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Abstract

We consider a modification of ordinal status games of Haagsma and von Mouche (2010). A number of agents make scalar choices, e.g., their levels of conspicuous consumption. The wellbeing of each agent is affected by her choice in three ways: internal satisfaction, expenses, and social status determined by comparisons with the choices of others. In contrast to the original model, as well as its modifications considered so far, we allow for some players not caring about comparisons with some others. Assuming that the status of each player may only be "high" or "low," the existence of a strong Nash equilibrium is shown; for a particular subclass of such games, the convergence of Cournot tatonnement is established. If an intermediate status is possible, then even Nash equilibrium may fail to exist in very simple examples.

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Key words: status game; strong equilibrium; Nash equilibrium; Cournot tatonnement.

1 Introduction

The objective of this paper is quite narrow. We study what happens if the assumption that "everybody watches everybody" implicit in the status game of Haagsma and von Mouche (2010), as well as its modifications in Kukushkin and von Mouche (2018) and Kukushkin (2019), is abandoned so that some players may be indifferent to the very existence of some others.

The place of all those models in the literature (Frank, 1985; Akerlof, 1997; Clark and Oswald, 1998; Becker et al., 2005; Bilancini and Boncinelli 2008; Arrow and Dasgupta, 2009) can be delineated by these characteristics: the number of players is finite and the notion of status is ordinal. Roughly speaking, the status of a player is determined by comparisons (rather than differences) between her choice and the choices of others.

Those features generate unpleasant discontinuities in the utility functions, which make inapplicable even recently obtained general theorems (Reny 1999, 2016; McLennan et al., 2011; Prokopovych, 2013; Prokopovych and Yannelis, 2017; Kukushkin, 2018). Unsurprisingly, Haagsma and von Mouche (2010) were only able to show the existence of a Nash equilibrium in the two-person case. Kukushkin and von Mouche (2018) showed the existence of Nash equilibrium and convergence of (consecutive) Cournot tâtonnement in somewhat modified models, with only two possible status levels and the top tier consisting of the players whose choice was maximal of all.

Kukushkin (2019) established the existence of a *strong* Nash equilibrium, which weakly Pareto dominates all other Nash equilibria, in a wider class of similar games where the line between the "top"

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and the “bottom” may be drawn anywhere (e.g., at the median choice). Besides, Cournot tâtonnement in such games always finds a Nash equilibrium in a finite number of steps. If there are three possible status levels, the existence of a Nash equilibrium is ensured under an additional assumption; with a greater number of possible status levels, there seems to be no reasonably general sufficient condition for Nash equilibrium existence.

In this paper, we consider two ways to define a network structure on the set of players. In a “simple” model, there is an undirected graph with players as vertices and edges between “neighbors”; each player only observes the choices of neighbors and her status is determined by the order rank of her choice among theirs. Thus, the status of a player may not be visible to anybody else, which is hardly consistent with the usual connotations of “social status.”

This inconsistency is (largely) avoided in a more general model: Instead of drawing edges between players, a set of *communities* is added such that each player, typically, belongs to several communities, her status in each community is determined separately and is visible to all members of the community. The “final,” global status of each player is an aggregate of her statuses in all the communities she belongs to; inevitable subjectivity of that aggregation diminishes the relevance of the question of whether anybody else observes all those local statuses.

Not surprisingly, extending the class of models under consideration, we lose some “positive” results. First of all, even in a simple model with a linear graph and quite nice utility functions, there may be no Nash equilibrium if more than two status levels are allowed. Consequently, we restrict attention in this paper to “dichotomic ” games: a player’s status may be either “top” or “bottom” throughout.

The main findings are as follows. Exactly as when “everybody is everybody’s neighbor,” there exists a strong Nash equilibrium; however, the set of equilibrium utility profiles, even in a simple model, may have a non-trivial Pareto border. Cournot tâtonnement always finds a Nash equilibrium after a finite number of steps in every simple model. Whether that claim holds in the general case remains an open question.

In Section 2, some standard definitions are recalled; Section 3 provides the formal descriptions of our class of games. Our main results are in Section 4: Theorem 4.4 asserts the existence of a strong equilibrium; Theorem 4.5, the convergence of Cournot tâtonnement. The proofs of those theorems are deferred to Sections 6 and 7, respectively. “Counterexemples” in Section 5 show the impossibility of easy generalizations.

2 Basic notions

As usual, a *strategic game* is defined by a finite set N of *players*, and, for each $i \in N$, a set X_i of *strategies* and a real-valued *utility function* u_i on the set $X_N := \prod_{i \in N} X_i$ of *strategy profiles*. We denote $\mathcal{N} := 2^N \setminus \{\emptyset\}$ and $X_I := \prod_{i \in I} X_i$ for each $I \in \mathcal{N}$. Given $i \in N$, we use notation X_{-i} instead of $X_{N \setminus \{i\}}$; given $I \in \mathcal{N}$, X_{-I} instead of $X_{N \setminus I}$.

The *best response correspondences* $\mathcal{R}_i \colon X_{-i} \to 2^{X_i}$ ($i \in N$) are defined in the usual way:

$$\mathcal{R}_i(x_{-i}) := \operatorname*{Argmax}_{x_i \in X_i} u_i(x_i, x_{-i}).$$

A strategy profile $x_N^0 \in X_N$ is a (pure strategy) *Nash equilibrium* if $x_i^0 \in \mathcal{R}_i(x_{-i}^0)$ for all $i \in N$.

A (consecutive) *Cournot path* in a strategic game is a finite or infinite sequence of strategy profiles $\langle x_N^k \rangle_{k=0,1,\ldots}$ such that, whenever x_N^{k+1} is defined, there is an $i \in N$ for which $x_{-i}^k = x_{-i}^{k+1}$ and $x_i^k \notin$

$\mathcal{R}_i(x^k_{-i}) \ni x^{k+1}_i$. Following Milchtaich (1996), we say that a strategic game has the *finite best response improvement property* (*FBRP*) if it admits no infinite Cournot path. If, additionally, $\mathcal{R}_i(x_{-i}) \neq \emptyset$ for every $x_{-i} \in X_{-i}$, then every Cournot path, if extended whenever possible, ends at a Nash equilibrium.

A *Cournot cycle* is a Cournot path $\langle x^k_N \rangle_{k=0,1,\ldots,K}$ such that $K > 0$ and $x^K_N = x^0_N$. In a *finite* game, the FBRP is equivalent to the absence of Cournot cycles; generally, there holds only a one-way implication.

Given $x_N \in X_N$ and $I \in \mathcal{N}$, $y_I \in X_I$ is called a *weak coalitional improvement* at x_N if $u_i(y_I, x_{-I}) \geq u_i(x_N)$ for all $i \in I$ while $u_i(y_I, x_{-I}) > u_i(x_N)$ for at least one $i \in I$. A strategy profile $x^0_N \in X_N$ is a *very strong equilibrium* if there is no weak coalitional improvement at x^0_N.

3 Network status games

A *network status game* is a strategic game with a finite set of players N (we assume $n := \#N \geq 2$), where strategy sets X_i and utility functions u_i satisfy a number of specific requirements, which need auxiliary definitions and notations. There is a closed subset $X \subseteq \mathbb{R}$ ("conceivable strategies") and there are $a_i \leq b_i$ in $\mathbb{R}$ for each $i \in N$ such that $X_i = [a_i, b_i] \cap X$; hence each X_i is compact. Concerning the utility functions u_i, there are a finite chain S (of potential "status levels"), a "status mapping" $\sigma_i \colon X_N \to S$ and a function $U_i \colon X_i \times S \to \mathbb{R}$ for each $i \in N$, such that $u_i(x_N) = U_i(x_i, \sigma_i(x_N))$ for all $i \in N$ and $x_N \in X_N$. Each function $U_i(x_i, s)$ is assumed to be strictly increasing in s, and upper semicontinuous in x_i; moreover, it is *single-peaked* in x_i, i.e., there are $\hat{x}^s_i \in X_i$ for all $i \in N$ and $s \in S$ such that $U_i(x_i, s)$ strictly increases in x_i when $x_i \leq \hat{x}^s_i$ and strictly decreases when $x_i \geq \hat{x}^s_i$.

For Haagsma and von Mouche (2010), the status $\sigma_i(x_N)$ of each player $i \in N$ was, roughly speaking, her *order rank* $\rho_i(x_N) := \#\{j \in N \mid x_i \geq x_j\}$. Kukushkin and von Mouche (2018) and Kukushkin (2019) had $\sigma_i(x_N) := q \circ \rho_i(x_N)$, where $q \colon \{1, \ldots, n\} \to S$ was increasing, but not necessarily *strictly* increasing.

We consider two approaches to the definition of the status mapping for a network of players: a "simple" and a "sophisticated" ones. In the "simple" version, for each $i \in N$, there is a subset $G(i) \subseteq N$ such that $i \in G(i)$, and $i \in G(j)$ whenever $j \in G(i)$. Given $x_N \in X_N$, we define the *order rank* of each player $i \in N$ by

$$\rho_i(x_N) := \#\{j \in G(i) \mid x_i \geq x_j\}, \tag{1}$$

and her status by

$$\sigma_i(x_N) := q_i \circ \rho_i(x_N), \tag{2}$$

with $q_i \colon \{1, \ldots, \#G(i)\} \to S$ increasing, but not necessarily strictly.

In the "sophisticated" version, there is a finite set C of "communities" (or reference groups); there are correspondences $\Phi \colon N \to C$ and $\Psi \colon C \to N$ such that $c \in \Phi(i) \iff i \in \Psi(c)$. Given $x_N \in X_N$, each player $i \in N$ gets a separate order rank in each community from $\Phi(i)$,

$$\rho^c_i(x_N) := \#\{j \in \Psi(c) \mid x_i \geq x_j\} \quad (i \in \Psi(c)), \tag{3}$$

which determines her "local status" $\sigma^c_i(x_N) := q^c \circ \rho_i(x_N)$, with increasing $q^c \colon \{1, \ldots, \#\Psi(c)\} \to S^c$. Those local statuses are then converted into a "global" status $\sigma_i(x_N) := q_i(\langle \sigma^c_i(x_N) \rangle_{c \in \Phi(i)})$, with increasing $q_i \colon S^{\Phi(i)} \to S$.

A model where each player's status is defined in the "simple" way is called a *simple network status game*. A model where each player's status is defined in the "sophisticated" way is called a *general* *network status game*, or just a *network status game*. The consistency of this terminology is shown by the following statement.

Proposition 3.1. *For every simple network status game, there is a general network status game with the same set of players N, the same sets of strategies X_i, and the same utility functions u_i.*

Proof. Retaining the same sets N and X_i ($i \in N$), we set $C := \{\{i,j\} \subseteq 2^N \mid j \in G(i) \setminus \{i\}\}$ and, for each $i \in N$, $\Phi(i) := \{\{i,j\}\}_{j \in G(i)\setminus\{i\}}$. For each $\{i,j\} \in C$, we set $S^{\{i,j\}} := \{0,1\}$ and $\Psi(\{i,j\}) := \{i,j\}$. Whenever $i \in N$ and $c \in \Phi(i)$, we set $\sigma_i^c(x_N) := \rho_i^c(x_N) - 1$. Obviously, we now have

$$\rho_i(x_N) = 1 + \sum_{c \in \Phi(i)} \sigma_i^c(x_N),$$

with ρ_i defined by (1), for all $i \in N$ and $x_N \in X_N$. Applying the same mappings q_i, we obtain the same utility functions u_i. □

Since the inequalities in (1) and (3) are non-strict, both ρ_i and ρ_i^c, and hence σ_i too, are upper semicontinuous in x_i; the upper semicontinuity of u_i in x_i easily follows. Therefore, $\mathcal{R}_i(x_{-i}) \neq \emptyset$ for every $x_{-i} \in X_{-i}$.

The interpretation of the whole construction is quite similar to that in Haagsma and von Mouche (2010). Several agents choose their levels of consumption of a certain good. Exactly as in Kukushkin (2019), an arbitrary closed set $X \subseteq \mathbb{R}$ is introduced to stress that our results are equally valid for discrete models, e.g., where X is the set of integers, and continuous models, e.g., where $X = \mathbb{R}$ and each X_i is just a closed interval. The wellbeing of an agent is affected by her choice in three ways. First, there is "internal satisfaction," the more, the better. Second, there are expenses, the more, the heavier. Assuming decreasing marginal satisfaction and increasing marginal expenses, we obtain the strict quasiconcavity of the utility function in own choice.

Thirdly, there are considerations of status: an agent may feel humiliated when noticing a greater choice made by somebody else. In contrast to Haagsma and von Mouche (2010), as well as Kukushkin (2019), we do not assume that "everybody watches everybody" here. In a "simple" model, subsets $G(i)$ define "who is whose neighbor"; we could talk of an undirected graph instead. The rank function (1) defines the position of player i among her neighbors; q_i in (2) may be injective, and hence superfluous, or it may assign the same status for some contingent ranks ("consolidated status" of Kukushkin, 2019). In a "general" model, each player is a member of one or more of "reference groups"; within a group c, "everybody is everybody's neighbor" and each player's local status $\sigma_i^c(x_N)$ is determined in the same way as in Kukushkin (2019). Then each player aggregates her status levels in relevant groups, obtaining her global status $\sigma_i(x_N)$.

Lemma 3.2. *Let $x_N^0 \in X_N$ and $Y_*(x_N^0) := \{\hat{x}_i^s\}_{i \in N, s \in S} \cup \{x_i^0\}_{i \in N} \subseteq X$. Let $\langle x_N^k \rangle_{k=0,1,\dots,K}$ be a finite Cournot path starting at x_N^0. Then $x_N^K \in (Y_*(x_N^0))^N$.*

Proof. We start with an auxiliary statement: If $x_N \in (Y_*(x_N^0))^N$, $i \in N$, and $y_i \in \mathcal{R}_i(x_{-i})$, then $y_i \in Y_*(x_N^0)$. Supposing the contrary and denoting $s := \sigma_i(y_i, x_{-i})$, we have either $y_i < \hat{x}_i^s$, or $y_i > \hat{x}_i^s$. In the first case, we have $\sigma_i^c(\hat{x}_i^s, x_{-i}) \geq \sigma_i^c(y_i, x_{-i})$ for all $c \in \Phi(i)$; hence $\sigma_i(\hat{x}_i^s, x_{-i}) \geq s$, and hence $u_i(\hat{x}_i^s, x_{-i}) \geq U_i(\hat{x}_i^s, s) > U_i(y_i, s) = u_i(y_i, x_{-i})$. In the second case, since $y_i \neq x_j$ for all $j \neq i$, there is $y_i' \in X_i$ such that $y_i' < y_i$, $y_i' \geq \hat{x}_i^s$, and $\sigma_i^c(y_i', x_{-i}) = \sigma_i^c(y_i, x_{-i})$ for all $c \in \Phi(i)$; therefore, $u_i(y_i', x_{-i}) > u_i(y_i, x_{-i})$. In either case, we have a contradiction with $y_i \in \mathcal{R}_i(x_{-i})$.

Now the statement of the lemma is proven with a straightforward recursion, exactly as in Lemma 3.1 of Kukushkin (2019). $\square$

Lemma 3.3. *A network status game has the FBRP if and only if it admits no Cournot cycle.*

Exactly as in Lemma 3.2 of Kukushkin (2019), immediately follows from Lemma 3.2 and the fact that $Y_*(x_N)$ is finite for every $x_N \in X_N$.

Lemma 3.4. *Let $x_N, y_N \in X_N$ and $i \in N$ in a network status game be such that $x_i = y_i$, while $y_j \neq x_j$ for any $j \neq i$ only if $x_j \leq x_i \geq y_j$. Then $\sigma_i(y_N) = \sigma_i(x_N)$ and $u_i(y_N) = u_i(x_N)$.*

Proof. Whenever $c \in \Phi(i)$, we have $\rho_i^c(y_N) = \rho_i^c(x_N)$; hence $\sigma_i^c(y_N) = \sigma_i^c(x_N)$. Therefore, $\sigma_i(y_N) = \sigma_i(x_N)$ and hence $u_i(y_N) = u_i(x_N)$ as well. $\square$

4 Dichotomic games

Henceforth, we assume that $S = \{\bot, \top\} = S^c$ for all $c \in C$, i.e., there are only two status levels (bottom and top, $\bot < \top$). We call such games *dichotomic network status games.* For $x_N \in X_N$ and $i \in N$, we define

$$\xi_i(x_N) := \min\{y_i \in X_i \mid \sigma_i(y_i, x_{-i}) = \top\}, \tag{4}$$

the minimal strategy choice that would ensure for player i (under strategy profile x_N, or rather x_{-i}) the top status; we assume $\min \emptyset := +\infty$ in (4).

Lemma 4.1. *Let $x_N \in X_N$, $i \in N$, and $\xi_i(x_N) < +\infty$ in a dichotomic network status game. Then either $\xi_i(x_N) = \min X_i$ or $\xi_i(x_N) = x_j$ for some $j \neq i$.*

Proof. We denote $J := \{j \in N \setminus \{i\} \mid x_j \leq \xi_i(x_N)\}$. If $J = \emptyset$, then $\xi_i(x_N) = \min X_i$; otherwise, we denote $y_i := \max_{j \in J} x_j \; [\leq \xi_i(x_N)]$. If $y_i = \xi_i(x_N)$, then we are home. Supposing the contrary, we would have $\{j \in N \mid x_j \leq \xi_i(x_N)\} = \{j \in N \mid x_j \leq y_i\}$; hence $\rho_i^c(y_i, x_{-i}) = \rho_i^c(\xi_i(x_N), x_{-i})$ for all $c \in \Phi(i)$ and hence $\sigma_i(y_i, x_{-i}) = \sigma_i(\xi_i(x_N), x_{-i}) = \top$: a contradiction. $\square$

Lemma 4.2. *Let $x_N \in X_N$ and $i \in N$ in a dichotomic network status game. Then $x_i \in \mathcal{R}_i(x_{-i})$ if and only if one of the following conditions holds:*

$$x_i = \hat{x}_i^\top \geq \xi_i(x_N); \tag{5a}$$

$$x_i = \xi_i(x_N) > \hat{x}_i^\top \;\&\; U_i(\hat{x}_i^\bot, \bot) \leq U_i(\xi_i(x_N), \top); \tag{5b}$$

$$x_i = \hat{x}_i^\bot < \xi_i(x_N) \;\&\; U_i(\hat{x}_i^\bot, \bot) \geq U_i(\xi_i(x_N), \top). \tag{5c}$$

Proof. The sufficiency is straightforward. To prove the necessity, a few alternatives have to be considered. Let $\sigma_i(x_N) = \bot$, i.e., $x_i < \xi_i(x_N)$; if $x_i \neq \hat{x}_i^\bot$, then $u_i(x_N) < U_i(\hat{x}_i^\bot, \bot) \leq u_i(\hat{x}_i^\bot, x_{-i})$; if $x_i = \hat{x}_i^\bot$ and the second inequality in (5c) does not hold, then $u_i(x_N) < U_i(\xi_i(x_N), \top) = u_i(\xi_i(x_N), x_{-i})$. Let $\sigma_i(x_N) = \top$, i.e., $x_i \geq \xi_i(x_N)$; if $x_i > \xi_i(x_N)$, then (5a) must hold since $\sigma_i(y_i, x_{-i}) = \top$ for all $y_i \in X_i$ close enough to x_i; if $x_i = \xi_i(x_N)$, then (5b) must hold. $\square$

Remark. By Lemma 4.2, $\#\mathcal{R}_i(x_{-i}) \leq 2$ for every $i \in N$ and $x_{-i} \in X_{-i}$; $\#\mathcal{R}_i(x_{-i}) = 2$ is only possible when the non-strict inequalities in (5b) and (5c) are equalities.

Lemma 4.3. *Let $x'_N, y'_N \in X_N$ and $i \in N$ in a dichotomic network status game be such that $\xi_i(x'_N) < +\infty$ and, whenever $j \neq i$ and $y'_j \neq x'_j$, there holds $x'_j < \xi_i(x'_N) \geq y'_j$. Then $\xi_i(y'_N) = \xi_i(x'_N)$.*

Proof. Since $\{j \in N \setminus \{i\} \mid x'_j \leq \xi_i(x'_N)\} = \{j \in N \setminus \{i\} \mid y'_j \leq \xi_i(x'_N)\}$, we have $\rho_i^c(\xi_i(x'_N), y'_{-i}) = \rho_i^c(\xi_i(x'_N), x'_{-i})$ for all $c \in \Phi(i)$; hence $\sigma_i(\xi_i(x'_N), y'_{-i}) = \top$ and hence $\xi_i(y'_N) \leq \xi_i(x'_N)$.

If $\xi_i(x'_N) = \min X_i$, then we are home; otherwise, we denote $J := \{j \in N \setminus \{i\} \mid x'_j \neq y'_j\}$. If $J = \emptyset$, then $x'_{-i} = y'_{-i}$ and we are home again; otherwise, we denote $w := \max_{j \in J} x'_j \, [< \xi_i(x'_N)]$. For every $z_i \in X_i$ such that $w \leq z_i < x'_i$, we have $\{j \in N \setminus \{i\} \mid y'_j \leq z_i\} \subseteq \{j \in N \setminus \{i\} \mid x'_j \leq z_i\}$; hence $\sigma_i(z_i, y'_{-i}) \leq \sigma_i(z_i, x'_{-i}) = \perp$. Since z_i was arbitrary, $\xi_i(y'_N) \geq \xi_i(x'_N)$. □

Theorem 4.4. *Every dichotomic network status game possesses a very strong equilibrium.*

Theorem 4.5. *Every simple dichotomic network status game has the FBRP.*

Both proofs are deferred to Sections 6 and 7, respectively.

Remark. Kukushkin (2019) contained a wrong claim about *simultaneous* Cournot tâtonnement, see Kukushkin (2020).

Open Problem 4.6. *What additional assumptions are needed for the FBRP in the general case?*

Remark. A plausible conjecture is that the symmetry of each mapping $q_i \colon S^{\Phi(i)} \to S$ in its arguments (local statuses of player i) would be sufficient.

5 "Counterexemples"

The introduction of a network into an ordinal status game inflicts two losses in comparison with Kukushkin (2019): In the case of two status levels, a strong equilibrium existing by Theorem 4.4 need not Pareto dominate all other Nash equilibria. In the case of three status levels, Nash equilibria may fail to exist at all.

Example 5.1. Let $N = \{1, 2, 3, 4\}$; $G(1) = G(2) = \{1, 2, 3\}$, $G(3) = N$, $G(4) = \{3, 4\}$; $S = \{\perp, \top\}$; for each $i \in N$, $X_i = [0, 4]$; $q_1(3) = q_2(3) = q_3(4) = q_4(2) = \top$, while $q_i(r) = \perp$ everywhere else (i.e., only the maximal choice confers the top status); $U_i(x, s) = \varphi_i(x) + \psi_i(s)$, where $\varphi_1(x) = \varphi_2(x) = \min\{x, 6 - x\}$, $\varphi_3(x) = \varphi_4(x) = \min\{x, 4 - x\}$, $\psi_i(\perp) = 0$ for all $i \in N$, $\psi_1(\top) = \psi_2(\top) = 2$, $\psi_3(\top) = 1.5$, and $\psi_4(\top) = 0.5$. The set of Nash equilibria consists of two components: $\{(3+\varepsilon, 3+\varepsilon, 3+\varepsilon, 2)\}_{\varepsilon \in [0,1/2]}$ and $\{(3+\varepsilon, 3+\varepsilon, 2, 2)\}_{\varepsilon \in [1/2,1]}$

A unique very strong equilibrium is $(3, 3, 3, 2)$, which gives the players this utilities vector: $\langle 5, 5, 2.5, 2 \rangle$; however, every Nash equilibrium in the second component gives player 4 the utility level 2.5.

Example 5.2. Let $N = \{1, 2, 3, 4\}$; $X_i = [0, 2]$ for each $i \in N$; $G(1) = \{1, 2\}$, $G(2) = \{1, 2, 3\}$, $G(3) = \{2, 3, 4\}$, $G(4) = \{3, 4\}$; $S = \{1, 2, 3\}$ (with the natural order); each q_i be an identity mapping, i.e., $\sigma_i(x_N) = \rho_i(x_N)$ for each $i \in N$ and all $x_N \in X_N$; $U_1(x, s) = x + s$, $U_2(x, s) = -2x + 3s$, $U_3(x, s) = -2x^2 + (s-1)^2$, and $U_4(x, s) = \min\{x, 2 - x\} + s$.

Assuming x^0_N to be a Nash equilibrium, we must have $x^0_1 = 2$ and $x^0_4 \geq 1$. Further, $u_2(x^0_N) \geq U_2(2, 3) = 5 > U_2(x_2, 2)$ whenever $x_2 > 1/2$. Therefore, either $x^0_2 = 2$, or $1/2 \geq x^0_2 \geq x^0_3$. We consider both alternatives.

Let $x_2^0 = 2$. Then $U_3(2,3) = -4 < 0 = U_3(0,1) > -1 \geq U_3(x_4^0, 2)$. Therefore, $x_3^0 = 0$; but then $u_2(x_N^0) = U_2(2,3) = 5 < 6 = U_2(0,2) = u_2(x_2 = 0, x_{-2}^0)$: a contradiction.

Now let $1/2 \geq x_2^0 \geq x_3^0$. We have $x_4^0 = 1$; hence $\sigma_3(x_N^0) \leq 2$, and hence $u_3(x_N^0) \leq 1$. On the other hand, $u_3(x_3 = 1, x_{-3}^0) = U_3(1,3) = 2 > u_3(x_N^0)$: a contradiction again.

Thus, there is no Nash equilibrium in the game.

Without the symmetry assumption, the extension of Theorem 4.5 to the general case is just wrong.

Example 5.3. Let $N = \{1,2,3\}$; $C = \{\alpha, \beta, \gamma\}$; $\Phi(1) = \{\alpha, \beta\}$, $\Phi(2) = \{\alpha, \gamma\}$, $\Phi(3) = \{\beta, \gamma\}$; $S = \{1,2\}$; $X_i = [0,1]$ for each $i \in N$; $\sigma_1(x_N) = \rho_1^\alpha(x_N)$, $\sigma_2(x_N) = \rho_2^\gamma(x_N)$, $\sigma_3(x_N) = \rho_3^\beta(x_N)$; $U_i(x,s) = -x + 2s$.

The set of Nash equilibria is $\{(x,x,x)\}_{x \in [0,1]}$; only $(0,0,0)$ is a very strong equilibrium. Meanwhile, there are Cournot cycles, e.g., this:

$$\begin{array}{ccccc} (0,1,0) & \rightarrow & (1,1,0) & \rightarrow & (1,0,0) \\ \uparrow & & & & \downarrow \\ (0,1,1) & \leftarrow & (0,0,1) & \leftarrow & (1,0,1) \end{array}$$

6 Proof of Theorem 4.4

First of all, we define an auxiliary mapping $r_i \colon X_N \to X_i$ by $r_i(x_N) := \min \mathcal{R}_i(x_{-i})$. In light of Lemma 4.2, $r_i(x_N)$ only depends on $\xi_i(x_N)$.

Claim 6.1. *Let $x_N, y_N \in X_N$ and $i \in N$ be such that $x_i = r_i(x_N)$, $y_i = x_i$, and $y_j \neq x_j$ for any $j \neq i$ only if $x_j < x_i \geq y_j$. Then $x_i = r_i(y_N)$.*

Proof. By Lemma 3.4, we have $\sigma_i(y_N) = \sigma_i(x_N)$. By Lemma 4.2, one of conditions (5) holds for i at x_N. If it is (5a), then it will hold at y_N as well. If (5b) holds for i at x_N, i.e., $\sigma_i(x_N) = \top$ and $x_i = \xi_i(x_N) > \hat{x}_i^\top$, then Lemma 4.3 applies with $x'_N = x_N$ and $y'_N = y_N$, implying $\xi_i(y_N) = \xi_i(x_N)$ and hence (5b) holds for i at y_N as well. Finally, if (5c) holds for i at x_N, then $x_i = \hat{x}_i^\perp < \xi_i(x_N)$ and Lemma 4.3 applies with $x'_N = (\xi_i(x_N), x_{-i})$ and $y'_N = (\xi_i(x_N), y_{-i})$, implying $\xi_i(y_N) = \xi_i(x_N)$ and hence (5b) holds for i at y_N as well. $\square$

Now we recursively construct a sequence of strategy profiles $\langle x_N^k \rangle_{k=0,1,\ldots}$, which are to reach an equilibrium after a finite number of steps. The construction of the sequence does not presume it to be a Cournot path (a player might replace one best response with another); however, eventually it turns out to be so.

We start with setting $x_i^0 := \hat{x}_i^\top$ for each $i \in N$. Having $x_N^k \in X_N$ already defined, we define a number of auxiliary subsets: $I_k^\uparrow := \{i \in N \mid r_i(x_N^k) > x_i^k\}$; $I_k^= := \{i \in N \mid r_i(x_N^k) = x_i^k\}$; $I_k^\downarrow := \{i \in N \mid r_i(x_N^k) < x_i^k\}$; $I_k^{\neq} := I_k^\uparrow \cup I_k^\downarrow$. If $N = I_k^=$, then the process must stop (and x_N^k is a Nash equilibrium). Otherwise, we define $t_i^k := \max\{x_i^k, r_i(x_N^k)\}$ for each $i \in N$, $H^k := \max_{i \in I_k^{\neq}} t_i^k$, and $J^k := \operatorname{Argmax}_{i \in I_k^{\neq}} t_i^k$. If $J^k \cap I_k^\downarrow \neq \emptyset$, then we pick $\mathfrak{i}(k) \in J^k \cap I_k^\downarrow$; otherwise, we pick $\mathfrak{i}(k) \in J^k \subseteq I_k^\uparrow$. Finally, we set $x_{\mathfrak{i}(k)}^{k+1} := r_{\mathfrak{i}(k)}(x_N^k)$ and $x_i^{k+1} := x_i^k$ for $i \neq \mathfrak{i}(k)$.

Claim 6.2. *Whenever x_N^k and x_N^{k+1} are defined, there hold:*

$$H^{k+1} \leq H^k; \tag{6a}$$

$$\text{if } J^k \subseteq I_k^\uparrow \text{ and } H^{k+1} = H^k, \text{ then } J^{k+1} \subseteq I_{k+1}^\uparrow \text{ too.} \tag{6b}$$

Proof. We argue by induction. To be more precise, assuming that statements (6a) and (6b) hold for all $h < k$, we prove them for k. When $k = 0$, this assumption holds vacuously.

To prove (6a), we show that the inequality $t_i^{k+1} > H^k = t_{\mathfrak{i}(k)}^k$ is only possible when $i \in I_{k+1}^{=}$. Thus, let either $x_i^{k+1} = x_i^k > t_{\mathfrak{i}(k)}^k$, or $r_i(x_N^{k+1}) > t_{\mathfrak{i}(k)}^k$. In the first case, we have $t_i^k > H^k$ and hence $x_i^k = r_i(x_N^k)$; therefore, Claim 6.1 applies with $x_N = x_N^k$ and $y_N = x_N^{k+1}$, implying $x_i^{k+1} = x_i^k = r_i(x_N^{k+1})$ and hence $i \in I_{k+1}^{=}$. In the second case, Claim 6.1 applies with $x_N = (r_i(x_N^{k+1}), x_{-i}^{k+1})$ and $y_N = (r_i(x_N^{k+1}), x_{-i}^k)$, implying $r_i(x_N^k) = r_i(x_N^{k+1}) > t_{\mathfrak{i}(k)}^k$; therefore, $x_i^{k+1} = x_i^k = r_i(x_N^k) = r_i(x_N^{k+1})$ and hence $i \in I_{k+1}^{=}$ again.

To prove (6b), we assume that $J^k \cap I_k^{\downarrow} = \emptyset$ (hence $H^k = t_{\mathfrak{i}(k)}^k = x_{\mathfrak{i}(k)}^{k+1} > x_{\mathfrak{i}(k)}^k$) and consider $i \in N$ such that $x_i^k = x_i^{k+1} = x_{\mathfrak{i}(k)}^{k+1}$. Since $J^k \cap I_k^{\downarrow} = \emptyset$, we have $x_i^k \le r_i(x_N^k)$; on the other hand, $r_i(x_N^k) \le H^k = x_i^k$. Therefore, $x_i^{k+1} = x_i^k = r_i(x_N^k)$. Now Claim 6.1 applies with $x_N = x_N^k$ and $y_N = x_N^{k+1}$, implying $x_i^{k+1} = x_i^k = r_i(x_N^{k+1})$ as well. Thus, if $J^k \cap I_k^{\downarrow} = \emptyset$, then either $H^{k+1} < H^k$ or $J^{k+1} \cap I_{k+1}^{\downarrow} = \emptyset$. □

Claim 6.3. *Let $i \in N$ and $i = \mathfrak{i}(m)$ for some m. Then $x_i^{m+1} = r_i(x_N^k) = x_i^k$ and $u_i(x_N^{m+1}) = u_i(x_N^k)$ for all $k > m$.*

Proof. Let $i = \mathfrak{i}(m)$ for the first time; then $x_i^m = \hat{x}_i^{\top}$. If $x_i^{m+1} = r_i(x_N^m) > x_i^m$, then $H^m = x_i^{m+1}$, so Claim 6.2 ensures that Lemma 4.3 and Claim 6.1 apply with $x_N = x_N^{m+1}$ and $y_N = x_N^k$, implying $x_i^k = r_i(x_N^k)$ and $u_i(x_N^k) = u_i(x_N^{m+1})$. If $x_i^{m+1} = r_i(x_N^m) < x_i^m = \hat{x}_i^{\top}$, then (5c) holds for i at x_N^{m+1} and hence $\xi_i(x_N^{m+1}) = \xi_i(x_N^m) > x_i^m = H^m$. Now Lemma 4.3 applies with $x'_N = x_N^{m+1}$ and $y'_N = x_N^k$, implying $\xi_i(x_N^k) = \xi_i(x_N^m)$; therefore, $x_i^k = x_i^{m+1} = r_i(x_N^k)$ by (5c) and $u_i(x_N^k) = u_i(x_N^{m+1})$ by Lemma 3.4. □

Remark. It easily follows from Claim 6.3 that $\langle x_N^k \rangle_k$ *is* a Cournot path after all.

Claim 6.3 implies that the process stops at some stage $K \le n$, i.e., $I_K^{=} = N$; we denote $\mathcal{K} := \{0, 1, \dots, K-1\}$. Claim 6.3 also allows us to define a converse mapping $\varkappa\colon \mathfrak{i}(\mathcal{K}) \to \mathcal{K}$ such that $\varkappa(\mathfrak{i}(k)) = k$ for all $k \in \mathcal{K}$. The way our construction is organized ensures that x_N^K is a Nash equilibrium. To complete the proof of the theorem, let us show that x_N^K is a very strong equilibrium as well.

Let $\emptyset \ne I \subseteq N$ and $y_I \in X_I$ be a weak coalitional improvement at x_N^K. First of all, we may, without restricting generality, assume that $y_i \ne x_i^K$ for each $i \in I$. We denote $y_N := (y_I, x_{-I}^K)$.

Claim 6.4. *For each $i \in I$, there is $j \in I$ such that $j \ne i$ and $x_j^K \ge \xi_i(x_N^K) > \xi_i(y_N)$.*

Proof. Since $x_i^K = r_i(x_N^K)$, one of conditions (5) must hold for i at x_N^K. It cannot be (5a) since no improvement would be possible; therefore, it must be either (5b) or (5c). In either case, for y_I to be an improvement, we must have $\sigma_i(y_N) = \top$ and hence $\xi_i(x_N^K) > \xi_i(y_N)$. An assumption that $x_j^K < \xi_i(x_N^K)$ for all $j \in I$ would, by Lemma 4.3, imply $\xi_i(y_N) = \xi_i(x_N^K)$. □

We denote $I^* := \operatorname{Argmax}_{i \in I} x_i^K$.

Claim 6.5. *For each $i \in I^*$, there hold:* (a) $i \in \mathfrak{i}(\mathcal{K})$; (b) $y_i < x_i^K$; (c) $x_i^{\varkappa(i)} < x_i^{\varkappa(i)+1} = \xi_i(x_N^{\varkappa(i)}) = x_i^K$.

Proof. An assumption that (5c) holds for i at x_N^K would lead to $x_j^K > x_i^K$ for $j \in I$ from Claim 6.4, which is impossible. Therefore, (5b) holds for i at x_N^K; both (a) and (b) immediately follow. Now (c) follows from the fact that $x_i^{\varkappa(i)} = \hat{x}_i^{\top}$, while $x_i^K = x_i^{\varkappa(i)+1} = \xi_i(x_N^{\varkappa(i)}) > \hat{x}_i^{\top}$ by (5b). □

We pick player $i \in I^*$ whose strategy is determined first, i.e., $\{i\} = \mathrm{Argmin}_{j \in I^*} \varkappa(j)$, and denote $k := \varkappa(i)$. Thus, $x_i^K = x_i^{k+1} = \xi_i(x_N^k) > x_i^k = \hat{x}_i^\top$. By the choice of i, we have $x_j^k = x_j^{k+1} < x_i^{k+1} = \xi_i(x_N^{k+1})$ for each $i \in I \setminus \{i\}$. Now, Claim 6.1 implies that $x_i^{k+1} = r_i(y_N)$, while Lemma 3.4 implies that $u_i(x_i^{k+1}, y_{-i}) = u_i(x_N^{k+1}) = u_i(x_N^K)$. Therefore, the inequality $u_i(y_N) > u_i(x_N^K)$ cannot hold.

Remark. In the case of a *complete* network, i.e., a simple model with $G(i) = N$ for each $i \in N$, the proof of Theorem 4.4 reduces to that of Theorem 1 from Kukushkin (2019).

7 Proof of Theorem 4.5

In light of Lemma 3.3, it is enough to show the impossibility of Cournot cycles. Supposing, to the contrary, $\langle x_N^k \rangle_{k=0,1,\ldots,K}$ to be a Cournot cycle, we denote $\mathcal{K} := \{0, 1, \ldots, K\}$ and $\mathcal{K}(i) := \{k \in \mathcal{K} \mid x_{-i}^k = x_{-i}^{k+1}\}$ for $i \in N$. Then, we partition N into those players participating in the cycle, $N^* := \{i \in N \mid \mathcal{K}(i) \neq \emptyset\}$, and everybody else, $N^0 := N \setminus N^* = \{i \in N \mid \forall k, h \in \mathcal{K}\, [x_i^k = x_i^h]\}$. Whenever $i \in N^*$, we have $\#\mathcal{K}(i) \geq 2$ and $G^*(i) := G(i) \cap N^* \setminus \{i\} \neq \emptyset$ (otherwise, there could be no reason for player i to change her strategy back and forth).

We denote $m_k := \max_{i \in N^*} \xi_i(x_N^k)$ for each $k \in \mathcal{K}$, $M^+ := \max_{k \in \mathcal{K}} m_k$, and $I^+ := \{i \in N^* \mid \exists k \in \mathcal{K}\, [x_i^k = M^+]\}$.

Claim 7.1. *$m_k = M^+$ for all $k \in \mathcal{K}$.*

Proof. Supposing the contrary, we may, without restricting generality, assume that $\xi_i(x_N^0) \leq m_0 < m_1 = M^+ = \xi_i(x_N^1)$ for an $i \in N^*$. Then there must be $j \in G^*(i)$ such that $0 \in \mathcal{K}(j)$, i.e., $x_{-j}^1 = x_{-j}^0$, and $x_j^0 \leq m_0 < m_1 \leq x_j^1$. Since $\xi_j(x_N^1) = \xi_j(x_N^0) \leq m_0 < m_1$, we must have $\sigma_j(x_N^1) = \top$ and $x_j^1 = \hat{x}_i^\top$. Thus, the utility of player j attains its global maximum at x_N^1. Moreover, $x_j^1 \geq M^+ \geq \xi_j(x_N^k)$ for all k; hence the choice of x_j^1 ensures the maximal utility of player j forever. Therefore, x_j^1 could not be replaced with $x_j^K = x_j^0$ at any stage: a contradiction. $\square$

Claim 7.2. *If $x_i^k > M^+$ for some $i \in N$ and $k \in \mathcal{K}$, then $i \in N^0$.*

Proof. If $i \in N^*$, $h \in \mathcal{K}(i)$, and $x_i^{h+1} = x_i^k > M^+$, then $u_i(x_N^{h+1}) = \max_{x_N \in X_N} u_i(x_N)$ and hence no change of strategy x_i could happen later, exactly as in the proof of Claim 7.1. $\square$

For each $i \in I$, we denote $g_i := \#G(i)\ [\leq n]$ and $\beta_i := \min q_i^{-1}(\top)$ (the minimal order rank that ensures the top status for player i). Note that $\xi_i(x_N)$ is uniquely defined by these two inequalities:

$$\#\{j \in G^*(i) \mid x_j \geq \xi_i(x_N)\} > g_i - \beta_i; \tag{7a}$$

$$\#\{j \in G^*(i) \mid x_j > \xi_i(x_N)\} \leq g_i - \beta_i. \tag{7b}$$

Claim 7.3. *$I^+ \neq \emptyset$.*

Proof. Let $\xi_i(x_N^k) = M^+$ for some $i \in N^*$ and $k \in \mathcal{K}$ (by Claim 7.1, such an i exists for each k). We denote $J := \{j \in G(i) \mid x_j^k \geq M^+\}$. If $J \subseteq N^0$, then $\xi_i(x_N^h) \geq M^+$ for each $h \in \mathcal{K}$; hence $\xi_i(x_N^h) = M^+$ for all h by Claim 7.1; hence x_i^h cannot change all along by Lemma 4.2: a contradiction. Otherwise, $\emptyset \neq N^* \cap J \subseteq I^+$ by Claim 7.2. $\square$

For each $i \in I^+$, we denote $\nu_i := \#\{j \in G(i) \cap N^0 \mid \forall k \in \mathcal{K}\, [x_j^k \geq M^+]\}$ (the number of neighbors of i whose choice is always M^+ or greater). For each $i \in I^+$ and $k \in \mathcal{K}$, we set $\chi_i^k := 0$ if $x_i^k < M^+$, $\chi_i^k := 1$ if $x_i^k = M^+$, and $s_i^k := \sum_{j \in G^*(i) \cap I^+} \chi_j^k$.

Claim 7.4. *Let $i \in I^+$ and $k \in \mathcal{K}(i)$. If $x_i^{k+1} = M^+$, then $\nu_i + s_i^k + \beta_i > g_i$. If $x_i^k = M^+$, then $\nu_i + s_i^k + \beta_i \leq g_i$.*

Proof. Since $i \in I^+$, there must be $h \in \mathcal{K}(i)$ such that $x_i^{h+1} = M^+$. By the definition of M^+, we have $\sigma_i(x_N^{h+1}) = \top$. If $\hat{x}_i^\top > M^+$, then $x_i^{h+1} = M^+$ could not be optimal; if $\hat{x}_i^\top = M^+$, then player i would obtain at x_N^{h+1} her maximal utility, which would imply the same contradiction as in the proof of Claim 7.1. Thus, $\hat{x}_i^\top < M^+$ and $u_i(M^+, \top) \geq u_i(\hat{x}_i^\perp, \perp)$.

Now, if $x_i^{k+1} = M^+$, then $x_i^k < M^+ = \xi_i(x_N^k)$; hence $g_i - \beta_i < \#\{j \in G(i) \mid [x_j^k \geq M^+]\} = \nu_i + s_i^k$ by (7a). If $x_i^k = M^+$, then $x_i^{k+1} = \xi_i(x_N^k) < M^+$; hence $g_i - \beta_i \geq \#\{j \in G(i) \setminus \{i\} \mid [x_j^k \geq M^+]\} = \nu_i + s_i^k$ by (7b). $\square$

Finally, we define a function $H \colon \mathcal{K} \to \mathbb{R}$ by

$$H(k) := \frac{1}{2} \sum_{i \in I^+} \sum_{j \in G^*(i) \cap I^+} \chi_i^k \cdot \chi_j^k \; + \sum_{i \in I^+} \chi_i^k \cdot (\nu_i + \beta_i - g_i - 1/2). \tag{8}$$

Claim 7.5. *For each $k \in \mathcal{K}$, there holds $H(k+1) \geq H(k)$. If $k \in \mathcal{K}(i)$ with $i \in I^+$, and $x_i^k = M^+$ or $x_i^{k+1} = M^+$, then $H(k+1) > H(k)$.*

Proof. If $k \in \mathcal{K}(i)$ with $i \notin I^+$, then nothing changes in either sum in the right-hand side of (8). Let $k \in \mathcal{K}(i)$ with $i \in I^+$. Then $H(k)$ can be re-written as

$$H(k) = \chi_i^k \cdot \left(s_i^k \; + \; (\nu_i + \beta_i - g_i - 1/2)\right) \; + \; \mathcal{H}_i(x_{-i}^k), \tag{9}$$

where $\mathcal{H}_i(x_{-i}^k)$ is the sum of all terms in the right-hand side of (8) that do not contain χ_i^k. Quite similarly,

$$H(k+1) = \chi_i^{k+1} \cdot \left[s_i^{k+1} \; + \; (\nu_i + \beta_i - g_i - 1/2)\right] \; + \; \mathcal{H}_i(x_{-i}^{k+1}). \tag{10}$$

Note that $s_i^k = s_i^{k+1}$ and $\mathcal{H}_i(x_{-i}^k) = \mathcal{H}_i(x_{-i}^{k+1})$ since $x_{-i}^k = x_{-i}^{k+1}$.

We consider three alternatives. If $x_i^k < M^+ > x_i^{k+1}$, then $\chi_i^k = \chi_i^{k+1} = 0$; hence $H(k) = \mathcal{H}_i(x_{-i}^k) = \mathcal{H}_i(x_{-i}^{k+1}) = H(k+1)$ and we are home. If $x_i^k < M^+ = x_i^{k+1}$, then $H(k) = \mathcal{H}_i(x_{-i}^k)$, while

$$H(k+1) = s_i^{k+1} \; + \; (\nu_i + \beta_i - g_i - 1/2) \; + \; \mathcal{H}_i(x_{-i}^{k+1}). \tag{11}$$

By Claim 7.4, we have $H(k+1) > \mathcal{H}_i(x_{-i}^{k+1}) = \mathcal{H}_i(x_{-i}^k) = H(k)$ (note that $\nu_i + \beta_i - g_i$ is an integer). If $x_i^k < M^+ = x_i^{k+1}$, then

$$H(k) = s_i^k \; + \; (\nu_i + \beta_i - g_i - 1/2) \; + \; \mathcal{H}_i(x_{-i}^k). \tag{12}$$

By Claim 7.4, we have $H(k) < \mathcal{H}_i(x_{-i}^k) = \mathcal{H}_i(x_{-i}^{k+1}) = H(k+1)$. $\square$

Remark. An attentive reader will undoubtedly recognize the trick working in the proof of Claim 7.5 (Huang, 2002; Dubey, Haimanko, and Zapechelnyuk, 2006; Kukushkin, 2005; Jensen, 2010).

The obvious contradiction between Claim 7.3 and Claim 7.5 proves Theorem 4.5.

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