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Externalities and Optimal Taxation: A Progressive Tax Case*

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Abstract

The paper studies the optimal income taxation by adding utility interdependence over labour choice. Both theoretically and numerically, it is shown that the optimal marginal tax schedule could be progressive with this additional feature. Previous studies on optimal redistributive income taxation consider the consumption externalities but ignore the labour interdependency. Specifically, if disutility depends on the average working hour, the increase in an agent's working hour creates positive externality on other agents as it lowers the disutility of others. It is shown that as the degree of utility interdependence increases, the tax schedule becomes more progressive. Moreover, the paper analyses the effect of having a more dispersed skill distribution on the marginal income tax rates. By using their wage distribution data as a proxy for their ability distribution, the optimal marginal tax rates in the United Kingdom and the Czech Republic are examined. Considering the more unequal wage distribution in the UK, there should be a more progressive tax schedule.

Keywords: Optimal Income Tax; Externalities

JEL Codes: H21; H23

1 Introduction

Since the seminal work of Mirrlees (1971), there has been a series of studies on the modern optimal income taxation. Mirrlees indicates that a high marginal tax rate implemented to the high productive workers distorts the labour supply decision and leads to a disincentive for

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working. Considering the efficiency loss, optimal income taxation might follow a regressive pattern which means lower marginal tax rates for high-income earners. Even more strikingly, Sadka (1976) and Seade (1977) show that the optimal income tax rate at the top of the income (skill) distribution should be zero if there exists a finite maximum to the skill distribution.

There is an ongoing conflict between the optimal income taxation theory and the current tax policies as there is almost no country with a regressive income tax schedule. Even Mirrlees himself admitted he found the results of the study surprising¹. Furthermore, Atkinson (1973), Tuomala (1983,1990) verified Mirrlees (1971) regressive marginal tax result. However, Diamond (1998) has shown that optimal tax schedule could be progressive at the upper tail of the income distribution if the skill distribution is unbounded. Saez(2001) mentioned the progressivity of the tax schedule when the link between optimal tax formulas and elasticities of earnings are considered. It is important to note that these results are very sensitive to the skill distribution assumptions. While Mirrlees and Tuomala use log-normal skill distribution, Diamond and Saez assume Pareto distribution for the upper tail of the skill (income) distribution.

Theoretically, the optimal income tax has a very complex structure, and it is hard to get clear-cut analytical results without making strong assumptions. According to Dahan and Strawczynski (2000), when the utility of consumption is concave, the optimal marginal tax for the high-income earners is unclear. Because a concave utility implies weaker income effects for rich individuals, which is lowering the tax rates. However, a concave utility of consumption also affects the inequality aversion and leads to a rise in tax rates. Due to these opposite forces, Dahan and Strawczynski(2000) mentions that under the concave utility of consumption, one can only use simulations to determine the shape of the marginal tax rates.

Mirrlees (1971) paper also relies on simulations to obtain tax schedules. In these simulations, Mirrlees finds a slightly decreasing pattern for marginal taxes over the skill distribution. This almost linear decreasing pattern for marginal taxes is criticised by Atkinson (1973) and Tuomala (1984). They claim that this linear and almost constant tax rates are due to the specific functional forms used by Mirrlees (1971). By using a maximin objective function Atkinson (1973) shows that there will be a non-linear pattern for the marginal tax schedule. Tuomala (1984,1990,1994) find an inverted U-shaped profile for marginal taxes which exhibits a non-linear pattern but again declining marginal tax rates for high-income earners. Dahan and Strawczynski (2000) shows that rising marginal tax rates in Diamond (1998) depend on the assumption of utility for consumption. In their simulations, they find that with a linear utility for consumption; it is possible to have an increasing marginal tax pattern for high-income earners even with a log-normal ability distribution. Their paper shows the importance of utility for consumption in determining the tax schedule.

In this study, we consider optimal marginal income taxation by adding utility interde-

¹“I must confess that I had expected the rigorous analysis of income-taxation in the utilitarian manner to provide an argument for high taxes. It has not done so.”, Sir James A. Mirrlees, “An Exploration in the Theory of Optimum Income Taxation”.

pendence over labour choice. Both theoretically and numerically, we show that the optimal marginal tax schedule could be progressive with this additional feature. Previous studies on optimal redistributive income taxation consider the consumption externalities but ignore the labour interdependency of agents. Samano (2009) investigates the consumption externalities and mentions the progressivity effect of the consumption externality over tax schedules. Oswald (1983), Tuomala (1990), Kanbur and Tuomala (2010) look at the tax schedule when agents value their consumptions relative to the average consumption. Tuomala (1990), Kanbur and Tuomala (2010) find support for greater progressivity in the tax structure as relative consumption concern increases. According to Oswald (1983) if there is utility interdependence then zero marginal tax at the top of the skill (income) distribution result does not hold. He claims that if utility is separable and the agent's behaviour (consumption-labour choice) is not affected by the relative concern, then this additional relative consumption concern leads to a progressive marginal income tax schedule. Also, he mentions that marginal taxes could be negative in this environment.

To the best of our knowledge, the only study with labour externalities under a Mirrleesian framework is Oztek (2011). This study analyses the tax schedule by adding utility interdependence over labour choice and shows that optimal marginal income taxation could be progressive depending on the parameters of the model. There are two separate forces that are at work in determining the optimal tax schedule. First, due to the informational problems, there is a usual Mirrleesian force that works towards the regressivity of taxes. The second effect is a novel force that arises from labour externality and has a progressive effect on the income tax. This effect could be called as Pigouvian tax. Labour externality requires subsidies for agents which are asymmetric according to productivities. Due to this asymmetry, there should be higher subsidies for low types which has a progressive effect on the optimal tax schedule. Additionally, when labour interdependence is added to the model, zero tax at the top of the skill distribution result is no longer valid.

Oztek (2011) analyses the tax schedule under a quasi-linear utility in an N-type model. With a quasi-linear utility form, it is possible to solve the tax functions analytically. However, when we consider Dahan and Strawczynski (2000) results, this type of utility could also give a progressive tax schedule without labour interdependence. Therefore, using a quasi-linear utility function may represent a limited idea about the effect of labour externality. Moreover in Oztek (2011), labour externality does not affect agent's behaviour, so it is not possible to see the effect of labour externality when agent's behaviour is changed. Also, Oztek (2011) does not have any comparable simulations with the existing tax literature which holds a crucial role in optimal taxation studies.

In this study, first, we set up an N-type model with a separable utility form and show the progressive effect of labour externality. To interpret the results clearly, we used a specific (log utility) form of utility function which is used by Tuomala (1990) for consumption interdependence. Tuomala(1990) and Oswald(1983) analyse the cases where the agent's behaviour is

not affected by the additional relative consumption concern. However, we show that even the agent's behaviour is changed, it is possible to have a progressive tax schedule. The change in the agent's behaviour solely shows up in the Mirrleesian part of the tax function as an additional regressive force.

Simulations constitute an important part of this study as explicit solution is not possible with a log-utility. By assuming a log-normal skill distribution with parameters $(\mu, \sigma) = (-1, 0.39)^2$, we show that as the degree of utility interdependence increases, the tax schedule becomes more progressive. Also, we analysed the effect of having a more dispersed skill distribution on the marginal income tax rates. We conduct various tests in the numerical examples part, but the most important examples are summarized in Table 1. Mirrlees(1971) regressive result is based on a logarithmic utility and log-normal skill distribution. Diamond(1998) changes the utility of consumption to a linear form and uses a Pareto distribution for skills. By doing these changes he reverses the regressive pattern of Mirrlees to an increasing pattern for the high-income earners. However, Dahan and Strawczynski(2000) suggests that under linear utility for consumption there is no need to assume a Pareto distribution for a progressive tax schedule. A rising pattern for high-income earners is also possible with a log-normal distribution. The last two rows of Table 1 show our numerical examples by adding utility interdependence to the model. With this additional concern it could be said that even under a logarithmic utility for consumption, it is possible to have a rising pattern for high-income earners.

Table 1: Summary of Results

	Utility of Leisure	Utility of Consumption	Distribution of Skills	Optimal Tax at High Levels of Income
Mirrlees(1971)	Logarithmic	Logarithmic	Log-normal	Declining
Diamond(1998)	Logarithmic	Linear	Pareto	Rising
D&S(2000) Example 1	Logarithmic	Logarithmic	Pareto	Declining
D&S(2000) Example 2	Logarithmic	Linear	Log-normal	Rising
Our Example 1	Logarithmic	Logarithmic	Log-normal	Weakly Rising
Our Example 2	Logarithmic	Linear	Log-normal	Rising

Finally, in the last part of the numerical examples, by using their wage distribution as a proxy for their ability distribution, we have analysed the marginal tax rates in the United Kingdom and the Czech Republic. According to OECD data and the log-normal estimations of wage distributions, United Kingdom has a more unequal wage distribution than the Czech Republic. By using Cowell(2011) and Mala and Nedved(2011) parameter estimates for log-normal wage distribution, we show that the United Kingdom should have a more progressive tax schedule than the Czech Republic.

This study considers utility interdependence among agents over their labour supply decisions. Unlike most studies which claim people derive disutility from their own work only,

²Parameters of Mirrlees(1971).

this paper suggests that individuals also care about other people working hours. Therefore, in our setup, we use the combination of their own and other people's labour choice $v(l, L)$ as the form of utility(disutility) for leisure(labour), where L denotes the average working hour in the society. Specifically, if disutility depends on the average work hour, the increase in an agent's working hour has a positive externality on other agents because it lowers the disutility of others.

Labour interdependence among people has been an interesting topic for economists over the decades. Economists like Veblen and Pigou have mentioned this fact in their works: people are affecting each other's labour decisions. Not only the income and consumption but people also consider and compare their leisure (labour) hours with the others. Veblen (1899) found a great term for this fact as "conspicuous consumption and conspicuous leisure".³ Arthur Pigou(1920) on the other hand, found a more straightforward way to explain it: "men do not desire to be rich, but richer than other men".

Several micro-level studies in labour economics pay attention to direct interactions between the agents. Grodner and Kniesner (2006,2009) show that labour interdependency has a significant effect on the agent's labour supply decision. Aromsson, Blomquist, and Sacklen (1999) analyses people's choices for working hours. They mention that people are influenced by each other's hours of work and neglecting this could lead to serious underestimates of the labour supply effects of income taxes. As an empirical example, Weinberg et al. (2004) finds that an extra hour worked by the social reference group of an individual can increase the individuals total working hours by 0.6 in the United States. Moreover, Pingle and Mitchell (2002) finds evidence of leisure positionality in a questionnaire-based study. They mention that most income is derived from allocating time toward labour and leisure; therefore, any observed positional concern for income is potentially confounded with a positional concern for leisure.⁴

The rest of the paper proceeds as follows: section 2 presents the model, section 3 presents the separable utility case, section 4 assumes a specific form of utility, section 5 presents the case when behaviour is changed due to utility interdependence, section 6 shows the numerical examples, section 7 conducts the simulations for United Kingdom and Czech Republic, finally section 8 concludes.

³"...the utility of both (conspicuous leisure and conspicuous consumption) alike for the purposes of reputability lies in the waste that is common to both. In the one case it is a waste of time and effort, in the other it is a waste of goods. Both are methods of demonstrating the possession of wealth, and the two are conventionally accepted as equivalents." The Theory of the Leisure Class, 1899.

⁴For other studies about labour interdependency see: Baskaya, Y., and Kilinc, M. (2010), Woittiez and Kapteyn (1998), Becker and Murphy (2000) Glaeser, Sacerdote, and Scheinkman (2003), Elster (1989), Fryer and Payne (1986), Jackson and Warr (1987), Feather (1990), Platt and Kreitman (1990) and Platt, Micciolo and Tansella (1992).

2 Model

For the theoretical discussion, first, we start with a general separable utility form. Afterwards, we utilize a specific form of utility function which is used in Mirrlees (1971) and Tuomala(1990). However, in this type of utility, the agent's behaviour is not affected by additional labour externality concern. In the incentive compatibility constraint, labour externality shows up in both sides of the equation in the same form, and these cancels out each other. Therefore, in the next stage, we use a specific form of utility in which the agent's behaviour is affected by the labour interdependence. Even in this case, we show the progressive effect of labour externality.

The paper utilises an N-type model, and agents are heterogeneous about their privately known productivity levels. An agent with a productivity level θ has a separable utility function in the form of consumption and labour;

$$U(c, y, L, \theta) = u(c) - v\left(\frac{y}{\theta}, L\right)$$

where c is consumption, y is agent's income and $L = \sum_{i=1}^N \pi_i \frac{y_i}{\theta_i}$ is the average working hour of the society. Production function is $y = \theta l$, thus labour equals to $l = \frac{y}{\theta}$ in terms of working hours. There could be many alternative interpretations of the variable L . In this paper, we use the Tuomala(1990) approach. While Tuomala(1990) uses the average consumption of people in the economy for testing the effect of utility interdependence, we use the average working hour of the society to examine the effects of labour interdependence on the optimal tax rates.

Assumptions of the model are as follows:

- i) Preferences satisfy the usual assumptions that; $u'(c) > 0, u''(c) < 0$ and $v_1(\frac{y}{\theta}, L) > 0, v_{11}(\frac{y}{\theta}, L) > 0$.
- ii) There are two additional assumptions where the second one is optional;
 - 1-) $v_2(\frac{y}{\theta}, L) < 0$ which means disutility decreases when L increases.
 - 2-) $v_{21}(\frac{y}{\theta}, L) > 0$ which means the agent who works more is getting a higher disutility decrease from a rise in L .

While agents derive utility from consumption, as usual, working is a source of disutility. In this setup, they also derive utility from an increase in average working hour, because agent's disutility is decreasing with the average working hour of the society.

3 Separable Utility Case

The aim of the social planner is to maximize the overall welfare of the society while evaluating all agents equally. π_i is the proportion of the different productive agents in the society and it is normalised to 1. In the presence of information asymmetry, which arises when the productivity is private information of agent and cannot be observed by the social planner, the social planner problem is as follows;

$$\begin{aligned}
& \max_{c_i, y_i} \left\{ \sum_{i=1}^N \pi_i \left[u(c_i) - v\left(\frac{y_i}{\theta_i}, L\right) \right] \right\} \\
& \quad s.t. \\
& \quad \sum_{i=1}^N \pi_i c_i \leq \sum_{i=1}^N \pi_i y_i \\
& \quad u(c_i) - v\left(\frac{y_i}{\theta_i}, L\right) \geq u(c_{i-1}) - v\left(\frac{y_{i-1}}{\theta_i}, L\right) \text{ for all } i \\
& \quad L = \sum_{i=1}^N \pi_i \frac{y_i}{\theta_i}, \theta_{i+1} > \theta_i \text{ for all } i \text{ and } \mu_1 = 0
\end{aligned}$$

Letting λ and μ_i be the multipliers on the feasibility and incentive compatibility constraints respectively, FOCs are as follows:

$$(c_i): \pi_i u'(c_i) - \lambda \pi_i + \mu_i u'(c_i) - \mu_{i+1} u'(c_i) = 0$$

$$(c_N): \pi_N u'(c_N) - \lambda \pi_N + \mu_N u'(c_N) = 0$$

$$\begin{aligned}
(y_i): \pi_i \left[-v_1\left(\frac{y_i}{\theta_i}, L\right) \frac{1}{\theta_i} \right] - \frac{\pi_i}{\theta_i} \left[\sum_{i=1}^N \pi_i v_2\left(\frac{y_i}{\theta_i}, L\right) \right] + \lambda \pi_i + \frac{\pi_i}{\theta_i} \left[\sum_{i=1}^N \mu_i \left[v_2\left(\frac{y_{i-1}}{\theta_i}, L\right) - v_2\left(\frac{y_i}{\theta_i}, L\right) \right] \right] \\
- \mu_i \left[v_1\left(\frac{y_i}{\theta_i}, L\right) \frac{1}{\theta_i} \right] + \mu_{i+1} \left[v_1\left(\frac{y_i}{\theta_{i+1}}, L\right) \frac{1}{\theta_{i+1}} \right] = 0
\end{aligned}$$

$$\begin{aligned}
(y_N): \pi_N \left[-v_1\left(\frac{y_N}{\theta_N}, L\right) \frac{1}{\theta_N} \right] - \frac{\pi_N}{\theta_N} \left[\sum_{i=1}^N \pi_i v_2\left(\frac{y_i}{\theta_i}, L\right) \right] + \lambda \pi_N + \frac{\pi_N}{\theta_N} \left[\sum_{i=1}^N \mu_i \left[v_2\left(\frac{y_{i-1}}{\theta_i}, L\right) - v_2\left(\frac{y_i}{\theta_i}, L\right) \right] \right] \\
- \mu_N \left[v_1\left(\frac{y_N}{\theta_N}, L\right) \frac{1}{\theta_N} \right] = 0
\end{aligned}$$

While $u'(c_i)$ is the marginal utility of consumption, $v_1\left(\frac{y_i}{\theta_i}, L\right) \frac{1}{\theta_i}$ is the marginal cost of working for an additional unit of consumption. These terms would be equal if there were not externalities. $\frac{\pi_i}{\theta_i} \left[\sum_{i=1}^N \pi_i v_2\left(\frac{y_i}{\theta_i}, L\right) \right]$ term is the summation of all marginal benefits when type- i agent increase his working hour. Therefore, each agent creates an externality asymmetric to his productivity. A low ability agent is creating much more externality than a high ability agent in order to increase his consumption by one unit. Other terms in the FOCs are the usual terms arises from asymmetric information.

In order to implement optimal taxes, the social planner should consider the market optimality conditions of agents. In the market, agents know that they derive utility from the average working hour, but they are not aware of the fact that they can affect the average working hour. Agents are maximizing their utility subject to their budget constraint and their problem is as follows:

Agent's Problem;

$$\begin{aligned} \max_{c,y} c - v\left(\frac{y}{\theta}, L\right) \\ \text{s.t.} \\ c \leq y - \tau(y) \end{aligned}$$

Letting λ be the multiplier on the budget (resource) constraint, FOCs are as follows:

$$\begin{aligned} (c) : \quad u'(c) - \lambda &= 0 \\ (y) : \quad -v_1\left(\frac{y}{\theta}, L\right)\frac{1}{\theta} + \lambda(1 - \tau'(y)) &= 0 \end{aligned}$$

which gives;

$$(1 - \tau'(y)) = \frac{v_1\left(\frac{y}{\theta}, L\right)\frac{1}{\theta}}{u'(c)}$$

where if there were no taxes, agents would equalize their marginal costs and benefits.

Marginal tax is an important public policy instrument that mainly cares about the distributional concerns. The following proposition presents the optimal non-linear marginal tax function of each agent.

Proposition 1. *Optimal Marginal Income Tax;*

$$\text{for type-}i : (1 - \tau'(y_i)) = \left(1 + \frac{1}{\lambda\theta_i}\Upsilon\right)\Psi_i$$

$$\text{for type-}N : (1 - \tau'(y_N)) = 1 + \frac{1}{\lambda\theta_N}\Upsilon$$

$$\text{where } \Upsilon = \left\{ \left[\sum_{i=1}^N \mu_i \left[v_2\left(\frac{y_{i-1}}{\theta_i}, L\right) - v_2\left(\frac{y_i}{\theta_i}, L\right) \right] \right] - \left[\sum_{i=1}^N \pi_i v_2\left(\frac{y_i}{\theta_i}, L\right) \right] \right\}$$

$$\Psi_i = \left[\frac{\pi_i + \mu_i - \mu_{i+1}}{\pi_i + \mu_i - \phi_i \mu_{i+1}} \right] \text{ and } \phi_i = \frac{v_1\left(\frac{y_i}{\theta_{i+1}}, L\right)\frac{1}{\theta_{i+1}}}{v_1\left(\frac{y_i}{\theta_i}, L\right)\frac{1}{\theta_i}}$$

Proof. Marginal Tax for type- N :

$$\begin{aligned} \text{From } (y_N) : v_1\left(\frac{y_N}{\theta_N}, L\right)\frac{1}{\theta_N} [\pi_N + \mu_N] &= \lambda\pi_N + \frac{\pi_N}{\theta_N} \left[\sum_{i=1}^N \mu_i \left[v_2\left(\frac{y_{i-1}}{\theta_i}, L\right) - v_2\left(\frac{y_i}{\theta_i}, L\right) \right] \right] \\ &\quad - \frac{\pi_N}{\theta_N} \left[\sum_{i=1}^N \pi_i v_2\left(\frac{y_i}{\theta_i}, L\right) \right] \end{aligned}$$

$$\text{From } (c_N) : u'(c_N) [\pi_N + \mu_N] = \lambda\pi_N$$

Dividing both side gives;

$$(1 - \tau'(y_N)) = 1 + \frac{1}{\lambda\theta_N} \left\{ \left[\sum_{i=1}^N \mu_i \left[v_2\left(\frac{y_{i-1}}{\theta_i}, L\right) - v_2\left(\frac{y_i}{\theta_i}, L\right) \right] \right] - \left[\sum_{i=1}^N \pi_i v_2\left(\frac{y_i}{\theta_i}, L\right) \right] \right\}$$

Marginal Tax for type- i

$$\begin{aligned} \text{From } (y_i) : v_1\left(\frac{y_i}{\theta_i}, L\right)\frac{1}{\theta_i} \left[\pi_i + \mu_i - \mu_{i+1} \frac{v_1\left(\frac{y_i}{\theta_{i+1}}, L\right)\frac{1}{\theta_{i+1}}}{v_1\left(\frac{y_i}{\theta_i}, L\right)\frac{1}{\theta_i}} \right] &= \lambda\pi_i + \frac{\pi_i}{\theta_i} \left[\sum_{i=1}^N \mu_i \left[v_2\left(\frac{y_{i-1}}{\theta_i}, L\right) - v_2\left(\frac{y_i}{\theta_i}, L\right) \right] \right] \\ &\quad - \frac{\pi_i}{\theta_i} \left[\sum_{i=1}^N \pi_i v_2\left(\frac{y_i}{\theta_i}, L\right) \right] \end{aligned}$$

From (c_i) : $u'(c_i) [\pi_i + \mu_i - \mu_{i+1}] = \lambda \pi_i$

Dividing both side gives;

$$(1 - \tau'(y_i)) \left[\frac{\pi_i + \mu_i - \mu_{i+1} \frac{v_1\left(\frac{y_i}{\theta_{i+1}}, L\right) \frac{1}{\theta_{i+1}}}{v_1\left(\frac{y_i}{\theta_i}, L\right) \frac{1}{\theta_i}}}{\pi_i + \mu_i - \mu_{i+1}} \right] = 1 + \frac{1}{\lambda \theta_i} \left\{ \left[\sum_{i=1}^N \mu_i \left[v_2\left(\frac{y_{i-1}}{\theta_i}, L\right) - v_2\left(\frac{y_i}{\theta_i}, L\right) \right] \right] - \left[\sum_{i=1}^N \pi_i v_2\left(\frac{y_i}{\theta_i}, L\right) \right] \right\}$$

or $(1 - \tau'(y_i)) = (1 + \frac{1}{\lambda \theta_i} \mathcal{Y}) \Psi_i$

where $\mathcal{Y} = \left\{ \left[\sum_{i=1}^N \mu_i \left[v_2\left(\frac{y_{i-1}}{\theta_i}, L\right) - v_2\left(\frac{y_i}{\theta_i}, L\right) \right] \right] - \left[\sum_{i=1}^N \pi_i v_2\left(\frac{y_i}{\theta_i}, L\right) \right] \right\}$

$$\Psi_i = \left[\frac{\pi_i + \mu_i - \mu_{i+1}}{\pi_i + \mu_i - \phi_i \mu_{i+1}} \right] \text{ and } \phi_i = \frac{v_1\left(\frac{y_i}{\theta_{i+1}}, L\right) \frac{1}{\theta_{i+1}}}{v_1\left(\frac{y_i}{\theta_i}, L\right) \frac{1}{\theta_i}}$$

□

In the tax function of type- i agent, while Ψ_i term is the usual Mirrlees component, $(1 + \frac{1}{\lambda \theta_i} \mathcal{Y})$ term is the labour externality component. By assumption $v_{11}(\frac{y}{\theta}, L) > 0$ so that $\phi_i < 1$. With this condition we can come up with $\Psi_i < 1$. Without labour externality there will be a positive tax for all types. However, when we add the labour externality, tax schedule is not clear. If $(1 + \frac{1}{\lambda \theta_i} \mathcal{Y}) > 1$ (or $\mathcal{Y} > 0$) then labour externality has a progressive effect on tax schedule. $\left[\sum_{i=1}^N \pi_i v_2\left(\frac{y_i}{\theta_i}, L\right) \right]$ term in \mathcal{Y} is always negative and with the minus sign, it will always be a positive term. For the sign of the first term in \mathcal{Y} , we can use the optional assumption $v_{21}(\frac{y}{\theta}, L) > 0$ which says the agent who works more is getting a higher disutility decrease from an increase in L . This assumption implies $v_2(\frac{y_{i-1}}{\theta_i}, L) > v_2(\frac{y_i}{\theta_i}, L)$, and because both terms are negative, their difference will be a positive term. Then one can conclude that \mathcal{Y} term will be always positive under this assumption. Then labour externality term will be a progressive force on the marginal tax schedule. Also, if agents who work different numbers of hours get the same benefit from an increase in L then $v_2(\frac{y_{i-1}}{\theta_i}, L)$ and $v_2(\frac{y_i}{\theta_i}, L)$ terms will be equal and the first bracket in \mathcal{Y} will be zero. Then, \mathcal{Y} will be a positive term which leads to a progressive tax schedule. If the optional assumption is violated it is not possible to claim that labour externality has always a progressive effect on marginal taxes. For all cases, the multiplication of two forces will identify the tax schedules. In the cases where labour externality progressive effect exceeds regressive Mirrleesian component, there will be a progressive tax schedule.

When we consider the effect of labour externality between any two agents, we should compare their tax functions: $(1 - \tau'(y_i)) = (1 + \frac{1}{\lambda \theta_i} \mathcal{Y}) \Psi_i$ and $(1 - \tau'(y_{i+1})) = (1 + \frac{1}{\lambda \theta_{i+1}} \mathcal{Y}) \Psi_{i+1}$

Comparison of Ψ_i and Ψ_{i+1} is the main concern of the literature in which there is no utility interdependence. In absence of utility interdependence this comparison identifies the marginal tax schedule which depends on the form of utility function and ability distribution. Leaving

this discussion behind, we focus on the effect of labour externality on the tax rates. We know the Υ term will be same for all agents, then $\theta_{i+1} > \theta_i$ implies $\frac{1}{\lambda\theta_i}\Upsilon > \frac{1}{\lambda\theta_{i+1}}\Upsilon$ and shows the progressive effect of labour interdependence on the agents' tax rates.

By using a general separable utility function, it is hard to interpret the results in a clear way. Therefore, in the following section, we assume a particular form of utility and examine the marginal tax schedule.

4 Log-Utility Case

In this section, we assume a particular log-utility function used in Mirrlees(1971) and Tuomala(1990) papers. We change the consumption interdependence form of Tuomala(1990) to a labour interdependence form. In order to implement optimal taxes, the social planner solves the following maximization problem:

$$\max_{c_i, y_i} \sum_{i=1}^N \pi_i \left[\log(c_i) + (1 - \beta) \log \left(1 - \frac{y_i}{\theta_i} \right) + \beta \left(\log \left(1 - \frac{y_i}{\theta_i} \right) + \log L \right) \right]$$

s.t.

$$\sum_{i=1}^N \pi_i c_i \leq \sum_{i=1}^N \pi_i y_i$$

$$\log(c_i) + \log \left(1 - \frac{y_i}{\theta_i} \right) \geq \log(c_{i-1}) + \log \left(1 - \frac{y_{i-1}}{\theta_i} \right) \text{ for all } i$$

$$L = \sum_{i=1}^N \pi_i \frac{y_i}{\theta_i}, \theta_{i+1} > \theta_i \text{ for all } i \text{ and } \mu_1 = 0$$

Where β reflects the degree of concern, namely shows how much an agent cares about other people working hours. The problem turns out to be a standard Mirrleesian tax problem when $\beta = 0$ and the degree of utility interdependence will increase as β increases. Letting λ and μ_i be the multipliers on the feasibility and incentive compatibility constraints respectively, FOCs are as follows:

$$(c_i) : \quad \frac{\pi_i}{c_i} - \lambda \pi_i + \frac{\mu_i}{c_i} - \frac{\mu_{i+1}}{c_i} = 0$$

$$(c_N) : \quad \frac{\pi_N}{c_N} - \lambda \pi_N + \frac{\mu_N}{c_N} = 0$$

$$(y_i) : \quad \pi_i \left[\frac{-\frac{1}{\theta_i}}{1 - \frac{y_i}{\theta_i}} \right] + \beta \frac{1}{L} \frac{\pi_i}{\theta_i} + \lambda \pi_i + \mu_i \left[\frac{-\frac{1}{\theta_i}}{1 - \frac{y_i}{\theta_i}} \right] + \mu_{i+1} \left[\frac{\frac{1}{\theta_{i+1}}}{1 - \frac{y_i}{\theta_{i+1}}} \right] = 0$$

$$(y_N) : \quad \pi_N \left[\frac{-\frac{1}{\theta_N}}{1 - \frac{y_N}{\theta_N}} \right] + \beta \frac{1}{L} \frac{\pi_N}{\theta_N} + \lambda \pi_N + \mu_N \left[\frac{-\frac{1}{\theta_N}}{1 - \frac{y_N}{\theta_N}} \right] = 0$$

In order to implement optimal rates, the social planner should use market condition. For this utility form market condition becomes;

$$(1 - \tau'(y)) = \frac{v_1(\frac{y}{\theta}, L)^{\frac{1}{\theta}}}{u'(c)} \Rightarrow (1 - \tau'(y)) = \frac{c}{\theta - y}$$

(See Appendix Part A for derivation)

Then tax functions will get the forms in proposition 2.

Proposition 2. *Optimal Marginal Income Tax;*

$$\text{for type-}i: (1 - \tau'(y_i)) = (1 + \frac{1}{\lambda\theta_i} \frac{\beta}{L}) \Psi_i$$

$$\text{for type-N: } (1 - \tau'(y_N)) = 1 + \frac{1}{\lambda\theta_N} \frac{\beta}{L}$$

$$\text{where } \Psi_i = \frac{\pi_i + \mu_i - \mu_{i+1}}{\pi_i + \mu_i - \mu_{i+1} \frac{\theta_i - y_i}{\theta_{i+1} - y_i}}$$

(See Appendix Part B for derivations)

In this utility form, for the top agent, there is a certain subsidy because $\frac{1}{\lambda\theta_N} \frac{\beta}{L}$ term is greater than zero. Therefore, the marginal tax rate for the top ability worker is strictly negative under labour interdependence. In the tax function for type- i , while the first term in parentheses is the externality component, Ψ_i term is the usual Mirrlees term.

Proposition 3. *Tax schedule becomes more progressive as the degree of concern β increases.*

As β increases the externality component $1 + \frac{1}{\lambda\theta_i} \frac{\beta}{L}$ will increase. Also, as β goes up, marginal subsidies for all agents will increase. However, because these subsidies are allocated asymmetric to their abilities, low able agents will get more subsidies than the high able ones. This asymmetry creates a progressive effect on the tax schedule. Note that in these comments we put aside the changes in Mirrlees effect Ψ_i . The magnitude of Ψ_i will be affected by the difference between θ_i and θ_{i+1} . Assume that θ_{i+1} is excessively greater than θ_i , then the denominator of Ψ_i will be greater, therefore Ψ_i term will get closer to zero. Obviously, this will have a regressive effect on tax schedule. For this reason, the ability distribution assumption is very crucial in determining the tax schedule. A very dispersed ability distribution in the upper tail of the distribution will create a regressive force on the tax schedule. However, this high difference will create a higher progressive force over the labour externality component. In the end, the structure of the tax rates will be identified by these opposing forces.

In this utility form, interdependence does not affect individuals' labour supply decisions. Because of the form of adding labour externality to the question, the incentive compatibility constraint does not change. The following section changes the way of adding labour externality to the problem and it shows that labour externality has a progressive effect on the tax schedule even if the agent's behaviour is changed.

5 Log-Utility Form with Changing Behaviour

In this section, we assume a particular form of utility in which the labour externality affects agents' labour supply decisions. The only difference between section 4 is the way of adding

L to the utility function. In this case of a utility form, the social planner problem will be as follows:

$$\max_{c_i, y_i} \sum_{i=1}^N \pi_i \left[\log(c_i) + \log\left(1 - \frac{y_i}{\theta_i}\right) + \beta \log\left(\frac{L}{\frac{y_i}{\theta_i}}\right) \right]$$

s.t.

$$\sum_{i=1}^N \pi_i c_i \leq \sum_{i=1}^N \pi_i y_i$$

$$\log(c_i) + \log\left(1 - \frac{y_i}{\theta_i}\right) + \beta \log\left(\frac{L}{\frac{y_i}{\theta_i}}\right) \geq \log(c_{i-1}) + \log\left(1 - \frac{y_{i-1}}{\theta_i}\right) + \beta \log\left(\frac{L}{\frac{y_{i-1}}{\theta_i}}\right) \text{ for all } i$$

$$L = \sum_{i=1}^N \pi_i \frac{y_i}{\theta_i}, \theta_{i+1} > \theta_i \text{ for all } i \text{ and } \mu_1 = 0$$

In this utility form agents are getting utility when they are working less than the society, however, if they are working more than other people $\frac{L}{y_i/\theta_i}$ term will be less than one which gives a negative value for logarithm, so reduces the utility.

Letting λ and μ_i be the multipliers on the feasibility and incentive compatibility constraints respectively, FOCs are as follows:

$$(c_i) : \quad \frac{\pi_i}{c_i} - \lambda \pi_i + \frac{\mu_i}{c_i} - \frac{\mu_{i+1}}{c_i} = 0$$

$$(c_N) : \quad \frac{\pi_N}{c_N} - \lambda \pi_N + \frac{\mu_N}{c_N} = 0$$

$$(y_i) : \quad \pi_i \left[\frac{-\frac{1}{\theta_i}}{1 - \frac{y_i}{\theta_i}} \right] + \frac{\beta \pi_i}{L \theta_i} - \beta \pi_i \left[\frac{\frac{1}{\theta_i}}{\frac{y_i}{\theta_i}} \right] + \lambda \pi_i + \mu_i \left[\frac{-\frac{1}{\theta_i}}{1 - \frac{y_i}{\theta_i}} - \beta \frac{\frac{1}{\theta_i}}{\frac{y_i}{\theta_i}} \right] + \mu_{i+1} \left[\frac{\frac{1}{\theta_{i+1}}}{1 - \frac{y_i}{\theta_{i+1}}} + \beta \frac{\frac{1}{\theta_{i+1}}}{\frac{y_i}{\theta_{i+1}}} \right] = 0$$

$$(y_N) : \quad \pi_N \left[\frac{-\frac{1}{\theta_N}}{1 - \frac{y_N}{\theta_N}} \right] + \frac{\beta \pi_N}{L \theta_N} - \beta \pi_N \left[\frac{\frac{1}{\theta_N}}{\frac{y_N}{\theta_N}} \right] + \lambda \pi_N + \mu_N \left[\frac{-\frac{1}{\theta_N}}{1 - \frac{y_N}{\theta_N}} - \beta \frac{\frac{1}{\theta_N}}{\frac{y_N}{\theta_N}} \right] = 0$$

For this type of utility market condition becomes;

$$(1 - \tau'(y)) = \frac{v_1(\frac{y}{\theta}, L)^{\frac{1}{\theta}}}{u'(c)} \Rightarrow (1 - \tau'(y)) = \frac{[y + \beta(\theta - y)]c}{y(\theta - y)}$$

(See Appendix Part C for derivation)

By using this condition, marginal taxes under additional incentive problem with labour externalities are in proposition 4.

Proposition 4. *Optimal Marginal Income Taxes;*

$$\text{for type-}i : (1 - \tau'(y_i)) = \left(1 + \frac{1}{\lambda \theta_i} \frac{\beta}{L}\right) \Psi_i$$

$$\text{for type-}N : (1 - \tau'(y_N)) = 1 + \frac{1}{\lambda \theta_N} \frac{\beta}{L}$$

$$\text{where } \Psi_i = \frac{\frac{\pi_i + \mu_i - \mu_{i+1}}{y_i + \beta(\theta_{i+1} - y_i)}}{\frac{\pi_i + \mu_i - \mu_{i+1}}{\frac{y_i(\theta_{i+1} - y_i)}{y_i + \beta(\theta_{i+1} - y_i)}}}$$

(See Appendix Part D for derivations)

Tax function for the top agent is exactly the same with the previous case. Due to the no distortion at the top, additional incentive problem does not create any change in the tax for the top agent. Note that, this is true for tax function forms not for the rates, because the problems are different, Lagrange multipliers could be different, so the marginal tax rates. The following proposition shows that the conditions are not the same for other agents.

Proposition 5. *Marginal tax function for type- i person for both utility functions are as follows:*

$$i-) \text{ When behaviour is not changed; } (1 - \tau'(y_i)) = (1 + \frac{1}{\lambda\theta_i} \frac{1}{L}) \frac{\pi_i + \mu_i - \mu_{i+1}}{\frac{\theta_i - y_i}{\pi_i + \mu_i - \mu_{i+1}} \frac{1}{\theta_{i+1} - y_i}} (\beta = 1)$$

$$ii-) \text{ When behaviour is changed; } (1 - \tau'(y_i)) = (1 + \frac{1}{\lambda\theta_i} \frac{1}{L}) \frac{\pi_i + \mu_i - \mu_{i+1}}{\frac{(\theta_i - y_i)\theta_{i+1}}{\pi_i + \mu_i - \mu_{i+1}} \frac{1}{(\theta_{i+1} - y_i)\theta_i}} (\beta = 1)$$

Again, tax functions are not directly comparable as the problems are different. However, the labour externality component is not affected by additional concern for behavioural change. This additional concern shows up in the Mirrleesian component and it increases the regressive effect of the Mirrleesian component since $\theta_{i+1} > \theta_i$. Note that this result might be specific for assumed utility forms. The labour externality can be added to the model in many ways. If the utility function is not separable, it could be possible to have a case where both components are affected.

Equations for marginal taxes contain endogenous variables. Therefore, it is not possible to give a precise condition that makes the income schedule progressive. As Dahan and Strawczynski(2000) mentioned, under the concave utility of consumption, literature is bound to use simulations to determine the shape of marginal tax rates. The following section presents various numerical examples under log-normal skill distribution.

6 Numerical Examples

In this part of the paper, we conduct numerical examples and analyse tax schedules under the presence of labour interdependence. For the sake of comparison with the literature, the logarithmic utility of leisure is assumed for all cases. In order to compare with Mirrlees, first, we assume logarithmic utility of consumption, and then to compare with Diamond(1998) we utilize linear utility of consumption to analyse the effect of labour externality.

Since the model and the skill distributions are discrete, zero tax at the top result holds for the Mirrlees case. As the tax rate turns out to be zero, there is a big break in graphs, therefore, we exclude the last person from some of the figures. However in our examples with labour externality, there is a subsidy for the last agent.

For simulations, we use the utility form presented in section 4, which is a reformed version of Tuomala(1990,2010). All simulations are performed for the strict utilitarian case.

Kanbur and Tuomala(2010) focuses specifically on the progressive impact of consumption relativity on tax schedules and concludes that as the degree of relative concern increases marginal tax rate increases at all levels of income. They consider consumption externalities and because high-income earners consume more, they create higher negative externality than the others. While the negative externality is the reason for the progressive tax schedule in their paper, in our analysis the reason for progressivity is the positive externality. Low able agents work more, create more positive externality and thus, get more subsidies than high-able agents.

Figure 1 shows the tax rates under Mirrlees and labour externality. Both calculations are based on logarithmic utility and log-normal skill distribution with parameters $(\mu, \sigma) = (-1, 0.39)$. While the left axis shows the tax rates for Mirrlees, the right axis shows the tax rates under labour externality for $\beta = 1$. Mirrlees case has an inverse-U shaped tax schedule and the tax rate of the top agent is zero. By using the same parameters, adding labour externality to the problem changes the tax pattern. As it is shown in the analytical solution, tax is negative and there is a subsidy for all agents. However, these subsidies are asymmetric to the agent's abilities. Therefore, while the ability increases the subsidy decreases, which forms a progressive tax schedule.

By assuming a log-utility and log-normal skill distribution, Mirrlees example has a slightly decreasing tax pattern, however, Diamond(1998) changes the utility of consumption from logarithmic to a linear form and assumed a Pareto skill distribution. As a result, he claimed that the optimal tax schedule has a U-shaped pattern. The study of Dahan and Strawczynski(2000) stated that this result depends on the assumed utility form. In other words, when the utility form is logarithmic even if the skill distribution is Pareto there is a regressive tax schedule for high-ability agents. According to their study, the factor that determines the shape of tax schedule at high incomes is the assumed utility form.

Most often literature focuses on high-income earner's tax ratios; actually, theoretically the interesting result is the diminishing tax ratios in this part of the distribution. Figure 2 gives a summary of Table 1. In Mirrlees case, if utility of consumption is linear under log-normal skill distribution, regressive tax schedule turns to progressive. However, if the labour externality is considered, the tax schedule also becomes more progressive under the logarithmic utility of consumption. When we assume linear consumption under the presence of labour externality, the tax schedule becomes significantly progressive.

In this analysis, the linear consumption case is normalized with Mirrlees(1971) tax rates⁵. From these simulations, one can say that the structure of the optimal marginal income tax rates at high levels of income is sensitive to the assumed form of the utility for consumption. The

⁵Dahan and Strawczynski(2000) normalized their simulation results with Mirrlees $F(0.9) = 19\%$ tax. Our simulation results are very close to those Mirrlees(1971) found. Therefore we do not need to do any normalization for most of the cases. However, only for Figure 2, in order to compare exactly with Mirrlees(1971) and Dahan & Strawczynski(2000), we normalize the taxes with Mirrlees $F(0.9) = 19\%$.

assumed utility form changes the Mirrlees and Diamond's results completely. However in our case, when we consider the utility interdependence over labour decisions, there is a progressive tax schedule for both utility forms of consumption. Whether it is logarithmic or linear, the optimal tax for the high-income earners is rising. However, it could be seen that under both Mirrlees and labour externality cases, the tax schedule is progressive when we assume linear utility for consumption. According to Dahan and Strawczynski(2000), these changes are due to income effects. The logarithmic utility of consumption implies the presence of income effects and a concave utility of consumption implies that income effects are weaker for rich individuals, which calls for lower taxes at high levels of incomes. However in the linear consumption case, there is no income effect. Therefore under linear utility for consumption, rising tax in high-income levels is straightforward. However, present study shows that it is possible to get a rising tax schedule at the upper tail even the utility for consumption is logarithmic.

The tax schedules according to the increasing standard deviation σ are displayed in Figure 3. In Mirrlees case when the skill distribution becomes more dispersed, regressivity increases, as shown in the model section by comparing the Mirrleesian component in the tax functions. Analytically, change in tax structure is not clear when we add labour interdependence. For all levels of σ tax schedule turns to a progressive fashion, however it is not clear that which one is more progressive. While more dispersed skill distribution creates a regressive force, it also creates a progressive force over the labour externality component in the tax functions. As mentioned, subsidies are asymmetric according to the productivities. If agents' skills become more dispersed, then their subsidies become distant to each other. This will increase the progressivity effect of labour externality. On the other hand, a more dispersed skill distribution leads to a regressive effect on tax schedules via Mirrleesian component in the tax function. For all levels of σ , the structure of the tax schedule will be identified by these two opposing forces. Therefore, under which case we have a more progressive tax schedule is not clear and could change for different cases. For our particular example, the second panel in Figure 3 presents the marginal tax rates under labour externality. Since it is hard to perceive which one is more progressive, we normalized the tax rates with $\sigma = 0.39$ in the third panel. It is clear that a higher standard deviation implies a more progressive tax schedule for our example. Apparently, additional concern about labour externality reverses the regressive structure of marginal taxes to a progressive one.

We expect that while the degree of labour interdependence increases, the progressive effect is getting more powerful and convert the tax schedule from a regressive form to a progressive one. Figure 4 is showing the effects of different β (externality concern) parameters on the tax schedule which highlights the main idea of the present study. The point where β is zero is the pure Mirrlees result. If we go down on the graph, parameter β increases. When β increases, tax ratio decreases, and there is a transformation from a regressive schedule to a progressive schedule. This result is consistent with the theoretical part and with the results of Tuomala(1990) and Kanbur and Tuomala(2010), because as the agents' concerns about society's

working hour increases, the tax schedule becomes more progressive.

7 Optimal Marginal Taxes for United Kingdom and Czech Republic

In order to compare the hypothetical log-normal skill distribution example with real life, we examine the case of the United Kingdom and the Czech Republic. For the United Kingdom, we use Cowell(2011) log-normal wage distribution estimation as a proxy for skill distribution. This estimation is based on the New Earnings Survey(2002) and considers the distribution of weekly earnings of UK male manual workers for the year 2002. Parameters of the fitted log-normal distribution are $(\mu, \sigma) = (5.84, 0.36)$. And for the Czech Republic, we used the estimation of Mala and Nedved(2011) for hourly wage distribution. They used the data from the Average Earnings Information System from 2000 to 2010 and estimate the parameters of the log-normal distribution as $(\mu, \sigma) = (5.33, 0.24)$ for the non-business sector. These two data sets differ in time periods; however, they give an idea about wage distributions.

Figure 5 shows the earnings profile of these two countries. Apparently, the UK has more dispersed wage distribution than the Czech Republic. The Czech Republic's wage distribution exhibits a more linear and slightly increasing pattern. This is also verified by the calculations of OECD for these countries.

According to the OECD earnings deciles data⁶, the UK has higher rates in all ratios for 90th-to-10th, 90th-to-50th and 50th-to-10th. From these data sets, one can conclude that UK earnings distribution is more unequal than the Czech Republic. By using these log-normal distributions, the marginal tax rates under Mirrlees and labour externality are presented in Figure 6.

At the first panel of Figure 6, when there is no labour interdependence, a more dispersed ability distribution means a more regressive tax schedule. Therefore, the UK has a more regressive tax schedule. When we add the labour externality to the problem, both tax schedules turn to a progressive fashion. However, additional externality concern reverses the tax patterns. Second and third panel (normalized) of Figure 6 show that the UK should have a more progressive tax structure due to the higher progressive force over the externality component in the tax function. This is an example where the progressivity effect of labour externality exceeds the regressive force of the Mirrleesian component in the tax function.

This pattern for tax rates implies a more progressive income tax schedule in societies with a more unequal skill distribution. According to the Tax Database of OECD, the European countries like Germany, France, Netherlands, Belgium have more progressive marginal tax

⁶OECD Earnings Deciles Study - www.oecd.org/dataoecd/9/59/39606921.xls

schedules than the US⁷. And it is also known that according to the studies that are mostly using the International Adult Literacy Survey shows that the skills are more unequally distributed in the US than in the EU. In the absence of labour externality, optimal tax theory and tax policies seem inconsistent within each country (since theory implies regressive marginal rates). However, the existing theory explains the differences in tax structures across countries. When the labour interdependence is considered, the actual tax policy of countries is understandable and the theory could rationalise the expectation of higher tax progressivity for more unequal societies.

8 Conclusion

Discussion on tax rates is not only a subject for economists, but also a subject for ordinary people. However, while most people believe that the government should implement higher marginal tax rates for the high-income earners, optimal tax literature has difficulty explaining the progressive fashion of the actual tax systems. In this study, we have used the separable utility form in an N-type model and show the progressive effect of labour externality. We show that even if the agent's behaviour is affected by this additional feature, it is possible to have a progressive tax schedule.

In the tax literature, most of the papers rely on simulations to obtain tax schedules. By assuming the same utility form and skill distribution as Mirrlees(1971) it is possible to have a progressive tax under the presence of labour externality. Therefore, to have a progressive tax schedule, it is not necessary to assume a Pareto skill distribution or quasi-linear utility form as in Diamond(1998). We have shown that as the degree of utility interdependence increases, the tax schedule becomes more progressive. Numerical examples show that a more dispersed skill distribution does not mean a more regressive tax schedule, because as the skill distribution gets dispersed, the progressive effect of labour externality increases.

By using their wage distribution data as a proxy for their ability distribution, we have analysed the marginal tax rates in the United Kingdom and the Czech Republic. We show that the United Kingdom should have a more progressive tax structure than the Czech Republic because the UK ability distribution is more dispersed than the Czech Republic ability distribution.

We understood from this study that corrective concerns about the model make the model closer to real life and the expectation of the society. We have shown theoretically and numerically that actual tax schedules could be rationalised by changing the model features.

⁷OECD Tax Database, or the marginal tax calculation in Mankiw et al. (2009).

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Figures

Figure 1: Marginal Tax Rates under Mirrlees and Labour Externality

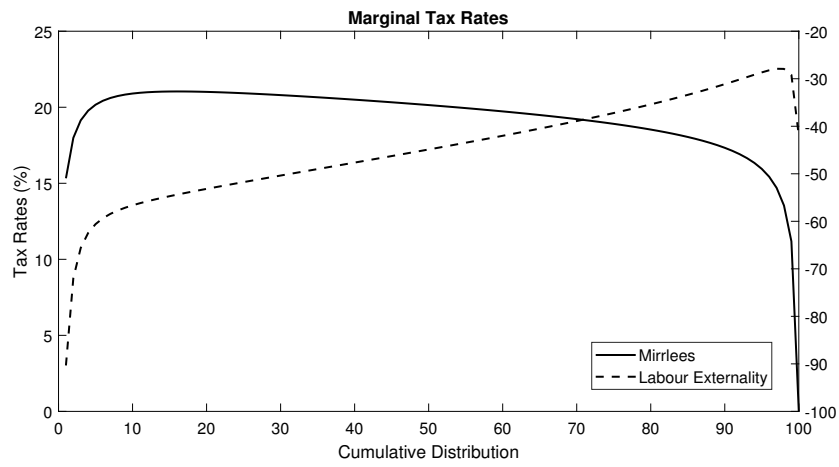


Figure 2: Marginal Tax Rates under Log and Linear Utilities for Consumption

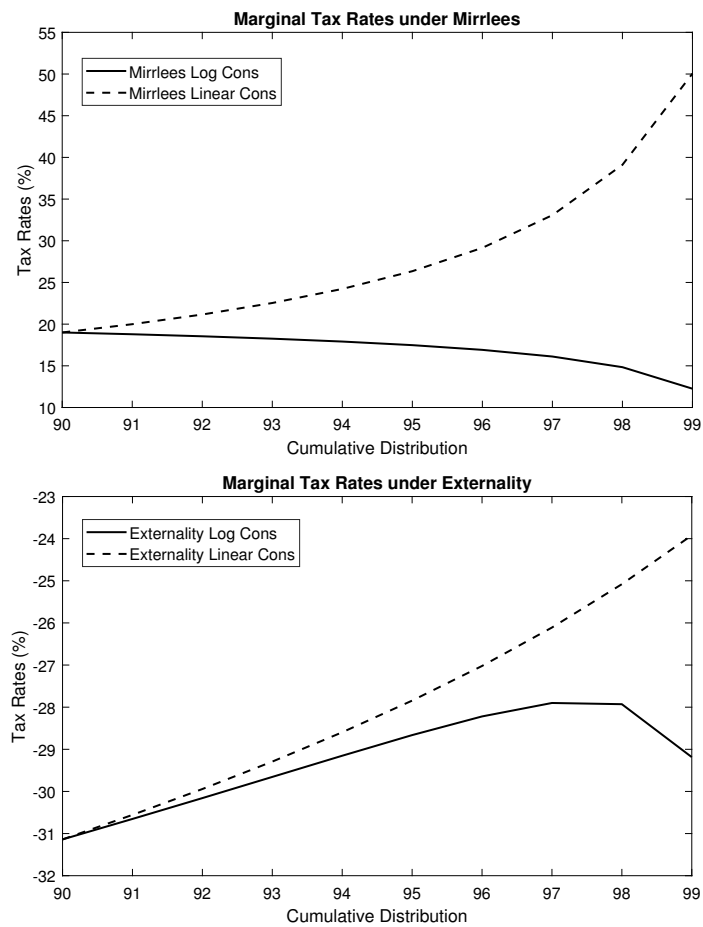


Figure 3: Marginal Tax Rates under Different Standard Deviations

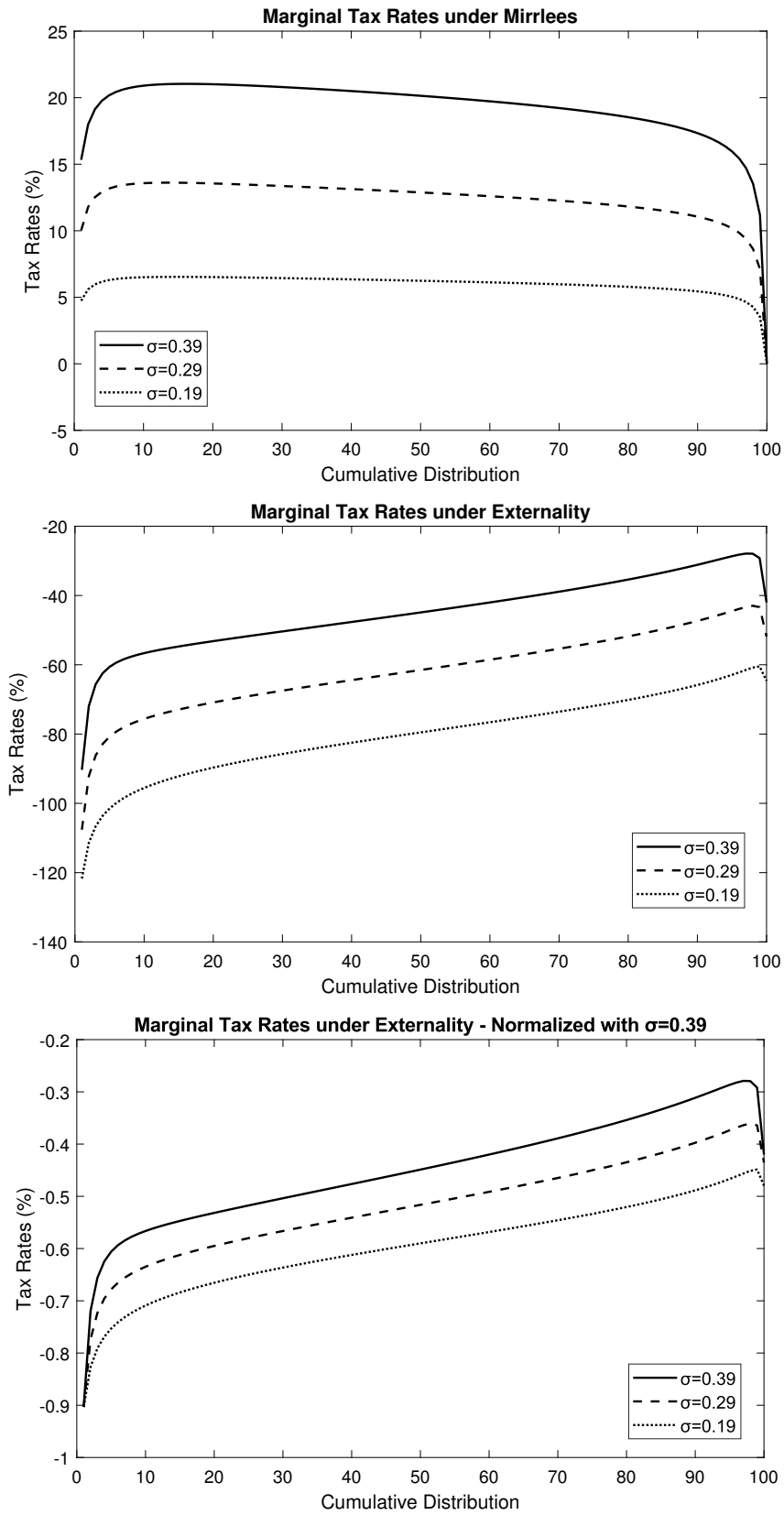


Figure 4: Marginal Tax Rates for Different β

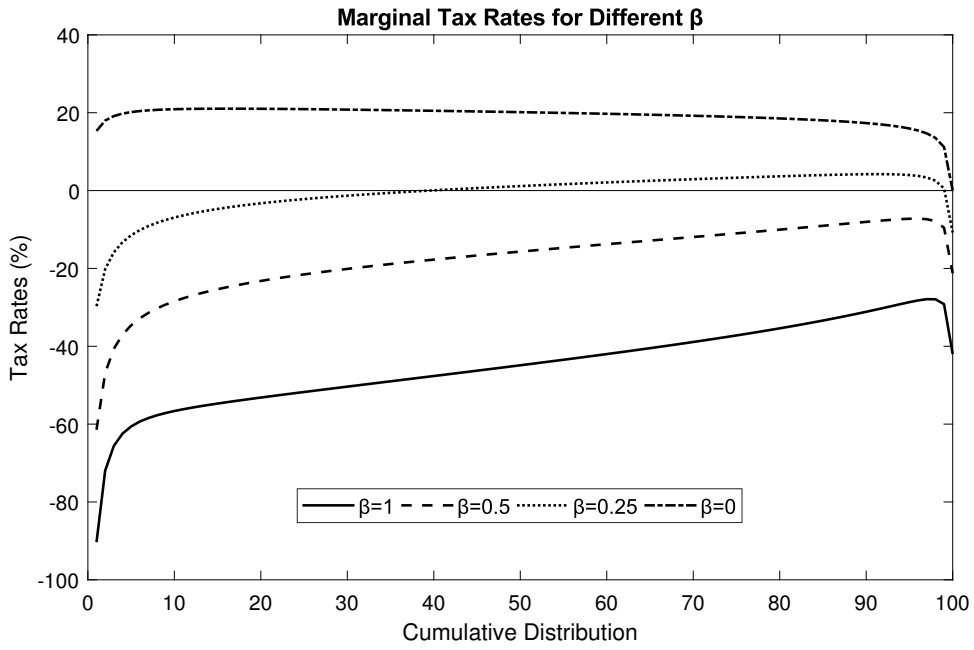


Figure 5: Earnings in United Kingdom and Czech Republic

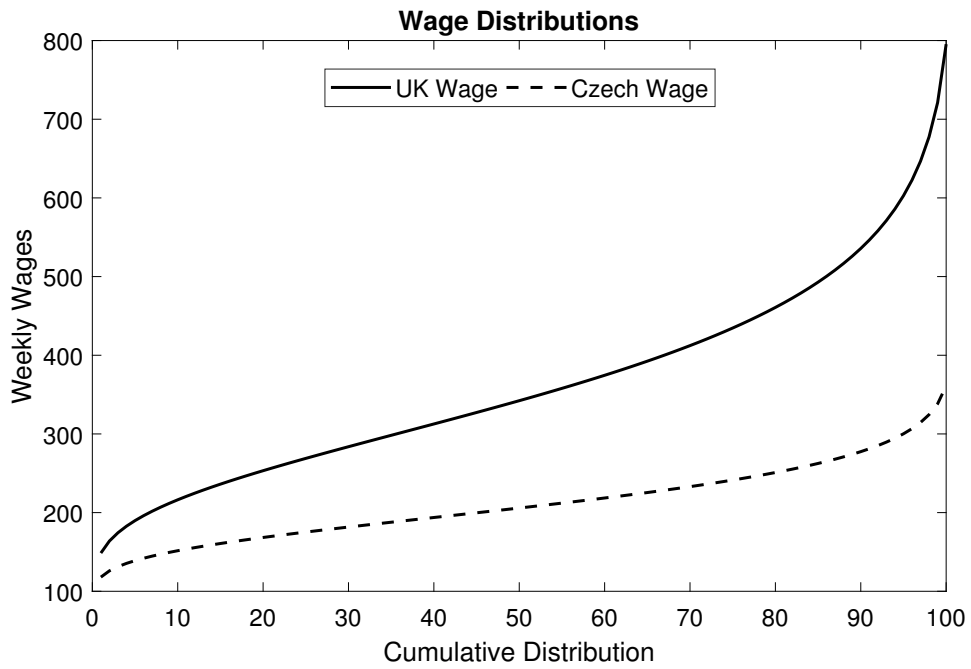
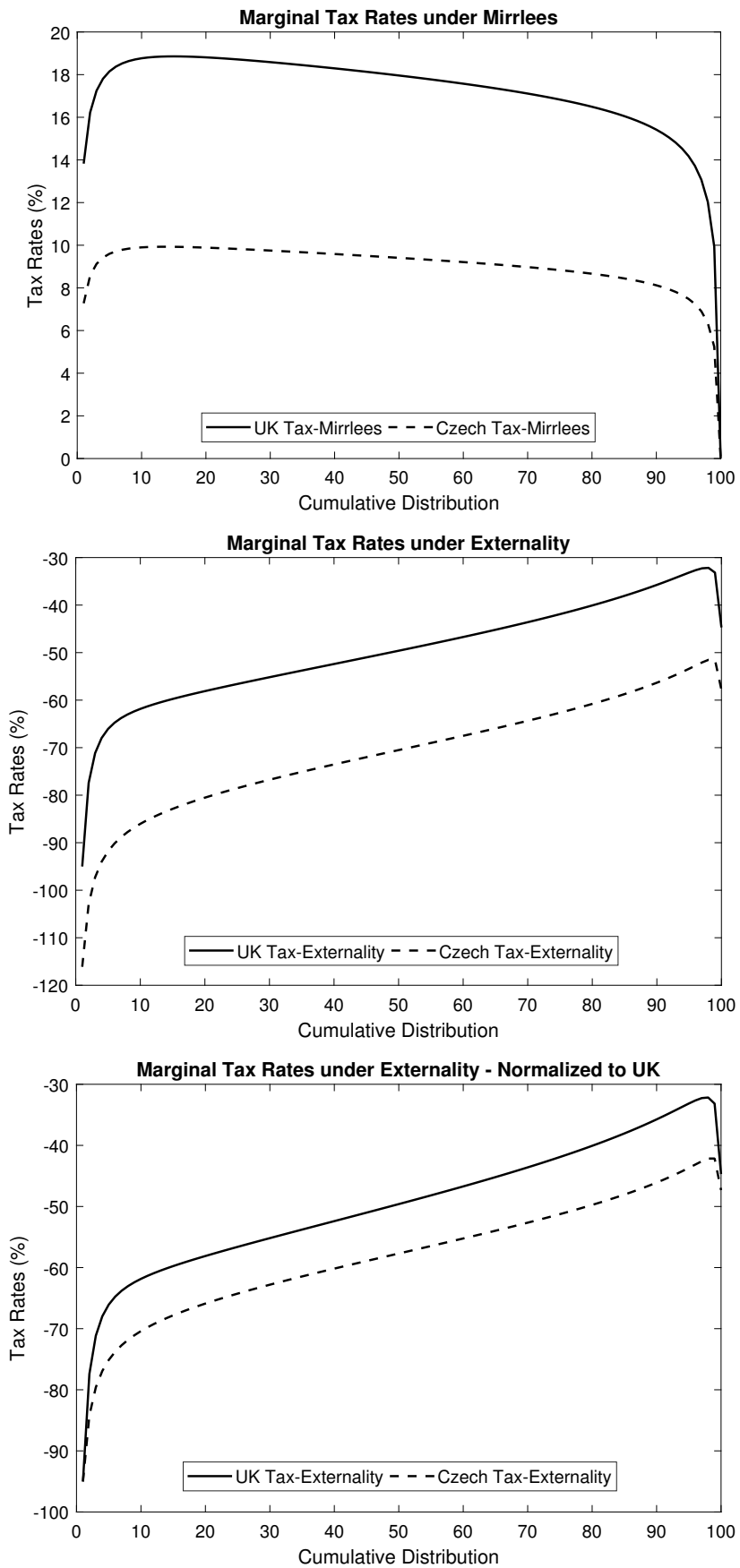


Figure 6: Marginal Taxes for the United Kingdom and the Czech Republic



Appendix

A - Derivation of Market Condition for Log-Utility

Agents maximize their utility subject to their budget constraint.

Agent's Problem;

$$\begin{aligned} \max_{c,y} \log(c) - (1 - \beta)\log\left(1 - \frac{y}{\theta}\right) + \beta \left[\log\left(1 - \frac{y}{\theta}\right) + \log L \right] \\ \text{s.t.} \\ c \leq y - \tau(y) \end{aligned}$$

Letting λ be the multiplier on the resource (feasibility) constraint, FOCs are as follows:

$$(c) : \quad \frac{1}{c} - \lambda = 0$$

$$(y) : \quad \frac{-\frac{1}{\theta}}{1 - \frac{y}{\theta}} + \lambda(1 - \tau'(y)) = 0$$

which gives;

$$(1 - \tau'(y)) = \frac{c}{\theta - y}$$

B - Derivation of Tax Rates for Log-Utility Case

$$\text{Market Condition: } (1 - \tau'(y)) = \frac{c}{\theta - y}$$

Marginal Tax for type- i :

$$\text{From } (y_i) : \frac{1}{\theta_i - y_i} \left[\pi_i + \mu_i - \mu_{i+1} \frac{\theta_i - y_i}{\theta_{i+1} - y_i} \right] = \lambda \pi_i + \beta \frac{1}{L} \frac{\pi_i}{\theta_i}$$

$$\text{From } (c_i) : \frac{1}{c_i} [\pi_i + \mu_i - \mu_{i+1}] = \lambda \pi_i$$

$$\text{Dividing both side gives; } (1 - \tau'(y_i)) = \left(1 + \frac{1}{\lambda \theta_i} \frac{\beta}{L}\right) \frac{\pi_i + \mu_i - \mu_{i+1}}{\pi_i + \mu_i - \mu_{i+1} \frac{\theta_i - y_i}{\theta_{i+1} - y_i}} = \left(1 + \frac{1}{\lambda \theta_i} \frac{\beta}{L}\right) \Psi_i$$

$$\text{Similar manipulations for type-}N \text{ gives; } (1 - \tau'(y_N)) = 1 + \frac{1}{\lambda \theta_N} \frac{\beta}{L}$$

C - Derivation of Market Condition for Log-Utility Form With Changing Behaviour

Agents maximize their utility subject to their budget constraint.

Agent's Problem;

$$\max_{c,y} \log(c) - \log\left(1 - \frac{y}{\theta}\right) + \beta \log\left(\frac{L}{\frac{y}{\theta}}\right)$$

s.t.

$$c \leq y - \tau(y)$$

Letting λ be the multiplier on the resource (feasibility) constraint, FOCs are as follows:

$$(c) : \quad \frac{1}{c} - \lambda = 0$$

$$(y) : \quad \frac{-\frac{1}{\theta}}{1-\frac{y}{\theta}} - \beta \frac{\frac{1}{\theta}}{\theta} + \lambda(1 - \tau'(y)) = 0$$

which gives;

$$(1 - \tau'(y)) = \frac{[y + \beta(\theta - y)] c}{y(\theta - y)}$$

D - Derivation of Tax Rates for Log-Utility Form With Changing Behaviour

$$\text{Market Condition: } (1 - \tau'(y)) = \frac{[y + \beta(\theta - y)] c}{y(\theta - y)}$$

Marginal Tax for type- i :

$$\text{From } (y_i) : \frac{y_i + \beta(\theta_i - y_i)}{y_i(\theta_i - y_i)} \left[\pi_i + \mu_i - \mu_{i+1} \frac{\frac{y_i + \beta(\theta_{i+1} - y_i)}{y_i(\theta_{i+1} - y_i)}}{\frac{y_i + \beta(\theta_i - y_i)}{y_i(\theta_i - y_i)}} \right] = \lambda \pi_i + \frac{\beta}{L} \frac{\pi_i}{\theta_i}$$

$$\text{From } (c_i) : \frac{1}{c_i} [\pi_i + \mu_i - \mu_{i+1}] = \lambda \pi_i$$

$$\text{Dividing both side gives; } (1 - \tau'(y_i)) = \left(1 + \frac{1}{\lambda \theta_i} \frac{\beta}{L}\right) \frac{\pi_i + \mu_i - \mu_{i+1} \frac{y_i + \beta(\theta_{i+1} - y_i)}{y_i(\theta_{i+1} - y_i)}}{\pi_i + \mu_i - \mu_{i+1} \frac{y_i + \beta(\theta_i - y_i)}{y_i(\theta_i - y_i)}}$$

$$\text{Similar manipulations for type-}N \text{ gives; } (1 - \tau'(y_N)) = 1 + \frac{1}{\lambda \theta_N} \frac{\beta}{L}$$