How much Keynes and how much Schumpeter? An Estimated Macromodel of the US Economy

Cozzi, Guido and Pataracchia, Beatrice and Ratto, Marco and Pfeiffer, Philipp

3 February 2017

How much Keynes and how much Schumpeter?*

Guido Cozzi   Beatrice Pataracchia   Philipp Pfeiffer
Marco Ratto

November 4, 2020

Abstract

This paper quantitatively assesses the relative importance of demand and supply-side factors in the recent slowdown of US growth. For this purpose, we estimate a DSGE model with heterogeneous firms and endogenous Schumpeterian growth. We find that Keynesian fluctuations in risk premia and savings behavior drive the recession. However, our results challenge the view that the slump is a pure demand-side phenomenon. Adverse supply-side factors such as reduced technological spillovers from frontier innovations have also shaped growth dynamics and emerged well before the financial turmoil.

JEL classification: E3 · O3 · O4
Keywords: Endogenous growth · R&D · Schumpeterian Growth · Bayesian Estimation

*Corresponding author: Guido Cozzi: guido.cozzi@unisg.ch, Institute of Economics (FGN-HSG), University of St.Gallen, Varnbuelstrasse 19, CH-9000 St.Gallen. Pataracchia: European Commission, JRC, beatrice.pataracchia@ec.europa.eu, Pfeiffer: TU Berlin & European Commission, DG ECFIN, E-mail: philipplepfeiffer@gmail.com, Ratto: European Commission, JRC, E-mail: marco.ratto@ec.europa.eu. We thank Michael Burda, Diego Comin, Frank Heinemann, Marco del Negro, and Janos Varga for very useful suggestions, and seminar participants in Canterbury, Edinburgh (SED), Cologne (EEA), Ispra, Milan (CEF), St. Gallen, Tilburg, and Villa del Grumello, Como for useful comments. The research leading to these results has received funding from the European Community's Seventh Framework Programme (FP7/2007-2013) under grant agreement no. 612796. The views expressed in this paper are those of the authors and should not be attributed to the European Commission.

1 Introduction

Following the Great Recession of 2008-2009, medium-term growth prospects in the US and many developed countries have deteriorated substantially. The crisis seems to have generated a persistent downward shift in the GDP trend displayed in Figure 1.

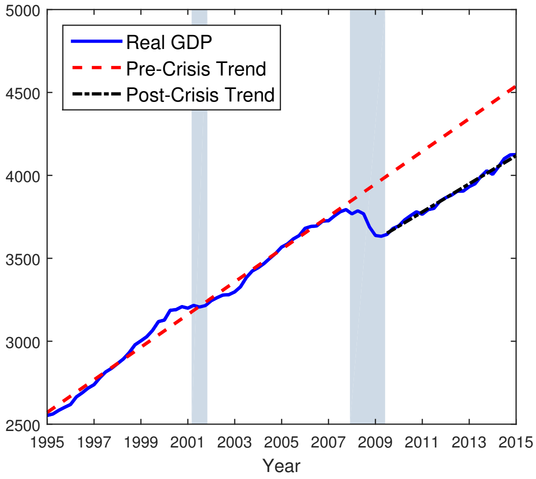

Figure 1: Real GDP and R&D Investment

Notes: Left panel: The blue continuous line depicts real GDP in Billions of Chained 2010 dollars in the US. Shaded areas denote NBER recessions. The dashed red line (dashed-dotted black line) fits a linear trend from 1995Q1 until 2007Q3 (from 2009Q3 until 2015Q1). Right panel: The blue continuous line depicts real R&D investment in Billions of Chained 2010 Dollars (excluding software expenses). Shaded areas denote NBER recessions. Source: Bureau of Economic Analysis (NIPA tables).

Since this has happened after the 2008 financial crisis, it suggests that the sudden unavailability of credit to investment resulting from the crisis has led to a GDP decline below its long-term trend. Can a temporary financial shock generate such a persistent effect? The financial collapse reduced physical capital accumulation during the crisis and the resulting credit crunch. However, since standard macroeconomics assumes decreasing returns to physical capital, GDP should go back to trend after the financial trouble has ended. Alternatively, we could think of an adverse exogenous technology shock, perhaps co-occurring with the financial crisis, or even causing it. Yet, the above evidence would suggest a very high persistence of such an exogenous technological shock.

Our paper emphasizes that technology does not evolve exogenously. Instead, an essential driver of technology is research and development (R&D) activity. Endogenous technological growth opens the possibility that slowing innovation is the result of insufficient demand. In fact, measured R&D investment features a significant temporary negative deviation of its trend during the financial crisis (right panel of Figure 1). If TFP growth results from R&D-driven innovations, then the contraction in R&D at least partially explains the protracted productivity and GDP growth slowdown. However, Cette et al. (2016) suggest that adverse innovation dynamics have started well before the crisis and that the TFP slowdown preceded the Great Recession. This observation also squares with the hypothesis of Gordon (2017) that advances in information and communications technology (ICT) have exhausted a large part of their growth potential and were already widely adopted in the early 2000s. More generally, while innovation-related channels may be central, Keynesian channels such as insufficient private and government demand or ineffective monetary policy close to the zero lower bound (ZLB) are important as well.

The precise question is: How much of the observed growth slowdown is due to supply-side factors such as a pre-crisis decline of the innovative capacity and how much is it due to Keynesian demand-side factors? Importantly, endogenous growth links supply and demand: The demand slump can reduce R&D activities and technology adoption, leading to protracted low TFP growth.

If these factors are jointly at work, they cannot be studied in isolation. Therefore, a complete macroeconomic explanation of what has happened should involve innovation, growth, *and* business cycle analysis. To this aim, we construct an integrated growth and business cycle model. The framework incorporates Schumpeterian creative destruction, based on Aghion and Howitt (1992) and Nuño (2011), into a medium-scale New Keynesian model, based on Smets and Wouters (2003) and Kollmann et al. (2016). We estimate the model using US data from 1984Q1 to 2017Q2. This period covers many important events involving innovation and growth - the burst of the dot-com bubble, the 2009 collapse in R&D - and business cycle and monetary policy events - financial crisis build-up and explosion, unprecedented fiscal stimulus, ZLB hit by the Fed interest rate.

Despite the importance of such an analysis, there are only a few estimated models that integrate business cycle and medium-run growth. Bianchi et al. (2019) estimate a model with R&D capital exerting a positive spillover on the economy like in Frankel (1962) and Romer (1986). Anzoategui et al. (2019) adapt the knowledge diffusion model of Comin and Gertler (2006) to show how demand-driven slumps lead to business cycle persistence via endogenous R&D activity. Varga et al. (2016) extend this framework to a semi-endogenous growth model calibrated to US and EA data. Guerron-Quintana and Jinnai (2019) highlight the role of micro founded financial frictions. Despite their insightful contributions to an emerging integrated macroeconomics[1] literature, none of these papers considers creative destruction, which is consistent with the micro evidence that innovation and growth correlate positively with firm entry *and* firm exit.[2] Variety expansion models predict firm exit to harm growth. By contrast, Schumpeterian models argue that reallocation from less productive exiting firms to more productive firms is an engine of productivity growth in line with industry evidence (Kogan et al. 2017).[3]

In our model, growth is endogenously driven by Schumpeterian R&D activities and knowledge accumulation. As in Aghion and Howitt (1992) and Nuño (2011), innovations are the outcome of a sectoral patent-race. Each innovation is a new intermediate good of enhanced quality. Entrepreneurs collect funds from households to invest in R&D aimed at capturing monopoly rents. In each period, there is a probability that the firm jumps to the technological frontier. If the innovation occurs, the entrepreneur earns monopoly profits until a new innovator replaces the firm. Growth of the technological frontier results from positive knowledge spillovers of R&D activities. These spillovers are subject to shocks that alter the basic research content of R&D. The model assumes a semi-endogenous growth structure in the innovative frontier evolution, while its adoption follows a purely endogenous growth mechanism. Unlike existing stylized Schumpeterian growth models, the DSGE structure allows an estimation of the main innovation and growth parameters.

On the Keynesian side, we incorporate monopolistic competition in product and labor markets, as well as price and wage stickiness. Only a few papers have considered creative destruction with price stickiness (see, in particular, Benigno and Fornaro (2017), Oikawa and Ueda (2015, 2018, 2019), Pinchetti (2020) and Rozsypal (2016)). The model also includes a potentially important role of stabilization policies: A rich fiscal policy block includes government debt accumulation and estimated fiscal rules governing distortionary taxation on labor, capital income, and consumption. The central bank follows an estimated Taylor rule subject to a ZLB constraint.[4]

We then proceed to take the model to the data. Generally speaking, our analy-

[1]Integrating growth and business cycle in a unified way.

[2]See for example Foster et al. (2001) influential evidence that the ongoing replacement of less productive with more productive plants is a central driver of industry multifactor productivity

[3]For the importance of reallocation as an engine of growth, see also Acemoglu et al. (2018).

[4]We include other standard aspects commonly used in estimated DSGE models such as habit formation, real wage rigidities, and flow adjustment costs in investment. Time-varying capacity utilization captures the cyclical use of the capital stock. Moreover, a previous version of our paper (Cozzi et al. 2017) presented an even richer model environment, including liquidity-constrained households, labor habits, as well as partially backward-looking price and wage setting. These features slightly improved the model fit but do not affect any of the main results.

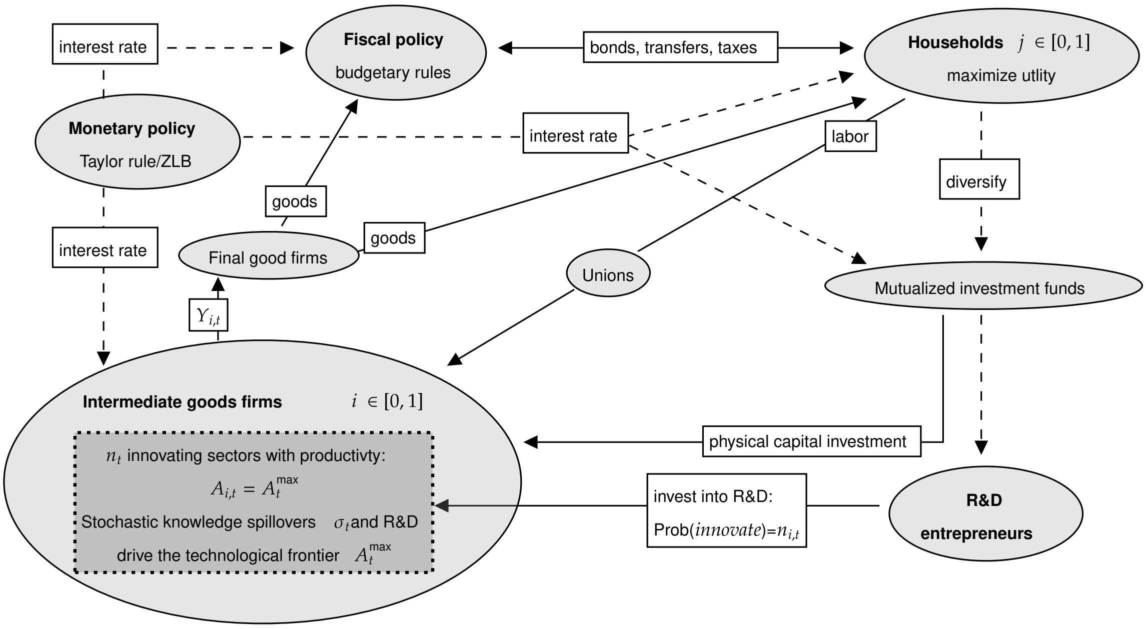

Figure 2: Model overview

Notes: This figure provides a graphical overview of the main flows in the model (omitting, for example, compensation flows).

sis builds on interactions between innovation dynamics and macroeconomic conditions. We therefore include R&D data as an observable among the main macro data series. A Bayesian estimation quantifies the relative contribution of the various shocks in explaining the low growth environment since the financial crisis of 2008-09. Nonlinear methods account for the occasionally binding ZLB. The estimation suggests that fluctuations in investment risk premia are a crucial driver of the Great Recession, alongside the consumer saving shock. Their interaction characterizes the joint decline in physical capital, R&D investment, and consumption.

Despite the importance of the 2008 financial crisis, pre-crisis factors play a central role in explaining the persistent downturn. The estimated model identifies a downward turning point of frontier technology dynamics following the dot-com bubble burst in 2001. The slowdown of innovation dynamics led to gradual exhaustion of unadopted technologies. According to the model estimation, these adverse dynamics and reduced knowledge spillovers are partially responsible for the sclerotic growth following the financial crisis.

2 Model

This section lays out the economic environment. Figure 2 provides a graphical overview of our unified growth and business cycle framework. The endogenous growth part is based on Schumpeterian theory. R&D Entrepreneurs collect funds from household-owned investment funds. They invest into R&D to increase their probabilities of innovation. If the innovation occurs, the firm jumps to the frontier and earns monopoly rents. Positive knowledge spillovers of R&D push the aggregate frontier further. The Keynesian channels owing to price and wage rigidities highlight the role of monetary and fiscal stabilization policy and stochastic household demand.

2.1 Households and unions

There is a continuum of households indexed by $j \in [0,1]$. Households work, consume, receive nominal transfers $TR_{j,t}$ from the government and hold assets. Their risky portfolio consists of government bonds ($B^B_{j,t}$) and firm shares ($S_{j,t}$) with nominal returns $(1+i^B_t)$ and $(1+i^S_{t+1})$, respectively.[5] The return on shares is derived below in equation (17). Households have access risk-free bonds $B^{rf}_{j,t}$ with return $(1+i_t)$.[6] P^S_t denotes the share price.

The real per-period budget constraint is

$$\left(1-\tau^N\right)\frac{W_t}{P_t}N_{j,t}+(1+i^B_{t-1})\frac{B^B_{j,t-1}}{P_t}+(1+i_{t-1})\frac{B^{rf}_{j,t-1}}{P_t}+(1+i^S_t)\frac{S_{j,t-1}P^S_{t-1}}{P_t}$$
$$+\frac{TR_{j,t}}{P_t}+\Pi_t=(1+\tau^C)C_{j,t}+\frac{B^B_{j,t}}{P_t}+\frac{B^{rf}_{j,t}}{P_t}+\frac{P^S_t}{P_t}S_{j,t}+\exp(\varepsilon^T_t)\frac{T_{j,t}}{P_t}+\Gamma^W_t/P_t, \tag{1}$$

where P_t denotes the GDP deflator. τ^C and τ^N are tax rates levied on consumption $C_{j,t}$ and wages W_t, respectively. Lump-sum taxes $T_{j,t}$ are subject to a tax shock ε^T_t. Γ^w_t are nominal wage adjustment costs. $N_{j,t}$ and Π_t denote hours worked and real profits of all firms other than intermediate goods producers, respectively.

[5] Bond returns are pre-determined whereas the stock market return is uncertain indicated by the timing $t+1$.

[6] Note that we distinguish government bonds B^B_t and risk-free bonds $B^{rf}_{j,t}$. The former feature a stochastic risk premium (see below) and are used to finance government deficits. We will use data on government interest payments to estimate the shock. By contrast, risk-free assets are in the zero net supply and do not feature a stochastic preference.

Households maximize their lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \Theta_t \left\{ \frac{(C_{j,t} - hC_{t-1})^{1-\theta}}{1-\theta} - \omega_t^N \frac{\left(N_{j,t}\right)^{1+\theta^N}}{1+\theta^N} - \mathcal{U}_{j,t}^B \times \bar{\lambda}_t / P_t \right\}, \tag{2}$$

where $0 < \theta, \theta^N$. h governs external consumption habits. β is the discount factor. $\Theta_t \equiv \exp\left(\sum_{\tau=0}^{t-1} \varepsilon_\tau^C\right)$ is a (cumulative) saving shock.[7] ω_t^N is a labor disutility term.[8] $P_t^{C,vat}$ denotes the prices including taxes, i.e. $P_t^{C,vat} = (1+\tau^C)P_t$.

$\mathcal{U}_{j,t}^B / P_t$ introduces preferences for (real) financial assets into the utility function. This approach is in the spirit of Sidrauski (1967) and follows Krishnamurthy and Vissing-Jorgensen (2012). Integrating stochastic preferences for financial assets allows to mimic dynamics of financial shocks without formally incorporating a financial sector.[9] Our results show these shocks drive a large share of cyclical variation in output growth, in particular during the financial crisis 2008-09. $\mathcal{U}_{j,t}^B$ is scaled by an exogenous marginal utility term $\bar{\lambda}_t$ to ensure a balance growth path. It is defined as

$$\mathcal{U}_{j,t}^B = \left(\alpha^B + \varepsilon_t^B\right) B_{j,t}^B + \left(\alpha^S + \varepsilon_t^S\right) P_t^S S_{j,t}. \tag{3}$$

Given this specification, the asset-specific risk premium depends on two elements. Estimated exogenous autocorrelated shocks $\varepsilon_{t-1}^{\mathcal{Q}}$ with $\mathcal{Q} \in \{B, S\}$ capture cyclical variation in preferences for financial assets. The intercept terms α^S and α^B imply a positive equity premium and a treasury bill convenience yield in the steady state, respectively.[10] For example, in the linearized model, returns on shares are a function of the risk-free rate and a risk premium (featuring a constant and a stochastic component):

$$i_{t+1}^S = i_t + \underbrace{\alpha^S + \varepsilon_t^S}_{\text{risk premium}} . \tag{4}$$

Appendix A.1 presents the household optimality conditions.

[7]The term Θ_t implies that the households' intertemporal optimality conditions equations feature a time t shock ε_t^C, i.e. the discount factor β is replaced by $\beta \exp(\varepsilon_t^C)$. See Appendix A.1.

[8]As we discuss below, we estimate the model while preserving trends in the data. To ensure a balance growth path with separable utility, labor disutility includes an exogenous multiplicative term featuring aggregate consumption, $C_t^{1-\theta}$, such that $\omega_t^N = \omega^N C_t^{1-\theta}$. Note that ω_t^N only depends on aggregate consumption, i.e. it is not internalized by households.

[9]See, e.g. Christiano et al. (2015), Del Negro et al. (2016), Vitek (2017), and Gust et al. (2017).

[10]Fisher (2015) uses a simpler formulation. He gives a "flight-to-quality" interpretation, micro founded as a preference for risk-free bonds. We generalize this idea by calibrating different marginal utilities for different assets.

Household j sells its labor service to a monopolistically competitive trade union which sets wages at a markup (μ^w) over the marginal rate of substitution between working and consuming (mrs_t):

$$\left(1-\tau^N\right)\frac{W_t}{P_t}+\Gamma_t^w(\cdot)+u_t^U\frac{W_t}{P_t}=\left(\mu^w mrs_t\right)^{1-\gamma^{wr}}\left(\left(1-\tau^N\right)\frac{W_{t-1}}{P_{t-1}}\right)^{\gamma^{wr}}, \tag{5}$$

where Γ_t^W summarizes nominal wage adjustment costs parametrized by γ^W. Real wages adjust sluggishly as in Blanchard and Galí (2007) governed by parameter γ^{wr}. u_t^U is a labor supply shock. See Appendix A.2 for further details.[11]

2.2 Goods production

A representative perfectly competitive final good producer bundles intermediate goods, indexed $i \in [0,1]$, using a constant-returns technology

$$Y_t=\left[\int_0^1 Y_{i,t}^{\frac{\sigma^y-1}{\sigma^y}}\,di\right]^{\frac{\sigma^y}{\sigma^y-1}}, \tag{6}$$

where $\sigma^y > 1$ denotes the elasticity of substitution. Demand for intermediate goods follows from profit maximization:

$$Y_t(i)=\left(\frac{P_t(i)}{P_t}\right)^{-\sigma^y}Y_t. \tag{7}$$

Each differentiated intermediate good is produced by a monopolist using total capital $K_{i,t-1}^{tot}$ and labor $N_{i,t}$. Total capital is the sum of perfectly substitutable private and public capital (see below). The production function is Cobb-Douglas:

$$Y_{i,t}=A_{i,t}\left(N_{i,t}\right)^a\left(cu_{i,t}\frac{K_{i,t-1}^{tot}}{A_t}\right)^{1-a}, \tag{8}$$

where a denotes the labor share, $A_{i,t}$ is the endogenous sector-specific productivity level and $cu_{i,t}$ is firm-specific level of capital utilization. With more sophisticated technologies, production becomes more capital-intensive. $A_t=\int_0^1 A_{i,t}d\,i$ is the average productivity across all sectors in the differentiated goods production. Total Factor Productivity is $TFP_t=\left(A_t\right)^a$.

As a consequence of (8), sectors with higher relative technological sophistica-

[11]The Online Appendix provides a full derivation.

tion benefit more from the average technological level across sectors:[12]

$$Y_{i,t} = \underbrace{\left(\frac{A_{i,t}}{A_t}\right)}_{\text{rel. technology level}} \left(A_t N_{i,t}\right)^a \left(cu_{i,t} K^{tot}_{i,t-1}\right)^{1-a}. \tag{9}$$

Intermediate good firms choose prices, employment, and capacity utilization to maximize dividends subject to the production technology (9) and the demand from final goods producers (7). Dividends are given by

$$d_{i,t} = \left(1 - \tau^K\right) \left(\frac{P_{i,t}}{P_t} Y_{i,t} - \frac{W_t}{P_t} N_{i,t}\right) - adj_{i,t}, \tag{10}$$

where τ^K is a corporate income tax. $adj_{i,t}$ denotes total adjustment costs associated with quadratic price and factor adjustments.[13] We assume that a fully mutualized investment fund invests on behalf of firms. Capital accumulation follows $K_{i,t} = (1-\delta)K_{i,t-1} + I_{i,t}$, where δ is the capital depreciation rate, and $I_{i,t}$ denotes gross investment in physical capital. Public capital $K^g_{i,t}$ follows an analogous law of motion. The Phillips curve linking output growth and inflation is standard:

$$\begin{aligned}
\mu_t =& (1-\tau^K)\frac{(\sigma^y - 1)}{\sigma^y} + \gamma^P \frac{P_t}{P_{t-1}}\left(\pi_t - \bar{\pi}\right) \\
& - \gamma^P \left[\Lambda_{t,t+1} \frac{P_{t+1}}{P_t} \frac{Y_{t+1}}{Y_t}\left(\pi_{t+1} - \bar{\pi}\right)\right] + \varepsilon_t^{MUY},
\end{aligned} \tag{11}$$

where μ_t denotes the multiplier associated with the production function (9) and $\pi_t \equiv P_t/P_{t-1}$. $\bar{\pi}$ is steady state inflation. γ^P governs quadratic price adjustment costs. $\Lambda_{t,t+1}$ is the stochastic (stock market) discount factor of households, derived in equation (A.5).

Price-setting and labor demand are subject to stochastic disturbances, ε_t^{MUY} and ε_t^{ND}, respectively. Since the optimality conditions are standard, we relegate the details to Appendix A.3.

2.3 Endogenous innovation

Next, we describe the endogenous technological progress feature of our model.

[12]This formulation allows us to avoid keeping track of a distribution of firms and instead only look at average firms and frontier firms.

[13]The functional forms are standard and reported in Appendix A.3.

Innovations. Innovations generate growth. In each period t, the productivity of a sector $A_{i,t}$ jumps to the technology frontier $A_t^{\max}$ with probability $n_{i,t-1}$. The frontier is publicly available. It represents the most advanced technological level across all sectors defined as $A_t^{\max} \equiv \max\{A_{i,t} | i \in [0,1]\}$. Productivity in each sector i evolves as:

$$A_{i,t} = \left\{ \begin{array}{lll} A_t^{\max}, & \text{with probability} & n_{i,t-1} \\ A_{i,t-1}, & \text{with probability} & 1 - n_{i,t-1} \end{array} \right\}. \tag{12}$$

Entrepreneurs. Innovations are the result of entrepreneurial investments into R&D. The probability of reaching the frontier is itself endogenous. In each sector i and in each period t, an entrepreneur is randomly selected with the opportunity to try to innovate.[14] If such entrepreneur's R&D firm invests R&D cost $X_{i,t}^{RD}$, it will produce a probability $n_{i,t}$ of a successful innovation, which entails the discovery of a new intermediate good with next period's frontier productivity $A_{t+1}^{\max}$. We assume that research is more difficult if the overall technology frontier is more advanced, i.e. per unit research costs increase with the frontier productivity $A_t^{\max}$. The probability of innovation is independent across sectors. In each sector i, it follows production function:

$$n_{i,t} = \left\{ \begin{array}{ll} \left(\frac{X_{i,t}^{RD}}{\lambda^{RD} A_{i,t}^{\max}} \right)^{\frac{1}{(\eta+1)}}, & \text{if} \quad X_{i,t}^{RD} < \lambda^{RD} A_{t+1}^{\max} \\ 1, & \text{if} \quad X_{i,t}^{RD} \geq \lambda^{RD} A_t^{\max} \end{array} \right\}, \tag{13}$$

where $\lambda^{RD} > 0$ is an R&D difficulty parameter and $\eta > 0$ accounts for decreasing returns of R&D. An innovation occurring at time t will permit production in period $t+1$. Moreover, the second line of equation (13) guarantees that the probability of innovation per period is no larger than 1.[15]

The new good is by definition of the highest quality. By Bertrand competition and following a price war, the new patent holder will replace the existing incumbent monopolist and appropriate all profits in $t+1$. Hence, the prospects of becoming the next period's incumbent create the incentives to invest in R&D at time t. Since the new entrant does not internalize the loss incurred by the previous incumbent, creative destruction can imply both, too much or too little R&D investment.[16]

[14] Hence, innovative ideas are scarce as they "arrive to random agents at random times" (Erkal and Scotchmer, 2011, p.1).

[15] In a continuous time framework, this could be any non-negative number. In our simulations n_t always remains strictly below one.

[16] A horizontal innovation framework, by missing this business-stealing externality, implies too

Similar to capital investment, we assume adjustment costs in R&D investment. The R&D adjustment cost function follows

$$adj_{i,t}^{RD}(X_{i,t}^{RD}) \equiv \frac{\gamma^{RD}}{2Y_t}(X_{i,t}^{RD} - X_{t-1}^{RD} g_Y)^2, \tag{14}$$

where g_Y denotes output trend which makes adjustment costs stationary.[17]

Innovation at time t is like a static lottery. Given individual risk aversion, we assume that the R&D firm in each sector finances risky innovations via a mutual fund. Households pool resources to completely diversify innovation risk across the continuum of sectors (by the law of large numbers). The fund requires a stochastic R&D-specific investment risk premium, ε_t^{AY}, which helps fit the observed R&D series. The R&D entrepreneurial problem is then a simple expected profit maximization:

$$\max_{X_{i,t}^{RD}} = n_{i,t}\left[\frac{P_{i,t}^{S\max}}{P_t}\right] - \left[X_{i,t}^{RD} + adj_{i,t}^{RD}(X_{i,t}^{RD})\right]\left(1+\varepsilon_t^{AY}\right), \tag{15}$$

where $P_t^{S\max}$ is the nominal stock market value of the firm at the technological frontier.

The value of becoming the incumbent is the same across sectors. Thus, in equilibrium the R&D investment cost is symmetric, $X_{i,t}^{RD} = X_t^{RD}$, as is the probability of success $n_{i,t} = n_t$. The R&D optimality condition, after making use of equation (13), then becomes:

$$\frac{n_t}{(\eta+1)}\left[\frac{P_t^{S\max}}{P_t}\right] = X_t^{RD}\left(\frac{\gamma^{RD}}{Y_t}(X_t^{RD} - X_{t-1}^{RD} g_Y)\right)(1+\varepsilon_t^{AY}). \tag{16}$$

Note that R&D firms earn positive profits as long as $\eta > 0$.[18] By the law of large numbers, n_t also measures the fraction of sectors which innovate each period, as well as the fraction of firms that exit and enter the market. The higher the equilibrium value of n_t, the stronger innovation and creative destruction, and the more dynamic the set of innovative industries.

---

little R&D in equilibrium.

[17]The interpretation of adjustment costs in a creative destruction environment requires explanation: we are implicitly assuming that if a new entrant undertakes R&D, the previous period R&D laboratory size (magnified by trend GDP growth) sets the benchmark for the current R&D investment size, penalizing departures from it.

[18]The case $\eta = 0$ nests a linear R&D technology and also a free-entry case (as long as the equilibrium innovation probability is less than 1). In this last case ideas are not scarce.

Stock Market Value. Households own intermediate good produces with total value $P_t^S S_t$. We normalize the total number of stocks to 1. Firms are heterogeneous and firm turnover follows innovation. Due to Schumpeterian creative destruction, a fraction n_{t-1} of obsolete firms belonging to time $t-1$ portfolio is lost at time t, replaced by new entrants with higher stock market value $P_t^{S\max}$. Taking firm exit into account, the gross nominal return on the aggregate time t stock market portfolio is given by:

$$1 + i_t^S \equiv \frac{d_t P_t + P_t^S - n_{t-1} P_t^{S\max}}{P_{t-1}^S}. \tag{17}$$

The return depends on the value of the average dividend payments d_t. It also takes into account capital gains and losses. The average time t portfolio, P_t^S, includes the innovative firms that have replaced fraction n_{t-1} of time t industry. Hence, we subtract their aggregate value, $n_{t-1} P_t^{S\max}$, in equation (17).

Frontier Value. The frontier technology net growth rate $g_{A_t^{\max}}$ is defined as

$$g_{A_t^{\max}} = \frac{A_t^{\max} - A_{t-1}^{\max}}{A_{t-1}^{\max}}. \tag{18}$$

Entrepreneurs collect funds from households. They invest into R&D to reach the technological frontier, patent its adaptation to their sector, and appropriate the resulting production monopoly. Hence, each entrepreneur at the frontier earns the monopoly profits resulting from offering the highest quality intermediate good. These profits (and the resulting dividends) are $(A_t^{\max})/(A_t)$ times bigger than those of the average technology firm. Therefore, the stock market value as of time t, $P_t^{S\max}$, of a firm that will start producing at the technology

$$\frac{P_t^{S\max}}{P_t} = E_t \left[\Lambda_{t,t+1} \left(d_{t+1} \left(\frac{A_{t+1}^{\max}}{A_{t+1}} \right) + \frac{P_{t+1}^{S\max}}{P_{t+1} g_{A_{t+1}^{\max}}} (1 - n_{t+1}) \right) \right]. \tag{19}$$

Notice that patents of the latest and most advanced technology require one period of implementation. Therefore, for an innovation developed in period t, production and dividend flows only start in $t+1$. Furthermore, the continuation value in the stock market in $t+1$ takes into account that competitors may successfully innovate. With probability n_{t+1} a new entrant will replace the monopolist in $t+2$. With probability $1 - n_{t+1}$ no innovation is found in the sector in $t+1$. In this case, the firm's value (along with its dividends) remains positive but lower than that of a generic newly entered innovator by a factor equal to $g_{A_{t+1}^{\max}}$. This explains the last part of the numerator of equation (19).

Frontier and diffusion. The growth of the technological frontier, $g_{A_t^{\max}}$, is the outcome of positive knowledge spillovers from the aggregate innovation efforts as in Howitt and Aghion (1998). According to this Schumpeterian view, R&D activities have an appropriable applied content, i.e. the patentable sectoral adoption of the technological frontier and a basic aspect that firms cannot appropriate. The latter pushes the aggregate technological frontier further: Basic content of aggregate R&D freely spills over to all sectors.

Following Nuño (2011), R&D spillover is time-varying and stochastic. This captures the potentially volatile basic research content of R&D. In reduced form, this reflects, for example, the research orientation of scientists and engineers, university policies, or regulatory aspects of intellectual property rights (IPRs). Formally, we assume that

$$A_t^{\max} = A_{t-1}^{\max} + \left(A_{t-1}^{\max}\right)^{\varphi} \left(\frac{X_{t-1}^{RD}}{Y_{t-1}} N_{t-1}\right)^{\lambda^A} \sigma_t^{RD}, \tag{20}$$

where $\varphi < 1$ reflects decreasing returns to the intertemporal knowledge spillover as in Jones (1995) and in Bloom et al. (2020). λ^A is an externality parameter. It captures, for example, research duplication or knowledge theft but also positive externalities from research efforts (Jones and Williams, 2000).

The instantaneous knowledge spillover follows an exogenous process given by

$$\sigma_t^{RD} = \sigma^{RD} \exp(\varepsilon_t^{\sigma}), \tag{21}$$

where ε_t^{σ} is an R&D spillover shock and σ^{RD} denotes steady-state spillovers. According to equation (20), the technological frontier depends on its previous period's value, and on the fraction of the employees indirectly working in R&D (given by $\frac{X_{t-1}^{RD}}{Y_{t-1}} N_{t-1}$) in the past period. We can then express the growth rate of the technological frontier as:

$$g_{A_t^{\max}} = \frac{\left(\frac{X_{t-1}^{RD}}{Y_{t-1}} N_{t-1}\right)^{\lambda^A} \sigma_t^{RD}}{\left(A_{t-1}^{\max}\right)^{1-\varphi}}. \tag{22}$$

On a balanced growth path, $g_{A_t^{\max}}$ and $\frac{X_{t-1}^{RD}}{Y_{t-1}}$ are constant. Log-differencing (22) implies

$$g_{A_t^{\max}} = \frac{\lambda^A g_{POP}}{1-\varphi}. \tag{23}$$

As in Jones (1995), the growth rate of population g_{POP} governs the long-run growth rate of the frontier. The factor of proportionality depends on the extent of decreasing returns, inversely represented by λ^A.[19]

Unlike in Nuño (2011), the evolution of the technological frontier in the model is semi-endogenous, as in Jones (1995). Its adoption, however, remains fully endogenous, as in Comin and Gertler (2006) and Anzoategui et al. (2019). This formulation permits to eliminate the counterfactual strong scale effects that plagued the early generation endogenous growth models. It also avoids steady-state growth rate effects of R&D policy variables and shocks, which facilitates the comparison with the standard exogenous growth DSGE models.

The average technological progress results from the adoption of the frontier technology. Endogenous R&D activity determines the average intermediate goods productivity in the economy:

$$\begin{aligned} A_t &= \int_0^1 \left\{ n_{i,t-1} A_t^{\max} + (1-n_{i,t-1}) A_{i,t-1} \right\} di \\ &= n_{t-1} \left(A_t^{\max} - A_{t-1} \right) + A_{t-1}, \end{aligned} \tag{24}$$

using $n_{i,t-1} = n_{t-1}$. On a balanced growth path, the frontier $A_t^{\max}$ grows at the same rate as the average technological level defined in equation (24).

2.4 Monetary and fiscal authorities

The monetary authority sets the notional nominal interest rate i_t^{not} according to a Taylor rule:

$$i_t^{not} = \rho^i \left(i_{t-1} - \bar{i} \right) + \left(1 - \rho^i \right) \left(\eta^{i,\pi} \left(\pi_t - \bar{\pi} \right) + \eta^{i,y} \tilde{y}_t \right) + \bar{i}, \tag{25}$$

where the steady-state nominal interest rate ($\bar{i}$) equals the sum of the steady-state real interest rate and steady-state inflation, i.e. $\bar{i} = r + \bar{\pi}$. $\tilde{y}_t$ is the output gap and

[19] Note that unlike in Jones (1995) and Varga et al. (2016) the growth of $A_t^{\max}$ does not refer to the growth rate of patents but rather to the growth rate of a frontier productivity index. While this is derived from the flow of new patents invented, its numerical value already accounts for the effect of patents on productivity. Therefore, we should expect a lower value of λ^A than in Jones and Williams (2000) and Varga et al. (2016).

$\eta^{i\pi} > 1$ and $\eta^{iy} > 0$. [20]

Following the Great Recession, at least through late 2015, the ZLB on nominal interest rates was effectively binding. It hampered the Fed's ability to stimulate the economy by further lowering the policy rate. Formally, we impose the ZLB constraint on the net nominal interest rate by requiring the effective policy rate to satisfy:

$$i_t = \max\{i_t^{not}, i^{LB}\} + u_t^i, \tag{26}$$

where u_t^i is a white noise shock and i^{LB} parametrizes the ZLB.

Government consumption and physical capital investment follow estimated rules:

$$c_t^G - c^G = \rho^G(c_{t-1}^G - c^G) + u_t^G, \tag{27}$$
$$i_t^G - i^G = \rho^{IG}(i_{t-1}^G - i^G) + u_t^{IG}, \tag{28}$$

where $c_t^G \equiv \frac{G_t}{Y_t}$ and $i_t^G \equiv \frac{I_t^G}{Y_t}$ are government consumption and investment as shares of GDP, respectively.[21] u_t^G and u_t^{IG} are white noise disturbances. Government transfers react to deviations from deficit and debt targets, denoted df and b, respectively.

$$tr_t - tr = \rho^{tr}(tr_{t-1} - tr) + \eta^{df}\left(\frac{B_t^B - B_{t-1}^B}{Y_t} - df\right) + \eta^b\left(\frac{B_t^B}{Y_t} - b\right) + u_t^{tr}, \tag{29}$$

where $tr_t \equiv \frac{TR_t}{Y_t}$ are net nominal transfers normalized by GDP. B_t^B denotes total nominal government debt owned by households, η^{df} is a deficit coefficient, η^b is a debt coefficient. u_t^{tr} is a white noise transfer shock. The government budget constraint is $B_t^B = (1 + i_t^B)B_{t-1}^B - Rev_t + P_tG_t + P_tI_t^G + TR_t$, where Rev_t is the nominal revenue of the government.

2.5 Market clearing

Labor markets clear. Financial market clearing requires $S_t = 1$ (normalization) and $B_t = B_t^B$. Finally, the aggregate budget identity takes R&D investment into account

$$Y_t = C_t + I_t + G_t + I_t^G + adj_t + X_t^{RD}. \tag{30}$$

[20] The output gap is measured by $\tilde{y}_t = log\,(Y_t) - \overline{y}_t$ where $\overline{y}_t$ is (log) output trend.

[21] Lower case letters without time subscript denote steady-state values.

2.6 Exogenous processes

All exogenous shock processes of a generic type x (unless specified explicitly) follow autoregressive processes of order one with an autocorrelation coefficient $|\rho^x| < 1$ and innovation u_t^x. Thus,

$$\varepsilon_t^x = \rho^x \varepsilon_{t-1}^x + u_t^x. \tag{31}$$

3 Results

3.1 Data and estimation approach

Data and non-stationarity. Our estimation approach does not require any transformation of the data. We assume the data follow a stochastic trend, induced by the effects of random shocks on endogenous productivity. Productivity also features a constant (drift) term common to all variables. This econometric approach preserves all relative trends in the data. It allows us to meaningfully decompose the growth slowdown into technological (supply) and demand factors. In total, we use data on 16 macroeconomic time series ranging from 1983Q1 until 2017Q2 taken from the Bureau of Economic Analysis (BEA) and the Federal Reserve.[22] Appendix B provides additional details.

Nonlinearity. To account for the occasionally binding constraint on nominal interest rates, we build on the piecewise linear OccBin algorithm (Guerrieri and Iacoviello 2015). This method handles the constraints as two regimes of the model in which the ZLB constraint is either slack or binding. The dynamics within any of the two regimes depend on the endogenous length of that regime. The expected duration, in turn, depends on the state variables and exogenous disturbances. As emphasized in Guerrieri and Iacoviello (2015), the interaction of the expected regime length and state variables can result in highly nonlinear dynamics. Following Giovannini et al. (2020), we integrate the nonlinear solution into a specially adapted Kalman filter and estimate the model with the occasionally binding ZLB. Appendix C provides additional details on the estimation and shock decomposition procedure.

Calibration and estimation. Following the literature on estimated DSGE models, we set the values for a subset of parameters *a priori* by using the steady-state

[22] The relatively large number of observables is due to our detailed fiscal policy block. Results are robust to the exclusion of details on fiscal policy.

restrictions. We estimate the remaining parameters with Bayesian methods using the piecewise linear model approximation using first a parallelized Slice sampler (150 draws). We then run an additional Metropolis Hastings algorithm with length 200.000. The Online Appendix provides detailed convergence statistics.

The calibration and estimation procedures are interdependent. The estimated parameters affect the calibrated parameters and vice versa. A notable example of this interdependence is λ^A, the R&D externality parameter. Since,

$$TFP_t = (A_t)^a, \tag{32}$$

it follows that the steady-state TFP growth rate is $g_{TFP} = ag_A = ag_{A^{\max}}$. The steady-state restriction imposed by equation (23) then determines λ^A as a function of the estimated parameter $\hat{\varphi}$ and the labor share a:

$$\lambda^A = (1 - \hat{\varphi})^{-1} \frac{g_{TFP}}{a g_{POP}}. \tag{33}$$

Model fit. Table 1 evaluates the capability of the model to fit the data. The first columns compare sample and model-implied moments for real GDP, private consumption, private (capital) investment, R& D investment, and employment. The volatility of real variables slightly exceeds its empirical counterparts, particularly for R& D investment. Nonetheless, the estimated model captures the volatility of GDP and its components reasonably well. Also, the relative magnitudes preserve the data patterns (columns "Rel. std."). The model underpredicts the persistence of GDP growth, but the first-order autocorrelation of key GDP components is broadly in line with the data. The correlation between output growth and the growth rates of consumption, investment, and hours worked is close to the data. The last column in Table 3 reports the R^2 of the 1-year ahead forecast. The positive value indicates that the forecast errors from the model are not very large. Overall, the theoretical moments in Table 1 indicate the model's ability to replicate business cycle features. More elaborate versions of the model have shown further improvement in the fit while preserving our main results.[23]

3.2 Model parameters

This section provides the calibrated and estimated parameters. Tables 2 reports prior distributions and posterior estimation results of model parameters and exogenous process, respectively. Table 3 presents calibrated parameters.

[23] See also footnote 4.

Variable	Std.		Std(x)/Std(g^Y)		AR(1)		Corr(x,g^Y)		R^2
	Data	Model	Data	Model	Data	Model	Data	Model	1y ahead
GDP	0.62	0.60	1.00	1.00	0.49	0.30	1.00	1.00	0.33
Consumption	0.54	0.63	0.88	1.06	0.44	0.63	0.68	0.55	0.20
Investment	2.93	3.10	4.71	5.21	0.12	0.11	0.72	0.71	0.03
R&D	1.35	1.53	2.17	2.57	0.32	0.44	0.32	0.24	0.16
Hours worked	0.57	0.60	0.92	1.00	0.70	0.52	0.63	0.69	0.47
GDP deflator	0.25	0.49	0.41	0.83	0.62	0.85	0.19	-0.18	0.40
Real wages	0.70	0.69	1.12	1.16	-0.09	0.11	0.05	0.14	0.04

Table 1: Theoretical moments and model fit.

Notes: Rel. std. and AR(1) refer to the standard deviation relative to output growth (g^Y) and first order autorcorrelation, respectively. Theoretical moments are reported for the linear model. We define the R^2 as 1 - the ratio of the 4-step-ahead (one year) forecast error and the deviation of the observed time series from the model-implied steady state. This definition implies that our R^2 has an upper bound at 1 (no forecast errors) and is unbounded from below (the volatility of the forecast error can exceed the volatility of the time series). R^2 declines monotonically as the forecast error increases. Consumption and investment refer to private GDP components. All variables are expressed in growth rates.

The growth literature has mostly relied on simple calibration procedures. By contrast, the rich macroeconomic model allows estimating important parameters for endogenous growth, such as the parameter determining intertemporal spillover, φ. Its mode estimate (0.93) is below one despite a positive prior density in $\varphi = 1$. Hence, the estimation supports semi-endogenous growth in the US economy but implies significant persistent of movements in endogenous productivity.

This finding is important for policy evaluations. It predicts that R&D policy shocks cannot permanently affect the per capita GDP growth rate, as originally predicted by Jones (1995). However, $\hat{\varphi}$ implies that R&D policy shocks, or any other shocks directly or indirectly affecting the R&D/GDP ratio, can have relatively long-lasting effects and change the macroeconomic behavior in the medium term. Despite the sizeable posterior distribution, this estimate suggests that the policy predictions of the estimated model are, in the short-run, not completely distinguishable from those of a fully endogenous growth model.[24] For example, changes in IPR policy or legal norms influence incentives to conduct basic or applied research. A shift away from basic research towards a more narrow focus on marketable products can have lasting adverse effects on frontier productivity growth. Following this argument, the persistence of the spillover shock could relate to the US common law regime, which implies a gradual transition to new IPR regimes. In like manner, Cozzi and Galli (2014) argue that a legal change only un-

[24]A previous version of the paper (Cozzi et al., 2017) considered alternative specifications of the growth engine including fully endogenous growth.

		Prior distribution			Posterior distribution		
		Distr.	Mean	Std.	Mode	10%	90%
Steady state innovation prob.	$\bar{n}$	Beta	0.03	0.01	0.18	0.17	0.18
Intertemporal spillover	φ	Beta	0.70	0.20	0.93	0.70	0.98
Habit persistence	h	Beta	0.50	0.20	0.89	0.84	0.92
Risk aversion[a]	θ	Gamma	1.50	0.20	1.61	1.26	1.91
Inverse Frisch elasticity	θ^N	Gamma	2.50	0.50	2.63	1.68	3.00
Price adj. cost	γ^P	Gamma	40.00	10.00	67.01	54.23	85.24
Wage adj. cost	γ^w	Gamma	5.00	2.00	3.23	1.85	4.69
Real wage rigidity	γ^{wr}	Beta	0.50	0.20	0.97	0.96	0.98
Labor adj. cost	γ^N	Gamma	60.00	40.00	3.35	2.02	4.56
Capacity util. adj. cost	$\gamma^{u,2}$	Gamma	0.02	0.01	0.01	0.01	0.01
Investment adj. cost, const.	$\gamma^{I,1}$	Gamma	60.00	40.00	66.39	46.96	84.36
Investment adj. cost, slope	$\gamma^{I,2}$	Gamma	60.00	40.00	9.15	3.27	25.66
R&D adj. cost	γ^{RD}	Gamma	60.00	40.00	197.84	158.77	238.87
Interest rate persistence	ρ^i	Beta	0.70	0.12	0.87	0.85	0.91
Response to inflation	$\eta^{i,\phi}$	Beta	2.00	0.40	2.79	2.19	2.90
Response to GDP	$\eta^{i,y}$	Beta	0.25	0.10	0.12	0.07	0.17
Response to deficit	η^b	Beta	0.01	0.01	0.00	0.00	0.00
Response to debt	η^{df}	Beta	0.03	0.01	0.01	0.01	0.01
Expenditure rule	ρ^G	Beta	0.70	0.10	0.99	0.98	0.99
Investment rule	ρ^{IG}	Beta	0.70	0.10	0.99	0.98	0.99
Transfer rule	ρ^{tr}	Beta	0.70	0.10	0.99	0.99	1.00
R&D risk premium	ρ^{AY}	Beta	0.50	0.20	0.97	0.95	0.98
Government risk premium	ρ^B	Beta	0.50	0.20	0.91	0.84	0.94
Discount factor	ρ^C	Beta	0.50	0.20	0.69	0.60	0.78
Price mark-up	ρ^{MUY}	Beta	0.50	0.20	0.79	0.68	0.84
Labor demand	ρ^{ND}	Beta	0.50	0.20	0.84	0.79	0.88
Investment risk premium	ρ^S	Beta	0.50	0.20	0.97	0.94	0.98
Knowledge spillover	ρ^σ	Beta	0.50	0.20	0.97	0.95	0.98
Lump-sum tax	ρ^T	Beta	0.50	0.20	0.93	0.92	0.97
Monetary policy	u^i	Gamma	1.00	0.40	0.12	0.11	0.14
Government investment	u^{IG}	Gamma	1.00	0.40	0.05	0.04	0.05
Government consumption	u^G	Gamma	1.00	0.40	0.07	0.07	0.08
Government transfers	u^{tr}	Gamma	1.00	0.40	0.11	0.10	0.13
R&D risk premium	u^{AY}	Gamma	1.00	0.40	5.37	4.90	5.90
Government risk premium	u^B	Gamma	1.00	0.40	0.16	0.15	0.18
Discount factor	u^C	Gamma	1.00	0.40	2.15	1.23	3.10
Price mark-up	u^{MUY}	Gamma	2.00	0.80	4.33	3.43	6.35
Labor demand	u^{ND}	Gamma	0.50	0.20	2.14	1.76	2.60
Investment risk premium	u^S	Gamma	1.00	0.40	0.29	0.28	0.48
Knowledge spillover	u^σ	Gamma	1.00	0.40	5.90	5.83	5.90
Lump-sum tax	u^T	Gamma	1.00	0.40	0.72	0.65	0.79
Wage mark-up	u^U	Gamma	1.00	0.40	1.79	1.20	2.54

Table 2: Estimated parameters and shocks

[a]Not accounting for the labor margin. See footnote 26.

Innovation and growth		
R&D decreasing returns	η	11.530
R&D externality	λ^A	0.128
Knowledge spillovers (steady state)	σ^{RD}	0.006
Preferences		
Preference for govt bonds	α^B	0.002
Preference for stocks	α^S	0.012
Intertemporal discount factor	β	0.990
Weight of disutility of labor	ω^N	17.584
Production		
Cobb-Douglas labor share	a	0.650
Depreciation of capital stock	δ	0.012
Linear capacity utilization adj. costs	$\gamma^{u,1}$	0.025
Wage markup (steady state)	μ^w	1.200
Demand elasticity	σ^y	-29.452
Fiscal policy		
Consumption tax	τ^C	0.200
Corporate profit tax	τ^K	0.300
Labor tax	τ^N	0.227
Debt target	b	3.400
Deficit target	df	0.039
Steady state ratios		
Private consumption share	C/Y	0.692
Private investment share	I/Y	0.126
Govt interest payment share	$(1+i^B)B^B/Y$	0.045
Govt consumption share	C^G/Y	0.153
Govt investment share	I^G/Y	0.042
Transfers share	TR/Y	0.118
Wage share	W/Y	0.595
R&D investment share	X^{RD}/Y	0.015

Table 3: Calibrated parameters.

folds its full economic consequences after a long enough series of court precedents have been ruled.[25]

Quite striking is also the estimate of the R&D adjustment costs parameter, γ^{RD}. Its estimated posterior mode, 197.84, is the highest adjustment cost parameter of the whole model. This value suggests that the R&D and growth models in the academic literature so far, by ignoring R&D adjustment costs, lead to potentially misleading predictions on the effect of policies on growth. High R&D adjustment costs indicate that the R&D response to policies could be much more sluggish than usually thought.

The estimated business cycle parameters imply slow adjustment of employment and capital, and significant nominal rigidities. The high estimated habit persistence (0.89) mirrors the sluggish response of consumption to income. The estimates of risk aversion (1.61) and inverse Frisch elasticity (2.63) are in line with other macro models.[26] The estimated persistence of fiscal rules is high.

3.3 Model dynamics

This section briefly describes impulse response functions of main structural shocks. Investment risk premium and savings disturbances are Keynesian demand shocks. By contrast, R&D spillover shocks capture supply-side factors. All shocks are assumed temporary, i.e. lasting only one term.

Consider first an exogenous shock to the investment risk premium shown in Figure 3 (dashed, red). This risk premium does affect not only physical capital investment, but also entail direct and indirect consequences on R&D investment and ultimately on innovation and productivity growth. The direct effects could be negative due to less aggregate investment in general. However, they could also be positive for R&D, if the shock liberates savings that would otherwise go to a different form of investment. Indirect effects are likely negative due. By reducing investment, lower physical capital investment will decrease the market size of innovative intermediate products, thereby decreasing the profitability of R&D. In the estimated model, the adverse effects dominate. The increase in the investment risk premium reduces GDP and R&D investment. Initially, the shock crowds in consumption. In the medium run, the consumption response turns negative owing to lower wage income. The strong adverse effects on output suggest that fluctuations in investment risk premia generate responses that resemble key

[25] R&D spillover shocks are very persistent. The 90 percent interval of the R&D spillover shock persistence, ρ^{σ}, ranges from 0.95 to 0.98.

[26] Strictly speaking, a measure of risk aversion would need to account for the labor margin (Swanson, 2012).

elements of the slowdown. However, the mild fall in productivity and the consumption response indicates that this shock cannot explain all salient facts. By contrast, consumer savings shocks (yellow, dotted) reduce consumption directly. However, higher savings increase investment and R&D expenditure and the negative on effect on output is relatively short-lived. Below we show that the joint behavior of investment risk premium and consumer savings shocks captures the rapid contraction during the crisis.

The presence of R&D in the model identifies another potential source of the slow recovery: stochastic technology spillover. The blue solid lines in Figure 3 depict the IRFs following a negative temporary shock to R&D spillovers. Even though R&D expenditure recovers faster than in the case of investment risk premia shock, the drop in productivity is equally persistent. The adverse effects on R&D investment and protracted slowdown of productivity suggests that the spillover shock is a good candidate for explaining the slow recovery. However, the fall in GDP remains smaller than for investment risk premia shocks. Spillover shocks cannot explain the rapid contraction in demand components observed during the Great Recession.

3.4 Historical decomposition

This section quantifies the relative contribution of exogenous shocks in explaining the data through the lens of the estimated DSGE model with endogenous Schumpeterian growth. Figure 4 presents a historical shock decomposition of real GDP growth. The continuous line displays the observed time series. Stacked vertical bars indicate the estimated relative contribution of different shocks in a given period. The contribution of all shocks recovers the observed time series.

According to the results, fluctuations in investment risk premia (pink) and private savings (black) contribute most to the Great Recession. The decline in GDP growth associated with the financial crisis is closely related to these shocks. Their joint occurrence reflects a sudden and sharp deterioration of financial intermediation. The credit channels from household savings to private firms investing in physical capital and R&D became less reliable than before. Production and R&D could not get funding comparable to the pre-crisis trend. At the same time, households became more pessimistic about the future. The increased propensity to save led to a drop in consumption expenditure. As a result of the simultaneous reductions in consumption and investment, aggregate private demand and GDP growth fell.

At first, monetary policy turned to an expansionary stance, reflected by the

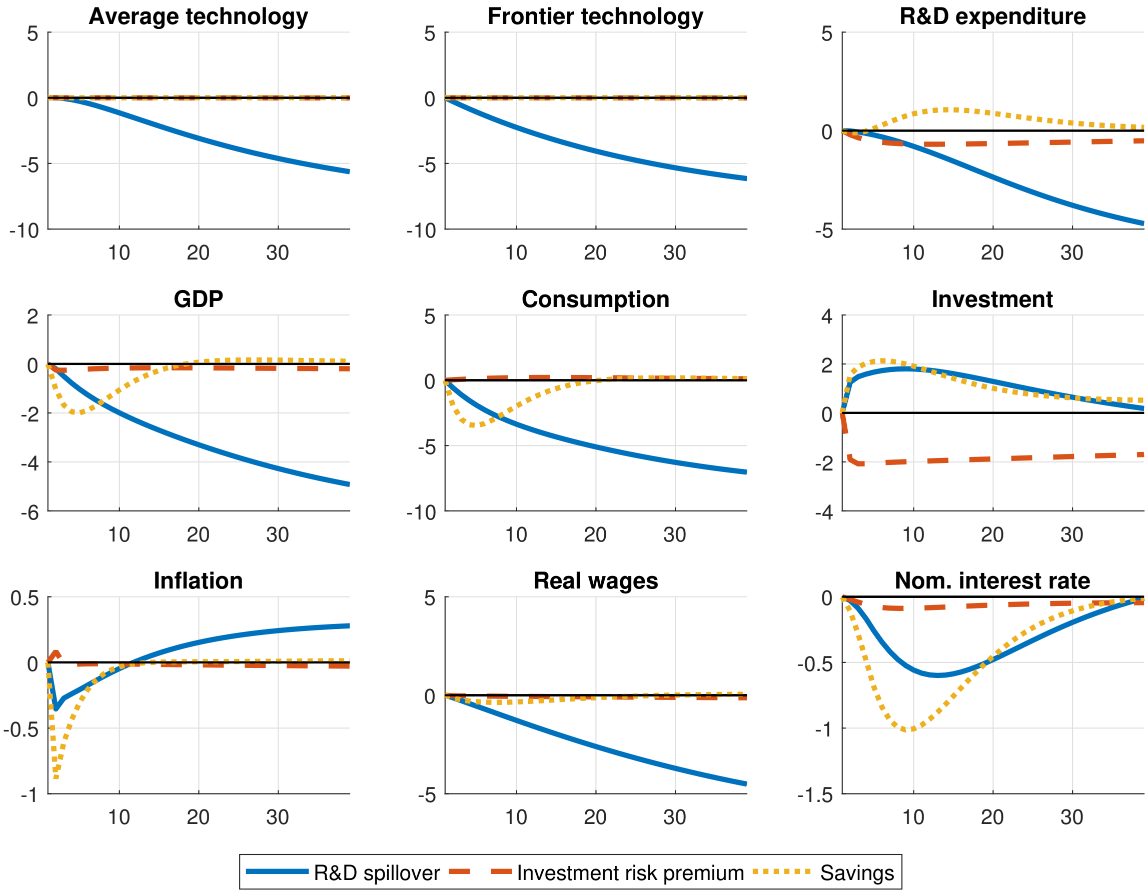

Figure 3: Impulse response functions of selected shocks

Notes: This figure displays the dynamic responses to exogenous shocks of an one estimated standard deviation. Blue solid (red dashed) [yellow dotted] lines show the effects of a negative R&D spillover (positive investment risk premium) [consumption savings] shocks. Variables are displayed in percentage deviations from their steady-state value. The figure reports the interest rate and inflation in annualized percentage points. Impulse responses are displayed for 40 periods, corresponding to 10 years.

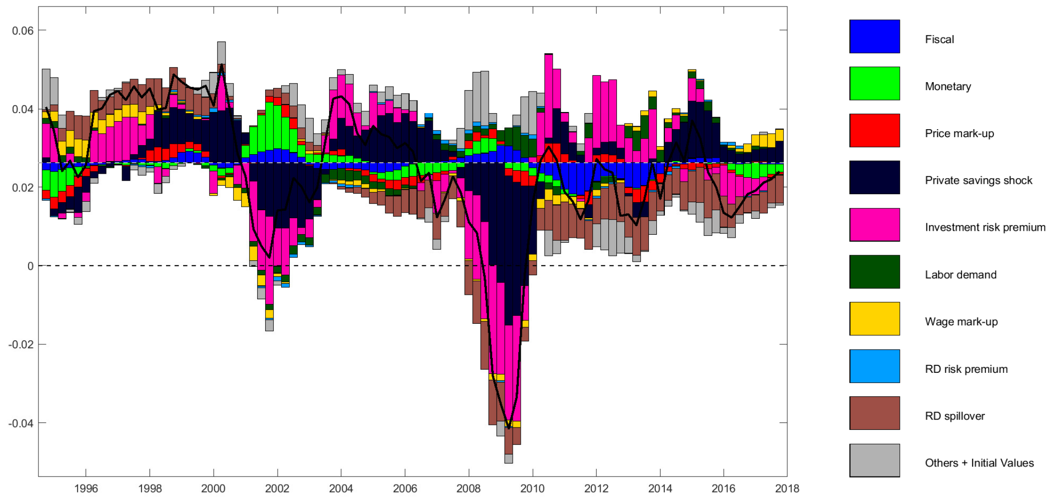

Figure 4: Historical decomposition of real GDP growth (year-on-year)

Notes: This figure shows a historical shock decomposition of real GDP growth. The black continuous line displays demeaned series of real GDP growth (year-on-year). Vertical bars indicate the relative contribution of each (group of) shocks to (i) fiscal policy rules (blue), (ii) monetary policy (light green), (iii) price markups (red), (iv) saving preferences (black) (v) investment risk premia (pink), (vi) labor demand (dark green), (vii) wage markups (yellow), as well as innovation-specific shocks, such as shocks to (viii) the R&D risk premium (light blue) and (ix) R&D spillovers (brown). All remaining shocks (x) are grouped in Others (gray). Appendix C provides additional technical details on the shock decomposition.

positive contribution of innovations to the Taylor rule (green). However, when nominal interest rates hit the ZLB, the Fed became unable to lower the policy rate further into the negative territory, as would have been dictated by the pre-crisis Taylor rule. Therefore, monetary policy failed to give enough relief to financially strained firms and households. Other aspects of the Fed's policy, however, have helped repair the post-crisis financial intermediation. In fact, the negative contribution of the investment risk premium and saving shocks vanishes from 2010 onwards. The model estimation also suggests that discretionary fiscal policy (blue) stabilized the US economy only initially. Its cumulative contribution to GDP growth became negative in the second part of 2009.[27]

Following the crisis, firms have been subject to a more competitive environment. Reduced price markup (red) exerted a positive GDP effect. By contrast, the labor market appears to have suffered rigidities in the two years after the crisis, represented by wage markup shocks (yellow).

[27]This observation may also reflect the expectation of higher future taxes associated with the persistently higher government spending.

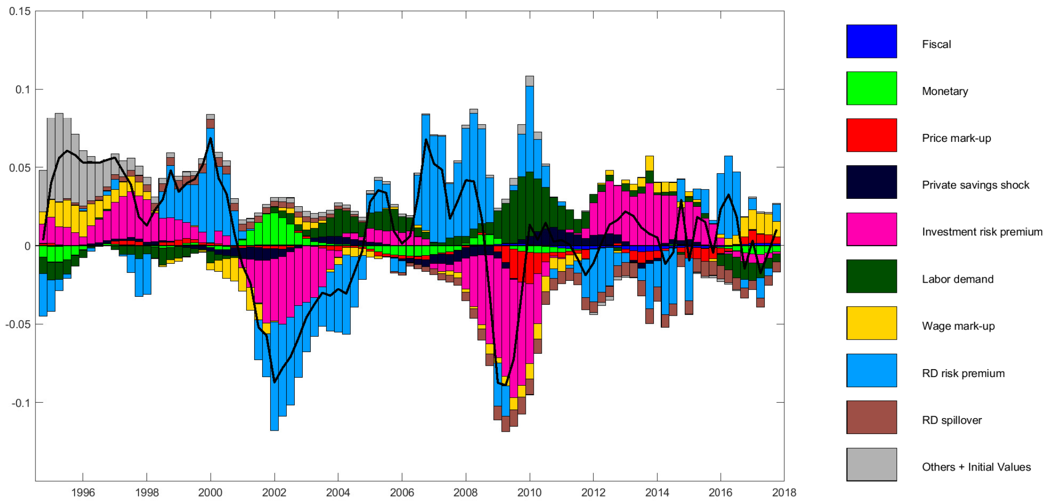

Figure 5: Historical decomposition of R&D investment growth (year-on-year)

Notes: This figure shows a historical shock decomposition of real R&D investment growth. The black continuous line displays demeaned series of real R&D investment growth (year-on-year). Vertical bars indicate the relative contribution of each (group of) shocks to (i) fiscal policy rules (blue), (ii) monetary policy (light green), (iii) price markups (red), (iv) saving preferences (black) (v) investment risk premia (pink), (vi) labor demand (dark green), (vii) wage markups (yellow), as well as innovation-specific shocks, such as shocks to (viii) the R&D risk premium (light blue) and (ix) R&D spillovers (brown). All remaining shocks (x) are grouped in Others (gray). Appendix C provides additional technical details on the shock decomposition.

The period around the dot-com bubble burst is also noteworthy. A strongly expansionary monetary policy stance and supportive fiscal policy at least partially offset the adverse financial conditions associated with persistent investment and saving shocks.

The estimated model identifies the importance of R&D and innovation for fluctuations in GDP growth. We observe a persistently negative contribution of the R&D spillover shock (brown) starting in 2002. According to the Schumpeterian growth model, basic aspects of R&D push the aggregate technological frontier further. The model estimation suggests that R&D activities since this time have had a lower basic content, which has reduced technological spillovers. The strength of this supply-side channel is remarkable, given the rich Keynesian demand features of the model. While our model cannot pin down the precise underlying drivers of this shock, it complements views expressed in other studies. For example, the shock contribution could reflect that new technologies do not share the same general applicability in a wide range of economic sectors. This interpretation squares with Gordon's (2017) hypothesis that advances in ICT have exhausted a large part of their growth potential and were already widely adopted around 2004. It could also relate to changes in the institutional or legal landscape, in which innovation takes place. Cozzi and Galli (2014) emphasize the harmful effects of too strict IPRs after the Madey *vs* Duke Supreme Court verdict of 2002. This court decision has likely negatively affected academic and basic research in the US innovation system. It formally ended the "research exemption" doctrine, which previously permitted researchers to use patented discoveries without incurring the risk of patent infringement.

Except for the technology boom from 1995-2001, contributions from Schumpeterian technology shocks are negligible before the 2000s. This finding suggests that the slow recovery contains essential Schumpeterian elements: Technology dynamics are crucial to understanding why the recovery after the Great Recession was different from previous recessions.

How does the model interpret the time series on R&D investment? Figure 5 presents a decomposition of R&D investment. The estimation explains the drop in R&D investment mainly by effects of financial distress. The malfunctioning of financial markets has a strong adverse effect on R&D investment. The constrained ability of financial markets to channel savings thus helps explain the low growth following the Great Recession. Apart from the dot-com bubble and the short pre-crisis boom, R&D spillovers affect R&D investment mostly negative. Moreover, labor market and monetary policy shocks contribute to fluctuations in R&D investment.

3.5 Adverse innovation dynamics

This section analyzes the Schumpeterian features of our model estimation in more detail. An important hypothesis in the debate on low growth is that the US economy's innovation capacity has slowed down before the Great Recession. Gordon (2017) argues that the growth effects of postwar technological innovation and business re-organization have been harvested, and spillover effects become smaller. In a similar vein, Bloom et al. (2020) ask whether ideas are getting harder to find as a range of studies and data suggest that more and more research effort needs to be devoted to discoveries. Given its endogenous innovation structure, our approach contributes to this discussion.

Figure 1 shows that R&D investment dropped after the 2001 dot-com bubble burst. However, this was not (directly) followed by the downward trend shift in output as observed in the aftermath of the 2008 financial crisis. To explore this important difference through the lens of our model, Figure 6 tracks the implied

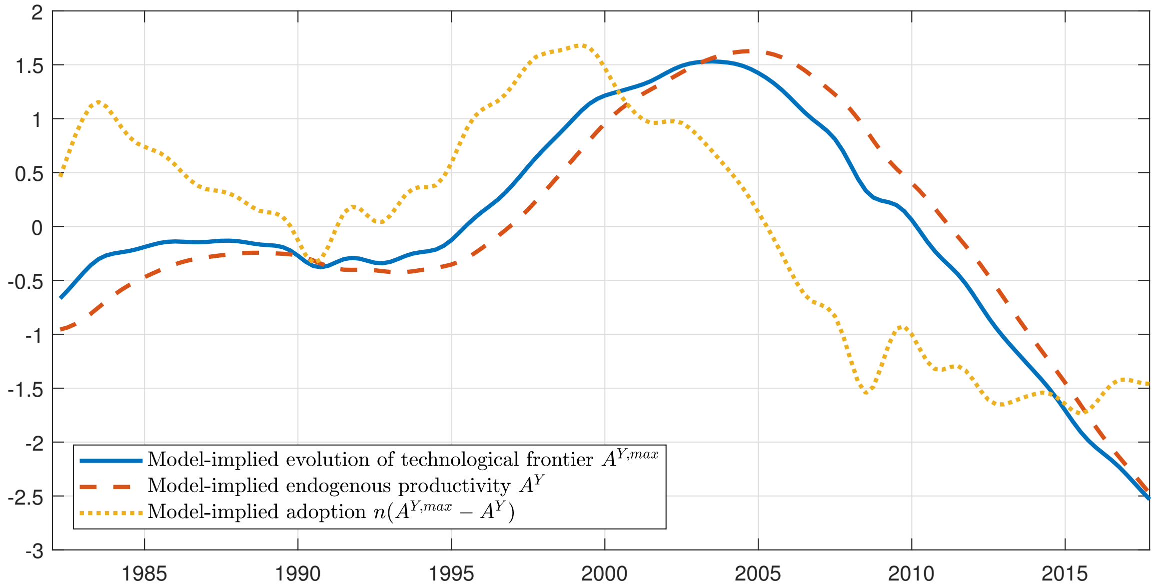

Figure 6: Estimated innovation dynamics

Notes: This figure depicts (detrended and standardized) smoothed estimates of endogenous variables related to innovation.

evolution of frontier technology, $A_t^{Y,\max}$, labor-augmenting productivity, A_t^Y, and adoption, $n_{t-1}\left(A_t^{Y,\max} - A_{t-1}^Y\right)$.[28]

We find model-based evidence of a slowdown of frontier growth at around 2001. The technology boom before the burst of the dot-com bubble led to an increase in frontier technology. Average productivity increased following the adoption of innovations to the general economy. The 2001 crisis generated a strong drop in R&D. Reduced technology adoption and the downward turning point of the (detrended) frontier technology reflect this contraction. Weaker innovation gradually exhausted the stock of available unadopted technologies, $\left(A_t^{Y,\max} - A_{t-1}^Y\right)$. At the time of the financial crisis, this stock was substantially lower than in 2001. Gradually, the deterioration of the frontier led to a slowdown of the average technological pace. As a consequence, aggregate productivity remained persistently below trend. Therefore, the interaction of (semi-)endogenous frontier growth and endogenous adoption suggests that technology dynamics in the early 2000s play an important role in the slow recovery.

[28]Notice that all variables in Figure 6 are shown as *standardized* deviations from their long-term value. This explains why productivity seems to get higher than frontier technology (which is of course impossible) and adoption to get negative.

4 Conclusion

The macroeconomic experience of the last decade has stressed the importance of jointly studying the growth and fluctuations behavior of the economy. Medium-run dynamics and the business cycle are intertwined, suggesting the need to quantify drivers of these quite complex dynamics. To that aim, we have developed an integrated medium-scale DSGE model featuring a rich New Keynesian part in the spirit of Smets and Wouters (2003) as well as a Schumpeterian (semi-)endogenous growth engine. As in Aghion and Howitt (1992) and Nuño (2011), innovations are the outcome of a patent-race, with each innovation improving upon existing goods. Innovating firms replace the incumbent monopolist and earn higher profits until the next innovation occurs. Knowledge spillovers push the technological frontier further.

Results from a Bayesian estimation underline the importance of Keynesian demand shocks. Fluctuations in investment risk premia and saving behavior explain the depth of the Great Recession. In line with recent literature (e.g. Gust et al. 2017), we interpret this finding as malfunctioning financial intermediation. The endogenous innovation channel further amplifies the effects of financial shocks. Endogenous growth links supply and demand as demand-side shocks also drive dynamics of R&D and productivity.

Despite the strength of Keynesian channels, the model estimation challenges the claim that the slump is a pure demand-side phenomenon. We identify adverse supply-side factors such as a decline of the innovative capacity and frontier technology well before the financial turmoil. Indeed, the estimation suggests that adverse supply-side developments have been partially responsible for the long-lasting slowdown. The slow-moving adverse innovation dynamics then led to gradual exhaustion of remaining (unadopted) innovations. When the financial crisis arrived, it struck the innovative system in an already weak state, and forced the economy into a period of protracted low growth.

References

Acemoglu, D., U. Akcigit, H. Alp, N. Bloom, and W. Kerr (2018). Innovation, reallocation, and growth. *American Economic Review 108*(11), 3450–91.

Aghion, P. and P. Howitt (1992). A model of growth through creative destruction. *Econometrica 60*(2), 323–51.

Anzoategui, D., D. Comin, M. Gertler, and J. Martinez (2019). Endogenous technology adoption and r&d as sources of business cycle persistence. *American Economic Journal: Macroeconomics 11*(3), 67–110.

Benigno, G. and L. Fornaro (2017). Stagnation traps. *The Review of Economic Studies 85*(3), 1425–1470.

Bianchi, F., H. Kung, and G. Morales (2019). Growth, slowdowns, and recoveries. *Journal of Monetary Economics 101*, 47 – 63.

Blanchard, O. and J. Galí (2007). Real wage rigidities and the New Keynesian model. *Journal of Money, Credit and Banking 39*, 35–65.

Bloom, N., C. I. Jones, J. Van Reenen, and M. Webb (2020). Are ideas getting harder to find? *American Economic Review 110*(4), 1104–44.

Cette, G., J. Fernald, and B. Mojon (2016). The pre-great recession slowdown in productivity. *European Economic Review 88*, 3 – 20. SI: The Post-Crisis Slump.

Christiano, L. J., M. S. Eichenbaum, and M. Trabandt (2015). Understanding the Great Recession. *American Economic Journal: Macroeconomics 7*(1), 110–167.

Comin, D. and M. Gertler (2006). Medium-term business cycles. *American Economic Review 96*(3), 523–551.

Cozzi, G. and S. Galli (2014). Sequential R&D and blocking patents in the dynamics of growth. *Journal of Economic Growth 19*(2), 183–219.

Cozzi, G., B. Pataracchia, M. Ratto, and P. Pfeiffer (2017). How much Keynes and how much Schumpeter? An estimated macromodel of the US economy. MPRA Paper 77771, University Library of Munich, Germany.

Del Negro, M., G. B. Eggertsson, A. Ferrero, and N. Kiyotaki (2016). The great escape? a quantitative evaluation of the Fed's liquidity facilities. *American Economic Review*.

Erkal, N. and S. Scotchmer (2011). Scarcity of ideas and R&D options: Use it, lose it or bank it. *Mimeo*.

Fisher, J. D. (2015). On the structural interpretation of the Smets-Wouters "risk premium" shock. *Journal of Money, Credit and Banking 47*(2-3), 511–516.

Foster, L., J. C. Haltiwanger, and C. J. Krizan (2001). *Aggregate Productivity Growth: Lessons from Microeconomic Evidence*, pp. 303–372. University of Chicago Press.

Frankel, M. (1962). The production function in allocation and growth: A synthesis. *American Economic Review 52*(5), 996–1022.

Giovannini, M., P. Pfeiffer, and M. Ratto (2020). Efficient and robust inference of models with occasionally binding constraints. *Mimeo*.

Gordon, R. J. (2017). *The rise and fall of American growth: The US standard of living since the civil war*, Volume 70. Princeton University Press.

Guerrieri, L. and M. Iacoviello (2015). OccBin: A toolkit for solving dynamic models with occasionally binding constraints easily. *Journal of Monetary Economics 70*, 22 – 38.

Guerron-Quintana, P. A. and R. Jinnai (2019). Financial frictions, trends, and the Great Recession. *Quantitative Economics 10*(2), 735–773.

Gust, C., E. Herbst, D. López-Salido, and M. E. Smith (2017). The empirical implications of the interest-rate lower bound. *American Economic Review 107*(7), 1971–2006.

Hall, R. E. (2011). The high sensitivity of economic activity to financial frictions*. *The Economic Journal 121*(552), 351–378.

Howitt, P. and P. Aghion (1998). Capital Accumulation and Innovation as Complementary Factors in Long-Run Growth. *Journal of Economic Growth 3*(2), 111–130.

Jones, C. I. (1995). R&D-based models of economic growth. *Journal of Political Economy 103*(4), 759–84.

Jones, C. I. and J. C. Williams (2000). Too much of a good thing? The economics of investment in R&D. *Journal of Economic Growth 5*(1), 65–85.

Kogan, L., D. Papanikolaou, A. Seru, and N. Stoffman (2017). Technological innovation, resource allocation and growth. *The Quarterly Journal of Economics*. forthcoming.

Kollmann, R., B. Pataracchia, R. Raciborski, M. Ratto, W. Roeger, and L. Vogel (2016). The post-crisis slump in the Euro Area and the US: Evidence from an estimated three-region DSGE model. *European Economic Review 88*(C), 21–41.

Krishnamurthy, A. and A. Vissing-Jorgensen (2012). The aggregate demand for treasury debt. *Journal of Political Economy 120*(2), 233–267.

Maih, J. (2015). Efficient perturbation methods for solving regime-switching DSGE models. Working Paper 2015/01, Norges Bank.

Nuño, G. (2011). Optimal research and development and the cost of business cycles. *Journal of Economic Growth 16*(3), 257–283.

Oikawa, K. and K. Ueda (2015). State-Dependent Pricing, Firm Entry and Exit, and Non-Neutrality of Money. *CAMA Working Papers* (2015-03).

Oikawa, K. and K. Ueda (2018). The optimal inflation rate under schumpeterian growth. *Journal of Monetary Economics 100*, 114 – 125.

Oikawa, K. and K. Ueda (2019). Short- and long-run tradeoff of monetary easing. *Journal of the Japanese and International Economies 52*, 189 – 200.

Pinchetti, M. (2020). What is driving the TFP slowdown? Insights from a Schumpeterian DSGE model. *MPRA Working Paper*.

Romer, P. M. (1986). Increasing returns and long-run growth. *Journal of Political Economy 94*(5), 1002–1037.

Rozsypal, F. (2016). Schumpeterian business cycles. *Mimeo*.

Sidrauski, M. (1967). Rational Choice and Patterns of Growth in a Monetary Economy. *American Economic Papers and Proceedings 57*(2), 534–544.

Smets, F. and R. Wouters (2003). An estimated dynamic stochastic general equilibrium model of the euro area. *Journal of the European Economic Association 1*(5), 1123–1175.

Swanson, E. T. (2012). Risk aversion and the labor margin in dynamic equilibrium models. *American Economic Review 102*(4), 1663–91.

Varga, J., W. Roeger, and J. in 't Veld (2016). Financial crisis and TFP growth in the euro area. *Mimeo*.

Vitek, F. (2017). Policy, risk and spillover analysis in the world economy. a panel dynamic stochastic general equilibrium approach. IMF Working Papers 17/89, International Monetary Fund.

A Model details

The notation follows the main text.

A.1 Households

Denoting the Lagrangian multiplier on the budget constraint (1) by $\lambda_{j,t}$, the cumulative discount factor shock $\Theta_t \equiv \exp\left(\sum_{\tau=0}^{t-1} \varepsilon_\tau^C\right)$, and thus

$$\frac{\Theta_{t+1}}{\Theta_t} = \frac{\exp\left(\sum_{\tau=0}^{t} \varepsilon_\tau^C\right)}{\exp\left(\sum_{\tau=0}^{t-1} \varepsilon_\tau^C\right)} = \varepsilon_t^C.$$

The first-order optimality conditions of the household problem in a symmetric equilibrium are as follows.

Consumption C_t:

$$\lambda_t = \Theta_t \frac{(C_t - hC_{t-1})^{-\theta}}{(1+\tau^C)} \tag{A.1}$$

Risk-free assets B_t^{rf}:

$$1 = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} (1+i_t) \right] \tag{A.2}$$

Government bonds B_t^B:

$$1 = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} \frac{1}{1+\alpha^B+\varepsilon_t^B} \left((1+i_t^B) \right) \right] \tag{A.3}$$

Firm shares S_t:

$$1 = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{P_t}{P_{t+1}} \frac{1}{1+\alpha^S+\varepsilon_t^S} \left((1+i_{t+1}^S) \right) \right] \tag{A.4}$$

For latter reference, we define the (stock market) stochastic discount factor as:

$$\beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{1}{1+\alpha^S+\varepsilon_t^S} \right] \equiv E_t \left[\Lambda_{t,t+1} \right]. \tag{A.5}$$

The term $\frac{1}{1+\alpha^S+\varepsilon_t^S}$ creates an exogenous wedge between the intertemporal marginal rate of substitution of household and the (stock market) stochastic discount factor. This approach is a short-cut for capturing financial frictions facing the firm. It can, e.g., be interpreted as a "principal agent friction" between the owner and the management of the firm (Hall (2011)). Equation (A.4) can be written as:

$$1 = E_t \left[\Lambda_{t,t+1} \frac{P_t}{P_{t+1}} \left((1+i_{t+1}^S) \right) \right]. \tag{A.6}$$

A.2 Wage Setting

Labor packers. Labor packers have access to a CES production technology:

$$N_t = \left(\int_0^1 N_{j,t}^{\frac{\sigma^n - 1}{\sigma^n}} dj \right)^{\frac{\sigma^n}{\sigma^n - 1}} \tag{A.7}$$

where σ^n denotes the substitution elasticity. The labor packers maximize output

$$\max_{\{N_{j,t}\}} W_t N_t - \int_0^1 W_{j,t} N_{j,t} dj = W_t \left(\int_0^1 N_t^{\frac{\sigma^n - 1}{\sigma^n}} dj \right)^{\frac{\sigma^n}{\sigma^n - 1}} - \int_0^1 W_{j,t} N_{j,t} dj. \tag{A.8}$$

Combining the first-order condition with a zero-profit condition gives their labor demand

$$N_{j,t} = \left(\frac{W_{j,t}}{W_t} \right)^{-\sigma^n} N_t. \tag{A.9}$$

Unions. The Online Appendix provides the detailed algebra. Trade unions maximize a discounted future stream of the household's utility:

$$\max_{W_{j,t}} U_{j0} = \sum_{t=0}^{\infty} \beta^t u(C_{j,t}, N_{j,t}, \cdot). \tag{A.10}$$

Utility maximization is subject to the demand from labor packers (A.9) and the household budget constraint. Nominal wage adjustment costs follow $\Gamma_t^W = \frac{(\sigma^n - 1)\gamma^w}{2} W_t N_t \left(\pi_{j,t}^W - \pi^w \right)^2$, where $\pi_t^W = \frac{W_{j,t}}{W_{j,t-1}} - 1$ denotes quarterly wage inflation. This specification implies that the costs of deviating from steady state wage inflation (π^w) are scaled by the wage bill and a constant factor.

This optimization problem gives rise to the following Lagrangian:

$$\begin{aligned}
\mathcal{L}^W = E_0 \sum_{t=0}^{\infty} \beta^t \Theta_t \Bigg\{ & \frac{(C_{j,t} - hC_{t-1})^{1-\theta}}{1-\theta} - \omega_t^N \frac{(N_{j,t})^{1+\theta^N}}{1+\theta^N} - \mathcal{U}_{j,t}^B / P_t^{C,vat} \times (C_t - hC_{t-1})^{-\theta} \\
& - \frac{\lambda_t}{P_t^{C,vat}} \Bigg[P_t^{C,vat} C_{j,t} + B_{j,t}^B + B_{j,t}^{rf} + P_t^S S_t + \exp(\varepsilon_t^T) T_{j,t} + \Gamma_t^W - \left(1 - \tau^N\right) \\
& W_{j,t} N_{j,t} + (1 + i_{t-1}^B) B_{j,t-1}^B + (1 + i_{t-1}) B_{j,t-1}^{rf} + (1 + i_t^S) S_{j,t-1} + TR_{j,t} + \Pi_t P_t \Bigg] \Bigg\},
\end{aligned} \tag{A.11}$$

with $\Gamma_t^W = \frac{(\sigma^n - 1)\gamma^w}{2} W_t N_t \left(\frac{W_{j,t}}{W_{j,t-1}} - 1 - \pi^w \right)^2$.

Define $V_{j,t}^N \equiv \Theta_t \omega^N (C_t)^{1-\theta} N_{j,t}^{\theta^N}$, the first-order condition reads:

$$\begin{aligned}
\frac{\mathcal{L}^W}{\partial W_{j,t}} = & -V_{j,t}^N (-\sigma^n) \left(\frac{W_{j,t}}{W_t} \right)^{-\sigma^n - 1} \frac{N_t}{W_t} + \frac{\lambda_t}{P_t^{C,vat}} \Big[(1 - \tau^N) W_{j,t} (-\sigma^n) \left(\frac{W_{j,t}}{W_t} \right)^{-\sigma^n - 1} \frac{N_t}{W_t} \\
& + \left(1 - \tau^N\right) N_{j,t} - (\sigma^n - 1)\gamma^w W_t N_t \frac{1}{W_{j,t-1}} \left(\frac{W_{j,t}}{W_{j,t-1}} - 1 - \pi^w \right) \Big] \\
& + \beta E_t \left[\frac{\lambda_{t+1}}{P_{t+1}^{C,vat}} \left((\sigma^n - 1)\gamma^w W_{t+1} N_{t+1} \frac{W_{j,t+1}}{W_{j,t}^2} \left(\frac{W_{j,t+1}}{W_{j,t}} - 1 - \pi^w \right) \right) \right] = 0,
\end{aligned} \tag{A.12}$$

dropping j subscripts due to symmetry (same hours and wages across households $N_{j,t} = N_t$ and

$W_{j,t} = W_t$), defining $\mu^W \equiv \frac{\sigma^n}{(\sigma^n - 1)}$, and $mrs_t \equiv \frac{V_t^N}{\lambda_t}$, and allowing for real wage rigidity as in Blanchard and Galí (2007), governed by parameter γ^{wr}, labor supply follows

$$\left(\mu^w mrs_t\right)^{1-\gamma^{wr}} \left[\frac{(1-\tau^N)W_{t-1}}{P_{t-1}}\right]^{\gamma^{wr}} = \frac{W_t}{P_t}\left[(1-\tau^N) + \gamma^w(\pi_t^W + 1)\left(\pi_t^W - \pi^w\right)\right]$$
$$- \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{P_t^{C,vat}}{P_{t+1}^{C,vat}} 1/P_t\left(\gamma^w W_{t+1}\frac{N_{t+1}}{N_t}(\pi_{t+1}^W + 1)\left(\pi_{t+1}^W - \pi^w\right)\right)\right] + \frac{W_t}{P_t}u_t^U, \tag{A.13}$$

where we include the labor supply shock u_t^U.

A.3 Intermediate good firms

Firms maximize a stream of future dividends $d_{i,t}$ equal to a discounted stream of future dividends, $V_t = d_t + E_t\left[\Lambda_{t,t+1}V_{t+1}\right]$ where $\Lambda_{t,t+1}$ corresponds to the households discount factor as derived in equation (A.5). Firms choose labor, capacity utilization, and prices.

Period t dividends in real terms are:

$$d_{i,t} = \left(1-\tau^K\right)\left(\frac{P_{i,t}}{P_t}Y_{i,t} - \frac{W_t}{P_t}N_{i,t}\right) - adj_{i,t}, \tag{A.14}$$

where W_t is the wage rate, τ^K is the profit tax, δ is capital depreciation rate and $adj_{i,t}$ are total adjustment costs associated with adjustment of prices $P_{i,t}$, capacity utilization $cu_{i,t}$ and labor input $N_{i,t}$ adjustment.

For tractability, we make two more assumptions. (i) When a new incumbent starts production, she inherits the previous stocks (employment, capital) and costs of the previous firm that dropped out. (ii) As discussed in the main text, moving closer to the frontier implies higher adjustment costs due to sophistication. As a consequence, adjustment costs reflect the relative technological position $\left(\frac{A_{i,t}^Y}{A_t^Y}\right)$ of the sector i. Thus,

$$adj_{i,t} = adj_{i,t}^P + adj_{i,t}^N + adj_{i,t}^{cu} \tag{A.15}$$

with

$$adj_{i,t}^P = \frac{\gamma^p}{2}Y_t\left(\frac{P_{i,t}}{P_{i,t-1}} - \exp(\overline{\pi})\right)^2\left(\frac{A_{i,t}^Y}{A_t^Y}\right) \tag{A.16}$$

$$adj_{i,t}^N = \frac{\gamma^n}{2}Y_t\left(\frac{N_{i,t}}{N_{i,t-1}} - \exp(\overline{g^{pop}})\right)^2\left(\frac{A_{i,t}^Y}{A_t^Y}\right) \tag{A.17}$$

$$adj_{i,t}^{cu} = K_{t-1}^{tot}\left(\gamma^{u,1}\left(cu_{i,t}-1\right) + \frac{\gamma^{u,2}}{2}(cu_{i,t}-1)^2\right)\left(\frac{A_{i,t}^Y}{A_t^Y}\right) \tag{A.18}$$

where $\overline{\pi}$ and $\overline{g^{pop}}$ denote steady state inflation and population growth. Firms maximize dividends subject to the production technology (8) and the demand schedule for final goods, equation (A.28), $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\sigma^y} Y_t$. μ_t will denote the multiplier on the production technology (8). We allow for shocks to labor demand and price mark-ups, denoted ϵ_t^{ND} and ϵ_t^{MUY}, respectively. In a symmetric equilibrium, the first-order conditions are:

$$N_t: \quad (1-\tau^K)\frac{W_t}{P_t} = \left(\mu_t - \varepsilon_t^{ND}\right)\alpha\frac{Y_t}{N_t} - \frac{\partial adj_t^N}{\partial N_t} + E_t\left[\Lambda_{t,t+1}\frac{\partial adj_{t+1}^N}{\partial N_t}\right], \tag{A.19}$$

$$cu_t: \quad K_{t-1}^{tot}\left[\gamma^{u,1} + \gamma^{u,2}(cu_t - 1)\right] = \mu_t(1-\alpha)\frac{Y_t}{cu_t} \tag{A.20}$$

$$\begin{aligned} P_t: \quad \mu_t\sigma^y &= (1-\tau^K)(\sigma^y - 1) + \sigma^y\gamma^P\frac{P_t}{P_{t-1}}\left(\pi_t - \bar{\pi}\right) \\ &- \sigma^y\gamma^P\left[\Lambda_{t,t+1}\frac{P_{t+1}}{P_t}\frac{Y_{t+1}}{Y_t}\left(\pi_{t+1} - \bar{\pi}\right)\right] + \sigma^y\varepsilon_t^{MUY}. \end{aligned} \tag{A.21}$$

A.3.1 Investment

Investment is carried out by an investment fund owned by households. The fund perfectly internalizes choices of the firms. The (private) capital stock at each firm i accumulates according to

$$K_{i,t} = (1-\delta)K_{i,t-1} + I_{i,t}, \tag{A.22}$$

and is subject to investment adjustment costs:

$$adj_{i,t}^I = \left(\frac{\gamma^{I,1}}{2}K_{t-1}\left(\frac{I_{i,t}}{K_{t-1}^{tot}} - \delta_t\right)^2 + \frac{\gamma^{I,2}}{2}\frac{\left(I_{i,t} - \exp(g^Y + \pi)I_{i,t-1}\right)^2}{K_{t-1}^{tot}}\right)\left(\frac{A_{i,t}^Y}{A_t}\right), \tag{A.23}$$

Optimal choices of capital and investment imply in a symmetric equilibrium:

$$Q_t = E_t\left[\Lambda_{t,t+1}\left(\tau^K\delta - \frac{\partial adj_t^{CU}}{\partial K_{t-1}} + Q_{t+1}(1-\delta) + (1-\alpha)\mu_{t+1}\frac{Y_{kt+1}}{K_t^{tot}}\right)\right], \tag{A.24}$$

and

$$\begin{aligned} Q_t &= \left[1 + \gamma^{I,1}\left(\frac{I_t}{K_{t-1}} - \delta_t^K\right) + \gamma^{I,2}\frac{(I_t - I_{t-1})}{K_{t-1}}\right] \\ &\quad - E_t\left[\Lambda_{t,t+1}\exp(g^Y + \pi)\gamma^{I,2}\frac{\left(I_{t+1} - I_t\exp(g^Y + \pi)\right)}{K_t}\right]. \end{aligned} \tag{A.25}$$

Equation (A.24) and (A.25) define Tobin's Q, the replacement cost of capital (the multiplier on (A.22)). g^Y denotes steady-state growth.

A.4 Final good producers

Output (Y_t) is produced by perfectly competitive firms by combining a large number of differentiated goods, $Y_{i,t}$, produced by monopolistically competitive firms. Final good producers have access to a CES production technology:

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\sigma^y - 1}{\sigma^y}}di\right]^{\frac{\sigma^y}{\sigma^y - 1}} \tag{A.26}$$

Firms choose $Y_{i,t}$, taking $P_{i,t}$ as given.

$$\max_{Y_{i,t}} P_t \Big[\int_0^1 Y_{i,t}^{\frac{\sigma^y - 1}{\sigma^y}} di \Big]^{\frac{\sigma^y}{\sigma^y - 1}} - \int_0^1 P_{i,t} Y_{i,t} di \tag{A.27}$$

The resulting demand for a differentiated good i is:

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\sigma^y} Y_t \tag{A.28}$$

where σ^y is inversely related to the steady state price mark-up. The corresponding price index is:

$$P_t = \Big[\int_0^1 (P_{i,t})^{1-\sigma^y} di \Big]^{\frac{1}{1-\sigma^y}} \tag{A.29}$$

A.5 Entrepreneurs

The R&D entrepreneurial problem is to maximize profits:

$$\max_{X_{i,t}^{RD}} = n_{i,t} \left[\frac{P_{i,t}^{S\max}}{P_t} \right] - \left[X_{i,t}^{RD} + \frac{\gamma^{RD}}{2Y_t} (X_{i,t}^{RD} - X_{t-1}^{RD} g_Y)^2, \right] \left(1 + \varepsilon_t^{AY} \right) \tag{A.30}$$

R&D investment optimality:

$$\frac{n_t}{(\eta + 1)} \left[\frac{P_t^{S\max}}{P_t} \right] = X_t^{RD} \left(\frac{\gamma^{RD}}{Y_t} (X_t^{RD} - X_{t-1}^{RD} g_Y) \right) (1 + \varepsilon_t^{AY}) \tag{A.31}$$

Probability of innovation:[29]

$$n_{i,t} = \left(\frac{X_{i,t}^{RD}}{\lambda^{RD} A_{i,t}^{\max}} \right)^{\frac{1}{(\eta+1)}} \tag{A.32}$$

[29] We verify that $X_{i,t}^{RD} < \lambda^{RD} A_{t+1}^{\max}$.

B Estimation data

All data sets (unless otherwise noted) are post-war US data observed from 1983Q1 on. Data of macroeconomic observables come from the Bureau of Economic Analysis (BEA) and the Federal Reserve. We use quarterly R&D data from the US. Bureau of Economic Analysis (NIPA Table 5.3.5. Private Fixed Investment by Type). The data are seasonally adjusted at annual rates. In particular, we employ the series on intellectual property rights Y001RC1 from which we subtract the investment in Entertainment, literary, and artistic originals (Y020RC1). The data are available at http://www.bea.gov/national/nipaweb/DownSS2.asp.

Observables include time series of:

1. Real GDP
2. GDP deflator
3. Nominal interest rate
4. Consumption
5. Investment
6. Wage share
7. Population
8. R&D investment
9. Hours worked
10. Trend TFP
11. Activity rate.

We also include a set of fiscal variables as observables:

12. Government consumption
13. Government investment
14. Government interest payments
15. Government transfers
16. Public debt

C Nonlinear estimation

C.1 Overview

We build on the OccBin toolkit (Guerrieri and Iacoviello 2015) to account for the occasionally binding constraint on nominal interest rates. This method handles the constraints as different regimes of the same model in which the constraints are either slack or binding. Consequently, our model consists of two regimes: an unconstrained baseline, and a regime in which the ZLB constraint binds. Importantly, the dynamics in both regimes depend on the endogenous length of that regime. The expected duration, in turn, depends on the state variables and exogenous disturbances. As emphasized in Guerrieri and Iacoviello (2015), this interaction can result in highly nonlinear dynamics.

Following Giovannini et al. (2020), we integrate the nonlinear solution into a specially adapted Kalman filter and estimate the model with the occasionally binding constraint. This approach is based on the so-called Piecewise Kalman filter (PKF), a particular form of nonlinear filter where the state-space representation becomes time-varying. At each time period, the filter proceeds in two steps. The first, the prediction step, is standard in the filtering of non-linear models. The second (update) step is tailored to the piecewise-linear model. In a nutshell, the update step entails an iterative convergence procedure for the temporary binding regime materializing in each period, that ensures that occasionally binding constraints are not violated. Giovannini et al. (2020) embed this iterative algorithm into a diffuse Kalman filter.

C.2 Details of the algorithm

The algorithm follows Giovannini et al. (2020). Let us define the local linear representation of the policy function of the DSGE model featuring OBC solved with the piecewise linear approach:

$$x_t = \mathbf{T}(x_{t-1}, \varepsilon_t) x_{t-1} + \mathbf{C}(x_{t-1}, \varepsilon_t) + \mathbf{R}(x_{t-1}, \varepsilon_t)\varepsilon_t \tag{C.1}$$

where x_t is the vector of endogenous variables in deviation from the steady state of the 'baseline' (normal time) regime. ε_t is the vector of shocks. The reduced form matrices $\mathbf{T}$ $\mathbf{C}$ and $\mathbf{R}$ are state-dependent. They are functions of the lagged states and the current period shocks (note that $\mathbf{C}(0,0) = 0$ under the 'baseline' regime equilibrium).

In every period t, the piecewise linear solution ensures that, given the lagged states and the current shocks, the constraints are never violated for all periods $s \in [t, \infty)$. The state matrices need to be updated in the new period $t+1$. A new shock in $t+1$ can change the future sequence of state matrices expected given the shock in t, $\mathbf{T}_s(x_{t-1}, \varepsilon_t)$ $\mathbf{C}_s(x_{t-1}, \varepsilon_t)$ and $\mathbf{R}_s(x_{t-1}, \varepsilon_t)$. The one step recursion of the solution algorithm implies, in general, that:

$$\begin{aligned}
\mathbf{T}(x_{t-1}, \varepsilon_t) &\neq \mathbf{T}(x_t, \varepsilon_{t+1}) \\
\mathbf{C}(x_{t-1}, \varepsilon_t) &\neq \mathbf{C}(x_t, \varepsilon_{t+1}) \\
\mathbf{R}(x_{t-1}, \varepsilon_t) &\neq \mathbf{R}(x_t, \varepsilon_{t+1})
\end{aligned}$$

To ease notation, let us re-define the state matrices as:

$$\begin{aligned}
\mathbf{T}_{t|t} &= \mathbf{T}(x_{t-1}, \varepsilon_t) \\
\mathbf{C}_{t|t} &= \mathbf{C}(x_{t-1}, \varepsilon_t) \\
\mathbf{R}_{t|t} &= \mathbf{R}(x_{t-1}, \varepsilon_t)
\end{aligned}$$

$$\begin{aligned}
\mathbf{T}_{t|t-1} &= \mathbf{T}(x_{t-1}, 0) \\
\mathbf{C}_{t|t-1} &= \mathbf{C}(x_{t-1}, 0) \\
\mathbf{R}_{t|t-1} &= \mathbf{R}(x_{t-1}, 0)
\end{aligned}$$

State filtering and the likelihood. Assume we want to estimate the deep parameters of the model, given a set of observables y_t linked to x_t by the observation equation

$$y_t = \mathbf{H}x_t \tag{C.2}$$

where, for simplicity and without loss of generality, we assume no observation error. Let denote as z_t, the observations for y_t.[30]

Given the initial state mean and variance:

$$x_0, \mathbf{P}_0$$

and denoting their 'best' estimate thereof at any time $t-1$ as

$$x_{t-1|t-1}, \mathbf{P}_{t-1|t-1}, \tag{C.3}$$

the prediction step for states and observables reads:

$$\begin{aligned}
x_{t|t-1} &= \mathbf{T}_{t|t-1} \cdot x_{t-1|t-1} + \mathbf{C}_{t|t-1} \\
\mathbf{P}_{t|t-1} &= \mathbf{T}_{t|t-1} \cdot \mathbf{P}_{t-1|t-1} \cdot \mathbf{T}'_{t|t-1} \\
y_{t|t-1} &= \mathbf{H}x_{t|t-1} \\
\mathbf{F}_t &= \mathbf{H} \cdot \mathbf{P}_{t|t-1} \cdot \mathbf{H}'
\end{aligned} \tag{C.4}$$

The prediction step (C.4) is standard in filtering of nonlinear models, e.g. it is the same as for the extended Kalman Filter.

The update step is the critical element of the piecewise linear Kalman filter (PKF). It is tailored to the piecewise linear solution. The update step entails the following iterative procedure, which mimics the analog iterative procedure applied to simulate the model with the piecewise linear approach.

We initialize the guess of the updated state matrices as:

$$\begin{aligned}
\mathbf{T}(\mathbf{0})_{t|t} &= \mathbf{T}_{t|t-1} \\
\mathbf{R}(\mathbf{0})_{t|t} &= \mathbf{R}_{t|t-1} \\
\mathbf{C}(\mathbf{0})_{t|t} &= \mathbf{C}_{t|t-1}
\end{aligned}$$

[30] We initialize the filter with the unconditional mean and variance of the 'baseline' regime with diffuse priors.

Then, we iterate until convergence. Each iteration j follows the algorithm:

1. take the prediction step for the guess matrices $\mathbf{T}(j-1)_{t|t}$,$\mathbf{R}(0)_{t|t}$, $\mathbf{C}(0)_{t|t}$:

$$\begin{aligned} x(j)_{t|t-1} &= \mathbf{T}(j-1)_{t|t} \cdot x_{t-1|t-1} + \mathbf{C}(j-1)_{t|t} \\ \mathbf{P}(j)_{t|t-1} &= \mathbf{T}(j-1)_{t|t} \cdot \mathbf{P}_{t-1|t-1} \cdot \mathbf{T}'(j-1)_{t|t} + \mathbf{R}(j-1)_{t|t} \cdot \mathbf{Q} \cdot \mathbf{R}'(j-1)_{t|t} \\ y(j)_{t|t-1} &= \mathbf{H}x(j)_{t|t-1} \\ \mathbf{F}(j)_t &= \mathbf{H} \cdot \mathbf{P}(j)_{t|t-1} \cdot \mathbf{H}' \\ v_t(j) &= z_t - y(j)_{t|t-1} \end{aligned} \tag{C.5}$$

2. update state and covariance given the guess matrices

$$\begin{aligned} \mathbf{K}_t(j) &= \mathbf{P}(j)_{t|t-1} \cdot \mathbf{H}' \\ x(j)_{t|t} &= x_{t|t-1} + \mathbf{K}_t(j)\mathbf{F_t}(j)^{-1}v(j)_t \\ \mathbf{P}(j)_{t|t} &= \mathbf{P}(j)_{t|t-1} - \mathbf{K}(j)_t\mathbf{F_t}(j)^{-1}\mathbf{K}(j)'_t \end{aligned} \tag{C.6}$$

3. perform a one step backward iteration (a smoother step) to also update the state in $t-1$ given t and estimate the shock in t, i.e. for $s = t, t-1$

$$\begin{aligned} L_s &= \mathbb{I} - \mathbf{K}(j)_s\mathbf{F_s}(j)^{-1}\mathbf{H} \\ r(j)_s &= \mathbf{H}'\mathbf{F_s}(j)^{-1}v_s(j) + L'_s\mathbf{T}'(j-1)_{s|s}r(j)_{s+1} \\ x(j)_{s|t} &= x_{s|s-1} + \mathbf{P}(j)_{s|s-1} \cdot r(j)_s \\ \varepsilon(j)_{s|t} &= \mathbf{Q} \cdot \mathbf{R}'(j-1) \cdot r(j)_s \end{aligned} \tag{C.7}$$

   where the backward one step recursion is initialized by $r_{t+1} = 0$.

4. project the piecewise linear model given the initial condition $x(j)_{t-1|t}$ and shock $\varepsilon(j)_{t|t}$ for $s \in (t, \infty)$ and obtain the updated matrices $\mathbf{T}(j)_{t|t}$,$\mathbf{R}(j)_{t|t}$, $\mathbf{C}(j)_{t|t}$

   (a) if the updated state matrices are different from the guessed ones, update the guess matrices to $\mathbf{T}(j)_{t|t}$,$\mathbf{R}(j)_{t|t}$, $\mathbf{C}(j)_{t|t}$ and restart from 1) with $j+1$

   (b) otherwise, proceed to $t+1$ and until T, by setting updated state matrices

$$\begin{aligned} \mathbf{T}_{t|t} &= \mathbf{T}(j)_{t|t} = \mathbf{T}(j-1)_{t|t} \\ \mathbf{R}_{t|t} &= \mathbf{R}(j)_{t|t} = \mathbf{R}(j-1)_{t|t} \\ \mathbf{C}_{t|t} &= \mathbf{C}(j)_{t|t} = \mathbf{C}(j-1)_{t|t}, \end{aligned}$$

   as well as states and covariances

$$\begin{aligned} x_{t|t} &= x(j)_{t|t} \\ \mathbf{P}_{t|t} &= \mathbf{P}(j)_{t|t} \end{aligned}$$

Note that the updating algorithm applies one backward smoothing step for each period t. Each step of the algorithm is simple since it applies standard Kalman filter formula, using the guess state matrices. For the piecewise linear solution method, this filtering and updating algorithm is optimal in the least-squares sense. Typically one iteration is sufficient for the convergence of the updating step. In case of failed convergence, we give a penalty to the likelihood and try a new proposal for the deep parameters.

Given the prediction error

$$v_t(j) = z_t - y(j)_{t|t-1} \tag{C.8}$$

we can compute the log-likelihood density of the data at time t:

$$\mathcal{L}_t = \log(\det(\mathbf{F}(\mathbf{j})_\mathbf{t})) + v(j)_t' \cdot \mathbf{F}(\mathbf{j})_\mathbf{t}^{-1} \cdot v(j)_t + n_t \log(2\pi) \tag{C.9}$$

where j denotes the updated state matrices at the end of the update step and n_t denotes the number of observables available in time t.

C.3 Shock decomposition

This section provides additional details on how we obtain an additive shock decomposition for the nonlinear model.

To fix ideas, consider a piecewise-linear Kalman smoother, which estimates the sequence of regimes in the historical time interval. The sequence of regimes triggers a sequence of state-space matrices:

$$y_t = C(t) + T(t)y_{t-1} + R(t)e_t, \tag{C.10}$$

where y_t stacks all endogenous variables in deviation from steady state, e_t are the smoothed shocks, and $C(t)$ is a constant which is triggered by the regime. $C = 0$ in the unconstrained regime.[31]

The algorithm estimates the additive shock decompositions in two steps. The first step exploits the piecewise-linear Kalman smoother, that provides the smoothed estimate of the sequence of regimes together with the historical series of exogenous shocks. Given the sequence of regimes, the shocks are propagated individually though the sequence of state-space matrices $T(t)$ and $R(t)$. The array $C(t)$ is treated as an additional exogenous process, labelled "regime effect".[32]

A second step extends this procedure to obtain an additive shock decomposition. Note first that the regime effect results from the interaction of all shocks simultaneously hitting the system $\forall \tau \leq t$. Hence, the regime effect is a function of exogenous shocks. For each t and variable j, we compute the absolute value of the contribution of each shock $e_{i,t}$ onto a variable y_j^R:

$$w_{j,i,t} = |y_t^R(e_{i,t})|, \tag{C.11}$$

where $w_{j,i,t}$ is a set of weights which apportion the regime affect all shocks and y_j^R is a variable determining the regime sequence. In case of the ZLB constraint on the Taylor rule, $y^R = [\pi_t, \tilde{y}_t]$, i.e. the arguments of the nominal interest rate rule. Both are functions of shocks $y_t^R(e_{i,t})$ and can be used to calculate the weights $w_{j,i,t}$. The intuition behind this procedure is the following. Any shock causing a change in the Taylor rule (and the associated monetary policy regime) matters for the regime effect. For example, a large enough expansionary demand shock shortens the duration of the ZLB constrained regime and increases inflation and the output gap.

[31] Note that at the ZLB, the Taylor rule $i_t = i^{lb}$ violates the steady state condition $(i^{lb} < \bar{i})$.

[32] The RISE Toolbox (Maih 2015) follows a similar approach.

Declaration(s) of interest

All authors: None.