Regimes, Non-Linearities, and Price Discontinuities in Indian Energy Stocks

Shaw, Charles

17 December 2020

Regimes, Non-Linearities, and Price Discontinuities in Indian Energy Stocks

Charles SHAW

Abstract

We construct a representative index of largest Indian energy companies listed on the National Stock Exchange (NIFTY 50). We test for presence of regimes, non-linearities, and jumps in the price signal. We benchmark performance against alternative models, including single-regime models and models with no jumps. We then benchmark the quality of regime identification against other indices examined in the literature, such as Nikkei 225 and FTSE 100. Overall, find that our regime-switching model performs well in identifying the regimes in this comparative setting. Based on our model selection criteria, we prefer a regime-augmented model to a model that allows no regime identification. But overall, we prefer a model with jumps and regimes over those that do not allow for jump-diffusion and Markov regime-switching.

Keywords: regime switching, non-linear equilibrium asset pricing models, jumps.
JEL Classification:: G11, G12.

Email address: `cshaw11@mail.bbk.ac.uk` (Charles SHAW)

Contents

List of Figures

List of Tables

1. Introduction

We construct an index of energy stocks using daily data from NIFTY50, a benchmark Indian stock market index that represents the largest Indian companies listed on the National Stock Exchange. Motivated by recent expository work on Levy models (e.g. Cont / Tankov: Financial Modelling with Jump Processes, Second Edition, 2019) we examine this newly-constructed index for regime-switching behaviour, non-linearities, and evidence of jump-diffusion processes. The calibration of regime switching models will be done using the EM-algorithm, which appears to be a standard approach in the applied literature.

We examine this newly-constructed index for regime-switching behaviour, non-linearities, and evidence of jump-diffusion processes, deploying a Markov Regime-switching Lévy Model. We further use the expectation–maximization algorithm to calculate the log-likelihood values. We then introduce alternative specification from competing models in an attempt to benchmark the performance of the regime-switching model with jumps, versus a model without jumps, and versus a model with no regime switching.

We then use regime classification measures in order to identify our model performance, and compare results with other indices, such as Dow Jones Industrial Average, EURO STOXX, Russell 2000, Nikkei 225, NASDAQ, and FTSE 100. Overall, we can see that our framework performs well in identifying the regimes. It is also able to identify regimes more sharply using our data than some (but not all) benchmark indices.

Based on our model selection criteria, we can say that regime-augmented models are preferred to a model that allows no regime identification. But overall, we prefer a model with jumps and regimes over those that do not allow for jump-diffusion and Markov regime-switching.

There is an extensive and active literature on regime switching models – both from a theoretical point of view and from applied. The applied literature includes seminal work on analysis of energy prices [26, 35], exchange rates [9], stock returns [23, 17], systemic risk [10], and other areas. See [3] and [24] for surveys.

Jump-diffusion models have been popularised by Merton et al around 1070s [33], and further developed by [6]. More recently, there has been cross-polination from the applied probability literature to applied finance where improved models (at least in the sense of continuous-time specification, finite moments of all orders, long tailedness, and good empirical fit) have been introduced. Such models include the variance gamma model of Madan and Seneta [32], the CGMY model (named after of Carr, Geman, Madan and Yor [12]), and many others. The CGMY model [12], for instance, examines the Madan-Seneta variance-gamme framework and the Normal Inverse Gaussian (NIG) as examples of Lévy processes.[1]

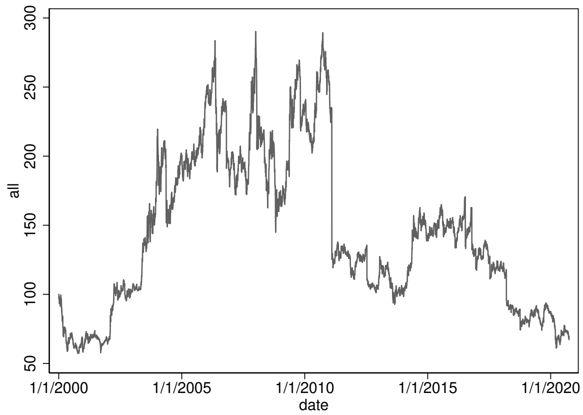
On the other hand, the literature on switching regime Lévy processes is less well-developed. Some seminal works include Deaton [16] demonstrated that price signals in commodity markets exhibit jumps. Analogous evidence has been examined in a range of asset classes [11], indices [27], interest rates ([7, 8, 13]. Numerous applications have been made other areas of finance, most notably to the theory of contingent claims ([28, 29, 30, 38]. See [22] for a survey.

2. Data

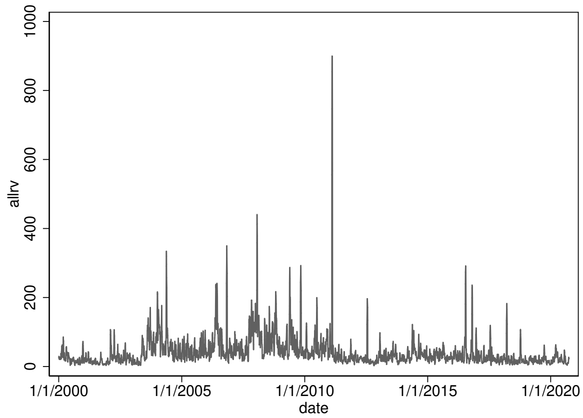
The data is the price history (at day-level) of stocks in the index NIFTY 50 from India's National Stock Exchange. The data spans from 3rd January, 2000 to 30 September, 2020 (5153 observations) [25].

We then construct an index from the closing prices of those stocks in the sample that are listed by the Exchange as being in the 'Energy' industry. we focus on stocks that are listed for our entire time-period of interest, which is last two decades.[2] Table 1 lists the companies which are included in our index. We plot the Index closing price, returns, and (realised) volatility. in Figure 1.

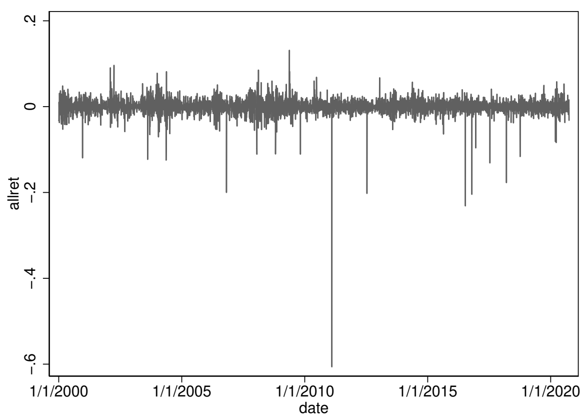
[1]The NIG (VG) are obtained by replacing Brownian motion component with the inverse Gaussian (gamma) process.
[2]We discard for example, Oil & Natural Gas Corporation due to not having a complete data sample.



(a) Index closing price 03.01.2000 - 30/09/2020 (03/01/2000 = 100)



(b) Index volatility, 03.01.2000 - 30.09.2020.



(c) Index log returns, 03.01.2000 - 30.09.2020.

Figure 1: Index closing price, returns, and (realised) volatility.

Company Name	Industry	Symbol	ISIN Code
Bharat Petroleum Corporation Ltd.	ENERGY	BPCL	INE029A01011
Indian Oil Corporation Ltd.	ENERGY	IOC	INE242A01010
Oil & Natural Gas Corporation Ltd.	ENERGY	ONGC	INE213A01029
Reliance Industries Ltd.	ENERGY	RELIANCE	INE002A01018

Table 1: Stocks composition of our index, dates: 03/01/2000 to 30/09/2020

Some initial observations are as follows. Volatility clustering can be observed around the time of the North Atlantic financial crisis from Q2 2007 to Q2 2009. There is a strong shock around 2011 which clearly has a profound negative affect on the market. After some further diagnostics, we note a slow autocorrelation function: i.e. the absolute values of the daily log returns remain positive for time lag upto 30 days. This slow decay of autocorrelation in absolute returns indicates non linear dependence amongst the daily log return values in the index. Data is leptokurtic: i.e. showing having greater kurtosis than the normal distribution. We can observe extreme daily log returns (greater than 5 per cent). We also observe the presence of heavy tails: i.e. the unconditional distribution of returns appears to display a tail resembling a power law. We can observe a significant shift downwards in price around 2011. To explain this anomaly, we quote from a World Bank report which examined this issue ([31]):

"At the end of 2011, the Indian power sector found itself in financial crisis—just a decade after the 2001 bailout of state electricity boards (SEBs) by the central government. Bankrupt state power distribution utilities in several states were unable to pay their bills or repay their debts. Despite the passage of the landmark 2003 Electricity Act and implementation of a broad set of reforms over the past decade, the sector today is looking at another rescue from the center, four times larger than before. This financial rescue scheme amounts to about Rs 1.9 trillion ($42 billion) and was instigated by the nonperforming assets of the banks and other financial institutions."

We suspect that the presence of non-linearities and jumps plays an important role in this market. Looking at this are is of interest, both to policy makers and practitioners, for the following reasons.

First, India is an important world player in energy policy. India has rapidly growing economy with a large population where the government (perhaps unusually for an economy of its size) set itself very aggressive climate change and green energy targets. Notably, the country has pledged a 33-35% reduction in the 'emissions intensity' of its economy by 2030, compared to 2005 levels – this, if achieved, would put the country on the lowest carbon emission per-capita basis than EU, US, or China.

Second, whilst there is an abundance of studies on similar topics using developed markets indices – especially US, European, and Australian data – there are surprisingly few studies on Indian energy data using jump-robust models. However, there exist a number of tractable tools for modelling financial time series which are superiour to the standard Wiener process-based models. Such tools can be used to model discontinuous variations in the price data (eg spikes/jumps) that are empirically observable.

3. Empirical strategy

3.1. Markov-switching models

We already noted the volatility clustering, the spikes, and especially the apparent discontinuity around 2011. Given the evident discontinuities, the assumption of stationarity may not hold for these data and classic time series analysis techniques may be partially or completely inadequate in this regard. For example, financial time-series can often exhibit periods of high/low volatility and also periods of fast/slow mean growth. Standard GARCH models tend to be ill-equipped to tackle such irregularities. Now, one potential solution may be to make use of Markov-switching models which allow an applied researcher to address the non-stationarity of time series under some mild assumptions. This is the approach we take in this chapter. We set up a general Markov Regime Switching (MRS) model as follows:

$$\begin{cases} S_t \in \Lambda \\ y_t = f(S_t, \theta, \psi_{t-1}) \\ S_t = g\left(\tilde{S}_{t-1}, \psi_{t-1}\right) \end{cases} \tag{1}$$

where S_t corresponds to the state of the model at time t, $\tilde{S}_t := \{S_1, ..., S_t\}$ irefers to observed states up to t, $\psi_t := \{y_k : k = 1, \ldots, t\}$ refers to observations up to t, $\Lambda = \{1, ..., M\}$ is the set of possible states, θ corresponds to the vector of parameters in our model, and g is the so-called transition function which controls the transitions between states. Then, function f corresponds to how our observations at each point of time t depend on $S_t, \theta,$ and ψ_{t-1}. And $t \in \{0, 1, ..., T\}$, where $T \in \mathbb{N}$, $T < +\infty$, indicates terminal time. This can be a fruitful approach since realisations of 1 enable the modeller to tackle specific problems that may not be subject to meaningful analysis in a single regime.

3.2. Levy-process regime-switching models

We follows [14] and combine jump-diffusion Levy framework with a Markov-switching model in order to model our index of NIFTY50-listed Indian energy equities. This combined jump-diffusion Markov-switching model offers a way to model stochastic jumps, capture different market regimes, and potentially offer further insight into discontinuities. and non-linearities in our data of interest. Before we set up our model let us provide some definiions.

Definition 3.1 (Stochastic Process)**.** *A stochastic process X on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ is a collection of r.v. s $(X_t)_{0 \leq t < \infty}$.*

If $X_t \in \mathcal{F}_t$, the process X is adapted to the filtration $\mathcal{F}$, or equivalently, $\mathcal{F}_t$-measurable.

Definition 3.2 (Brownian Motion)**.** *Standard Brownian motion $W = (W_t)_{0 \leq t < \infty}$ has the following three properties:*
(i) $W_0 = 0$
(ii) W has independent increments: $W_t - W_s$ is independent of $\mathcal{F}_s$, $0 \leq s < t < \infty$
(iii) $W_t - W_s$ is a Normal r.v.: $W_t - W_s \sim N(0, t-s)$ $\forall$ $0 \leq s < t < \infty$

Property (ii) implies the Markov property i.e. conditional probability distribution of future states of the process depend only on the present state. Property (iii) indicates that knowing the distribution of W_t for $t \leq \tau$ provides no predictive information about the state of the process when $t > \tau$. We can also define Poisson Process, another stochastic process as follows.

Definition 3.3 (Poisson Process)**.** *A Poisson process $N = (N_t)_{0 \leq t < \infty}$ satisfies theseproperties:*
(i) $N_0 = 0$
(ii) N has independent increments: $N_t - N_s$ is independent of $\mathcal{F}_s$, $0 \leq s < t < \infty$
(iii) N has stationary increments: $P(N_t - N_s \leq x) = P(N_{t-s} \leq x)$, $0 \leq s < t < \infty$

Now, a stochastic differential equation written as only as a standard Wiener process or Poisson process is of limited use in applied research, especially where complex and evolving financial dynamics dictate a need for a more general approach. We can combine their properties in a more fruitful manner as follows.

Definition 3.4 (Lévy Process)**.** *Let L be a stochastic process. Then L_t is a Lévy process if the following conditions are satisfied:*
(i) $L_0 = 0$
(ii) L_t it is stochastically continuous (continuous in probability): $\lim_{t \to s} L_t = L_s$
(iii) L has stationary and independent increments: $\mathbb{P}(L_t - L_s \leq x) = \mathbb{P}(L_{t-s} \leq x)$, $0 \leq s < t < \infty$
(iv) L has independent increments: $L_t - L_s$ is independent of $\mathcal{F}_s$, $0 \leq s < t < \infty$

Definition 3.5. *A real valued r.v.* Θ *has an infinitely divisible distribution when for each* $n = 1, 2, \ldots$*, there exists a i.i.d. sequence of r.v. s* $\Theta_1, \ldots, \Theta_n$ *such that*

$$\Theta \stackrel{d}{=} \Theta_{1,n} + \ldots + \Theta_{n,n}$$

In other words, the law μ of a real valued r.v. is infinitely divisible if for each $n = 1, 2, \ldots$ there is another law μ_n of a real valued r.v. such that $\mu = \mu_n^{*n}$, the n-fold convolution of μ_n.

We characterise infinitely divisible distributions via their characteristic function[3] and the Lévy-Khintchine formula.

Theorem 3.6 (Lévy-Khintchine)**.** *Let* $\mu \in \mathbb{R}$*,* $\sigma \geq 0$*, and* Π *be a measure concentrated on* $\mathbb{R}/\{0\}$ *such that* $\int_{\mathbb{R}} \min(1, x^2)\Pi(dx) < \infty$*. A probability law* μ *of a real-valued r.v.* L *has characteristic exponent* $\Psi(u) := -\frac{1}{t}\log \mathbb{E}[e^{iuL_t}]$ *stated as follows*

$$\Phi(u; t) = \int_{\mathbb{R}} e^{iux}\mu(dx) = e^{-t\Psi(u)} \quad \text{for} \quad u \in \mathbb{R}, \tag{2}$$

iff there exists a triple (γ, σ, Π)*, where* $\gamma \in \mathbb{R}, \sigma \geq 0$ *and* Π *is a measure supported on* $\mathbb{R} \setminus \{0\}$ *satisfying* $\int_{\mathbb{R}}(1 \wedge x^2)\Pi(dx) < \infty$*, such that*

$$\Psi(\lambda) = i\gamma u + \frac{\sigma^2 u^2}{2} + \int_{\mathbb{R}} \left(1 - e^{(iux)} + iux\mathbf{1}_{|x|<1}\right)\Pi(dx) \tag{3}$$

for all $u \in \mathbb{R}$*.*

From 3.6 we are able to see that a probability space exists when $L = L^{(1)} + L^{(2)} + L^{(3)}$; $L^{(1)}$ is standard one-dimensional Wiener process with drift, $L^{(3)}$ is a square integrable martingale[4], $L^{(2)}$ is a (compound) Poisson process. This is the the Lévy-Itô decomposition, which we now phrase as follows:

$$L_t = \eta t + \sigma W_t + \int_0^t \int_{|x|\geq 1} x\mu^L(ds, dx) + \int_0^t \int_{|x|<1} x(\eta^L - \Pi^L)(ds, dx). \tag{4}$$

Definition 3.7 (Markov-Switching)**.** *Let* $(Z_t)_{t\in[0,T]}$ *be a continuous time Markov chain on finite space* $S := \{1, \ldots, K\}$*. Let* $\mathcal{F}_t^Z := \{\sigma(Z_s); 0 \leq s \leq t\}$ *be the natural filtration generated by the continuous time Markov chain* Z*. The generator matrix of* Z*, denoted by* Π^Z*, is stated as follows*

$$\Pi_{ij}^Z \begin{cases} \geq 0, & \text{if } i \neq j \\ -\sum\limits_{j\neq i} \Pi_{ij}^Z, & \text{otherwise} \end{cases} \tag{5}$$

And now we define the Regime-switching Lévy model:

Definition 3.8 (Lévy Regime-switching model)**.** *For all* $t \in [0, T]$*, let* Z_t *be a continuous time Markov chain on finite space* $S := \{1, \ldots, K\}$ *defined as per 3.7. A regime-switching model is a stochastic process* (X_t) *which is solution of the following SDE*

$$dX_t = k(Z_t)(\theta(Z_t) - X_t)dt + \sigma(Z_t)dY_t \tag{6}$$

[3] alternatively, via the Fourier transform of their law

[4] with countable number of jumps of magnitude less than 1 (almost surely)

where $k(Z_t)$, $\theta(Z_t)$, and $\sigma(Z_t)$ are functions of the Markov chain Z. They are scalars which take values in $k(Z_t)$, $\theta(Z_t)$, and $\sigma(Z_t)$: $k(Z_t) := \{k(1), \dots, k(K)\} \in \mathbb{R}^{K^*}$, $\theta(S) := \{\theta(1), \dots, \theta(K)\}$, $\sigma(S) := \{\sigma(1), \dots, \sigma(K)\} \in \mathbb{R}^{K^*}$. where Y is a Wiener or a Lévy process. Here, k denotes the mean reverting rate, θ denotes the long run mean, and σ denotes the volatility of X.

Here we get two sources of 'variation' (stochasticity): the stochastic process Y which follows the dynamics of X, and the Markov chain Z. What this means is that there is variation due to the Markov chain Z, $\mathcal{F}^Z$, and variation from market information which is the filtration $\mathcal{F}$ generated by the stochastic process Y.

3.3. Normal Inverse Gaussian (NIG)

Barndorff-Nielsen (see [4, 19, 20] and [5] for a survey) introduced the Normal Inverse Gaussian (NIG) distribution, which "is defined as a variance-mean mixture of a normal distribution with the inverse Gaussian as the mixing distribution.". It is a versatile model for heavy-tailed stochastic processes, where the it is completely specified by four real valued parameters that have appealing interpretations in terms of the resulting probability density function. Following [14], we let our Levy process L follows the Normal Inverse Gaussian distribution. The density function of a $NIG(\alpha, \beta, \delta, \mu)$ is stated as

$$f_{NIG}(x; \alpha, \beta, \delta, \mu) = \frac{\alpha}{\pi} e^{\delta \sqrt{\alpha^2 - \beta^2 + \beta(x-\mu)}} \frac{K_1(\alpha\delta\sqrt{1 + (x-\mu)^2/\delta^2})}{\sqrt{1 + (x-\mu)^2/\delta^2}}, \tag{7}$$

where $\delta > 0$, $\alpha \geq 0$. The parameters in the Normal Inverse Gaussian distribution can be interpreted as follows: α is the tail heaviness of steepness, β is the skewness, δ is the scale, μ is the location. The NIG distribution is the only member of the family of general hyperbolic distributions to be closed under convolution. K_v is the Hankel function with index v. This can be represented by

$$K_v(z) = \frac{1}{2} \int_0^\infty y^{v-1} e^{\left(-\frac{1}{2}z\left(y + \frac{1}{y}\right)\right)} dy \tag{8}$$

For any real v, the function K_v satisfies the differential equation

$$v^2 y'' + xy' - (x^2 + v^2)y = 0. \tag{9}$$

The log cumulative function of a NIG variable is

$$\phi^{NIG}(z) = \mu z + \delta \left(\sqrt{\alpha^2 - \beta^2} - \sqrt{\alpha^2 - (\beta + z)^2}\right) \quad \text{for all } |\beta + z| < \alpha.$$

We can state the first moments as $\mathbb{E}[X] = \mu + \frac{\delta\beta}{\gamma}$, and $Var[X] = \frac{\delta\alpha^2}{\gamma^3}$, where $\gamma = \sqrt{\alpha^2 - \beta^2}$. Then, the Levy measure of a Normal Inverse Gaussian $NIG(\alpha, \beta, \delta, \mu)$ law is

$$F_{NIG}(dx) = e^{\beta x} \frac{\delta\alpha}{\pi|x|} K_1(\alpha|x|)dx.$$

4. Estimation

We use the expectation–maximisation algorithm to estimate our RS Levy model, namely the SDE given in 6, by estimating the stochastic process Y which is a Levy process that follows a Normal Inverse Gaussian distribution. After discretising the model, we estimate the following parameters in a in a two-step procedure as per [14]:

$$\hat{\Theta} := (\hat{k}_i, \hat{\theta}_i, \hat{\sigma}_i, \hat{\alpha}_i, \hat{\beta}_i, \hat{\delta}_i, \hat{mu}_i, \hat{\Pi}) \quad i \in S.$$

Apart from the transition matrix of the Markov chain Z, we have we have 3 parameters of the dynamics of X, and 4 parameters of the density of the Levy process L, *4.1. Discretisation the model*

Consider a stochastic process Y and Wiener process W. Γ is the increasing time index:

$$\Gamma = \{t_j; 0 = t_0 \leq t_1 \leq \dots t_{M-1} \leq t_M = T\},\ with\ \Delta_t = t_j - t_{j-1} = 1.$$

In this specification, $M+1$ denotes to the size of historical data. The discretized version of the SDE given in 6 is

$$X_{t+1} = k(Z_t)\theta(Z_t) + (1 - k(Z_t))X_t + \sigma(Z_t)\epsilon_{t+1}. \tag{10}$$

Since Y is a Wiener process, $\epsilon_{t+1} \sim N(0,1)$. Let $\mathcal{F}^X_{t_k}$ be the vector of historical values of the process X until time $t_k \in \Gamma$. Then $\mathcal{F}^X_{t_k}$ is a vector of $k+1$ values of the discretized model. Thus $\mathcal{F}^X_{t_k} = (X_{t_0}, X_{t_1}, \dots, X_{t_k})$.

Next, we proceed with estimating our model in two stages.

- **Stage 1**: Estimation of the regime-switching model 6 in the Wiener case. To estimate the parameters of the discretised model 10 we rely on the EM-algorithm. First, the estimate $\hat{\Theta}$ is divided thus $\hat{\Theta}_1 := (\hat{k}_i, \hat{\theta}_i, \hat{\sigma}_i, \hat{\Pi}_i)$ for $i \in S$.
- **Stage 2**: Estimation of the parameters of the Levy process fitted to each regime. Using the regime classification from Step 1, we calculate parameters $\hat{\Theta}_2 := (\hat{\alpha}_i, \hat{\beta}_i, \hat{\delta}_i, \hat{\mu}_i)$ for $i \in S$. This corresponds to the NIG distribution parameters of the Levy jump-diffusions fitted for each regime.

4.2. Augmented Dickey-Fuller test

First, we conduct an Augmented Dickey-Fuller test. This tests the null hypothesis that our index log returns follow a unit root process. We conclude that our index log returns has neither a trend nor seasonality.

Whilst the Normal Inverse Gaussian distribution, will allow us to model for skewness ad heavier tails, we also estimate. other benchmark models used in the financial econometrics literature. We estimate models by maximum likelihood estimates, where neccessary these estimates are determined by the EM algorithm.

4.3. Lilliefors test

We examine the data using the Lilliefors test for normality, where the null is that the data have been drawn from a normal distribution[5]. Using MATLAB's function *lillietest*, we observe that the test statistic k is greater than the critical value c, so the test indicates a rejection of the null hypothesis at the default 5% significance level.

We then use *MS Regress*, a MATLAB toolbox specially designed for the estimation of a general markov regime switching model [36]. We first estimate the following for $S = 1, 2$:

$$E(\Delta y_t | S_t) = \mu(S_t) \begin{cases} \mu_0 + e_t \\ \mu_1 + e_t \end{cases} \tag{11}$$

where $e_t \sim N(0, \sigma^2)$. Then we retrieve the residuals vector of the dependent variable $\hat{e}_t$ and regress it on k lagged variables

$$\hat{e_t} = \sum_{i=1}^{k} \beta_i \Delta o^+_{t-i} + \nu_t \tag{12}$$

where $\nu_t \sim N(0, \sigma^2)$. We plot the smoothed state probabilities in Figure 2.

[5]Lilliefors test is a version of the classic Kolmogorov-Smirnov test which has been corrected for the possible presence of parameter uncertainty. We avoid omnibus tests a-la Jarque–Bera since they tend to be underpowered.

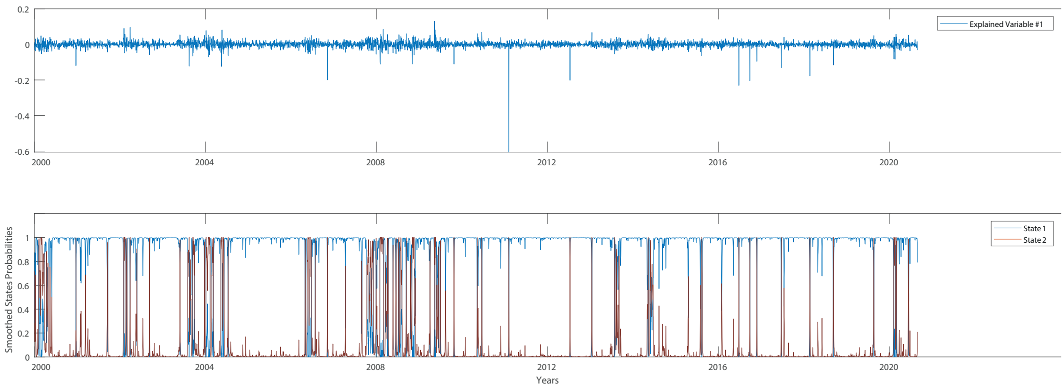


Figure 2: Regime identification - smoothed state probabilities (bottom), and realised volatility (top).

4.4. Regime classification measures

We assess how well our regime fitting exercise has been conducted in two following ways. First, we use the Regime classification measure (RCM) introduced by Ang and Bekaert (2002) [1, 2]. The RCM statistic takes values between 0 and 100. Low values indicate sharp regime classification. High values indicate that the regimes have not been identified well.

$$RCM(K) = 100 \times \Big(1 - \frac{K}{K-1}\frac{1}{T}\sum_{k=1}^{N}\sum_{Z_{t_k}}^{N}\Big(P(Z_{t_k} = i|\mathcal{F}_{t_M}^X; \hat{\Theta}_1^{(n)}) - \frac{1}{K}\Big)^2\Big), \tag{13}$$

where $\hat{\Theta}_1^{(n)}$ corresponds to the vector of estimated parameters in question, and $P(Z_{t_k} = i|\mathcal{F}_{t_M}^X; \hat{\Theta}_1^{(n)})$ corresponds to the smoothed probability. The role of the constant 100 is to normalise the statistic to be between 0 and 100.

Second, we use the smoothed probability indicator [21]. A good classification is when the smoothed probability is greater than 0.9 or less than than 0.1. This then means that the data at time $t \in [0, T]$ is, with a probability higher than 90%, in one of the regimes at the 10% error level.

5. Results

5.1. Regime classification

We start with regime classification. After estimating our model, we obtain RCM Statistic: 12.13 and smoothed probability indicator 85.85%. This then means that the data in our sample is in one of the regimes with a probability close to 90%.

This type of model estimation has precedent in the literature. For example, [15] estimated jumps and regime switches in international stock markets returns for all major developed markets using data from June, 2004 to July 11, 2014. We can compare this model with their as follows:

Table 2: Regime classification measures for Regime Switching Levy Model

	RCM	P%
India Energy Index (NIFTY50)	12.13	85.85

Overall, we can see that our regime classification performs well. Not only do we see that our regime-switching model performs well in identifying the regimes, it is also able to identify regimes more sharply using our data (i.e. energy equities in India) than some (but not all) benchmark indices reported in Tables 3 and 2.

Table 3: Regime classification measures for benchmark indices, in [15]

	RCM	P%
Dow Jones Industrial Average	12.51	86.88
EURO STOXX European 600 Index	13.92	85.71
Russell 2000 Index	18.33	80.43
Nikkei 225 Index	9.07	90.35
NASDAQ Composite Index	18.38	81.13
FTSE 100 Index	13.12	86.33
Dow US Global Dow Jones	19.22	78.92

Figure 4 graphically shows the presence of two identified regimes, which we can plausibly describe as "normal volatility" and "high volatility". The high volatility regimes seem to correspond to crisis periods, such as the 2007-08 North Atlantic crisis and the 2011 Indian energy crisis.

	Regime 1	Regime 2
α	0.001803	1.026199
β	-0.000282	0.006025
Σ	1.362165	6.089931
E_{tt}	-6.386800	170.309453
q_{ii}	0.973255	0.902871
$P(R=i)$	0.784094	0.215906
θ	-6.386800	170.309453
κ	-0.000282	0.006025
σ	1.855494	37.087262

Table 4: Two state regime switching model with mean-reverting process

	Regime 1	Regime 2
α	0.640648	0.986841
β	-0.002221	-0.177631
δ	0.293492	0.643627
μ	0.001018	0.117776

Table 5: Estimating parameters of our NIG model

5.2. Comparison against other models (incl. single-regime)

We now introduce alternative specification from competing models in an attempt to benchmark the performance of the regime-switching model with jumps, versus a model without jumps, and versus a model with no regime switching.

1. Regime-switching Lévy model. This is our main specification as described in model 5. Here, $Y = L$ is a Lévy process such that $L_1 \sim NIG(\alpha, \beta, \mu)$.

$$dX_t = k(Z_t)(\theta(Z_t) - X_t)dt + \sigma(Z_t)dY_t \tag{14}$$

2. Regime-modified Ornstein–Uhlenbeck. Here $Y = W$ is a Brownian motion.

$$dX_t = k(Z_t)(\theta(Z_t) - X_t)dt + \sigma(Z_t)dY_t dW_t \tag{15}$$

3. Ornstein-Uhlenbeck with constant volatility. Here the process $Y = W$ is a Brownian motion without regime switches. The SDE does not depend on the Markov chain Z.

$$dX_t = k(Z_t)(\theta(Z_t) - X_t)dt + \sigma dW_t \tag{16}$$

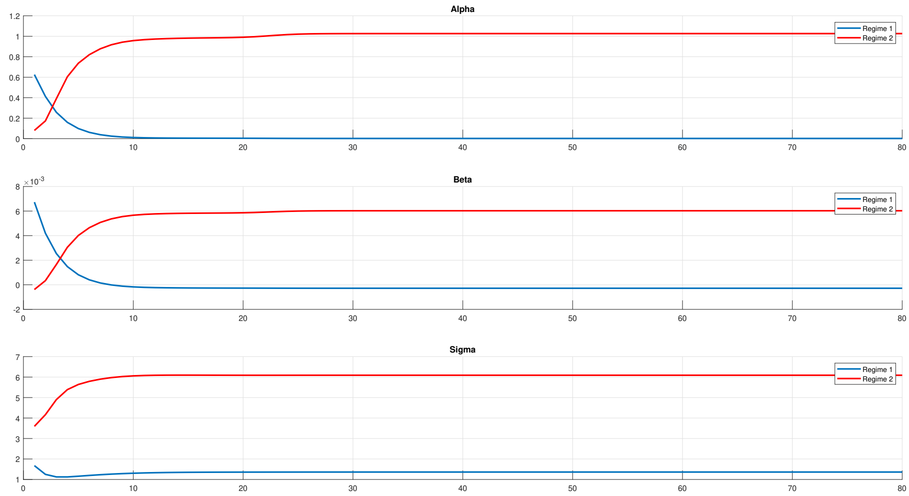


Figure 3: Plotting evolution of parameters

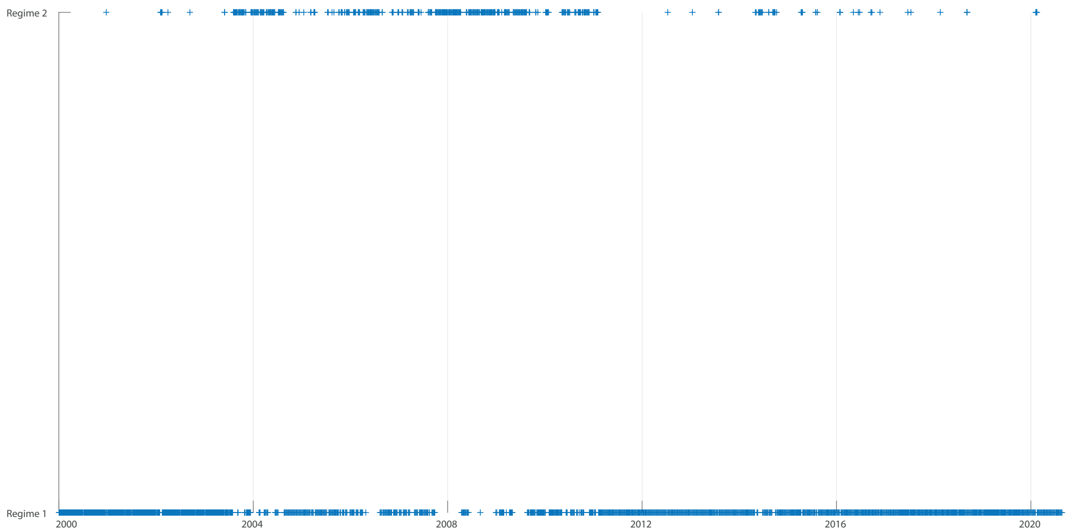


Figure 4: Regime identification

Total number of observations	5163
maximum	290.22
minimum	57.32
mean	144.29
variance	3221.63
standard deviation	56.75
Skewness	82250.83
Kurtosis	22848585.21

Table 6: Summary statistics

Model	BIC:	AIC:
Regime-switching Lévy model	4787.50	4735.10
Regime-modified Ornstein–Uhlenbeck	20891.14	20838.75
Ornstein-Uhlenbeck with constant volatility	26287.07	26267.42

Table 7: Model comparison

5.3. Model comparison

We use the EM-algorithm to calculate the log-likelihood values. We use standard model selection criteria for each specification (AIC and BIC). The results are presented in Table 7

Based on our model selection criteria, we can say that regime-augmented models are preffered to a model that allows no regime identification. But overall, we prefer a model with jumps and regimes over those that do not allow for jump-diffusion and Markov regime-switching.

6. Conclusion

In this study we constructed an index of energy stocks using daily data from NIFTY50, a benchmark Indian stock market index that represents the largest Indian companies listed on the National Stock Exchange. We examined this newly-constructed index for regime-switching behaviour, non-linearities, and evidence of jump-diffusion processes, deploying a Markov Regime-switching Lévy Model.

We made use of the expectation–maximization algorithm, an iterative method to find maximum likelihood or maximum a posteriori estimates of parameters, to calculate the log-likelihood values.

We then introduced alternative specification from competing models in an attempt to benchmark the performance of the regime-switching model with jumps, versus a model without jumps, and versus a model with no regime switching. We use Regime classification measures in order to identify our model performance, and compare results with other indices, such as Dow Jones Industrial Average, EURO STOXX, Russell 2000, Nikkei 225, NASDAQ, and FTSE 100.

Overall, the findings suggest that our regime classification framework performs to an adequate standard, both in terms of theoretical expectations [33] and in terms of comparison with related literature [34].

Based on our model selection criteria, we can say that regime-augmented models are preferred to a model that allows no regime identification. But overall, we prefer a model with jumps and regimes over those that do not allow for jump-diffusion and Markov regime-switching. An interesting line of research is to pursue more advanced models, such as Levy rough paths.

Bibliography

[1] Ang,A.,Bekaert,G. (2002) Regime Switching in Interest Rates. *Journal of Business and Economic Statistics* 20:00 163-182.
[2] Ang, A., Timmermann, A. (2012) Regime Changes and Financial Markets, *Annual Review of Financial Economics*, 4, 313–337.
[3] Ang,A.,Timmermann, A. (2012) Regime Changes and Financial Markets, *Annual Review of Financial Economics*, 4, 313–337.
[4] Barndorff-Nielsen, O. E. (1997). Normal Inverse Gaussian Distributions and Stochastic Volatility Modelling. Scand. J. Statist., 24:1-13.
[5] Barndorff-Nielsen, O. E., T. Mikosch, and S. I. Resnick. (2001). Levy Processes: Theory and Applications. Berlin, Germany: Springer Science & Business Media.
[6] Bates, D. S. (1991). The crash of 87 Was it expected? The evidence from options markets. *J. Finance* 46 1009-1044.
[7] Bansal, R. and Zhou, H. (2002). *Term structure of interest rates with regime shifts*. Journal of Finance, 57 (4), 1997-2043.
[8] Beaglehole D. and Tenney, M. (1991). *General Solutions of Some Interest Rate- Contingent Claim Pricing Equations*. Journal of Fixed Income 1, 69-83.
[9] Bekaert, G., Hodrick R., (1993) On Biases in the Measurement of Foreign Exchange Risk Premium. *Journal of International Money and Finance* 12, 115-138.
[10] Billio, M., Pelizzon. L., (2000) Value-at-Risk: a Multivariate Switching Regime Approach. *Journal of Empirical Finance*, 7, 531-554.
[11] Brooks, C., Prokopczuk, M., (2013) The Dynamics of Commodity Prices. *Quantitative Finance* 13 (4): 527-542.
[12] Carr, P., Geman H., Madan, D. B., Yor, M., (2003) Stochastic Volatility for Lévy Processes. *Mathematical Finance* 13:00 345-382.
[13] Chen L., Filipovic, D. and Poor, H.V. (2004). *Quadratic Term Structure Models for risk-free and defaultable rates.* Mathematical Finance, 14(4), 515-536.
[14] Chevallier, J., Goutte, S. (2016) On the estimation of regime-switching Lévy models. *Studies in Nonlinear Dynamics & Econometrics*, 21(1), pp. 3-29.
[15] Chevallier, J, Goutte, S. (2015). Detecting jumps and regime switches in international stock markets returns. Applied Economics Letters. 22 1011-1019.
[16] Deaton, A., Laroque, G., (1992) On the Behaviour of Commodity Prices. *Review of Economic Studies* 59: 1-23.
[17] Driffill, J., Sola, M., (1998), Intrinsic Bubbles and Regime Switching. *Journal of Monetary Economics* 42, 357-373.
[18] Engle, R., Hamilton, JD., (1990) Long Swings in the Dollar: Are They in the Data and Do Markets Know It? *American Economic Review* 80: 689-713.
[19] Eriksson, A., Ghysels, E., Wang W. (2009) The Normal Inverse Gaussian Distribution and the Pricing of Derivatives, *The Journal of Derivatives*, 16, 23-37
[20] Fragiadakis, K., Karlis, D., and Meintanis S.G. (2009) Tests of fit for normal inverse Gaussian distributions, *Statistical Methodology*, 6,553-564
[21] Goutte S., Zou B. (2013) Continuous time regime switching model applied to foreign exchange rate. Mathematical Finance Letters 8, 1-37.
[22] Hainaut D. (2010) Switching Lévy processes : a toolbox for financial applications.
[23] Hamilton JD., Susmel, R. (1994), Autoregressive Conditional Heteroskedasticity and Changes in Regime. *Journal of Econometrics* 64 307-333.
[24] Hamilton JD., (2016), Macroeconomic Regimes and Regime Shifts, in *Handbook of Macroeconomics*, eds. Taylor, J. B. and Uhlig, H. Elsevier, vol. 2, chap. 3.
[25] https://www.kaggle.com/rohanrao/nifty50-stock-market-data, accessed 1 October 2020
[26] Huisman, R., Mahieu, R., (2003) Regime Jumps in Electricity Prices. *Energy Economics* 25 (5):425-434.
[27] Kaeck, A. (2013) Asymmetry in the Jump-Size Distribution of the S&P 500 Evidence from Equity and Option Markets. *Journal of Economic Dynamics and Control* 37 (9): 1872-1888.
[28] Kallsen, J. and Pauwels, A. (2011). *Variance-optimal hedging for time-changed Lévy processes,* Appl. Math. Finance 18, no. 1, 1-28.
[29] Kallsen, J. and Pauwels, A. (2010). *Variance-optimal hedging in general affine stochastic volatility models*, Advances in Applied Probability **20** no. 1, 83-105.
[30] Kallsen, J., Muhle-Karbe, J., Shenkman, N. and Vierthauer, R. (2009). *Discrete-time variance-optimal hedging in affine stochastic volatility models*. In R. Kiesel, M. Scherer, and R. Zagst, editors, Alternative Investments and Strategies. World Scientific, Singapore.
[31] Khurana, M., and Banerjee, S. G. (2014). Beyond Crisis: The Financial Performance of India's Power Sector. The World Bank.
[32] Madan, D. B., Seneta, E., -1990 The VG Model for Share Market Returns. *Journal of Business* 36. 364-419.
[33] Merton, R. C. (1976) Option pricing when underlying stock returns are discontinuous. *Journal of Financial Economics* 3 125-144.
[34] Merton, R. C. (2001) Continuous- Time Finance, 1st revised edn (Blackwell).
[35] Monfort, A. Feron, O., (2012) Joint Econometric modelling of Spot Electricity Prices, Forwards and Options. *Review of Derivatives Research*, 15, 217-256.
[36] Perlin M. (2015) MS Regress, The MATLAB Package for Markov Regime Switching Models.
[37] Revuz, D., Yor, M. (2005). *Continuous martingales and brownian motion*. Springer 3rd edition Vol 293.

[38] Rüdiger, B. and Tappe, S. (2009). *Stability results for term structure models driven by Lévy processes.* Working paper No. 17, Wirtschaft Universität Wien.