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Ogawa, Shogo

Kyoto University

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Monetary Growth with Disequilibrium: A Non-Walrasian Baseline Model*

Shogo Ogawa†

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Abstract

In this study, we present a baseline monetary growth model of disequilibrium macroeconomics similar to the existing Keynes-Wicksell model. However, we highlight a characteristic of disequilibrium (non-Walrasian) macroeconomics, specifically the regime dividing in the static model. In addition, because we synthesize demand-side factors (Keynesian) and supply-side factors (neo-classical), we find a new effect on the dynamic feedback loops; that is, the dual-decision effect. This new effect stabilizes/destabilizes an unstable/stable feedback loop when the regime switches from the demand-side to the supply-side. Moreover, this dual-decision effect partly works in the real wage adjustment process and enhances instability if the economy is in a Keynesian regime. We implement numerical experiments to confirm these results, and find that the Walrasian equilibrium itself is not always stable.

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†Graduate School of Economics, Kyoto University, Yoshida Honmachi, Sakyo-ku, Kyoto (Email: ogawa.shougo.54e@st.kyoto-u.ac.jp).

1 Introduction

Monetary economics is currently one of the most important areas in macroeconomics. Many researchers analyzed monetary economics in terms of the "Keynesian" features of macrodynamics, and thus developed New-Keynesian economics. The core model of New-Keynesian economics is usually called the Dynamic Stochastic General Equilibrium (DSGE) model, which typically has "a core structure that corresponds to a Real Business Cycle (RBC) model" (Galí, 2015, p.2). New-Keynesian researchers introduced nominal rigidities into the Real Business Cycle framework and proved the non-neutrality of monetary policy, at least in the short term.[1]

Equilibrium monetary macroeconomics is currently popular, leaving research on disequilibrium (or non-Walrasian) economics relatively less examined.[2] This is not because this perspective has some crucial flaw, but because research interest shifted from disequilibrium to equilibrium models before conducting a well-developed discussion in the literature (Backhouse and Boianovsky, 2012). In the early days of monetary growth research, however, the difference between the equilibrium and disequilibrium schools was not so distinct. Since Tobin (1965) specified the portfolio mechanism in the neo-classical growth framework, the relationship between capital intensity in the steady state and the existence of money has been an important issue.[3] In contrast to neo-classical monetary growth, in which planned savings and planned investment always match, Stein (1969) constructs a "Keynes-Wicksell" monetary growth model, in which the speed of the price adjustment is finite and the gap between saving and investment determines the price dynamics.[4] This short-run disequilibrium adjustment often leads to a growth cycle dynamic, while the neo-classical (equilibrium) model usually has a unique path that converges to a steady state. However, the coexistence of these approaches does not mean a separation between equilibrium and disequilibrium dynamics; for example, Villanueva (1971) explores the disequilibrium dynamics of a goods market using a neo-classical monetary growth approach. As Bénassy (1986, Chapter. 1) argues, disequilibrium dynamics is an expansion of the equilibrium model rather than an opposing counterpart.

Although disequilibrium economics has been forgotten for a long time, some researchers recently referred to disequilibrium economics to explore secular stagnation. Dupor et al. (2019) and Schoder (2020) use sticky wage models that allow labor market disequilibrium (unemployment). Eggertsson et al. (2019) refers to Barro and Grossman (1971) as a quantity constraint model on labor supply in their secular stagnation analysis with the Zero Lower Bound (ZLB) of the interest rate. Although Stiglitz (2018, Appendix) introduces a disequilibrium model as an alternative to the DSGE model, these

[1] See Christiano et al. (2005). For a brief summary of DSGE analyses, see Christiano et al. (2018).

[2] Strictly speaking, researchers often analyze disequilibrium macrodynamics from the Keynesian economics perspective; see Chiarella and Flaschel (2000), Chiarella et al. (2000), Chiarella et al. (2005), Asada et al. (2006), and Asada et al. (2011). However, the non-Walrasian economic perspective, which synthesizes Keynesian and neo-classical regimes, is scarce. The exceptions are works by Chiarella et al. (2012, Chapter. 8, 9), Böhm (2017), and Ogawa (2019a). This seems to be because Flaschel (1999) and Malinvaud (1980) show the dominance of the Keynesian regime in the disequilibrium dynamics.

[3] Tobin refers to it in his earlier work (Tobin (1955)). For works on neo-classical monetary growth, see Sidrauski (1967a,b), Levhari and Patinkin (1968), Hadjimichalakis (1970), Benhabib and Miyao (1981), and Hayakawa (1984).

[4] On Keynes-Wicksell monetary growth, see Stein (1966), Rose (1967, 1969), Fischer (1972), Franke (1992), and Flaschel and Sethi (1996). Burmeister and Dobell (1970, Chapter. 6), Orphanides and Solow (1990), and Chiarella and Flaschel (2000) summarize the neo-classical- and Keynes-Wicksell-type analyses.

works are not complete as (non-Walrasian) disequilibrium economics since they lack the core concept of the disequilibrium model (*dual decision hypothesis*), which we explain below. Thus, in this study, we construct a baseline disequilibrium monetary growth model and show how persistent Keynesian unemployment (which could be interpreted as secular stagnation) occurs in a disequilibrium macroeconomic model. The important point is that persistent unemployment could occur due to the dual decision effect, not to ZLB or the lower bound of the nominal wage.[5] Before the model analysis, we will first discuss "disequilibrium" economics.

Non-Walrasian economics, as influenced by Clower (1965), treats the quantity-constrained transactions of goods under the prevailing prices. Many researchers worked on the basic general disequilibrium model created by Barro and Grossman (1971).[6] Since goods-related transactions occur under the prevailing prices, the model must adjust the demand and supply for each market. This adjustment induces demand-supply gaps (and then a quantity constraint for individuals providing excess supply or excess demand) in the markets. For the quantity constraint, individuals reconsider the demand or supply in other markets. This dual decision hypothesis is at the core of disequilibrium models, and is expressed mathematically as

$$\tilde{x}_i = \min\{x_i^s(P, \tilde{x}_{-i}), x_i^d(P, \tilde{x}_{-i})\}, \quad \forall i,$$

where $\tilde{x}_i$ is the realized transaction of good i, $-i$ is a set of indexes for goods except i, and P is the prevailing price vector. The subscripts s and d denote supply and demand, respectively. This expression explicitly shows the strong spillover effect among the realized transactions. Non-Walrasian economists highlight this characteristic to distinguish the effective demand derived from the dual decision from the notional demand derived from normal optimization problems without quantity constraints.

To analyze disequilibrium monetary growth, Azam (1980) uses the IS-LM framework and portfolio equilibrium suggested by Tobin (1969) and finds that the slow price adjustment stabilizes the convergence to the steady state. However, his dynamic analysis is limited because he shows only some example paths that converge to the steady state. Sgro (1984) compares the disequilibrium growth without money to the other with money, and demonstrates the non-neutrality of money in the steady state, and that the steady state becomes the saddle-point that normal (equilibrium) monetary growth models often refer to. Although Sgro (1984) conducts a detailed dynamic analysis, the goods market in that model should always be in equilibrium; thus, the paper does not analyze disequilibrium dynamics completely. Therefore, the literature is missing a model that allows for disequilibria in both the goods and labor markets, and is thus missing a complete dynamic analysis.

Our framework is based on the classical monetary model in Sargent (1987, Chapter. 1). We extend his model by adding the possibility of a demand-supply gap in each market. We adopt the dual decision mechanism in the static model and formulate the static transactions in (dis)equilibrium. Thus, the model in this paper is a further generalization of

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[5] As Palley (2019) argues, ZLB economics and sticky price (or wage) models are classical economic models rather than Keynesian. In this study, persistent unemployment is induced by the quantity shortage of aggregate demand and therefore the stickiness is not the core concept of unemployment.

[6] For early studies using this macroeconomic model, see Korliras (1975), Malinvaud (1977, 1980), Hildenbrand and Hildenbrand (1978), and Muellbauer and Portes (1978). In particular, Böhm (1978), Ito (1980), Honkapohja and Ito (1980, 1982), Blad and Zeeman (1982), and Picard (1983) study the dynamics of disequilibrium macroeconomics. For microeconomic features such as exchange and money, see Younès (1974), Bénassy (1975), and Grandmont and Laroque (1976). For the history of disequilibrium analyses, see Backhouse and Boianovsky (2012).

the (dis)equilibrium dynamics of Tobinian or Keynes-Wicksell monetary growth. We note that our model is also very similar to the sophisticated monetary growth model in Chiarella et al. (2000, Chapter. 5), which extends the non-Walrasian monetary growth model in Picard (1983). However, we omit the inventory dynamics and flexible workforce (they allow the possibility of overtime work), which act as buffers and weaken the potential disequilibria. Our simplifications specify the characteristics of non-Walrasian economics such as regime switching, thus revealing new characteristics of the dynamic feedback loops. Therefore, this paper proposes a baseline model that treats non-Walrasian monetary growth rather than developing existing Keynesian dynamics models.

The rest of this paper proceeds as follows. In Section 2, we construct the static model that defines the regimes of the temporary equilibria. The temporary equilibrium uniquely exists under the given fixed price vectors and the stock variables. In Section 3, we formulate the dynamics of the stock variables and the adjustment processes for wages, prices, and expectations. We demonstrate the feedback loops of these variables and how the dynamics become (de)stabilized. Additionally, we discuss the new dual-decision effect on the feedback loops. As the dynamic system is five dimensional in our model, we implement a numerical experiment in Section 4. In Section 5, we summarize the results of our disequilibrium monetary growth model and offer our concluding remarks.

2 The Model

In this section, we construct a static model. Before the analysis, we set the following mathematical conditions and notations; unless specifically mentioned, all functions in this paper are at least twice continuously differentiable. Let $\dot{x}$ denote the time derivative of x, or $\dot{x} = dx/dt$. Let f_i denote the partial derivative of function f with respect to the i-th variable; that is, $f_1 = \partial f(x_1, x_2, x_3)/\partial x_1$, where the double partial derivative is $f_{ij} = \partial^2 f/\partial x_j \partial x_i$.

The model consists of identical households, the representative firm, and the government. These economic agents trade labor, goods, and assets (money, bonds, and equity) under fixed prices and fixed wages. The nominal interest rate responds to the disequilibrium immediately, thus always ensuring the asset market equilibrium, unlike in real markets. As we fix the capital stock K in the static model, we describe the static equilibrium (temporary equilibrium) in the intensive form by dividing the quantity variables by K.

2.1 The firm

The representative firm produces goods Y using the labor E and their own capital stock K. We express the firm's production technology as the following neo-classical type production function F:

$$\begin{aligned} Y = F(K, E), \text{where } & F(0,0) = 0, \\ & F_1, F_2 > 0, \\ & F_{11}, F_{22} < 0, \\ & F(\lambda K, \lambda E) = \lambda F(K, E), \ \forall \lambda > 0. \end{aligned} \tag{2.1}$$

Each period, the firm gains net real revenues of $F(K, E) - \delta K$ and pays the real wages w to its employees and dividends to its shareholders. We suppose that the firm does not reserve money, so the net real dividend flow ρK is

$$\rho K = F(K, E) - wE - \delta K, \tag{2.2}$$

where $\delta > 0$ is the constant positive depreciation rate. The firm intends to maximize the real dividend flow ρK in each period, and therefore the firm solves the following profit maximization problem, which has a quantity constraint:

$$\max_E F(K, E) - wE \ \text{ subject to } F(K, E) \leq Y^d \text{ and } w, K \text{ are given.} \tag{2.3}$$

Note that when the demand quantity constraint $F(K, E) \leq Y^d$ is bounded, the solution is different from the usual maximum that follows the first order condition. The solution of E is the labor demand function L^d, which consists of two different labor demand functions:

$$L^d = \min\{L^{d*}, \tilde{L}^d\}, \text{ where } L^{d*} = (F')^{-1}(w; K) \text{ and } \tilde{L}^d = F^{-1}(Y^d; K) \tag{2.4}$$

The first function L^{d*}, which is an interior solution and derived from the first-order condition without the demand constraint, is the *notional* labor demand. Since the production function should be linear homogeneous, we can rearrange L^{d*} as $v(w)K$, where $v' < 0$. The second function (corner solution) is the *effective* labor demand since it depends on the quantity of the goods demand.

For the goods supply, we use the variable Y^s, as follows:

$$Y^s = \min\{F(K, L^{d*}),\ F(K, L^s)\}. \tag{2.5}$$

The firm purchases the produced goods for investment by issuing equities:[7]

$$P\dot{K} = \dot{V} - V\pi, \tag{2.6}$$

where V is the total nominal equity value of the firm, P is the price of the goods, and π is the expected inflation rate. The real equity is equal to the firm's existing capital according to the market valuation:

$$V/P = qK, \tag{2.7}$$

where q is the real market-valued price of the firm's existing capital.[8] The firm invests following the investment function below:

$$I = \dot{K} + \delta K = \psi(q-1)K + (n+\delta)K, \quad \psi(0) = 0, \quad \psi > -(n+\delta), \quad \psi' > 0. \tag{2.8}$$

This investment function implies that the capital and the population grow at the same rate when $q = 1$, as in Chiarella and Flaschel (2000). q is Tobin's (average) q, which depends on the (expected) net cash flow stream to dividend payments in future. If we can predict the quantity constraint on goods demand, the q at moment t would be

$$q(t)K(t) = \int_t^\infty \mathbb{E}_t \left[\rho(\tau)K(\tau)e^{-(r(t)-\pi(t))\tau}\right] d\tau,$$

[7] For the case in which the firm uses debt financing, see Picard (1983) and Chiarella et al. (2000, Chapter. 5).

[8] We could also use a notation for the price of the issued equities such as $P_e E$ in Chiarella and Flaschel (2000) and Asada et al. (2011). Although this notation is more correct and intuitive, we use the notation in equation (2.7) to simplify the calculations.

where $\mathbb{E}_t$ is the expectation operator at t and r is the nominal interest rate.

However, the calculable forward-looking expectation of the goods demand does not seem suitable in "Keynesian" disequilibrium models (Murakami, 2016). As Neary and Stiglitz (1983) show, a pessimistic expectation of the goods demand in the future might shrink the actual goods demand in both the present and future. This is a kind of self-fulfilling prophecy or sun-spot equilibrium[9]. We should consider that the excess goods demand today affects the expectations of the future and that ample goods demand would lead to an optimistic expectation, following Malinvaud (1980).

In this study, we use the following ad-hoc function:[10]

$$q = q(\rho, Y^d/Y^s, r - \pi), \ q_1 > 0, \ q_2 > 0, \ q_3 < 0. \tag{2.9}$$

The term $q_2 > 0$ argues that today's excess demand ratio Y^d/Y^s is a criterion for the expected goods demand in the future. As in equation (2.8), q affects the investment directly. Our formulation implies that the investment depends both on the return rate terms $r - \pi$ and ρ and on the goods demand expectation term Y^d/Y^s. This is a kind of reconciliation of Wicksellian and Keynesian investments.

As our study is an extension of equilibrium models, we suppose that the q function in equation (2.9) is equal to the normal q function in equilibrium theories such as those by Yoshikawa (1980) and Hayashi (1982), as long as the situation is "Walrasian."

Assumption 1. The q function in equation (2.9) satisfies the following condition:

$$q(\rho, 1, r - \pi) = 1 \Leftrightarrow \rho = r - \pi + \xi, \tag{2.10}$$

where $\xi > 0$ is a constant risk premium.

This assumption states that when the goods market is in equilibrium (which implies a stationary expectation of excess goods demand), the condition for $q = 1$ is equivalent to the normal condition in equilibrium theory, such as those by Sargent (1987, Chapter. 1).

2.2 Households

The homogeneous households supply labor and buy goods for consumption. Households supply labor inelastically, such that the labor supply L^s is equal to the population, which grows constantly:[11]

$$\dot{L}^s/L^s = n > 0, \quad n = \text{const.} \tag{2.11}$$

Households hold assets consisting of money, bonds, and equity. The real asset holdings A is

$$A = (M + B + V)/P, \tag{2.12}$$

[9] For a discussion of this phenomenon in equilibrium theory, see Azariadis (1981), Woodford (1986), and Farmer (1999). Howitt and McAfee (1985) refer to the sun-spot equilibrium as the business cycle driven by *Animal Spirits*. For simplicity, they often utilize the Markov process, which means that the state variable today is the most important factor to determine how optimistic (pessimistic) the expectation is. Our formulation is suitable to extend the investment function of Bénassy (1984), which utilizes the adoptive expectation.

[10] For a simple example of a formulation of q in this paper, see AppendixA.

[11] If the quantity constraint on the goods purchases affects labor supply (if the labor supply becomes the effective supply), then the multiplier effect emerges for both the goods supply and goods demand (Barro and Grossman, 1971).

where M is the money held and B represents government-issued bonds.

The households plan their consumption and saving and express them under the budget constraint of the perceived real disposable income concept. The perceived real disposable income Y_{di} is equal to the real wage payment on the *realized* employment wE *plus* dividend payments ρK *minus* total real tax collection T *plus* the real return on the bond rB/P *minus* the anticipated capital loss on the real value of government debt $(M+B)P^{-1}\pi$ *plus* the real value of equities $\dot{V}/P$ *minus* the rate at which the firm issues equities to finance investment $\dot{K}$.

$$\begin{aligned} C^d + \dot{A}^d = Y_{di} &\equiv wE + \rho K - T + rB/P - (M+B)\pi/P + \dot{V}/P - \dot{K} \\ &= Y - \delta K - T + rB/P - A\pi, \end{aligned} \tag{2.13}$$

where C^d is consumption demand and $\dot{A}^d$ is equal to the *ex ante* saving.

In this paper, we omit the utility-maximization problem and suppose that the consumption demand function follows Azam (1980) and Sargent (1987, Chapter. 1):

$$C^d = C^d(Y_{di}, A, r-\pi) > 0, \quad 0 < C^d_1 < 1, \; C^d_2 > 0, \; C^d_3 < 0, \tag{2.14}$$

and the aggregate consumption function C^d is linear homogeneous with the aggregate variables Y_{di} and A. The first term of the partial derivative C^d_1 shows that consumption is increasing in the perceived real disposable income and the marginal propensity to consume out of Y_{di} is positive, but less than unity.[12] The fact that the planned consumption depends on the realized income implies that C^d is an effective demand function. The term C^d_2 reflects the real balance effect (*Pigou* effect) on consumption. The third term C^d_3 indicates the substitution effect of future consumption.

To clarify the implications induced in the latter sections, we formulate the consumption demand function as follows:

$$C^d = f^c(A, r-\pi, Y_{di})Y_{di}, \; 0 < f^c < 1, \; f^c_1 > 0, \; f^c_2 < 0, \; -f^c/Y_{di} < f^c_3 \le 0, \tag{2.15}$$

and the propensity-to-consume function f^c is homogeneous with degree zero with A and Y_{di}. The negativity of f^c_3 represents the substitution effect, but this effect is not stronger than the income effect.

For simplicity, we suppose that the consumption demand function is increasing in A and π.

Assumption 2. When $\pi > 0$ holds, the following condition holds:

$$\epsilon_j > (1 + \epsilon_{Ydi})\pi A/Y_{di}, \; j = A, \pi, \tag{2.16}$$

where $\epsilon_j = (\partial f^c/j) \cdot (j/f^c)$, or j is the elasticity of consumption propensity.

This assumption implies that the effect of capital loss $A\pi$ in the perceived disposable income does not have a strong effect on consumption demand.

[12]As in Sargent (1987, Chapter. 1), it is in line with Clower (1965), who shows that consumption demand is a function of the realized income. This is an interpretation of the "dual decision hypothesis."

2.3 The government

The government purchases goods G and pays net real interest rB/P by collecting real tax T and issuing bonds and money.

$$G + rB/P = T + \dot{B}/P + \dot{M}/P. \quad \dot{M}/M = \mu > 0, \quad \mu = \text{const.} \tag{2.17}$$

In this study, we suppose that the money supply grows at a constant rate for simplicity. Following Sargent (1987) and Asada et al. (2011), we suppose that the government purchases are proportional to the existing capital:

$$G = gK, \quad g = \text{const} > 0. \tag{2.18}$$

The government taxes the household's net real income cash flow for the positive constant rate plus the same amount of the net real interest:

$$T = \tau_w wE + \tau_\rho \rho K + rB/P.$$

We suppose that the tax rate is common $\tau_w = \tau_\rho = \tau > 0$ such that the taxation is proportional to the realized income.

$$T = \tau(Y - \delta K) + rB/P. \tag{2.19}$$

For the static analysis, the following condition is satisfied:

$$dM = -dB,$$

which states that the government or the central bank implements open market operations. Finally, we define the effective goods demand as follows:

$$Y^d = C^d + I + G = C^d + \dot{K} + \delta K + G. \tag{2.20}$$

Note that the investments are financed by issuing equities and the government purchase is not quantity rationed: if $Y < Y^d$, then consumption is rationed.[13]

2.4 Asset market

Equation (2.12) shows that the aggregate asset consists of money M, bonds B, and equity V. Following Sargent (1987, Chapter. 1), the households want to divide their assets between M and $B + V$, where the latter two assets are perfectly substitutable. We describe this division using the following functions:

$$M^d/P = f^m(r, Y, A) \tag{2.21}$$
$$(B^d + V^d)/P = f^b(r, Y, A) \tag{2.22}$$
$$(M^d + B^d + V^d)/P = A. \tag{2.23}$$

Then, we characterize the portfolio equilibrium condition by the following money balance condition:

$$M/P = M^d(r, Y)/P, \quad M_1^d < 0, \quad M_2^d > 0, \quad M^d(r, Y/K) = M^d(r, Y)/K \tag{2.24}$$

[13]Böhm (1978) also adopts this assumption. Ogawa (2019b) analyzes the case in which the investment is quantity constrained using a two-sector framework.

where M^d/P is the real money demand function.[14]. The partial derivative conditions express the speculative motive and the transaction motive for holding money. For the boundedness and positiveness of r, we suppose that

$$\forall X > 0, \quad \exists r > 0, \quad \lim_{Y \to 0} M^d(r, Y) = X. \tag{2.25}$$

2.5 Temporary equilibrium

From the formulations above, we define a temporary equilibrium of goods, labor, and money. As the capital K is given in the short term, we adopt the intensive form description by dividing the variables by K. Note that $y = Y/K$, $c^d = C^d/K$, $i = I/K$, $l^j = L^j/K$, $e = E/K$, $f(e) = F(1, e)$, $m = M/(PK)$, and $b = B/(PK)$.

Definition 1. A temporary equilibrium is the solution $(y, e, m) \in (0, f(l^s)] \times (0, l^s] \times \mathbb{R}_{++}$ for the following system:[15]

$$y = \min\{y^d, f(l^{d*}), f(l^s)\}, \tag{2.26}$$

$$e = \min\{l^{d*}, \tilde{l}^d, l^s\}, \tag{2.27}$$

$$m = m^d(r, y), \tag{2.28}$$

where $(l^s, m, b, g, w, \pi) \in \mathbb{R}^5_{++} \times \mathbb{R}$ is given and $y^d = c^d + i + g$.

Proposition 1. When the consumption propensity is not too large, and the consumption and investment is not too sensitive to q, the temporary equilibrium $(y, e, m) \in (0, f(l^s)] \times (0, l^s] \times \mathbb{R}_{++}$ is uniquely determined for any given $(l^s, m, b, g, w, \pi) \in \mathbb{R}^5_{++} \times \mathbb{R}$.

Proof. Obviously, employment e is uniquely determined when production y is determined because $y = f(e)$ always holds and f is monotonically increasing. When we omit the exogenous variables, the realized transaction-of-goods function reduces to $y = y(y, q, r)$ since $y_{di} = Y_{di}/K = (1 - \tau)(y - \delta) - \pi(m + b + q)$. As q is a function of y, y^d, $y^s = \min\{f(l^{d*}), f(l^s)\}$ and r, the endogenous variables can reduce to the two variables (r, y).

To prove the unique determination of (r, y), we use the IS-LM framework following Azam (1980) and Sargent (1987). First, we can check that $y^s = y^s(w, l^s)$ is determined exogenously in the short term. Therefore, we should prove that the production level that satisfies $y = y^d(y; r)$ uniquely exists for all $r > 0$. From equations (2.8) and (2.14), $y^d(0; r) > 0$. The solution to $y = y^d$ exists when the two curves $y = y$ and $y = y^d(y; r)$ cross uniquely, as in a Keynesian Cross, and the sufficient condition for it is

$$1 > (1 - \tau)f^c - (q_1 + q_2)(c^d_q + i_q), \tag{2.29}$$

where $c^d_q = (\partial c^d/\partial q)$ and $i_q = (\partial i/\partial q)$.[16] This condition is satisfied when the consumption propensity is not too large and the consumption and investment are not too sensitive to

[14]The last equation states that we can rearrange the money balance condition with the real balances ratio $M/(PK)$. For a brief discussions of the money demand function M^d, see Burmeister and Dobell (1970, Chapter. 6)

[15]We also determine the dynamics of each real asset. However, the accumulation demand, such as $\dot{m}^d$, does not directly appear in the system; the actual dynamics of money and bond holdings are determined by the supply side. However, note that the accumulation demand works indirectly in the static model since the asset accumulation demand works inversely in the expressed consumption demand c^d.

[16]Strictly speaking, the slope of $c^d(y) + i(y, y^d(y)) + g$ under $y = y^d$ is $(1 - \tau)(f^c_3 y_{di} + f^c) + (q_1(1 - w/f') + q_2)(f^c_1 y_{di} - \pi f^c + \psi')$.

q. We suppose that this stability condition holds hereinafter. From equation (2.14), the "IS" curve $r_{IS}(y)$ that satisfies $y = \min\{y^d(r), y^s\}$ slopes downward when $y = y^d$ and vertically when $y = y^s$ on the y - r plane (see Figure 1).

The LM curve $r_{LM}(y)$ satisfies equation (2.24) and slopes upward. From the inequality $y^d(0; r) > 0$, there exists a $\underline{y} > 0$ that satisfies $r_{IS} \to \infty$ for $y \to \underline{y}$. Therefore, the IS and LM curves cross uniquely, and the solution to the temporary equilibrium (r, y) is the crossing point.

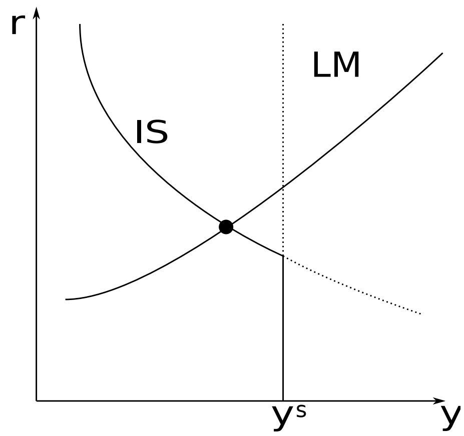

Figure 1: IS-LM interpretation of the temporary equilibrium

□

The figure shows the mechanism that determine the economy's regime, as well as the existence of a temporary equilibrium.[17] We should note that this IS-LM model includes the employment determination. The downward sloping segment of the IS curve contains the region in which $y = y^d$ and $e = \tilde{l}^d \leq l^s$. When the LM curve crosses the IS curve on this segment, the output and employment are determined by the goods demand. On the other hand, employment is determined following the goods supply constraint when the LM curve crosses the vertical segment of the IS curve. The exogenous variables affect how the two curves cross.

From the budget constraints of the three economic agents, we derive the extended "Walras' law." Aggregating equations (2.2), (2.13), and (2.17),

$$Y^d - Y = C^d - C = \dot{A} - \dot{A}^d, \tag{2.30}$$

which means that the excess goods demand is equal to the difference between the realized (or *ex post*) saving and the expressed (or *ex ante*) saving.[18]

2.6 The regimes and comparative statics

The realized production and realized employment are determined by the magnitude of the correlations among y^d, $f(l^{d*})$, and $f(l^s)$.

We define the regimes of the economy following Malinvaud (1977):

[17] Bénassy (1983) and Sneessens (1984) provide the IS-LM description for disequilibrium macroeconomics.

[18] This equation is an extension of the rearranged Walras' laws in Azam (1980, equation (11)) and Ogawa (2019b, equation (2.20)).

◦ **Keynesian unemployment (KU)**
In this regime, the effective demand y^d constrains the goods production, and the effective labor demand is less than the supply. Involuntary unemployment occurs due to insufficient goods demand, and the conditions are

$$y^d \leq y^s = \min\{f(l^{d*}), f(l^s)\}. \tag{2.31}$$

◦ **Classical unemployment (CU)**
Although this regime also has involuntary unemployment it has a different unemployment mechanism. The firm restricts employment due to the high wage w:

$$y = f(l^{d*}) \leq y^d, f(l^s). \tag{2.32}$$

◦ **Repressed inflation (RI)**
In this regime, the production is constrained by the insufficient labor supply. Both markets have excess demand and

$$y = f(l^s) < y^d, f(l^{d*}). \tag{2.33}$$

◦ **Equilibrium (EQ)**
When both markets are in equilibrium, the economy is in an equilibrium regime.

$$y = y^d = f(l^s). \tag{2.34}$$

In particular, the *Walrasian equilibrium* (WE) is the regime in which the economy is in the EQ regime and $f(l^s) = f(l^{d*})$. In the WE, the notional and effective demand are equal.

The economy always belongs to one of the regimes above, and the set of five exogenous variables (l^s, m, b, w, π) in the static model determines the realized regime. To simplify the comparative statics, we use some natural assumptions.

Assumption 3. The slope of the LM curve is not too steep and the goods demand is not too sensitive to the interest rate. $|m_2^d/m_1^d|$, f_2^c, and q_2 are sufficiently small that $(\partial y^d)/(\partial y) > 0$ always holds.

Assumption 4. The effective goods demand is strongly affected by the realized income, so $(\partial y^d)/(\partial y) + (\partial y^d)/(\partial y^s) > 0$ holds when $y = y^s < y^d$.

The large goods supply decreases the effective goods demand as the investment demand weakens. However, a large goods supply also means a high realized income when $y = y^s < y^d$. Assumption 4 states that the latter impact is stronger for the effective goods demand.

We now move to the comparative statics. We again apply the IS-LM framework; see Appendix B for the detailed calculations.

When $y = y^d$, the IS curve slopes downward and shifts according to the change in the exogenous variables, including policy parameters g and τ. Using differentiation, we calculate the equation for the IS-LM temporary equilibrium as follows:

$$r_{IS}(y; \underbrace{m, b, g, \pi}_{\oplus}, \underbrace{w, y^s, \tau}_{\ominus}) = r_{LM}(y; \underbrace{m}_{\ominus}) \tag{2.35}$$

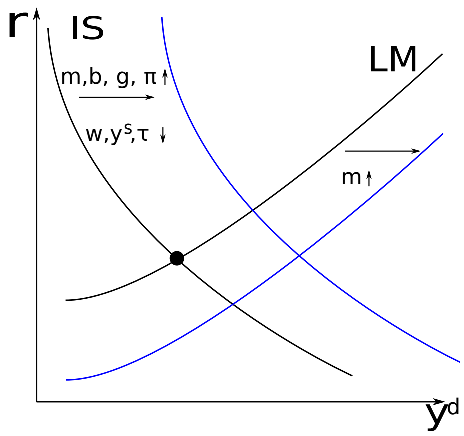

Figure 2: Comparative statics when $y = y^d$

The notations below the exogenous variables show the signs of the partial differentiation. Figure 2 depicts this equation.

When $y = y^s$, the IS curve becomes a vertical line. The amount of production is determined by w or l^s, and any other changes in the exogenous variables do not affect production.

Therefore, we can formulate the realized production as

$$y^d = y^d(\underbrace{m, b, g, \pi}_{\oplus}, \underbrace{w, y^s, \tau}_{\ominus}) \tag{2.36}$$

$$f(l^{d*}) = v(\underbrace{w}_{\ominus}) \tag{2.37}$$

$$f(l^s) = f(\underbrace{l^s}_{\oplus}) \tag{2.38}$$

Using these equations, we can illustrate the divisions of the regime on a plane. Figure 3 illustrates the simple version of the regime dividing because there are too many exogenous variables.

Note that a high real wage always induces both types of unemployment, while other disequilibrium models, such as those by Böhm (1978), Weddepohl and Yildirim (1993), and Ogawa (2019b), show that a lower real wage induces KU. In our model, the income distribution does not affect the consumption demand and the investment function is decreasing in w since the high wage rate lowers profitability. These formulations make y^d decreasing in w.[19]

Intuitively, an increasing g and decreasing τ remedy the KU regime since y^d increases. However, note that these fiscal policies increase production only when the economy is trapped in a Keynesian regime. The expansion of real assets m and b stimulates the real economy through the *Pigou* effect channel (y^d increases), and an increasing m also stimulates the money market. The equilibrium regime is on the border between the KU and RI regimes, and the Walrasian equilibrium is in the center of the regimes. These regional characteristics are common in disequilibrium models; see, for instance, Bénassy (1986).

[19]Several Keynesian monetary growth models, like those of Chiarella and Flaschel (2000) and Asada et al. (2011), adopt an income distribution effect on goods demand by establishing two classes: workers and asset holders.

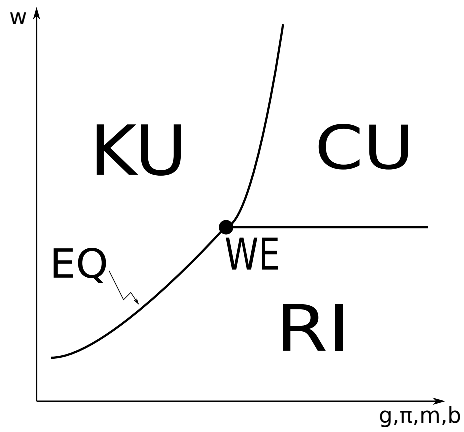

Figure 3: Regime divisions on the g, π, m, b - w plane

3 Dynamic analysis

In the previous section, we showed how the statically given exogenous variables (l^s, m, b, w, π) determine the regime of the temporary equilibrium. In this section, we analyze the dynamics of these variables to check the stability of the balanced growth path, determine how the regime changes in the growth path, and ascertain the dominant regime in the dynamics.

To complete the dynamic system, we must formulate the dynamics of the wage, price, and expectations of the price change. We apply the Phillips curve, which is a common method in Keynesian monetary growth studies.[20] We also refer to Fischer (1972), who presents the wage-price dynamics for Keynes-Wicksell models. We formulate the change in price P and the nominal wage W as following Walrasian adjustments:

$$\dot{P}/P = \pi + \nu_P(y^d - y^s), \;\; \nu_P = \text{const} > 0, \tag{3.1}$$

$$\dot{W}/W = \pi + \nu_W(l^{d*} - l^s), \;\; \nu_W = \text{const} > 0. \tag{3.2}$$

As Orphanides and Solow (1990) point out, these Fischer relations enable the market-clearing steady state with price inflation. Note that the nominal wage adjustment matches the labor supply and the notional labor demand. This formulation implies that the wage dynamics would adjust employment to the "potential" level in Chiarella et al. (2000, Chapter. 5).[21]

We describe the development of the inflation expectation, or the dynamics of π, as a combination of the adaptive- and forward-looking development:

$$\dot{\pi} = \beta[\alpha(\dot{P}/P - \pi) + (1-\alpha)(\pi_0 - \pi)], \quad \beta > 0, \quad 0 \le \alpha \le 1, \tag{3.3}$$

where π_0 is the steady-state value of π. If $\alpha = 0$, then the economy is characterized by myopic perfect foresight. If α becomes unity, on the other hand, the expectation adjustment is completely adaptive.

[20]See Chiarella and Flaschel (2000), Asada et al. (2006), Proaño et al. (2007), and Asada et al. (2011). For the details of wage-price modules, see Chiarella et al. (2005, Chapter. 5). Proaño et al. (2011) discuss the relationship between the income distribution and wage-price modules.

[21]Chiarella et al. (2000, Chapter. 5) use the term "potential" as the level at which the production occurs under the marginal profit principle. Ogawa (2019a, Appendix B) uses the bargaining framework to justify the nominal wage adjustment at which the employment would become the notional demand level.

3.1 Dynamic system

From Equations (2.8), (2.11), (2.17)-(2.19), and (3.1)-(3.3), the dynamics of (l^s, m, b, w, π) are

$$\dot{l}^s = l^s(n - \psi - n) = -l^s\psi \tag{3.4}$$
$$\dot{m} = m\{\mu - \pi - \nu_P(y^d - y^s) - \psi - n\} \tag{3.5}$$
$$\dot{b} = \{g - \tau(y - \delta) - \mu m\} - b\{\pi + \nu_P(y^d - y^s) + \psi + n\} \tag{3.6}$$
$$\dot{w} = w\{\nu_W(l^{d*} - l^s) - \nu_P(y^d - y^s)\} \tag{3.7}$$
$$\dot{\pi} = \beta[\alpha\nu_P(y^d - y^s) + (1 - \alpha)(\mu - n - \pi)]. \tag{3.8}$$

Note that the dynamic system consists of differential equations with discontinuous right-hand sides (Filippov, 1988) since the goods supply y^s and the realized production y (and then y^d) is determined through the minimum function. In the dynamics, the economic regime switches several times and the right-hand sides of the dynamic equations above also switch. Therefore, the common analytical tools will sometimes be invalid for our system.

The (usual) steady state condition of this dynamic system is

$$y^d = y^s \tag{3.9}$$
$$l^{d*} = l^s \tag{3.10}$$
$$q = 1 \tag{3.11}$$
$$\pi = \mu - n \tag{3.12}$$
$$0 = g - \tau(y - \delta) - \mu(m + b) \tag{3.13}$$

The first two equations say that the steady state is in the Walrasian equilibrium regime.[22] Furthermore, the real interest rate plus the risk premium $r - \pi + \xi$ equals the real rate of return on held capital ρ^* from Equation (2.10). Therefore, the "Wicksellian" equilibrium condition $r - \pi + \xi = \rho$ and the real equilibrium conditions are satisfied at the steady state.

The last equation in the steady state conditions states that $\dot{B}/B = \dot{M}/M = \mu$, or the nominal bond is issued at the same rate as the government prints money. Then, the net government deficit $G - \tau(Y - \delta K)$ grows at the same rate as n at the steady state.

Definition 2. The steady-state value of the dynamic variables is the set $(l_0^s, m_0, b_0, w_0, \pi_0) \in \mathbb{R}^4_{++} \times \mathbb{R}$, which satisfies Equations (3.9)-(3.13).

Proposition 2. The steady-state value $(l_0^s, m_0, b_0, w_0, \pi_0)$ uniquely exists.

Proof. Obviously, $\pi_0 = \mu - n$ is uniquely determined. Let x_0 denote the value of x when the exogenous variables are $(l_0^s, m_0, b_0, w_0, \pi_0)$. The real equilibrium Equations (3.9) and (3.10) imply that $y_0^d = v(w_0) = f(l_0^s)$, which generates two independent equations. From Equation (2.10), $\rho_0 = y_0 - l_0^s f'(l_0^s) - \delta = r_0 - \pi_0 + \xi$ holds. We thus have four independent equations for the four variables (l_0^s, m_0, b_0, w_0). $\square$

[22] Note that the regime in the steady state often depends on the formulations of the dynamic system in the disequilibrium school. If we adopt the Walrasian adjustment process for both the wage and price dynamics, as in Equation (3.7), the persistent existence of the demand-supply gaps in both markets are easily enabled. The other dynamic equations determine the regime in the steady state, and our system ensures goods market clearing because we base our model on neo-classical monetary growth, as in Sargent (1987). If we adopt frictions such as the searching process, we can obtain another result easily. We utilize the adjustment to an ideal equilibrium point for simplicity.

Note that our dynamic system is discontinuous and therefore the dynamics could stop elsewhere, which Filippov (1988) calls the pseudo-equilibrium. Our system also has the possibility of this pseudo-steady state.

3.2 Stability and basic feedback loops

When checking the local stability condition for the dynamic system, we should examine the Jacobian matrix **J**, which is the coefficient matrix of the linearized dynamic system at the steady state. However, note that the values of the factors of **J** change for each regime because the steady state is located at the intersection of the regime boundaries, where the vector field becomes discontinuous. Due to this discontinuity, the local stability analysis with the Jacobian matrix is difficult for high dimensional systems; that is, we cannot apply a graphical analysis to detect the shapes of the (un)stable manifolds. [23]

Instead, we concentrate on how the stabilizing-destabilizing feedback loops of each economic variable work globally. Chiarella et al. (2000), Chiarella et al. (2005), and Asada et al. (2006) summarize each feedback channel. We should note that these channels might work differently in each disequilibrium regime; for instance, one feedback loop stabilizes in the *Keynesian* regime, but destabilizes in other regimes.

1. *The Keynes (and Pigou) effect.* When the price level increases, the nominal (and then the real) interest rate in the LM market become high. The high real interest rate decreases q and the current consumption demand. Furthermore, the low q weakens the investment and consumption demand again. This is the Keynes effect. The high price level also depreciates real asset holdings A, which weakens the consumption demand. This is the result of the Pigou effect. The decline in the effective demand induces low price inflation, and therefore the Keynes and Pigou effects stabilize the price dynamics.

2. *The Mundell effect.* As our model formulation adopts the IS-LM framework, this effect works as usual. When the economy expects higher inflation, the incentive to hold money decreases and the capital accumulation increases because q increases. The increase in q leads to higher goods demand (note that the actual production rises only if $y = y^d$), thereby stimulating actual price inflation. This price inflation pulls up the inflation expectation as long as $\alpha \neq 0$, or the expectation has an adoptive characteristic. Thus, the Mundell effect destabilizes the expectation dynamics.

3. *The real wage effect.* As the static model analysis shows, both the effective goods demand and supply are decreasing in the real wage w. This negativity induces ambiguity in price inflation against the real wage dynamics. Note that if the goods demand is more sensitive to the real wage than supply is, the high real wage lowers the price inflation pressure. This unstable feedback is called the *Rose effect*, as in Flaschel and Sethi (1996). Furthermore, instability becomes stronger in the Keynesian regime ($y = y^d$); see Appendix B. As we assume a Walrasian adjustment

[23] However, the following theorem is worthwhile for our system. When the two-dimensional dynamic system is locally stable for all three regimes, the whole system is in our model is also locally stable (Eckalbar, 1980). However, this sufficient condition could be violated by the unstable expectation dynamics and our model is five-dimensional. Therefore, we do not apply this method. On the Jacobian matrix, see Appendix B.

process in the labor market, the nominal wage moves in the opposite direction against the real wage. Summing up, the direction of the real wage adjustment is ambiguous when the price dynamics are not too slow. Therefore, the real wage feedback loop in the two Walrasian adjustments in Equations (3.1) and (3.2) could be both stabilizing and destabilizing the dynamics.

The feedback loops above are the same as in the ordinal Keynesian dynamic models. The next one is characteristic of the non-Walrasian regime switching phenomenon.

4. *The dual-decision effect.* To specify this effect, we must examine two feedback loops of y^s: the dynamics of w and l^s. y affects their feedback loops, so we should check the cases $y = y^d$ and $y = y^s$. First, suppose that $y = y^d$. The first loop $y^s = v(w)$ is included in the loop in *the real wage effect*. As a large y^s (low wage) decreases y^d and the price declines, the real wage tends to increase. Therefore, the feedback loop of $v(w)$ seems stable, as long as we consider the real economy and $y = y^d$ holds. The second loop $y^s = f(l^s)$ is in contrast. When $y = y^d$, the high y^s directly lowers y^d/y^s and decreases the investment (and then increases l^s). As the excess demand term works in the investment function, y^s destabilizes the feedback loop of l^s. This is a kind of Harrodian instability.
   The (in)stability of the y^s feedback loops are dampened when $y = y^s$ compared with the case $y = y^d$. In the first case $y^s = v(w)$, the large y^s obviously increases y, and the increased y enlarges y^d through the effective demand principle $y^d = y^d(y)$. This new path increases price inflation pressure, which is in opposite to the stable feedback when $y = y^d$. This *dual-decision effect* works similarly for $y^s = f(l^s)$. The large y^s (and then large y) increases investment and puts negative pressure on l^s.

As the descriptions in the item are mathematical, we should discuss how the dual-decision effect works in detail. We adopt the dual-decision hypothesis, so the goods demand is *effective* in the sense it depends on the realized income y. In ordinal Keynesian models, production and income are always determined by the effective goods demand. The supply side is usually regarded as the criterion for the potential production and the gap between the potential and realized production (e.g., the capital utilization rate) is an important issue in macrodynamics, though not the goods supply itself. In contrast, production is always determined by the supply side in typical neo-classical models since full capacity is realized.

Non-Walrasian models synthesize the two perspectives. Compared to ordinal Keynesian model, we can treat the supply and demand sides directly. In our model, the effective demand principle always works; thus, $y^d = y^d(y)$. However, the supply side could determine the realized production, and $y^d = y^d(y^s)$ holds in this case. Certainly, the (relatively) large y^s leads to the large gap between the potential and realized production, and it usually decreases the effective demand since the investment demand declines. When $y = y^s$, however, it also means a high realized income and the *effective* demand increases. This composite effect complicates the pure feedback in the Keynesian case of $y = y^d$. The dual-decision effect works as both a stabilizer and a destabilizer in the feedback loops.

Although the stability analysis of our high-dimensional dynamic system is difficult, we can derive the sufficient condition for instability, which is so limited, but similar to that of Chiarella et al. (2000, Chapter. 5).

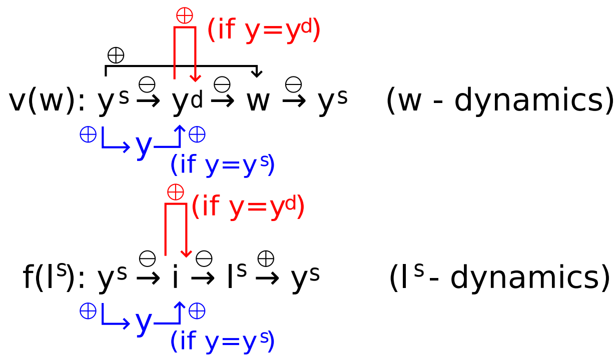

Figure 4: The y^s feedback loops with the dual-decision effect

Proposition 3. The steady state is not asymptotically locally stable when the speed of the expectation adjustment β is high, the expectation adjustment is near to adaptive, and the real wage effect is moderately unstable.

Proof. Take the 2 by 2 principal minor consisting of the fourth and fifth rows (and columns) $\mathbf{J}_{45}$ of the Jacobian matrix. If $\nu_W v'(w_0)(\nu_P w_0)^{-1} < (d/dw)(y^d - y^s) < 0$ and $\alpha\nu_P(\partial y^d/\partial\pi) > 1-\alpha$ hold, then $\det\mathbf{J}_{45} = \beta\ominus$, where $\ominus < 0$; see Appendix B. When β is sufficiently large, the sum of all 2 by 2 principal minors would be negative. Then, the Routh-Hurwitz stability condition is violated in every regime. $\square$

From this proposition, we confirm that our dynamics overlap with the standard Keynesian models. The sensitive adoptive adjustment of the inflation expectation usually destabilizes the monetary dynamics(Burmeister and Dobell, 1970; Chiarella and Flaschel, 2000). However, the instability condition hardly holds in our model, as we show in the next section.

4 Numerical experiments

In this section, we conduct numerical experiments of the canonical disequilibrium monetary growth model and simulate the dynamic system presented in the previous section. As we cannot use graphical or analytical deductions for the five-dimensional dynamic system, we need a method to detect the discontinuity accurately and address it properly in the simulation. Therefore, we use the `DISODE45` algorithm of MATLAB produced by Calvo et al. (2016).

In the beginning, we should specify the parameter values and functional forms. In this study, we utilize the empirical study by Flaschel et al. (2001), who construct a disequilibrium monetary growth model and specify the parameter values using US quarterly time series data 1960-1995.

First, we formulate the functions in our system. We suppose a Cobb-Douglas-type production function: $F(K, E) = K^a E^{1-a}$, $a > 0$. Following Karabarbounis and Neiman (2014), we set $a = 0.34$; that is, the profit share rate is near one-third of the steady state. We consider E as the efficient labor that includes the labor productivity and therefore n is the sum of the population growth rate and the labor productivity growth rate. We next formulate the q function, as follows: $q = (y^d/y^s)^\gamma(\rho/(r-\pi+\xi))$, $\gamma > 0$. This formulation is compatible with Appendix A. We estimate the consumption demand function c^d from

US postwar data, as in Appendix C, and obtain $c^d = 0.6483\exp(0.9044(r-\pi))((m+b+q)/y_{di})^{0.1866}y_{di}$. In this estimation, we arbitrarily set $\tau = 0.15$ and $\nu_P = 0.010$,[24]

The remaining functions are the same as the linearized functions in Flaschel et al. (2001): $m^d = h_1 y + h_2(r_0 - r)$; $\psi = i_1(\rho - r + \pi - \xi) + i_2((y^d/y^s) - 1)$. The parameters also follow Flaschel et al. (2001): $h_1 = 0.1769$, $h_2 = 2.1400$, $i_1 = 0.1363$, $i_2 = 0.0340$ and $\nu_W = 0.0958$.

Second, we set the residual parameters to make the steady-state values of y and r compatible with the empirical study. Flaschel et al. (2001) show that $n = 0.0081$, $\mu = 0.0154$, $\delta = 0.0468$, and $\xi = 0.1500$, and we utilize them for calculation.

The steady-state value of y_0 and r_0 (and $(l_0^s, m_0, b_0, w_0, \pi_0)$) now depend on the undecided parameter g. Since Flaschel et al. (2001) find that their values are $y_0 = 0.5091$ and $r_0 = 0.0221$, we set g equal to 0.1250, which indicates

$$y_0 = 0.6276,\ r_0 = 0.0239,\ l_0^s = 0.4937,\ m_0 = 0.1110,\ b_0 = 2.3491,\ w_0 = 0.8390,\ \pi_0 = 0.0073.$$

Using these results, we estimate the value of γ. The Taylor expansion around the steady state implies

$$\psi \simeq \psi'\rho_0^{-1}(\rho - r + \pi - \xi) + \psi'\gamma((y^d/y^s) - 1).$$

Thus, we have $\gamma = i_1/(i_2\rho_0) = 1.4976$.

4.1 Two examples: Persistent KU and cyclical growth

As we have not set the parameters α and β, we should examine their effects on the stability of the steady state. However, we first introduce two characteristic examples here.

First, we set the initial values at $(l^s, m, b, w, \pi) = (0.5159, 0.1157, 2.2440, 0.8590, 0.0071)$ and the adjustment parameters at $\alpha = 0.400$ and $\beta = 0.280$. Figures 5 and 6 illustrate the simulated path. The dashed line in Figure 5 shows the steady-state value, and the dots in Figure 5 and the vertical lines in Figure 6 correspond to the discontinuous points of the dynamic system. These figures show that the economy is initially in the CU regime and moves to the persistent KU regime until it reaches the steady state. When the regime switches from classical to Keynesian, the low goods demand leads to pessimistic expectations of goods sales, and thus the real value of capital q quickly falls, even though the nominal interest rate r remains low. The low investment and consumption demand cause further shortages of effective demand, with the multiplier effect in the short run. This instability induces consistent KU. The low goods demand, the sticky low interest rates and the low real wages seem to reflect "secular stagnation."[25] Note that the wage rate w remains at a low level, even though the CU is resolved. This is because in the KU regime, investment is not stimulated and l^s is consistently high. As the production factors are substitutable, the relatively ample labor supply lowers the wage. Therefore, the equilibrium real wage is underestimated around the first switching point $t = 5$ and the

[24]The results of the simulations are not sensitive to these values. The value of τ affects the scale of y, and the value of g compensates for this effect to make y_0 the same as the empirical results. The value of ν_P affects c^d and the adjustment speed. Their effects are small and the qualitative results remain when it changes; therefore, we use this value in the following simulation.

[25]As this is a continuous-time model, it is difficult to identify the length of secular stagnation. If we measure time as in the estimation in Appendix C, the length of one period is three months. This setting implies that CU is solved in one year but KU continues over one century without any policy.

wage overshoots the steady-state value. The dual decision effect affects this overshoot. In the KU regime, the wage adjusts quickly to the underestimated wage rate, and its cause (high l^s) adjusts slowly. This example shows that the dual decision effect exacerbates the KU, even though it stabilizes the feedback loop for w.

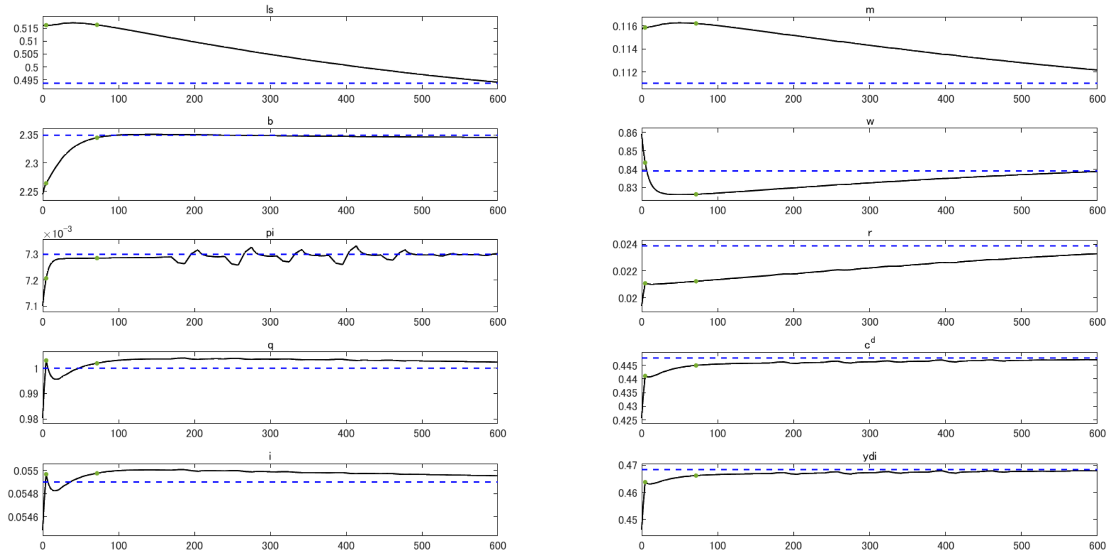

Figure 5: The dynamics of the variables in Example 1

The second example is a cyclical dynamic. We suppose a mostly adoptive expectation adjustment $\alpha = 0.9920$ with an adjustment speed of $\beta = 0.2800$, which is the same as in Example 1. We set the initial values at $(l^s, m, b, w, \pi) = (0.4755, 0.1159, 2.4562, 0.8453, 0.0070)$.[26] As Figures 7 and 8 show, the (long-run) cyclical dynamics occur. When $y^s < y^d$, the dynamics of y^s and y^d lead to similar patterns (the effect is stabilizing). CU occurs in the latter half of the term $y^s < y^d$, but the employment rate does not decline so much. When $y^d < y^s$, on the other hand, y^s moves upward excessively and the KU becomes more serious than the classical one. The cyclical regime switching WE $\to$ RI $\to$ CU $\to$ KU $\to$ WE $\to \cdots$ continues. Appendix D shows that the adaptive expectation adjustment causes cyclical dynamics and the cycle becomes unstable as α increases. This example is difficult to adopt to the real world economy because y^d becomes higher in KU than in the other regimes. The aggregated disposable income grows in the KU regime, even though the employment and the wage become seriously low. This result implies that the ample incomes of asset holders support goods demand and the need for future studies to analyze the income distribution in disequilibrium models further.

4.2 Fiscal policy on the persistent KU path

Does fiscal policy remedy persistent KU? We check whether the permanent fiscal stimulus remedies persistent KU in the example above. Suppose that the government decides to expand its purchase parameter g to offset the shortage of goods demand by increasing

[26] In fact, the cyclical dynamics seem not to depend on the initial value: the initial value in Example 1 also leads to the cycle with a high value of α.

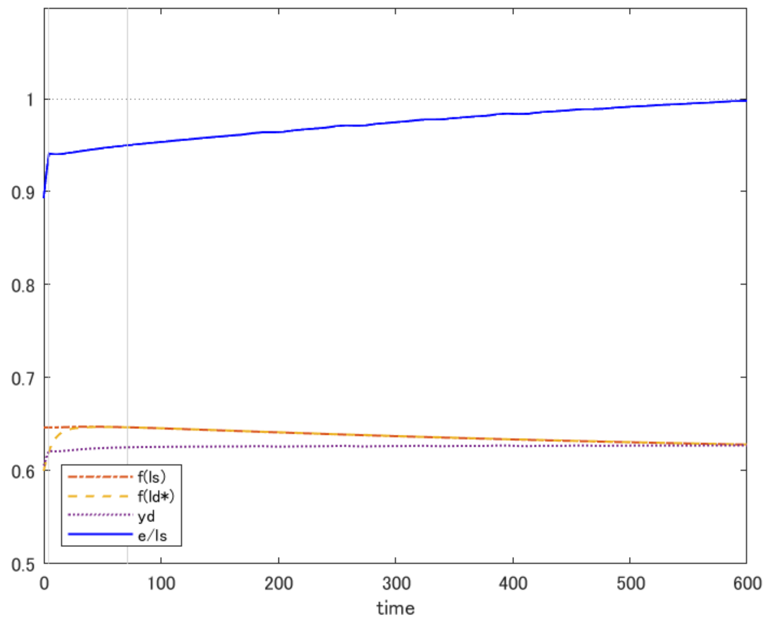

Figure 6: The dynamics of employment and production in Example 1

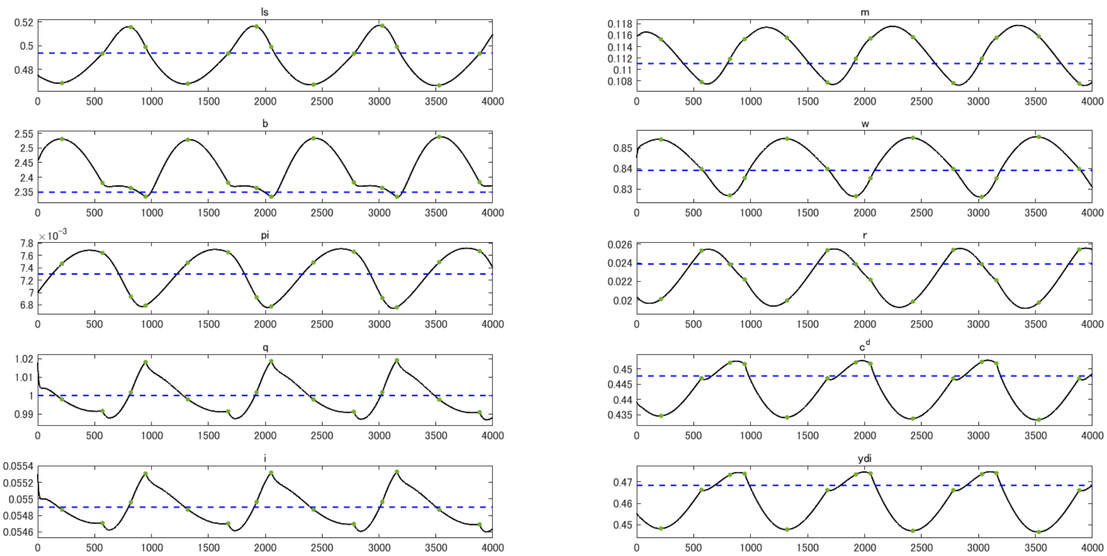

Figure 7: The dynamics of the variables in Example 2

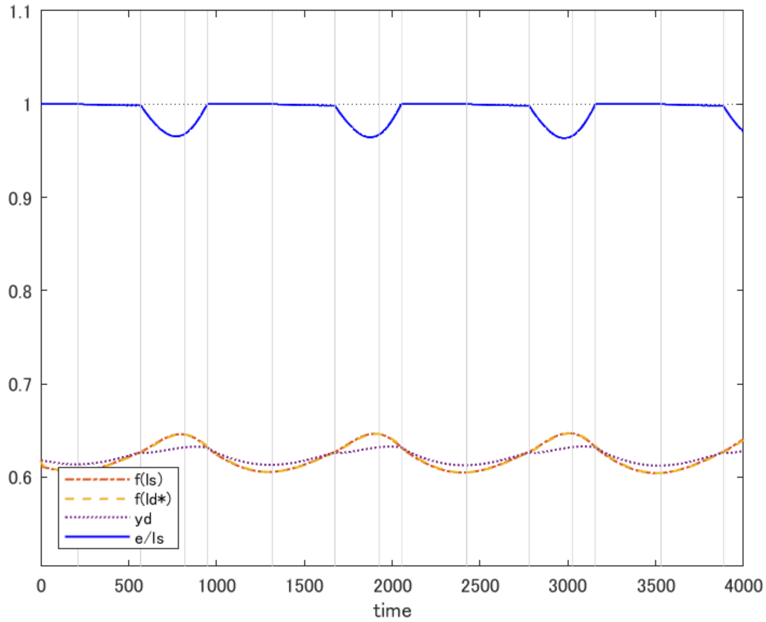

Figure 8: The dynamics of employment and production in Example 2

tax rate τ at $t = 100$. Since the two parameters are exogenous variables, this fiscal policy is permanent in that the steady-state values of the variables change. We assume that g increases from 0.1250 to 0.1300 and that the tax rate increases 0.5% points from 0.1500 to 0.1550. Then, the new steady-state values are

$$y_0 = 0.6410,\ r_0 = 0.0285,\ l_0^s = 0.5098,\ m_0 = 0.1134,\ b_0 = 2.3471,\ w_0 = 0.8299,\ \pi_0 = 0.0073.$$

Since the production y_0 increases, the steady-state value of the wage must decrease to satisfy $f(l^{d*}) = v(w) = y_0$. The higher tax compensates for the government deficit, so the government bond balance b_0 decreases. Figure 9 shows how the fiscal stimulus works on the persistent KU path. The dashed line shows the new steady state.

In our model, fiscal stimulus has a remarkable effect on KU because the quantity of goods demand increases and its effect is enhanced by the multiplier effect. The expansion of realized production y directly increases the income y_{di} and then increases the consumption demand c^d, so that the goods demand y^d (and y) increases. This positive feedback quickly remedies *Keynesian* unemployment.

Note that the consumption demand c^d increases even though the wage rate w remains low. This result implies that the effect of the fiscal stimulus might be overestimated since the consumption demand might not be stimulated without increase in wage income. Furthermore, the government has other policy choices such as temporary stimulus and wage regulations. The detailed policy analysis and the evaluation with the income distribution model are important issues for future research.

4.3 The strong local stability of the steady state

We should check how much the expectation adjustment parameters α and β affect the local stability of the steady state. Figure 10 implies the strong local stability. In Figure 10, the point at which the full Jacobian matrix of the system corresponds with all six possible

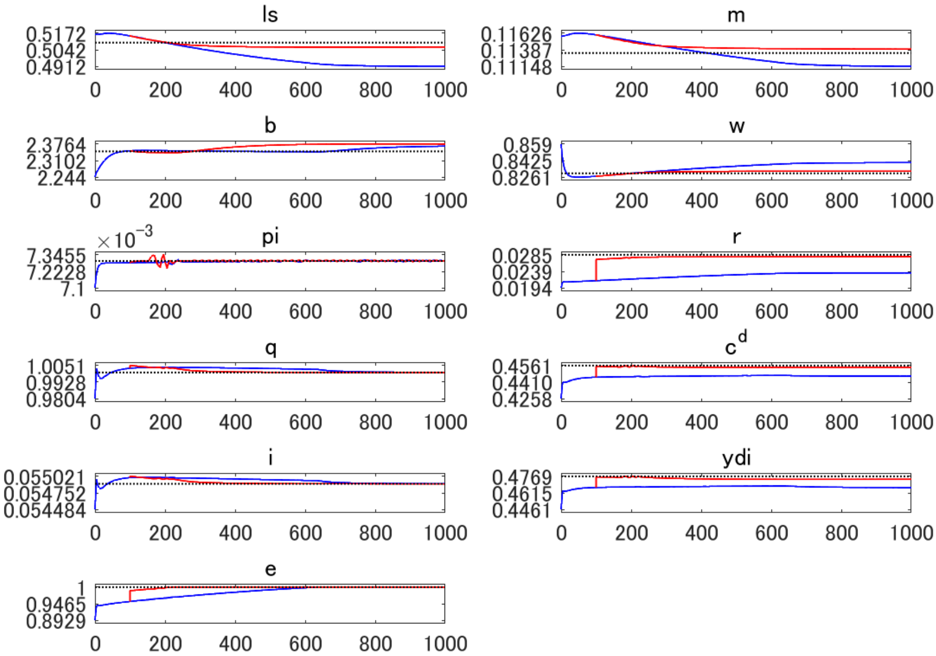

Figure 9: Path without policy (blue) and path with fiscal stimulus at $t = 100$ (red)

cases (e.g., $y^d < f(l^s) < v(w)$) has only negative eigenvalues under the combination (α, β) that is rationed.

The figure shows that the steady state of the dynamic system would be locally (and maybe asymptotically) stable unless the expectation adjustment is completely adaptive. This result implies the strong local stability of the steady state. However, the global stability is not ensured, as the second example above shows.

In addition, the second example in the simulation shows that the Walrasian equilibrium (notional equilibria in all markets) is not always stable. The steady state in our system lies in the set of all possible Walrasian equilibria, but the economy might move to a disequilibrium regime unless it reaches the steady state. This result implies that the assumption that the normal state of the economy is a (Walrasian) equilibrium is doubtful. We should reconsider how to justify (excessive) equilibrium models as the starting point of macroeconomics.

5 Concluding remarks

In this study, we analytically explore a non-Walrasian monetary growth model and reach two important conclusions from the dynamic perspective:

- In the goods supply feedback loops, the dual-decision effect is both a stabilizer and destabilizer, and causes persistent KU.
- Although the locally stable steady state belongs to the Walrasian regime, the economy at another point in the Walrasian regime might move into a disequilibrium regime.

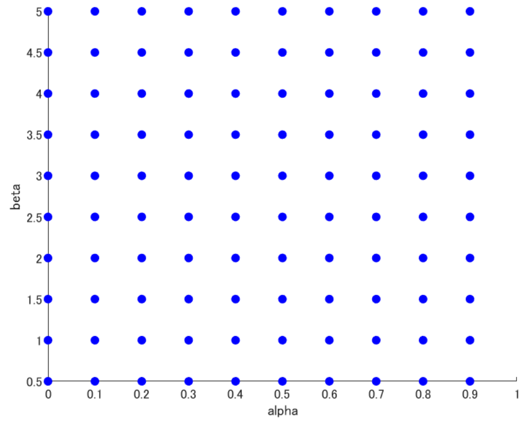

Figure 10: The point at which the steady state is locally stable

Our model specifies the dual-decision effect on economic dynamics, in which individuals determine the demand or supply depending on current realized transactions. In our formulation, this effect *stabilizes* the adjustment of the price system. Interestingly, the superficially ideal price adjustment leads to persistent KU. The real wage adjusts to the underestimated level and overshoots the steady-state value when the regime switches.

The persistent KU path shows that we should consider the relationships among the *realized* current transactions of goods, labor, and money when studying the cause of a recession (or secular stagnation). In this study, the consistent KU comes from the regime switching from classical to Keynesian, which means that the cause of the recession consists of a classical factor (the wage-price system) and a Keynesian factor (quantity constraints). Furthermore, the dual-decision effect leads to unemployment without the ZLB constraint, which New-Keynesian economics currently emphasizes. Palley (2019) argues that both the ZLB and nominal rigidities are classical issues rather than Keynesian. Keynesian unemployment in this study is definitely Keynesian because solving the ZLB constraint and rigidities do not remedy it. Our model shows that macroeconomic studies that refer to "Keynesian unemployment" must consider the dual-decision effect and how individuals respond to quantity constraints.

We propose a baseline model, though it is too crude to function as a direct approximation for an actual economy. To create a more sophisticated non-Walrasian macrodynamic model, we should consider several issues. The first is friction in markets. Almost all non-Walrasian models ignore friction to simplify the model analyses. However, frictions such as searching processes are important issues for unemployment, which is the main problem in macroeconomics. We should unify the frictions and the dual decision hypothesis. The second consideration is inventory dynamics. Ordinal Keynesian models such as in Chiarella et al. (2000) often adopt the inventory dynamics stimulated by Metzler (1941). As Green and Laffont (1981) and Honkapohja and Ito (1980) formulate the issue of inventory in disequilibrium macroeconomics, we should extend the model by referring to their works. This study is just the first step. We hope that our model will contribute to the further development of disequilibrium dynamics.

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A A simple example of q

We present a simple example of the formulation of q that suggests the function in equation 2.9. We follow the calculation in Sargent (1987).

The nominal value of firm V at time t is determined by the stream of the net cash flows:

$$V(t) = \int_t^\infty \mathbb{E}_t \left[\{P(\tau)F(K(\tau), E(\tau)) - W(\tau)E(\tau) - \delta P(\tau)K(\tau)\} e^{-\left(\int_t^\tau (r(s)+\xi)\, ds\right)} \right] d\tau. \tag{A.1}$$

We assess the value with the constant positive risk premium ξ.

We suppose that all individuals expect that the nominal interest rate will be constant and that the price inflates at a constant rate of $\pi(t)$ in the long run. Then,

$$V(t) = P(t) \int_t^\infty \mathbb{E}_t \left[F(K(\tau), E(\tau)) - w(\tau)E(\tau) - \delta K(\tau) \right] e^{-(r(t)-\pi(t)+\xi)(\tau - t)}\, d\tau. \tag{A.2}$$

Furthermore, we introduce the following arbitrary assumption:

$$\mathbb{E}_t \left[F(K(\tau), E(\tau)) - w(\tau)E(\tau) - \delta K(\tau) \right] = \Theta(Y^d(t)/Y^s(t)) \left[F(K(t), E(t)) - W(t)E(t) - \delta K(t) \right], \tag{A.3}$$

where $\Theta' > 0$ and $\Theta(1) = 1$. This assumption implies that the future expected value of ρ consists of today's value of ρ and the measure of optimism Θ. If the goods market has excess supply, future profitability is underestimated relative to today's profitability because the individuals become pessimistic about future sales. If we adopt this assumption, then the value of V becomes

$$V(t) = P(t)K(t)\Theta(Y^d(t)/Y^s(t))[\rho(t)/(r(t) - \pi(t) + \xi)]. \tag{A.4}$$

Therefore,

$$q(t) \equiv V(t)/(P(t)K(t)) = \Theta(Y^d(t)/Y^s(t))\rho(t)/(r(t) - \pi(t) + \xi)$$

. This equation is compatible with assumption 1.

B Comparative statics of the model and Jacobian matrix

We characterize the temporary equilibrium with the following seven simultaneous equations:

$$y = \min\{y^d, y^s\} \tag{B.1}$$
$$e = f^{-1}(y) \tag{B.2}$$
$$m = m^d(r, y) \tag{B.3}$$
$$y^d = f^c(m + b + q, r - \pi, y_{di})y_{di} + \psi(q - 1) + n + \delta + g \tag{B.4}$$
$$y^s = \min\{v(w), f(l^s)\} \tag{B.5}$$
$$y_{di} = (1 - \tau)(y - \delta) - \pi(m + b + q) \tag{B.6}$$
$$q = q(y - we - \delta, y^d/y^s, r - \pi) \tag{B.7}$$

As is in the proof of Proposition 1, this system has a unique solution as long as the variables (l^s, m, b, w, π) are exogenous and the parameters (g, τ, δ, n) are given.

For the dynamic analysis, we should know how the exogenous and other endogenous variables affect the scale of the effective goods demand term y^d. We use the total difference approach for equation (B.4), as follows:

$$\begin{aligned}
(1 - q_2\phi/y^s)dy^d = {} & (\phi - \psi' + (f_2^c y_{di} + q_3\phi)/m_1^d)dm + (\phi - \psi')db + dg - (f_3^c y^d i + f^c)(y - \delta)d\tau \\
& - [(f_3^c y_{di} + f^c)(m + b + q) + (f_2^c y_{di} + q_3\phi)]d\pi - eq_1\phi dw - (y^d/y^s)q_2\phi/y^s dy^s \\
& + [1 - G_y - q_2\phi/y^s - (f_2^c y_{di} + q_3\phi)m_2^d/m_1^d]dy, \\
\text{where } \phi = {} & f_1^c y_{di} + \psi' - \pi(f_3^c y_{di} + f^c) = (\partial c^d/\partial q) + (\partial i/\partial q) > 0, \\
G_y = {} & 1 - (1 - \tau)(f_3^c y_{di} + f^c) - [q_1(1 - w/f') + q_2/y^s]\phi > 0.
\end{aligned} \tag{B.8}$$

Note that this equation is not valid when $y = y^d = y^s$ (the equation is not totally differentiable and we should therefore use the limitation calculation) and still has indeterminate terms dy^s and dy. When $y = y^d < y^s = f(l^s) < v(w)$, for instance, dy in equation (B.8) becomes dy^d and dy^s becomes $f'dl^s$.

This discontinuity makes the correlations between the variables complicated. Therefore, we use differential equations with discontinuous-righthand-side techniques for the dynamic analysis. We now move to the Jacobian matrix analyses.

To see the signs of the factors in the first row of $\mathbf{J}$, we check how the other variables affect Tobin's q:

$$dq = [q_1(1 - w/f') + q_3 m_2^d/m_1^d]dy + (q_2/y^s)dy^d - (y^d/y^s)(q_2/y^s)dy^s - eq_1 dw + (q_3/m_1^d)dm - q_3 d\pi. \tag{B.9}$$

As $(dq)/(dy^d) > 0$, the signs of the partial derivatives of q with respect to (l^s, m, b, π) are the same as those of y^d. We summarize (dq/dw) as follows:

$$\begin{aligned}(dq/dw)|_{y=y^d} &= [-eq_1 - (y^d/y^s)(q_2/y^s)(\partial y^s/\partial w)]\oplus, \\ (dq/dw)|_{y=y^s} &= [-eq_1 - (y^d/y^s)(q_2/y^s)(\partial y^s/\partial w) + \ominus]\oplus,\end{aligned} \tag{B.10}$$

where $\oplus > 0$ and $\ominus < 0$. The magnitude relation between the two cases is ambiguous due to the nonlinear term y^d/y^s, but $(dq/dw)|_{y=y^d} < 0$ always holds from Assumption 4. In the steady state $y^d = y^s$, $(dq/dw) < 0$ is ensured in each regime.

We next implement the comparative statics for the Walrasian price adjustment term $\nu_P(y^d - y^s)$. As m, b, and π do not affect y^s, we can adopt the results of y^d directly. For l^s, there is not direct effect on y^d; thus,

$$dy^d/dl^s = (\partial y^d/\partial y^s)(\partial y^s/\partial l^s) \tag{B.11}$$

holds. Therefore the partial derivative of $\nu_P(y^d - y^s)$ with respect to l^s is 0 if $y^s = v(w)$ and negative if $y^s = f(l^s)$. By contrast, the effect of the real wage w on the Walrasian price adjustment is complicated. Using equation (B.8), we summarize $(d/dw)(y^d - y^s)$ as follows:

$$\begin{aligned}(d/dw)(y^d - y^s)|_{y=y^d} &< [-eq_1\phi - (\partial y^s/\partial w)]/[G_y + (f_2^c y_{di} + q_3\phi)m_2^d/m_1^d] \\ (d/dw)(y^d - y^s)|_{y=y^s} &= [-eq_1\phi - (\partial y^s/\partial w)(1 + \ominus)]/(1 - q_2\phi/y^s) \\ &\geq [-eq_1\phi - (\partial y^s/\partial w)]/(1 - q_2\phi/y^s),\end{aligned} \tag{B.12}$$

where the denominators in the above equations are positive because $G_y > 0$. Note that the sign of the numerator $-eq_1\phi - (\partial y^s/\partial w)$ is ambiguous when $y^s = v(w)$.[27] We conclude only that the real wage effect would be more unstable in a Keynesian regime than in a classical regime.

As we know the signs of the important factors, we can summarize the Jacobian matrix $\mathbf{J}$:

$$\mathbf{J} = \begin{bmatrix} \oplus(\partial y^s/\partial l^s) & \ominus & \ominus & \ominus & \ominus \\ \oplus(\partial y^s/\partial l^s) & \ominus & \ominus & -\nu_P\rho_w + \oplus & \ominus \\ -\tau(\partial y/\partial l^s) + \oplus(\partial y^s/\partial l^s) & \ominus & \ominus & \ominus - \nu_P\rho_w & \ominus \\ -w_0\nu_W + \nu_P \oplus (\partial y^s/\partial l^s) & \ominus & \ominus & \nu_W v' - w_0\nu_P\rho_w & \ominus \\ \ominus(\partial y^s/\partial l^s) & \oplus & \oplus & \beta\alpha\nu_P\rho_w & \beta[\alpha\nu_P(\partial y^d/\partial\pi) - (1-\alpha)], \end{bmatrix} \tag{B.13}$$

where $\rho_w = (d/dw)(y^d - y^s)$. Note that this expression of the Jacobian matrix is common among the regimes.

[27] As $1 - q_2\phi/y^s > 0$ holds, $-eq_1 - \phi^{-1}(\partial y^s/\partial w) > -eq_1\phi - (q_2/y^s)(\partial y^s/\partial w)$ holds. Assumption 3 ensures only the negativity of the right-hand side of the inequality.

C Estimation of the consumption demand function

In this appendix, we conduct a rough estimation of the consumption demand function $c^d = f^c y_{di}$ using postwar (1982Q1 − 2017Q4) US data. We collected the data on private consumption expenditure from the NIPA Table, (realized) inflation rate from the OECD, the (10 year-) expected inflation rate and expected real interest rate from the Federal Reserve Bank of Cleveland, and the remaining data from FRED Economy Data, published by the Federal Reserve Bank.

We use net national product $Y - \delta K$, net worth (households and nonprofit organizations) A, private consumption expenditure C, and capital stock K. We use the geometric mean of the monthly data of π and $r - \pi$, and directly use the annual capital stock data for every quarter of each year.

Note that the observed consumption C is not always the same as the consumption demand C^d in our model.[28] We estimate the value of the consumption demand using equation (3.1):

$$f^c = c^d / y_{di} = c / y_{di} + y_{di}^{-1} \max\{(\dot{P}/P - \pi)/\nu_P, 0\}. \tag{C.1}$$

We use the average propensity to consume f^c calculated from above equation as the explained variable. From the assumptions, we set the following equation for the OLS estimation:

$$\ln f_t^c = c_0 + c_1(r_t - \pi_t) + c_2 \ln(A_t / Y_{di\ t}) + \varepsilon_t, \tag{C.2}$$

where $\varepsilon_t \sim N(0, \sigma)$. In this equation, the rate of change in the average propensity to consume is determined by the change on the real rate of return on the safe asset and the rate of change in the asset-disposable income ratio. From the usual OLS estimation, we obtain the following table 1. Therefore, we set the following consumption demand function:

$$f^c = 0.6483 \exp(0.9044(r - \pi))((m + b + q)/y_{di})^{(0.1866)} \tag{C.3}$$

Variables	Coefficient (standard error)
Intercept	$-0.4334(0.0389)$
$r - \pi$	$0.9044(0.0938)$
$\ln(A/Y_{di})$	$0.1865(0.0201)$
$\bar{R}^2$	0.4676

Table 1: Estimation results

D Expectation parameters and cyclical dynamics

In this appendix, we check the causes of the high α (mostly adaptive expectation adjustment) in the cyclical dynamics using a numerical experiment.

First, we see that the enhanced and dampened cycle occurs as α varies. Figure D.1 shows the results of a simulations with the same parameters as example 2, excepted α.

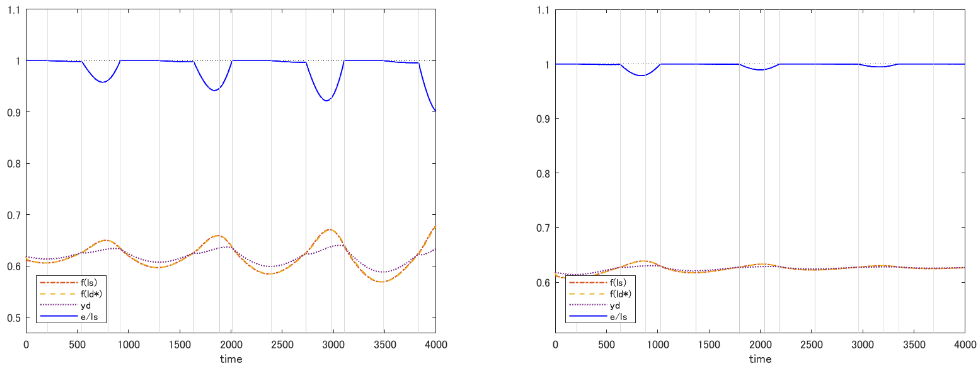

Figure D.1: Cycle with $\alpha = 0.9950$ (left) and $\alpha = 0.9850$ (right)

The figure implies that the value of α determines the stability of cycle. Therefore, we check how α affects the dynamics.

We set the same initial value as that of example 2 and simulate the dynamics varying α and β. We define the cyclical dynamics as having the same pattern of regime switching continuing to occur. We detect this using the **ydis** function of the `DISODE45` package. When the continuous regime switching is undetected, we define the dynamics as immediate convergence. Sometimes, regime switching continues and is irregular around a steady state. This occurs is because the system is discontinuous and we should exclude it from the cyclical dynamics. We therefore define this case as perturbation convergence. When the cycle occurs, we check whether the scale of cycle changes over time. When the rate of change between the maximum value of l^s in the first cycle and that in the second cycle is less than 1%,[29] we define the scale of the cycle as unchanged or persistent. When the scale of the second cycle is larger than that of the first, the cycle is enhanced. In the other case, the cycle is dampened.

Figure D.2 shows that a high (above about 0.94) value of α leads to cyclical economic dynamics. Furthermore, when $\alpha = 1$ (completely adaptive expectation), we see an enhanced cycle. Note that the stabilization effect of the value of β is ambiguous. When α is around 0.97, a low value of β stabilizes the dynamics since the cycle changes from persistent to dampened as β decreases. When α is around 0.99, by contrast, a lower β destabilizes the cycle. These results imply that the Mundel effect does not work uniformly in a disequilibrium model. This result implies the need for future research because the numerical approximation of disequilibrium dynamics is complicated and the calculation methods are currently underdeveloped.

[28] For the estimation methods, see Quandt (1988).

[29] Of course, we could use another definition for a persistent cycle, but the result is not significantly different.

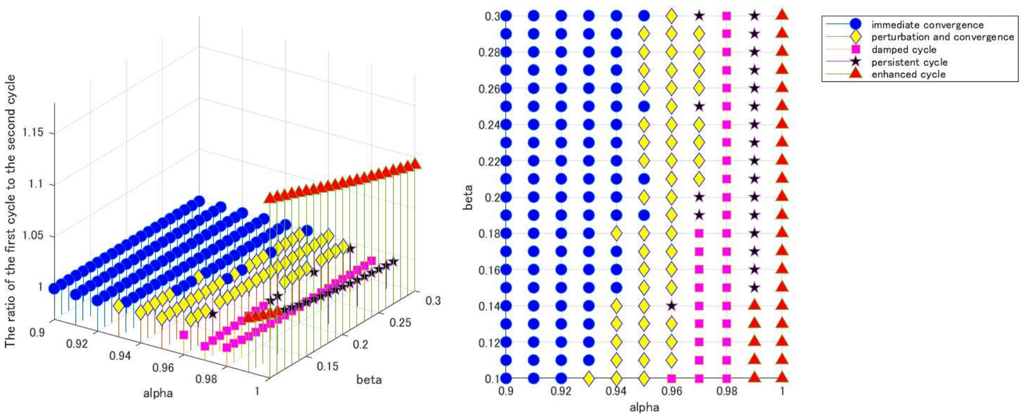

Figure D.2: Dynamic properties with α and β