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# How Do Volatility and Return Series Interact?

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Literature in the last forty years is swamped with a myriad of studies on the relationship between asset returns and volatility. Although the correlation between these two variables is already well-documented, our knowledge regarding their causal relationship remains limited. This study formally investigates the true dynamic relationship between the VIX implied volatility index and the S&P500 returns. Innovation accounting results indicate strong influence of S&P500 returns on VIX but not vice versa. Plus, unexpected S&P500 losses tend to increase VIX temporarily, while return shocks in general have permanent impact on VIX in the adverse direction of the shock.

*Key Words:* Volatility feedback hypothesis; Leverage effect; Endogeneity.

*JEL Classification Numbers:* C01, G10, G14.

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# 1. INTRODUCTION

Volatility should affect average stock returns positively according to one of the basic premises of the theory of finance, i.e. markets have to compensate high-risk-taking rational investors with higher returns. In an efficient marketplace, this kind of a relationship that is running from volatility to return should occur without delay. In order to understand the rationale behind this immediacy, let us assume that investors' volatility expectations for time $t_1$ go up at time $t_0$. Under normal circumstances and the assumption of rational expectations, investors should correlate higher expected future volatility with higher future required rates of return. Since the higher required returns in future necessitates an adjustment today, stock prices go down immediately at time $t_0$ without delay. By the way, this precise line of thought is simply known as the 'volatility feedback hypothesis' in literature and it will be revisited in more detail in section three.

In line with the volatility feedback hypothesis, French, Schwert and Stambaugh (1987) report positive relationship between the current levels of the expected excess returns on a stock portfolio and the predicted future volatilities of stock returns. According to their findings, only the unexpected stock returns are negatively related to the unexpected change in volatility. In sum, the study of French et. al (1987) claims a stable and positive relationship between the current returns and current predicted levels of ex-ante volatility. In fact, many of the earlier papers on this subject have similarly attempted to relate the current returns and some form of current volatility ranging from historical volatilities to realized volatilities and from conditional volatilities to implied volatilities. Unfortunately, there is no consensus in the empirical studies on which volatility measure to use (Sung and Wu, 2018). Banerjee, Doran and Peterson (2006) expand on prior studies by examining the relationship between future returns (not current returns) and current levels of implied volatility. They study the relationship between the stock market returns and the VIX volatility index - an implied model-free measure of market expectations of stock market volatility for the next 30 days derived from S&P stock index options. They find that VIX can be used as a strong predictor for future returns. Needless to say, this finding stands as a clear violation of the efficient market hypothesis.

Therefore, the first aim of this paper is going to be checking these alleged relationships in the literature between current and future levels of expected volatility and future returns with a larger dataset. A model establishing both contemporaneous and lagged links between the variables would therefore be a good fit for this purpose and that is why a structural vector autoregression model is going to be employed in this paper, among other reasons explained in the following parts.

Secondly, VIX is going to be employed as the volatility measure in this paper like in many other papers in the literature since VIX, as a model-free implied volatility measure, should be perfectly reflecting the assumptions of the market participants about volatility by definition for

VIX is estimated from the weighted market prices of the as many 'reliable' out-of-the-money put and call S&P500 index options as possible. This reference to 'reliability' is particularly important since the Chicago Board Options Exchange (CBOE) does not include all the options in the calculation of VIX. Options with zero bid prices are automatically excluded as it should be the case. Plus, CBOE excludes all the call options with a higher call strike price than the call strike prices of two consecutive zero-bid-price options, too. Similarly, all the put options with a lower put strike price than the put prices of two consecutive zero-bid-price options are also excluded. This cut-off mechanism is also highly reasonable for excluding unrealistic prices from the calculation of VIX. In sum, VIX should really reflect the true market assumption for the expected volatility without significant distortions.

As explained in the opening paragraph, the volatility feedback mechanism assumes that future volatility expectations affect current returns. The 'leverage effect hypothesis' on the other hand, a well-known hypothesis for explaining the volatility asymmetry phenomenon, argues for the possibility of a reverse causality between volatility and return series. Both the leverage effect and the volatility asymmetry phenomenon are going to be discussed in detail in section three. For the time being, what matters is the claim of the leverage effect, i.e. returns might be affecting volatility as well. The idea behind this claim is simple: Stock prices decline, financial leverage increases, risk and volatility go up. Since volatility feedback and leverage effect hypotheses propose two opposing possibilities for the direction of the causality between returns and volatility, the second aim of this paper is going to be formally testing the relevant direction of the causal relationship between return and volatility. As aforesaid, if it exists, the relationship can either be from market returns to VIX or from VIX to market returns, or in both ways.

The main contribution of such a study would be twofold. Initially, volatility is a central concept both in the theory and practice of asset pricing, asset allocation, and risk management. Determining the true dynamics of the relationship between the current market returns and future volatility, if there exists any such relationship, could be helpful to improve volatility forecasts.

On the other hand, if the opposite relationship holds, i.e. if future market returns are systematically related to the present volatility, it would be a counter-argument to the market efficiency concept.

This paper is organized as follows. Section two introduces VIX implied volatility index and provides a brief explanation about the volatility estimation techniques. Paper's dataset is described in section three. Section four discusses methodology issues and proposes an appropriate VAR specification, which is in conformity with the well-established findings of the volatility asymmetry literature. Estimation results are presented in section five. Concluding remarks are made in section six.

## 2. VOLATILITY ESTIMATION AND VIX

There exist two popular approaches for volatility estimation. The first approach is the time series modeling of volatility, based on historical asset prices. This approach encompasses GARCH and stochastic volatility (SV) models, and their variants. Once the parameters of the time series model are estimated, future volatility can be forecasted.

The second approach involves the calculation of implied volatilities. In option pricing framework, the only input that cannot be directly observed is the future volatility. For example, let us take the Black and Scholes pricing model. Parameters and features such as the call/put feature, time-to-maturity, and strike price are clearly written down on the option contract. Spot price of the underlying asset is revealed at time $t_0$. The risk-free interest rate and dividend payments are easy to agree upon. Volatility, however, remains the only input in the formula that needs to be estimated. In a market with actively traded options, one can solve for the volatility that equates the observed option price with the calculated price from a given option pricing model. This sort of volatility derived from backward solution of an option pricing model is known as the implied volatility and it reflects the expectations (or the estimations) of the market participants for the unobserved future volatility. The problem with this approach is the possibility of relying on a misspecified pricing model. In response to this problem, one can exploit the model-free implied volatility concept as exemplified by Britten-Jones and Neuberger (2000), whose information content capacity is well discussed by Jiang and Tian (2005). VIX volatility index is a widely used model-free implied volatility measure. VIX is calculated and announced in real-time by Chicago Board Options Exchange (CBOE). It is a weighted average of the prices for a range of out-of-the-money put and call options in the near- and next-term on the S&P500 index, i.e. usually in the first and second SPX contract months (SPX: the symbol for S&P500 index options) of the S&P500 index over the next 30 days. However, whenever 8 days are left to expiration, VIX rolls to the second and third SPX contract months to minimize pricing anomalies that might occur close to expiration.

## 3. DATASET

Dataset in this paper contains observations on the daily changes in the closing values of the S&P500 and the VIX indices. In order to control for the asymmetric volatility phenomenon, the days that ended up with a negative change in the S&P500 market and the days with high rates of change in the VIX index are recorded separately. The reason for this sort of classification of the data, as aforesaid, is the well-known volatility asymmetry phenomenon, whose first empirical observation dates back to 1976.

Volatility asymmetry simply refers to the common empirical finding in the existing literature that negative returns tend to have greater effect on volatility than positive returns do. One of the staple explanations for the volatility asymmetry is the leverage effect hypothesis. Leverage effect was first proposed by Black (1976) and Christie (1982) when both researchers reported similar empirical findings, i.e. volatility increases when stock prices fall. The idea they put forward was simple: The falling value of a stock would increase the financial leverage, making the stock riskier and thus more volatile. The leverage effect became a cornerstone explanation for the volatility asymmetry phenomenon as increasing number of empirical studies began to report that it was significant only when the stock values were declining but not so significant in the opposite direction. “Is the ‘Leverage Effect’ a Leverage Effect”, a study by Figlewski and Wang (2001), is an influential paper, among some others, documenting this asymmetric response of volatility to negative and positive returns. As the title of their paper suggests, Figlewski and Wang also question the validity of the leverage effect explanation. They do not reject the existence of leverage effect but they argue against the existing literature by saying that the leverage effect might be not so powerful as generally thought. In fact, they are not the first researchers who questioned the strength of the leverage effect. French et. al (1987) and Campbell and Hentschel (1992) had proposed a completely different explanation for the volatility asymmetry, known as the volatility feedback hypothesis, long time ago. According to the proponents of volatility feedback hypothesis, anticipated increases in future volatility would foster the required rate of return, thus causing an immediate decline in stock price to allow for higher future returns. Clearly, the direction of causal relationship runs from volatility to returns according to the volatility feedback hypothesis, and from returns to volatility according to the leverage effect hypothesis. It is obvious that one needs to consider the possibility of endogeneity between these two variables.

Time coverage of this paper’s dataset extends from 03 January 1990 to 01 July 2020. CBOE introduced VIX index in 1993 and this first model was dependent on Black-Scholes pricing model. In 2003, CBOE - together with Goldman Sachs - updated the index and switched to the current model-free calculation methodology for VIX. CBOE also created a historical record for this new model-free VIX Index dating back to 1990. The following table presents the basic descriptive statistics about the two main variables in this study.

**TABLE 1.**

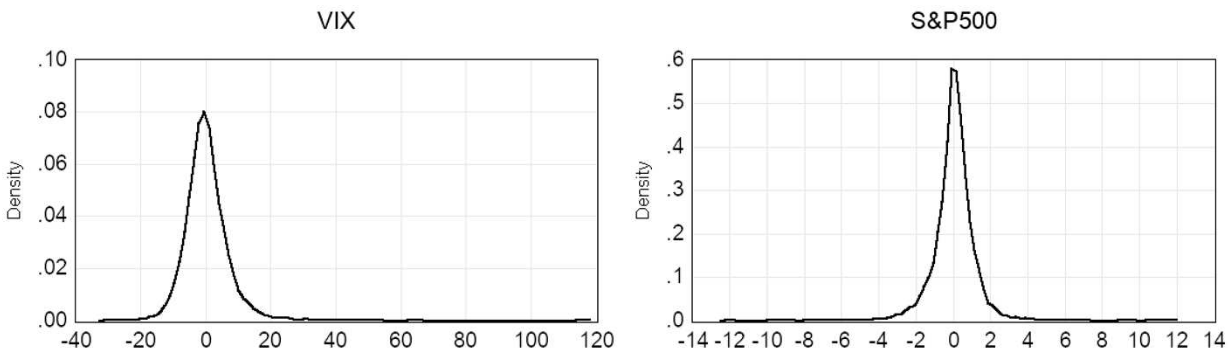
Summary Statistics of Variables

| Variable | Obs. | Median | Mean | Max. | Min. | Std. Deviation | Coeff. of Variation |
|---|---|---|---|---|---|---|---|
| S&P500 Returns | 7684 | 0.055 | 0.035 | 11.58 | -11.98 | 1.149 | 0.030 |
| VIX Changes | 7684 | -0.343 | 0.233 | 115.60 | -29.57 | 6.944 | 0.034 |

As it can be seen from the table, both variables have comparable coefficients of variation. Nevertheless, the non-parametric distributions of the series indicate a wider dispersion of VIX changes. Furthermore, although the median value of S&P500 returns are slightly greater than the mean, signaling some degree of leftward skewness, S&P500 returns follow a more balanced distribution around the central value of zero as would be expected from a return series of that many observations. Any centralization around a different value other than zero would simply mean the existence of systematic failures in the expectation processes (rational or adaptive) of investors and would be major point of concern in a study like that. VIX series, on the other hand, has more pronounced skewness profile towards right. Nevertheless, the big mass of its unimodal distribution also seems to be centralized around zero. That said, both series are stationary as stationarity test results indicate in the following section.

**FIG. 1.**

Non-Parametric Distributions of VIX Changes and S&P500 Returns




# 4. MODEL

Since the series are introduced, it is time to build the proper model for analyzing the true dynamics between them. As mentioned before, this paper questions whether the time path of volatility is affected by market returns and/or vice versa. As well-known, an appropriate way of modeling this kind of possible interdependencies between multiple time series is to use vector autoregressions (VAR).

A VAR model might essentially be specified in two forms, namely the reduced and structural forms. Although it is possible to include recursive VAR modeling as a third option, it is essentially a hybrid form in between these two. Structural form is a description of the relationship between the variables as they are in the real economy, hence the model of interest in a paper like this. However, obtaining good estimators of the parameters in the structural form is usually a difficult task. In a reduced form VAR, each variable is expressed as a linear function of its own past values, past values of all other variables being considered, and a serially uncorrelated error term. Each equation in the system then can be estimated by simple linear estimation techniques.

In this study, we will begin with a reduced form VAR specification involving the following volatility and return equations.

$$VIX_t = \alpha_1 + \sum_{i=1}^{n} \beta_{1i} VIX_{t-i} + \sum_{j=1}^{n} \gamma_{1j} r_{t-j} + \sum_{k=1}^{n} \delta_{1k} r_{t-k}^{(-)} + e_{1t}$$

$$r_t = \alpha_2 + \sum_{i=1}^{n} \beta_{2i} VIX_{t-i} + \sum_{j=1}^{n} \gamma_{2j} r_{t-j} + \sum_{k=1}^{n} \delta_{2k} VIX_{t-k}^{(high)} + e_{2t}$$

$VIX$ and $r$ in these notations stand for the daily percentage changes in the values of the VIX volatility index and S&P500 index, respectively. $VIX^{high}$ and $r^{(-)}$ are interaction terms, where $VIX$ and S&P500 series are multiplied with dummies for high levels of VIX and negative returns in accordance with the well-established findings of the literature on volatility asymmetry. In this study, a high rate of change in the $VIX$ index is considered to be any observation above the median rate of change. Since the full sample includes 7684 observations, there are 3842 observations in the $VIX^{high}$ series and all of them are non-negative observations. Negative returns are simply the negative daily returns in the S&P500 index and thanks to almost symmetric distribution of S&P500 returns around zero, as seen in the non-parametric distribution chart of returns in Fig. 1., there are 3562 observations on the negative side.

The reason for including an interaction dummy for negative returns in the model is the volatility asymmetry phenomenon recorded by the existing empirical literature. It was already stated in section three that the relationship between volatility and negative returns seems to be stronger than the relationship between volatility and positive returns. The reason of including higher rates of change in the VIX index as an interaction dummy comes from the earlier works of Giot (2002, 2003), which consider the possibility of a stable relationship between the current volatility and future returns. He finds that very high levels of VIX signal an imminent increase in stock indices for a short term. Besides, many professional market participants seem to believe that high levels of implied volatility signal attractive entry points for opening long positions. According to Giot, this belief must be based on the observation that high levels of volatility tend to be accompanied by a period of financial turmoil where investors over-react and sell their financial assets indiscriminately to raise cash or limit losses. The VIX implied volatility index of Chicago Board Options Exchange (CBOE) is frequently cited in leading financial news outlets such as Baron's and the Wall Street Journal. For example, Giot (2002) quotes the following line from an article in the July 29, 2002 issue of Baron's: "a big VIX spike indicates the kind of extreme fear contrarians associate with market bottoms". Similar views are posted on the popular website *seekingalpha.com* on March 05, 2009 with the following self- revealing title: “Market will bottom when VIX finds a new top.” Parallel examples are numerous in non-academic writing and easily accessible on the World Wide Web.

If we go back to the reduced form VAR model above, this system might be our starting point but it falls short of serving our purpose completely. This system will be helpful in recovering a structural system, i.e. a structural VAR system, which takes the contemporaneous links between the variables into consideration. The error terms in such a structural model would be uncorrelated with each other if the model is correctly specified. Although structural models are not suitable for estimation and forecasting since contemporary values of endogenous variables are used as explanatory variables in the model, and a reduced form VAR is strictly advised for estimation and forecasting purposes, structural models are preferred for establishing the true dynamic relationships between variables and that is why a structural model is more suitable for our purpose in this paper.

It is possible to reduce any structural model to a corresponding reduced form model using simple matrix algebra. The real problem is the opposite; i.e. recovering an unknown structural model from a reduced form model like the one presented above. That problem is known as the 'identification problem' and results from the fact that the number of parameters estimated by a reduced form model is always less than the number of parameters in its structural form. The remedy is to impose certain restrictions. There are two ways to impose them on a VAR model. Restrictions can be placed on contemporaneous effects or on the long-run impacts.

In this study, restrictions will be placed on the long-run effects for two reasons. First, the paper investigates the possible short-run dynamics of the relationship between the changes in VIX and the returns. Imposing restrictions on the short- run dynamics is therefore unwanted. Second, there exists no clue at hand to decide on the ordering of the short-run effects. On the other hand, placing a long-run restriction seems reasonable from a pure economic point of view as well since there is supporting evidence in the literature for the long-run relationship between volatility and stock prices. The persistence of stock market volatility was an active area of research in the 1980s – changing risk premium hypothesis – in an attempt to explain the findings at the time that stock market volatility in general cannot be explained by movements in the rational expectation of future dividends and interest rates. In Poterba and Summers (1986), it was shown that shocks to volatility decay rapidly and therefore can affect market index level only for a limited time. To be clearer, changing risk premium hypothesis was an attempt for explaining the relationship between the volatility and the market returns over very long-time horizons. For that reason, it should not be confused with the question in this study, which is interested in explaining the relationship between the volatility and the market returns for much shorter time period. What Poterba and Lawrence (1986) reported and is useful for us is that volatility increases cannot explain prospective market fluctuations over the long run. However, they did not discard the possibility for shorter time periods.

A well-known methodology for restricting long-run impacts was developed by Blanchard and Quah. Blanchard-Quah methodology (BQ decomposition) is a useful technique when the researcher assumes that one of the variables has no long-run effect on the others. As explained above, VIX is a measure of volatility that shows the magnitude of expected changes in the S&P500 index over the next 30-day period and should not be expected to have a lasting influence on market returns in a deep market like S&P500.

Poterba and Summers' results are supportive of no-long-run-effect on market index level, too. Therefore, VIX is assumed to have no long-run effect on market index level. BQ decomposition requires working with stationary series. The unit root in both variables is checked and the variables are found to be stationary. The results from unit root tests are reported in Table 2 but before moving on to the table, stationarity issue within BQ context needs further clarification. The assumption in this paper with regard to the long-run restriction is that there exists no long-run effect on market index level rather than market return, although the model in this paper uses market returns. Imposing the restriction on the index level itself, rather than its change is in line with the changing risk premium literature and it serves a technical reason at the same time, i.e. BQ methodology requires to have at least one non-stationary variable since stationary variables would not have permanent effects. However, in order to use BQ method, variables should be made stationary. The usual application in the literature is to get the differences of the non-stationary series and then use these differenced stationary series in the estimations. Estimation results, such as impulse responses, then show the responses of these differenced stationary series to shocks. In this study, the variables are the daily changes in the S&P500 and VIX indexes which are stationary as aforesaid and presented in Table 2 along with jumps in the VIX changes and S&P500 losses for controls. For the results from a structural VAR model recovered with BQ method to be meaningful, at least one of the level series of the variables had to be non-stationary. The level of S&P500 index is non-stationary with trend, with trend and intercept and without trend and intercept terms. Level of VIX is non- stationary without time trend and intercept terms.

**TABLE 2.**

Augmented Dickey Fuller Unit Root Test Results

| | No Exogenous Variable | Intercept | Intercept & Trend |
|---|---|---|---|
| $VIX$ | -67.57<br>(0.0001) | -67.69<br>(0.0001) | -67.70<br>(0.0000) |
| $r$ | -95.67<br>(0.0001) | -95.76<br>(0.0001) | -95.75<br>(0.0001) |
| $VIX^{high}$ | -13.15<br>(0.0000) | -22.94<br>(0.0000) | -23.47<br>(0.0000) |
| $r^{(-)}$ | -19.84<br>(0.0000) | -27.19<br>(0.0000) | -27.23<br>(0.0000) |

$H_0$: Unit root exists.
MacKinnon one-sided p-values in parantheses.
Optimal lag lengths in ADF test equation are determined by SIC. 1 lag for VIX, no lag for S&P500, 9 lags for high VIX jumps and 4 lags for negative S&P500 returns.

Another important issue that needs careful attention in this study is the possibility of structural breaks in the dataset. As aforementioned, this paper's dataset covers daily observations for three decades, from 03 January 1990 to 01 July 2020. The relationship between the S&P500 returns and VIX might have been strongly influenced by many powerful forces during a long data period of that extent. That is why, before covering the structural model, the reduced form model, which is more suitable for estimation and forecasting purposes, is used to identify the possible structural break periods in the dataset. This is done in an iterative way using the Chow break-point and Chow sample-split methodologies. In order to check for the existence of possible breaks, first the reduced form model with lags up to seven periods is estimated. Optimal lag length was decided by Schwarz Information Criteria.

Then the following tests are conducted on the entire system rather than the individual equations. Test statistics and p-values are established according to Candelon and Lütkepohl (2001). To be more precise, the break-point and sample split test statistics are specified as follows.

$$\lambda_{BP} = (T_1 + T_2)logdet\left(SSR_{1,2}\right) - T_1 logdet(SSR_1) - T_2 logdet(SSR_2) \approx \chi^2(k)$$

$$\lambda_{SS} = (T_1 + T_2)[logdet\left(SSR_{1,2}\right) - logdet[(T_1 + T_2)^{-1}(T_1 SSR_1 - T_2 SSR_2)]] \approx \chi^2(k^-)$$

In these notations, $T$ stands for the full sample of observations, i.e. $T = 7684$ in this study. $T_1$ is a subset of the first $T_1$ observations, while $T_2$ is a subset of the last $T_2$ observations. Assuming that a structural break occurred at $T_B$, $T_1 < T_B$ and $T_2 \leq T - T_B$. This is how we split the full sample into two as the observations before the break and after the break. Let us assume that we estimated the system first on the full sample of observations (i.e. $T$) and then on $T_1$ and $T_2$ and the resulting residuals are $\hat{u}_t$, $\hat{u}_t^{(1)}$, and $\hat{u}_t^{(2)}$, respectively. Then, $SSR_1 = T_1^{-1} \sum_{t=1}^{T_1} \hat{u}_t^{(1)} \hat{u}_t^{(1)\prime}$, $SSR_2 = T_2^{-1} \sum_{t=T-T_2+1}^{T} \hat{u}_t^{(2)} \hat{u}_t^{(2)\prime}$, and $SSR_{1,2} = (T_1 + T_2)^{-1}(\sum_{t=1}^{T_1} \hat{u}_t^{(1)} \hat{u}_t^{(1)'} + \sum_{t=T-T_2+1}^{T} \hat{u}_t^{(2)} \hat{u}_t^{(2)'})$. In these notations, $k$ refers to the difference between the total number of the estimated parameters in the first and the last subperiods and the number of parameters estimated in the full sample model. $k^-$ equals $k$ minus the parameters in the white noise covariance matrix (i.e. only counting the coeeficients estimated). Although these test statistics practically follow a $\chi^2$ distribution, Candelon and Lütkepohl (2001) recommend bootstrapping for the sake of more precise p-values.

Searching over the data points by increasing the number of observations included in $T_1$ and $T_2$ one by one, and conducting 1000 bootstrap replications in order to set the bootstrapped p-values, we find the following 19 observations as the potential breakpoints in our dataset of 7684 observations.

**TABLE 3.**

Potential Breakpoints in the Dataset

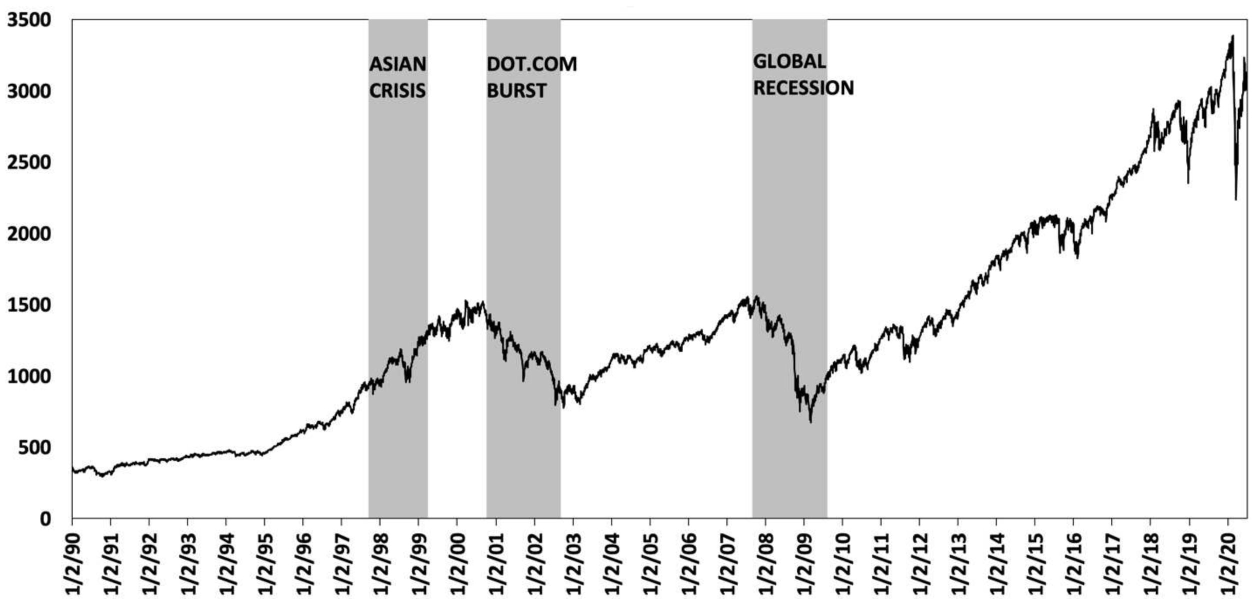
| Potential Breakpoint Dates | Chow Breakpoint Test | | | | Chow Sample Split Test | | | |
|---|---|---|---|---|---|---|---|---|
| | Test stat. | Bootstrap p-val | $\chi^2$ p-val | df | Test stat. | Bootstrap p-val. | $\chi^2$ p-val | df |
| 02/28/1996 | 1660.96 | 0.0000 | 0.0000 | 56 | 271.54 | 0.0000 | 0.0000 | 46 |
| 12/04/1996 | 1905.51 | 0.0000 | 0.0000 | 56 | 299.55 | 0.0000 | 0.0000 | 46 |
| **09/10/1997** | **1612.86** | **0.0000** | **0.0000** | **56** | **271.71** | **0.0000** | **0.0000** | **46** |
| **06/17/1998** | **1384.89** | **0.0000** | **0.0000** | **56** | **248.12** | **0.0000** | **0.0000** | **46** |
| **03/24/1999** | **1248.70** | **0.0000** | **0.0000** | **56** | **218.96** | **0.0000** | **0.0000** | **46** |
| 12/29/1999 | 1087.16 | 0.0000 | 0.0000 | 56 | 222.82 | 0.0000 | 0.0000 | 46 |
| **10/04/2000** | **1062.35** | **0.0000** | **0.0000** | **56** | **224.12** | **0.0000** | **0.0000** | **46** |
| **07/11/2001** | **1022.25** | **0.0000** | **0.0000** | **56** | **236.02** | **0.0000** | **0.0000** | **46** |
| **04/17/2002** | **952.749** | **0.0000** | **0.0000** | **56** | **239.53** | **0.0000** | **0.0000** | **46** |
| **09/04/2002** | **959.91** | **0.0000** | **0.0000** | **56** | **271.34** | **0.0000** | **0.0000** | **46** |
| 10/29/2003 | 1281.89 | 0.0000 | 0.0000 | 56 | 344.82 | 0.0000 | 0.0000 | 46 |
| 08/04/2004 | 1534.05 | 0.0000 | 0.0000 | 56 | 368.99 | 0.0000 | 0.0000 | 46 |
| 05/11/2005 | 1736.04 | 0.0000 | 0.0000 | 56 | 374.38 | 0.0000 | 0.0000 | 46 |
| 02/15/2006 | 1881.92 | 0.0000 | 0.0000 | 56 | 383.79 | 0.0000 | 0.0000 | 46 |
| 11/22/2006 | 1928.10 | 0.0000 | 0.0000 | 56 | 376.00 | 0.0000 | 0.0000 | 46 |
| **08/29/2007** | **1843.13** | **0.0000** | **0.0000** | **56** | **341.01** | **0.0000** | **0.0000** | **46** |
| **06/04/2008** | **1181.40** | **0.0000** | **0.0000** | **56** | **352.05** | **0.0000** | **0.0000** | **46** |
| **03/11/2009** | **1680.49** | **0.0000** | **0.0000** | **56** | **484.51** | **0.0000** | **0.0000** | **46** |
| **12/16/2009** | **1622.77** | **0.0000** | **0.0000** | **56** | **488.80** | **0.0000** | **0.0000** | **46** |

Boldened dates are selected as the periods of major economic influences on S&P500 for the reasons explained below. The first group of boldened dates are associated with the Asian Crisis, the second group with Dot.Com Burst, and the third group with the Global Recession.

Although the dataset in this paper covers three decades from 1990 to 2020, Table 3 suggests us to focus only on the economic events from 1996 to 2009. The most significant economic events that might have influenced the S&P500 market within these years are the Asian Crisis, Dot.Com Burst, and the Global Recession. According to the FederalReserveHistory.org, a webpage administered by the St. Louis Fed, Asian Crisis started in July 1997 and terminated at the end of 2018, leading to spill-over shocks shortly in Latin America and Eastern Europe. That is why I tend to take 09/10/1997 in Table 3 as the starting point of a potential break for S&P500 and set the ending time of that break at 03/24/1999. Secondly, I relate 10/04/2000 in the table to the start of the Dot.Com Burst. As we know, Nasdaq Composite reached its peak at March 2000 after rising by 400% from 1995 to March 2000. However, the gains of these five years were completely lost in between March 2000 and October 2002. That is why I choose 09/04/2002 from Table 3 as the end of Dot.Com Burst. Thirdly, the Global Recession technically lasted from July 2007 to December 2009. Hence, I tend to relate the possible structural breaks starting from 08/29/2007 to 12/16/2009 in Table 3 to the Global Recession. In short, table 3 provides us with a strong clue for the necessity of taking the important economic events of the period from 1996 to 2009 into account when estimating our system. That is why, three dummy variables are incorporated into the model to capture the influences of the Asian Crisis (from 09/10/1997 to 03/24/1999), Dot.Com Burst (from 10/04/2000 to 09/04/2002), and the Global Recession (from 08/29/2007 to 12/16/2009). S&P500 index over the entire dataset and the three dummies highlighted in grey are provided on a graph below for visual inspection.

**FIG. 2.**

S&P500 INDEX AND THE MAJOR GLOBAL ECONOMIC EVENTS




## 5. ESTIMATION RESULTS

After including the dummy variables for the important economic events, optimal lag length is searched once again. Schwarz Information Criteria decided on seven endogenous lags like it did formerly for the model without economic event dummies. As a result, the model is re-estimated using EGLS in its structural form with seven lags and the economic event dummies added. Long-run restrictions for the structural model are imposed using the BQ decomposition methodology in an ordering of variables assuming no long-run impact of volatility on market index. As explained in the previous section, this assumption conforms to the well-established empirical finding of the existing literature, which claims only short-run impact of volatility on market index.

Following table presents the results for the forecast error variance decomposition of the structural errors over a period of 30 days. Errors from the estimated model are tested for serial and cross-correlation using collelograms and for ARCH effects at 1,7, 10 and 20 lags using

Portmanteau, univariate and multivariate ARCH-LM tests to make sure that the model parameter estimates do not suffer from efficiency problem since using inefficient estimates would render variance estimates unreliable. The following forecast error variance decomposition results from reliable errors indicate us that 51% of the forecast error variations for the daily changes of VIX are attributable to return shocks in S&P500 index. Transfer of S&P500 shocks to VIX variance series seem to be imminent and stable over 30 days. Shocks to VIX, however, do not have similar impact on the forecast error variances of S&P500 returns. VIX does not influence the forecast error variances of S&P500 returns until day four and then accounts for only 1% of the variances of S&P500, indicating a delayed miniscule impact.

**TABLE 4.**

Forecast Error Variance Decompositions at Different Forecast Horizons

| Proportions of Forecast Error in | VIX | | S&P500 | |
|---|---|---|---|---|
| Accounted for by Shocks in | S&P500 | VIX | S&P500 | VIX |
| Forecast Horizon: 1 day | 0.51 | 0.49 | 1.00 | 0.00 |
| 2 days | 0.51 | 0.49 | 1.00 | 0.00 |
| 3 days | 0.51 | 0.49 | 1.00 | 0.00 |
| 4 days | 0.51 | 0.49 | 0.99 | 0.01 |
| 5 days | 0.51 | 0.49 | 0.99 | 0.01 |
| 10 days | 0.51 | 0.49 | 0.99 | 0.01 |
| 15 days | 0.51 | 0.49 | 0.99 | 0.01 |
| 20 days | 0.51 | 0.49 | 0.99 | 0.01 |
| 25 days | 0.51 | 0.49 | 0.99 | 0.01 |
| 30 days | 0.51 | 0.49 | 0.99 | 0.01 |

Results are for the structural model with seven lags and economic event dummies.

As the second part of the innovation accounting, I constructed the impulse responses of these two variables to one standard deviation shocks in errors. Responses, both accumulated and periodic, of S&P500 returns to shocks in S&P500 returns, VIX volatility changes and high jumps in VIX levels are presented in Fig. 3., while Fig. 4. presents responses of VIX to itself, S&P500 returns and S&P500 losses. Since both series are stationary, periodic responses should die out and, as we see from the graphs, they all die out between 10 and 15 days. Secondly, all the periodic response tables indicate overshooting effects. For example, when we look at the periodic response of S&P500 returns to a shock in S&P500 return errors, we see an imminent increase in returns, which immediately swoops down in the following period. This sharp decline happens in an overshooting manner and the impacts in the following days gradually lose their strength day by day until the impact of the shock dies away around 10 days.

Accumulated response graphs reveal more intriguing facts. According to the accumulated responses of S&P500 returns, S&P500 is permanently moved only by itself. Shocks to VIX errors, temporarily depress S&P500 returns but the impact is not permanent. It almost totally dies out after 7 days. Jumps in Volatility changes seem to have more lasting impact on S&P500 but eventually their impact also dies out around 12 days later. As for the responses of VIX, a shock to S&P500 errors and shock to VIX errors both seem to have permanent impacts on VIX, while the impact of shocks in S&P500 losses eventually dies out. Since error shocks reflect the unexpected innovations in the series, we can argue that the impact of unexpected S&P500 losses first increases volatility but this impact is destined to die off eventually. On the other hand, unexpected S&P500 return innovations decrease the volatility and this impact is permanent.

**FIG. 3.**

Responses to One Standard Deviation Structural Innovations in Errors

| Accumulated Responses | Periodic Responses |
| --- | --- |

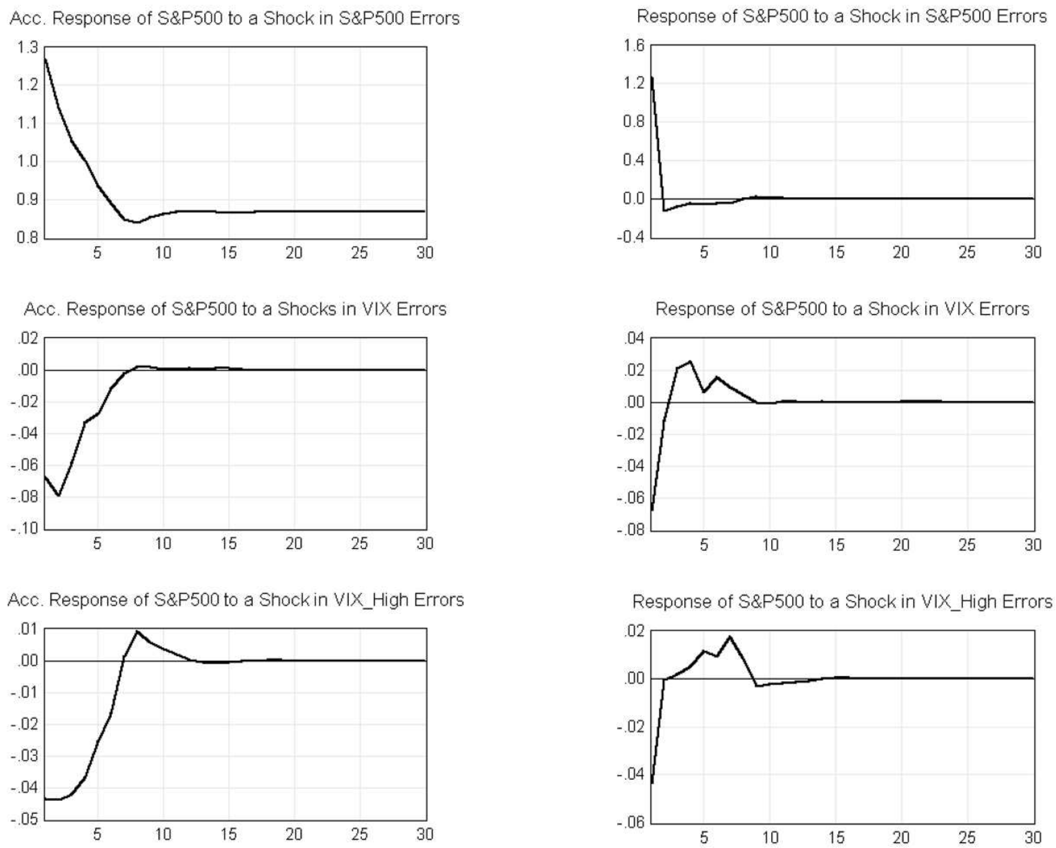
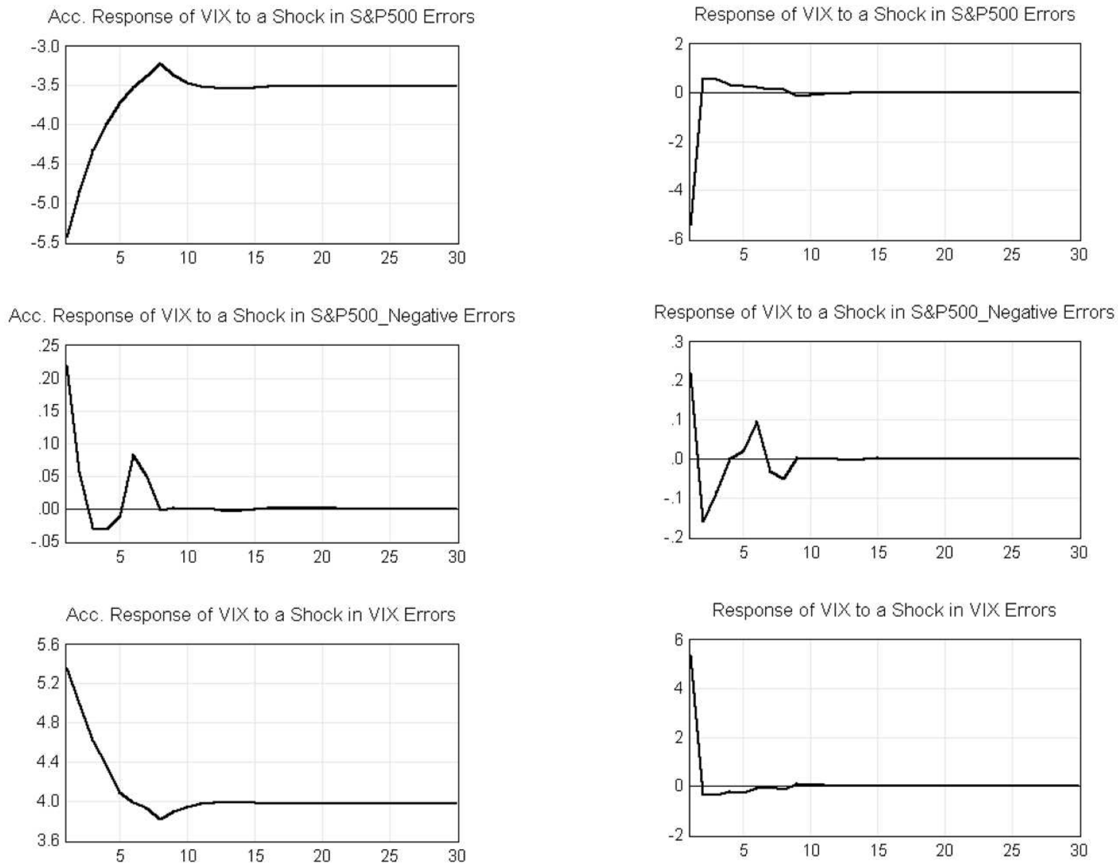


**FIG. 4.**

Responses to One Standard Deviation Structural Innovations in Errors

| Accumulated Responses | Periodic Responses |
|---|---|




## 6. CONCLUSIONS

This paper is interested in establishing the true dynamic relationship between the return and volatility series in S&P500, one of the leading stock exchange markets in the world. To be honest, there already exists a highly rich literature on return and volatility relationship in stock markets and this literature has already established some stylized facts. For example, volatility asymmetry phenomenon is a well-documented empirical fact. Volatility asymmetry simply refers to the situation that poorer returns tend to correlate strongly with higher volatilities but there is no comparable strong correlation between positive returns and lower volatilities. Although this asymmetric correlation is well-documented, there is no clear explanation in the literature for the underlying causation. According to the leverage effect hypothesis, one of two staple hypotheses for explaining volatility asymmetry phenomenon, the relationship between volatility and return series runs from return to volatility, while volatility feedback hypothesis, the second important hypothesis attempting to explain volatility asymmetry phenomenon, claims a reverse causation.

Using a structural vector autoregression model accounting for the volatility asymmetry phenomenon and controlling for the effects of major economic events during the estimation period, this paper reports innovation accounting results between the daily S&P500 returns and changes in VIX volatility index. According to the forecast error variance decompositions, S&P500 returns have strong and stable impact on the variations in the VIX series. However, the impact of VIX on the variations in the S&P500 returns is almost nil. Similarly, the impulse – response analysis indicates that the only permanent impact on S&P500 returns comes from unexpected innovations to S&P500 returns. On the other hand, both S&P500 returns and VIX changes seem to have permanent impact on VIX. In sum, the results of this paper indicate a strong influence of S&P500 returns on VIX but not the other way around.

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