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Abstract

The article considers the synthesis of an optimal discrete control algorithm for the conveyor belt speed, based on the use of Time-Of-Use tariffs. Methods have been investigated that reduce the consumption of electricity required to transport material from the extracted place to the place of processing. It is shown that the uneven distribution of material along the transportation route for long multi-section conveyors leads to a significant increase in the share of transportation costs among the total costs of material extraction. The systems for controlling the flow parameters of the transport system are analyzed to ensure uniform distribution of material along the transport route and reduce the cost of transporting material. It has been demonstrated that energy management methodology is an effective tool to reduce the cost of material transportation. Analyzed the common classes of tariffs using electricity by industrial enterprises, which can be successfully used in the design of control systems for flow parameters of the conveyor line. When synthesizing algorithms for regulating the speed of the belt, the model of primary friction and the assumption of the absence of a stress wave in the belt during instantaneous switching of the speed modes were used. An analytical model of the conveyor is presented in a dimensionless form, taking into account the transport delay. The problem of optimal control of the speed of the conveyor belt at fixed energy consumption for the considered control interval is formulated. The ranges of variation of the model parameters are estimated. An algorithm for optimal regulation of the belt speed using the coefficients Ukraine – TOU periods is synthesized. The influence of the initial conditions on the belt speed control modes is analyzed.

Keywords
Transport conveyor, distributed transport system, energy management, conveyor belt speed control, transport delay, uneven material distribution

1. Introduction

For mining enterprises, an important issue associated with reducing the cost of extracting material is the issue of reducing the cost of electricity for transporting material. The cost of transporting material from the place of extraction to the place of distribution and processing at the standard loading of the conveyor system is 20\% of the cost of extracting material \cite{1}. A typical mode of operation of a transport conveyor is a mode with an uneven supply of material at the entrance of the transport system. This leads to a decrease in the load on the conveyor line. The coefficient of loading the conveyor section with the material can be 0.5–0.7 of the full loading of the conveyor section \cite{2}. This mode leads to a nonlinear increase in energy consumption for the transportation of material of a unit weight due to a decrease in the material load factor of the conveyor section \cite{3}. For low loaded conveyor sections, the energy consumption for transporting material of a unit weight can increase several times. The uneven distribution of material along the conveyor section is especially important for extended transport systems \cite{4, 5, 6}. A common solution to reduce energy costs for material transportation is the division of an extended conveyor into separate sections \cite{7, 8, 9}. The development of the mining industry is associated with a further increase in the length of the transport route. The length of a separate section has exceeded ten km \cite{10} and continues to increase. The presence of the fact of uneven distribution of material along the transport route for long multi-section conveyors leads to the fact that the cost of transporting material can reach a significant share in the total cost of extracting material. To increase
the loading factor of the conveyor sections, systems for controlling the flow of material from between sectional bunker [11, 12], the speed control systems of conveyor belts [13, 14] or combined control systems are used. The division into sections increases the control efficiency of the transport route section within a separate section and the reliability of the transport system functioning as a whole. Reducing energy costs for material transportation is due to the fact that the conveyor belt speed for these sections is selected depending on the optimizing transport costs conditions for a separate section, is different for each section of the conveyor. In the absence of dividing the transport route into sections, the speed of the conveyor belt is the same for each section, which leads to additional electricity costs compared to multi-section conveyor systems. The main control element in conveyor-type transport systems is an asynchronous motor, which uses an electronic motor controller. Most control systems for transport system parameters are based on the use of embedded systems that require the development of high-performance algorithms for optimal control of transport system parameters with minimal use of computing resources. An additional effective method for reducing unit energy costs is the use of energy management methodologies [15]. An overview and analysis of the use of various types of electricity consumption tariffs are given in [16, 17]. Among them, three common tariff classes should be distinguished: tariff with a fixed price for electricity consumption (FPT); tariff with the price for electricity consumption, depending on the time of use (TOU); real-time pricing (RTT) tariffs.

The FPT tariff defines a constant energy price for all 24 hour periods throughout the year. The TOU tariff has different prices depending on the time of use but is the same for the year or season of the year (for example, winter or summer period). A dynamic RTT tariff has a variable price throughout the day depending on expected demand and generator availability. The price changes every hour. This period of time is consigned for making a decision on changing the electricity consumption regime at the enterprise. The paper [18] analyzes the factors that determine the choice of a specific tariff for the use of electricity by the company, presents the structure of the distribution of tariff types by industry. The structure of TOU periods for South Africa, Ukraine and Great Britain is shown in Fig.1-Fig.3. The coefficient $k_t$ characterizes the ratio of the price of electricity at a given tariff to the price of electricity at a fixed tariff. The value $k_t = 1$ in the figures defines the line with the coefficient value for the price of the fixed tariff. Long-distance transportation systems using the TOU tariff can greatly benefit from scheduling the electrical equipment of the transportation system during periods of time with low electricity costs. The mining enterprises may have additional costs if transport systems are not properly scheduled. This is especially true of conveyor lines in South Africa, where the price of electricity is very different during the peak and off-peak periods (Figure 1), [19, 20].

![Figure 1: Eskom–TOU periods and coefficients](image)

The price for electricity consumption depending on the tariff and recommendations for the use of tariffs are given in [21,22,23]. The electricity price for peak periods in the high demand season is almost six times higher than the price for off-peak periods and more than four times higher than the price at a fixed tariff. Due to such a strong difference in prices for peak and off-peak periods, transport systems using Eskom TOU-tariff (Figure 1) require speed control systems taking into account the TOU-tariff price schedule.
The structure of TOU tariffs for different countries is qualitatively similar. Figure 2 shows the structure of the TOU tariff in force in Ukraine [24]. The TOU tariff structure for Great Britain is shown in Figure 3, which demonstrates a comparison of the price coefficients for the TOU-tariff and for the RTT–tariff. The use of the RTT–tariff in speed control systems is of practical interest for conveyor-type transport systems, which to move material during the day consume almost 2 times less electricity than the maximum allowable consumption.

![Figure 2: Ukraine–TOU periods and coefficients](image)

![Figure 3: Great Britain–TOU periods and coefficients](image)

It should be noted that when using the RTT–tariff, the time that can be allocated for calculating the optimal speed control algorithm and changeover of production operations is limited to 30 minutes (Figure 3). This imposes a strong constraint on the choice of conveyor system models used to design conveyor belt speed control systems.

2. Formal problem statement

Reducing the cost of mining by using the TOU–tariff structure in the control speed algorithms of the conveyor belt is an actual problem for mining enterprises. Using the TOU–tariff structure to reduce the total cost of electricity consumption is not always successful in practice. A statistical analysis of the data set on the successful and unsuccessful use of the TOU-tariff structure for 12000 enterprises in 44 industrial sectors is presented in [18]. This clearly demonstrates the fact that the successful use of the TOU–tariff structure requires the development of an optimal speed control algorithm, taking into account the TOU–tariff structure. The continuous operation of the transport system and a significant share of the cost of material transportation in the cost of the extracted material determines the relevance of designing conveyor control algorithms, taking into account the use of the TOU strategy. When
constructing an algorithm for optimal speed control taking into account the structure of TOU–tariffs, the following assumptions were used in this work: a) the conveyor system consumes a constant amount of electricity during the day, which allows using the control quality criterion formulated for a period of 24 hours; b) in accordance with DIN 22101, the primary friction model is used [27, 28]; c) the effects of disturbances associated with instantaneous switching of speed modes are not taken into account. The duration of the modes of acceleration (deceleration) of the belt is negligible compared to the total transportation time.

3. Model of the conveyor line

To construct an optimal control algorithm of the conveyor belt speed, taking into account the structure of TOU–tariffs, the analytical PiKh–model of the conveyor [29] was used. The analytical PiKh–model for calculating the parameters of a separate one has the form:

\[
\begin{align*}
\frac{\partial [\chi_0(t,S)]}{\partial t} + \frac{\partial [\chi_1(t,S)]}{\partial S} &= \delta(S)\lambda_1(t), \\
[\chi_0](0,S) &= \Psi(S), \\
[\chi_1](t,S) &= a(t)[\chi_0](t,S),
\end{align*}
\]

where \([\chi_0](t,S)\leq[\chi_0]_{\text{max}}\); \([\chi_1](t,S)\) are the linear density of the material and the material flow at the moment in time \(t\in[0,T_d]\) at the point of the transport route with the coordinate \(S\in[0,S_d]\); \(\Psi(S)\leq[\chi_0]_{\text{max}}\) is the distribution of material along the transport route at the initial moment of time \(t=0\); \([\chi_0]_{\text{max}}\) is maximum permissible material density; \(T_d\) is the characteristic time of the transportation process; \(a(t)\) is speed of the conveyor belt; \(\lambda_1(t)\) is the material flow at the input of the conveyor section; \(\delta(S)\) is Dirac function; \(H(S)\) is Heaviside function:

\[
H(S) = \begin{cases} 
0, & S < 0, \\
1, & S \geq 0,
\end{cases}
\int_{-\infty}^{\infty} \delta(S) dS = 1
\]

The force required to move the conveyor belt with material distributed along the transport route with density \([\chi_0](t,S)\) at the specific mass of the belt \([\chi_{0C}]\) and the linear loading from the rotating parts \([\chi_{0R}]\) [27] is determined by the expression [28]:

\[
T = C_{\text{f,c}} g_m \int_0^{S_d} \left( 2[\chi_{0R}] + [\chi_{0C}] + [\chi_0](t,S) \right) dS
\]

which takes into account the model of primary resistances. \(f_{\text{c}}\) is coefficient of resistance to belt indentation and rolling resistance of driving rollers [27]; \(g_m = 9.81\ (\text{m/sec}^2);\ C\) is secondary resistance coefficient [27]; \(\eta_{\text{c}}\) is efficiency. The doubled value of the parameters \([\chi_{0C}]\); \([\chi_{0R}]\) takes into account the upper and lower conveyor belt.

To model the transport conveyor let’s use dimensionless parameters [29]:

\[
\tau = \frac{t}{T_d}, \quad \xi = \frac{S}{S_d}, \quad \delta(\xi) = S_d \delta(S),
\]

\[
\psi(\xi) = \frac{\Psi(S)}{[\chi_0]_{\text{max}}}, \quad \gamma_1(\tau) = \lambda_1(t) \frac{T_d}{S_d [\chi_0]_{\text{max}}}, \quad g(t) = a(t) \frac{T_d}{S_d}, \quad \theta_0(\tau,\xi) = \frac{[\chi_0](t,S)}{[\chi_0]_{\text{max}}}
\]

which will make it possible to represent the solution of equation (1)–(3) in the form

\[
\theta_0(\tau, \xi) = \left( H(\xi) - H(\xi - G(\tau)) \right) \frac{\gamma_1(\tau, \xi)}{g(\tau, \xi)} + H(\xi - G(\tau)) \psi(\xi - G(\tau))
\]

\[
\tau_\xi = G^{-1}(G(\tau) - \xi), \quad G(\tau) = \int_0^\tau g(\alpha) d\alpha,
\]

where \( \Delta \tau_\xi(\tau) = \tau - \tau_\xi \) is the transport delay. The transport delay determines the time interval during which the material moves from the input of the conveyor section to the point of the transport route with the coordinate \( \xi \) at the moment in time \( \tau \). The value \( \theta_0(\tau, 1) \) and \( \theta_1(\tau, 1) \) at the output from the conveyor section (6) is determined through the value of the transport delay \( \Delta \tau_1(\tau) = \tau - G^{-1}(G(\tau) - 1) \):

\[
\theta_0(\tau, 1) = \begin{cases} \frac{\gamma_1(\tau - \Delta \tau_1)}{g(\tau - \Delta \tau_1)}, & G(\tau) \geq 1, \tau \geq G^{-1}(1), \\ \psi(1 - G(\tau)), & G(\tau) < 1, \tau < G^{-1}(1), \\ \theta_1(\tau, 1) = \theta_0(\tau, 1) g(\tau). \end{cases}
\]

Dimensionless electrical power \( n_e(\tau) \) (8), required to move the conveyor belt with the material is determined by the expression:

\[
n_e(\tau) = g(\tau) m(\tau), \quad m(\tau) = \int_0^\tau \left( 2\theta_{0R} + 2\theta_{0C} + \theta_0(\tau, \xi) \right) d\xi
\]

The obtained expressions (10), (11) are used to determine the optimal control algorithm of the conveyor belt speed.

4. Optimal belt speed control

4.1. Statement of the control problem

The optimal control problem of the conveyor belt speed is formulated as follows: To determine the modes of switching the conveyor belt speed during a period of time \( \tau = [0, \tau_{24}] \) with the value of the price coefficients of the cost of electricity \( z(\tau) \) (Figure 1, Figure 2, Figure3) with stepwise the speed belt control \( g(\tau) = u(\tau) = (u_1, u_2) \), \( 0 < u_1 < u_2 < \infty \), \( u_j = \text{const} \), which leads to a minimum of functional:

\[
\int_0^{\tau_{24}} z(\tau) u(\tau) m(\tau) d\tau \rightarrow \min,
\]

with differential connections

\[
\frac{d m(\tau)}{d\tau} = \gamma_1(\tau) - \theta_1(1, \tau) = \gamma_1(\tau) - \gamma_1(\tau - \Delta \tau_1) \frac{u(\tau)}{u(\tau - \Delta \tau_1)},
\]
\[ m(0) = 2(\theta_{0R} + \theta_{0C}) + \int_0^1 \psi(\xi) d\xi, \]

and limitation on the total amount of energy consumed per day \((\tau = [0, \tau_{24}])\)

\[ \int_0^{\tau_{24}} u(\tau)m(\tau) d\tau = b = \text{const}, \quad u(\tau) = (u_1, u_2). \] (14)

The equation (13) is written taking into account dependence (10) for the material output flow. The choice of the stepped control mode is due to its prevalence in the control of transport systems [30–33]. The equation (14) can be replaced by the differential equation

\[ \frac{dx_b}{d\tau} = u(\tau)m(\tau), \quad x_b(0) = 0, \quad x_b(\tau_{24}) = b. \] (15)

The Hamilton function and the conjugate system of equations for the considered problem have the form:

\[ H = (\psi_b - z(\tau))u(\tau)m(\tau) + \psi_m \left( \gamma_1(\tau) - \gamma_1(\tau - \Delta \tau_1) \frac{u(\tau)}{u(\tau - \Delta \tau_1)} \right), \] (16)

\[ \frac{d\psi_b}{d\tau} = -\frac{\partial H}{\partial x_b} = 0, \] (17)

\[ \frac{d\psi_m}{d\tau} = (z(\tau) - \psi_b)u(\tau), \quad \psi_m(\tau_{24}) = 0. \] (18)

From equation (18) follows \( \psi_b = C_b = \text{const} \). For a two-stage control mode \( u(\tau) = (u_1, u_2) \) [33] \( 0 < u_1 < u_2 < \infty \) the optimal belt speed corresponds to the maximum value of the Hamilton function (16). Switching points of control modes are determined by solving equations (13), (15), (17) and (18).

### 4.2. Selecting the range of the parameters

For quantitative calculations, let us take the values of the characteristic time and the characteristic length as values \( T_d = 1 \text{ (hour)} \), \( S_d = 20.5 \text{ (km)} \). The choice of the value of the characteristic time \( T_d \) allows you to conveniently display the change in parameters during the day \( \tau \in [0; 24] \), and the choice of the value of the characteristic length corresponds to the consideration of extended transport conveyors (Sasol–Shondoni Overland [18] (20.5 km single flight overland conveyor with multiple horizontal curves)). Then the control modes \( u(\tau) = (u_1, u_2) \), at the speed \( a_1 = 1 \text{ (meter/second)} \), \( a_1 = 5 \text{ (meter/second)} \) will correspond to dimensionless values of the belt speed:

\[ u_1 = 0.176, \quad u_2 = 0.878. \] (19)

For values of the specific mass of the belt \( \theta_{0C} \) and the linear load from rotating parts \( \theta_{0R} \), \( (\theta_{0R} + \theta_{0C}) = 0.2 \text{ mass } m(\tau) \) (11), (13) will vary in the range

\[ 0.4 = m_1 \leq m(\tau) \leq 1.4 = m_2, \] (20)
Consider the case of the initial distribution of material along the conveyor section
\[
\psi(\xi) = 0.5 + 0.5 \sin\left(2\pi\xi\right),
\]  
(21)
for which the mass \( m(0) \) at the initial moment of time determined by the expression
\[
m(0) = 2(\theta_0R + \theta_0C) + \int_0^1 \psi(\xi) d\xi = 0.4 + \int_0^1 (0.5 + 0.5 \sin(2\pi\xi)) d\xi = 0.9.
\]

The values \( u_1, u_2 \) (19) and inequality (20) set the range of variation of the value \( x_b(24) = b \)
\[
1.69 = 24u_1m_1 = b_1 \leq b \leq b_2 = 24u_2m_2 = 29.50.
\]
(22)

The condition for the inadmissibility of exceeding the maximum permissible specific load from the incoming material on the conveyor belt can be written as
\[
\frac{\lambda_1(t)}{a(t)} \leq [\chi]_{\max}.
\]
(23)

This condition imposes a limitation on the maximum value of the input flow at the current belt speed \( a(t) \)
\[
\frac{\lambda_1(t)}{[\chi]_{\max}} \leq a(t).
\]
(24)

If inequality (24) is satisfied, there will be no excess of the maximum permissible belt load. Using dimensionless notation (7), condition (24) can be written as follows:
\[
\gamma_1(\tau) \leq \varsigma(\tau) \leq u_2 = 0.878,
\]
(25)
\[
\gamma_1(\tau) = \frac{\lambda_1(t)}{[\chi]_{\max}} \frac{T_d}{S_d} \leq u(\tau), \quad u(\tau) = (u_1, u_2).
\]
(26)

The condition (25) allows us to carry out an important conclusion: if for any moment of time the inequality
\[
u_1 \leq \gamma_1(\tau) \leq u_2,
\]
(27)
then the transport section will operate only in speed mode \( u(\tau) = u_2 \), without switching speed modes.

The equation (18) provides a boundary range for \( \psi_b \)
\[
z_{\min} \leq \psi \leq z_{\max}, \quad z_{\min} \leq z(\tau) \leq z_{\max}.
\]
(28)

Conditions (19)–(21), (25)–(28) determine the existence of a solution to the system of equations (13), (15)–(18).
5. Analysis of results

Let us consider the construction of a schedule for switching the belt speed modes for the tariff coefficients Ukraine – TOU periods (Figure 2), when the intensity of the material input flow is constant \( \gamma_1(\tau) = 0.15 \) with daily energy consumption \( h = 6.5 \) (14), (22). The selected value of the intensity of the input flow \( \gamma_1(\tau) \) in accordance with inequality (26) allows the transport system to operate in a two-speed mode \( u(\tau) = (0.176, 0.878) \) (19). An increase in belt speed \( g(\tau) \) leads to an increase in the power consumption of the transport system \( n_e(\tau) \) (11). On the other hand, an increase in the belt speed leads to a decrease in the linear density of the material \( \theta_0(\tau, 0) \), at the input the section (8) and, possibly, to a decrease in the mass \( m(\tau) \) (11). The connection between the belt speed and the linear density of the material along the transport route (and, as a consequence, the connection between the belt speed and mass) is a characteristic feature of the transport systems functioning. The next feature is that the conveyor belt is an accumulator of the material incoming the section input. Constraint (23) does not allow both the excess of the specific density \( \theta_0(\tau, \xi) \) and the overflow of the accumulator. The presence of an upper and a lower limit for the material amount (20) in the accumulator for a sufficiently long period of operation of the transport system determines the ratio between the average intensity of the incoming flow and the average power consumption of the transport system for the period under consideration. An increase in the value of the intensity \( \gamma_1(\tau) \) of the incoming material over a sufficiently long time period leads to an increase in the power consumption of the transport system. These features significantly complicate the synthesis of the optimal control algorithms of the flow parameters of the transport system. In order to simplify the qualitative analysis in this paper, a constant value for the intensity of the incoming flow is taken \( \gamma_1(\tau) \).

The solution of equations (15), (18) for the parameters \( x_b(\tau) \) and \( \psi_m(\tau) \) are presented in Figure 4. The parameter \( x_b(\tau) \) determines the amount of energy used by the transport system during the period of the time \([0, \tau]\), satisfies the conditions (15) for the initial and final moments of the time. The quasi-linear dependence of energy consumption \( x_b(\tau) \) as a function of time indicates a quasi-stationary value of the power consumption of the transport system with a variable value of the transport delay \( \Delta \tau_1(\tau) \). Изломы функции \( \Delta \tau_1(\tau) \) соответствуют точкам переключения скорости ленты \( u(\tau) = (u_1, u_2) \). The quasi-stationary change in the amount of energy consumed by the transport system is explained by the considered feature, which is characteristic of a conveyor-type transport system (8).
The belt speed regulation modes $u(t) = (u_1, u_2)$ for tariff coefficient values $k_\tau(t) = (0.35, 1.8)$ are shown in Figure 5. The value of the tariff coefficient $k_\tau = 1.8$ corresponds to the minimum belt speed $u_1$. There are several switching points of the speed mode for the tariff coefficient $k_\tau = 0.35$. The belt speed switching modes on the interval $\tau \in [0.01.0]$ are related to the type of function (21), which determines the initial distribution of material along the route. The points of the belt speed switching for the interval $\tau \in [4.0, 5.0]$ are repeated points of the belt speed switching for the interval $\tau \in [0.0, 1.0]$ with a shift along the time scale equal to the value of the transport delay $\Delta \tau = 3.711$ (Figure 4). With a uniform distribution of material at the initial moment of time

$$\psi(\xi) = 0.5 + 0.5 \sin(2\pi \xi). \quad (29)$$

there are no the speed modes switching points in the interval $\tau \in [0.0, 5.0]$ (Figure 6), which leads to a decrease in the value of the power consumption of the transport system.

The density of the material $\theta_0(t, 0)$ at the input to the conveyor section is shown in Figure 7 and corresponds to the condition

$$\psi(\xi) = \theta_0(12, 1) = 0.8523, \quad (30)$$
The speed modes switching points correspond to the density jumps of the material \( \theta_0(\tau, 0) \) at the input to the section. The mass \( m(\tau) \) of the belt with the material correlates with the transport delay function \( \Delta T_1(\tau) \). For times when the transport delay is high, the mass of the belt with the load being moved also has high values (Figure 4, Figure 7). An increase in the transport delay leads to an increase in the mass of material on the belt.

6. Conclusion

The article deals with the problem of synthesis of the optimal belt speed control algorithms for a distributed conveyor-type transport system. A method for constructing an algorithm based on the Pontryagin maximum principle with the use of an analytical PiKh–model for a conveyor section is proposed. The use of the Pontryagin maximum principle in conjunction with the analytical PiKh–model allows providing acceptable accuracy for calculating the switching points of speed modes. The software for the synthesis of algorithms for optimal discrete control of the belt speed has been developed and used for analysing the belt conveyor flow parameters. The influence of the initial distribution of the material on the choice of modes for controlling the belt speed is demonstrated. The relationship between the transport delay for a conveyor system and the mass of material that moves with the belt is presented.

The originality and novelty of the obtained results consist in improving the analytical model by including additional parameters that characterize the state of the transport system at an arbitrary moment in time (the total material mass and the power consumption of the transport system). This made it possible to form a new criterion for the quality of control based on Time-Of-Use tariffs. When synthesizing the optimal control algorithm, additional differential connections were added that determine the change in the mass of the material being moved and the power consumption of the transport system required to move the material.

The prospect of further research is the determination of algorithms for the belt speed optimal control for the time interval when the influence of the initial conditions of material distribution becomes insignificant.

7. References


