A Multivariate GARCH-Jump Mixture Model

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December 2020

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November 2020

Abstract

This paper proposes a new parsimonious multivariate GARCH-jump (MGARCH-jump) mixture model with multivariate jumps that allows both jump sizes and jump arrivals to be correlated among assets. Dependent jumps impact the conditional moments of returns as well as beta dynamics of a stock. Applied to daily stock returns, the model identifies co-jumps well and shows that both jump arrivals and jump sizes are highly correlated. The jump model has better predictions compared to a benchmark multivariate GARCH model.

Key words: Multivariate GARCH, Jumps, Multinomial, Co-jump, beta dynamics, Value-at-Risk

*Maheu thanks SSHRC of Canada for financial support.

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1 Introduction

There is a large literature that has focused on the estimation and importance of jumps in asset returns. Most of this work has focused on jumps in individual assets. How jump arrivals and jump sizes affect each other among several assets remains unclear. This paper proposes a new model for jumps in multiple assets. It allows both jump arrival and jump sizes to be contemporaneously correlated. We find that although jumps arrive infrequently in daily data, assets display dependence in jump arrival. When assets jump together, their jump size is strongly positively correlated. Thus co-jumps among assets can change the benefits of diversification and present an additional source of systematic risk.

This paper builds on the literature that models jumps in discrete time and combines them with stochastic volatility or GARCH volatility dynamics. The latter specifications capture the smooth predictable component of volatility while jumps capture abnormal moves in stock prices. This literature began with the compounded Poisson model introduced by Press (1967). Numerous extensions include ARCH (Jorion, 1988) and GARCH (Vlaar and Palm, 1993; Nieuwland et al., 1994) and stochastic volatility (Bates, 1996, 2000; Pan, 2002). Time-varying jump intensities has been consider by Chan and Maheu (2002); Maheu and McCurdy (2004), and the implications of jumps for asset pricing by Duan et al. (2006); Maheu et al. (2013); Christoffersen et al. (2012); Bates (2000); Pan (2002) among others.

Introducing jumps will affect the conditional mean, conditional variance as well as higher-order conditional moments such as skewness and kurtosis (Das and Sundaram, 1997). This captures the empirical fact that the unconditional distribution of stock returns is skewed and leptokurtic relative to a normal distribution. Jumps are especially helpful in explaining large extreme return changes like market crushes.

Although this literature has focused on univariate jumps, there is empirical evidence of co-jumps among several assets. Bollerslev et al. (2008) identify the existence of co-jumps and provide a test for co-jumps in multiple assets. Gilder et al. (2014) confirm co-jumps and provide another test. Additional papers exploit high frequency data to perform jump tests.[1]

Papers modelling multivariate jumps are sparse. Laurini and Mauad (2015) propose a bivariate SV model with built-in co-jumps, but idiosyncratic jumps are not allowed in the model while Chua and Tsiaplias (2019) introduce another model with correlated jump sizes but independent jump arrivals and autocorrelated jump intensities. Aït-Sahalia et al. (2015) use a mutually exciting jump processes to model contagion among assets but have independent jump sizes and homoskedastic diffusive components.

[1] Mancini and Gobbi (2012) suggest a nonparametric estimator based on realized covariation. Similarly, Aït-Sahalia and Xiu (2016) decompose quadratic variation into continuous and discontinuous component to estimate co-jumps. Bibinger and Winkelmann (2015); Winkelmann et al. (2016) also concentrate on extracting co-jump from quadratic covariation and introduce a truncated estimator. Caporin et al. (2017) further apply this estimator in a higher dimensional experiment. Other attempts are Gobbi and Mancini (2007) to derive a bivariate parametric co-jump estimator, and Novotnỳ and Urga (2017) to introduce a new approach to test the existence of co-jumps.

This paper proposes a new model (MGARCH-jump) in which one stochastic component of returns follows a multivariate GARCH (MGARCH) specification while the second is a jump innovation component. The jump component allows for all possible combinations of jumps to occur from a single individual jump to co-jumps among several or all assets. Each of the probabilities of these jump events are allowed to differ and admits deviations from purely independent jumps among all assets. The jump size is multivariate normal and potentially correlated. The conditional moments of returns are derived and show how multivariate jumps impact returns. We design a Markov chain Monte Carlo sampler to simulate from the posterior density of the model. The estimation approach allows for inference on both jump arrival and jump size.

Several applications of the model to daily return data are reported. There is strong evidence of co-jumps among assets and the jump size distribution displays strong positive correlation among assets. In applications of a stock with its industry portfolio and the market, co-jumps are the most likely jump event. Co-jumps are the result of dependence in jump arrival between assets and are not independent.

In applications with the market portfolio, including multivariate jumps changes beta dynamics. Dependent jumps generally lowers beta compared to that from an MGARCH model. Log-Bayes factors favour the new model compared to an MGARCH model without jumps.

Applied to five large firms from very different industries, jump arrivals are mostly individual firm events unless it is a market-wide event in which a five assets jump together. Even in this setting of diverse firms we find strong evidence of jump dependence in both arrival and through a correlated jump size distribution. These jump dependencies result in different conditional correlations and risk measures compared to an MGARCH model with no jumps.

This paper is organized as follows. Section 2 describes the MGARCH-jump model and how dependent jumps affect the conditional moments of returns. Section 3 outlines a posterior simulation method to estimate the model parameters and jumps sizes and jump arrivals. Computation of the predictive density and predictive likelihood are reviewed. Section 4 presents the data. Section 5 presents a series of trivariate applications of a firm, its corresponding industry portfolio and the market portfolio. Beta dynamics for the model is discussed and illustrated in Section 6. Section 7 shows that the model can be used in higher dimensions in this case five assets. Section 8 concludes. An Appendix collects additional derivations.

2 Model

In this section, we present the discrete time MGARCH-jump model for financial returns. The model has a multinomial jump arrival and a multivariate normal jump size component. Let $\boldsymbol{r}_t = (r_{t,1},\ r_{t,2},\ \cdots,\ r_{t,N})'$ be a $N \times 1$ vector of returns at time t. $\boldsymbol{r}_t$ is specified as

$$\boldsymbol{r}_t = \boldsymbol{\mu} + \boldsymbol{\epsilon}_t, \tag{1}$$

$$\boldsymbol{\epsilon}_t = \boldsymbol{\epsilon}_{1,t} + \boldsymbol{\epsilon}_{2,t}, \tag{2}$$

where $\boldsymbol{\mu} = (\mu_1, \mu_2, \ldots, \mu_N)'$ is a $N \times 1$ vector of constant drift terms, $\boldsymbol{\epsilon}_{1,t}$ is a $N \times 1$ return innovation with $\mathrm{E}\left(\boldsymbol{\epsilon}_{1,t} | \boldsymbol{r}_{1:t-1}\right) = 0$, where $\boldsymbol{r}_{1:t-1} = \{\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_{t-1}\}$. In particular,

$$\boldsymbol{\epsilon}_{1,t} = \boldsymbol{H}_t^{1/2} \boldsymbol{z}_t, \qquad \boldsymbol{z}_t \sim NID\left(\boldsymbol{0}, \boldsymbol{I}\right), \tag{3}$$

where $\boldsymbol{H}_t^{1/2}$ is the Cholesky decomposition of a $N \times N$ conditional covariance matrix following a multivariate GARCH structure. Define $\boldsymbol{J}_t = (J_{t,1}, J_{t,2}, \ldots, J_{t,N})'$ as a $N \times 1$ vector of jumps, with $J_{t,i}$ being the jump for asset i, which further is a product of a jump arrival indicator and a jump size variable. The second stochastic component of returns is $\boldsymbol{\epsilon}_{2,t}$, a $N \times 1$ vector of jump innovations,

$$\boldsymbol{\epsilon}_{2,t} = \boldsymbol{J}_t - \mathrm{E}\left(\boldsymbol{J}_t | \boldsymbol{\Theta}, \boldsymbol{r}_{1:t-1}\right), \tag{4}$$

where $\boldsymbol{\Theta}$ is the union set of all parameters and $\mathrm{E}\left(\boldsymbol{\epsilon}_{2,t} | \boldsymbol{r}_{1:t-1}\right) = 0$. Note that the conditional expectation of jumps is removed from the model so $\mathrm{E}\left(\boldsymbol{r}_t | \boldsymbol{r}_{1:t-1}\right) = \boldsymbol{\mu}$ for all t. This feature provides a constant drift without jump effecting the conditional (Merton, 1976). $\boldsymbol{\epsilon}_{1,t}$ and $\boldsymbol{\epsilon}_{2,t}$ are contemporaneously independent from each other.

2.1 Vector-Diagonal GARCH (VD-GARCH)

We use a slightly modified version of the vector diagonal GARCH (VD-GARCH) model introduced by Ding and Engle (2001):

$$\boldsymbol{H}_t = \boldsymbol{C}\boldsymbol{C}' + \boldsymbol{\alpha}\boldsymbol{\alpha}' \odot \boldsymbol{\epsilon}_{t-1}\boldsymbol{\epsilon}_{t-1}' + \boldsymbol{\beta}\boldsymbol{\beta}' \odot \boldsymbol{H}_{t-1}, \tag{5}$$

where $\odot$ is the Hadamard product operator that performs element-by-element multiplication, $\boldsymbol{C}$ is an $N \times N$ lower triangular matrix and both $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $N \times 1$ vectors of parameters. $\boldsymbol{\epsilon}_{t-1}$ includes both MGARCH component shocks $\boldsymbol{\epsilon}_{1,t-1}$ and jump shocks $\boldsymbol{\epsilon}_{2,t-1}$. This means that both shocks will propagate into the MGARCH structure and impact future covariances. Although it is natural to consider only $\boldsymbol{\epsilon}_{1,t-1}$ entering the MGARCH recursion, this requires the separation of the two shocks making inference much more difficult.

The VD-GARCH specification is a simplified version of the BEKK model (Engle and Kroner, 1995) and inherits the property that guarantees $\boldsymbol{H}_t$ to be positive definite if the startup value $\boldsymbol{H}_0$ is positive definite. Each element $h_{t,ij}$ in matrix $\boldsymbol{H}_t$ follows,

$$h_{t,ij} = \omega_{ij} + \alpha_i \alpha_j \epsilon_{t-1,i} \epsilon_{t-1,j} + \beta_i \beta_j h_{t-1,ij}, \tag{6}$$

where $\omega_{ij} = \left(\boldsymbol{C}\boldsymbol{C}'\right)_{ij}$. Given positive definite coefficient matrices, covariance stationarity holds if of $\alpha_i^2 + \beta_i^2 < 1 \; \forall i$ (Ledoit et al., 2003) in a standard no-jump VD-GARCH model.

2.2 A Multinomial Jump Structure

Most of the past univariate jump models parameterize jumps as a compound Poisson process follow Press (1967). Although a Poisson process fits well in univariate continuous-time models, it is not easily extended to higher dimension with sufficient flexibility and dependence. While empirically observed data is discrete in time, a Bernoulli jump is a good discrete approximation of a Poisson process over a small time interval (Ball and Torous, 1983). One convenient feature of a Bernoulli jump is that it's much easier to generalize into the multivariate setting. We use a multinomial distribution jump indicator with one trial to index all possible jump/co-jump combination patterns among stocks. Therefore, define

$$\boldsymbol{J}_t = \boldsymbol{Y}_t \odot \boldsymbol{B}_t, \tag{7}$$

$$\boldsymbol{Y}_t \sim N\left(\boldsymbol{\mu}_J, \boldsymbol{\Sigma}_J\right), \tag{8}$$

where $\boldsymbol{Y}_t$ is a $N \times 1$ vector of jump sizes that are multivariate normally distributed with mean vector $\boldsymbol{\mu}_J$ and covariance matrix $\boldsymbol{\Sigma}_J$. $\boldsymbol{B}_t = (B_{t,1}, B_{t,2}, \ldots, B_{t,N})'$ is a $N \times 1$ vector of jump indicators with each element $B_{t,i} \in \{0, 1\}$ and 1 being a jump and otherwise being no jump for all $i = 1, \ldots, N$. Let $L = 2^N$ denotes the number of all possible jump events among the N assets then,

$$\boldsymbol{B}_t \sim multinomial\left(1, p_1, \ldots, p_L\right), \tag{9}$$

where $\sum_{j=1}^{L} p_j = 1$. $multinomial\left(n, q_1, \ldots, q_l\right)$ denotes a multinomial distribution with n trials and event probabilities $q_1, \ldots, q_L$. The parameter p_j is the jump/co-jump probability. Unlike univariate models where the jump intensity parameter represents the probability of jump arrivals, in this specification, the jump/co-jump probability p_j is a separate probability assigned to each possible jump/co-jump $\boldsymbol{B}_t$ outcome. To be more specific, define a $2^N \times N$ matrix $\boldsymbol{\Omega}_B$ that contains all possible outcomes of $\boldsymbol{B}_t$, with each row being one exclusive possible value of $\boldsymbol{B}_t'$, and $\boldsymbol{p} = (p_1, p_2, \ldots, p_L)'$ is a vector of corresponding jump probabilities. For example, in a trivariate case, there are $2^3 = 8$ possible outcomes of $\boldsymbol{B}_t$: one trivariate co-jump $(1,1,1)'$; three bivariate co-jumps $(1,1,0)'$, $(1,0,1)'$, and $(0,1,1)'$; three idiosyncratic jumps $(1,0,0)'$, $(0,1,0)'$, and $(0,0,1)'$; and one no jump outcome $(0,0,0)'$. This covers all possible jump patterns including all-asset co-jumps and subset co-jumps. Each outcome is associated with one probability element in $\boldsymbol{p}$.

One merit of this specification is that one can easily verify whether the jumps are cross-sectionally independent through these probabilities. Our empirical results show that the jump arrivals are clearly correlated cross-sectionally.

Besides jump arrivals, the multivariate normal structure naturally connects jump sizes among assets through $\boldsymbol{\mu}_J$ and $\boldsymbol{\Sigma}_J$. As a result, in this model, one can easily extract the correlation of jump arrivals and that of jump sizes separately, so question like "whether and when do they jump together" and "how do they jump together" can be answered explicitly.

2.3 Conditional Moments

The first two conditional moments of jump $\boldsymbol{J}_t$ are[2]

$$\mathrm{E}\left(\boldsymbol{J}_t|\boldsymbol{\Theta},\boldsymbol{r}_{1:t-1}\right)=\boldsymbol{\mu}_J\odot\boldsymbol{\Omega}_B{}'\boldsymbol{p}=\boldsymbol{\mu}_J\odot\left(\sum_{j=1}^{2^N}\boldsymbol{\Omega}_j p_j\right), \tag{10}$$

and

$$\mathrm{Cov}\left(\boldsymbol{J}_t|\boldsymbol{\Theta},\boldsymbol{r}_{1:t-1}\right)=\left(\boldsymbol{\Sigma}_J+\boldsymbol{\mu}_J\boldsymbol{\mu}_J{}'\right)\odot\left(\sum_{j=1}^{2^N}p_j\boldsymbol{\Omega}_j\boldsymbol{\Omega}_j{}'\right)-\boldsymbol{\mu}_J\boldsymbol{\mu}_J{}'\odot\boldsymbol{\Omega}_B{}'\boldsymbol{p}\boldsymbol{p}'\boldsymbol{\Omega}_B, \tag{11}$$

where $\boldsymbol{\Theta}=\{\boldsymbol{\mu},\boldsymbol{\theta}_H,\boldsymbol{p},\boldsymbol{\mu}_J,\boldsymbol{\Sigma_J}\}$, and $\boldsymbol{\Omega}_j$ is the jth row of $\boldsymbol{\Omega}_B$. Similarly, the first two conditional moments of return are

$$\mathrm{E}\left(\boldsymbol{r}_t|\boldsymbol{\Theta},\boldsymbol{r}_{1:t-1}\right)=\boldsymbol{\mu}, \tag{12}$$

$$\mathrm{Cov}\left(\boldsymbol{r}_t|\boldsymbol{\Theta},\boldsymbol{r}_{1:t-1}\right)=\boldsymbol{H}_t+\mathrm{Cov}\left(\boldsymbol{J}_t|\boldsymbol{\Theta},\boldsymbol{r}_{1:t-1}\right). \tag{13}$$

Now jumps impact not only the conditional variance of returns but also the conditional covariance and correlations through jumps arrival dependence and jump size dependence – something missing in univariate jump applications.

How jumps impact moments ex-post can be seen from the following conditional moments given the jump event $\boldsymbol{B}_t$. Note that conditional on $\boldsymbol{B}_t$, returns follow a multivariate normal distribution. The first two conditional moments are:

$$\mathrm{E}\left(\boldsymbol{r}_t|\boldsymbol{B}_t,\boldsymbol{\Theta},\boldsymbol{r}_{1:t-1}\right)=\boldsymbol{\mu}+\boldsymbol{\mu}_J\odot\left(\boldsymbol{B}_t-\boldsymbol{\Omega}_B{}'\boldsymbol{p}\right) \tag{14}$$

$$\mathrm{Cov}\left(\boldsymbol{r}_t|\boldsymbol{B}_t,\boldsymbol{\Theta},\boldsymbol{r}_{1:t-1}\right)=\boldsymbol{H}_t+\boldsymbol{B}_t\boldsymbol{B}_t{}'\odot\boldsymbol{\Sigma}_J \tag{15}$$

Because $\boldsymbol{B}_t\boldsymbol{B}_t{}'$ is positive semi-definite, and both $\boldsymbol{H}_t$ and $\boldsymbol{\Sigma}_J$ are positive definite, the conditional covariance of $\boldsymbol{r}_t$ is also positive definite. To be more specific,

$$\mathrm{E}\left(\boldsymbol{r}_t|\boldsymbol{B}_t,\boldsymbol{\Theta},\boldsymbol{r}_{1:t-1}\right)=\boldsymbol{\mu}+\begin{pmatrix}B_{t,1}\mu_{J,1}\\ B_{t,2}\mu_{J,2}\\ \vdots\\ B_{t,N}\mu_{J,N}\end{pmatrix}-\boldsymbol{\mu}_J\odot\boldsymbol{\Omega}_B{}'\boldsymbol{p} \tag{16}$$

[2]Derivations can be found in Appendix A.

$$\mathrm{Cov}\left(\boldsymbol{r}_t|\boldsymbol{B}_t,\boldsymbol{\Theta},\boldsymbol{r}_{1:t-1}\right)=\boldsymbol{H}_t+\begin{pmatrix} B_{t,1}^2\sigma_{J,1}^2 & B_{t,1}B_{t,2}\sigma_{J,12} & \cdots & B_{t,1}B_{t,N}\sigma_{J,1N} \\ B_{t,2}B_{t,1}\sigma_{J,21} & B_{t,2}^2\sigma_{J,2}^2 & \cdots & B_{t,2}B_{t,N}\sigma_{J,2N} \\ \vdots & \vdots & \ddots & \vdots \\ B_{t,N}B_{t,1}\sigma_{J,N1} & B_{t,N}B_{t,2}\sigma_{J,N2} & \cdots & B_{t,N}^2\sigma_{J,N}^2 \end{pmatrix}. \tag{17}$$

Clearly,

$$B_{t,i}B_{t,j}=\begin{cases} 1 & \text{if } B_{t,i}=B_{t,j}=1 \\ 0 & \text{otherwise.} \end{cases} \tag{18}$$

This determines which element(s) in $\boldsymbol{\mu}_J$ and $\boldsymbol{\Sigma}_J$ are turned on from a co-jump. The corresponding element $\mu_{J,i}$ and $\sigma_{J,i}^2$ will be turned on if and only if asset i jumps, and $\sigma_{J,ij}$, where $i\neq j$, will be turned on if and only if asset i and asset j both jump at the same time. This property helps to capture the co-jump behaviour among assets and reflect it directly to return covariances. If there's no jump for all N assets, then $\boldsymbol{B}_t=(0,0,\ldots,0)'$, so $\mathrm{E}\left(\boldsymbol{r}_t|\boldsymbol{B}_t,\boldsymbol{\Theta},\boldsymbol{r}_{1:t-1}\right)=\boldsymbol{\mu}-\boldsymbol{\mu}_J\odot\boldsymbol{\Omega}_B{}'\boldsymbol{p}$ and $\mathrm{Cov}\left(\boldsymbol{r}_t|\boldsymbol{B}_t,\boldsymbol{\Theta},\boldsymbol{r}_{1:t-1}\right)=\boldsymbol{H}_t$. If all N assets jump, then $\boldsymbol{B}_t=(1,1,\ldots,1)'$, so $\mathrm{E}\left(\boldsymbol{r}_t|\boldsymbol{B}_t,\boldsymbol{\Theta},\boldsymbol{r}_{1:t-1}\right)=\boldsymbol{\mu}+\boldsymbol{\mu}_J-\boldsymbol{\mu}_J\odot\boldsymbol{\Omega}_B{}'\boldsymbol{p}$ and $\mathrm{Cov}\left(\boldsymbol{r}_t|\boldsymbol{B}_t,\boldsymbol{\Theta},\boldsymbol{r}_{1:t-1}\right)=\boldsymbol{H}_t+\boldsymbol{\Sigma}_J$. In other cases, only a sub-block of $\boldsymbol{\Sigma}_J$ is turned on. For instance, in a trivariate case with a bivariate co-jump occurring, say $\boldsymbol{B}_t=(1,1,0)'$, two elements in $\boldsymbol{\mu}_J$ and four elements in $\boldsymbol{\Sigma}_J$ are turned on:

$$\mathrm{E}\left(\boldsymbol{r}_t|\boldsymbol{B}_t,\boldsymbol{\Theta},\boldsymbol{r}_{1:t-1}\right)=\boldsymbol{\mu}+\begin{pmatrix}\mu_{J,1}\\ \mu_{J,2}\\ 0\end{pmatrix}-\boldsymbol{\mu}_J\odot\boldsymbol{\Omega}_B{}'\boldsymbol{p}$$

$$\mathrm{Cov}\left(\boldsymbol{r}_t|\boldsymbol{B}_t,\boldsymbol{\Theta},\boldsymbol{r}_{1:t-1}\right)=\boldsymbol{H}_t+\begin{pmatrix}\sigma_{J,1}^2 & \sigma_{J,12} & 0\\ \sigma_{J,21} & \sigma_{J,2}^2 & 0\\ 0 & 0 & 0\end{pmatrix}.$$

This is consistent with the intuition that conditional mean and variance can only be affected when the corresponding asset jumps, and conditional covariance can only be affected when the two corresponding assets jump together. Obviously, this model supports all jump/co-jump possibilities and channels the jump/co-jump effects into conditional moments. As for conditional correlations, they are $\frac{h_{t,ij}+B_{t,i}B_{t,j}\sigma_{J,ij}}{\sqrt{\left(h_{t,ii}+B_{t,i}^2\sigma_{J,i}^2\right)\left(h_{t,jj}+B_{t,j}^2\sigma_{J,j}^2\right)}}$. For a co-jump between asset i and j this is $\frac{h_{t,ij}+\sigma_{J,ij}}{\sqrt{\left(h_{t,ii}+\sigma_{J,i}^2\right)\left(h_{t,jj}+\sigma_{J,j}^2\right)}}$ and the impact on the correlation depends on the size of the MGARCH components $h_{t,ii}$, $h_{t,jj}$, and $h_{t,ij}$. Clearly, the co-jumps can but do not necessarily increases conditional correlations. Thus jumps can have important effects on the diversification benefits in a portfolio.

3 Estimation

This model consists of two latent variables, $\boldsymbol{Y}_t$ and $\boldsymbol{B}_t$. We estimate the model from a Bayesian perspective using Markov chain Monte Carlo (MCMC) methods to sample the parameters and latent variables. We select proper uninformative priors for all parameters. This facilitates Gibbs sampling steps for some parameters. The prior choices are:

$$\begin{aligned}\boldsymbol{\mu} &\sim N(\mathbf{0}, 100\boldsymbol{I})\\ \boldsymbol{\theta}_H &\sim N(\mathbf{0}, 100\boldsymbol{I})\\ \boldsymbol{p} &\sim Dir(1,\ldots,1)\\ \boldsymbol{\mu_J} &\sim N(\mathbf{0}, 100\boldsymbol{I})\\ \boldsymbol{\Sigma_J} &\sim IW(N+2, \boldsymbol{I}).\end{aligned}$$

A full MCMC run contains $M_0 + M$ iterations, where the first $M_0 = 10000$ are burn-in samples, and the rest $M = 10000$ are posterior draws. Each MCMC iteration samples from the following conditional distributions:

1. $\boldsymbol{\mu}|\boldsymbol{r}_{1:t}, \boldsymbol{\mu}_J, \boldsymbol{\Sigma}_J, \boldsymbol{B}_{1:t}, \boldsymbol{p}$.
2. $\boldsymbol{\theta}_H|\boldsymbol{r}_{1:t}, \boldsymbol{\mu}, \boldsymbol{\mu}_J, \boldsymbol{\Sigma}_J, \boldsymbol{B}_{1:t}, \boldsymbol{p}$, where $\boldsymbol{\theta}_H = (\boldsymbol{C}, \boldsymbol{\alpha}, \boldsymbol{\beta})'$.
3. $\boldsymbol{B}_{1:t}|\boldsymbol{r}_{1:t}, \boldsymbol{\mu}, \boldsymbol{\mu}_J, \boldsymbol{\Sigma}_J, \boldsymbol{p}$,
4. $\boldsymbol{p}|\boldsymbol{r}_{1:t}, \boldsymbol{\mu}, \boldsymbol{\mu}_J, \boldsymbol{\Sigma}_J, \boldsymbol{B}_{1:t}$.
5. $\boldsymbol{Y}_{1:t}|\boldsymbol{r}_{1:t}, \boldsymbol{\mu}, \boldsymbol{\mu}_J, \boldsymbol{\Sigma}_J, \boldsymbol{B}_{1:t}, \boldsymbol{p}$.
6. $\boldsymbol{\mu}_J|\boldsymbol{r}_{1:t}, \boldsymbol{\mu}, \boldsymbol{\Sigma}_J, \boldsymbol{Y}_{1:t}, \boldsymbol{B}_{1:t}, \boldsymbol{p}$.
7. $\boldsymbol{\Sigma}_J|\boldsymbol{\mu}_J, \boldsymbol{Y}_{1:t}$.

Steps 3, 5, 7 are simply Gibbs samplers, and steps 1, 2, 4, 6 are Metropolis-Hastings (MH) due to unknown type of posterior distributions. Although sampling $\boldsymbol{p}$ and $\boldsymbol{\mu}_J$ are often a Gibbs step in a univariate jump model here they require an MH step from the condition $E(\boldsymbol{\epsilon}_{2,t}|\boldsymbol{r}_{1:t-1}) = 0$. Similarly, $\boldsymbol{\mu}$ enters both the conditional mean and the MGARCH recursion necessitating a MH step. Details of each sampling step can be found in Appendix B.

From the posterior draws $\{\boldsymbol{\mu}^{(i)}, \boldsymbol{\theta}^{(i)}, \boldsymbol{B}_{1:t}^{(i)}, \boldsymbol{p}^{(i)}, \boldsymbol{Y}_{1:t}^{(i)}, \boldsymbol{\mu}_J^{(i)}, \boldsymbol{\Sigma}^{(i)}\}_{i=1}^{M}$, posterior quantities of interest can be estimated. For instance, simulation consistent estimates of jump arrivals ($\boldsymbol{B}_t$) and jump sizes ($\boldsymbol{Y}_t$) can be estimated as:

$$\mathrm{E}(\boldsymbol{B}_t|\boldsymbol{r}_{1:t}) \approx \frac{1}{M}\sum_{i=1}^{M}\boldsymbol{B}_t^{(i)}$$

and

$$\mathrm{E}(\boldsymbol{Y}_t|\boldsymbol{r}_{1:t}) \approx \frac{1}{M}\sum_{i=1}^{M}\boldsymbol{Y}_t^{(i)}.$$

3.1 Predictive Likelihood

From the posterior simulation, it is straightforward to compute the predictive density of returns and the predictive likelihood which evaluates the predictive density at the realized data. Recall that $\boldsymbol{\Theta} = \{\boldsymbol{\mu}, \boldsymbol{\theta}_H, \boldsymbol{p}, \boldsymbol{\mu}_J, \boldsymbol{\Sigma_J}\}$ and the predictive likelihood is computed by integrating out all parameters $\boldsymbol{\Theta}$ and unobserved variables. From equation (14) and (15), the conditional distribution of returns conditional on jump arrivals is simply a multivariate normal distribution.

The predictive likelihood for $\boldsymbol{r}_{t+1}$ integrates out all future jump possibilities and parameter uncertainty as

$$\begin{aligned} p\left(\boldsymbol{r}_{t+1}|\boldsymbol{r}_{1:t}\right) &= \int\int p\left(\boldsymbol{r}_{t+1}|\boldsymbol{r}_{1:t}, \boldsymbol{\Theta}, \boldsymbol{B}_{t+1}\right) p\left(\boldsymbol{B}_{t+1}|\boldsymbol{r}_{1:t}, \boldsymbol{\Theta}\right) p\left(\boldsymbol{\Theta}|\boldsymbol{r}_{1:t}\right) \mathrm{d}\boldsymbol{\Theta}\mathrm{d}\boldsymbol{B}_{t+1} \\ &\approx \frac{1}{M}\sum_{i=1}^{M}\sum_{j=1}^{L} N\left(\boldsymbol{r}_{t+1}|\boldsymbol{\mu}^{(i)} + \boldsymbol{\mu}_J^{(i)} \odot \left(\boldsymbol{\Omega}_j - \boldsymbol{\Omega}_B{}'\boldsymbol{p}^{(i)}\right), \boldsymbol{H}_{t+1}^{(i)} + \boldsymbol{\Omega}_j\boldsymbol{\Omega}_j{}' \odot \boldsymbol{\Sigma}_J^{(i)}\right) p_j^{(i)}, \end{aligned} \quad (19)$$

where $N(\boldsymbol{r}_{1+1}|\boldsymbol{a}, \boldsymbol{A})$ denotes the multivariate normal density with mean $\boldsymbol{a}$, covariance $\boldsymbol{A}$ and evaluated at $\boldsymbol{r}_{t+1}$. From each of the posterior draws $\{\boldsymbol{\Theta}^{(i)}\}_{i=1}^{M}$, we integrate out all possible jump events $\boldsymbol{\Omega}_j$, $j = 1, \ldots, L$, with $\boldsymbol{\Omega}_j$ a row in $\boldsymbol{\Omega}_B$ and associated jump probability p_j.

The log-predictive likelihood for $\boldsymbol{r}_{s:t}$, $s < t$ is

$$\log p\left(\boldsymbol{r}_{s:t}|\boldsymbol{r}_{1:t-1}\right) = \sum_{l=s}^{t} \log p\left(\boldsymbol{r}_l|\boldsymbol{r}_{1:l-1}\right). \quad (20)$$

From this we can formally compare models based on log-predictive Bayes factors, which is the difference of log-predictive likelihoods for two models.

It is straightforward to simulate draws $\{\boldsymbol{r}_{t+1}^{(i)}\}_{i=1}^{M}$ from the predictive density $p\left(\boldsymbol{r}_{t+1}|\boldsymbol{r}_{1:t}\right)$. For every MCMC parameter draw $\boldsymbol{\Theta}^{(i)}$, we simulate a jump event followed by the return as,

$$\boldsymbol{B}_{t+1}^{(i)}|\boldsymbol{r}_{1:t}, \boldsymbol{\Theta}^{(i)} \sim Multinomial\left(1, p_1^{(i)}, \ldots, p_L^{(i)}\right) \quad (21)$$

$$\boldsymbol{r}_{t+1}^{(i)}|\boldsymbol{r}_{1:t}, \boldsymbol{\Theta}^{(i)}, \boldsymbol{B}_{t+1}^{(i)} \sim N\left(\boldsymbol{\mu}^{(i)} + \boldsymbol{\mu}_J^{(i)} \odot \left(\boldsymbol{B}_{t+1}^{(i)} - \boldsymbol{\Omega}_B{}'\boldsymbol{p}^{(i)}\right), \boldsymbol{H}_{t+1}^{(i)} + \boldsymbol{B}_{t+1}^{(i)}\boldsymbol{B}_{t+1}^{(i)}{}' \odot \boldsymbol{\Sigma}_J^{(i)}\right). \quad (22)$$

Repeating this yields a set of samples $\{\boldsymbol{r}_{t+1}^{(i)}\}_{i=1}^{M}$ from the predictive density of returns.

4 Data

We consider two main sets of applications. The first is to several trivariate systems for an individual stock and its corresponding industry and market portfolio. This allows us to consider the impact of jumps on the industry and the market. Daily returns from General Electric (GE), Exxon (XOM), Wal-Mart (WMT), Microsoft (MSFT) and American Express (AXP) are selected from the Center for Research in Security Prices (CRSP) database. The value-weighted market portfolio (MKT) is used as the market portfolio while industry portfolios are from the Fama-French 49 industry portfolios and the risk-free rate from Kenneth French's website. In order to match each stock with its corresponding industry portfolio, SIC codes of the above stocks are also acquired from CRSP. Data ranges from January 1, 1990 to December 31, 2016, with 6805 observations in total. Earning announcement dates are gathered from I/B/E/S database. The second application is to the five individual stocks GE, XOM, WMT, MSFT and AXP.

Table 1 illustrates descriptive statistics of daily continuously compounded returns in percent for the selected stocks as well as the value-weighted market portfolio.

5 Individual Stocks, Industry and the Market Co-Jumps

5.1 Estimation

The first example is to estimate the trivariate model each for GE, XOM, WMT, MSFT and AXP stock coupled with their corresponding industry and the market respectively. Table 2 reports the results for these trivariate estimates. All the posteriors are in reasonable regions with small intercepts μ_i (0.02 – 0.05), low MGARCH α_i parameters (0.15 – 0.20), and high β_i parameters (0.97 – 0.98). This results in a volatility persistence measure of $\alpha_i^2 + \beta^2$ of about 0.98 for each group of stocks.

All trivariate models indicates that "no jump" is the most likely outcome. No-jump probabilities ($p_{\overline{STK},\overline{IND},\overline{MKT}}$) ranges from 0.82 to 0.88. The jump size variances are large, often in excess of the sample variances of individual stocks in Table 1. The other jump size variances are substantial as well with industries ($\sigma^2_{J,IND}$) ranging from 1.61 to 3.66 and the market ($\sigma^2_{J,MKT}$) ranging from 1.00 to 1.52. Jump size covariances are all positive and also relatively large, with covariance for stocks and corresponding industry ($\sigma_{J,STK,IND}$) ranging from 1.83 to 4.25, for stocks and the market ($\sigma_{J,STK,MKT}$) ranging from 1.03 to 3.08, for industries and the market ($\sigma_{J,IND,MKT}$) ranging from 1.09 to 2.43. This confirms the fact that jumps are rare but extreme movements in stock returns.

To investigate jump dependence among assets, we report the co-jump joint probability which allows for dependence along with the co-jump probability derived from the marginal jump probabilities. This latter quantity is derived from summing over all jump events in which the stock jumps. From the basic probability rules, if jumps are cross-sectionally independent, a co-jump joint probability should be equal to the product of marginal jump probabilities for the corresponding assets.

Panel A of Table 3 compares the co-jump joint probabilities with the product of its marginal probabilities. The co-jump probabilities range from 0.0595 to 0.0984, while the product of marginal probabilities ranges from 0.0007 to 0.0250. Clearly, jump arrivals are strongly correlated as the joint probabilities and product of marginal probabilities are very different from each other. The differences are even greater when the number of assets in a co-jump is greater. For example, the bivariate co-jump probabilities of GE and its industry, GE and the market, GE's industry and the market are 0.0984, 0.0980, 0.0986 respectively, while the products of marginal jump probabilities are 0.0181, 0.0159, 0.0121, respectively. They are very different but still the same magnitude. In contrast, the joint probability of a trivariate co-jump with GE, its industry and the market jump all together is 0.0954, while the product of marginal jump probabilities is 0.0019, 50 times less than the corresponding co-jump probability.

Panel B further computes the co-jump probabilities conditional on different univariate jumps. This indicates the proportion of co-jumps an asset has given that the asset jumps. The results show that if the market jumps, each selected stock and its industry will most likely jump as well. GE, WMT, and AXP are more likely to jump along with the market when unusual conditions occur, more than half of jumps in XOM and MSFT coincide with market jumps. For XOM, WMT, and MSFT, their industries are most likely to jump together with them, with probabilities of co-jump with their industries conditional on stock jumps being 0.9496, 0.9517 and 0.9783 respectively (see Stk,Mkt column of table). When WMT jumps, the whole market is very likely to follow, with a probability of co-jump with the market conditional on stock jumps being 0.8102. GE and AXP also have strong influence on their industry when they jump, with co-jump probability conditional on stock jumps of 0.6392 and 0.5457 respectively.

Figure 1 plots the posterior probability of jumps for each of the five stocks with their corresponding industry and the market. Most of the jump arrivals are aligned together, which confirms the results in panel B of Table 3.

Figure 2 plots jump size realizations over time. The figure shows jump size realizations are relatively large (up to 10% and −10%) and infrequent. The results are more clear if we focus on a small time span. Take AXP from January 1, 2007 to December 31, 2009 as an example shown in Figure 3. The jump probability is usually high around quarterly earnings announcement dates (vertical lines). Beyond that, progression of sub-prime mortgage crisis plays an important role on jump dynamics. For instance: on March 13, 2007, reacting to the potential risk of sub-prime mortgages, causes a −2.93% jump in AXP, a −2.73% jump in the banking industry and a −1.86% jump in the market. On November 1, 2007, after a previous interest rate cut, the Federal Reserve injected 41 billion dollars into the money supply with a response of −3.14% AXP jump, −3.49% industry jump and −2.12% market jump. On September 29, 2008, the House of Representatives rejected the bailout plan, accompanying with a −5.24% jump in AXP, a −3.85% jumps of the industry and a −3.18% market jump. All the above jumps have posterior jump probabilities greater than 0.9.

The top panel of Figure 4 plots pairwise posterior jump probabilities of two assets from the AXP trivariate model. The second panel displays a scatter plot of posterior jump sizes for two assets also from the AXP model. The top three plots of the jump probabilities include a 45-degree line. Independent jump arrivals would display a random pattern along vertical and horizontal lines. Instead we see clear dependence of jump arrival in all three plots. Between AXP and the industry, there are many cases in which they jump together but also many jumps in AXP with no jump in the industry portfolio. As for AXP and the market, if the market jumps, AXP jumps almost all the time as well. The strongest dependence is found in the industry and the market. The bottom three plots display jump sizes among assets with the linear regression line of the vertical axis variable against the horizontal axis variable. In all three cases, the points cluster quite close along the regression line, indicating the jump sizes are highly correlated.

Table 4 reports the jump size correlations for the five selected stocks with their industry and the market. The first observation is that for each trivariate system all jump sizes are positively correlated. All the five stocks are highly correlated with their corresponding industry, and each of the five industries is also highly correlated with the market when co-jump arrives. GE, XOM, and AXP strongly follows the market in jump sizes, while WMT and MSFT are just moderately correlated with the market. The relatively low jump size correlation between WMT and the market is probably because of the defensive nature of WMT in business cycle, while that between MSFT and the market is more likely due to the comparably lower stock market co-jump probability. The high jump size correlations imply that when extreme events, for example crisis, occur, diversification benefits may be greatly affected as the overall correlation among asset returns could be significantly altered by jumps. Details are further discussed in Section 7.

5.2 Prediction

This subsection compares the forecasts between the MGARCH-jump model and a benchmark MGARCH model with no jumps by computing their predictive likelihood respectively. These predictive likelihoods are computed for the five trivariate systems. The last 100 observations (Aug 10, 2016 – Dec 30, 2016) are used for out-of-sample density forecast evaluation and prediction is implemented by one period ahead recursive forecasting, following equation (19) to (20).

Log-Bayes factor is computed by subtracting the log-predictive likelihoods of the benchmark MGARCH model from that of the MGARCH-jump model. A rule of thumb of this measure is that if log-Bayes factor is greater than 5, then the evidence for the MGARCH-jump is considered as very strong. Table 5 lists the log-predictive likelihoods and log-Bayes factors from different cases. The MGARCH-jump model dominates the benchmark MGARCH model in all five cases, with log-Bayes factors from around 12.70 to 61.81,

Figure 5 plots the log-predictive likelihood contribution $\log p\left(\boldsymbol{r}_t|\boldsymbol{r}_{1:t-1}\right)$ at each t in the out-of-sample period. During normal days, both model performs very similarly due to the same VD-GARCH component; while in days with large returns, the predictive likelihood is significantly greater for the MGARCH-jump model.

6 Beta Dynamics

Consider a bivariate volatility model for excess returns of an individual stock and the market. We derive the dynamic beta from the associated conditional covariance matrix of returns. Compared to the dynamic beta from an MGARCH model (Engle, 2016), in the presence of jumps, we can compute an ex-ante and ex-post beta.

Based on results from Section 2.3, let $\tilde{\boldsymbol{r}}_t = (\tilde{r}_{t,i}, \tilde{r}_{t,m})'$, where $\tilde{r}_{t,i}$ is the excess return of an arbitrary asset i and $\tilde{r}_{t,m}$ is the excess return of the market. The ex-ante beta can be derived directly from the appropriate conditional covariance and conditional variance in (13). An ex-post version uses

$$\mathrm{Cov}\left(\tilde{\boldsymbol{r}}_t | \boldsymbol{B}_t, \boldsymbol{\Theta}, \boldsymbol{r}_{1:t-1}\right) = \begin{pmatrix} h_{t,ii} + B_{t,i}^2 \sigma_{J,i}^2 & h_{t,im} + B_{t,i} B_{t,m} \sigma_{J,im} \\ h_{t,im} + B_{t,i} B_{t,m} \sigma_{J,im} & h_{t,mm} + B_{t,m}^2 \sigma_{J,m}^2 \end{pmatrix}. \tag{23}$$

So an ex-post beta is

$$\beta_{t,i} = \begin{cases} \frac{h_{t,im}+\sigma_{J,im}}{h_{t,mm}+\sigma_{J,m}^2} & \text{both jump} \\ \frac{h_{t,im}}{h_{t,mm}+\sigma_{J,m}^2} & \text{only market jumps} \\ \frac{h_{t,im}}{h_{t,mm}} & \text{otherwise} \end{cases} \tag{24}$$

This definition agrees with how beta relates to systematic risk. When the market does not jump, there's no change in systematic risk, so beta, which measures the exposure to systematic risk, is not affected. If only the market jumps, then the stock's relative exposure to the market decreases and so does beta. If there is a co-jump, both market risk and stock risk increase, and the effect on beta depends on values in the jump size covariance matrix. Now systematic risk transfers through h_{im} and $\sigma_{J,im}$ when co-jumps occur. Since a single stock is usually riskier than the market, ex-post beta is more likely to increase when co-jump occurs.

As seen in the last Section, co-jumps are the dominate jump event and therefore the ex-post beta should be mostly greater than ex-ante beta when jumps arrive. Figure 6 plots beta dynamics computed from bivariate models with excess returns of AXP and the market. The MGARCH-jump model separates the variance into the two components and results in a generally smaller ex-ante beta compared to the benchmark MGARCH model as seen in the figure.

7 Co-jumps among Individual Stocks

The next application is to estimate a 5-dimensional model with GE, XOM, WMT, MSFT, and AXP all together. Table 6 lists posterior results for the MGARCH-jump model. Again, the posterior estimates are in reasonable regions with low intercept ($\boldsymbol{\mu}$ of 0.03–0.06), low $\boldsymbol{\alpha}$ parameter (0.11–0.15) and high $\boldsymbol{\beta}$ parameter (0.98). As shown in Panel B of Table 6, the jump probabilities strongly favour "no jump" ($p_{\overline{GE},\overline{XOM},\overline{WMT},\overline{MSFT},\overline{AXP}} = 0.7102$) while in Panel C, and jump size variances are large (all greater than 4.6). Furthermore, the probability of only one stock jumping while others do not is higher than that of any co-jumps, as the former are all above 0.024 and the latter are generally below 0.01 with the only exception of a 5-asset mutual jump probability of 0.017. This suggests that systematic co-jumps and individual idiosyncratic jumps are important to these stocks.

Panel A of Table 7 compares the joint co-jump probability and the product of corresponding marginal univariate jump probabilities. Even for this diverse set of stocks there is clear evidence of jump dependence. In many cases, the joint probability differs from the jump probability from the marginals by at least an order of magnitude. For instance for GE, XOM, WMT, and MSFT, the joint and marginal probability of jumps is 0.0022 vs 0.0001, respectively. The most likely co-jump event is when all stocks jump together. Following this bivariate jumps are most likely from XOM, WMT; and GE, MSFT.

Table 8 lists jump size correlations among the five stocks. The jump size correlations are high with XOM having smaller correlations. As mentioned before, these jump size correlations could significantly change the overall return correlations.

To see how jumps can impact correlations, Figure 7 plots the differences in correlations between the full jump model and the MGARCH component from the jump model. This difference is computed based on covariances of ex-ante and ex-post jumps. These differences are usually around zero (no jump or very low probability of jump), but they can also go up to 0.4 and down to -0.4 as a result of jumps. As seen in the figure, jumps mostly reduce ex-ante correlations among assets compared to the MGARCH component ($\boldsymbol{H}_t$) of the model. This is generally consistent with the ex-ante beta from the jump model being lower than the beta from a MGARCH model with no jumps as seen in Figure 6. However, there are a substantial number of days in which jumps do increase ex-ante correlations. This appears to occur when correlation levels are low.

The last row of Table 5 reports the log-Bayes factor for the MGARCH-jump model relative to the MGARCH model is 67.43. This is strong evidence for the presence of jumps. The bottom right plot in Figure 5 shows several influential observations for the jump model.

7.1 Impact on Value-at-Risk

The value-at-risk (VaR_α) at level α, or the α quantile of a portfolio can be easily computed from the multivariate predictive density of returns as follows. Simulate a set of draws $\{\boldsymbol{r}_{t+1}^{(i)}\}_{i=1}^{M}$ from the predictive density following Section 3.1 and for each draw form $r_{p,t+1}^{(i)} = \boldsymbol{w}'\boldsymbol{r}_{t+1}^{(i)}$. In our example we consider an equally-weighted portfolio with $\boldsymbol{w} = (1/N, \ldots, 1/N)'$, $N = 5$. The $\lfloor M\alpha \rfloor$-th smallest value of $\{\boldsymbol{r}_{p,t+1}^{(i)}\}_{i=1}^{M}$ is an estimate of the VaR with significance level α.

Figure 8 plots the VAR for $\alpha = 0.10$, 0.05, and 0.01 for an equally-weighted portfolio from the predictive density one day ahead for both the MGARCH-jump model and the MGARCH model.

In these 100 out-of-sample days, the main difference in the models is seen with $\alpha = 0.01$ where the MGARCH-jump model VaR is always lower than the MGARCH model. This is a result of the fatter tails the jump model generates.

8 Conclusion

This paper proposes a new multivariate GARCH-jump mixture model that allows for dependent jumps among the set of assets. The model allows for all possible jump combinations and explicitly allows for different jump probabilities for each jump event. Jump sizes are allowed to be correlated as well.

We show how dependent jumps impact the conditional moments of returns. A posterior simulation method is presented that allows for estimation of parameters and jump events and jump sizes. In several applications, we show that jumps are generally infrequent but strongly dependent when they occur. For instance, all stocks jumping is one of the more common jump events as are bivariate jumps. This model provides superior density forecasts and we discuss how a stock's beta and value-at-risk is affected from multivariate jumps.

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Table 1: Descriptive Statistics for Daily Returns

	GE	XOM	WMT	MSFT	AXP	MKT
Means	0.0372	0.0409	0.0421	0.0737	0.0379	0.0353
Std. Dev.	1.7608	1.4729	1.6679	2.0451	2.2313	1.1099
Skewness	0.0338	0.0657	0.1050	0.0217	0.0039	-0.3445
Ex. Kurtosis	8.4730	8.7859	3.9253	5.7624	7.8700	8.6008
Min	-13.6841	-15.0271	-10.5811	-16.9577	-19.3523	-9.4059
Max	17.9844	15.8631	10.5018	17.8692	18.7711	10.8753

Data is from January 1, 1990 to December 31, 2016, 6805 observations.

Table 2: Posterior Estimates of Stock, Corresponding Industry and the Market

$$\boldsymbol{r}_t = \boldsymbol{\mu} + \boldsymbol{\epsilon}_{1,t} + \boldsymbol{\epsilon}_{2,t},\ \boldsymbol{\epsilon}_{1,t} = \boldsymbol{H}_t^{1/2}\boldsymbol{z}_t,\ \boldsymbol{z}_t \sim NID\left(\boldsymbol{0}, \boldsymbol{I}\right),\ \boldsymbol{\epsilon}_{2,t} = \boldsymbol{J}_t - \boldsymbol{\mu}_J \odot \boldsymbol{\Omega}_B{}'\boldsymbol{p}$$

$$\boldsymbol{H}_t = \boldsymbol{C}\boldsymbol{C}' + \boldsymbol{\alpha}\boldsymbol{\alpha}' \odot \boldsymbol{\epsilon}_{t-1}\boldsymbol{\epsilon}_{t-1}{}' + \boldsymbol{\beta}\boldsymbol{\beta}' \odot \boldsymbol{H}_{t-1},\ \boldsymbol{\epsilon}_{t-1} = \boldsymbol{r}_{t-1} - \boldsymbol{\mu}$$

$$\boldsymbol{J}_t = \boldsymbol{Y}_t \odot \boldsymbol{B}_t,\ \boldsymbol{Y}_t \sim N\left(\boldsymbol{\mu}_J, \boldsymbol{\Sigma}_J\right),\ \boldsymbol{B}_t \sim multinomial\left(1, \boldsymbol{p}\right)$$

Parameter	GE		XOM		WMT		MSFT		AXP	
	Mean	0.95 DI	Mean	0.95 DI	Mean	0.95 DI	Mean	0.95 DI	Mean	0.95 DI
C_{11}	0.0604	(0.0307, 0.0873)	0.0432	(0.0120, 0.0683)	0.0261	(0.0068, 0.0440)	0.0844	(0.0567, 0.1098)	0.0654	(0.0446, 0.0877)
C_{21}	0.0312	(0.0008, 0.0547)	0.0180	(-0.0074, 0.0392)	0.0434	(0.0186, 0.0608)	0.0582	(0.0422, 0.0729)	0.0671	(0.0504, 0.0823)
C_{22}	0.0551	(0.0334, 0.0718)	0.0158	(0.0015, 0.0302)	0.0210	(0.0011, 0.0395)	0.0388	(0.0234, 0.0506)	0.0216	(0.0014, 0.0449)
C_{31}	0.0413	(0.0245, 0.0568)	0.0205	(-0.0023, 0.0402)	0.0295	(0.0061, 0.0461)	0.0426	(0.0304, 0.0550)	0.0537	(0.0386, 0.0669)
C_{32}	0.0203	(0.0028, 0.0330)	0.0217	(-0.0153, 0.0410)	0.0040	(-0.0194, 0.0256)	0.0234	(0.0036, 0.0365)	-0.0022	(-0.0290, 0.0262)
C_{33}	0.0230	(0.0032, 0.0335)	0.0194	(0.0012, 0.0362)	0.0159	(0.0010, 0.0285)	0.0245	(0.0109, 0.0330)	0.0235	(0.0021, 0.0368)
α_{STK}	0.2032	(0.1900, 0.2165)	0.1934	(0.1808, 0.2069)	0.1543	(0.1425, 0.1664)	0.1809	(0.1670, 0.1956)	0.1909	(0.1754, 0.2069)
α_{IND}	0.2022	(0.1894, 0.2152)	0.1901	(0.1802, 0.2010)	0.1773	(0.1676, 0.1872)	0.1853	(0.1738, 0.1970)	0.1951	(0.1836, 0.2070)
α_{MKT}	0.2002	(0.1877, 0.2130)	0.1984	(0.1856, 0.2122)	0.1898	(0.1772, 0.2035)	0.1933	(0.1799, 0.2067)	0.1915	(0.1777, 0.2057)
β_{STK}	0.9716	(0.9678, 0.9750)	0.9760	(0.9726, 0.9790)	0.9839	(0.9816, 0.9860)	0.9762	(0.9722, 0.9799)	0.9746	(0.9704, 0.9783)
β_{IND}	0.9732	(0.9697, 0.9764)	0.9775	(0.9749, 0.9797)	0.9791	(0.9769, 0.9813)	0.9770	(0.9740, 0.9797)	0.9739	(0.9707, 0.9769)
β_{MKT}	0.9727	(0.9691, 0.9759)	0.9752	(0.9717, 0.9783)	0.9772	(0.9738, 0.9801)	0.9754	(0.9719, 0.9787)	0.9742	(0.9705, 0.9778)
μ_{STK}	0.0298	(0.0008, 0.0587)	0.0239	(-0.0011, 0.0496)	0.0243	(-0.0047, 0.0533)	0.0520	(0.0176, 0.0858)	0.0311	(-0.0043, 0.0656)
μ_{IND}	0.0314	(0.0067, 0.0560)	0.0205	(-0.0033, 0.0444)	0.0320	(0.0101, 0.0540)	0.0393	(0.0131, 0.0652)	0.0376	(0.0134, 0.0618)
μ_{MKT}	0.0307	(0.0127, 0.0490)	0.0321	(0.0140, 0.0502)	0.0336	(0.0156, 0.0513)	0.0364	(0.0186, 0.0543)	0.0376	(0.0192, 0.0561)
$p_{STK,IND,MKT}$	0.0954	(0.0716, 0.1234)	0.0943	(0.0695, 0.1217)	0.0958	(0.0735, 0.1213)	0.0595	(0.0419, 0.0781)	0.0787	(0.0602, 0.0997)
$p_{STK,IND,\overline{MKT}}$	0.0030	(0.0003, 0.0075)	0.0570	(0.0360, 0.0815)	0.0180	(0.0067, 0.0309)	0.0313	(0.0189, 0.0450)	0.0064	(0.0009, 0.0138)
$p_{STK,\overline{IND},MKT}$	0.0026	(0.0002, 0.0069)	0.0009	(0.0000, 0.0033)	0.0011	(0.0000, 0.0034)	0.0006	(0.0000, 0.0022)	0.0012	(0.0000, 0.0040)
$p_{\overline{STK},IND,MKT}$	0.0032	(0.0001, 0.0097)	0.0026	(0.0001, 0.0079)	0.0056	(0.0003, 0.0153)	0.0054	(0.0003, 0.0146)	0.0023	(0.0001, 0.0074)
$p_{STK,\overline{IND},\overline{MKT}}$	0.0529	(0.0370, 0.0715)	0.0071	(0.0016, 0.0151)	0.0047	(0.0003, 0.0120)	0.0014	(0.0000, 0.0049)	0.0696	(0.0521, 0.0887)
$p_{\overline{STK},IND,\overline{MKT}}$	0.0161	(0.0082, 0.0252)	0.0029	(0.0004, 0.0075)	0.0030	(0.0001, 0.0085)	0.0140	(0.0069, 0.0224)	0.0110	(0.0038, 0.0198)
$p_{\overline{STK},\overline{IND},MKT}$	0.0018	(0.0001, 0.0051)	0.0031	(0.0001, 0.0095)	0.0013	(0.0000, 0.0047)	0.0016	(0.0001, 0.0053)	0.0011	(0.0000, 0.0037)
$p_{\overline{STK},\overline{IND},\overline{MKT}}$	0.8250	(0.7843, 0.8605)	0.8320	(0.7936, 0.8678)	0.8705	(0.8423, 0.8962)	0.8862	(0.8648, 0.9057)	0.8297	(0.7974, 0.8575)
$\mu_{J,STK}$	0.0597	(-0.0843, 0.1985)	-0.1648	(-0.2880,-0.0476)	-0.2137	(-0.3785,-0.0529)	-0.0007	(-0.2192, 0.2242)	-0.0939	(-0.2521, 0.0635)
$\mu_{J,IND}$	-0.3825	(-0.5380,-0.2395)	-0.2500	(-0.3698,-0.1406)	-0.4222	(-0.5419,-0.3072)	-0.3899	(-0.5268,-0.2587)	-0.2864	(-0.4345,-0.1428)
$\mu_{J,MKT}$	-0.4654	(-0.5853,-0.3536)	-0.5963	(-0.7381,-0.4580)	-0.4652	(-0.5739,-0.3599)	-0.4947	(-0.6175,-0.3782)	-0.4888	(-0.6176,-0.3642)
$\sigma^2_{J,STK}$	3.7909	(3.0470, 4.6388)	2.1384	(1.7064, 2.6669)	4.3986	(3.5767, 5.3886)	9.4538	(7.7795,11.4719)	5.9235	(4.9261, 7.0697)
$\sigma_{J,STK,IND}$	2.8112	(2.2472, 3.4705)	1.8279	(1.4466, 2.2952)	2.1381	(1.6897, 2.6688)	4.0416	(3.2766, 4.9676)	4.2522	(3.4928, 5.1996)
$\sigma^2_{J,IND}$	2.5340	(2.0338, 3.1581)	1.8704	(1.4839, 2.3497)	1.6121	(1.2676, 2.0193)	2.5496	(2.0470, 3.1482)	3.6641	(2.8846, 4.6942)
$\sigma_{J,STK,MKT}$	2.2686	(1.8405, 2.7742)	1.2118	(0.9410, 1.5322)	1.0306	(0.7271, 1.3770)	1.7621	(1.2730, 2.3417)	3.0789	(2.5331, 3.7240)
$\sigma_{J,IND,MKT}$	1.8730	(1.5009, 2.3271)	1.2394	(0.9736, 1.5563)	1.0919	(0.8466, 1.3901)	1.5444	(1.2142, 1.9376)	2.4323	(1.9411, 3.0436)
$\sigma^2_{J,MKT}$	1.5230	(1.2214, 1.8870)	1.0451	(0.8021, 1.3480)	1.0099	(0.7873, 1.2793)	1.2506	(0.9653, 1.5910)	1.9048	(1.5145, 2.3804)

This table reports the posterior mean and 0.95 density intervals in parentheses for the joint MGARCH-jump model

Table 3: Jump Probabilities for Stocks with Corresponding Industry and Market

Panel A: marginal and joint probabilities

Probabilities		GE	XOM	WMT	MSFT	AXP
Marginal	Stock	0.1539	0.1593	0.1196	0.0928	0.1560
	Industry	0.1176	0.1569	0.1224	0.1102	0.0984
	Market	0.1030	0.1010	0.1038	0.0672	0.0833
Joint	Stock and Industry	0.0984	0.1513	0.1139	0.0908	0.0851
		(0.0181)	(0.0250)	(0.0146)	(0.0102)	(0.0153)
	Stock and Market	0.0980	0.0953	0.0969	0.0602	0.0800
		(0.0159)	(0.0161)	(0.0124)	(0.0062)	(0.0130)
	Industry and Market	0.0986	0.0969	0.1014	0.0650	0.0810
		(0.0121)	(0.0158)	(0.0127)	(0.0074)	(0.0082)
	Stock, Industry and Market	0.0954	0.0943	0.0958	0.0595	0.0787
		(0.0019)	(0.0025)	(0.0015)	(0.0007)	(0.0013)

Numbers in parentheses below the joint probabilities are the product of corresponding marginal probabilities.

Panel B: conditional probabilities

Stock	Probabilities	Stk,Ind,Mkt	Stk,Ind	Stk,Mkt	Ind,Mkt
GE	p (co-jump|mkt-jump)	0.9261	—	0.9516	0.9568
	p (co-jump|ind-jump)	0.8110	0.8363	—	0.8378
	p (co-jump|stk-jump)	0.6198	0.6392	0.6369	—
XOM	p (co-jump|mkt-jump)	0.9338	—	0.9432	0.9598
	p (co-jump|ind-jump)	0.6012	0.9647	—	0.6179
	p (co-jump|stk-jump)	0.5918	0.9496	0.5977	—
WMT	p (co-jump|mkt-jump)	0.9233	—	0.9337	0.9770
	p (co-jump|ind-jump)	0.7829	0.9300	—	0.8284
	p (co-jump|stk-jump)	0.8011	0.9517	0.8102	—
MSFT	p (co-jump|mkt-jump)	0.8855	—	0.8950	0.9664
	p (co-jump|ind-jump)	0.5402	0.8240	—	0.5896
	p (co-jump|stk-jump)	0.6414	0.9783	0.6483	—
AXP	p (co-jump|mkt-jump)	0.9452	—	0.9602	0.9723
	p (co-jump|ind-jump)	0.8004	0.8657	—	0.8233
	p (co-jump|stk-jump)	0.5046	0.5457	0.5126	—

Each column indicates a particular type of co-jumps. For example, column 3 shows conditional probabilities of stock-industry-market co-jumps for each stock.

Table 4: Jump Size Correlations for Stocks with Corresponding Industry and Market

	GE	IND	MKT		XOM	IND	MKT
GE	1.0000	—	—	XOM	1.0000	—	—
IND	0.9070	1.0000	—	IND	0.9140	1.0000	—
MKT	0.9441	0.9534	1.0000	MKT	0.8106	0.8865	1.0000
	WMT	IND	MKT		MSFT	IND	MKT
WMT	1.0000	—	—	MSFT	1.0000	—	—
IND	0.8029	1.0000	—	IND	0.8232	1.0000	—
MKT	0.4890	0.8557	1.0000	MKT	0.5125	0.8649	1.0000
	AXP	IND	MKT				
AXP	1.0000	—	—				
IND	0.9127	1.0000	—				
MKT	0.9166	0.9207	1.0000				

This table reports correlations from the posterior mean of $\mathbf{\Sigma}_J$.

Table 5: Log-predictive Likelihoods Comparison

	MGARCH-jump	MGARCH	Log-Bayes factor
GE,IND,MKT	-1190.2726	-1211.7256	21.4529
XOM,IND,MKT	-1234.9129	-1264.9846	30.0717
WMT,IND,MKT	-1193.7295	-1206.4306	12.7011
MSFT,IND,MKT	-1148.3214	-1181.2845	32.9631
AXP,IND,MKT	-1234.9474	-1296.7611	61.8137
GE,XOM,WMT,MSFT,AXP	-1586.1071	-1653.5347	67.4276

This table reports log-predictive likelihood values for the last 100 observations (Aug 10, 2016 – Dec 30, 2016) in the sample for the MGARCH-jump model and a MGARCH model without jumps. A positive log-Bayes factor favours the MGARCH-jump model.

Table 6: Posterior Estimates for GE, XOM, WMT, MSFT and AXP

Panel A: drift and GARCH parameters

Parameter		GE	XOM	WMT	MSFT	AXP
$\boldsymbol{\mu}$		0.0313 (0.0015, 0.0613)	0.0328 (0.0043, 0.0605)	0.0364 (0.0064, 0.0666)	0.0672 (0.0284, 0.1049)	0.0372 (-0.0010, 0.0756)
$\boldsymbol{\alpha}$		0.1507 (0.1381, 0.1644)	0.1516 (0.1389, 0.1654)	0.1166 (0.1054, 0.1502)	0.1363 (0.1244, 0.1493)	0.1503 (0.1391, 0.1627)
$\boldsymbol{\beta}$		0.9832 (0.9797, 0.9860)	0.9825 (0.9791, 0.9853)	0.9898 (0.9829, 0.9916)	0.9855 (0.9827, 0.9879)	0.9839 (0.9812, 0.9862)
		1	2	3	4	5
$\boldsymbol{C}$	1	0.0868 (0.0668, 0.1049)				
	2	0.0532 (0.0371, 0.0737)	0.0717 (0.0515, 0.0898)			
	3	0.0317 (0.0207, 0.0573)	0.0113 (-0.0223, 0.0225)	0.0196 (0.0029, 0.0347)		
	4	0.0521 (0.0366, 0.0713)	0.0123 (-0.0057, 0.0275)	0.0334 (0.0019, 0.0593)	0.0206 (0.0010, 0.0481)	
	5	0.0600 (0.0446, 0.0768)	0.0120 (-0.0067, 0.0274)	0.0333 (-0.0049, 0.0597)	0.0079 (-0.0403, 0.0507)	0.0249 (0.0014, 0.0535)

This table reports the posterior mean and 0.95 density intervals in parentheses for the joint MGARCH-jump model of five stocks.

Table 6: Posterior Estimates for GE, XOM, WMT, MSFT and AXP (Continued)

Panel B: jump/co-jump probabilities

Parameter	Mean	0.95 DI	Parameter	Mean	0.95 DI
$p_{GE,XOM,WMT,MSFT,AXP}$	0.0170	(0.0076,0.0266)	$p_{\overline{GE},\overline{XOM},\overline{WMT},MSFT,AXP}$	0.0018	(0.0001,0.0059)
$p_{\overline{GE},XOM,WMT,MSFT,AXP}$	0.0018	(0.0001,0.0051)	$p_{\overline{GE},\overline{XOM},WMT,\overline{MSFT},AXP}$	0.0041	(0.0001,0.0143)
$p_{GE,\overline{XOM},WMT,MSFT,AXP}$	0.0020	(0.0001,0.0070)	$p_{\overline{GE},\overline{XOM},WMT,MSFT,\overline{AXP}}$	0.0023	(0.0001,0.0076)
$p_{GE,XOM,\overline{WMT},MSFT,AXP}$	0.0024	(0.0001,0.0074)	$p_{\overline{GE},XOM,\overline{WMT},\overline{MSFT},AXP}$	0.0040	(0.0003,0.0105)
$p_{GE,XOM,WMT,\overline{MSFT},AXP}$	0.0034	(0.0002,0.0093)	$p_{\overline{GE},XOM,\overline{WMT},MSFT,\overline{AXP}}$	0.0038	(0.0001,0.0126)
$p_{GE,XOM,WMT,MSFT,\overline{AXP}}$	0.0022	(0.0001,0.0071)	$p_{\overline{GE},XOM,WMT,\overline{MSFT},\overline{AXP}}$	0.0087	(0.0008,0.0231)
$p_{\overline{GE},\overline{XOM},WMT,MSFT,AXP}$	0.0017	(0.0000,0.0056)	$p_{GE,\overline{XOM},\overline{WMT},\overline{MSFT},AXP}$	0.0037	(0.0001,0.0129)
$p_{\overline{GE},XOM,\overline{WMT},MSFT,AXP}$	0.0010	(0.0000,0.0036)	$p_{GE,\overline{XOM},\overline{WMT},MSFT,\overline{AXP}}$	0.0067	(0.0006,0.0208)
$p_{\overline{GE},XOM,WMT,\overline{MSFT},AXP}$	0.0022	(0.0001,0.0074)	$p_{GE,\overline{XOM},WMT,\overline{MSFT},\overline{AXP}}$	0.0020	(0.0001,0.0065)
$p_{\overline{GE},XOM,WMT,MSFT,\overline{AXP}}$	0.0018	(0.0001,0.0058)	$p_{GE,XOM,\overline{WMT},\overline{MSFT},\overline{AXP}}$	0.0031	(0.0001,0.0106)
$p_{GE,\overline{XOM},\overline{WMT},MSFT,AXP}$	0.0030	(0.0001,0.0108)	$p_{\overline{GE},\overline{XOM},\overline{WMT},\overline{MSFT},AXP}$	0.0342	(0.0185,0.0496)
$p_{GE,\overline{XOM},WMT,\overline{MSFT},AXP}$	0.0020	(0.0001,0.0064)	$p_{\overline{GE},\overline{XOM},\overline{WMT},MSFT,\overline{AXP}}$	0.0621	(0.0385,0.0849)
$p_{GE,\overline{XOM},WMT,MSFT,\overline{AXP}}$	0.0021	(0.0001,0.0068)	$p_{\overline{GE},\overline{XOM},WMT,\overline{MSFT},\overline{AXP}}$	0.0471	(0.0300,0.0660)
$p_{GE,XOM,\overline{WMT},\overline{MSFT},AXP}$	0.0030	(0.0001,0.0084)	$p_{\overline{GE},XOM,\overline{WMT},\overline{MSFT},\overline{AXP}}$	0.0298	(0.0122,0.0521)
$p_{GE,XOM,\overline{WMT},MSFT,\overline{AXP}}$	0.0021	(0.0001,0.0064)	$p_{GE,\overline{XOM},\overline{WMT},\overline{MSFT},\overline{AXP}}$	0.0241	(0.0115,0.0357)
$p_{GE,XOM,WMT,\overline{MSFT},\overline{AXP}}$	0.0042	(0.0003,0.0108)	$p_{\overline{GE},\overline{XOM},\overline{WMT},\overline{MSFT},\overline{AXP}}$	0.7102	(0.6728,0.7454)

Panel C: jump size parameters

Parameter		GE	XOM	WMT	MSFT	AXP
$\boldsymbol{\mu}_J$		0.0409 (-0.1949,0.2692)	-0.2288 (-0.4219,-0.0489)	0.0713 (-0.1186,0.2588)	0.4148 (0.1857, 0.6384)	0.0279 (-0.2911,0.3439)
$\boldsymbol{\Sigma}_J$	GE	6.7958 (4.6207, 8.8016)				
	XOM	3.0414 (0.1584, 5.4529)	4.6635 (3.1805, 6.5901)			
	WMT	5.9970 (4.2706, 7.4735)	2.1946 (0.4840, 4.0247)	6.8466 (5.4093, 8.4263)		
	MSFT	6.6374 (1.7305, 9.3556)	4.5465 (1.7684, 7.0416)	6.3338 (3.1860, 8.4018)	10.1653 (8.3923,12.2379)	
	AXP	7.1474 (2.6935,10.0838)	4.1921 (1.1525, 6.6495)	6.3004 (2.1256, 9.2238)	8.9655 (2.5645,11.3860)	10.9217 (7.8769,14.2148)

This table reports the posterior mean and 0.95 density intervals in parentheses for the joint MGARCH-jump model of five stocks.

Table 7: Jump Probabilities among GE, XOM, WMT, MSFT and AXP

marginal and joint probabilities

Stock	GE	XOM	WMT	MSFT	AXP
Marginal probs	0.0832	0.0907	0.1047	0.1140	0.0875

Co-jump	Joint Pr	Product	Co-jump	Joint Pr	Product
All jump	0.0170	0.0000	XOM,WMT,AXP	0.0022	0.0008
GE,XOM,WMT,MSFT	0.0022	0.0001	XOM,MSFT,AXP	0.0010	0.0009
GE,XOM,WMT,AXP	0.0034	0.0001	WMT,MSFT,AXP	0.0017	0.0010
GE,XOM,MSFT,AXP	0.0024	0.0001	GE,XOM	0.0031	0.0075
GE,WMT,MSFT,AXP	0.0020	0.0001	GE,WMT	0.0020	0.0087
XOM,WMT,MSFT,AXP	0.0018	0.0001	GE,MSFT	0.0067	0.0095
GE,XOM,WMT	0.0042	0.0008	GE,AXP	0.0037	0.0073
GE,XOM,MSFT	0.0021	0.0009	XOM,WMT	0.0087	0.0095
GE,XOM,AXP	0.0030	0.0007	XOM,MSFT	0.0038	0.0103
GE,WMT,MSFT	0.0021	0.0010	XOM,AXP	0.0040	0.0079
GE,WMT,AXP	0.0020	0.0008	WMT,MSFT	0.0023	0.0119
GE,MSFT,AXP	0.0030	0.0008	WMT,AXP	0.0041	0.0092
XOM,WMT,MSFT	0.0018	0.0011	MSFT,AXP	0.0018	0.0100

The column "Product" is product of corresponding marginal probabilities.

Table 8: Jump Size Correlations among GE, XOM, WMT, MSFT and AXP

Stock	GE	XOM	WMT	MSFT	AXP
GE	1.0000	—	—	—	—
XOM	0.5403	1.0000	—	—	—
WMT	0.8792	0.3884	1.0000	—	—
MSFT	0.7986	0.6603	0.7592	1.0000	—
AXP	0.8296	0.5874	0.7286	0.8509	1.0000

This table reports correlations from the posterior mean of $\boldsymbol{\Sigma}_J$.

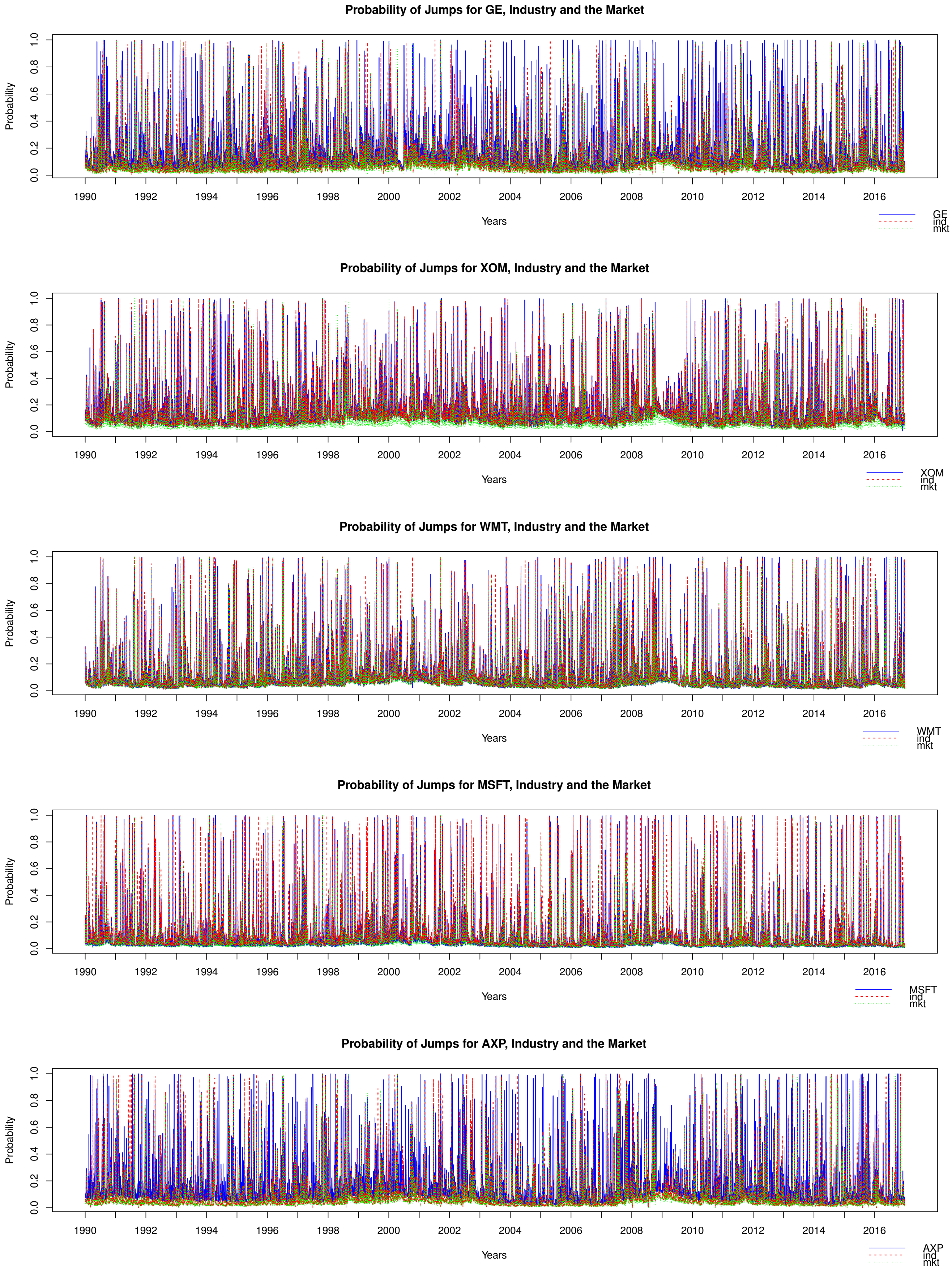

Figure 1: Posterior Jump Probability for Stock, Industry and Market

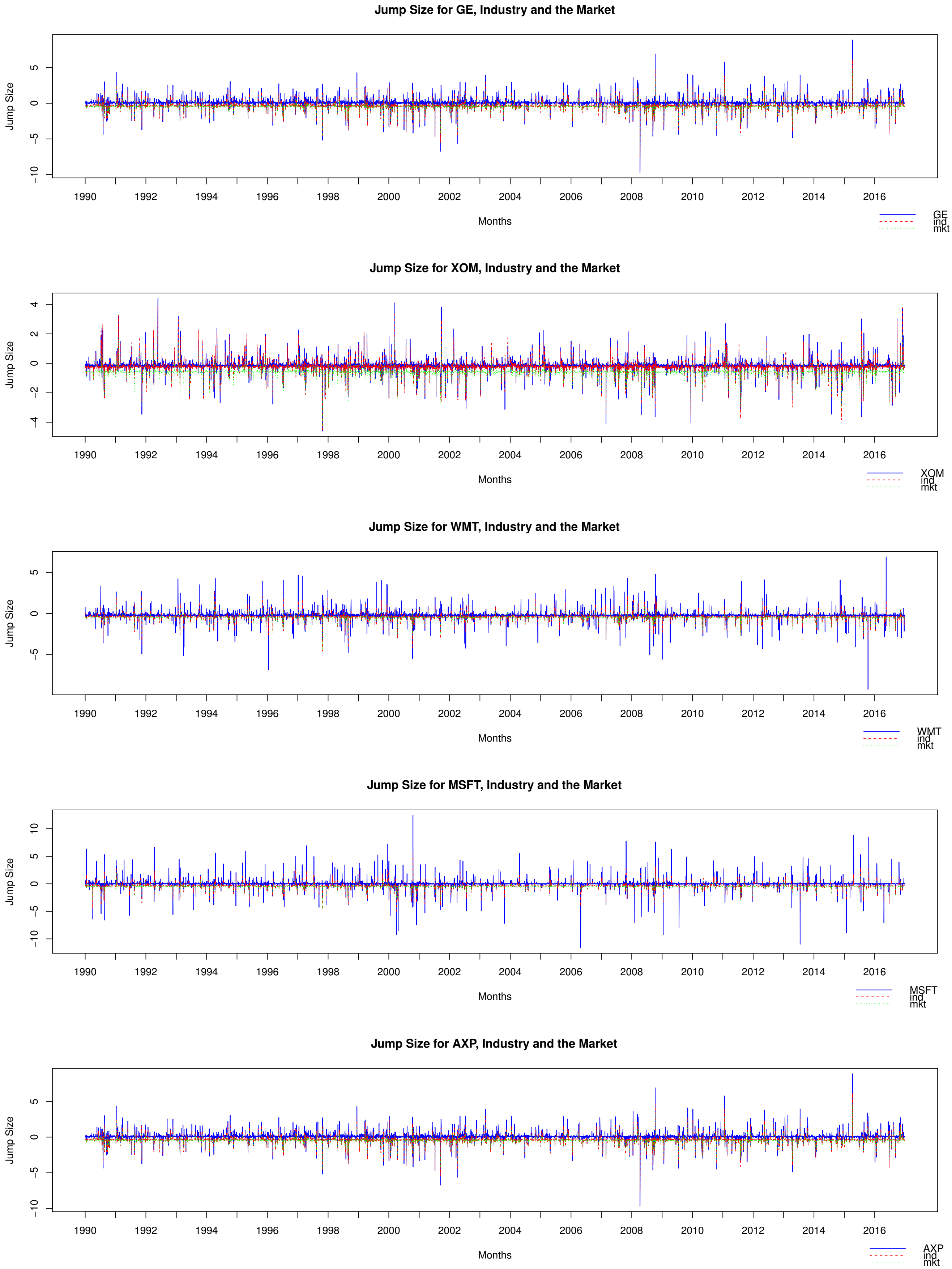

Figure 2: Posterior Jump Sizes for Stock, Industry and Market

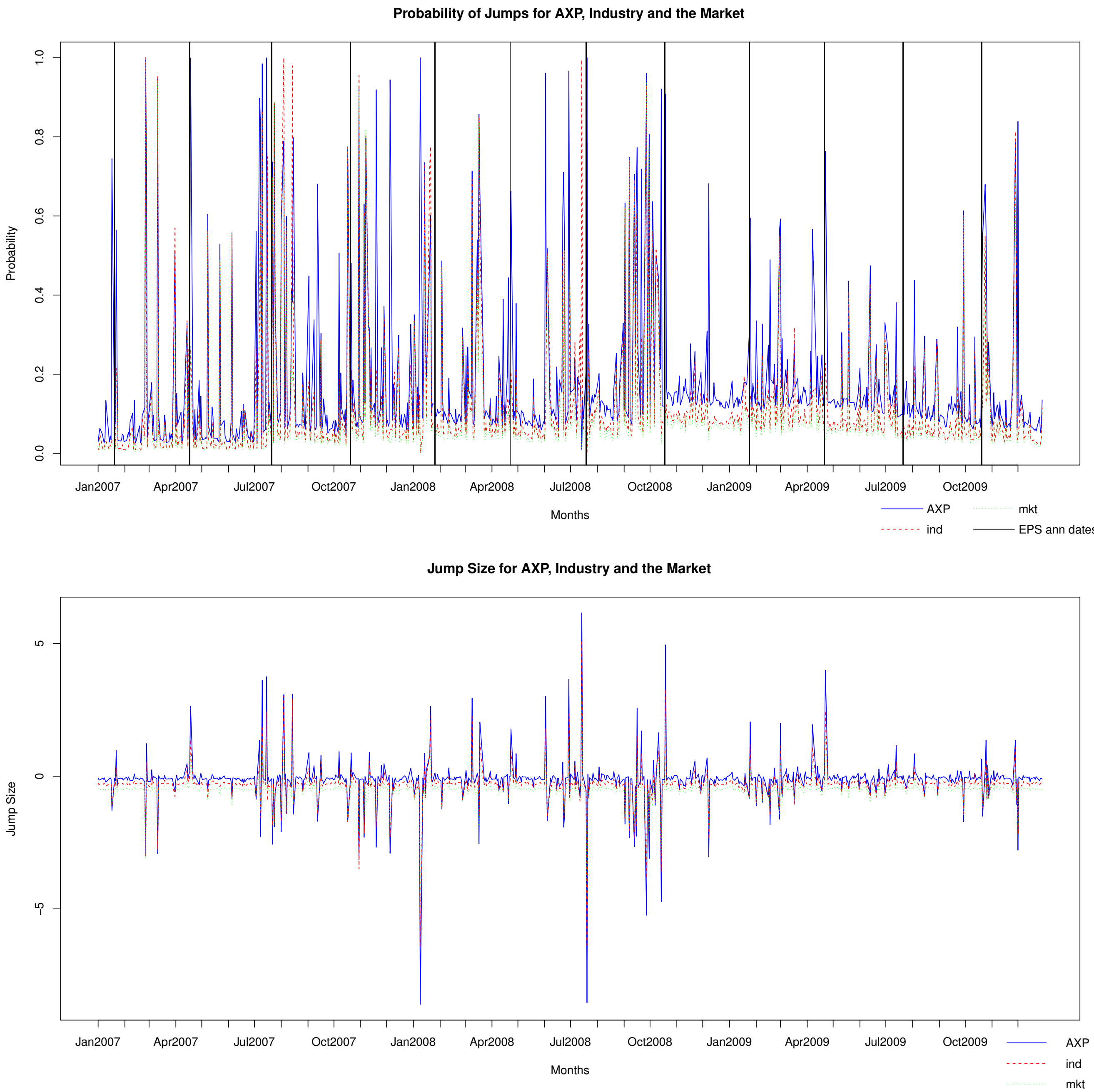

Figure 3: Posterior Jump Probability and Jump Size for AXP, Jan 1, 2007 to Dec 31, 2009

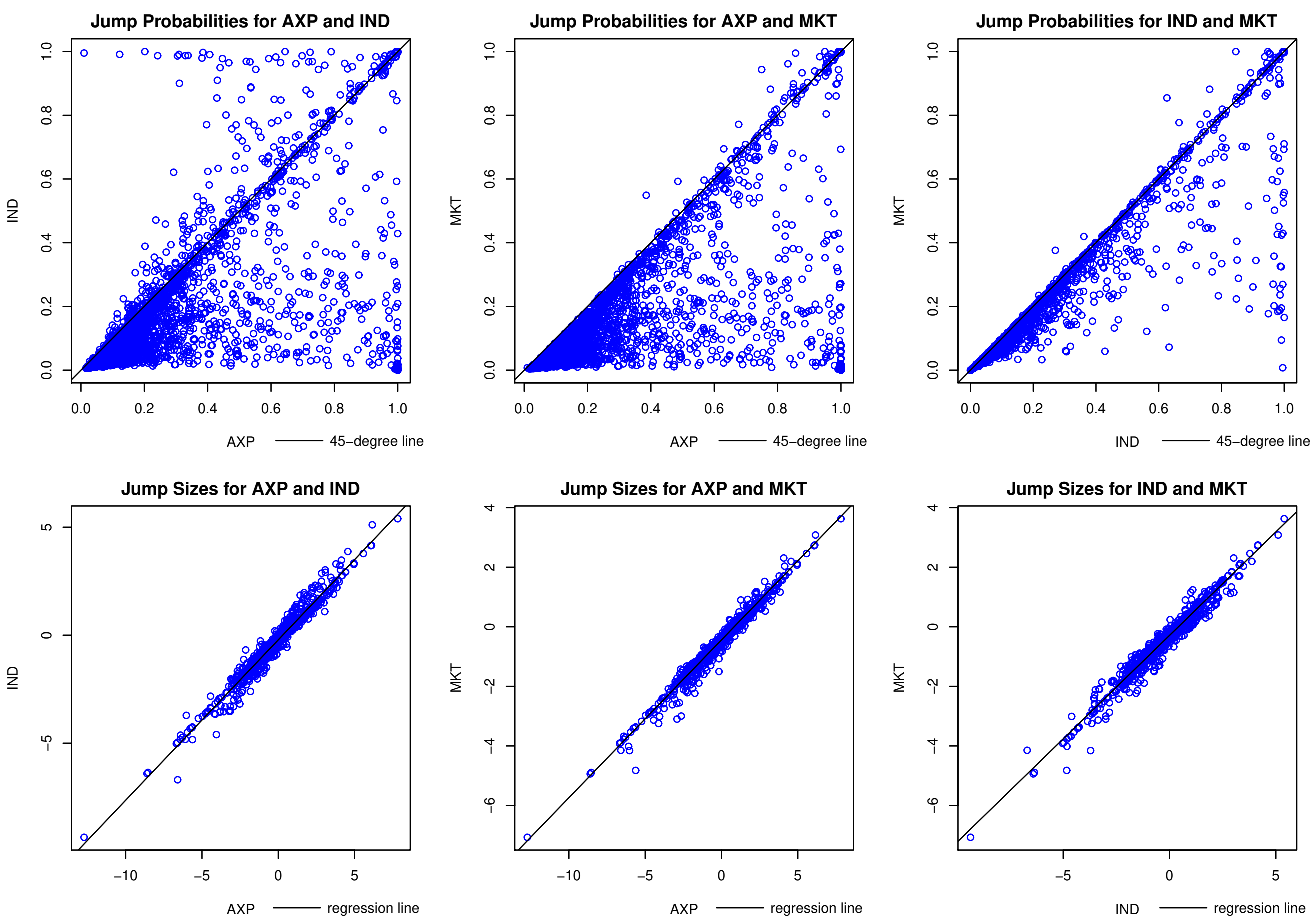

Figure 4: Scatter Plots for Jump Probability and Jump Size for AXP Model

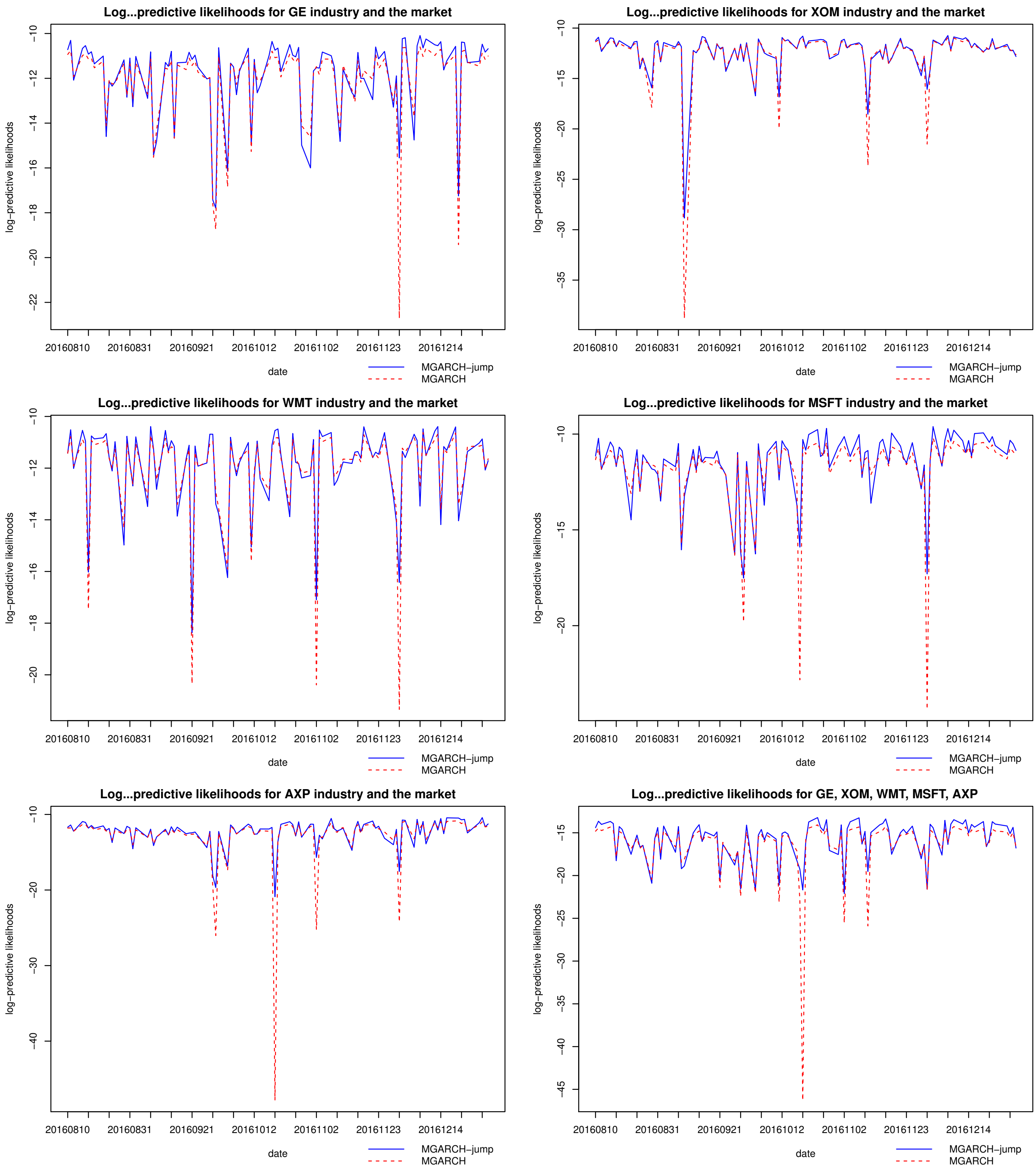

Figure 5: Log-predictive Likelihoods for the Out-of-Sample Period

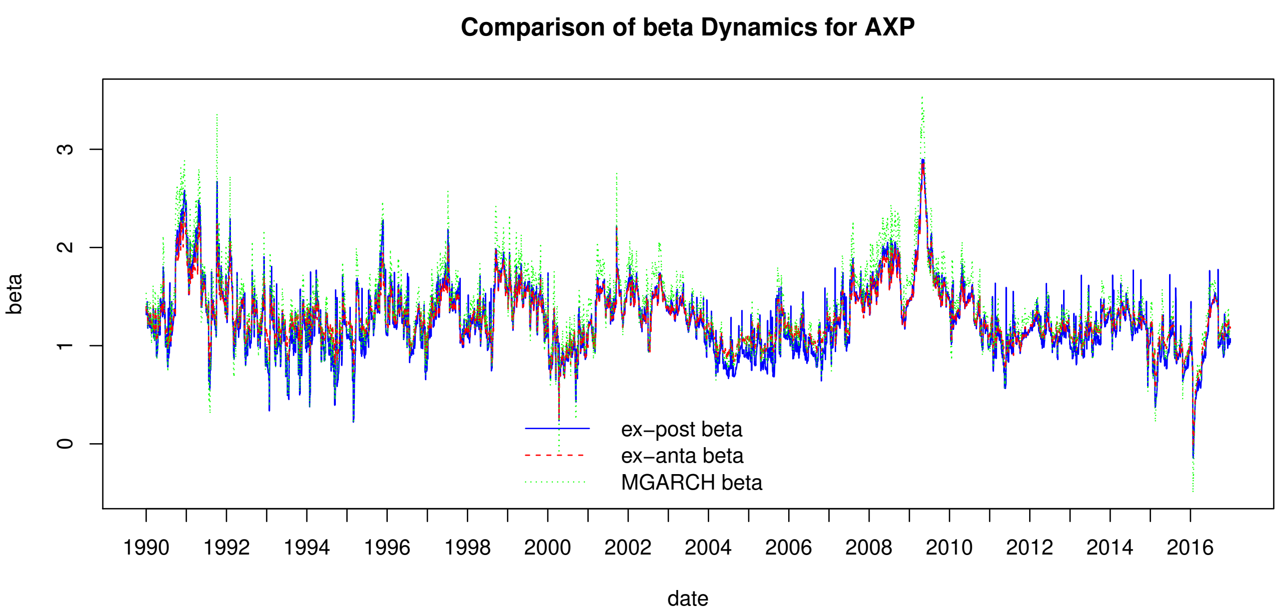

Figure 6: Beta Estimates for AXP from MGARCH-Jump Model and MGARCH Model

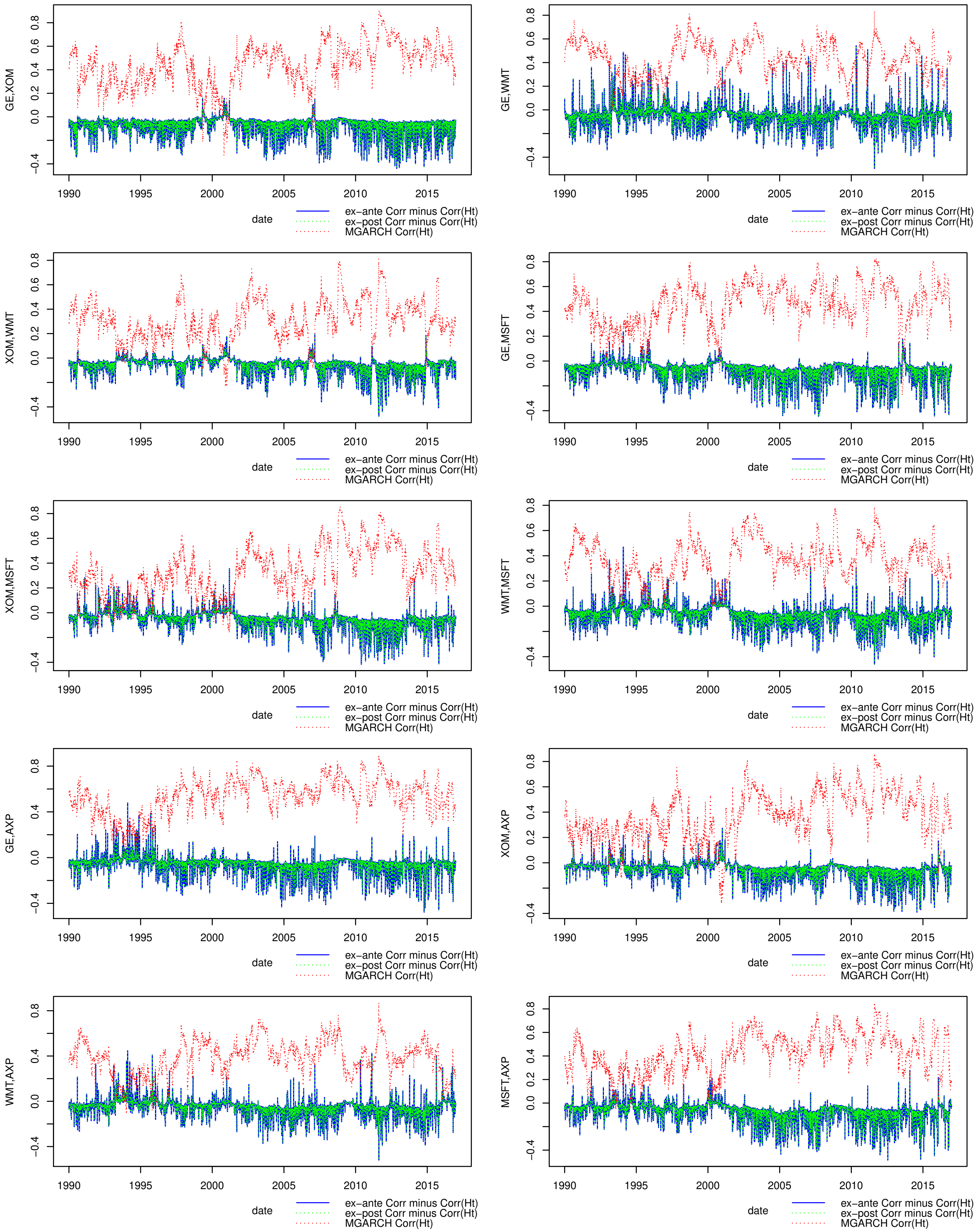

Figure 7: Correlation from Jump Components vs MGARCH Component.

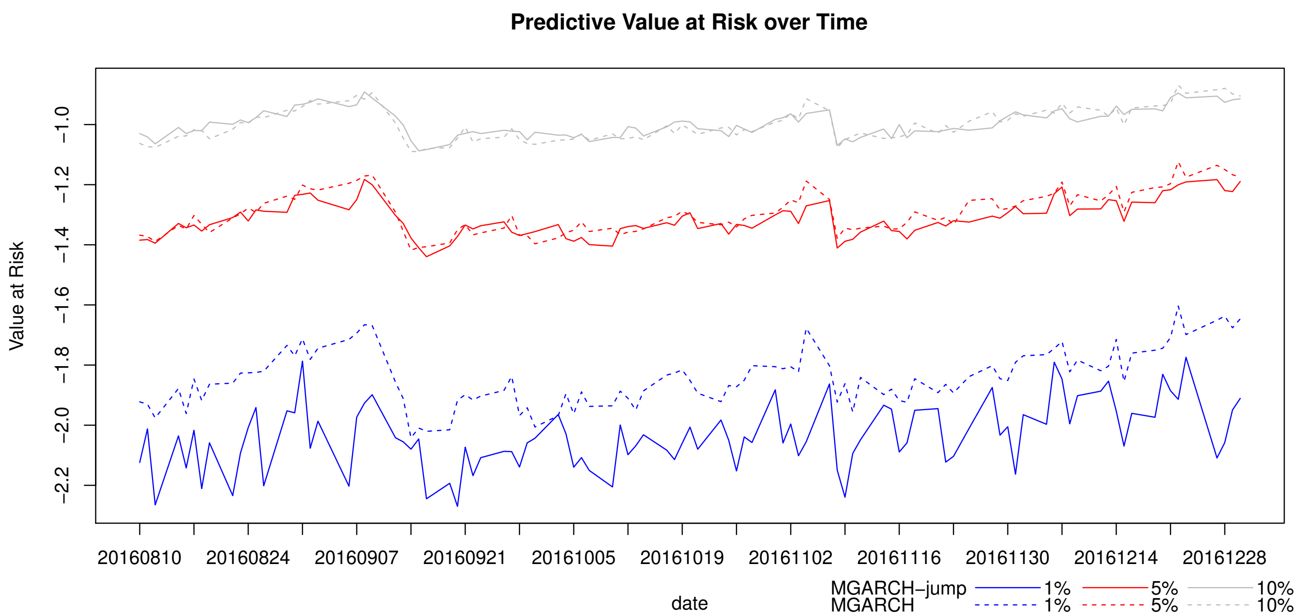

Figure 8: Predictive Value-at-risk for a Five-Asset Equally-Weighted portfolio

A Proof of Conditional Moments of $\boldsymbol{J}_t$

Proof. First, prove $\mathrm{E}\left(\boldsymbol{J}_t|\boldsymbol{\Theta},\boldsymbol{r}_{1:t-1}\right)$:

$$\mathrm{E}\left(\boldsymbol{J}_t|\boldsymbol{\Theta},\boldsymbol{r}_{1:t-1}\right)=\boldsymbol{\mu}_J\odot\mathrm{E}\left(\boldsymbol{B}_t|\boldsymbol{\Theta},\boldsymbol{r}_{1:t-1}\right)$$
$$\mathrm{E}\left(\boldsymbol{B}_t|\boldsymbol{\Theta},\boldsymbol{r}_{1:t-1}\right)=\boldsymbol{\Omega}_B{}'\boldsymbol{p}=\sum_{j=1}^{2^N}\boldsymbol{B}_t^{(j)}p_j$$

Then, prove $\mathrm{Cov}\left(\boldsymbol{J}_t|\boldsymbol{\Theta},\boldsymbol{r}_{1:t-1}\right)$:

$$\begin{aligned}
\mathrm{Cov}\left(\boldsymbol{J}_t|\boldsymbol{\Theta},\boldsymbol{r}_{1:t-1}\right)&=\mathrm{E}\left[\left(\boldsymbol{Y}_t\odot\boldsymbol{B}_t\right)\left(\boldsymbol{Y}_t\odot\boldsymbol{B}_t\right)'|\boldsymbol{\Theta},\boldsymbol{r}_{1:t-1}\right]\\
&\quad-\mathrm{E}\left(\boldsymbol{Y}_t\odot\boldsymbol{B}_t|\boldsymbol{\Theta},\boldsymbol{r}_{1:t-1}\right)\mathrm{E}\left(\boldsymbol{Y}_t\odot\boldsymbol{B}_t|\boldsymbol{\Theta},\boldsymbol{r}_{1:t-1}\right)'\\
&=\mathrm{E}\left(\boldsymbol{Y}_t\boldsymbol{Y}_t'\odot\boldsymbol{B}_t\boldsymbol{B}_t'|\boldsymbol{\Theta},\boldsymbol{r}_{1:t-1}\right)-\left(\boldsymbol{\mu}_J\odot\boldsymbol{\Omega}_B{}'\boldsymbol{p}\right)\left(\boldsymbol{\mu}_J\odot\boldsymbol{\Omega}_B{}'\boldsymbol{p}\right)'\\
&=\mathrm{E}\left(\boldsymbol{Y}_t\boldsymbol{Y}_t'\odot\boldsymbol{B}_t\boldsymbol{B}_t'|\boldsymbol{\Theta},\boldsymbol{r}_{1:t-1}\right)-\boldsymbol{\mu}_J\boldsymbol{\mu}_J{}'\odot\boldsymbol{\Omega}_B{}'\boldsymbol{p}\boldsymbol{p}'\boldsymbol{\Omega}_B\\
\mathrm{E}\left(\boldsymbol{Y}_t\boldsymbol{Y}_t'\odot\boldsymbol{B}_t\boldsymbol{B}_t'|\boldsymbol{\Theta},\boldsymbol{r}_{1:t-1}\right)&=\mathrm{E}\left[\mathrm{E}\left(\boldsymbol{Y}_t\boldsymbol{Y}_t'\odot\boldsymbol{B}_t\boldsymbol{B}_t'|\boldsymbol{B}_t,\boldsymbol{\Theta}\right)|\boldsymbol{r}_{1:t-1}\right]\\
&=\mathrm{E}\left[\mathrm{Cov}\left(\boldsymbol{Y}_t\odot\boldsymbol{B}_t|\boldsymbol{B}_t,\boldsymbol{\Theta}\right)+\mathrm{E}\left(\boldsymbol{Y}_t\odot\boldsymbol{B}_t|\boldsymbol{B}_t,\boldsymbol{\Theta}\right)\mathrm{E}\left(\boldsymbol{Y}_t\odot\boldsymbol{B}_t|\boldsymbol{B}_t,\boldsymbol{\Theta}\right)'|\boldsymbol{r}_{1:t-1}\right]\\
&=\mathrm{E}\left[\boldsymbol{\Sigma}_J\odot\boldsymbol{B}_t\boldsymbol{B}_t'+\boldsymbol{\mu}_J\boldsymbol{\mu}_J{}'\odot\boldsymbol{B}_t\boldsymbol{B}_t'|\boldsymbol{\Theta},\boldsymbol{r}_{1:t-1}\right]\\
&=\left(\boldsymbol{\Sigma}_J+\boldsymbol{\mu}_J\boldsymbol{\mu}_J{}'\right)\odot\mathrm{E}\left(\boldsymbol{B}_t\boldsymbol{B}_t'|\boldsymbol{\Theta},\boldsymbol{r}_{1:t-1}\right)\\
&=\left(\boldsymbol{\Sigma}_J+\boldsymbol{\mu}_J\boldsymbol{\mu}_J{}'\right)\odot\left(\sum_{j=1}^{2^N}p_j\boldsymbol{\Omega}_j\boldsymbol{\Omega}_j{}'\right)
\end{aligned}$$

□

B Sampling Details

In each MCMC iteration,

1. $\boldsymbol{\mu}|\boldsymbol{r}_{1:T},\boldsymbol{H}_{1:T},\boldsymbol{\mu}_J,\boldsymbol{\Sigma}_J,\boldsymbol{B}_{1:T},\boldsymbol{p}$. Assuming $\boldsymbol{\mu}$ has a normal prior $N\left(\boldsymbol{b}_\mu,\boldsymbol{B}_\mu\right)$, let $\boldsymbol{T}_\mu=\boldsymbol{B}_\mu^{-1}$, then the posterior is

$$p\left(\boldsymbol{\mu}|\boldsymbol{r}_{1:T},\boldsymbol{H}_{1:T},\boldsymbol{\mu}_J,\boldsymbol{\Sigma}_J,\boldsymbol{B}_{1:T},\boldsymbol{p}\right)\propto\prod_{t=1}^{T}p\left(\boldsymbol{r}_t|\boldsymbol{\mu},\boldsymbol{H}_t,\boldsymbol{\mu}_J,\boldsymbol{\Sigma}_J,\boldsymbol{B}_{1:T},\boldsymbol{p}\right)p\left(\boldsymbol{\mu}\right)$$

   The model is not linear given the MGARCH component but a standard conjugate Gibbs result can be used as the asymmetric proposal for the MH step: $\boldsymbol{\mu}'\sim N\left(\boldsymbol{M}_\mu,\boldsymbol{V}_\mu\right)$

$$\boldsymbol{M}_\mu=\boldsymbol{V}_\mu\left[\sum_{t=1}^{T}\left(\boldsymbol{H}_t+\boldsymbol{B}_t\boldsymbol{B}_t'\odot\boldsymbol{\Sigma}_J\right)^{-1}\boldsymbol{r}_t^*+\boldsymbol{T}_\mu\boldsymbol{b}_\mu\right]$$
$$\boldsymbol{V}_\mu=\left[\sum_{t=1}^{T}\left(\boldsymbol{H}_t+\boldsymbol{B}_t\boldsymbol{B}_t'\odot\boldsymbol{\Sigma}_J\right)^{-1}+\boldsymbol{T}_\mu\right]^{-1}$$

where $\boldsymbol{r}_t^* = \boldsymbol{r}_t - \boldsymbol{\mu}_J \odot (\boldsymbol{B}_t - \boldsymbol{\Omega}_B{}'\boldsymbol{p})$. Then accept $\boldsymbol{\mu}'$ with probability

$$\alpha\left(\boldsymbol{\mu}^{(i)}, \boldsymbol{\mu}'\right) = \min\left\{1, \frac{p\left(\boldsymbol{\mu}'|\boldsymbol{r}_{1:T}, \boldsymbol{H}_{1:T}, \boldsymbol{\mu}_J, \boldsymbol{\Sigma}_J, \boldsymbol{B}_{1:T}, \boldsymbol{p}\right) p\left(\boldsymbol{\mu}^{(i)}\right)}{p\left(\boldsymbol{\mu}^{(i)}|\boldsymbol{r}_{1:T}, \boldsymbol{H}_{1:T}, \boldsymbol{\mu}_J, \boldsymbol{\Sigma}_J, \boldsymbol{B}_{1:T}, \boldsymbol{p}\right) p\left(\boldsymbol{\mu}'\right)}\right\}$$

2. $\boldsymbol{\theta}_H|\boldsymbol{r}_{1:T}, \boldsymbol{\mu}, \boldsymbol{\mu}_J, \boldsymbol{\Sigma}_J, \boldsymbol{B}_{1:T}, \boldsymbol{p}$, where $\boldsymbol{\theta}_H = (\boldsymbol{C}, \boldsymbol{\alpha}, \boldsymbol{\beta})'$. The posterior is

$$p\left(\boldsymbol{\theta}_H|\boldsymbol{r}_{1:T}, \boldsymbol{\mu}, \boldsymbol{\mu}_J, \boldsymbol{\Sigma}_J, \boldsymbol{B}_{1:T}, \boldsymbol{p}\right) \propto \prod_{t=1}^{T} p\left(\boldsymbol{r}_t|\boldsymbol{\mu}, \boldsymbol{H}_t, \boldsymbol{\mu}_J, \boldsymbol{\Sigma}_J, \boldsymbol{B}_t, \boldsymbol{p}\right) p\left(\boldsymbol{\theta}_H\right)$$
$$\boldsymbol{r}_t|\boldsymbol{\mu}, \boldsymbol{H}_t, \boldsymbol{\mu}_J, \boldsymbol{\Sigma}_J, \boldsymbol{B}_t, \boldsymbol{p} \sim N\left(\boldsymbol{\mu} + \boldsymbol{\mu}_J \odot \left(\boldsymbol{B}_t - \boldsymbol{\Omega}_B{}'\boldsymbol{p}\right), \boldsymbol{H}_t + \boldsymbol{B}_t\boldsymbol{B}_t{}' \odot \boldsymbol{\Sigma}_J\right)$$

where $\boldsymbol{H}_t$ follows equation (5). Apply a standard random-walk Metropolis-Hastings (MH) algorithm.

3. $\boldsymbol{B}_t|\boldsymbol{r}_t, \boldsymbol{\mu}, \boldsymbol{H}_t, \boldsymbol{\mu}_J, \boldsymbol{\Sigma}_J, \boldsymbol{p}$. There are 2^N different possible realizations of $\boldsymbol{B}_t$, and the posterior is

$$p\left(\boldsymbol{B}_t|\boldsymbol{r}_t, \boldsymbol{\mu}, \boldsymbol{H}_t, \boldsymbol{\mu}_J, \boldsymbol{\Sigma}_J, \boldsymbol{p}\right) = \frac{p\left(\boldsymbol{r}_t|\boldsymbol{\mu}, \boldsymbol{H}_t, \boldsymbol{\mu}_J, \boldsymbol{\Sigma}_J, \boldsymbol{B}_t, \boldsymbol{p}\right) p\left(\boldsymbol{B}_t|\boldsymbol{p}\right)}{\int p\left(\boldsymbol{r}_t|\boldsymbol{\mu}, \boldsymbol{H}_t, \boldsymbol{\mu}_J, \boldsymbol{\Sigma}_J, \boldsymbol{B}_t, \boldsymbol{p}\right) p\left(\boldsymbol{B}_t|\boldsymbol{p}\right)\, \mathrm{d}\boldsymbol{B}_t}$$
$$p\left(\boldsymbol{B}_t|\boldsymbol{p}\right) = \prod_{i=1}^{2^N} p_{t,i}^{x_i}$$

where $x_i = \delta\left(\boldsymbol{B}_t, \boldsymbol{\Omega}_i\right)$. Here, x_i indicates whether the ith row of $\boldsymbol{\Omega}_B$, $\boldsymbol{\Omega}_i$, is realized.

4. $\boldsymbol{p}|\boldsymbol{r}_{1:T}, \boldsymbol{\mu}, \boldsymbol{H}_{1:T}, \boldsymbol{\mu}_J, \boldsymbol{\Sigma}_J, \boldsymbol{B}_{1:T}$. Assuming $\boldsymbol{p}$ has a Dirichlet prior $Dir(a_1, \ldots, a_{2^N})$, the posterior is

$$p\left(\boldsymbol{p}|\boldsymbol{r}_{1:T}, \boldsymbol{\mu}, \boldsymbol{H}_{1:T}, \boldsymbol{\mu}_J, \boldsymbol{\Sigma}_J, \boldsymbol{B}_{1:T}\right) \propto \prod_{t=1}^{T} p\left(\boldsymbol{r}_t|\boldsymbol{\mu}, \boldsymbol{H}_t, \boldsymbol{\mu}_J, \boldsymbol{\Sigma}_J, \boldsymbol{B}_t, \boldsymbol{p}\right) p\left(\boldsymbol{B}_{1:T}|\boldsymbol{p}\right) p\left(\boldsymbol{p}\right)$$

An asymmetric MH sampler instead of Gibbs need be applied. Since $B_{t,i}$'s are iid conditional on p_i, one asymmetric proposal density is the conjugate posterior of multinomial distribution:

$$\boldsymbol{p}' \sim Dir\left(a_i + \sum_{t=1}^{T} x_{t,i}\right), \quad i \in \left\{1, \ldots, 2^N\right\}$$

and accept $\boldsymbol{p}'$ with probability

$$\alpha\left(\boldsymbol{p}^{(i)}, \boldsymbol{p}'\right) = \min\left\{1, \frac{\prod_{t=1}^{T} p\left(\boldsymbol{r}_t|\boldsymbol{\mu}, \boldsymbol{H}_t, \boldsymbol{\mu}_J, \boldsymbol{\Sigma}_J, \boldsymbol{B}_t, \boldsymbol{p}'\right)}{\prod_{t=1}^{T} p\left(\boldsymbol{r}_t|\boldsymbol{\mu}, \boldsymbol{H}_t, \boldsymbol{\mu}_J, \boldsymbol{\Sigma}_J, \boldsymbol{B}_t, \boldsymbol{p}^{(i)}\right)}\right\}$$

5. $\boldsymbol{Y}_t|\boldsymbol{r}_t, \boldsymbol{\mu}, \boldsymbol{H}_t, \boldsymbol{\mu}_J, \boldsymbol{\Sigma}_J, \boldsymbol{B}_t, \boldsymbol{p}$. After simple transformation, conjugate Gibbs result can be ap-

plied:

$$\boldsymbol{Y}_t|\boldsymbol{r}_t, \boldsymbol{\mu}, \boldsymbol{H}_t, \boldsymbol{\mu}_J, \boldsymbol{\Sigma}_J, \boldsymbol{B}_t, \boldsymbol{p} \sim N\left(\boldsymbol{M}_{Y,t}, \boldsymbol{V}_{Y,t}\right)$$

where

$$\begin{aligned}
\boldsymbol{M}_{Y,t} &= \boldsymbol{V}_{Y,t}\left[\boldsymbol{B}_t \odot \boldsymbol{H}_t^{-1}\left(\boldsymbol{r}_t - \boldsymbol{\mu} + \boldsymbol{\mu}_J \odot \boldsymbol{\Omega}_B{}'\boldsymbol{p}\right) + \boldsymbol{\Sigma}_J^{-1}\boldsymbol{\mu}_J\right] \\
\boldsymbol{V}_{Y,t} &= \left(\boldsymbol{B}_t\boldsymbol{B}_t{}' \odot \boldsymbol{H}_t^{-1} + \boldsymbol{\Sigma}_J^{-1}\right)^{-1}
\end{aligned}$$

6. $\boldsymbol{\mu}_J|\boldsymbol{r}_{1:T}, \boldsymbol{\mu}, \boldsymbol{H}_{1:T}, \boldsymbol{\Sigma}_J, \boldsymbol{Y}_{1:T}, \boldsymbol{B}_{1:T}, \boldsymbol{p}$. Assume a prior of $\boldsymbol{\mu}_J \sim N(\boldsymbol{b}_{\mu_J}, \boldsymbol{B}_{\mu_J})$, then the posterior is

$$\begin{aligned}
&p\left(\boldsymbol{\mu}_J|\boldsymbol{r}_{1:T}, \boldsymbol{\mu}, \boldsymbol{H}_{1:T}, \boldsymbol{\Sigma}_J, \boldsymbol{Y}_{1:T}, \boldsymbol{B}_{1:T}, \boldsymbol{p}\right) \\
&\qquad \propto \prod_{t=1}^{T} p\left(\boldsymbol{r}_t|\boldsymbol{\mu}, \boldsymbol{H}_t, \boldsymbol{\mu}_J, \boldsymbol{\Sigma}_J, \boldsymbol{B}_t, \boldsymbol{p}\right) p\left(\boldsymbol{Y}_{1:T}|\boldsymbol{\mu}_J, \boldsymbol{\Sigma}_J\right) p\left(\boldsymbol{\mu}_J\right)
\end{aligned}$$

Similarly, a conjugate proposal density can be applied:

$$\begin{aligned}
\boldsymbol{\mu}_J{}' &\sim N\left(\boldsymbol{M}_{\mu_J}, \boldsymbol{V}_{\mu_J}\right) \\
\boldsymbol{M}_{\mu_J} &= \boldsymbol{V}_{\mu_J}\left(\boldsymbol{\Sigma}_J^{-1}\sum_{t=1}^{T}\boldsymbol{Y}_t + \boldsymbol{B}_{\mu_J}^{-1}\boldsymbol{b}_{\mu_J}\right) \\
\boldsymbol{V}_{\mu_J} &= \left(T\boldsymbol{\Sigma}_J^{-1} + \boldsymbol{B}_{\mu_J}^{-1}\right)^{-1}
\end{aligned}$$

accept $\boldsymbol{\mu}_J{}'$ with probability

$$\alpha\left(\boldsymbol{\mu}_J{}^{(i)}, \boldsymbol{\mu}_J{}'\right) = \min\left\{1, \frac{\prod_{t=1}^{T} p\left(\boldsymbol{r}_t|\boldsymbol{\mu}, \boldsymbol{H}_t, \boldsymbol{\mu}_J{}', \boldsymbol{\Sigma}_J, \boldsymbol{B}_t, \boldsymbol{p}\right)}{\prod_{t=1}^{T} p\left(\boldsymbol{r}_t|\boldsymbol{\mu}, \boldsymbol{H}_t, \boldsymbol{\mu}_J{}^{(i)}, \boldsymbol{\Sigma}_J, \boldsymbol{B}_t, \boldsymbol{p}\right)}\right\}$$

7. $\boldsymbol{\Sigma}_J|\boldsymbol{\mu}_J, \boldsymbol{Y}_{1:T}$. Assume a prior of $\boldsymbol{\Sigma}_J \sim IW\left(\nu_p, \boldsymbol{V}_p\right)$, then apply the standard conjugate Gibbs result

$$\begin{aligned}
\boldsymbol{\Sigma}_J|\boldsymbol{\mu}_J, \boldsymbol{Y}_{1:T} &\sim IW\left(\nu_J, \boldsymbol{V}_J\right) \\
\nu_J &= T + \nu_p \\
\boldsymbol{V}_J &= \sum_{t=1}^{T}\left(\boldsymbol{Y}_t - \boldsymbol{\mu}_J\right)\left(\boldsymbol{Y}_t - \boldsymbol{\mu}_J\right)' + \boldsymbol{V}_p
\end{aligned}$$