Are large national debt and ultra-low inflation harmful? —— S-shape Phillips curve: the inflation-unemployment relationship of a low profit rate model

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Are large national debt and ultra-low inflation harmful?
——S-shape Phillips curve: the inflation-unemployment relationship of a low profit rate model

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Abstract: This paper, through a neo-Kaleckian model of a closed industrialized economy, shows a large scale of national debt and an ultra-low inflation rate are not dangerous but necessary if the profit rate of capitalists is low. The S-shape Phillips curve in the static analyses (in the long-run perspective then) shows, when inflation rate is low (which is called semi-classical situation), unemployment rate increases with inflation rate. In the semi-classical situation, the ratio of national debt to GDP decreases with inflation rate while deficit ratio increases with inflation rate. The dynamic analyses show, if the government can fix inflation rate on a target level, an industrialized economy can be dynamically stable.
Key words: employment, inflation, deficit, national debt, profit rate

1. Introduction

Many post-Keynesian models have shown state intervention is necessary, like Cédric Rogé (2019). For economic stability the budget deficit of a government is often an indispensable thing. But a large scale of national debt is still regarded as dangerous and harmful as far as I know. As a result, Japan, with the large ratio of national debt to GDP (larger than 100%), is seen as an example of intervention failure. Besides, the ultra-low inflation rate (lower than 1%) of Japan is also blamed by mainstream economics. However, this paper shows it is not a nightmare for an industrialized economy to have large national debt and ultra-low inflation rate.

This paper, with a post-Keynesian model (a neo-Kaleckian model to be specific), focuses on employment, inflation and money. Like Cédric Rogé (2019), the present paper pays much attention to the money's role, but differentiates by considering inflation and national debt ratio. Inflation is taken into account since this paper assumes market clearing realizes in each period like mainstream economics. For more inflation mechanism see, e.g., Lavoie (2003).

Inspired by the B-M model (Bhaduri and Marglin, 1990), a famous post-Keynesian study, this paper assumes the investment is influenced not by the ratio of profit to total capital but by the ratio of profit to total product which, for simplicity, is called "profit rate" in this paper. For a closed industrialized economy, due to lack of surplus labor, the wage level is high, leading to the low profit rate. For reviews about post-Keynesian economics see, e.g., Serrano (2018) and Lavoie (2016). The crucial role of budget deficit in macroeconomic analyses has been shown by Sraffians (e.g. Serrano, 1995) and neo-Kaleckians (e.g. Allain, 2015; Godley and Lavoie, 2007).

The inflation-unemployment relationship can be depicted by the Phillips curve. Besides the famous original Phillips curve, there are the vertical Phillips curve, representing the NAIRU (non-accelerating inflation rate of unemployment) theories, and the backward bending (C-shape) Phillips curve, representing the MURI (minimum unemployment rate of inflation) theories (Palley, 2012). However, the static analyses (in the long-run perspective then) in the present paper show the S-shape Phillips

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curve for a closed industrialized economy where the profit rate for capitalists as a whole is low. Since the government has no motive to preserve a high inflation rate, after assuming the inflation rate is low (which is called semi-classical situation in this paper), unemployment rate increases with inflation rate.

For a typical industrialized economy, in the long-run perspective the economic growth rate is the independent variable determined by the factors in supply side (Gowans, 2014). But the scale of national debt Able to be directly controlled by the government. The static analyses show, in the semi-classical situation, the ratio of national debt at the end of a period to the nominal GDP (national debt ratio) decreases with inflation rate.

So a high national debt ratio and an ultra-low inflation rate can be blameless. Moreover, the economy can be dynamically stable, and even if an equilibrium of the economy is not dynamically stable, it is dynamically saddle-point stable.

This paper finds there exist dynamically stable equilibriums with low deficit ratio, low inflation rate and high employment rate, and large ratio of national debt which is not welcomed in mainstream economics. In the paper, the deficit, or the increased money, plays the role of the non-capacity creating autonomous expenditure (for more details see, e.g., Lavoie, 2016). So this paper can be seen as a neo-Kaleckian work. Some of the famous researches in the domain are, e.g., Rowthorn (1981), Dutt (1984, 1990, 1994, 1997), Amadeo (1986), Blecker (1989), Taylor (1991), Lavoie (1992, 1995), Naastepad and Storm (2006), Stockhammer (2012), and Onaran and Galanis (2013).

The article is organized as follows. Section 2 provides the frame of the model. Section 3 gives some assumptions about money multiplier. Section 4 and section 5 respectively present the static analyses and the dynamic analyses. Section 6 discusses stagnation situation and semi-classical situation. Section 7 draws conclusions from the results.

2. A post-Keynesian model

The logic of the model is not complicated. In a closed industrialized economy, due to the small scale of surplus labor, the wage level is high and correspondingly the profit rate is low, leading to a weak investment motive. To preserve economic stability, a large scale of monetary base is necessary.

2.1 Frame of model

The production function in a closed industrialized economy is

$$Y = K^{\alpha}(AL)^{1-\alpha} \tag{1}$$

where Y is the total product, L the labor employed, A the technical index, K the capital. If employment rate and $k=K/(AL)$ are given in the long run, the economic growth rate depends on the technical progress rate (noted by g_A) and the population growth rate (noted by g_N). This paper assumes the technical progress rate and the population growth rate are both low and stable, and focuses mainly on employment. Note

$$g_K \equiv \frac{K_{+1} - K}{K}. \tag{2}$$

The subscript "+1" means the next period (similarly "-1" means the last period). Note the natural economic growth rate by

$$g_n \equiv (1+g_A)(1+g_N) - 1. \tag{3}$$

Assume at the equilibrium state, k is on an unchanged level like what the famous Kaldor facts shows, which means $g_K=g_n$.

The total money at the beginning of a period, regarded in the paper as being equivalent to the national debt (last period) of the closed economy for simplicity, is noted by M. The increased money, corresponding to the exogenous purchase, is noted by D, equivalent to the government deficit in the closed economy. Assume the increased money is launched to the market at the end of a period. The total money then is the sum of the total money last period and the increased money last period, shown as follows.

$$M = M_{-1} + D_{-1}\,. \tag{4}$$

The money growth rate then is D/M, noted by g_m. Note the price at the end of the period by p. The inflation rate is $i=p/p_{-1}-1$.

The profit of the capitalist class is

$$rY = Y - bY - wL - \delta K \tag{5}$$

where b is the tax rate, w the realized real wage, r the profit rate (like B-M model, see Bhaduri and Marglin, 1990), δ the depreciation rate.

The total product is

$$Y = bY + cY + D/p + K_{+1} - K + \delta K \tag{6}$$

where bY is the tax which is expended on the public sector's consumption at all, D is the increased money, $bY+D/p$ is the government purchase, cY is the private sector's consumption.

The income of a capitalist household can be divided into two parts, the one as the wage paid by himself for his management work and the other as the profit. At first assume the consumption of a capitalist household is equivalent to his wage. If the class mobility is considered, the consumption of the working class can be larger than the total wage of it. Because when a capitalist household becomes a worker household with a big fortune (called a pre-capitalist worker household then), its consumption is larger than its wage. Like famous Kalecki formula, this paper further assumes the consumption of working class is equivalent to the total wage of it, which means there is no class mobility and no pre-capitalist worker household either. Thus the total consumption of the capitalism economy is equivalent to the total wage.

If the class mobility is considered, the discussion might be more complicated since the consumption is larger than the total wage. It is possible that the consumption of the pre-capitalist worker households plays the role of non-capacity creating autonomous expenditure and no extra source of profits is needed for preserving economic stability. However, Yang (2019) shows there is little difference relating to the following analyses after taking class mobility into account, since the appearance of new products due to product innovations can drain the wealth of the pre-capitalist worker households faster.

Total profit is the sum of the quantity of increased capital and the quantity of increased money which is the budget deficit of the government in the closed economy.

The wage includes two parts: the one obtained by the workers and the other obtained by the capitalists for their management works. According to the assumptions above, obtain

$$wL = cY\,. \tag{7}$$

2.2 Realized real wage and Profit rate

It is easy to obtain the nominal profit is

$$rpY = pY - bpY - WL - \delta pK \tag{8}$$

or

$$rpY = D + p(K_{+1} - K)\ . \tag{9}$$

of which W is the nominal wage decided at the beginning of a period.

Name w/A the unit wage realized, noted by w_{er}. The expected unit wage, w_e, increases with employment rate e, of which the function is shown as below.

$$w_e = w_e(e)\ . \tag{10}$$

Note unemployment rate by u=1-e. For simplicity, in the dynamic analysis part, assume $ew_e'(e)/w_e(e)$ (the reciprocal of wage elasticity of labor supply) is small, which is named reasonable wage assumption. It is not strange to assume the expected unit wage is elastic when employment rate is very close to 1 and inelastic otherwise. In other words, wage elasticity of labor supply is large unless u is very close to 0.

The expected real wage is Aw_e. The expected price level of the current period, predicted through the price level and the inflation rate last period, is p_e=p_{-1}(1+i_{-1}). The nominal wage, determined at the beginning of the current period, is

$$W = p_{-1}(1+i_{-1})Aw_e(e)\ , \tag{11}$$

The ratio of total wage to GDP is

$$\frac{WL}{pY} = \frac{1+i_{-1}}{1+i}\frac{ALw_e(e)}{Y} = \frac{1+i_{-1}}{1+i}\frac{w_e(e)}{k^{\alpha}}\ , \tag{12}$$

The realized real wage is

$$w = \frac{p_{-1}(1+i_{-1})Aw_e(e)}{p} = = \frac{1+i_{-1}}{1+i}Aw_e(e)\ , \tag{13}$$

The realized unit wage is

$$w_{er} = \frac{p_{-1}(1+i_{-1})w_e(e)}{p} = \frac{1+i_{-1}}{1+i}w_e(e)\ , \tag{14}$$

Capitalists decide the amount of capital and labor with the cost minimization strategy. The cost of the capital is equivalent to the depreciation part of it. Then it is easy to obtain

$$\frac{\delta K}{w_e(e)AL} = \frac{\alpha}{1-\alpha}\ , \tag{15}$$

or

$$k = \frac{\alpha w_e(e)}{\delta(1-\alpha)}\ . \tag{16}$$

It is obvious that k increases with e. Then the wage ratio is

$$\frac{WL}{pY} = \frac{1+i_{-1}}{1+i}\frac{\delta(1-\alpha)}{\alpha}k^{1-\alpha} = \frac{1+i_{-1}}{1+i}\frac{[\delta(1-\alpha)]^{\alpha}[w_e(e)]^{1-\alpha}}{\alpha^{\alpha}}\ . \tag{17}$$

The profit rate is

$$r = 1 - b - \left(1 + \frac{1+i_{-1}}{1+i}\frac{1-\alpha}{\alpha}\right)\delta k^{1-\alpha}\ , \tag{18}$$

or

$$r = \frac{D}{pY} + g_K k^{1-\alpha}\ . \tag{19}$$

In a closed industrialized economy, the scale of rural surplus labor force is very small, meaning a large e. Then the wage level is high and the capital-output ratio is large. As a result, its profit rate is low. Besides, given r is 0, it is obvious that the equilibrium employment rate decreases with b, the tax rate.

According to the details above, r and e (or k) change inversely while r and u change in the same direction in the equilibrium state in which i=i_{-1} always holds. When u is large (in the equilibrium state), meaning the idle labor scale is large, the capitalists can screw down the wage level, leaving a high profit rate.

The quantity of the employed labor is

$$L = \frac{(1-\alpha)\delta K}{\alpha A w_e(e)}. \tag{20}$$

Then obtain

$$e = \frac{L}{N} = \frac{(1-\alpha)\delta K}{\alpha A N w_e(e)} \tag{21}$$

and

$$e / e_{-1} = \frac{1+g_{K-1}}{(1+g_A)(1+g_N)} \frac{w_e(e_{-1})}{w_e(e)}. \tag{22}$$

Note

$$e = f_{-w}(e w_e(e)). \tag{23}$$

Obtain

$$e = f_{-w}\left(\frac{1+g_{K-1}}{(1+g_A)(1+g_N)} e_{-1} w_e(e_{-1})\right). \tag{24}$$

3. Money multiplier

3.1 Endogenous purchase

Given the total money, the enterprises in the closed industrialized economy choose to spend a part of it on production and save the rest, which is similar to the deposit reserves of banks. The workers have no money at the beginning of a period and cost all of their wages, leaving no money leakage. Then the "voluntary purchase", or the endogenous purchase *pY-D*, is a function of *M*, which also depends on the profit rate and the inflation rate, defined below.

$$pY - D = h(r, i, M),\ h_1 > 0,\ h_2 > 0,\ h_3 > 0. \tag{25}$$

Assume *h*(•), called purchase function in this paper, is first order homogeneous on *M* for simplicity, then define the money multiplier as

$$h_M(r,i) \equiv \frac{h(r,i,M)}{M},\ h_{M1} > 0,\ h_{M2} > 0. \tag{26}$$

Then money multiplier depends on the inflation rate and the employment rate (or the capital-output ratio) as follows.

$$h_M(r,i) = h_M\left(1-b-\left(1+\frac{1+i_{-1}}{1+i}\frac{1-\alpha}{\alpha}\right)\delta k^{1-\alpha}, i\right) = h_M\left(1-b-\left(1+\frac{1+i_{-1}}{1+i}\frac{1-\alpha}{\alpha}\right)\delta\left[\frac{\alpha w_e(e)}{\delta(1-\alpha)}\right]^{1-\alpha}, i\right). \tag{27}$$

It is not strange to assume, for the equilibrium with low profit rate and low inflation rate, h_M is very small (smaller than 1).

Furthermore assume, given *r* is small, h_{M2} increases at first and then decreases with *i*, and for *i* being very small or high enough h_{M2} is so small (or $h_M/h_{M2}-i$ is so large) that $h_M/h_{M2}-i-g_n/(1+g_n)$ is positive.

The former part of the assumption means, given *r*, the multiplier function being convex when *i* is low but concave when *i* is high. See figure 1. In the coordinate system of *i* and h_M respectively as x-axis and y-axis, the thick S-shape curve shows the multiplier as the function depending on inflation rate given r. Name the point ($-\frac{g_n}{1+g_n}$,0) threshold point. The point ($i-h_M \Big/ \frac{\partial h_M}{\partial i}$,0) is the intersection of the tangent line of the multiplier function curve and the x-axis. The latter part of the assumption means for i being very small or high enough, the intersection is on the left side of the threshold point.

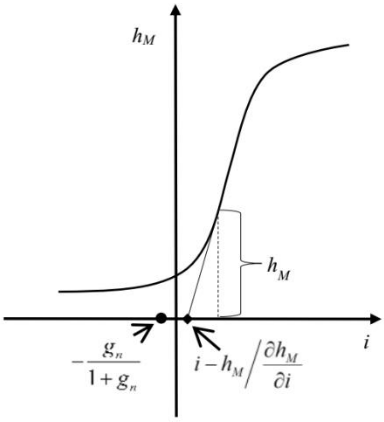

Figure 1.

This assumption can be depicted strictly as follows.

Given r (which is small), there exist $i_0>0$ and $i_1>0$ which make the following condition hold:

$$i-h_M \Big/ \frac{\partial h_M}{\partial i} < -\frac{g_n}{1+g_n} \quad \text{for any } i<i_0 \text{ or } i>i_1.$$

According to the assumption above, the intersection is on the left side of the threshold point when i is 0. And as inflation rate increases, it shifts towards right at first and then towards left.

This assumption, named inflation-multiplier assumption, plays a crucial rule in the model. It means as the money multiplier rises the sensitivity of money multiplier to the change of inflation rate increases at first and then decreases, leaving the sensitivity is small when inflation rate is very low or high enough.

The sensitivity is small when inflation rate is very high because enterprises must hold some cash or feasible money at any time and cannot react sensitively to the change of inflation rate. The sensitivity is small when inflation rate is very low is easier to explain. There is a floor of the multiplier function. At first, money multiplier must be nonnegative, meaning at least zero is a potential floor level. Moreover, because of adventurous spirit, some entrepreneurs always conduct positive investments of which the value sometimes might be just equivalent to a little fraction of the total money they hold. Given profit rate is small, if inflation rate is also low, the investment motive of entrepreneurs is weak and mainly depends on the adventurous spirit. Thus the money multiplier is insensitive to the change of inflation rate.

In a regular situation, given r is small, the intersection, as inflation rate increases, is on the left side of the threshold point at first, then on the right side for some time, and on the left side again at last.

All of the regular conditions can be depicted strictly as follows.

For a small r, there exist $i_0(r)$ and $i_1(r)$ and the following conditions hold:

1) $0<i_0(r)<i_1(r)$;
2) i is nonnegative and for any $i>0$, $i-h_M \Big/ \frac{\partial h_M}{\partial i} < -\frac{g_n}{1+g_n}$ if $i<i_0(r)$ or $i>i_1(r)$;

   $i-h_M \Big/ \frac{\partial h_M}{\partial i} > -\frac{g_n}{1+g_n}$ if $i_0(r)<i<i_1(r)$;

3) $i_0(r)$ and $i_1(r)$ are monotonic decreasing functions;

4) for i=0 $i - h_M \Big/ \frac{\partial h_M}{\partial i} < -\frac{g_n}{1+g_n}$ holds.

Condition 3) is reasonable since a larger r can lead to a larger money multiplier and r has some substitution effect on i. Besides, the regular conditions mean r is small enough, otherwise it is possible that $i_0(r)$ is not positive, which matters in the analyses below.

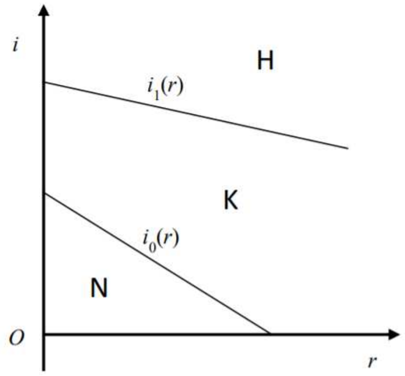

Figure 2.

In the coordinate system of r and i respectively as x-axis and y-axis as figure 2 shows, two curves (depicted by two straight lines for simplicity) representing respectively $i_0(r)$ and $i_1(r)$ divide the first quadrant into three domains, noted by "S", "K" and "H", respectively representing three situations: semi-classical situation, Keynesian situation and hyperinflation situation. It is obvious that for a large r the semi-classical situation disappears. That is why this paper emphasizes the low profit rate mode.

Of course, it is possible that the intersection is always on the left side of the threshold point (meaning unemployment rate and inflation rate always change in the same direction according to the conclusions of the next section). This may occur when the multiplier is always insensitive enough to the changes of inflation rate. This situation is not taken into account in following analyses since the inflation-unemployment relationship in the situation is just the same with that in the semi-classical situation of which the details are shown in section 4 and section 6.

Besides, in the dynamic stability analysis part, assume h_M/h_{M1} is large for small r given i is small. It is named profit-multiplier assumption there. The background of it is like what is mentioned about inflation's influence on money multiplier above.

3.2 Dynamic system

The total capital at the next period is as follows.

$$pK_{+1} = pK - \delta pK - WL - bpY + Mh_M(r,i) \,. \tag{28}$$

Then obtain the investment equation

$$\frac{K_{+1}-K}{K}\frac{K}{Y} = \frac{Mh_M(r,i)}{pY} - [1+\frac{(1-\alpha)}{\alpha}\frac{1+i_{-1}}{1+i}]\delta\frac{K}{Y} - b \,, \tag{29}$$

or

$$g_K k^{1-\alpha} = \frac{h_M(r,i)}{g_m + h_M(r,i)} - [1+\frac{(1-\alpha)}{\alpha}\frac{1+i_{-1}}{1+i}]\delta k^{1-\alpha} - b \,, \tag{30}$$

according to the price equation

$$p = \frac{Mg_m + Mh_M(r,i)}{Y}. \tag{31}$$

It is also easy to obtain the inflation equation

$$1+i = \frac{1+g_{m-1}}{(1+g_{K-1})^{\alpha}[(1+g_A)(1+g_N)e/e_{-1}]^{1-\alpha}} \frac{g_m + h_M(r,i)}{g_{m-1} + h_M(r_{-1}, i_{-1})}. \tag{32}$$

Note the total quantity of money, or national debt ratio, by

$$m \equiv \frac{D+M}{pY} = \frac{D+M}{D+Mh_M(r,i)} = \frac{g_m + 1}{g_m + h_M(r,i)}. \tag{33}$$

Note the increased quantity of money, or deficit ratio, by

$$d \equiv \frac{D}{pY} = \frac{D}{D+Mh_M(r,i)} = \frac{g_m}{g_m + h_M(r,i)}. \tag{34}$$

It is easy to obtain

$$m > 1 \Leftrightarrow h_M(r,i) < 1. \tag{35}$$

At last obtain the whole dynamic system:

$$r = 1 - b - [1 + \frac{1-\alpha}{\alpha}\frac{1+i_{-1}}{1+i}]\delta[\frac{\alpha w_e(e)}{\delta(1-\alpha)}]^{1-\alpha}, \tag{36}$$

$$e/e_{-1} = \frac{1+g_{K-1}}{1+g_n}\frac{w_e(e_{-1})}{w_e(e)}, \tag{37}$$

$$g_K[\frac{\alpha w_e(e)}{\delta(1-\alpha)}]^{1-\alpha} = \frac{h_M(r,i)}{g_m + h_M(r,i)} - b - [1 + \frac{(1-\alpha)}{\alpha}\frac{1+i_{-1}}{1+i}]\delta[\frac{\alpha w_e(e)}{\delta(1-\alpha)}]^{1-\alpha}, \tag{38}$$

$$i = \frac{1+g_{m-1}}{(1+g_{K-1})^{\alpha}[(1+g_n)e/e_{-1}]^{1-\alpha}} \frac{g_m + h_M(r,i)}{g_{m-1} + h_M(r_{-1}, i_{-1})} - 1. \tag{39}$$

At the equilibrium state, it is easy to obtain

$$g_n k^{1-\alpha} + b + \frac{\delta k^{1-\alpha}}{\alpha} = \frac{h_M}{g_m + h_M} \tag{40}$$

The equation above is named balance equation in the paper which plays a crucial role in the following analyses. The balance equation just shows such a fact in a market: the price level for the government purchase and the autonomous purchase is the same one at any period, which is also the reason for its name.

4. Static analyses

Because of low profit rate, capitalists are not sensitive to inflation rate when it is low, leading to the crowding-out effect of a rise of inflation rate being strong. As a result, employment rate decreases with inflation rate. The details are presented below.

4.1 S-shape Phillips curve

The inflation equation, at the equilibrium state, is as follows

$$g_m = (1+g_n)(1+i) - 1. \tag{41}$$

Money growth rate is 0 given g_n=0 and i=0.

Define

$$q_g \equiv \frac{h_M}{g_m} = \frac{h_M}{(1+g_n)(1+i)-1} = \frac{1}{1+g_n}\frac{h_M}{i+1-1/(1+g_n)}. \tag{42}$$

It is easy to know q_g increases with r and

$$\frac{\partial q_g}{\partial i} < 0 \Leftrightarrow h_M \Big/ \frac{\partial h_M}{\partial i} - i - \frac{g_n}{1+g_n} > 0 . \tag{43}$$

According to the balance equation, obtain

$$r(1+\frac{\alpha}{\delta} g_n) - \frac{1}{1+q_g} = g_n(1-b)\frac{\alpha}{\delta} . \tag{44}$$

It is easy to know given i, r decreases with b.

Given g_n, according to the equation above, combined with the relationship between r and e, it is easy to obtain

$$\frac{\partial e}{\partial i} < 0 \Leftrightarrow \frac{\partial r}{\partial i} > 0 \Leftrightarrow \frac{\partial q_g}{\partial i} < 0 \Leftrightarrow h_M \Big/ \frac{\partial h_M}{\partial i} - i - \frac{g_n}{1+g_n} > 0 . \tag{45}$$

According to the inflation-multiplier assumption, for i being very small or high enough, equilibrium inflation rate and equilibrium employment rate change inversely since h_M/h_{M2}-i-$g_n/(1+ g_n)$ is positive.

If the regular conditions hold, some interesting conclusions can also be obtained.

In the semi-classical situation and hyperinflation situation, profit rate and inflation rate change in the same direction, while in the Keynesian situation, they change inversely, as figure 3 shows.

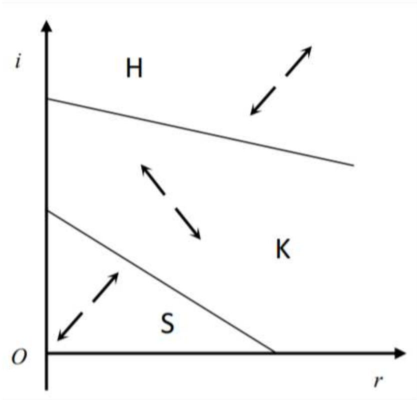

Figure 3.

Based on the regular conditions, it is easy to obtain the inflation-profit relationship depicted in figure 4. As the equilibrium inflation rate rises, the equilibrium profit rate increases at first, decreases then and increases at last.

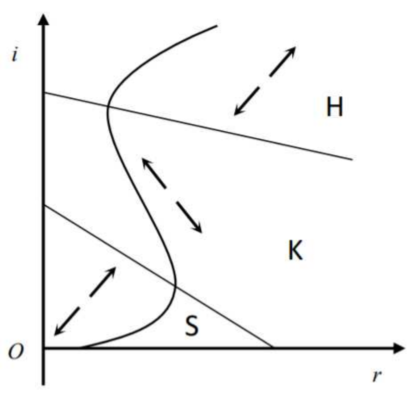

Figure 4.

After taking the regular condition 4) out of consideration, the inflation-profit relationship as figure 5 shows is also possible. For example, since r decreases with b given i, for a small tax rate, the profit rate can be large and the semi-classical situation does not appear. Thus as the equilibrium inflation rate rises, the equilibrium profit rate decreases at first and increases at last. This kind of inflation-profit relationship is not considered, in the following analyses since this paper focuses on the low profit rate mode. Besides, the macroscopic tax rate of an industrialized economy is often high, leaving a low social average profit rate.

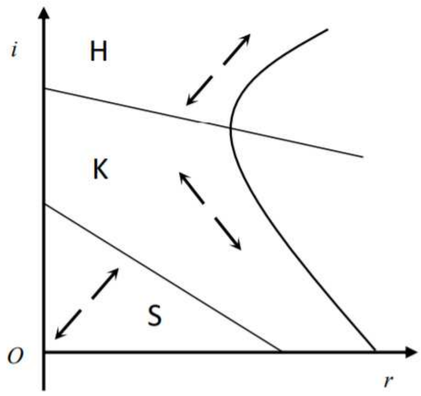

Figure 5.

Because r and u change in the same direction in the equilibrium state, the inflation-unemployment relationship can easily be obtained according to the inflation-profit relationship, as figure 6 shows. As inflation rate rises, unemployment rate increases, then decreases and at last increases. Naturally as the equilibrium inflation rate increases, the equilibrium employment rate decreases, then increases and at last decreases.

Compared with the famous original Phillips curve, the inflation-unemployment relationship of a closed industrialized economy can be depicted by the S-shape curve in figure 6. Only in Keynesian situation, do unemployment rate and inflation rate change inversely like what the original Phillips curve shows. In semi-classical situation and hyperinflation situation, unemployment rate and inflation rate change in the same direction.

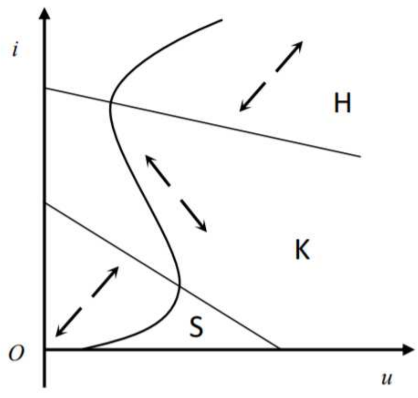

Figure 6.

In a closed industrialized economy, the profit rate is low. Moreover, it is obvious that the government has no incentive to preserve a high inflation level except that there is a big problem about its fiscal revenue. For simplicity assume inflation rate is low in the following analyses. That means the industrialized economy is in the semi-classical situation.

Since the inflation rate is low, the employment rate decreases with the inflation rate. And the unemployment rate and the profit rate both increases with inflation rate. As the inflation rate of the industrialized economy rises, behind which are a larger government deficit and a larger government purchase than before, capitalists are not sensitive enough for correspondingly increasing the scale of voluntary purchase. Thus the crowding-out effect appears, leading to the unemployment rate increasing.

4.2 Money's role

In an industrialized economy, profit rate is low. Since inflation rate is also small according to the assumption at the end of the last part, the money multiplier is small. For simplicity assume the money multiplier is smaller than 1.

Then it is not difficult to show, as equilibrium inflation rate increases, deficit ratio increases and national debt ratio decreases.

4.2.1 Deficit ratio—increased money quantity

The following analysis shows deficit ratio increases with inflation rate. Since unemployment rate increased with inflation rate, at the equilibrium state a high deficit ratio—the quantity of increased money—can be regarded as a negative thing.

The increased money quantity is

$$d = 1 - g_n k^{1-\alpha} - b - \frac{\delta k^{1-\alpha}}{\alpha} = r - \frac{\alpha}{\delta} g_n (1 - r - b) \ . \tag{46}$$

Given g_n, since r increases with i, it is easy to know d increases with i.

Thus equilibrium inflation rate and increased money quantity are positively related.

Besides, it is easy to know $D = 0$ and $d = 0$ given $g_m = 0$ according to their definitions. Thus given $g_n = 0$ and $i = 0$, which means $g_m = 0$, obtain

$$r = d + g_n k^{1-\alpha} = 0, \ k^{1-\alpha} = \frac{\alpha(1-b)}{\delta}, \ \frac{WL}{pY} = (1-\alpha)(1-b) \ \text{and} \ w_e(e) = (1-\alpha)(1-b)[\frac{\alpha(1-b)}{\delta}]^{\frac{\alpha}{1-\alpha}} \ .$$

4.2.2 National debt ratio—total money quantity

The national debt ratio, different from deficit ratio, decreases with inflation rate. Since unemployment rate increases with inflation rate, the big scale of national debt is a necessary and positive thing for an industrialized economy.

It is easy to know $m = 1/h_M$ given $g_m = 0$. Thus given $g_n = 0$ and $i = 0$, which means $g_m = 0$, m is very large since h_M is small due to low inflation rate (being 0) and low profit rate (being 0).

According to the definition of m, obtain

$$1 + g_n = \frac{m(1-h_M)}{(m-1)(1+i)} \ . \tag{47}$$

Given g_n, it is easy to obtain $h_M < 1 \Leftrightarrow m > 1 \Rightarrow \frac{\partial m}{\partial i} < 0$.

According to the assumption that the money multiplier is smaller than 1, it is obvious that equilibrium inflation rate and national debt ratio (total money quantity) are negatively related.

5. Dynamic analyses

The increased quantity and the total quantity of money are mainly determined, exogenously, by the government (and the central bank). So it is necessary to "image" a rule of money supply for the dynamic analyses.

Assume the money supply policy is to maintain the inflation rate on the target level. It is not only for simplicity. On the one hand, the government has much incentive to preserve price stability. On the other hand, this rule is actually followed in many economies. Since the inflation rate must be maintained at last on a desirable level after all, it is reasonable to consider the scenario where it is always fixed on a target level. Besides, this rule is operable. Even taking international trade into consideration, the government can adjust its deficit scale and control the money growth rate, to obtain the target inflation level.

Note the (nonnegative) target level of inflation rate by i^*, which is the inflation rate in each period. Then rewrite the dynamic system. See appendix A1 for detail.

Note

$$r = f_r(e) \equiv 1 - b - \frac{\delta}{\alpha}[\frac{\alpha w_e(e)}{\delta(1-\alpha)}]^{1-\alpha}. \tag{48}$$

Consider a system containing two variables: employment rate and capital growth rate.

Note

$$e = f_e(e_{-1}, g_{K-1}) \equiv f_{-w}(\frac{1+g_{K-1}}{1+g_n} e_{-1} w_e(e_{-1})), \tag{49}$$

It is easy to obtain

$$g_m = f_{gm}(e, g_K) \equiv h_M(f_r(e), i^*)\{\frac{1}{g_K[\frac{\alpha w_e(e)}{\delta(1-\alpha)}]^{1-\alpha} + \frac{\delta}{\alpha}[\frac{\alpha w_e(e)}{\delta(1-\alpha)}]^{1-\alpha} + b} - 1\} \tag{50}$$

Then note

$$\begin{aligned} g_K &= f_{gK}(e_{-1}, g_{K-1}) \\ &\equiv \frac{\{(g_{K-1} + \frac{\delta}{\alpha})[\frac{\alpha w_e(e_{-1})}{\delta(1-\alpha)}]^{1-\alpha} + b\}\frac{1+f_{gm}(e_{-1}, g_{K-1})}{1+i^*}}{(1+g_{K-1})^{\alpha}[(1+g_n)\frac{f_e(e_{-1}, g_{K-1})}{e_{-1}}]^{1-\alpha}} \frac{\frac{h_M(f_r(f_e(e_{-1}, g_{K-1})), i^*)}{h_M(f_r(e_{-1}), i^*)}}{[\frac{\alpha w_e(f_e(e_{-1}, g_{K-1}))}{\delta(1-\alpha)}]^{1-\alpha}} \\ &\quad - \frac{b}{[\frac{\alpha w_e(f_e(e_{-1}, g_{K-1}))}{\delta(1-\alpha)}]^{1-\alpha}} - \frac{\delta}{\alpha} \end{aligned} \tag{51}$$

The dynamic system's Jacobian matrix is

$$J_{(e, g_K)} = \begin{pmatrix} f_{e1}(e, g_K) & f_{e2}(e, g_K) \\ f_{gK1}(e, g_K) & f_{gK2}(e, g_K) \end{pmatrix} = \begin{pmatrix} \frac{\partial f_e(e, g_K)}{\partial e} & \frac{\partial f_e(e, g_K)}{\partial g_K} \\ \frac{\partial f_{gK}(e, g_K)}{\partial e} & \frac{\partial f_{gK}(e, g_K)}{\partial g_K} \end{pmatrix} \tag{52}$$

and then note the determinant of the matrix by

$$\mathrm{D}(J) = |J_{(e, g_K)}| = \begin{vmatrix} f_{e1}(e, g_K) & f_{e2}(e, g_K) \\ f_{gK1}(e, g_K) & f_{gK2}(e, g_K) \end{vmatrix} \tag{53}$$

and the trace of it by

$$\mathrm{Tr}(J) = f_{e1}(e, g_K) + f_{gK2}(e, g_K) \tag{54}$$

of which e and g_K refer respectively two variables' equilibrium values.

It is easy to know $\mathrm{D}(J) < 1$ and $\mathrm{D}(J) > \mathrm{Tr}(J) - 1$. For the details of the proof see A2 in appendixes. Thus an equilibrium in the dynamic system is dynamically stable or dynamically saddle-point stable. Note that neither of the reasonable wage assumption nor the profit-multiplier assumption is necessary for this conclusion.

The value of D(J)+Tr(J)+1, if the reasonable wage assumption and the profit-multiplier assumption hold, is positive when the total money quantity is large. For details see A2 in appendixes. When it is positive, the corresponding equilibrium is dynamically stable. In comparison, when it is negative, the corresponding equilibrium is a saddle point and only for some specific initial values of employment rate and capital growth rate can the equilibrium realize autonomously.

The conclusion can provide a special perspective to review the economic stability in some developed economies. However, since more works are needed relating to the reasonability of the assumptions and it might be difficult for a government to fix the inflation rate on a level all the time, it is necessary to be cautious in applying this conclusion.

It is possible that an equilibrium is dynamically saddle-point stable. To preserve economic stability, given the employment rate of a period, it is necessary to put the capital growth rate of the period on a specific level. That means the government needs to focus on not only the target inflation rate, but also the target capital growth rate. If the adjustments are conducted by the central bank, these two targets can be obtained through open market operations, reserve requirement ratio changes and so on.

6. Other discussions

If, in an industrialized economy, the equilibrium with economic growth rate, inflation rate and then nominal growth rate all being 0 realizes, this paper says that the economy is in the stagnation situation. It has been shown that, in the stagnation situation, deficit ratio, profit rate and money growth rate are all 0. Unemployment rate is low if the tax rate is not very high. Besides, enlarging the money scale is unnecessary. It seems to be in the world like what classical economics show, but an important and crucial difference is the total money quantity is very large since the money multiplier is very low due to low inflation rate (being 0) and low profit rate (being 0).

In the semi-classical situation, as the expansion of the stagnation situation, like what the classical economics show, equilibrium unemployment rate, economic growth rate and nominal growth rate are all low. Moreover, money growth rate, profit rate, deficit ratio and money multiplier are also low. But national debt ratio, or total money quantity, is large, which is why the situation is called "semi-classical situation".

In the semi-classical situation, the following conclusions (obtained in the parts above) hold. Unemployment rate increases with inflation rate. This conclusion suggests the low inflation policy is blameless and necessary. On the other hand, given that profit rate and inflation rate are both low, there are $h_M < 1$ and then $m > 1$. Thus total money quantity decreases with inflation rate. The conclusion suggests the money quantity should be large. Besides, government budget deficit and unemployment rate change in the same direction. The conclusion suggests deficit ratio should be low.

Thus except that total money quantity should be large, other conclusions are similar with opinions of classical or neo classical economics—government deficit should be small and inflation rate should be low. The conclusions above can be applied in analyzing what happen in developed economies where the real economic growth rate is restrained by low population growth rate and low technological progress rate.

The large scale of national debt is blameless and necessary when faced with low profit rate. Furthermore, a low profit rate, due to high wage level, is not a dangerous thing for the social as a whole. And the high wage level, due to high employment rate and low ratio of idle labor, is also blameless.

7. Conclusions

This paper mainly focuses on the relationship between unemployment and inflation in a closed industrialized economy through a neo-Kaleckian model. If the profit rate of capitalists is low, the static analyses obtains the S-shape Phillips curve which means, in the long-run perspective, unemployment rate increases with inflation rate when inflation rate is low (which is called semi-classical situation in this paper). So an ultra-low inflation rate is not harmful since the trade-off relationship between unemployment and inflation does not exist in the semi-classical situation.

In the semi-classical situation, deficit ratio, as the quantity of increased money, increases with inflation rate but national debt ratio, as the quantity of total money, decreases with inflation rate. Thus a high national debt ratio has positive influence on both employment and price stability. The large scale of national debt is not dangerous but necessary.

Relating to dynamic stability, discussed are the results of the rule of inflation rate being fixed on its target level. Equilibriums are dynamically stable or dynamically saddle-point stable. For an industrialized economy, an equilibrium can be dynamically stable, but this conclusion should be applied prudently since strict conditions are needed.

Appendixes

A1. After replacing the inflation rate in each period by i^*, the dynamic system can be rewritten as follows.

Rewrite the dynamic system in section 3 and obtain

$$r = 1 - b - \frac{\delta}{\alpha}[\frac{\alpha w_e(e)}{\delta(1-\alpha)}]^{1-\alpha}, \tag{55}$$

$$e w_e(e) = \frac{1+g_{K-1}}{1+g_n} e_{-1} w_e(e_{-1}), \tag{56}$$

$$g_K[\frac{\alpha w_e(e)}{\delta(1-\alpha)}]^{1-\alpha} = \frac{h_M(r,i^*)}{g_m + h_M(r,i^*)} - b - \frac{\delta}{\alpha}[\frac{\alpha w_e(e)}{\delta(1-\alpha)}]^{1-\alpha}, \tag{57}$$

and

$$i^* = \frac{1+g_{m-1}}{(1+g_{K-1})^{\alpha}[(1+g_n)e/e_{-1}]^{1-\alpha}} \frac{g_m + h_M(r,i^*)}{g_{m-1} + h_M(r_{-1},i^*)} - 1. \tag{58}$$

A2. Proof of $\mathrm{D}(J) < 1$ and $\mathrm{D}(J) > \mathrm{Tr}(J) - 1$.

Proof. For classifying the variables at the last period, at the current period and those at equilibrium state, symbolize each variable at equilibrium state by an asterisk.

It is easy to know $f_{e1}>0$ and $f_{e2}>0$.

It is easy to know $f_{gm1}<0$ and $f_{gm2}<0$.

It is easy to obtain

$$\begin{aligned} f_{e1}(e_{-1}, g_{K-1}) &= \frac{\partial f_e(e_{-1}, g_{K-1})}{\partial e_{-1}} \\ &= f'_{-w}(\frac{1+g_{K-1}}{1+g_n} e_{-1} w_e(e_{-1})) \frac{1+g_{K-1}}{1+g_n}[w_e(e_{-1}) + e_{-1} w'_e(e_{-1})] \end{aligned} \tag{59}$$

And

$$f_{e2}(e_{-1}, g_{K-1}) = \frac{\partial f_e(e_{-1}, g_{K-1})}{\partial g_{K-1}} = f'_{-w}(\frac{1+g_{K-1}}{1+g_n} e_{-1} w_e(e_{-1})) \frac{e_{-1} w_e(e_{-1})}{1+g_n}. \tag{60}$$

Then obtain

$$f_{e1}(e^*, g_K^*) = f'_{-w}(e^* w_e(e^*))[w_e(e^*) + e^* w'_e(e^*)], \tag{61}$$

and

$$f_{e2}(e^*, g_K^*) = f'_{-w}(e^* w_e(e^*)) \frac{e^* w_e(e^*)}{1+g_n}. \tag{62}$$

According to

$$f'_{-w}(e^* w_e(e^*)) = \frac{1}{w_e(e^*) + e^* w'_e(e^*)}, \tag{63}$$

obtain

$$f_{e1}(e^*, g_K^*) = 1, \tag{64}$$

and

$$f_{e2}(e^*, g_K^*) = \frac{1}{w_e(e^*) + e^* w'_e(e^*)} \frac{e^* w_e(e^*)}{1+g_n}. \tag{65}$$

It is easy to obtain

$$f_{gm1}(e_{-1}, g_{K-1}) = \frac{\partial f_{gm}(e_{-1}, g_{K-1})}{\partial e_{-1}}$$
$$= h_{M1}(f_r(e_{-1}), i^*) f_r'(e_{-1}) \{ \frac{1}{g_{K-1}[\frac{\alpha w_e(e_{-1})}{\delta(1-\alpha)}]^{1-\alpha} + \frac{\delta}{\alpha}[\frac{\alpha w_e(e_{-1})}{\delta(1-\alpha)}]^{1-\alpha} + b} - 1 \} \tag{66}$$
$$-h_M(f_r(e_{-1}), i^*) \frac{[g_{K-1} + \frac{\delta}{\alpha}][\frac{\alpha w_e(e_{-1})}{\delta(1-\alpha)}]^{1-\alpha}(1-\alpha)\frac{w_e'(e_{-1})}{w_e(e_{-1})}}{\{g_{K-1}[\frac{\alpha w_e(e_{-1})}{\delta(1-\alpha)}]^{1-\alpha} + \frac{\delta}{\alpha}[\frac{\alpha w_e(e_{-1})}{\delta(1-\alpha)}]^{1-\alpha} + b\}^2}$$

And

$$f_{gm2}(e_{-1}, g_{K-1}) = \frac{\partial f_{gm}(e_{-1}, g_{K-1})}{\partial g_{K-1}} = \frac{-[\frac{\alpha w_e(e_{-1})}{\delta(1-\alpha)}]^{1-\alpha} h_M(f_r(e_{-1}), i^*)}{\{g_{K-1}[\frac{\alpha w_e(e_{-1})}{\delta(1-\alpha)}]^{1-\alpha} + \frac{\delta}{\alpha}[\frac{\alpha w_e(e_{-1})}{\delta(1-\alpha)}]^{1-\alpha} + b\}^2}. \tag{67}$$

Then obtain

$$f_{gm1}(e^*, g_K^*) = h_{M1}(f_r(e^*), i^*) f_r'(e^*) \{ \frac{1}{g_K^*[\frac{\alpha w_e(e^*)}{\delta(1-\alpha)}]^{1-\alpha} + \frac{\delta}{\alpha}[\frac{\alpha w_e(e^*)}{\delta(1-\alpha)}]^{1-\alpha} + b} - 1 \}$$
$$-h_M(f_r(e^*), i^*) \frac{[g_K^* + \frac{\delta}{\alpha}][\frac{\alpha w_e(e^*)}{\delta(1-\alpha)}]^{1-\alpha}(1-\alpha)\frac{w_e'(e^*)}{w_e(e^*)}}{\{g_K^*[\frac{\alpha w_e(e^*)}{\delta(1-\alpha)}]^{1-\alpha} + \frac{\delta}{\alpha}[\frac{\alpha w_e(e^*)}{\delta(1-\alpha)}]^{1-\alpha} + b\}^2} \tag{68}$$

and

$$f_{gm2}(e^*, g_K^*) = \frac{-[\frac{\alpha w_e(e^*)}{\delta(1-\alpha)}]^{1-\alpha} h_M(f_r(e^*), i^*)}{\{g_K^*[\frac{\alpha w_e(e^*)}{\delta(1-\alpha)}]^{1-\alpha} + \frac{\delta}{\alpha}[\frac{\alpha w_e(e^*)}{\delta(1-\alpha)}]^{1-\alpha} + b\}^2}. \tag{69}$$

It is easy to obtain

$$f_{gK1}(e_{-1}, g_{K-1})$$
$$= \frac{\{(g_{K-1} + \frac{\delta}{\alpha})[\frac{\alpha w_e(e_{-1})}{\delta(1-\alpha)}]^{1-\alpha} + b\}\frac{1 + f_{gm}(e_{-1}, g_{K-1})}{1+i^*}}{(1+g_{K-1})^{\alpha}[(1+g_n)\frac{f_e(e_{-1}, g_{K-1})}{e_{-1}}]^{1-\alpha}} \frac{\frac{h_M(f_r(f_e(e_{-1}, g_{K-1})), i^*)}{h_M(f_r(e_{-1}), i^*)}}{[\frac{\alpha w_e(f_e(e_{-1}, g_{K-1}))}{\delta(1-\alpha)}]^{1-\alpha}}$$
$$\{ \frac{(g_{K-1} + \frac{\delta}{\alpha})(1-\alpha)[\frac{\alpha w_e(e_{-1})}{\delta(1-\alpha)}]^{1-\alpha}\frac{w_e'(e_{-1})}{w_e(e_{-1})}}{(g_{K-1} + \frac{\delta}{\alpha})[\frac{\alpha w_e(e_{-1})}{\delta(1-\alpha)}]^{1-\alpha} + b} + \frac{f_{gm1}(e_{-1}, g_{K-1})}{1 + f_{gm}(e_{-1}, g_{K-1})} \tag{70}$$
$$+ \frac{h_{M1}(f_r(f_e(e_{-1}, g_{K-1})), i^*) f_r'(f_e(e_{-1}, g_{K-1})) f_{e1}(e_{-1}, g_{K-1})}{h_M(f_r(f_e(e_{-1}, g_{K-1})), i^*)} - \frac{h_{M1}(f_r(e_{-1}), i^*) f_r'(e_{-1})}{h_M(f_r(e_{-1}), i^*)}$$
$$-(1-\alpha)\frac{w_e'(f_e(e_{-1}, g_{K-1})) f_{e1}(e_{-1}, g_{K-1})}{w_e(f_e(e_{-1}, g_{K-1}))} - (1-\alpha)[\frac{f_{e1}(e_{-1}, g_{K-1})}{f_e(e_{-1}, g_{K-1})} - \frac{1}{e_{-1}}]\}$$
$$- \frac{b}{[\frac{\alpha w_e(f_e(e_{-1}, g_{K-1}))}{\delta(1-\alpha)}]^{1-\alpha}} \{ -(1-\alpha)\frac{w_e'(f_e(e_{-1}, g_{K-1})) f_{e1}(e_{-1}, g_{K-1})}{w_e(f_e(e_{-1}, g_{K-1}))} \}$$

and

$$
\begin{aligned}
& f_{gK2}(e_{-1}, g_{K-1}) \\
& = \frac{\{(g_{K-1}+\frac{\delta}{\alpha})[\frac{\alpha w_e(e_{-1})}{\delta(1-\alpha)}]^{1-\alpha}+b\}\frac{1+f_{gm}(e_{-1},g_{K-1})}{1+i^*}}{(1+g_{K-1})^{\alpha}[(1+g_n)\frac{f_e(e_{-1},g_{K-1})}{e_{-1}}]^{1-\alpha}} \frac{\frac{h_M(f_r(f_e(e_{-1},g_{K-1})),i^*)}{h_M(f_r(e_{-1}),i^*)}}{[\frac{\alpha w_e(f_e(e_{-1},g_{K-1}))}{\delta(1-\alpha)}]^{1-\alpha}} \\
& \{\frac{[\frac{\alpha w_e(e_{-1})}{\delta(1-\alpha)}]^{1-\alpha}}{(g_{K-1}+\frac{\delta}{\alpha})[\frac{\alpha w_e(e_{-1})}{\delta(1-\alpha)}]^{1-\alpha}+b} + \frac{f_{gm2}(e_{-1},g_{K-1})}{1+f_{gm}(e_{-1},g_{K-1})} \\
& + \frac{h_{M1}(f_r(f_e(e_{-1},g_{K-1})),i^*)f_r'(f_e(e_{-1},g_{K-1}))f_{e2}(e_{-1},g_{K-1})}{h_M(f_r(f_e(e_{-1},g_{K-1})),i^*)} \\
& -\alpha\frac{1}{1+g_{K-1}} - (1-\alpha)\frac{f_{e2}(e_{-1},g_{K-1})}{f_e(e_{-1},g_{K-1})} - (1-\alpha)\frac{w_e'(f_e(e_{-1},g_{K-1}))f_{e2}(e_{-1},g_{K-1})}{w_e(f_e(e_{-1},g_{K-1}))}\} \\
& - \frac{b}{[\frac{\alpha w_e(f_e(e_{-1},g_{K-1}))}{\delta(1-\alpha)}]^{1-\alpha}}\{-(1-\alpha)\frac{w_e'(f_e(e_{-1},g_{K-1}))f_{e2}(e_{-1},g_{K-1})}{w_e(f_e(e_{-1},g_{K-1}))}\}
\end{aligned} \tag{71}
$$

Then obtain

$$
f_{gK1}(e^*, g_K^*) = -\frac{(1-\alpha)w_e'(e^*)}{m^* w_e(e^*)}\{\frac{\delta}{\alpha}\frac{h_{M1}(f_r(e^*),i^*)}{h_M(f_r(e^*),i^*)}d^*(1-d^*) + g_K^* + \frac{\delta}{\alpha}\} \tag{72}
$$

and

$$
\begin{aligned}
& f_{gK2}(e^*, g_K^*) \\
& = 1 - \frac{1}{1+g_n}[g_K^* + \frac{\delta}{\alpha} + \frac{b}{[\frac{\alpha w_e(e^*)}{\delta(1-\alpha)}]^{1-\alpha}}] - \frac{1}{m^*} + \frac{(1-\alpha)e^* w_e'(e^*)}{w_e(e^*)+e^* w_e'(e^*)}\frac{1}{1+g_n}\frac{b}{[\frac{\alpha w_e(e^*)}{\delta(1-\alpha)}]^{1-\alpha}} \\
& - \frac{(1-\alpha)e^* w_e'(e^*)}{w_e(e^*)+e^* w_e'(e^*)}\frac{1}{1+g_n}(1-d^*)\frac{\delta}{\alpha}\frac{h_{M1}(f_r(e^*),i^*)}{h_M(f_r(e^*),i^*)}
\end{aligned} \tag{73}
$$

Thus obtain

$$
\begin{aligned}
& f_{e2}(e^*, g_K^*) f_{gK1}(e^*, g_K^*) = \\
& -\frac{e^* w_e'(e^*)}{w_e(e^*)+e^* w_e'(e^*)}\frac{1-\alpha}{1+g_n}\frac{1}{m^*}\{\frac{\delta}{\alpha}\frac{h_{M1}(f_r(e^*),i^*)}{h_M(f_r(e^*),i^*)}d^*(1-d^*) + g_n + \frac{\delta}{\alpha}\}
\end{aligned} \tag{74}
$$

and

$$
\begin{aligned}
& \mathrm{D}(J) = f_{e1}(e^*, g_K^*) f_{gK2}(e^*, g_K^*) - f_{e2}(e^*, g_K^*) f_{gK1}(e^*, g_K^*) \\
& = [1 - \frac{1}{1+g_n}(g_n + \frac{\delta}{\alpha})][1 - \frac{1}{m^*}] \\
& -[1 - \frac{(1-\alpha)e^* w_e'(e^*)}{w_e(e^*)+e^* w_e'(e^*)}]\frac{1}{1+g_n}[\frac{1}{m^*}(g_n + \frac{\delta}{\alpha}) + \frac{b}{[\frac{\alpha w_e(e^*)}{\delta(1-\alpha)}]^{1-\alpha}}] \\
& - \frac{(1-\alpha)e^* w_e'(e)}{w_e(e^*)+e^* w_e'(e^*)}\frac{1}{1+g_n}\frac{\delta}{\alpha}\frac{h_{M1}(f_r(e^*),i^*)}{h_M(f_r(e^*),i^*)}(1-d^*)\frac{1}{(1+i^*)(1+g_n)}
\end{aligned} \tag{75}
$$

It is easy to know

$$
\begin{aligned}
\mathrm{D}(J) &= f_{e1}(e^*, g_K^*) f_{gK2}(e^*, g_K^*) - f_{e2}(e^*, g_K^*) f_{gK1}(e^*, g_K^*) \\
&= f_{gK2}(e^*, g_K^*) - f_{e2}(e^*, g_K^*) f_{gK1}(e^*, g_K^*)
\end{aligned} \tag{76}
$$

and

$$\mathrm{Tr}(J) = f_{e1}(e^*, g_K^*) + f_{gK2}(e^*, g_K^*) = 1 + f_{gK2}(e^*, g_K^*)\ . \tag{77}$$

Then obtain

$$\mathrm{D}(J) - \mathrm{Tr}(J) + 1 = -f_{e2}(e^*, g_K^*) f_{gK1}(e^*, g_K^*) \tag{78}$$

and

$$\mathrm{D}(J) + \mathrm{Tr}(J) + 1 = 2 + 2 f_{gK2}(e^*, g_K^*) - f_{e2}(e^*, g_K^*) f_{gK1}(e^*, g_K^*)\ . \tag{79}$$

It is easy to know $\mathrm{D}(J) < 1$ and $\mathrm{D}(J) > \mathrm{Tr}(J) - 1$.

Besides, obtain

$$\begin{aligned}
&\mathrm{D}(J) + \mathrm{Tr}(J) + 1 \\
&= 3 - \frac{2}{1+g_n}[g_K^* + \frac{\delta}{\alpha} + \frac{b}{[\frac{\alpha w_e(e^*)}{\delta(1-\alpha)}]^{1-\alpha}}] - \frac{2}{m^*} + \frac{(1-\alpha)e^* w_e'(e^*)}{w_e(e^*) + e^* w_e'(e^*)} \frac{2}{1+g_n} \frac{b}{[\frac{\alpha w_e(e^*)}{\delta(1-\alpha)}]^{1-\alpha}} \\
&- \frac{(1-\alpha)e^* w_e'(e^*)}{w_e(e^*) + e^* w_e'(e^*)} \frac{1}{1+g_n} \frac{\delta}{\alpha} \frac{h_{M1}(f_r(e^*), i^*)}{h_M(f_r(e^*), i^*)} (1-d^*)[\frac{1}{(1+i^*)(1+g_n)} + 1] \\
&+ \frac{1}{m^*} \frac{(1-\alpha)e^* w_e'(e^*)}{w_e(e^*) + e^* w_e'(e^*)} \frac{1}{1+g_n} (g_K^* + \frac{\delta}{\alpha})
\end{aligned} \tag{80}$$

The values of $ew_e'(e)/w_e(e)$ and h_{M1}/h_M are crucial for determining whether it is positive or not. These two values are both small according to assumptions in section 3. Thus it is not difficult to know it is positive if the total money quantity is large, inflation rate is low and economic growth rate is low.

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