# Class-crossing wealth circulation, profit rate and monetary remedy — an ideological experiment about capitalism system

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## —an ideological experiment about capitalism system

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**Abstract**
This paper, assuming the class-crossing wealth circulation in a capitalism economy can process in order, focuses on the influence of the wealth circulation speed on the profit rate in the capitalism system. Taking no account of the influence of the profit rate on the investment incentive or the money needed due to the economic growth, this paper finds, for preserving a desired profit rate the money quantity in the system should be larger when the circulation slows down. If the economic growth rate is positive, a slow wealth circulation will lead to a low equilibrium profit rate. Then for preserving the desired profit rate a large quantity of the increased money per unit time from outside is needed. In the transition period from a fast circulation to a slow circulation, the total money quantity also should jump upward to offset the money short before. If there is no economic growth, in the equilibrium state no increased money is needed, but in the transition period the total money quantity should be increased for preserving the profit rate unchanged. This paper also finds, the increased money per unit time needed for preserving the profit rate increases with the economic growth rate when it is very low but decreases with it when it is large, however the total money needed always decreases with the economic growth rate.

**Key words**: wealth circulation, profit rate, capitalist household, wealth, total money, increased money

## 1. Introduction

This paper focuses on the class-crossing wealth circulation across different households and its influence on profit rate. It is more like a thought experiment than a model reflecting real world, since often the wealth accumulation by some households, rather than the wealth circulation across different households, is observed if there is neither any big innovation nor big economic fluctuation, because a rich household is more like to be richer while a poor family often doesn't have enough start-up capital.

One may say some enterprises lose money and some earn money—leading to the wealth circulation. However, on one hand a smart owner would like to sell its enterprise when facing loss, on the other hand the existence of these enterprises itself can impede entrance of new enterprises and raise unemployment rate. That is, in the short run it is not sustainable for an enterprise to lose money because most capitalists are rational people, and in the long run this losing-earning process is not sustainable since the average profit rate is so low that people have little incentive to build new enterprises. Nevertheless, the analysis below have its enlightenment significance about the

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effect of wealth circulation on the profit rate and employment.

Besides, for simplicity, assume the wealth circulation is realized through the ownership transfer of enterprises across households rather than the gain or loss of enterprises. A capitalist household, in the scenario, runs an enterprise for some time and then sell it to a worker household (or some worker households), leaving the money received as the resource for high consumption level later until the money runs out (when this household becomes a normal worker household). And the worker becoming a capitalist will abandon the ownership some time later, leading to a circulation process. Assume a capitalist household's consumption is much larger than a worker household's wage and so are the consumptions of the rich households who have been capitalist households before (until their money runs out). Sometimes it is like the fact that a capitalist household loses its enterprise because its intelligent manager grows old or its descendants lack enough operation capacity. But the circulation often processes slowly. This paper shows, as one of its conclusions, if the circulation processes slowly or the speed becomes slower, like what often happens in the real world, the profit rate is low or will fall.

The real wealth circulation might be as follows. The circulation speed may be high when big technological innovations especially product innovations happen or the government actions or the beneficial exportation conditions stimulate the economic growth. Then lots of new enterprises are built and wealth flowing from the old enterprises to new enterprises and among new enterprises frequently happens. But after the fierce competition lasting for some time, the market structure tends to be stable—monopoly and oligopoly phenomena appear, leading to the wealth circulation slowing down. Slow wealth circulation, making the profit rate fall, may lead to an economic crisis or economic depression for long time. Then many old enterprises may become bankrupt but new enterprises hardly appear. This makes the wealth circulation almost stop since the existing old enterprises run with more prudence and the market structure is very stable, until big positive things happen, like a large quantity of government purchase or big inventions, which accelerate the wealth circulation and trigger a new cycle. In the long run, the wave of bankruptcies in the economic recession contributes to the wealth circulation since it clears lots of old enterprises and gives potential new enterprises opportunities. But in the short run, without positive stimuli the bankruptcy wave can stop the wealth circulation.

Without economic crises and the external interventions the "actual" wealth circulation can be very slow. So the speed of the wealth circulation across crises is faster than the "actual" speed, of which this paper concentrates on the "actual" speed. Because the purpose of this paper is to analyze the influence of the wealth circulation on the profit rate which is supposed to explain the reason of crises.

Here notice that the class-crossing wealth circulation, however, cannot be regarded as equivalent to the equality among all members in the system. The economy may run sustainably and most people still live in poverty all the time.

## 2. Basic logic

For an economy where there are some enterprises and each enterprise belongs to a capitalist household, the wealth circulation is determined mainly by the ownership transfer of the enterprises. For simplicity without loss of generality, considered below are one typical capitalist household, its enterprise and some relative workers, after assuming the ownership transfer just happens in this small group.

In his paper, Yang (2019) provides a neo-Kaleckian model for a profit-seeking economy where the increased money from outside is needed to remain a desired employment rate (or economic growth rate). Here are some important points. To realize the desired employment rate, the profit rate should be high enough. In the gross product perspective, since the cost consists of the wage and the capital depreciation, the profit consists of a part of savings from the worker households, the money from the rich households for their consumption, the money from the capitalist for the increased capital and the increased money from outside (as the government purchase or the net exportation). After assuming the normal workers spend all of their wages on consumption and have no debt, the profit consists of the money from the rich households for consumption, the money from the capitalist household for the increased capital and the money from outside, which will be equivalent to respectively the consumption of rich households, the increased capital and the increased cash. The number of rich households—the capitalist household and other households who can consume like the capitalist household—influences the profit of the capitalist. Yang (2019) assumes the number is 1 because there is no wealth circulation and wealth is accumulated by the capitalist household that runs the enterprise forever. While the current paper abandons this assumption and allows the ownership to be transferred across different households. Moreover, the ownership transfer processes in order, meaning a capitalist must sell its enterprise to a normal worker (or such workers) of whom the consumption is equivalent to the wage if he doesn’t get the ownership.

Assume there is a platform to lend money without interest. A normal worker household itself has no money, but for buying capital from the capitalist household then it borrows money (of which the value is equivalent to the total capital) from the platform. Then the worker becomes the new capitalist. The initial capitalist gets the money equivalent to the capital and divides it into two parts: one part to pay for the debt borrowed to buy his start-up capital before and the other part, equivalent to the increased capital, deposited on the platform. Besides the latter part, the initial capitalist also has the increased cash as his profit. Except the increased cash, the initial capitalist borrows all the cash from the platform for daily usage and the cash need be paid back to the platform when the ownership transfer happens. All of his profit is deposited on the platform and the value decreases when he spends some money for consumption until it is zero. The platform will lend to the new capitalist the cash for his usage in case, and he will pay back the cash to the platform and deposit the increased cash when the next ownership transfer happens. The increased cash, however, is from outside-sectors (e.g. from the government or the foreigners). The wealth of a normal worker household is zero while that of a capitalist household is large. The initial capitalist household’s wealth decreases until being zero when it becomes a normal worker household. The households with positive wealth are all rich households. The total consumption of the rich households contributes to the profit of the current capitalist.

It is somewhat explicit—if the economic growth rate is zero—that the wealth circulation can lead to any desired profit rate. Since the growth rate is zero, there is no increased capital, no increased consumption and no increased cash (after assuming that inflation rate is 0), leaving the consumption of the rich households providing all of the profit. So if a capitalist household obtains the profit equivalent to its consumption for some periods, then a new capitalist must obtain the same profit, the only condition for which is that the circulation processes with a constant speed. However, if the economic growth rate is positive, things are complicated.

But strictly speaking, the consumption function need be discussed at first. For simplicity, assume the consumption of a normal worker household, with no wealth, is equivalent to its wage, and the consumption of a rich household, with positive wealth, is equivalent to a unified value decided by the technology then, no matter how much wealth it has. Since the wealth is stock and the consumption is flow, if the wealth value is small, the consumption level can remains for only short time. The wealth of a rich household has no influence on its consumption level, which seems to be unreasonable, but, considering that this is for simplicity and it reflects a fact that the marginal propensity to consume decreases with wealth, it is excusable.

The rich household consumption's growth needs more explanation. The paper assumes the growth rate of the consumption of a rich household is equivalent to, if the capital-output ratio remains unchanged, the economic growth rate—approximately the sum of the technological progress rate and the labor growth rate. In other words, if the number of the family members of a rich household grows by the labor growth rate, the assumption above means the consumption of each member in the rich household grows by the technological growth rate by which the wage of a normal worker grows. Note the labor growth rate is not necessarily equivalent to the population growth rate. For example, many rural households can take part in the capitalism production in the industrialization process. Then if the family member of a rich households increases by a lower rate, which probably happens, but the consumption level above is remained, the total consumption of the rich households will be smaller, which can lead to a lower profit rate when there is no money from outside-sectors.

A necessary assumption here is the consumption of a family member of a rich household, like that of a normal worker household, grows by the technological progress rate. It seems to conflict with the theory of diminishing marginal propensity to consume. But from the horizontal perspective there is no conflict. At any time point the marginal propensity to consume diminishes with the income, but for a specific man it can remains unchanged even increases across different time, for which one reason is the new products brought by technological innovations can lead to a higher consumption level of him. Image such a scenario. A man A with 10000 dollars consumes 7000 dollars a year while another man B with 100000 dollars consumes 30000 dollars a year. Some years later the prices remains unchanged. The poor man, A, then having 100000 dollars, consumes 70000 dollars a year, not 30000. In comparison, B then having 1000000 dollars consumes 300000 a year. Time inconsistency exists about consumption choice.

To show the wealth circulation process in detail, image the mode below in which the number of workers in each enterprise remains unchanged and a capitalist household can have more than 1 enterprises, avoiding the explosion of the enterprise scale. A capitalist household's family member number grows by the worker growth rate. Then assume the manager number of the household is equivalent to the enterprise number it owns and grows by the total worker growth rate. So the number of workers attached to any manager remains unchanged and equivalent to the enterprise worker number. For one period a household has the enterprise ownership, and at the beginning the family member number of the capitalist household is $c$, the manager number $m$, the consumption of each family member $C$, the family member number of a normal worker household is also $c$, the enterprise worker number is $e$. In the period the total worker number grows by $w$, the population grows by $p$ (smaller than $w$ in the industrialization process), the technology index by $g$, so the manager number of the capitalist household grows by $w$ and becomes $m(1+w)$ with the worker number of an enterprise unchanged, the family member number becomes $c(1+w)$, the consumption of a member of the capitalist household $C$(1+$g$), the total consumption of the household $cC$(1+$w$)(1+$g$), the total worker number $em$(1+$w$) and the family member number of a normal worker household becomes $c$(1+$p$). At the end of the period, the capitalist household sells these $m$(1+$w$) enterprises to (1+$w$)/(1+$p$) worker households with the total family member number being $c$(1+$w$) and each worker household has $m$(1+$p$) managers and $m$(1+$p$) enterprises. Then at the beginning of the next period, the total consumption of the new capitalist households is $cC$(1+$w$)(1+$g$). In this way, the consumption of the capital households always grows by (1+$w$)(1+$g$)-1 which is exactly the economic growth rate. Assume the family member number of any rich household grows by $w$. Moreover, one can regard the worker households buying capital from one capitalist household as a new big worker household which can appear when, e.g., they unite by marriage.

# 3. Wealth circulation models

## 3.1 When the economic growth rate is 0

At first discuss the situation where the economic grow rate is 0. Then there is no technological progress and no population growth to be considered. Since there is no increased capital and no increased cash, the profit includes only the consumption of the rich households. Note the consumption of a capitalist household by $C$, the output of its enterprise(s) by $Y$.

The production function is

$$Y = K^{\alpha}(AL)^{1-\alpha} \tag{1}$$

where $Y$ is the total product, $L$ the labor employed—growing by $n$, $A$ the technical index—growing by $g$, $K$ the capital.

If a worker household can continuously be a capitalist household for time $t$, and the wealth circulation processes in order, then the total profit it receives is $pBt$ where $B$ is the total cost for production per unit time—the sum of the wage and the capital depreciation—and $pB$ is the profit per unit time. One household can continuously be a rich household for time $T$=$pBt$/$C$. The number of the rich households at any time, in the equilibrium state, is $j$=$T$/$t$ ($j$>1). So the profit function at any time is

$$pB = jC\ . \tag{2}$$

The total profit a capitalist household obtains is

$$tpB = tjC\ . \tag{3}$$

Besides the profit function, the consumption equation is needed as follows.

$$TC = tpB \tag{4}$$

And the rich household number is

$$j = T/t\ . \tag{5}$$

Given $t$, $C$ and $Y$, solution of $j$, $T$ and $p$ can be obtained. The capitalist time, $t$, can reflect the wealth circulation speed. A large $t$ means the circulation processes slowly. When t turns larger, the circulation slows down. One intention of this paper is to show how $t$ influences $p$ (see table 2 for results).

According to (3), (4) and (5), obtain

$$p = j\frac{C}{B}. \tag{6}$$

It means $p$ is a free variable when combined with (5), i.e., any regular profit rate can be obtained once in the equilibrium values of $T$ and $j$ are provided.

Rewrite the equations (3), (4) and (5) as

$$\int_{s=s_0}^{s_0+t(s_0)} p(s)B(s)ds = \int_{s=s_0}^{s_0+t(s_0)} j(s)C(s)ds\,, \tag{7}$$

$$\int_{s=s_0}^{s_0+T(s_0)} C(s)ds = \int_{s=s_0}^{s_0+t(s_0)} p(s)B(s)ds\,, \tag{8}$$

$$j(s_0+T(s_0)) = \int_{s=s_0}^{s_0+T(s_0)} \frac{1}{t(s)}ds \tag{9}$$

where $t$, $T$, $j$, $p$, $B$ and $C$ are functions of time $s$.

If $t$ remains unchanged, once the equilibrium with any specific $p$ level has remained for a long time, the profit rate will remain forever and so will $T$ and $j$. So the circulation speed has no influence on the equilibrium profit rate. Notice that $C$ and $B$ remain unchanged here. According to (7) and (8), obtain

$$\int_{s=s_0}^{s_0+t(s_0)} p(s)ds = \frac{C}{B}\int_{s=s_0}^{s_0+t(s_0)} j(s)ds\,, \tag{10}$$

$$\frac{C}{B}T(s_0) = \int_{s=s_0}^{s_0+t(s_0)} p(s)ds\,. \tag{11}$$

Further obtain

$$T(s_0) = \int_{s=s_0}^{s_0+t(s_0)} j(s)ds\,. \tag{12}$$

Given the path of $t$, the paths of $j$ and $T$ determine each other according to (9) and (12), shown more clearly below with figures. Given $C$ and $B$, the profit rate determined only by $j$. Then analyze $j$.

If the capital and the cash for daily usage of a capitalist household is obtained through borrowing the equivalent money from the platform, then the wealth of the capitalist household is the profit it received. In the equilibrium state, the total profit obtained by all capitalist households in the period with the length $T$ is $pBT$, or $jCT$, determined by $p$ or by $j$. The half of the profit is consumed by the rich households and the other half is saved as their wealth.

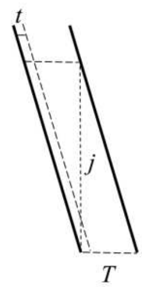


Figure 1.

In figure 1 showing an equilibrium of the wealth circulation without economic growth, a point on the left thick line means the time point when a household becomes a capitalist household, the point on the left thick line on the same horizontal level means the time point when the household ends its role as a rich household. The consumption per unit time of a rich household is unified to 1. The long horizontal line, with the length of $T$, means the wealth of a capitalist household at the beginning. The short horizontal line, with the length of *t*, means the consumption of a rich household in the period when it is a capitalist household. The vertical line means *j*. It is obvious, in the figure, that *j* value for a capitalist household is determined by *T* values of the rich households before and determines its own *T* value. The area of the parallelogram between two thick lines means the total profit of all capitalist households in the period with the length *T*. At the time point corresponding to the vertical line, the triangle on the left side means the wealth spent on consumption while the triangle on the right side means the wealth saved.

For a given profit rate, the total profit, *jCT*, should increase with *T* (then with *t*). Assume *C* and *Y* are constant values. The capitalist household at the beginning of the period provides *CT* through its consumption, and that at the end of the period provides 0, leaving the average consumption of rich households in time *T* is *CT/2*. Since the capitalist households before can provide some profit by their consumption for the current capitalist household, and the circulation processes with an unchanged speed in the equilibrium state, the total profit needed by the rich households as a whole is the half of *jCT*, because the rich households becoming capitalist in the period (with the length *T*) provide the other half of *jCT*. In the equilibrium state, the former half of *jCT* is provided by the ever-being capitalist households in the last period with the length being *T* through their consumption. At the same time, the wealth at the last period, equivalent to *jCT*/2, is transferred to the rich households at the current period. The wealth of the capitalist household at the end of the period is *CT*, while that at the beginning of the period is 0, leaving the average wealth of the household being *CT*/2. However, this part is determined by the profit obtained in the last period, i.e., the wealth at the current period equals to that at the last period if there is no new wealth introduced. So if the economy jumps from the equilibrium with small *t* and *T*, corresponding to small profit and small wealth in a period, to the equilibrium with large *t* and *T*, corresponding to large profit and large wealth in a period, the profit from outside is needed to increase the wealth quantity in the system and to preserve the desired profit rate. Otherwise even if a new equilibrium is realized, the profit rate is on a lower level.

Figure 2 shows, when the wealth circulation processes with a slower speed, the left thick line should be less steep, meaning *t* value becomes larger. If the money is provided as the shaded triangle depicts, the new equilibrium can be realized immediately. Compared with the old equilibrium, the new equilibrium has larger *T* values, but *j* values remains the same.

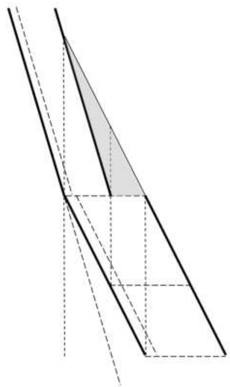

Figure 2.

But if the wealth is unchanged when the wealth circulation speed slows down, the new equilibrium in figure 2 will not realize. The old rich households cannot provide enough consumption to preserve the initial profit rate, i.e., *j* values are smaller than those of the old equilibrium, leading to the profit of a new capitalist being smaller than that to preserve the profit rate (but the total profit not necessarily being smaller). Taking no account of the spinoff effects of changes of profit rate—like that a capitalist may decrease the investment and output due to a lower profit rate—the new rich households will remain its high consumption level for a shorter time than that in figure 2, leading to a smaller profit rate than that of the old equilibrium again. Figure 3 shows this process.

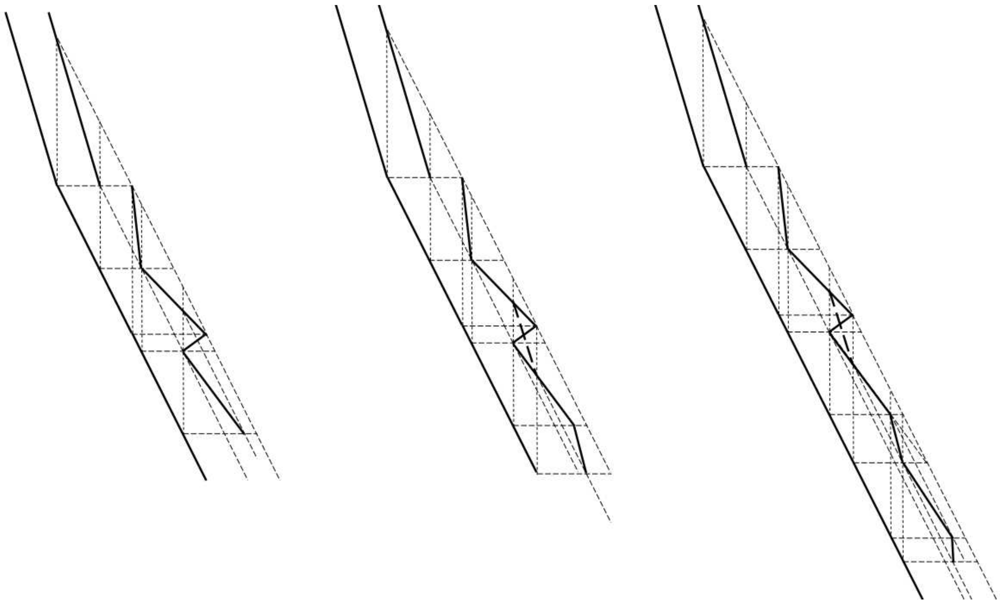

Figure 3.

Because there is not economic growth, the increased capital is not needed and there is no increased money from outside in the equilibrium state. Moreover the consumption of a rich household remains unchanged, leading to that the profit received have to be hoarded as money and as the wealth. On the other hand the money quantity to buy capital and for daily usage of the capitalist household is constant, leaving the total money quantity is only influenced by the quantity of the hoarded money. Given a desired profit rate, corresponding to an equilibrium with a specific circulation speed, $1/t$, is a specific quantity of hoarded money. So any wealth circulation speed can lead to a desired profit rate, if the corresponding money quantity is provided. But if the money quantity is given, a slower wealth circulation will lead to a lower profit rate. And naturally, given the wealth circulation speed, a larger money quantity can lead to a larger wealth quantity and a higher profit rate since the rich households can, ceteris paribus, preserve their high consumption level for longer time and then the rich household number increases, leading to the larger total consumption and the larger profit.

Note the wealth of the economy in the equilibrium state is $W= jCT/2$. According to (5) and (6), obtain

$$j = \sqrt{\frac{2W}{tC}}\ , \tag{13}$$

$$p = \frac{1}{B}\sqrt{\frac{2CW}{t}}\ . \tag{14}$$

It is clear that the equilibrium profit rate increases with the money (influencing the economic wealth) and with the wealth circulation speed. When the wealth circulation processes more slowly, new money is needed to preserve the initial profit rate. But once the new equilibrium has realized, no more money should be provided from outside.

## 3.2 When the economic growth rate is positive

When the economic growth rate is positive, there will be the increased money and the increased capital. But the analysis above have provided some important clues. Like (7), the profit function is

$$\int_{s=s_0}^{s_0+t(s_0)} p(s)B(s)ds = \int_{s=s_0}^{s_0+t(s_0)} j(s)C(s)ds + \int_{s=s_0}^{s_0+t(s_0)} \dot{K}(s)ds + \int_{s=s_0}^{s_0+t(s_0)} G(s)ds \tag{15}$$

where $s_0$ is any possible time point in the time line noted by $s$, $K$ is the total capital as the function of time $s$, and $G$ is the money from outside, like the government purchase, as the function of time. Note $I(s) = \dot{K}(s)$. The consumption function and the rich household number function are respectively (8) and (9).

In the equilibrium state, the consumption level of a rich household growing by the economic growth rate has been interpreted in the above part. Besides, it is easy to know or it is reasonable to assume the total cost, the total capital and the government purchase all grow by the economic growth rate. Then assume the rich household consumption function, the capital function, the increased capital function, the cost function and increased money function are respectively as follows.

$$C = C(0)e^{(g+n)s} \tag{16}$$

$$K = K(0)e^{(g+n)s} \tag{17}$$

$$I = I(0)e^{(g+n)s} = (g+n)K(0)e^{(g+n)s} \tag{18}$$

$$B = lw + \delta K = l(0)e^{ns}w(0)e^{gs} + \delta K(0)e^{(g+n)s} = [l(0)w(0) + \delta K(0)]e^{(g+n)s} \tag{19}$$

$$G = G(0)e^{(g+n)s} \tag{20}$$

So (15) and (8) can be rewritten as follows.

$$\begin{aligned} &B(0)\int_{s=s_0}^{s_0+t(s_0)} p(s)e^{(g+n)s}ds \\ &= C(0)\int_{s=s_0}^{s_0+t(s_0)} j(s)e^{(g+n)s}ds + I(0)\int_{s=s_0}^{s_0+t(s_0)} e^{(g+n)s}ds + G(0)\int_{s=s_0}^{s_0+t(s_0)} e^{(g+n)s}ds \end{aligned} \tag{21}$$

$$C(0)\int_{s=s_0}^{s_0+T(s_0)} e^{(g+n)s}ds = B(0)\int_{s=s_0}^{s_0+t(s_0)} p(s)e^{(g+n)s}ds\ , \tag{22}$$

According to the cost minimization theory, it is easy to obtain

$$\frac{l(0)w(0)}{\delta K(0)} = \frac{1-\alpha}{\alpha}\ . \tag{23}$$

Combining the assumption described before, assume $l(0)w(0)/C(0)=h$ remains unchanged no matter what the value of $(g+n)$ is. Besides, $G(0)/C(0)=x$ are often determined exogenously. Then obtain

$$C(0)=\delta K(0)\frac{1-\alpha}{h\alpha} \tag{24}$$

and

$$\frac{B(0)}{C(0)}=\frac{h}{1-\alpha}\,, \tag{25}$$

$$\frac{I(0)}{C(0)}=\frac{h\alpha(g+n)}{\delta(1-\alpha)}\,. \tag{26}$$

Since in this part the comparative static analysis is more important than the dynamic analysis, for simplicity mainly focus on the equilibrium state. In the equilibrium state, assume $p$, $t$, $T$ and $j$ all remain unchanged. So the first thing is to prove the existence of such an equilibrium, i.e., to prove once such an equilibrium have remained for a long time, then it can remain forever. The main work is to prove the beginning time, $s_0$, has no influence on the sustainability of the equilibrium, or the equilibrium can be depicted by parallel lines in figure 4, like in figure 1.

Regarding $p$, $t$, $T$ and $j$ as constant parameters, according to (21), (22) and (9), obtain the system as follows.

$$pB(0)\int_{s=s_0}^{s_0+t(s_0)}e^{(g+n)s}ds=[jC(0)+I(0)+G(0)]\int_{s=s_0}^{s_0+t(s_0)}e^{(g+n)s}ds \tag{27}$$

$$C(0)\int_{s=s_0}^{s_0+T(s_0)}e^{(g+n)s}ds=pB(0)\int_{s=s_0}^{s_0+t(s_0)}e^{(g+n)s}ds \tag{28}$$

$$j=\frac{T}{t} \tag{29}$$

It is easy to rewrite (27) as

$$pB(0)=jC(0)+I(0)+G(0)\,. \tag{30}$$

or

$$p=\frac{C(0)}{B(0)}[j+\frac{I(0)+G(0)}{C(0)}]=\frac{1-\alpha}{h}[j+\frac{h\alpha(g+n)}{\delta(1-\alpha)}+x]=\frac{1-\alpha}{h}j+\frac{\alpha}{\delta}(g+n)+\frac{1-\alpha}{h}x\,. \tag{31}$$

So the profit rate is determined by $j$. Then just focus on $j$.

According to (27) and (28), obtain the consumption function as

$$C(0)\int_{s=s_0}^{s_0+T(s_0)}e^{(g+n)s}ds=[jC(0)+I(0)+G(0)]\int_{s=s_0}^{s_0+t(s_0)}e^{(g+n)s}ds \tag{32}$$

or

$$e^{(g+n)T}-1=[j+\frac{I(0)+G(0)}{C(0)}][e^{(g+n)t}-1] \tag{33}$$

or

$$e^{(g+n)T}-1=[j+\frac{h\alpha(g+n)}{\delta(1-\alpha)}+x][e^{(g+n)t}-1] \tag{34}$$

where the beginning time, $s_0$, has no influence.

Of course there exist countless equilibriums determined by (30), (33) and (29). Given $t$ in an equilibrium state, $p$, $j$ and $T$ can be obtained. Like figure 1, describe the equilibrium state by parallels in figure 4. The most important difference from the zero-growth-rate figure is there are two more parts—the money from outside (noted by $x$) and the investment (noted by $I/C$)—as the source of the total profit which can make a capitalist household preserve the rich household consumption level for time $T$. The profit rate, $p$, is determined by the sum of three parts: $j$, $x$ and $I/C$. Given $I/C$, for preserving the profit rate unchanged the sum of $j$ and $x$ should remain unchanged. Besides, the figure for the equilibrium with positive economic growth rate should be regarded as a top view and the point height increases by the economic growth rate when shifting towards the right on a horizontal line but remains unchanged when shifting on a vertical line.

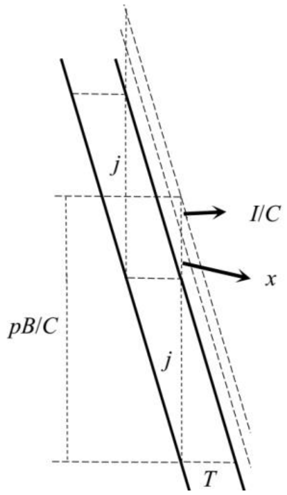


Figure 4.

It is easy to obtain

$$\frac{\partial j}{\partial t} < 0 \tag{35}$$

and then

$$\frac{\partial p}{\partial t} < 0 . \tag{36}$$

Note $a = e^{(g+n)t}$, and $b = \frac{h\alpha(g+n)}{\delta(1-\alpha)} + x$. In the equilibrium state, $b$ is the ratio of the sum of the increased capital and the increased money from outside on the consumption. Then rewrite (33) as

$$a^j - 1 = (j+b)(a-1) \tag{37}$$

or

$$b = \frac{a^j - 1}{a-1} - j . \tag{38}$$

It can be shown that $j$ increases with $b$ given $a$ and $b$ increases with $a$ given $j$. Thus given $b$, $j$ increases with $a$. So obtain $\frac{\partial j}{\partial a} < 0$ and then $\frac{\partial j}{\partial t} < 0$.

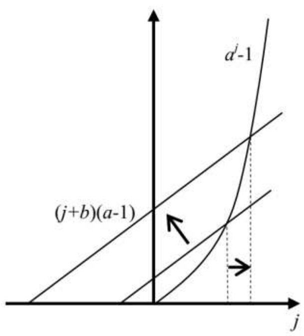


Figure 5.

Consider the left side and the right side of (37). After plotting them in a coordinate system with the horizontal axis representing *j*, if *b* increases, the curve of the left side function shifts towards the left and upward. Due to the shapes of the two curves, the intersection point shifts towards the right and upward, meaning *j*, as the solution, increases, as figure 5 shows. So obtain $\frac{\partial j}{\partial b} > 0$. According to $\frac{\partial j}{\partial b} > 0$, obtain $\frac{\partial j}{\partial x} > 0$ and then $\frac{\partial p}{\partial x} > 0$.

Consider (38) through figure 6. Given *j* (*j*>1), plot $a^j - 1$ (*a*>1) and $a-1$ (*a*>1) in a coordinate system with the horizontal axis representing *a*. It is obvious that, due to the shapes of the two curves, the ratio, $\frac{a^j - 1}{a-1}$, increases with *a*. Thus *b* increases with *a* given *j*. So obtain $\frac{\partial b}{\partial a} > 0$. Considering the ratio represents $\int_{s=s_0}^{s_0+T(s_0)} e^{(g+n)s} ds \Big/ \int_{s=s_0}^{s_0+t(s_0)} e^{(g+n)s} ds$, the figure shows the total profit required grows by a higher rate than the growth rate of the total profit provided when *t* increases given *j*.

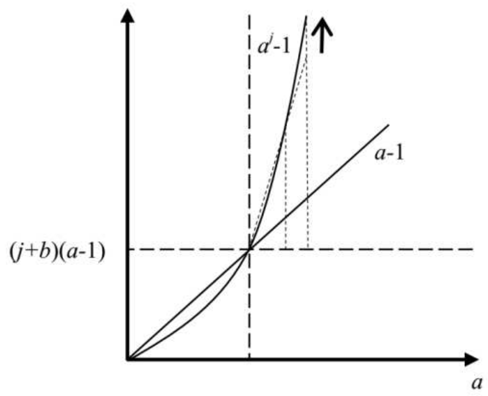


Figure 6.

The influence of the economic growth rate on the equilibrium profit rate is complicated. On the one hand, corresponding to a higher economic growth rate is a lower *j* level because for the given level of *j* the profit needed to preserve the high consumption level increases faster than the profit increases with the economic growth. On the other hand, a higher economic growth rate can lead to a larger investment quantity, having a positive effect on the profit rate. Moreover, given the quantity of the money from outside, if the economic growth rate is very low, *j* value can be very large given the wealth circulation processes in order. So if the economic growth rate is very low, the rise of the economic growth rate will lead to the fall of the profit rate. But if the economic growth rate is enough high, its rise will lead to a rise of the profit rate. However, it is not enough for showing the economic growth's effect on the profit rate. Here just give the result as

$$\frac{\partial p}{\partial (g+n)} = -\frac{\frac{1-\alpha}{h} ta(j+b-ja^{j-1}) + \frac{\alpha}{\delta} a^j \ln a}{a-1-a^j \ln a} \tag{39}$$

where $a-1-a^j \ln a < 0$ and $j+b-ja^{j-1} < 0$ hold, as shown below.

According to $\frac{\partial j}{\partial b} > 0$ and $a > 1$, obtain $a-1-a^j \ln a < 0$.

According to $\frac{\partial j}{\partial a} = -\frac{j+b-ja^{j-1}}{a-1-a^j \ln a} < 0$ and $a-1-a^j \ln a < 0$, obtain $j+b-ja^{j-1} < 0$.

And what has been mentioned is

$$\frac{\partial p}{\partial x}=\frac{1-\alpha}{h}\frac{\partial j}{\partial x}+\frac{1-\alpha}{h}>0\ . \tag{40}$$

Thus in an equilibrium state with a high value of *t*, its value of *x* should also be high to preserve the desired profit rate. Notice that in the paper the spinoff effects of the profit rate changes are not taken into consideration, e.g., the effects of changes of *p* on *b* due to the influence of the profit rate changes on the voluntary investment. Then more situations can be discussed. As shown above, the increase of *t* will lead to the decreases of *j* and *p*. For preserving the same value of *p*, *x* need increase and as a result *j* decreases compared to the initial value. For preserving the equilibrium profit rate being *p*, the value of *x* should be

$$x=\frac{h}{1-\alpha}p-\frac{h\alpha(g+n)}{\delta(1-\alpha)}-\frac{\ln[\frac{h}{1-\alpha}p(e^{(g+n)t}-1)+1]}{(g+n)t}\ . \tag{41}$$

Then obtain

$$\frac{\partial x}{\partial(g+n)}=\frac{\ln[\frac{h}{1-\alpha}p(e^{(g+n)t}-1)+1]}{(g+n)^2 t}-\frac{h\alpha}{\delta(1-\alpha)}-\frac{pe^{(g+n)t}/(g+n)}{p(e^{(g+n)t}-1)+(1-\alpha)/h}\ . \tag{42}$$

A simple numerical simulation can show, given *t*, *x* value increases at first and then decreases as the economic growth rate rises (see the appendix). Of course, when *t* is large, *x* value is large even if the economic growth rate is low. Notice that here only consider the quantity of the money for directly maintaining the profit rate and other functions of money are not considered. It is why negative values of *x* can appear. In fact, due to the necessity of the existence of the money for daily usage which is neglected in this paper, the increased money quantity should be positive if the economic growth rate is positive, see Yang (June 2019).

For an equilibrium, the increase of *x* means the increased money from outside per unit time should increase from very long time ago, i.e., the total money should increase by a large quantity to offset the total short before. Here assume the quantity of the total money from outside is equivalent to that of all money in the system since the money purely for a capitalist's daily usage (provided by the platform) is neglected for simplicity. And the money from outside is all hoarded by the rich households. Thus the total money at time $s_0$ is

$$M(s_0)=\int_{-\infty}^{s_0}G(0)e^{(g+n)s}ds=xC(0)\int_{-\infty}^{s_0}e^{(g+n)s}ds\ . \tag{43}$$

And the total wealth at time $s_0$ is

$$W(s_0)=\int_{-\infty}^{s_0}[I(0)e^{(g+n)s}+G(0)]e^{(g+n)s}ds=[\frac{h\alpha(g+n)}{\delta(1-\alpha)}+x]C(0)\int_{-\infty}^{s_0}e^{(g+n)s}ds\ . \tag{44}$$

The total profit obtained by the capitalist households from time $s_0$-*T* to $s_0$ is as follows.

$$\begin{aligned}P(s_0-T,s_0)&\equiv\int_{s_0-T}^{s_0}[pB(0)e^{(g+n)s}]ds\\&=\int_{s_0-T}^{s_0}[jC(0)e^{(g+n)s}+I(0)e^{(g+n)s}+G(0)e^{(g+n)s}]ds\\&=[j+\frac{h\alpha(g+n)}{\delta(1-\alpha)}+x]C(0)\int_{s_0-T}^{s_0}e^{(g+n)s}ds\end{aligned} \tag{45}$$

It is easy to show

$$P(s_0-T,s_0)=\int_{s_0-T}^{s_0}j\frac{s-(s_0-T)}{T}C(0)e^{(g+n)s}ds+W(s_0)\ . \tag{46}$$

According to

$$\int_{s_0-T}^{s_0} jC(0)e^{(g+n)s}ds = \int_{s_0-T}^{s_0} j\frac{s-(s_0-T)}{T}C(0)e^{(g+n)s}ds + \int_{s_0-T}^{s_0} j\frac{s_0-s}{T}C(0)e^{(g+n)s}ds \tag{47}$$

and

$$P(s_0-T, s_0) = \int_{s_0-T}^{s_0} j\frac{s-(s_0-T)}{T}C(0)e^{(g+n)s}ds + \int_{s_0}^{s_0+T} j\frac{(s_0+T)-s}{T}C(0)e^{(g+n)s}ds\,, \tag{48}$$

obtain

$$\begin{aligned}&\int_{s_0}^{s_0+T} j\frac{(s_0+T)-s}{T}C(0)e^{(g+n)s}ds - \int_{s_0-T}^{s_0} j\frac{s_0-s}{T}C(0)e^{(g+n)s}ds\\ &= P(s_0-T, s_0) - \int_{s_0-T}^{s_0} jC(0)e^{(g+n)s}ds\\ &= [\frac{h\alpha(g+n)}{\delta(1-\alpha)}+x]C(0)\int_{s_0-T}^{s_0} e^{(g+n)s}ds\end{aligned} \tag{49}$$

where the last equality sign is according to (45).

Similarly, for any nonnegative integer $i$, obtain

$$\begin{aligned}&\int_{s_0-iT}^{s_0+T-iT} j\frac{(s_0+T-iT)-s}{T}C(0)e^{(g+n)s}ds - \int_{s_0-T-iT}^{s_0-iT} j\frac{s_0-iT-s}{T}C(0)e^{(g+n)s}ds\\ &= [\frac{h\alpha(g+n)}{\delta(1-\alpha)}+x]C(0)\int_{s_0-T-iT}^{s_0-iT} e^{(g+n)s}ds\end{aligned} \tag{50}$$

which is depicted by figure 7.

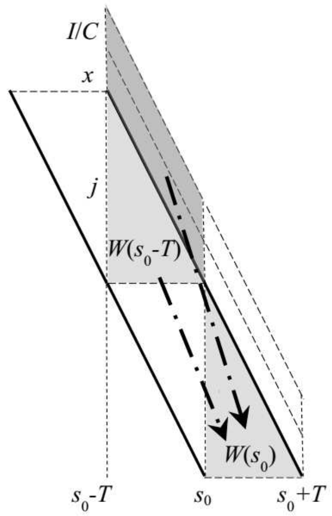


Figure 7.

According to (50), obtain

$$\int_{s_0}^{s_0+T} j\frac{(s_0+T)-s}{T}C(0)e^{(g+n)s}ds = [\frac{h\alpha(g+n)}{\delta(1-\alpha)}+x]C(0)\int_{-\infty}^{s_0} e^{(g+n)s}ds = W(s_0) \tag{51}$$

Thus (46) holds obviously according to (48) and (51). Shown by (46), the equilibrium profit rate is influenced by the total wealth and then by the total money.

So if there is a higher $t$ value than before, it is not enough to just increase $G$ value—the quantity of the increased money from outside per unit time (as a flow value)—for preserving the same profit rate, since the total money, as a stock value, is smaller than what is needed for the equilibrium profit rate. However, after making $G$ value on the equilibrium level for very long time, the money short due to the initial short of the total stock is negligible compared to the total money then, and the equilibrium profit rate can be realize approximately.

For preserving the profit unchanged and reaching the new equilibrium when $t$ becomes a larger value than before, the increased money from outside should be provided as figure 8 shows. New equilibrium values of $j$ and $x$ are respectively noted by $j$' and $x$'. At the new equilibrium state, the increased money per unit time divided by the rich household consumption level increases and the number of rich households decreases respectively compared to those values at the initial equilibrium state. In the first period with the time length being $T$', the increased money supply path is depicted by the shaded polygon. The quantity of the total increased money in the period is the sum of the equilibrium increased money and the increased wealth needed due to circulation slowing down.

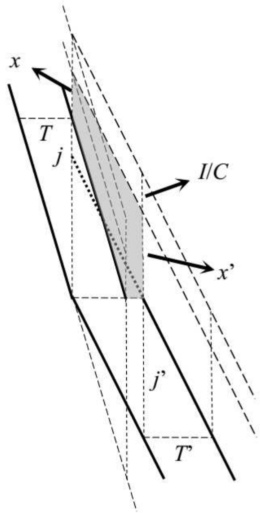


Figure 8.

At time 0, given the rich household consumption level is $C(0)$, the total money is

$$M(0) = xC(0)\int_{-\infty}^{0} e^{(g+n)s} ds = \frac{x}{g+n} C(0) . \tag{52}$$

Combining (41) and (52), obtain that the total money, for preserving the same profit rate when the economic growth rate changes at time 0, satisfies

$$\frac{M(0)}{C(0)} = \frac{h}{1-\alpha}\frac{p}{g+n} - \frac{\ln[\frac{h}{1-\alpha} p(e^{(g+n)t}-1)+1]}{(g+n)^2 t} - \frac{h\alpha}{\delta(1-\alpha)} . \tag{53}$$

The simple simulation in the appendix shows the equilibrium total money increases with $g+n$ and with $t$. So if the economic growth rate is very low, the quantity of the increased money per unit time from outside can be small but the total money have to be large to preserve the desired profit rate.

If the economic growth rate is 0, in the equilibrium state no increased money (as flow) is needed, but for an equilibrium with a slower wealth circulation speed, to remain the same profit rate the money from outside is necessary and the total money quantity (as a stock value) should increase. In comparison, if the economic growth rate is positive, in the equilibrium state the increased money from outside (as flow) is needed, and for an equilibrium with a slower wealth circulation, to preserve the same profit rate the quantity of the increased money from outside per unit time (as flow) should be larger and the quantity of the total money (as a stock value) should jump onto a higher level. When the economic growth rate is very low, the increased money quantity (as a flow value) compared to the rich household consumption level should be very small and so should the investment quantity, but the total money quantity (as a stock value) should be large, leaving the equilibrium being very similar with that without economic growth. In the transition period from fast circulation to slow circulation, for preserving the profit rate unchanged, more money should be provided in both situations. Moreover, the increased money path shown by figure 8 will be very similar with that shown by figure 2 if the economic growth rate is very low, with the polygon very similar with the triangle. The difference is, without the money quantity (as a stock value) jump, the new equilibrium with the same profit rate can realize approximately if there are economic growth but can never realize if the growth rate is 0. The main results are shown by table 1, table 2 and table 3.

**Table 1. The influence of the economic growth rate.**

| g+n>0 | | For preserving a specific profit rate | |
|---|---|---|---|
| | | The equilibrium increased money per unit time | The equilibrium total money |
| very small | ↑ | ↑ | ↓ |
| large | | ↓ | ↓ |

In his paper, Yang (June 2019) considers the models upon the assumption that there is no wealth circulation and suggests the necessity of the increased money from outside. By most the necessity of the money quantity increase is accepted since the economic growth demands the money growth. But it is also needed due to the request from the capitalists for the desired profit rate. Yang (June 2019) shows, if the economic growth rate is very low, the large quantity of money is needed. The current paper, upon the assumption that there can be the wealth circulation in order, considers the influence of the circulation speed on the profit rate and the money quantity needed to preserve a desired profit rate. When the wealth circulation processes very slowly, the large quantity of money is needed, corresponding to the conclusion of Yang (June 2019). The current paper does not pay much attention to the influence of economic growth rate. It finds, when the growth rate is very low, the quantity of the increased money (as flow) needed is very small but the total money (as stock) needed is very large. In comparison, Yang (June 2019) finds, when the growth rate is very low, the money quantity (as flow and as stock) needed is very large if the circulation stops forever. So the circulation matters a lot.

**Table 2. The influence of the wealth circulation speed**

| | | | |
|---|---|---|---|
| Wealth circulation speed | | | ↓ |
| *t* value | | | ↑ |
| The profit rate given the money path | | | ↓ |
| For preserving a specific profit rate | g+n=0 | The equilibrium increased money per unit time | 0 |
| | | The equilibrium total money | ↑ |
| | g+n>0 | The equilibrium increased money per unit time | ↑ |
| | | The equilibrium total money | ↑ |

**Table 3. The monetary remedy at the transition period for preserving a specific profit rate**

| | | | |
|---|---|---|---|
| Wealth circulation speed | | | ↓ |
| *t* value | | | ↑ |
| The increased money per unit time for preserving a specific profit rate | g+n=0 | In the transition period | ↑ |
| | | After the transition period | 0 |
| | g+n>0 | In the transition period | ↑ |

| | | | |
|---|---|---|---|
| | | After the transition period | ↑ |

(“ ↑ ” and “ ↓ ” respectively mean the values after the circulation speed change increase and decrease compared to those without the change.)

## 4. Conclusions

This paper considers the influence of the wealth circulation on the profit rate but takes no account of many other elements. So it can be regarded as just an interpretation about the wealth circulation for Yang (June 2019). A slow wealth circulation will lead to a low equilibrium profit rate if the economic growth rate is positive. Thus for preserving a desired profit rate a large quantity of the increased money per unit time from outside is needed. The increased money quantity decreases with the circulation speed. In the transition period from a fast circulation to a slow circulation, the money quantity (as a stock value) should also have an upward jump to offset the money short due to the small quantity of the increased money per unit time from outside before. If the economic growth rate is zero, in the equilibrium state no money from outside is needed, but in the transition period from the fast circulation to the slow circulation the total money quantity should be increased for preserving the initial profit rate. And given the circulation speed, corresponding to a smaller equilibrium quantity of total money is a lower profit rate. This paper also finds, the increased money per unit time from outside needed for preserving the profit rate increases with the economic growth rate when it is very small but decreases with it when it is large, but the total money needed always decreases with the economic growth rate. This paper does not consider the influence of the profit rate on the investment incentive in turn or the money for trade influenced by the economic growth. Moreover, this paper assumes the wealth circulation processes in order. These constrains should be noticed.

**Appendixes**

Set $h = 100$, $\alpha = 1/3$, $\delta = 0.05$, $p = 0.2$.

Just consider the increased money per unit time needed to maintain a desired profit rate and take no account of the increased money requested for trade due to the economic growth. Then obtain $x$ values corresponding to different values of $t$ and $g+n$, shown by figure 9, figure 10 and figure 11, and $x/(g+n)$ values by figure 13, figure 14 and figure 15.

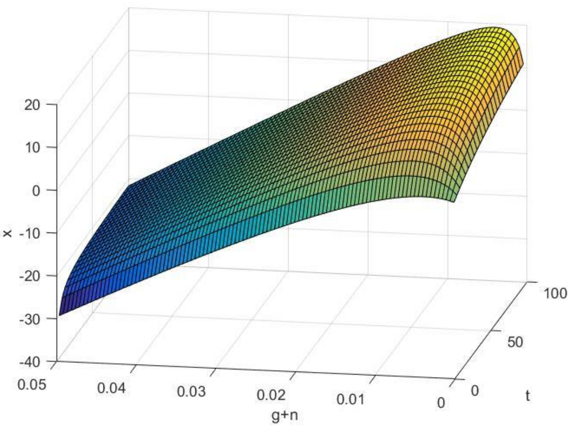


Figure 9.

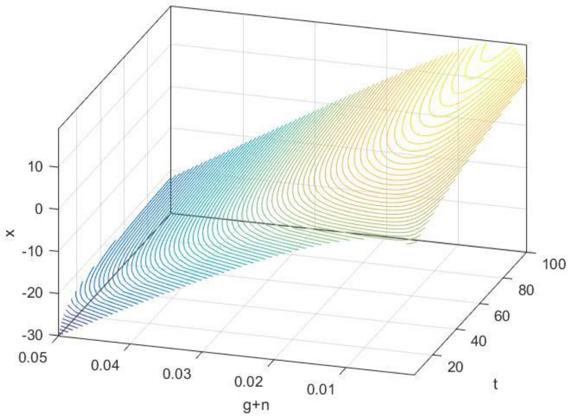


Figure 10.

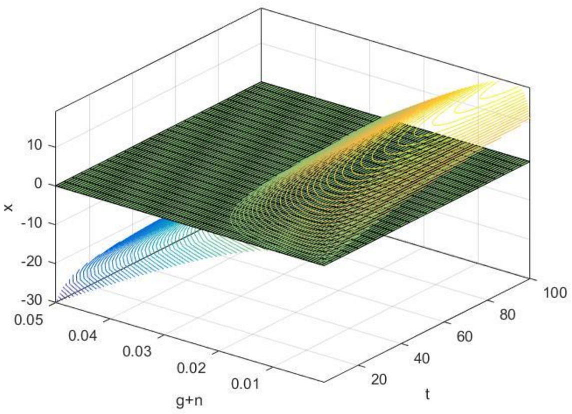


Figure 11.

Figure 9 shows the increased money from outside increases with $t$ value, and increases with $g+n$ when it is very small but decreases with it when it is large. Figure 10 provides the contour lines. Combined with 0-plane, figure 11 shows, there is the "dangerous" domain with small $g+n$ values, leaving the demand for positive quantity of the money from outside. After changing $h$ value to 20, obtain figure 12 with the smaller "dangerous" domain, suggesting a smaller $h$ is better for solve the low profit rate problem. The reason is obvious: a smaller $h$ means a lower wage cost compared to the rich household consumption level.

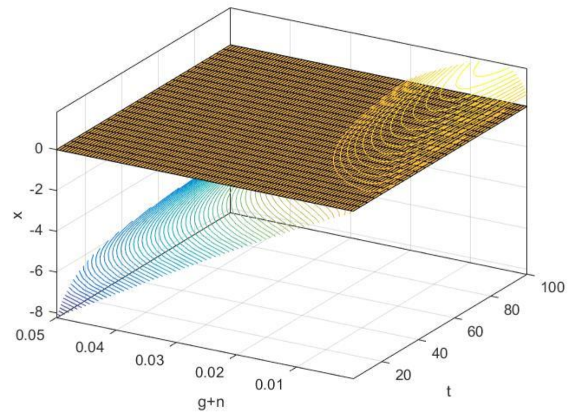


Figure 12.

Figure 13 shows the total money from outside increases with $t$ and with $g+n$. Figure 14 provides the contour lines. Combined with 0-plane, figure 15 shows, there is the "dangerous" domain with small $g+n$ values, leaving the demand for positive quantity of the money from outside.

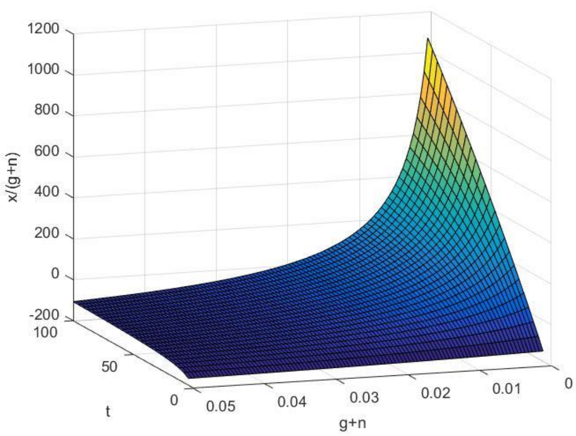


Figure 13.

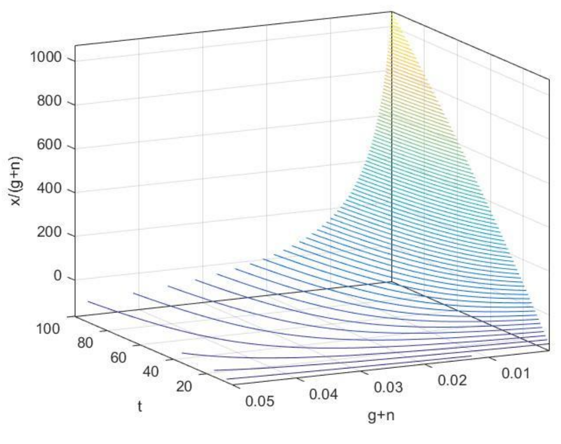


Figure 14.

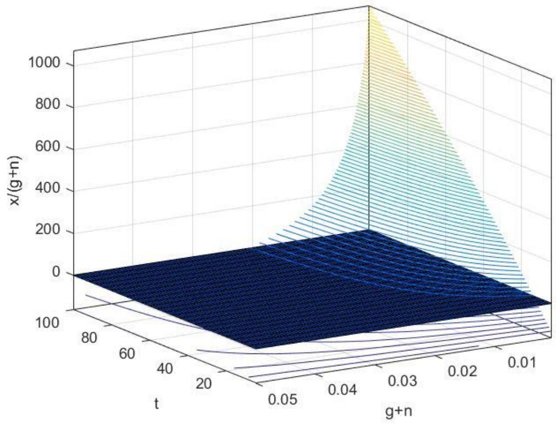


Figure 15.

## References

Yang, Jinrui. Growth, money and unemployment, inflation -a post-Keynesian model with profit rate. Working paper, School of applied economics, Renmin University of China, 2019.