Money, Human Capital and Endogenous Market Structure in a Schumpeterian Economy

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Abstract

We incorporate endogenous human capital accumulation into a scale-invariant Schumpeterian growth model with endogenous market structure. Endogenous human capital accumulation leads to continuous entry of firms. Therefore, continuous horizontal innovation is sustained by human capital accumulation in the absence of population growth and becomes a twin engine of long-run growth (together with vertical innovation). We then study monetary policy by considering a cash-in-advance constraint on consumption. We find that when the capital share in final good production is low (high), the effect of inflation on growth is positive (negative). We then use cross-country panel regressions to test the theoretical prediction and find that inflation and capital share have a significant, negative interaction effect on growth, which provides support for our theory.

JEL Classification: O30, O40, E41, I15

Keywords: Monetary policy; Human capital; Endogenous market structure; Economic growth

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1 Introduction

What drives long-run growth? Uzawa (1965) and Lucas (1988) highlight the role of human capital accumulation.[1] The later new growth theories (NGTs) use endogenized technological progress: the first-generation NGTs use either expanding varieties (Romer, 1990) or quality improvement (Aghion and Howitt, 1992), while the second-generation NGTs consider both expanding varieties and quality improvement (see e.g., Dinopoulos and Thompson, 1998; Peretto, 1998, 1999; Young, 1998; Howitt, 1999; Segerstrom, 2000). Our study incorporates human capital accumulation into second-generation NGTs to get a deeper understanding of the mechanism of long-run growth. This integrated framework also yields new findings on the effects of inflation on growth and welfare—the fundamental issues in monetary economics—that are supported by our cross-country panel regressions, as elaborated on below.

First, our model enriches our understanding of the roles of each of the three factors—the accumulation of human capital, the quality improvement of existing products, and the emergence of new products—in driving long-run growth. Specifically, based on Peretto (2007, 2011), we model variety-expanding as the entry of new firms (horizontal R&D), while vertical R&D (quality improvement) is conducted by incumbents.[2] As in Lucas (1988), we assume that a household devotes an endogenous fraction of its non-leisure time to current production (as unskilled labor), and the remaining to human capital accumulation (as skilled labor supplied to current production). In existing second-generation NGTs, long-run growth eventually depends on vertical innovation (see Peretto, 1998, 1999; Segerstrom, 2000). This is because a larger population leads to entry of more firms, leaving the per firm employment unchanged on the balanced growth path (BGP) and thereby offsetting the positive effect of a larger population on the returns to R&D. In several of these models, horizontal entry occurs at the growth rate of the population on the BGP. The key feature here is that this is an exogenous mechanism. Besides, it establishes a positive correlation between the long-run economic growth rate and the population growth rate, which does not seem to be backed up by the empirical evidence (see Strulik et al., 2013). By contrast, our mechanism (human capital accumulation) is endogenous. Endogenous human capital accumulation leads to continuous entry of firms.[3] Therefore, continuous horizontal innovation is sustained in the absence of population growth. Besides, it establishes a positive correlation between long-run growth and human capital accumulation, which is backed up by empirical studies (see, e.g., Glaeser et al., 2004).

Second, we study the effects of monetary policy on growth and welfare by considering a cash-in-advance (CIA) constraint on consumption. Researchers have also studied the effects of inflation on growth and welfare in NGTs (e.g., Marquis and Reffett, 1994; Funk and Kromen, 2010; Chu and Lai, 2013; Chu and Cozzi, 2014; Chu and Ji, 2016; Chu et al., 2019).

[1] Recently, Lucas (2015) shows that long-run growth can be sustained by human capital accumulation alone. Stokey (2017) highlights the complementarity between human capital accumulation and technological progress in sustaining long-run growth.

[2] In a series of papers by Howitt (1999), Li (2000), Cozzi and Spinesi (2006), Strulik (2007), Gil (2013), Gil et al. (2013), and Gil et al. (2017), both types of R&D are conducted by entrants.

[3] The setup in Gil (2013), Gil et al. (2013), and Gil et al. (2017) also allows for continuous entry of firms that relies on an endogenous mechanism: both types of R&D have a lab-equipment specification, and the prospect of future profits due to vertical innovation sustains horizontal entry even without population growth. This is a complementary mechanism to human capital accumulation.

Our integrated framework predicts a testable, new finding: when the capital share in final good production is low, the effect of inflation on growth is positive; when the capital share is high, the effect of inflation on growth is negative. Steady-state welfare is an increasing function of the nominal interest rate, when the capital share is low. By contrast, when the capital share is high, the sign of the effect of the nominal interest rate on steady-state welfare depends on structural parameters. The intuition concerning growth is as follows.

A higher nominal interest rate reduces labor supply through the consumption-leisure choice, which results in a decrease in human capital accumulation and thereby the growth rate of variety. However, the growth rate of output will increase (decrease) if the equilibrium firm size in terms of human capital increases (decreases), because the return on quality-improving innovation and the growth rate of quality positively depend on per firm human capital. The lower level of labor supply increases (decreases) the steady-state value of firm size when the capital share is low (high), and the higher (lower) level of firm size increases (decreases) the growth rate of quality and the growth rate of output. This is the typical scale economy/effect of a larger firm size in terms of human capital on innovation. Our modelling strategy is consistent with existing empirical findings that the return on R&D depends on the level of firm's human capital (e.g., firm-sponsored training); see Ballot et al. (2001). According to the review of Miyamoto (2003), researchers have found that a high level of human capital is one of the key ingredients for attracting foreign direct investment (FDI). Inward FDI in developing countries has been modeled as varieties expanding in Borensztein et al. (1998).

Third, we use cross-country panel data during 1970-2014 to test the growth implications of our theory. We find that inflation and capital share have a significant, negative interaction effect on growth. When the capital share lies on the sample interval $[0.15, 0.31)$ ($(0.31, 0.85]$), the marginal effect of inflation on growth would be positive (negative). The results indicate that the predictions of our theory are supported by the data. Additionally, we find that having a 1% increase in annual inflation rate would have allowed countries with the mean level of capital share (0.45) to experience a 0.06% decrease in annual growth rate of real GDP per employment, and the country with the lowest/highest value of capital share (0.15/0.85) would have experienced an annual growth rate increase/decrease of 0.07%/0.24% during 1970-2014. Our empirical results not only help to explain the substantial variations in growth rates across countries, but also has strong policy implications for the conduct of monetary policy. We also calibrate the model using the US data and simulate the effects of a 1% increase in the nominal interest rate on economic growth.

Our study relates to existing studies that introduce human capital accumulation into NGTs. Some researchers have introduced endogenous human capital accumulation into first-generation NGTs (see e.g., Arnold, 1998; Chu et al., 2013; Chu et al., 2019). These models feature human capital accumulation and only one type of innovation (either vertical or horizontal). Strulik (2007) considers human capital accumulation with both vertical and horizontal innovation in very different set-ups, but his focus is on the effect of population growth on long-run economic growth. This study contributes to this literature by introducing endogenous human capital accumulation into the second-generation NGTs with endogenous market structure (EMS) and exploring the role of human capital accumulation in the two dimensions of innovation and growth. The EMS has novel implications on the way that human capital accumulation affects innovation and growth.

Our study also relates to existing studies by exploring the effects of monetary policy on growth and welfare: from capital accumulation models (e.g., Sidrauski, 1967; Lucas, 1972; Stockman, 1981) to non-R&D-based endogenous growth models (e.g., Gomme, 1993; Jones and Manuelli, 1995; Dotsey and Sarte, 2000) and to NGTs (e.g., Chu et al., 2019).[4] This paper contributes to this literature by considering a CIA constraint on consumption in the second-generation Schumpeterian growth model with EMS and endogenous human capital accumulation to explore the effects of inflation on growth and welfare. The EMS has novel implications on the effects of inflation on innovation and growth through the endogenous human capital accumulation.

The rest of this paper is organized as follows. Section 2 presents the stylized facts. Section 3 sets up the model. Section 4 explores the dynamic effects of monetary policy on economic growth and social welfare. Section 5 concludes.

2 Stylized facts

In this section, we present the empirical findings from testing the theoretical prediction of our model—when the capital share in final good production is low, the effect of inflation on growth is positive; when the capital share is high, the effect of inflation on growth is negative.[5] We use the following empirical specification:

$$g_{it} = \varphi_0 + \varphi_1 \pi_{it} + \varphi_2 \pi_{it} \times \kappa_{it} + \Gamma \psi_{it} + \lambda_i + \lambda_t + \varepsilon_{it},$$

where g_{it} is the average annual growth rate of real GDP per employment for country i during period t; $\pi_{i,t}$ is the average annual inflation rate for country i during period t; κ_{it} is the average capital share in output for country i during period t; $\pi_{it} \times \kappa_{it}$ is the interaction term between inflation and capital share. ψ_{it} denotes a vector of control variables, namely, κ_{it}, a measure of human capital because our model features human capital accumulation, the role of government, and the degree of openness. λ_i and λ_t stand for the country and time fixed effects, respectively. As discussed, our theory predicts $\varphi_2 < 0$, which indicates that the marginal effect of inflation on growth would depend *negatively* on κ_{it}. Our theory also predicts that $\partial g_{it}/\partial \pi_{it} = \varphi_1 + \varphi_2 \kappa_{it} > (<)\, 0$ when κ_{it} is relatively low (high). Moreover, the capital share κ_{it} lies on the interval $(0, 1)$. As a result, our model predicts $\varphi_1 > 0$ and $\varphi_1 + \varphi_2 < 0$.

The recent Penn World Table (PWT), explained by Feenstra et al. (2015), provides the most complete growth accounting data for 182 countries during 1950–2014. It is a common practice in the empirical growth literature to take five-year averages of the data to smooth out business cycle fluctuations. Therefore, the time sample of 1950–2014 naturally delivers 13 non-overlapping five-year average subperiods, the first being 1950–1954 and the last being 2010–2014. There are missing data, especially before 1970. Therefore, we focus on the time

[4] See Gillman and Kejak (2005) for a survey of the literature on the effect of inflation on economic growth in capital-based growth models (including physical and human capital), and Chu (2020) for a survey in R&D-based growth models.

[5] As discussed in Walsh (2010), the Fisher equation gives rise to a positive long-run relationship between the inflation rate and the nominal interest rate, which is supported by empirical studies (e.g., Mishkin, 1992; Booth and Ciner, 2001). Therefore, we use the inflation rate instead of the nominal interest rate.

sample of 1970–2014. The inflation data is from the World Development Indicators (WDI) of the World Bank. After merging the two datasets and retaining the observations that appeared in both the PWT and the WDI, our final sample has 154 countries during 1970–2014. Taking five-year non-overlapping averages of the data yields 9 subperiods, the first being 1970–1974 and the last being 2010–2014. In summary, we have a balanced panel with 1,386 observations (the final number of observations are much smaller due to missing data).

Our dependent variable is the average annual growth rate of real GDP per employment g_{it} for each five-year subperiod. Following the existing literature (e.g., Aghion et al., 2009), we measure the inflation rate π_{it} as the percentage change in the consumer price index (CPI) from the WDI. According to the theoretical model in the subsequent section, our final good production function is a Cobb-Douglas one, in which the capital share in output is κ_{it} and the labor share from PWT equals $1 - \kappa_{it}$. The other control variables include log value of human capital index, log value of government consumption share in GDP, and log value of openness (we add together the ratio of export value to GDP and the absolute value of the ratio of import value to GDP) from PWT. We then compute the five-year averages of all the variables. Table B1 in Appendix B presents the summary statistics of the data.

We focus on the ordinary least squares (OLS) results.[6] The reason is double-fold. First, Aghion et al. (2009) discussed: "Endogeneity will be less of an issue with an interaction term than with single variables." Second, capital share changes very slowly in our five-year data sample, which means the feedback of growth and other confounding omitted factors/policies on capital share may be small. Table 1 presents the OLS results with country and year fixed effects. In regression (1) of Table 1, we use the full sample. The results indicate that the estimated coefficient on the interaction term $\pi_{it} \times \kappa_{it}$ is positive and significant at the 1% level, and the estimated coefficient on π_{it} is negative and significant at the 1% level. In our sample, some countries have average annual inflation rates over 8600% (see Table B1). Therefore, we need to remove the inflation rate outliers.[7] We find that when the inflation rate outliers are removed, the signs of the estimated coefficients become consistent with the prediction of our model. In regression (2) of Table 1, where we use the sample of average annual inflation rates below 60%, the estimated coefficient on the interaction term $\pi_{it} \times \kappa_{it}$ is negative and significant at the 1% level, and the estimated coefficient on π_{it} is positive and significant at the 10% level. In regression (3) of Table 1, where we use the sample of average annual inflation rates below 50%, the estimated coefficient on $\pi_{it} \times \kappa_{it}$ remains negative and

[6] Dealing with endogeneity is the hardest in country-level growth regressions with panel data. See the critical review in Bazzi and Clemens (2013) on long-run growth regressions, and Nakamura and Steinsson (2017) on regressions with a business cycle focus. Invalid or weak instruments may cause the bias of instrumental variables (IV) regressions to be larger than that of OLS regressions. Nevertheless, our results remain robust in IV estimation when we follow Chu et al. (2019) to use the first lags of the endogenous variables and the average annual inflation rate of the rest of the world and its square as two additional instruments. Our results also hold up when we follow Checherita-Westphal and Rother (2012) to use the following seven instruments: the average annual inflation rate of the rest of the world together with its square and cubic term, the average broad money growth rate of the rest of the world together with its square and cubic term, and the logarithm of the total population of the same country. The results are available upon request. As discussed in Murray (2006), because the two groups of instruments are not totally grounded on the same rationale, they help to dispel the cloud of endogeneity.

[7] See also discussions in existing empirical studies, e.g., Barro (1995), Bruno and Easterly (1996), and Khan and Senhadji (2001).

significant at the 1% level, and the estimated coefficient on π_{it} is positive and significant at the 5% level. Regressions (4) and (5) of Table 1 indicate that the results remain similar in the sample of average annual inflation rates below 40% and 30%, respectively. The results in Table B2 in Appendix B indicate that our results remain robust when we use robust standard errors.

The magnitudes of estimated coefficients on π_{it} and $\pi_{it} \times \kappa_{it}$ are similar across the regressions. Therefore, we use regression (3) in Table 1 as our benchmark. We have

$$\frac{\partial g_{it}}{\partial \pi_{it}} = 0.138 - 0.445 \times \kappa_{it},$$

where threshold value of capital share κ_{it} is 0.31 (0.138÷0.445). According to Table B1 in Appendix B, the sample range of capital share lies on the interval $[0.15, 0.85]$. Therefore, when the capital share lies on the sample interval $[0.15, 0.31)$, the marginal effect of inflation on growth would be positive; when the capital share lies on the sample interval $(0.31, 0.85]$, the marginal effect of inflation on growth would be negative. The results indicate that the predictions of our theory are supported by the data.

We can estimate how important inflation and capital share have been in affecting the growth rate of output per employment from Table 1. From regression (3), it turns out that having a 1% increase in annual inflation rate would have allowed countries with the mean level of capital share (0.45) to experience a 0.06% decrease in annual growth rate of real GDP per employment (0.138-0.445*0.45) during the 45-year-period 1970-2014, the country with the lowest value of capital share (0.15) to experience a 0.07% increase (0.138-0.445*0.15), and the country with the highest value of capital share (0.85) to experience a 0.24% decrease (0.138-0.445*0.85).

Table 1. Effects of inflation on economic growth

	Full sample	π_{it}<60	π_{it}<50	π_{it}<40	π_{it}<30
	(1)	(2)	(3)	(4)	(5)
π_{it}	-0.009***	0.107*	0.138**	0.115	0.105
	(0.002)	(0.063)	(0.068)	(0.083)	(0.090)
$\pi_{it} \times \kappa_{it}$	0.020***	-0.355***	-0.445***	-0.386**	-0.383**
	(0.004)	(0.137)	(0.146)	(0.172)	(0.187)
Control variables	Yes	Yes	Yes	Yes	Yes
Country fixed effects	Yes	Yes	Yes	Yes	Yes
Time fixed effects	Yes	Yes	Yes	Yes	Yes
R^2	0.29	0.33	0.34	0.34	0.33
Observations	787	742	736	724	712

Note: *** Significant at the 0.01 level, ** at the 0.05 level, * at the 0.10 level. Standard errors are in parentheses. π_{it} is the inflation rate (in percentage term), and κ_{it} is the capital share. The dependent variable is the average annual growth rate of real GDP per employment (in percentage term). Control variables include capital share, (log) human capital index, (log) government consumption share, and (log) openness.

3 A monetary Schumpeterian growth model with endogenous market structure and human capital accumulation

In this section, we develop a monetary Schumpeterian growth model with human capital accumulation based on Peretto (2007, 2011), which features two dimensions of innovation, i.e., the horizontal innovation (variety-expanding innovation) and the vertical innovation (quality-improving innovation). We introduce human capital accumulation as in Lucas (1988) and CIA constraint on consumption.[8] We provide a complete closed-form solution for the economy's transitional dynamics and its balanced growth path.

3.1 Household

There is a representative household endowed with L units of labor, which has a lifetime utility function as

$$U = \int_0^\infty e^{-\rho t} \left[\ln c_t + \theta \ln(L - l_t) \right] dt, \tag{1}$$

where c_t is the household's consumption of final good (numeraire) and l_t is the labor supply at time t. Population is normalized to 1. $\rho > 0$ is the rate of time preference and $\theta > 0$ governs the household's preference for leisure. The representative household maximizes its lifetime utility in (1) subject to the asset-accumulation equation given by

$$\dot{a}_t + \dot{m}_t = r_t a_t + w_{h,t} h_t + w_{l,t} \left(u_t l_t \right) - c_t - \pi_t m_t + \tau_t. \tag{2}$$

a_t is the real value of equity shares in monopolistic intermediate goods firms owned by the household, and r_t is the real interest rate of assets a_t. $w_{h,t}$ is the real wage rate of human capital h_t (skilled labor), and $w_{l,t}$ is the real wage rate of raw labor $u_t l_t$ (unskilled labor) used in the production of final good. m_t is the real money balance held by the household to facilitate purchases of final good for consumption, and π_t is the cost of holding money (i.e., the inflation rate). The household also receives a lump-sum transfer of the seigniorage revenue τ_t from the government (or pays a lump-sum tax if $\tau_t < 0$). As in Lucas (1988), the household devotes an endogenous fraction $u_t \in (0, 1]$ of its non-leisure time l_t to current production (as unskilled labor), and the remaining $(1 - u_t)$ to human capital accumulation (then as skilled labor supplied to current production). Therefore, the law of motion for human capital accumulation would be

$$\dot{h}_t = \xi \left(1 - u_t \right) l_t h_t, \tag{3}$$

where $\xi > 0$ is the productivity parameter for human capital investment. The cash-in-advance (CIA) constraint on consumption is given by

$$\varsigma c_t \leq m_t, \tag{4}$$

[8] Chu and Ji (2016) develop a monetary Schumpeterian growth model with CIA constraint on consumption without human capital accumulation in Peretto (2007, 2011).

where $\varsigma > 0$ captures the strength of the CIA constraint on consumption.

The optimality condition for consumption is (see Appendix A for derivation)

$$\frac{1}{c_t} = \mu_t \left(1 + \varsigma i_t\right), \tag{5}$$

where μ_t is the Hamiltonian costate variable on (2), and $i_t = r_t + \pi_t$ is the nominal interest rate. The Euler equation is given by

$$\frac{\dot{c}_t}{c_t} = -\frac{\dot{\mu}_t}{\mu_t} = r_t - \rho. \tag{6}$$

The optimality condition for labor supply is

$$l_t = L - \frac{\theta c_t \left(1 + \varsigma i_t\right)}{w_{l,t}}. \tag{7}$$

The no-arbitrage condition between investment in asset holding and that in human capital is

$$r_t = \xi h_t \frac{w_{h,t}}{w_{l,t}} + \frac{\dot{w}_{l,t}}{w_{l,t}}. \tag{8}$$

3.2 Final good

Final good sector is competitive. The production function of final good firms is given by

$$Y_t = \int_0^{N_t} X_t^{\gamma}(j) \left[Z_t^{\alpha}(j) Z_t^{1-\alpha} \frac{h_t^{\epsilon} \left(u_t l_t\right)^{1-\epsilon}}{N_t} \right]^{1-\gamma} dj, \tag{9}$$

where $\{\alpha, \gamma, \epsilon\} \in (0,1)$. $X_t(j)$ is the quantity of intermediate goods $j \in [0, N_t]$, h_t is the human capital (skilled labor), and $u_t l_t$ is the raw labor (unskilled labor).[9] The productivity of $X_t(j)$ is determined by its own quality $Z_t(j)$ and the average quality of all intermediate goods $Z_t = \int_0^{N_t} Z_t(j)\, dj / N_t$. Profit maximization of final good firms yields the following conditional demand functions for $X_t(j)$, h_t and $u_t l_t$:

$$X_t(j) = \left(\frac{\gamma}{p_t(j)}\right)^{1/(1-\gamma)} Z_t^{\alpha}(j) Z_t^{1-\alpha} \frac{h_t^{\epsilon} \left(u_t l_t\right)^{1-\epsilon}}{N_t}, \tag{10}$$

$$h_t = \epsilon \left(1-\gamma\right) Y_t / w_{h,t}, \tag{11}$$

$$u_t l_t = \left(1-\epsilon\right)\left(1-\gamma\right) Y_t / w_{l,t}, \tag{12}$$

where $p_t(j)$ is the price of $X_t(j)$ denominated in units of final good Y_t. Perfect competition in the final good sector implies that firms pay $\gamma Y_t = \int_0^{N_t} p_t(j) X_t(j)\, dj$ for intermediate

[9] Peretto (2007, 2011) considers a production function that replaces $h_t^{\epsilon} \left(u_t l_t\right)^{1-\epsilon} / N_t$ by $l_{x,t}(j)$. In Chu and Ji (2016), it is replaced by the ratio of l_t / N_t, which is equivalent to $l_{x,t}(j) = l_t / N_t$ in equililibrium. Therefore, the indirect specification $h_{x,t}^{\epsilon}(j)\, l_{x,t}^{1-\epsilon}(j)$ is equivalent to the direct specification $h_t^{\epsilon} \left(u_t l_t\right)^{1-\epsilon} / N_t$.

goods.

3.3 Intermediate goods and in-house R&D

The second-generation Schumpeterian growth model features two dimensions of innovation, specifically, the variety-expanding innovation (horizontal innovation) and the quality-improving innovation (vertical innovation). Quality improvement is conducted by incumbent firms, and the horizontal innovation is the entry of new firms that invent new types of intermediate goods.

The monopolistic incumbent firms produce intermediate goods as well as invest in improving the quality of their products. For production, an incumbent firm transforms one unit of final good into one unit of intermediate good and sells it to the final good firms. An incumbent firm also incurs a fixed operating cost in the amount of $\phi Z_t^{\alpha}(j) Z_t^{1-\alpha}$ units of final good, where ϕ is the operating cost parameter. Therefore, the before-R&D profit flow of the incumbent firm j is

$$F_t(j) = [p_t(j) - 1] X_t(j) - \phi Z_t^{\alpha}(j) Z_t^{1-\alpha}. \tag{13}$$

For quality improvement, the incumbent firm invests $R_t(j)$ units of final good to improve the quality of its products. The quality improvement process is

$$\dot{Z}_t(j) = R_t(j). \tag{14}$$

We denote $v_t(j)$ as the value of the monopolistic firm in industry j, which is given by

$$v_t(j) = \int_t^{\infty} \exp\left(-\int_t^s r_{\omega} d\omega\right) \Pi_s(j)\, ds, \tag{15}$$

where the profit flow $\Pi_t(j)$ is given by

$$\Pi_t(j) = F_t(j) - R_t(j). \tag{16}$$

The monopolistic firm maximizes its market value in (15) subject to (14) and (16). The equilibrium price of intermediate goods is

$$p_t(j) = 1/\gamma. \tag{17}$$

Following the assumption of symmetric equilibrium in the literature (e.g., Peretto, 1998, 2007; Segerstrom, 2000; Cozzi et al., 2007; Chu and Ji, 2016) in which $Z_t(j) = Z_t$, we have $X_t(j) = X_t$, which implies that the size of each intermediate goods firm is identical across all industries, and $R_t(j) = R_t$, $\Pi_t(j) = \Pi_t$, and $v_t(j) = v_t$. The quality-adjusted firm size is

$$\frac{X_t}{Z_t} = \gamma^{2/(1-\gamma)} \frac{h_t^{\epsilon} (u_t l_t)^{1-\epsilon}}{N_t}. \tag{18}$$

We define the following transformed variable:

$$x_t = \frac{1}{(u_t l_t)^{1-\epsilon}} \frac{X_t}{Z_t} = \gamma^{2/(1-\gamma)} \frac{h_t^\epsilon}{N_t}, \tag{19}$$

which is a state variable and increasing in h_t^ϵ / N_t.

Using current-value Hamiltonian (see Appendix A) yields the rate of return on in-house R&D r_t^I as

$$r_t^I = \alpha \left[\frac{1-\gamma}{\gamma} x_t (u_t l_t)^{1-\epsilon} - \phi \right]. \tag{20}$$

The rate of return on quality-improvement (in-house R&D) r_t^I is increasing in the quality-adjusted firm size $x_t (u_t l_t)^{1-\epsilon}$.

3.4 Entrants

Following previous studies, the new variety has the average quality level of existing products Z_t to ensure that symmetric equilibrium always holds. A new firm has to pay a setup cost in the amount of βX_t units of final good, where β is the entry cost parameter and X_t is the size of its initial production, to invent a new variety of product. In other words, the entry mechanism is $\dot{N}_t = R_t^N / (\beta X_t)$, where R_t^N is the units of final good used for entry. The free-entry condition for the entrants is

$$v_t = \beta X_t, \tag{21}$$

where v_t is the market value of incumbent firms given in (15). Using symmetry, we have the following no-arbitrage condition

$$r_t = \frac{\Pi_t}{v_t} + \frac{\dot{v}_t}{v_t}. \tag{22}$$

Substituting (14), (16), (19), (21) and (17) into (22), we have the rate of return on entry given by

$$r_t^E = \frac{1}{\beta} \left[\frac{1-\gamma}{\gamma} - \frac{\phi + z_t}{x_t (u_t l_t)^{1-\epsilon}} \right] + \frac{\dot{x}_t}{x_t} + z_t + (1-\epsilon) \left(\frac{\dot{u}_t}{u_t} + \frac{\dot{l}_t}{l_t} \right), \tag{23}$$

where $z_t \equiv \dot{Z}_t / Z_t$ is the growth rate of quality. The rate of return on entry r_t^E is also increasing in the quality-adjusted firm size $x_t (u_t l_t)^{1-\epsilon}$.

3.5 Monetary authority

The nominal money supply is denoted by M_t, and its growth rate is $\dot{M}_t / M_t$. The price of final good is denoted by P_t. The real money balance is $m_t = M_t / P_t$, which gives $\dot{m}_t / m_t = \dot{M}_t / M_t - \pi_t$. The seigniorage revenue τ_t is

$$\tau_t = \frac{\dot{M}_t}{P_t} = \frac{\dot{M}_t}{M_t} m_t = \dot{m}_t + \pi_t m_t, \tag{24}$$

which is conferred to household as a lump-sum transfer. The nominal interest rate i_t is the monetary policy instrument that we consider. Using Fisher equation $i_t = r_t + \pi_t$ and $\dot{c}_t/c_t = \dot{m}_t/m_t$, we have $i_t = \dot{M}_t/M_t + \rho$. As a result, i_t is determined by the growth rate of money supply $\dot{M}_t/M_t$.

3.6 General equilibrium

The general equilibrium is a time path of prices $\{p_t, r_t, w_{h,t}, w_{l,t}, v_t\}$, monetary policy i_t, and allocations $\{c_t, a_t, m_t, l_t, h_t, u_t, Y_t, X_t, R_t\}$ which satisfies the following conditions at each instance of time:

- the household maximizes utility taking prices $\{i_t, r_t, w_{h,t}, w_{l,t}\}$ as given;
- competitive final good firms maximize profits taking $\{p_t, w_{h,t}, w_{l,t}\}$ as given;
- intermediate goods firms choose $\{R_t, p_t\}$ to maximize v_t taking r_t as given;
- entrant firms make entry decisions taking v_t as given;
- the labor market clears;
- the final good market clears: $Y_t = c_t + N_t (X_t + \phi Z_t + R_t) + \dot{N}_t \beta X_t$;
- the CIA constraint binds: $\varsigma c_t = m_t$;
- the value of monopolistic firms adds up to the value of household's assets, i.e., $a_t = v_t N_t$.

Substituting (10) into (9), and using (17) and symmetry, we have the following aggregate production function:

$$Y_t = \gamma^{2\gamma/(1-\gamma)} Z_t h_t^{\epsilon} (u_t l_t)^{1-\epsilon}. \tag{25}$$

3.7 Dynamics of the economy

Lemma 1 *The consumption-output ratio must jump to its steady-state value given by*

$$\frac{c_t}{Y_t} = \rho\beta\gamma^2 + 1 - \gamma, \tag{26}$$

when $L > \rho\theta (\rho\beta\gamma^2 + 1 - \gamma)(1 + \varsigma i_t) / [(1-\gamma)\xi\epsilon]$ (to ensure $l_t > 0$).

Proof. See Appendix A. ■

Proposition 1 *The labor supply l_t and the households' non-leisure time allocated to current production $u_t l_t$ must jump to their steady-state values given by*

$$l = L - \frac{\theta\rho (\rho\beta\gamma^2 + 1 - \gamma)}{\epsilon\xi (1-\gamma)} (1 + \varsigma i), \tag{27}$$

$$ul = \frac{\rho (1-\epsilon)}{\epsilon\xi}, \tag{28}$$

in which l is decreasing in the nominal interest rate i, and ul is independent of i.

Proof. See Appendix A. ■

Lemma 1 shows that consumption c_t and output Y_t grow at the same rate, i.e., $\dot{c}_t/c_t = \dot{Y}_t/Y_t$. Proposition 1 shows that the household's non-leisure time allocated to current production $u_t l_t$ is a constant and independent of the nominal interest rate i_t. The labor supply l_t always jumps to its steady-state value, which is decreasing in the nominal interest rate i_t. Therefore, the fraction of the household's non-leisure time allocated to current production (as unskilled labor) u_t is increasing in i_t, and the remaining $1 - u_t$ to human capital accumulation is decreasing in i_t whenever $u_t \in (0, 1)$.

Intuitively, a higher nominal interest rate raises the cost of consumption relative to leisure under CIA constraint, which results in an increase in leisure and a decrease in labor supply of the household. The lower level of labor supply reduces the household's non-leisure time available to both production and human capital accumulation, which causes a negative effect on the labor supply to production. The lower level of labor reduces the marginal product of human capital in final good production for a given level of u_t and the return on human capital accumulation, which shifts the labor from human capital accumulation to production and causes a positive effect on the labor supply to production. The positive effect and the negative effect of lower level of labor supply on production labor cancel each other, such that the non-leisure time allocated to production is independent of total labor supply and the nominal interest rate.

When the labor supply l_t is low enough, u_t takes the value of its upper bound, i.e., $u_t = 1$, which happens when the nominal interest rate i_t is high enough or the productivity of human capital investment ξ is low enough. In this case, the level of human capital h_t remains at its initial level h_0 at any time t. In the following analysis, we focus on the cases of $u_t \in (0, 1)$, in which $\dot{h}_t > 0$. Using Lemma 1 and log differentiating (25) yields the output growth rate

$$g_t \equiv \frac{\dot{Y}_t}{Y_t} = z_t + \epsilon \frac{\dot{h}_t}{h_t}, \tag{29}$$

which is determined by the quality growth rate z_t and human capital growth rate $\dot{h}_t/h_t$.

The Euler equation in (6) and Lemma 1 imply that

$$g_t = \frac{\dot{Y}_t}{Y_t} = \frac{\dot{c}_t}{c_t} = r_t - \rho. \tag{30}$$

Substituting r_t^I in (20) into (30) yields the growth rate of output as

$$g_t = r_t^I - \rho = \alpha \left[\frac{1-\gamma}{\gamma} x_t \left(ul \right)^{1-\epsilon} - \phi \right] - \rho, \tag{31}$$

which is increasing in the quality-adjusted firm size x_t, i.e., the level of human capital per firm h_t^{ϵ}/N_t. Substituting (27) and (28) into (3) yields the growth rate of human capital given by

$$\frac{\dot{h}_t}{h_t} = \xi \left(l_t - ul \right), \tag{32}$$

which is decreasing in the nominal interest rate i_t through l_t. From (28), (29), (31) and (32), we can derive the growth rate of quality given by

$$z_t = \alpha \left[\frac{1-\gamma}{\gamma} x_t \left(ul\right)^{1-\epsilon} - \phi \right] - \epsilon \xi l_t - \rho \epsilon, \tag{33}$$

which is increasing in x_t, and is positive if and only if

$$x_t > \bar{x} \equiv \frac{\gamma}{1-\gamma} \left(\frac{\epsilon \xi l + \rho \epsilon}{\alpha} + \phi \right) \left(ul\right)^{\epsilon - 1}. \tag{34}$$

Intuitively, the quality-improving innovation is viable when the market size (human capital per firm) is large enough. Substituting (6), (29) and (33) into (23) and using the definition of x_t in (19), (28) and Lemma 1 yield the growth rate of variety $n_t \equiv \dot{N}_t / N_t$ given by

$$n_t = \frac{1}{\beta} \left[\left(1-\alpha\right) \frac{1-\gamma}{\gamma} - \frac{\left(1-\alpha\right)\phi - \rho\epsilon - \epsilon\xi l_t}{x_t \left(ul\right)^{1-\epsilon}} \right] - \rho, \tag{35}$$

which is also increasing in x_t. Substituting (35) and (32) into $\dot{x}_t / x_t = \epsilon \dot{h}_t / h_t - n_t$ yields

$$\dot{x}_t = \frac{\left(1-\alpha\right)\phi - \rho\epsilon - \epsilon\xi l_t}{\beta \left(ul\right)^{1-\epsilon}} - \left[\left(1-\alpha\right) \frac{1-\gamma}{\beta\gamma} - \rho\epsilon - \epsilon\xi l_t \right] x_t. \tag{36}$$

The dynamics of x_t is determined by the above one-dimensional differential equation.

Proposition 2 *When $\alpha < \min\left\{1 - \epsilon\left(\rho + \xi l\right)/\phi, 1 - \beta\gamma\epsilon\left(\rho + \xi l\right)/\left(1-\gamma\right)\right\}$, the dynamics of x_t is stable and x_t converges to its steady-state value given by*

$$x^* = \frac{\left(1-\alpha\right)\phi - \epsilon\left(\rho + \xi l\right)}{\left[\left(1-\alpha\right)\left(1-\gamma\right)/\gamma - \beta\epsilon\left(\rho+\xi l\right)\right]\left(ul\right)^{1-\epsilon}}, \tag{37}$$

and the steady-state value of g_t is given by

$$g^* = \alpha \left[\frac{1-\gamma}{\gamma} \left(ul\right)^{1-\epsilon} x^* - \phi \right] - \rho. \tag{38}$$

Proof. See Appendix A. ■

In previous studies, e.g., Peretto (1998, 2007) and Chu and Ji (2016), the return on horizontal innovation depends on the market size that is determined by the population size of the economy. The growth rate of the number of products (firms) is proportional to the growth rate of population on the BGP, such that the variety-expanding innovation is determined by the exogenous population growth. As a result, these studies establish a positive correlation between the long-run economic growth rate and the population growth rate, which does not seem to be backed up by the empirical evidence; see, e.g., Strulik et al. (2013). In addition, when population is constant, the entry of new firms will stop eventually. As a result, although the existing studies (e.g., Peretto, 1998, 2007; Chu and Ji, 2016) feature two-dimension R&D, long-run growth depends solely on quality improvement (i.e., long-run growth does not depend on variety-expanding) under fixed population.

By contrast, in our model, the return on horizontal innovation depends on the market size that is determined by the level of human capital of the economy. Human capital is accumulated by the household endogenously, which will not be limited by the population size. The growth rate of the number of products (firms) is proportional to the growth rate of human capital on the BGP, such that the variety-expanding innovation is determined by the endogenous human capital accumulation.[10] Using (29), the growth rate of the economy can also be expressed as $g^* = z_t + \epsilon \dot{h}_t/h_t = z_t + n_t$. Therefore, long-run growth depends on both quality improvement and variety-expanding even with fixed population. In other words, variety-expanding sustained by human capital accumulation, together with quality-improving innovation, becomes a twin-engine of long-run economic growth in our model. Moreover, our model establishes a positive correlation between the long-run economic growth rate and the human capital growth rate, which is backed up by the empirical evidence; see, e.g., Glaeser et al. (2004).

Our model is also consistent with the empirical findings in the industrial organization (IO) literature, in which the return on R&D depends on the human capital level of firms, e.g., firm-sponsored training; see Ballot et al. (2001), who find that the rates of return for R&D at the mean value are 38% for France and 32% for Sweden, and the rates of return for training capital are 288% for France and 441% for Sweden.

4 Growth and welfare effects of monetary policy

In this section, we explore the effects of monetary policy on economic growth and social welfare, specifically, the effects of a permanent increase in the nominal interest rate. Section 4.1 analyzes the effects of monetary policy on economic growth, Section 4.2 analyzes the effects of monetary policy on social welfare, and Section 4.3 performs a quantitative analysis.

4.1 Growth effects of monetary policy

An increase in the nominal interest rate i_t decreases the household's labor supply l_t in (27) but does not affect the non-leisure time allocated to current production $u_t l_t$ in (28) as shown in the Proposition 1, which results in a permanent decrease in the human capital growth rate $\dot{h}_t/h_t$ in (32). The lower level of l_t also decreases the growth rate of variety n_t in (35) but increases the growth rate of quality z_t in (33), leaving the growth rate of output g_t in (31) unchanged when the number of products is fixed in the short run (x_t is a state variable and $u_t l_t$ is independent of i_t).

The intuition of the above results can be explained as follows. Under the CIA constraint on consumption, an increase in the nominal interest rate raises the cost of consumption relative to leisure, which results in a decrease in consumption and an increase in leisure, or equivalently, a decrease in labor supply of the household. Proposition 1 shows that the non-leisure time allocated to current production is independent of the nominal interest rate. Therefore, the lower level of labor supply decreases the household's non-leisure time allocated to human capital accumulation and the growth rate of human capital permanently. The lower

[10] Recall that the steady-state value of firm size x^* is a constant $x^* = \gamma^{2/(1-\gamma)} h_t^{\epsilon}/N_t$, which yields $\epsilon \dot{h}_t/h_t = n_t$.

growth rate of human capital decreases the return on entry (horizontal innovation), leading to a decrease in the growth rate of variety in the short run. Therefore, more resources will be devoted to in-house R&D (vertical innovation), which increases the growth rate of quality. The positive effect on quality-improving innovation and the negative effect on human capital accumulation of an increase in the nominal interest rate cancel each other, leaving the growth rate of output unchanged in the short run.

The long-run effects of a higher nominal interest rate on economic growth is determined by parameter values. When the capital intensity γ is low (high), an increase in the nominal interest rate i_t increases (decreases) the long-run growth rate of output g_t in (31). This is because the lower level of labor supply l_t caused by a higher nominal interest rate increases (decreases) the steady-state value of firm size x^* in (37) when γ is low (high), and the higher (lower) level of firm size increases (decreases) the growth rate of quality z_t in (33) and the growth rate of output g_t.

The intuition of the above results can be explained as follows. When the capital intensity in the final good production is low (high), the intermediate goods firms will charge a higher (lower) markup as shown in (17), which results in higher (lower) profits and thus higher (lower) market values of the monopolistic firms for a given level of firm size x_t. Therefore, the decrease in the growth rate of human capital caused by a higher nominal interest rate lowers the market values of monopolistic firms by a relatively larger (smaller) extent when γ is low (high), resulting in a larger (smaller) decrease in the growth rate of variety than that of human capital in the short run. This larger (smaller) decrease in the growth rate of variety increases (decreases) the firm size x_t, raising (reducing) the returns on variety-expanding innovation and quality-improving innovation, and thus raising (reducing) the growth rates of variety and quality on the transition path of the economy.

Eventually, the growth rate of variety converges to its lower steady-state value and the growth rates of quality and output converge to their higher (lower) steady-state values in the long run, with the firm size converges to its higher (lower) steady-state value according to (36) when γ is low (high). The transitional dynamics of output growth rate following an increase in the nominal interest rate is summarized in Figure 1 and Proposition 3.

Proposition 3 *An increase in the nominal interest rate has the following effects on economic growth: (a) it does not affect the growth rate of output in the short run; and (b) it increases (decreases) the growth rate of output in the long run, when the capital share in final good production is low (high).*

Proof. See Appendix A. ■

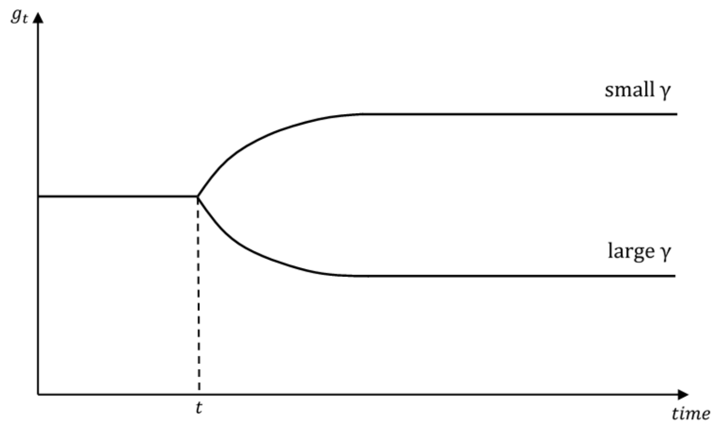

Figure 1. Transitional dynamics of the growth rate

4.2 Welfare effects of monetary policy

As discussed above, following an increase in the nominal interest rate, the firm size x_t adjusts gradually to its new steady-state value. On the transition path, the labor supply in (27) is constant as shown in Proposition 1, but the growth rate of output g_t adjusts gradually to its new steady-state value. For this reason, we discuss the welfare effects in two parts. The first compares the steady-state welfare, and the second takes into account the transition path.

To derive the steady-state welfare, we impose balanced growth on (1) to have

$$U = \frac{1}{\rho}\left[\ln c_0 + \frac{g^*}{\rho} + \theta \ln\left(L - l^*\right)\right], \tag{39}$$

where $c_0 = \left(\rho\beta\gamma^2 + 1 - \gamma\right) Y_0$. The changes of welfare can be decomposed into two parts. First, an increase in the nominal interest rate decreases labor supply through the consumption-leisure choice, which results in an increase in leisure and welfare. This effect is captured by $\theta \ln\left(L - l^*\right)$ in (39). Second, an increase in the nominal interest rate changes the steady-state value of firm size x^*, ending up changing the growth rate of output. This growth effect is captured by the g^*/ρ in (39). Therefore, the effect of the nominal interest rate on the steady-state social welfare depends on the sign and size of $\partial g^*/\partial i$.

When the capital share γ is low, following an increase in the nominal interest rate, firm size x_t increases to its higher steady-state value gradually, and the growth rate of output g_t also increases gradually on the transition path. Therefore, the increase in social welfare would be smaller than that in the steady-state studies in (39), because $g_t < g^*$ on the transition path. Nevertheless, the social welfare is still increasing in the nominal interest rate. When the capital share γ is high, following an increase in the nominal interest rate, firm size x_t decreases to its lower steady-state value gradually, and the growth rate of output g_t also decreases gradually on the transition path. Despite that the growth rate of output decreases both on the transition and balanced growth paths, the effect of an increase in the nominal interest rate on social welfare is ambiguous and depends on structural parameters, because there exists a positive effect from the increase in leisure. We summarize these results in Proposition 4.

Proposition 4 *When the capital share in final good production is low, the social welfare is increasing in the nominal interest rate. When the capital share in final good production is high, the effect of the nominal interest rate on social welfare is ambiguous and depends on structural parameters.*

Proof. Proven in text. ■

4.3 Quantitative analysis

We calibrate the model to simulate the effects of raising i by 1%.[11] The model features the following parameters: $\{\rho, \theta, \xi, \varsigma, \alpha, \epsilon, \gamma, \beta, \phi, L, i\}$. We normalize the labor endowment L to 1. Following Chu and Cozzi (2014), we set the baseline nominal interest rate i to its long-run average value of 8%, and the CIA constraint parameter ς to 0.2 to match the long-run M1 money-consumption ratio in the United States. We set the discount rate ρ to 0.04. We follow Iacopetta et al. (2019) to set the degree of technology spillovers $1-\alpha$ to 0.833. Then, we calibrate $\{\theta, \xi, \epsilon, \gamma, \beta, \phi\}$ by matching the following moments in the United States. First, the labor income share including skilled and unskilled labor is 60%. Second, the intensity of low-skill labor is 0.5; see Heathcote et al. (2010). Third, the consumption share of output is 64%. Fourth, the long-run growth rate of per capita GDP g_t is 2%. Fifth, the average annual TFP growth rate (the quality growth rate) is 1.33% (Chu, 2009), such that the growth rate of human capital multiplied by ϵ is 0.67%. Sixth, the labor supply as a fraction of labor endowment l is 0.3 as in Chu and Cozzi (2014).

Table 2. Calibration

ρ	α	ς	i	θ	ξ	ϵ	γ	β	ϕ
0.040	0.167	0.200	0.080	1.437	0.178	0.500	0.400	6.250	1.419

Table 2 presents the calibrated parameter values. Figure 2 presents the simulated path of g_t. Figure 3 presents the simulated path of z_t. Figure 4 presents the simulated paths of n_t and $\epsilon\dot{h}_t/h_t$. We find that raising the nominal interest rate i_t by 1% (from 8% to 9%) decreases the labor supply l_t permanently from 0.3 to 0.2986, which results in a decrease in the human capital growth rate $\dot{h}_t/h_t$ permanently from 1.34% to 1.315% and a decrease in the variety growth rate n_t from 0.67% to 0.668%, but increases the quality growth rate z_t from 1.33% to 1.342%, leaving the per capita GDP growth rate g_t unchanged on impact. This is because g_t is determined by x_t, which is a state variable and converges to its new steady-state value gradually. In the long run, the growth rate of variety n_t, the growth rate of quality z_t, and the growth rate of GDP per capita g_t gradually declines and converges to their new steady-state values 0.658%, 1.322%, and 1.979%, respectively.

[11] From the Fisher equation $i_t = r_t + \pi_t$ and the Euler equation $g_t = \dot{c}_t/c_t = r_t - \rho$ in (6), the relationship between the nominal interest rate and inflation rate can be expressed as $\partial\pi_t/\partial i_t = 1 - \partial g_t/\partial i_t$, which is positive if and only if $\partial g_t/\partial i_t < 1$. Our calibration results show the above condition holds.

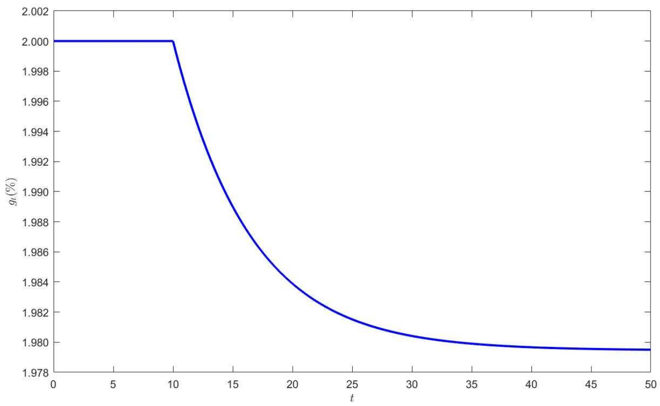

Figure 2. Simulated path of the output growth rate

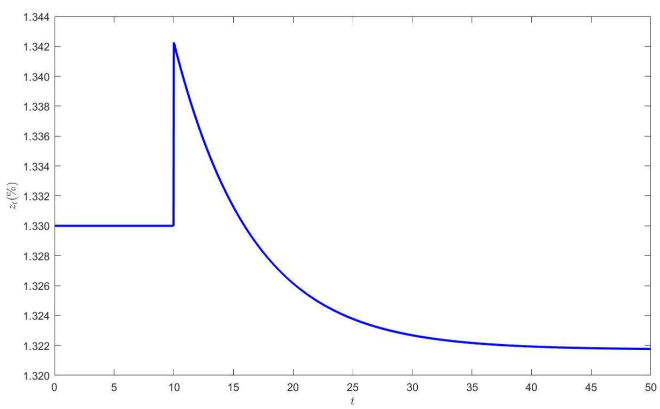

Figure 3. Simulated path of the quality growth rate

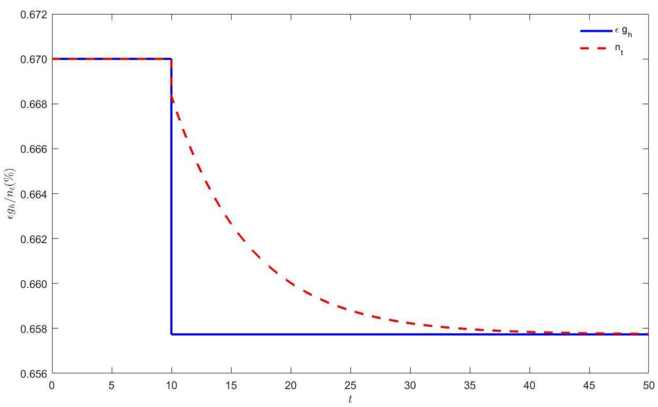

Figure 4. Simulated paths of the variety and human capital growth rates

5 Conclusion

We incorporate endogenous human capital accumulation into a scale-invariant Schumpeterian growth model with endogenous market structure. Endogenous human capital accumulation leads to continuous entry of firms. Therefore, continuous horizontal innovation is sustained by human capital accumulation in the absence of population growth and becomes a twin engine of long-run growth, together with vertical innovation. We then study monetary policy by considering a CIA constraint on consumption in this model. We find that when the capital share in final good production is low, the effect of inflation on economic growth is positive; when the capital share is high, the effect is negative. Then, we use cross-country panel regressions to test the prediction of our model and find that inflation and capital share have a significantly negative interaction effect on economic growth, which provides supportive empirical evidence for our model. Finally, we calibrate the model using the aggregate US data to perform a quantitative analysis. Within this integrated theoretical framework, researchers can re-investigate the macroeconomic implications of some important policy instruments, such as patent policy, R&D subsidies, other types of monetary policies, and fiscal policies. We leave these interesting studies to future research.

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Appendix A: Proofs

Dynamic optimization of household. The household's Hamiltonian is

$$H_t = \ln c_t + \theta \ln (L - l_t) + \mu_t \dot{a}_t + \nu_t \dot{h}_t + \eta_t (m_t - \varsigma c_t), \tag{A1}$$

where μ_t is the costate variable on $\dot{a}_t$ in (2), ν_t is the costate variable on $\dot{h}_t$ in (3) and η_t is the multiplier on the CIA constraint in (4). The first-order conditions include

$$\frac{\partial H_t}{\partial c_t} = \frac{1}{c_t} - \mu_t - \varsigma \eta_t = 0, \tag{A2}$$

$$\frac{\partial H_t}{\partial l_t} = -\frac{\theta}{L - l_t} + \mu_t w_{l,t} u_t + \nu_t \xi (1 - u_t) h_t = 0, \tag{A3}$$

$$\frac{\partial H_t}{\partial u_t} = \mu_t w_{l,t} l_t - \nu_t \xi l_t h_t = 0, \tag{A4}$$

$$\frac{\partial H_t}{\partial a_t} = \mu_t r_t = \rho \mu_t - \dot{\mu}_t, \tag{A5}$$

$$\frac{\partial H_t}{\partial m_t} = -\mu_t \pi_t + \eta_t = \rho \mu_t - \dot{\mu}_t, \tag{A6}$$

$$\frac{\partial H_t}{\partial h_t} = \mu_t w_{h,t} + \nu_t \xi (1 - u_t) l_t = \rho \nu_t - \dot{\nu}_t. \tag{A7}$$

Combining (A5) and (A6) yields $\eta_t = \mu_t (r_t + \pi_t) = \mu_t i_t$, where we define the nominal interest rate as $i_t = r_t + \pi_t$ (the Fisher equation). Substituting this equation into (A2) yields (5). Log differentiating (5) and combining it with (A5) yield (6). Substituting (5) and (A4) into (A3) yields (7). Using (A4), taking the logarithm and differentiating it with respect to t yields

$$\frac{\dot{\nu}_t}{\nu_t} = \frac{\dot{\mu}_t}{\mu_t} + \frac{\dot{w}_{l,t}}{w_{l,t}} - \frac{\dot{h}_t}{h_t}. \tag{A8}$$

Now substituting (3), (6), (A4) and (A8) into (A7) yields (8).

Dynamic optimization of incumbent firms. The current-value Hamiltonian of firm j in the intermediate-goods sector is

$$H_t(j) = \Pi_t(j) + q_t(j) \dot{Z}_t(j), \tag{A9}$$

where $q_t(j)$ is the costate variable on $\dot{Z}_t(j)$. Substituting (10), (13), (14) and (16) into (A9), we can derive the following first-order conditions:

$$\frac{\partial H_t(j)}{\partial p_t(j)} = \frac{\partial \Pi_t(j)}{\partial p_t(j)} = 0, \tag{A10}$$

$$\frac{\partial H_t(j)}{\partial R_t(j)} = 0 \Rightarrow q_t(j) = 1, \tag{A11}$$

$$\frac{\partial H_t(j)}{\partial Z_t(j)} = \alpha \left\{ [p_t(j) - 1] \left[\frac{\gamma}{p_t(j)} \right]^{1/(1-\gamma)} \frac{h_t^{\epsilon} (u_t l_t)^{1-\epsilon}}{N_t} - \phi \right\} Z_t^{\alpha-1}(j) \, Z_t^{1-\alpha} = r_t q_t(j) - \dot{q}_t(j) \, . \tag{A12}$$

(A10) yields $p_t(j) = 1/\gamma$ in (17). Substituting (17), (19) and (A11) into (A12) and imposing symmetry yield (20).

Proof of Lemma 1. Substituting (21) and $\gamma^2 Y_t = N_t X_t$ into $a_t = N_t v_t$, we have

$$a_t = N_t \beta X_t = \beta \gamma^2 Y_t. \tag{A13}$$

Substituting (A13), (6), (11), (12) and (24) into the household budget constraint in (2), we have

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{a}_t}{a_t} = r_t + \frac{w_{h,t} h_t + w_{l,t} (u_t l_t)}{a_t} - \frac{c_t}{a_t} = \rho + \frac{\dot{c}_t}{c_t} + \frac{1-\gamma}{\beta\gamma^2} - \frac{c_t}{\beta\gamma^2 Y_t}, \tag{A14}$$

which can be re-expressed as

$$\frac{\dot{c}_t}{c_t} - \frac{\dot{Y}_t}{Y_t} = \frac{1}{\beta\gamma^2} \frac{c_t}{Y_t} - \left(\frac{1-\gamma}{\beta\gamma^2} + \rho \right). \tag{A15}$$

Therefore, (A15) shows that the dynamics of the consumption-output ratio c_t/Y_t is characterized by saddle-point stability, and the consumption-output ratio c_t/Y_t must jump to its steady-state value given in (26).

Proof of Proposition 1. Combining (6) and (8) and using (11) and (12) yield

$$r_t = \rho + \frac{\dot{c}_t}{c_t} = \xi \frac{\epsilon (1-\gamma)}{(1-\epsilon)(1-\gamma)} u_t l_t + \frac{\dot{Y}_t}{Y_t} - \frac{\dot{(u_t l_t)}}{u_t l_t}. \tag{A16}$$

Using (26), (A16) can be re-expressed as

$$\frac{\dot{(u_t l_t)}}{u_t l_t} = \xi \frac{\epsilon (1-\gamma)}{(1-\epsilon)(1-\gamma)} u_t l_t - \rho, \tag{A17}$$

which shows that the dynamics of $u_t l_t$ is characterized by saddle-point stability. As a result, $u_t l_t$ must jump to its steady-state value given in (28). Substituting (12), (26) and (28) into (7) yields (27).

Proof of Proposition 2. The dynamics of x_t is stable and has a unique steady-state value if the following conditions hold:

$$\frac{(1-\alpha)\phi - \rho\epsilon - \epsilon\xi l_t}{\beta (ul)^{1-\epsilon}} > 0, \tag{A18}$$

$$\left[(1-\alpha) \frac{1-\gamma}{\beta\gamma} - \rho\epsilon - \epsilon\xi l_t \right] > 0, \tag{A19}$$

from which we can obtain the parameter restrictions in Proposition 2. Then we can obtain x^* in (37) from $\dot{x}_t = 0$. Substituting x^* in (37) into (29) yields g^* in (38).

Proof of Proposition 3. From (31), we know that the nominal interest rate i_t does not affect the growth rate of output g_t in the short run, because x_t is a state variable which does not change in the short run (both h_t and N_t are state variables) and ul is independent of i_t. Taking the log of x^* in (37) and differentiating it with respect to l yield

$$\frac{\partial \ln x^*}{\partial l} = -\frac{\epsilon\xi}{(1-\alpha)\,\phi - \epsilon\,(\rho+\xi l)} + \frac{\beta\epsilon\xi}{(1-\alpha)\,(1-\gamma)\,/\gamma - \beta\epsilon\,(\rho+\xi l)}. \tag{A20}$$

Rearranging (A20), we can see that

$$sign\left(\frac{\partial x^*}{\partial l}\right) = sign\left(\beta\phi - \frac{1-\gamma}{\gamma}\right), \tag{A21}$$

which implies that if $\gamma < 1/\,(1+\beta\phi) \equiv \bar{\gamma}$ $(\gamma > \bar{\gamma})$, then x^* is increasing (decreasing) in the nominal interest rate i, because l is decreasing in i. Therefore, the steady-state growth rate of output g^* is increasing in i if $\gamma < \bar{\gamma}$ $(\gamma > \bar{\gamma})$, because g^* is increasing in x^* from (38).

Appendix B: Data and robustness check

Table B1. Descriptive statistics

	Obs	Mean	Std Dev	Min	Max
growth of real GDP per employment (%)	1168	1.23	4.33	-30.08	30.59
inflation rate (%)	1094	44.71	339.73	-23.82	8603.28
capital share	1044	0.45	0.13	0.15	0.85
log human capital index	1068	0.72	0.35	0.01	1.31
log government consumption share	1303	2.90	0.50	0.86	5.20
log openness	1303	3.63	0.95	-1.75	6.98

Note: The average annual growth rate of real GDP per employment (in percentage term), capital share in GDP (measured as one minus labor share in GDP), (log) human capital index, (log) government consumption share in GDP, and (log) openness (measured as the sum of exports and imports as a share of GDP) are from the Penn World Table, covering 154 countries during 1970-2014. Inflation rate (in percentage term) is the CPI inflation rate from the World Development Indicators of the World Bank. We take five-year averages of all the data to avoid the influence from business cycles. Government consumption share and openness are multiplied by 100 (in percentage term) before taking logarithms.

Table B2. Effects of inflation on economic growth (robust standard errors)

	Full sample	$\pi_{it}<60$	$\pi_{it}<50$	$\pi_{it}<40$	$\pi_{it}<30$
	(1)	(2)	(3)	(4)	(5)
π_{it}	-0.009***	0.107*	0.138**	0.115	0.105
	(0.003)	(0.063)	(0.068)	(0.084)	(0.097)
$\pi_{it}\times\kappa_{it}$	0.020***	-0.355**	-0.445***	-0.386**	-0.383*
	(0.006)	(0.144)	(0.153)	(0.184)	(0.215)
Control variables	Yes	Yes	Yes	Yes	Yes
Country fixed effects	Yes	Yes	Yes	Yes	Yes
Time fixed effects	Yes	Yes	Yes	Yes	Yes
R^2	0.29	0.33	0.34	0.34	0.33
Observations	787	742	736	724	712

Note: *** Significant at the 0.01 level, ** at the 0.05 level, * at the 0.10 level. Robust standard errors are in parentheses. π_{it} is the inflation rate (in percentage term), and κ_{it} is the capital share. The dependent variable is the average annual growth rate of real GDP per employment (in percentage term). Control variables include capital share κ_{it}, (log) human capital index, (log) government consumption share, and (log) openness.