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Abstract

Why has capitalism prevailed as an institution in promoting economic growth despite its apparent unfairness? In this paper, we argue that within a neoclassical framework, capitalism is fairer compared to collectivism due to the absence of a rationally acceptable collective solution. This is demonstrated using a dynamic game with a vote-maximizing government(G) and profit-maximizing representative firm(F). In this GF game, collectivism or cooperation between the players appears to trump capitalism at the aggregate level. Developing countries operating below the steady state may be better off cooperating as they will enjoy positive long term economic growth and profit growth once their capital stock exceeds the steady state level. But this requires them to sacrifice short term growth and possible inequity as the firm's profits grow. Developed countries operating above the steady state will find the cooperative solution attractive since both economic growth and profit growth will be positive. So, from an aggregate level, collectivism or cooperation performs better than capitalism. However, a fair imputation of cooperative or collective solutions which is rationally acceptable for all players does not exist. In every stage of development, the firm always finds it rationally unacceptable to cooperate because the profits earned by the firm under the feedback Nash equilibrium always dominate the profits under cooperation. On the other hand, the government only finds the cooperative solution to be rationally acceptable when the economy is above the steady state. Hence, collectivist cooperation between the government and the firm are not rationally acceptable for both and a fair equilibrium cannot be attained with collectivism.

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1 Introduction

Since the collapse of communism, it is widely accepted that capitalism is the key to prosperity and even countries like China, which is communist in ideology, has adopted capitalism in practice. The raison d'être in China's economic policy since the 1980s can be regarded as one of maximum growth in the initial phase with postponed consumption for a later phase. Yet, such a strategy is not without its problems. Like most capitalist economies, such an intertemporal contract may be neither acceptable to the workers who form the core of political support for the party nor sustainable over the long term. According to World Bank estimates, while the real GDP grew at an annual average rate of 10 % in China during the last two decades of the last century, the income disparity has also widened. This has prompted many have argued for enforcing some collective solutions so as to curb the dynamic inefficiency of capitalism and bring about a "harmonious society".

This situation is not unique to China. Since Malthus, Ricardo and Marx, "the essence of capitalism (is perceived) to be centered on the problems of capital accumulation and the distribution of income between workers and capitalists"(Lancaster, 1973). Within such a paradigm, Lancaster(1973) shows that capitalism is dynamically inefficient when compared to a social optimum which can be achieved by both the workers and the capitalism cooperating together. The attractiveness of such cooperative or collective outcomes has achieved commanding heights in mid 20th century, with many countries adopting communism or some forms of socialism. Perhaps because of the scars from the Great Depression, capitalism was somewhat discredited and the idea of a "benevolent dictator" appealed to both influential economists and policymakers. In China, for instance, in the early years (1949-52) of Communist rule, private firms were allowed to continue their operations but beginning 1953, the capitalists were ordered to surrender their enterprises, "until they became only managers of the enterprise and had to follow government instructions if they were to remain part of it." (Chow 2002). The collapse of communism and the adoption of capitalism by Communist China, Russia and Vietnam attest that such collective or cooperative outcomes, while apparently attractive, is not sustainable. Countries which persist in collective solutions, such as North Korea and Myanmar(Burma), continue to suffer from dismal growth.

If capitalism is dynamically inefficient, why is it the choice of practically every economy in the world today? The answer offered here is that capitalism may be more rationally acceptable and fairer than collectivism. This is demonstrated using a government-firm (GF) dynamic game with a vote-maximizing government(G) and profit-maximizing representative firm(F). Capitalism in this paper is defined by the independent pursuit for profits by the firm whereas with "collectivism", this objective by the firm becomes subsumed under some cooperative optimization. In this GF game, a fair imputation of cooperative or collective solutions which is rationally acceptable for all players does not exist. Rationally acceptable strategies are the set of cooperative strategies with payoffs greater than or at least equal to the payoffs from pursuing non-cooperative strategies. Regardless of the stages of development, the firm always finds it rationally unacceptable to cooperate because the profits earned by the firm under the feedback Nash equilibrium always dominate the profits under cooperation. On the other hand, the government only finds cooperative solution to be rationally acceptable when the economy is above the steady state. Below the steady state, developing countries are trapped in low growth and political instability. Thus, while group rationality dictates cooperation, such cooperation is not realized because they are not rationally acceptable for both the government and the firms. As a result, a fair equilibrium attained is equivalent to the feedback Nash equilibrium because the trigger strategies which are employed to enforce cooperation degenerates easily into the feedback Nash equilibrium.

Our paper contributes to the game-theoretic literature on capitalism and economic growth. Phelps and Pollak (1968) were perhaps the first to consider a game-theoretic approach in economic growth. They modeled economic growth and distribution as an intergenerational conflict. In their model, the present generation derives its utility from the consumption pattern of infinitely many nonoverlapping generations but it can only control its own saving rate. As a result, the Nash equilibrium of this intergenerational game results in undersaving. Strictly speaking, they neither considered the issues of distributional conflict between different types of players within each generation and across generations nor the possibility of cooperation. These issues were explored by Lancaster (1973) who adopted a two player noncooperative dynamic game where the workers control the share of their consumption in total output while the capitalists control the share of investment in the surplus. Comparing the feedback Nash equilibrium with the cooperative solution (from maximizing a weighted sum of worker and capitalist consumption), Lancaster demonstrated that both players obtain more consumption under cooperation, hence demonstrating the dynamic inefficiency of capitalism. This has been extended by others (see Dockner et al, 2000 for a survey) in various degrees of sophistication but the basic conclusion is fundamentally the same. For example, Kaitala and Pohjola(1990) consider a variation on the original Lancaster model in which the politically powerful group of workers controls redistribution while the economically powerful group of capitalists controls accumulation. Grim trigger strategies are employed by both groups to sustain cooperation as an equilibrium. In their model, the workers and capitalists receive returns equivalent to the labor and capital share respectively. In all these models, it is implicitly assumed that some binding agreement can be accepted by all and enforced rigidly, without worrying whether such binding agreement can be achieved in the first place.

The present model thus departs from the literature in two respects. Firstly, the government in this present model is a vote-maximizer while the firm receives a return equivalent to the marginal product of capital. The characterization of the government as a vote-maximizer is a distinctive feature of the present model. This is a significant departure from conventional economic models, in which the government is typically characterized as a benevolent dictator which maximizes social welfare. The idea of a vote-maximizing government follows from Nordhaus(1975). However, the government in Nordhaus' political business cycle model faces the short-run Phillips inflation-unemployment tradeoff. In contrast, the government in this model deals with the long-run political "tradeoff" between economic growth and distributional equity. Vote-maximization is not to be taken literally to imply a democracy. Instead, the vote function in this paper can be interpreted as a function for political support. For instance, in the case of China, the Communist Party depends on the political support of the people, despite the absence of any democratic mechanisms. Hence, regardless of whether a government is democratic or authoritarian, we assume that its main objective is to maximize its political support or vote function.

Secondly, unlike Lancaster and his follower, we derive explicitly the set of fair imputations of the cooperative payoffs so as to determine whether the cooperative solution is rationally acceptable. Rationally acceptable strategies exist if a fair imputation of the cooperative solution offers players higher payoffs compared to those obtained in a feedback Nash equilibrium. Where rationally acceptable cooperative solution exists, the fair equilibrium is simply the set of rationally acceptable strategies and cooperation is enforced through a reciprocal punishment mechanism. Otherwise, the fair equilibrium is the set of feedback Nash equilibrium. In this way, our approach is distinct from those adopted in the recent literature on distributional fairness in growth and development, such as Alesina and Angeletos (2005) and Benabou and Tirole (2006). These papers incorporate the economists' judgment of fairness and equity and ignore the concerns for fairness and equity of the economic players under analysis. In contrast, the present paper derives the fair equilibrium explicitly from the endogenous and strategic interactions between the players.

The rest of this paper is organized as follows. Section 2 presents the GF game of economic growth. The feedback Nash equilibrium and the cooperative solutions are derived and discussed in Section 3. Section 4 explores the characterization of a fair equilibrium based on rational acceptability. Section 5 concludes.

2 Government-Firm Game of Economic Growth

Consider a dynamic game of economic growth with two players: a government(G) and a representative firm(F), henceforth referred to as the GF game. The economy has a neoclassical production function, which is represented in intensive form as $y = f(k), f'(k) > 0, f''(k) < 0, \lim_{k\to 0}[f'(k)] = \infty, \lim_{k\to\infty}[f'(k)] = 0$.The labor force receives an income equal to its marginal product $f(k) - kf'(k)$ while capital a rent equivalent to its marginal product $f'(k) - \delta$, where δ is the depreciation rate.

The firm in the model owns the capital and has to decide between retaining its capital earnings for investment and consuming the dividends payments. Its objective is to maximize the stream of dividends payments over time. The present game analysis suggests a more active role of the firm in the policy-making process. Not only will the investment strategy of the firms adjust dynamically to the tax policy of the government, the corporate tax strategy of the government will also change in response to changes in the firms' investment policy.

The government is a vote-maximizer: it will adopt policies that will best assure its continuation in power, increase its political support or improve its vote-getting power. This is represented using a vote function $v[k, x(\cdot), s(\cdot)]$, where $x(\cdot)$ represents the tax or social transfer within the government's control while $s(\cdot)$ represents the investment rate controlled by a representative firm.

The government's objective functional can thus be expressed as follows:

$$\max_{x(.)} J^G(k_0, x(\cdot), s(\cdot)) = \int_0^\infty e^{-\rho t} v[x(\cdot), s(\cdot)]\, dt.$$

Assume a balanced budget and that the policy instrument $x(t)$ used by the government to promote growth and effect redistribution is lump-sum. The government faces the long-run political tradeoff between long run growth economic growth and distributional equity. The conflict can be encapsulated in the vote function:

$$v\left[k,x(\cdot),s(\cdot)\right]=f\left(k\right)-kf'\left(k\right)+x.$$

The transfer or tax,$x(t)$, on the workers must be less than or equal to their incomes while redistribution disbursement cannot exceed the marginal product of workers. Similarly, the tax must be less than or equal to profits when it is imposed on the firm and the subsidy to the firm will not exceed its marginal product. Hence, the following constraint $-f(k)+k.f'(k)\leq x\leq f(k)-k.f'(k)$ is binding.

Assume a representative firm which owns the capital in the production process and controls investment. The objective of the firm is to maximize the flow of dividend payment $\pi\left[x(\cdot),s(\cdot)\right]$ for the planning horizon. Since the government may tax or subsidize the firm, its after tax/subsidy profit is given by $f'\left(k\right)-x$. Out of this after-tax profits, the firm must decide how much to pay out as dividends to shareholders and how much to retain for investing in capital by adjusting $s(\cdot)$, the rate of capital investment. The firm's objective functional is given by

$$\max_{s(.)}J^{F}\left(k_0,x(\cdot),s(\cdot)\right)=\int_0^{\infty}e^{-\rho t}\left(\pi\left[x(\cdot),s(\cdot)\right]\right)dt,$$

where $\rho>0$ is a positive discount rate ; $k(t_0)=k_0>0$ is an initial capital-labor ratio; $s(t)$ be the investment rate which is controlled by the firm and $\left[\pi\left[x(\cdot),s(\cdot)\right]\right)$ are the dividends payments, given by

$$\pi\left[k,x(\cdot),s(\cdot)\right]=(1-s)\left[f'(k)-x\right],0\leq s\leq 1.$$

Assume that labor consume fully its wage, thus only the firm contributes to the accumulation of capital. Capital accumulation then follows the dynamics

$$\dot{k}=g\left(x(.),s(.),k\right)=s\left[f'\left(k\right)-x\right]-\left(n+\delta\right)k,$$

where δ denotes effective depreciation for the capital-labor ratio k, n is the population growth rate and $n+\delta>0$;x is a per person lump sum which is controlled by the government, so that $f'\left(k\right)-x$ is the after tax/subsidy profit for the firm.

The GF game thus described involves the government and the private sector acting independently, affecting a common state variable and each other's payoffs through time and is hence a dynamic game. In this GF game, each player takes into account the other player's decision while making his own decisions. Since the game is dynamic, each player will take into account not only the current but also future decisions of the other player.

The complete GF game, $\Gamma(k)$, can be characterized as follows:

Government

$$\max_{x(.)}J^{G}\left(k_0,x(\cdot),s(\cdot)\right)=\int_0^{\infty}e^{-\rho t}\left[f\left(k\right)-k.f'\left(k\right)+x\right]dt,\tag{1}$$

Firm

$$\max_{s(.)} J^F\left(k_0, x(\cdot), s(\cdot)\right) = \int_0^\infty e^{-\rho t}\left(1-s\right)\left[f'(k) - x\right] dt, \tag{2}$$

subject to

$$\dot{k} = s\left[f'(k) - x\right] - (n+\delta)k, \qquad k(0) = k_0 \tag{3}$$

$$0 \leq \quad s \quad \leq 1 \tag{4}$$

$$-f(k) + k.f'(k) \leq \quad x \quad \leq f'(k) \tag{5}$$

The control function pair $(x(\cdot), s(\cdot))$ is such that $x(\cdot) : [0,\infty) \to X$ and $s(\cdot) : [0,\infty) \to S$ where X and S are the control sets of the government and the firm respectively. Assume information is complete.

Alternatively, a cooperative GF game can be defined as $\Gamma^c(k)$, which denotes a cooperative game between the government and the firm with the game structure of $\Gamma(k)$, given some initial state k_0. Group rationality dictates the joint maximization of the sum of payoffs for both players. Specifically,

$$\max_{x(.)} J^C\left(k_0, x(\cdot), s(\cdot)\right) = \int_0^\infty e^{-\rho t}\left\{v\left[k, x(\cdot), s(\cdot)\right] + \pi\left[k, x(\cdot), s(\cdot)\right]\right\} dt, \tag{6}$$

subject to (3) through (5)

3 Feedback Nash Equilibrium and Cooperative Solutions

In this section, the feedback Nash equilibrium to the GF game, $\Gamma(k)$, and the cooperative solutions to the cooperative GF game, $\Gamma^C(k)$, are derived. These solutions are then computed for a common specification of the neoclassical production function.

3.1 Feedback Nash Equilibrium

In the GF game, $\Gamma(k)$, the problem for the government is to take $s(\cdot)$ as given and choose a transfer/tax strategy, $x(\cdot)$, so as to maximize its political payoff (1). Taking $x(\cdot)$ as given, the firm chooses an investment strategy,$s(\cdot)$, so as to maximize its after tax profit flow (2). Both are subject to the constraints (3) through (5). This section discusses the feedback Nash equilibrium to the GF game $\Gamma(k)$.

Theorem 3.1 *A set of strategies $\{\bar{x}(k), \bar{s}(k)\}$ constitutes a feedback Nash equilibrium solution to the game $\Gamma(k)$ if there exists functionals, $J^G(k) : R^m \to R$ and $J^F(k) : R^m \to R$ satisfying the following set of partial differential equations:*

$$\rho J^G(k) = \max_x \left\{v(k, x, \bar{s}) + J_k^G(k) g(k, x, \bar{s})\right\} \tag{7}$$

$$\rho J^F(k) = \max_s \left\{\pi(k, \bar{x}, s) + J_k^F(k) g(k, \bar{x}, s)\right\} \tag{8}$$

where

$$J^G(k) = \int_{\tau}^{\infty} e^{-\rho(t-\tau)} \left[f(k) - k.f'(k) + x\right] dt \tag{9}$$

$$J^F(k) = \int_{\tau}^{\infty} e^{-\rho(t-\tau)} (1-s) \left[f'(k) - x\right] dt, \tag{10}$$

represents the current value payoffs of the government and the firm at time τ

Proposition 3.2 *The feedback Nash equilibrium* $(\bar{x}(k), \bar{s}(k))$ *for* $\Gamma(k)$ *is given by*

$$(\bar{x}(k), \bar{s}(k)) = \left(f'(k), 0\right) \tag{11}$$

Proof A noncooperative feedback Nash equilibrium solution to $\Gamma(k)$ is characterized by:

$$\rho J^G(k) = \max_x \left\{ f(k) - kf'(k) + x + J_k^G(k) \left(\bar{s}\left[f'(k) - x\right] - (n+\delta)k\right) \right\} \tag{12}$$

$$\rho J^F(k) = \max_s \left\{ (1-s)\left[f'(k) - \bar{x}\right) + J_k^F(k) \left(s\left[f'(k) - \bar{x}\right] - (n+\delta)k\right) \right\} \tag{13}$$

Performing the above maximizations yields:

$$\bar{x}(k) = f'(k), \bar{s}(k) = 0$$

If the tax is $\bar{x}(k) = f'(k)$, the whole rental income of the firm is effectively taxed away and the firm will have to stop investing, thus $\bar{s}(k) = 0$. The economic intuition is as follows. In the GF game, a government may postpone redistribution to later stages so as to facilitate the most rapid economic growth. But the firm predicts as much and being free to optimize on its investment decisions, will stop investing just before profits are being taxed. But the government is also aware of the firm's reaction: its best response is to impose taxes on capital income earlier. The process will then converge to the feedback Nash equilibrium $(\bar{x}(k), \bar{s}(k)) = (f'(k), 0)$.

Proposition 3.3 *The feedback Nash equilibrium* $(\bar{x}(k), \bar{s}(k))$ *for* $\Gamma(k)$ *is both* time consistent *and* subgame perfect.

Proof First denote any subgame of $\Gamma(k)$ by $\Gamma(0, k_\tau)$.

To establish time consistency, note that $\{\bar{x}_i(\tau, T,)\, \bar{s}_i(\tau, T,)\}, i = 1, \cdots, N$ constitutes a set of equilibrium for the subgame $\Gamma\left(\tau, \bar{k}(\tau)\right)$,where $\bar{k}(\tau) = k(0, t, \{\bar{x}_i(0, \tau), \bar{s}_i(0, \tau)\})$ for every $i = 1, \cdots, N$ and $\tau \in (0, \infty)$.

To establish subgame perfectness, note that for every $\tau \in (0, T)$, the feedback Nash equilibrium $\{\bar{x}_i(\tau, T), \bar{s}_i(\tau, T)\}$ constitutes a set of equilibrium for the subgame $\Gamma(t_1, \bar{k}_\tau)$ where $k_\tau \in R^m$ is an arbitrarily chosen state which is reachable from some initial state at $t_0 = 0$.

Substitution of these into (12) and (13) to solve for the value functions of the government and the firm respectively:

$$J^G(k) = \frac{1}{\rho}\left\{f(k) - kf'(k) + f'(k)\right\} \quad (14)$$

$$J^F(k) = -\frac{1}{\rho}(n+\delta)k \quad (15)$$

It is straightforward to derive the economic growth rate which is given by

$$\frac{\dot{y}}{y} = f'(k)\frac{\dot{k}}{f(k)} = \left[kf'(k)/f(k)\right]\left(\frac{\dot{k}}{k}\right) = -\left[kf'(k)/f(k)\right](n+\delta)$$

Similarly, the rate of growth of after-transfer profits, denoted by $\frac{\dot{\pi}}{\pi}$, can be easily obtained: $\frac{\dot{\pi}}{\pi} = -\frac{\dot{k}}{k} = (n+\delta)$.

Both the economic growth rate and the rate of growth of after tax profits depends on the level of breakeven investment, denoted by $(n+\delta)$, defined as the level of investment needed to keep k at its existing level. Given the feedback Nash equilibrium $(\bar{x}(k), \bar{s}(k))$ for the game $\Gamma(k)$, the breakeven investment will depend on the capital labor ratio k. Below steady state, $(n+\delta) < 0$, at steady state, $(n+\delta) = 0$ and above steady state, $(n+\delta) > 0$.

Thus, in the feedback Nash equilibrium, the economy in a developing country below the steady state has positive growth but will stop growing eventually because the firms are getting subnormal profits and will not be motivated to invest. Those firms in countries which have achieved steady state growth and beyond, will break even or enjoy supernormal profits and enter into higher levels of investment.

In short, the consequences of the feedback Nash equilibrium in terms of economic growth rate, the profitability of the firm and capital accumulation will be more adverse for the developing countries than relatively more developed countries.

The results so far are not controversial and are similar to Lancaster(1973) and Kaitala and Pohjola(1990). These authors went on to argue that cooperation between the government and the firm will be more beneficial compared to the dynamic inefficiency of capitalism. They assumed implicitly that the cooperative solution will be accepted by all. In the case of Kaitala and Pohjola(1990), the cooperation is enforced by trigger threats that force everyone to cooperate but whether such a cooperation is fair or not is not considered. In the next section, the cooperative solution to the game and the set of fair imputations of the cooperative payoff are explicitly derived so as to determine whether cooperation is indeed rationally acceptable compared to the feedback Nash equilibrium.

3.2 Cooperative Solutions

Consider a cooperative GF game $\Gamma^c(k)$.

Theorem 3.4 *A set of strategies $\{\hat{x}(k), \hat{s}(k)\}$ constitutes a solution to the game $\Gamma^c(k)$ if there exists functionals, $W^G(k) : R^m \to R$ satisfying the infinite horizon Bellman equation:*

$$\rho J^C(k) = \max_{x,s}\left\{v(k,x,s) + J_k^C(k)\, g(k,x,s)\right\}$$

where $J_k^C(k)\,g(k,x,s) = \int_\tau^\infty e^{-\rho(t-\tau)}\,[f(k)-(1-k)f'(k)-s(f'(k)-x)]dt.$

More specifically, the cooperative solution of the GF game $\Gamma^c(k)$ can be obtained by considering the optimization problem

$$\max_{x(.),s(.)} J^C(k_0,x(\cdot),s(\cdot)) = \int_0^\infty e^{-\rho t}\left[f(k)-(1-k)f'(k)-s\left(f'(k)-x\right)\right]dt, \qquad (16)$$

subject to (3) through (5).

Due to the simple linear structure of the model, the problem can be solved more directly by applying the Most Rapid Approach Path (MRAP) technique. First, use the state equation (3) to obtain

$$J^C(k_0,x(\cdot),s(\cdot)) = \int_0^\infty e^{-\rho t}\left[f(k)-(1-k)f'(k)-\left(\dot{k}+(n+\delta)k\right)\right]dt$$

.

Next, integrate the term containing $\dot{k}$ and use the initial condition $k(0)=k_0$ to obtain

$$J^C(k_0,x(\cdot),s(\cdot)) = k_0 + \int_0^\infty e^{-\rho t}\,[h(k)-((n+\delta+\rho)k)]dt.$$

where $h(k) = f(k)-(1-k)f'(k)$. The integrand in this representation of the objective functional J^C is strictly concave of the state variable k and attains its maximum at the unique steady state value $k=k^{SS}$ defined by the equation $h'(k^{SS}) = n+\delta+\rho$. It follows that to maximize J^C, the state trajectory must approach the steady state k^{SS} as fast as possible and remain there forever. It is trivial that this is the case if and only if the controls are selected as follows:

$$\hat{s}[f'(k)-\hat{x}] = \begin{cases} f(k)+(1-k)f'(k) & k<k^{SS} \\ (n+\delta)k & k=k^{SS} \\ 0 & k>k^{SS} \end{cases} \qquad (17)$$

From this, it is straightforward to derive the set of cooperative strategies to the game.

Proposition 3.5 *The set of cooperative strategies* $\{\hat{x}(k),\hat{s}(k)\}$ *to the game* $\Gamma^c(k)$ *is given by*

$$\hat{s}(k) = \begin{cases} 1 & k<k^{SS} \\ u,\, u\in(0,1] & k=k^{SS} \\ 0 & k>k^{SS} \end{cases} \qquad (18)$$

$$\hat{x}(k) = \begin{cases} kf'(k)-f(k) & k<k^{SS} \\ f'(k)-(n+\delta)u^{-1}k,\, u\in(0,1] & k=k^{SS} \\ f(k) & k>k^{SS} \end{cases} \qquad (19)$$

It is reasonable to define a *fair imputation* of the cooperative payoff for each player as one which includes their Nash payoffs and half of their cooperative gains. Hence, the imputed payoffs for each player in the cooperative game $J^C = \left\{J^{CG},J^{CF}\right\}$ are

$$J^{CG}(k) = \frac{1}{\rho}\left\{f(k) - kf'(k) + \frac{1}{2}\left[sf'(k) - (n+\delta)k\right]\right\} \tag{20}$$

$$J^{CF}(k) = \frac{1}{\rho}\left\{\frac{(s-2)}{2}f'(k) - \left(\frac{n+\delta}{2}\right)k\right\} \tag{21}$$

Accordingly, denote $\hat{v} \equiv \rho J^{CG}$ and $\hat{\pi} \equiv \rho J^{CF}$, the

$$\hat{v}(k) = \begin{cases} f(k) - kf'(k) + \frac{1}{2}\left[f'(k) - (n+\delta)k\right] & k < k^{SS} \\ f(k) + \left(\frac{1}{2}u - k\right)f'(k) - \left(\frac{n+\delta}{2}\right)k & k = k^{SS}, 0 < u \leq 1 \\ f(k) - kf'(k) - \frac{1}{2}(n+\delta)k & k > k^{SS} \end{cases} \tag{22}$$

$$\hat{\pi}(k) = \begin{cases} -\frac{1}{2}f'(k) - \left(\frac{n+\delta}{2}\right)k & k < k^{SS} \\ \frac{(u-2)}{2}f'(k) - \left(\frac{n+\delta}{2}\right)k & k = k^{SS}, 0 < u \leq 1 \\ -f'(k) - \left(\frac{n+\delta}{2}\right)k & k > k^{SS} \end{cases} \tag{23}$$

Similarly, the growth rate and the rate of growth for the after tax profit can be computed respectively.

$$\frac{\dot{y}}{y} = \begin{cases} -\left[kf'(k)/f(k)\right]\left(\left[f'(k) - f(k)\right]/k - f'(k) - (n+\delta)\right)(n+\delta) & k < k^{SS} \\ 0 & k = k^{SS} \\ \left[kf'(k)/f(k)\right](n+\delta)^2 & k > k^{SS} \end{cases}$$

$$\frac{\dot{\pi}}{\pi} = \begin{cases} \left(f(k) + (1-k)f'(k)\right)/k - (n+\delta) & k < k^{SS} \\ 0 & k = k^{SS} \\ (n+\delta) & k > k^{SS} \end{cases}$$

3.3 Solutions for Specific Neoclassical Production Function

To make concrete the solution concepts and allow easy comparison, it is useful to adopt a specific neoclassical function. The most common specification for this in the literature is the constant elasticity of substitution production function,given by:

$$y = f(k) = \tilde{A} \cdot \left[a \cdot (bk)^{\psi} + (1-a) \cdot (1-b)^{\psi}\right]^{\frac{1}{\psi}} \tag{24}$$

where $0 < a < 1, o < b < 1$ and $\psi < 1$. The marginal product of capital are given by

$$f'(k) = \tilde{A}ab^{\psi}\left[a \cdot b^{\psi} + (1-a) \cdot (1-b)^{\psi} \cdot k^{-\psi}\right]^{\frac{1-\psi}{\psi}} \tag{25}$$

Without loss of generality, this can be simplified to a Cobb-Douglas form by letting $\psi < 1 \rightarrow 0$ and applying l'Hôpital's rule to obtain $f(k) = Ak^a$, where $A = \tilde{A}b^a(1-b)^{1-a}$ and $0 < a < 1$.

The results are summarized in the table 1.

Nash	$\bar{x}(.)$	$\bar{s}(.)$	$\bar{v}(.)$	$\bar{\pi}(.)$
$k < k^{ss}$	Aak^{a-1}	0	$Ak^{a}\left(1-a+ak^{-1}\right)$	$(n+\delta)\,k$
$k = k^{ss}$	Aak^{a-1}	0	$Ak^{a}\left(1-a+ak^{-1}\right)$	$(n+\delta)\,k$
$k > k^{ss}$	Aak^{a-1}	0	$Ak^{a}\left(1-a+ak^{-1}\right)$	$(n+\delta)\,k$
Cooperative	$\hat{x}(.)$	$\hat{s}(.)$	$\hat{v}(.)$	$\hat{\pi}(.)$
$k < k^{ss}$	$(a-1)Ak^{a}$	1	$\bar{v}(k)-\frac{1}{2}\left[Aak^{a-1}+(n+\delta)\,k\right]$	$-\frac{1}{2}Aak^{a-1}-\left(\frac{n+\delta}{2}\right)k$
$k = k^{ss}$	$Aak^{a-1}-\frac{(n+\delta)}{u}k$	u	$\bar{v}(k)-\frac{1}{2}\left[(1-2u)\,Aak^{a-1}+(n+\delta)\,k\right]$	$\frac{(u-2)}{2}Aak^{a-1}-\left(\frac{n+\delta}{2}\right)k$
$k > k^{ss}$	Aak^{a-1}	0	$\bar{v}(k)-\frac{1}{2}\left[2Aak^{a-1}+(n+\delta)\,k\right]$	$-Aak^{a-1}-\left(\frac{n+\delta}{2}\right)k$

where $u \in (0,1]$

Table 1: Respective Strategies x, s and Payoffs v, π for the Government and the Firm under Cooperative and Feedback Nash Equilibria

From these, the economic growth rate and the rate of growth of after-tax profits for the firm can be derived for the cooperative solutions.

$$\frac{\dot{y}}{y} = \begin{cases} -a\left(Ak^{a-1}\left(k^{-1}-2\right)-(n+\delta)\right)(n+\delta) & k < k^{SS} \\ 0 & k = k^{SS} \\ a(n+\delta)^2 & k > k^{SS} \end{cases}$$

$$\frac{\dot{\pi}}{\pi} = \begin{cases} Ak^{a-1}\left[(1-a)+ak^{-1}\right] & k < k^{SS} \\ 0 & k = k^{SS} \\ (n+\delta) & k > k^{SS} \end{cases}$$

Table 2 summarizes summarizes the direction of change for both economic growth rates $\frac{\dot{y}}{y}$ and profit growth rates $\frac{\dot{\pi}}{\pi}$ under feedback Nash equilibrium and cooperative solution.

	$sgn\left(\frac{\dot{y}}{y}\right)$		$sgn\left(\frac{\dot{\pi}}{\pi}\right)$	
	Feedback Nash	Cooperative	Feedback Nash	Cooperative
Below k^{ss}	+	-	-	+
At k^{ss}	0	0	0	0
Above k^{ss}	-	+	+	+

Table 2: Economic Growth Rates $\frac{\dot{y}}{y}$ and Profit Growth Rates $\frac{\dot{\pi}}{\pi}$ under Feedback Nash Equilibrium and Cooperative Solution

From table 2, three key observations can be made:

1. Below the steady state, the economic growth rate under the feedback Nash equilibrium is positive while that under cooperation is negative.

2. The rate of economic growth and profit growth are both zero at the steady state.

3. Above the steady state, both the economic growth rate and the profit growth rate are positive under cooperation whereas only the profit growth rate is positive under the feedback Nash equilibrium.

An interpretation for these observations would be as follows. At the aggregate level, developing countries operating below the steady state may be better off cooperating as they will enjoy positive long term economic growth and profit growth once their capital stock exceeds the steady state level. But this requires them to sacrifice short term growth and possible inequity as the firm's profits grow. Developed countries operating above the steady state will find the cooperative solution attractive since both economic growth and profit growth will be positive. So, from an aggregate level, collectivism or cooperation performs better than capitalism. This is the dynamic inefficiency of capitalism.

However, the collective solution requires a short term sacrifice for the voters in developing countries. the logical question which follows would be: will such a sacrifice be rationally acceptable? To answer this question, one must derive the fair imputations for each players and compare these to the feedback Nash outcomes. This is what we set out to do in the next section.

4 Rational Acceptability and Fair Equilibrium

In this GF game of growth, the cooperative solution can be interpreted as a social contract between the voters, the government and the firm, whereby the voters curtailed their present consumption for economic growth with the expectation that at some point in the future, the government will ensure a transfer to them for the earlier sacrifice. On the other hand, in the context of a communist country, it can be interpreted as the firm subjecting itself to the collective will. In both cases, the idea is to prevent the dynamic inefficiency of capitalism. But is the cooperative solution, which is derived from a fair imputation of the collective maximization, necessarily better than the feedback Nash equilibrium? To achieve such a comparison, it is important to establish some criteria for comparison. In this case, the criteria considered is fairness: is the cooperative solution fair to every players in the GF game?

4.1 Rational Acceptability

The characterization of fair equilibrium is not easy. There are many definitions of fairness. One can focus the discussion of fair equilibrium here by invoking the idea of rationally acceptable strategies. It is reasonable to view any policies that are rationally acceptable to all parties as a pre-requisite to a fair equilibrium. Before proceeding to define the fair equilibrium, it is useful to define *rational acceptability*.

Definition A cooperative strategies pair $(\hat{x}, \hat{s})$ is *rationally acceptable* to both players at initial time t_0 and state k_0 if and only if the continuation of this strategy pair at $(\tau, k(\tau))$ yields payoffs that are greater than or at least equal to the payoffs obtained with the continuation of the feedback Nash solutions at $(\tau, k(\tau))$ for all possible $(\tau, k(\tau))$ such that $\tau > o$ and $k(\tau) \geq k_0$ *for all players*.

Substitute the set of cooperative controls $\{\hat{x}(k), \hat{s}(k)\}$ into (3) to obtain the dynamics of the optimal cooperative trajectory

$$\dot{k} = g[k, x(k), s(k)] = \hat{s}(k)\left[f'(k) - \hat{x}(k)\right] - (n+\delta)k,\ k(0) = k_0 \tag{26}$$

Let $\hat{k}$ denote the solution to (26). Denote $\xi^i(\hat{k}),\ i \in \{G, F\}$ to be an imputation of the payoffs of the ith player.

For the cooperative solution to be rationally acceptable, the following condition is required:

$$\xi^i(\hat{k}) \geq J^i(\hat{k}),\ \forall i \in \{G, F\} \tag{27}$$

If such an acceptable cooperative solution exists, then the next step involves determining whether it is time consistent or dynamically stable.

Theorem 4.1 *A time consistent solution to* $\Gamma^c(\hat{k}_0)$ *is given by the instantaneous payoff set at time* $\tau \in [0, T]$

$$J^{Ci}(\tau) = \xi^i\left(\hat{k}_\tau\right) - \xi^i_{\hat{k}_\tau}\left(\hat{k}_\tau \cdot g\left[\hat{k}_\tau, \hat{x}\left(\hat{k}_\tau\right), \hat{s}\left(\hat{k}_\tau\right)\right]\right)$$

for all $\tau \in [0, T], i \in \{G, F\}$

Proof See appendix

The intuition here is that $J^{Ci}(\tau)$ is the transitory or instantaneous payoff that sustains a time consistent solution to the cooperative game.

4.2 Fair Equilibrium

Having considered the concept of rational acceptability, the next step is to establish such a class of fair equilibrium. This paper establishes the fair equilibrium, in which players adopt trigger strategies that pose as threats to discourage players from deviating from their cooperative policies. In dynamic games, the characterization of such an equilibrium is necessarily complicated by the fact that the payoffs are contingent on the whole history of the game. Tolwinski et al (1986) proposed memory dependent trigger strategies and threat to maintain the agreed-upon cooperative path. This was adopted by Kaitala and Pohjola(1990) to enforce their cooperative equilibrium.

In this paper, a fair equilibrium is defined as follows:

1. If rationally acceptable cooperative solutions exist, the fair equilibrium will be the set of rationally cooperative solutions.
2. In the absence of such rationally acceptable cooperative solutions, the fair equilibrium is equivalent to the feedback Nash equilibrium.
3. The fair equilibrium must be time consistent.

More formally,

Definition The fair equilibrium for the game is the pair of strategies $(x^*(\cdot), s^*(\cdot))$, described by:

$$\begin{aligned} x^*(\tau_0) &= \hat{x}(\tau_0) \\ &\textit{and} \\ x^*(\tau_j) &= \begin{cases} \hat{x} & \text{if } x(\tau) = \hat{x}(\tau) \text{ for almost all } \tau \leq \tau_j, \\ \bar{x}(k(\tau_j)) & \text{otherwise} \end{cases} \end{aligned}$$

and

$$\begin{aligned} s^*(\tau_0) &= \hat{s}(\tau_0) \\ &\textit{and} \\ s^*(\tau_j) &= \begin{cases} \hat{s} & \text{if } s(\tau) = \hat{s}(\tau) \text{ for almost all } \tau \leq \tau_j, \\ \bar{s}(k(\tau_j)) & \text{otherwise} \end{cases} \end{aligned}$$

where $k(\tau_j)$ is the state observed at $\tau_j \in [0, \infty]$.

Can such the fair equilibrium be attained in the GF game? From figures 1 and 2, it is evident that in this GF game with a neoclassical production technology, there can be no rationally acceptable cooperative outcomes for countries in the initial stage of development or $k < k^{SS}$. For these countries, the cooperative payoff for both the government and the firm is worse than the feedback Nash outcome. Accordingly, the cooperative solution is rationally unacceptable, hence the fair equilibrium will be equivalent to the feedback Nash equilibrium. The consequences for the developing countries are vicious cycles of low growth and low capital accumulation and government end up with zero payoffs or very low political support.

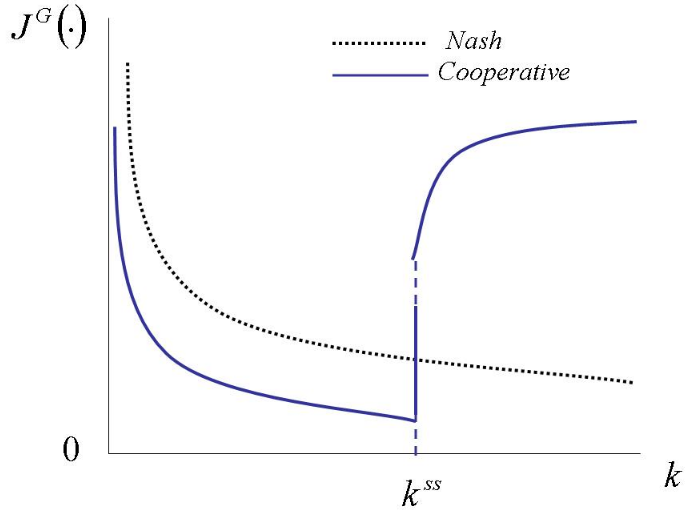

Figure 1: Payoffs for the Government under Cooperative and Feedback Nash Equilibrium

At a very advanced stage of development,$k > k^{SS}$, the government will find the cooperative solution to be rationally acceptable as its vote payoff increases but the firm will continue to find it rationally unacceptable to cooperate because the profits earned by the firm under the feedback Nash equilibrium still dominate the profits under cooperation. Cooperation breaks down and the feedback Nash equilibrium is adopted, but the consequences are not so dire in this case, because the firm can realize a positive and steady profit growth in this case while the economic growth for such countries are steady and positive.

At the steady state, the firm will not cooperate because doing so is put it at a disadvantage compared to its feedback Nash outcome. The government will find the cooperative solution to be rationally acceptable in some cases and unacceptable in others.

Hence, a fair equilibrium cannot be attained because the trigger strategies which are employed to enforce cooperation degenerates easily into the feedback Nash equilibrium. While group rationality may dictates cooperation, such cooperation is not realized even with fair imputations of the cooperative rewards because they are not

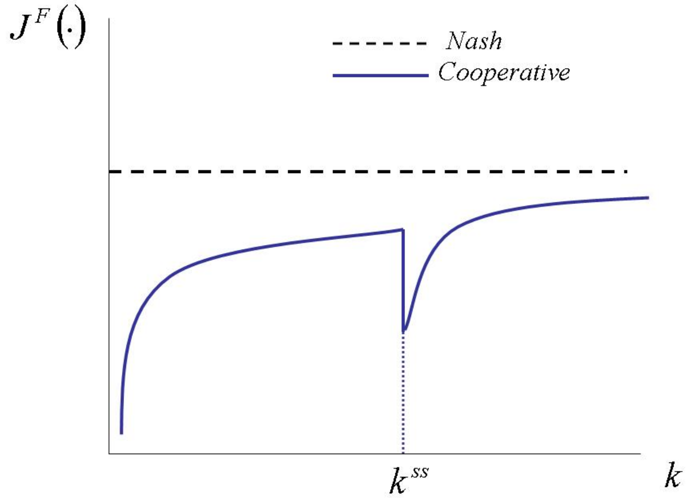

Figure 2: Payoffs for the Firm under Cooperative and Feedback Nash Equilibrium

rationally acceptable for both the government and the firms. Regardless of the stages of development, the firm always finds it rationally unacceptable to cooperate while the the government only finds cooperative solution to be rationally acceptable when the economy is above the steady state. Below the steady state, developing countries are trapped in low growth and political instability.

These results depart from Lancaster (1973) and Kaitala and Pohjola(1990). Both these papers argue that cooperation can help resolve the dynamic inefficiency of capitalism. The results here demonstrates unequivocally that such cooperation cannot be rationally acceptable even if fair imputations of the cooperative payoffs are awarded to each players. A fair equilibrium that is enforced by trigger strategies cannot even be attained because of the strong dominance of the feedback Nash payoffs for the players in most cases. Consequently, in a neoclassical growth model, capitalism may be more rationally acceptable than collectivism or other forms of cooperative solutions.

5 Concluding Remarks

Capitalism has prevailed as an institution in promoting economic growth. This paper argues that capitalism prevails as an institution as it is more rationally acceptable than collectivism.

The paper proposes a dynamic GF game with a vote-maximizing government and profit-maximizing representative firm. In the feedback Nash equilibrium, a government may postpone redistribution to later stages so as to facilitate the most rapid economic growth. But the firm predicts as much and being free to optimize on its investment decisions, will stop investing just before profits are being taxed. But the government is also aware of the firm's reaction: its best response is to impose taxes on capital income earlier. The process will then converge to the feedback Nash equilibrium, with the government taxing away all the profits of the firm and the firm will eventually stop investing altogether. The political support for the government falls. Thus, in the feedback Nash equilibrium, the economy in a developing country below the steady state will stop growing eventually because the firms are getting subnormal profits and will not be motivated to invest. This in turn perpetuates a vicious cycle of low capital accumulation level, low growth and political instability. Those in countries that have achieved steady state growth and beyond enjoyed normal or supernormal profits and enter into a virtuous cycle of higher levels of investment and positive growth. These results are consistent with the literature, which suggests that capitalism is dynamically inefficient.

The implication here appears to be that a cooperative or collective solution should be enforced through a "benevolent dictator". This concurs with Lancaster(1973) and Kaitala and Pohjola(1990). On the other hand, we also explicitly demonstrate in this paper that a cooperative or collective solution which is rationally acceptable to all players may not exist in a neoclassical growth model. This is because the firm will always finds it rationally unacceptable to cooperate since the profits earned by the firm under the feedback Nash equilibrium always dominate the profits under cooperation; the government only finds cooperative solution to be rationally acceptable when the economy is above the steady state. Developing countries are hence trapped in low growth and political instability. Generally, cooperation is not rationally acceptable for both the government and the firm. As a result, a fair equilibrium attained is equivalent to the feedback Nash equilibrium because the trigger strategies which are employed to enforce cooperation degenerates easily into the feedback Nash equilibrium. The significant insight here is that capitalism may be more rationally acceptable and fairer compared to collectivism.

Hence, this model serves to illustrate the importance of rational acceptability in obtaining fair equilibrium. Existing literature takes for granted that cooperative solutions are always preferred to non-cooperative solutions in a static context. In contrast, we use the rational acceptability criteria to demonstrate that the existence of rational acceptable cooperative equilibrium solutions in dynamic games is not trivial. As a result, even if cooperative solutions may trump the non-cooperative solutions in some truncated subgames, the failure of arriving at some rationally acceptable fair imputations of the cooperative outcomes for each players for the overall game may undermine cooperation.

The feedback solution in which the rent of the firm is totally taxed away is admittedly extreme and unrealistic. In practice, the political pressure to redistribute is always present for both developing and developed countries though it is unlikely that the government will completely tax away the rent of the firm. Our key insight on this issue is that the consequences for taxing the firm are less dire for developed countries than developing countries.

Another key assumption in this paper is that both the government and the firm represent the interests of their respective principals effectively. The government in the model depends very much on voters (workers) for political support while the firm depends on their shareholders for support. As such, the government is an agent for the voters (workers) while the firm is an agent for their shareholders. How do the principals ensure that their respective agents will exert efforts to represent their interests? In this principal-agent scenario, the rational acceptability of the outcome may be contingent on the performance of the government and the firm as perceived by their principals. Effectively, this would be a multi-principals-agents dynamic game. Establishing rationally acceptable outcomes for players with different objective functionals and possibly different state equations is challenging and best reserved for future research.

As a caveat, it should be emphasized that this paper should not be perceived as a carte blanche endorsement of capitalism. Indeed, the dynamic inefficiency of capitalism is a real one so that it is always easy and tempting to argue for some collective solution in the face of such inefficiency. This is the case for China, where embracing capitalism has brought about phenomenal growth and widening disparity, prompting recent debates about bringing about a "harmonious society" through a return to some forms of collectivism. However, if capitalism is more rationally acceptable than collectivism, such collective solutions are bound to fail, as they had in the past. A policy implication would be that policymakers have to work harder to come up with more creative solutions to achieve a "harmonious society" within a capitalist framework rather than imposing rationally unacceptable collective solutions.

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