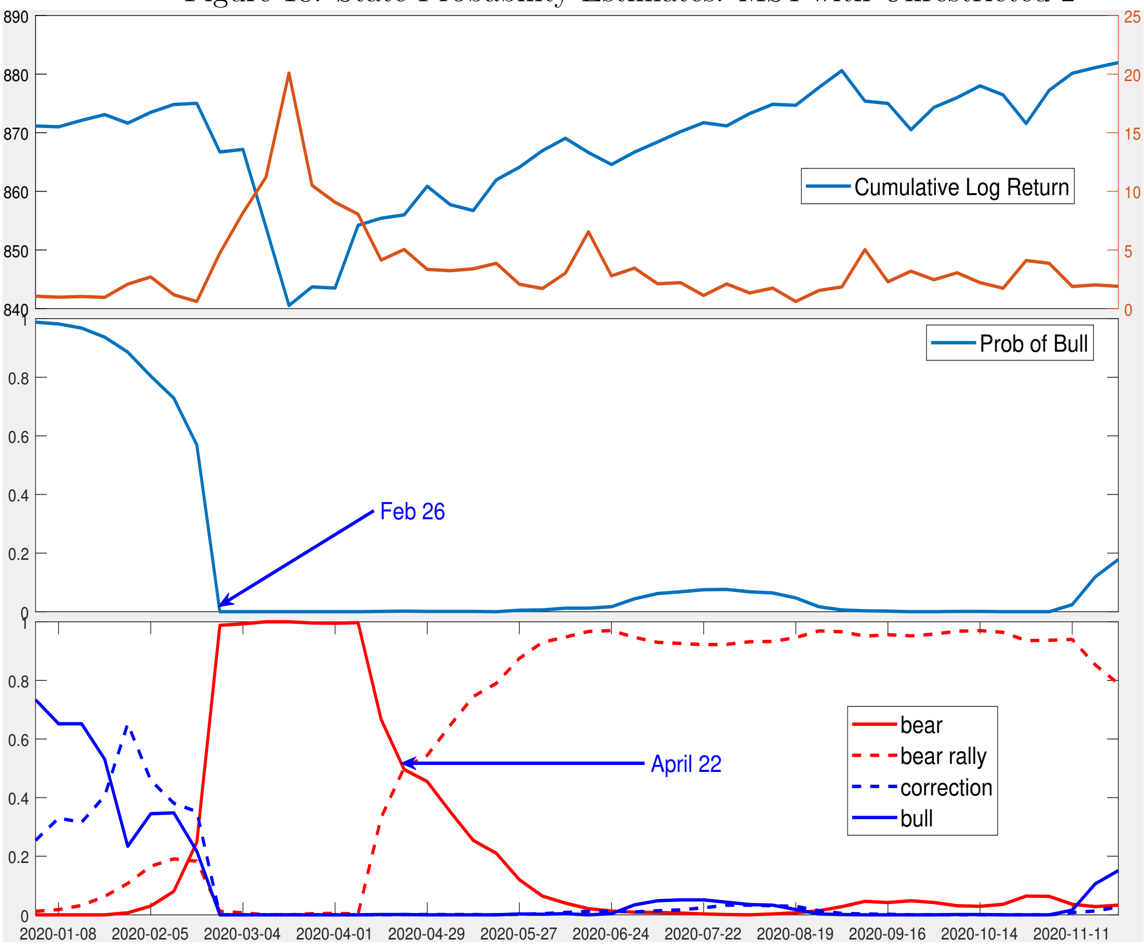


Figure 19: State Probability Estimates: MS4 with vs without P Restrictions