Capital Share, Consumption Volatility and Long-run Redistribution Risks

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Abstract

Capital return variability is a macroeconomic factor that exhibits significant explanatory power of long-run equity prices. In short-run, capital share risks create strong volatility effects on equity premium, as redistributive shocks that shift the share of income between the wealthy and the poor are not persistent. In long-run, capital return variability is positively correlated to stockholder consumption growth prospects and captures the long-term wealth redistribution trend. Exposure to capital return variability risks generates over a half of cross-sectional equity return variations.

Keywords: Asset Pricing, Capital Share, Consumption Volatility, Recursive Utility, Long-run Risks.

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1 Introduction

Leading asset pricing theories frequently assume a single representative agent when seeking to model expected returns. However, stock returns are considerably more volatile than aggregate consumption growth. This empirical observation is a cornerstone of the equity premium puzzle, see Mehra and Prescott (1985) and Breeden et al. (2014).

Seeking for explanations of this equity premium puzzle, Bansal and Yaron (2004) propose a long-run risk (LRR) framework to account for economic uncertainty and long-run growth prospects. Long-run risk literature frequently assumes a single representative agent when seeking to model expected returns, see Bansal and Yaron (2004), and shows strong explanatory power of key asset market phenomena. Recent works, such as Schorfheide et al. (2018), extend the LRR framework by investigating the data generation process (DGP) of aggregate consumption growth uncertainty. However, the DGP of consumption growth is analysed and tested in ad-hoc manners. Also, aggregate consumption growth performs poorly in asset pricing models. Other studies, for example, Campbell et al. (1993) emphasize a focus upon heterogeneous agent asset price model, since stockholder consumption is more representative than non-stockholder consumption for pricing equity returns. Toda and Walsh (2019) also highlight that rising wealth holdings of the richest one percent predict excess stock returns.

Therefore, in this paper, I use stockholder consumption instead of aggregate consumption. I incorporate stockholder consumption growth dynamics into LLR and explain stockholder consumption growth uncertainty as the elevated consumption growth volatility of the wealthy over the poor, as the wealth heterogeneity plays an important role in asset pricing models. Following Lettau et al. (2019), I call the wealthy as stockholders and the poor as labour workers. I use growth of capital share, which is defined as the capital income over the aggregate income, to portray stockholder consumption growth. Capital share fluctuations account for limited stock market participation and proxy the concentration of wealth. Also, changes of capital share capture the redistribution risks between the wealthy (stockholders) and the poor (non-stockholders), and, therefore is tightly linked to the difference between stockholder and labour worker consumption dynamics. Recent work by Lettau et al. (2019) adopts heterogeneous agents and proposes capital share growth as a risk factor. It shows that capital share growth explains expected equity returns, and empirically dominates aggregate consumption growth and the Fama and French's (1993) factors (Lettau et al., 2019).

The construction of the stockholder consumption growth is consistent with empirical evidences. Using United States wealth distribution data from Saez and Zucman (2016), Lettau et al. (2019) identify that stockholders' and labour workers' consumption behaviours respond differently to capital share growth. Based on Lettau et al. (2019), I model the elevated stockholder consumption growth volatility through capital share growth and test its asset pricing implications.

Firstly, this paper relaxes model assumptions in Lettau et al.'s (2019) capital share model and Bansal and Yaron 's (2004) long-run risk framework. Instead of assuming the stockholder consumption as capital share times the aggregate consumption as in Lettau et al. (2019), this paper constructs the stockholder consumption growth dynamics based on empirical facts. Also, this paper representative agent assumption in Bansal and Yaron (2004) by focusing on the consumption growth of stockholders and omitting labour worker consumption dynamics. Secondly, based on Epstein and Zin (1989) type recursive utility, this paper analyses the elevated stockholder consumption volatility impacts separately under short-run and long-run expectations. It shows that elevated stockholder consumption volatility generates short-run equity return volatilities, while captures long-run stockholder consumption growth prospects.

This paper finds that capital share growth has strong volatility effect on the short-run equity premium. By focusing on stockholder consumption growth, I find that elevated stockholder consumption volatility carries asset pricing powers through capital share growth. Intuitively, stockholders and labour workers are faced with identical long-run economic equilibrium path and a common consumption growth trend. However, the consumption growth volatility of stockholders differs from that of labour workers, while capital share growth is found to enter the elevated stockholder consumption volatility. Given that stockholders consume primarily out of their wealth, capital share growth serves an indicator of redistribution risks. In adjacent periods, individuals do not expect consumption shocks due to consumption smoothing. Stockholder consumption shocks associated with the wealth redistribution are unexpected in short-run, and, therefore, investors do not claim redistribution associated risk premiums. In this case, capital share growth only affects the volatility of the short-run equity premium.

I proposes a capital return variability (CRV) as a risk factor to capture the long-term risk-return relationship between redistribution risks and the equity premium. In the long-run, the volatility effect of capital share growth accumulates and generates significant risk prices. Intuitively, the wealth redistribution in the long-run falls within investor's expectations, as the wealth accumulation rate of richest cohorts is much higher than that of the poor (Saez and Zucman, 2016). The predictable capital return variability implies better long-run growth prospects of the stockholder consumption and, therefore, raises equity prices. Therefore, as a proxy for accumulated capital share volatility effects, CRV captures expected excess stockholder consumption growth and explains long-term redistribution risks. In asset pricing tests, CRV explains over a half of cross-sectional return variations. Also, CRV outperforms and strongly dominates the capital share growth factor of Lettau et al. (2019). Therefore, my theoretical predictions are justified by empirical evidences. My findings are also in line with Lettau et al. (2019) as the long-term capital share growth outperforms its short-term counterparts.

This paper is structured as follows. Section 2 presents a theoretical asset pricing model with heterogeneous agents, in which stockholder consumption volatility operates through capital share growth. Section 3 conducts an empirical analysis of the theoretical model. This section includes data, empirical methodologies and evidences on the asset pricing performance of capital share growth and the capital return variability factor. Finally, Section 4 provides concluding remarks of this paper.

2 Theoretical Framework

In this section, I present an asset pricing model that incorporates the capital share risks through elevated stockholder consumption volatility. This model combines the heterogeneous model by Lettau et al. (2019) and the long-run risk framework by Bansal and Yaron (2004).

This paper models a stock market equilibrium which only considers stockholders, since labour workers are absent from the market. I assume that stockholders have Epstein and Zin (1989) type utility function to disentangle risk attitude from the degree of intertemporal substitutability, and separate the utility impact instantaneous consumption from the long-run expectation of consumption (Epstein and Zin, 1989). Also, the assumption that stockholders consume all their capital income can be relaxed by only considering stockholder consumption growth. Intuitively, stockholders are highly likely to have non-zero reinvestment rate and self-finance their consumption, given the limited saving of labour workers and high wealth accumulation of stockholders (Saez and Zucman, 2016). Empirical evidences also show that the consumption-wealth ratio is a function of expected equity returns (Lettau and Ludvigson, 2001a). Therefore, stockholders do not consume all income and adjust their consumption share in account for expected equity returns.

2.1 The Stockholder Consumption Growth

In this subsection, I construct the stockholder consumption growth according to empirical evidences. Going beyond the homogeneous agent model of Bansal and Yaron (2004), the stockholder consumption growth g_t^s is underpinned by the particular consumption patterns of heterogeneous agents.[1] According to Lettau et al. (2019), the consumption growth of stockholders is more volatile than that of those labour workers who derive income from wages. Intuitively, the top of the wealth distribution has a larger discretionary consumption on

[1]In our paper, g_t^s and g_t^w denotes the consumption growth of stockholders and labour workers calculated from $g_t^n = C_{t+1}^n / C_t^n - 1, n \in \{s,\ w\}$, respectively. C_t^s and C_t^w are the time-t consumption of stockholders and labour workers, respectively.

luxury goods (Ait-Sahalia et al., 2004). Stockholders' large discretionary consumption linked to volatile asset prices, while labour workers spend a larger proportion on the same essential goods each month.

Capital share captures the proportion of stockholder income in the economy. Therefore, capital share growth is strongly and positively correlated with the stockholder consumption growth, while strongly negative correlated with the labour worker consumption growth (Lettau et al., 2019). Accordingly, I derive the aggregate consumption growth $\bar{g}_t$ as the weighted average of consumption growth of labour workers and stockholders:

$$\bar{g}_t = w^s g_t^s + (1 - w^s) g_t^w \tag{1}$$

w^s denotes the stockholder population weight. g_t^s and g_t^w are the consumption growths of stockholders and labour workers, respectively.

The consumption growth volatility of stockholders is higher than that of average households in economy. Therefore, I adopt the aggregate consumption as a benchmark for modelling stockholder consumption volatility. The relationship between stockholder (g_t^s) and aggregate ($\bar{g}_t$) consumption growths is defined as follows:[2]

$$g_t^s = \bar{g}_t + \frac{1}{w^s} g_t^k \xi_t \tag{2}$$

where g_t^k is capital share growth. The stochastic term $\xi_t \sim N_{i.i.d}(0, \Sigma)$ captures the elevated consumption growth volatility of the stockholder consumption compared to aggregate consumption, as the top wealth consumption is more volatile. $g_{KS,t}$ is capital share growth. Σ denotes a constant variance for ξ_t. The ξ_t term, therefore, also defines the variance of the consumption distribution of the economy. This paper does not make an autocorrelation assumption for ξ_t to avoid possible explosive growth. Equation (2) is consistent with Mankiw and Zeldes's (1991) finding that stockholder consumption is more volatile than the aggregate consumption.

[2]Based on the data correlations identified by Lettau et al. (2019), I adopt capital share growth g_t^k as a multiplier directly to capture its volatility effect on stockholder consumption growth. The robustness test of equation (2) is in Appendix. The labour worker consumption growth is defined as $g_t^w = \bar{g}_t(1 - \frac{1}{1-w^s} g_t^k \xi_t)$ according to equations (1) and (2).

A positive capital shock increases stockholder consumption growth. This shock is absorbed by the labour workers since labour workers adjust their consumption according to decreased relative labour income level. The overall volatility generated by stockholder and labour worker consumption growths must be consistent with the volatility pattern of aggregate consumption $\bar{g}_t$. I ignore the labour worker consumption growth in the theoretical model since labour workers do not invest. Overall, the aggregate consumption growth $\bar{g}_t$ remains on its equilibrium path. The stockholder population weight w^s does not affect the analysis in following sections, given any percentage at the top can be used to illustrate how the concentration of wealth affects the intensive margin of the stock market (Lettau et al., 2019).[3]

The stockholder consumption contains a persistent component, as aggregate consumption growth $\bar{g}_t$ enters equation (2)[4] Following Bansal and Yaron (2004), I define the aggregate consumption growth rate $\bar{g}_{t+1} = \mu + x_t + \sigma\eta_{t+1}$, where x_t is the predictable term. Therefore, the stockholder consumption growth in equation (2) can be rewritten as:

$$g^s_{t+1} = \mu + x_t + \frac{1}{w^s} g^k_t \xi_t + \sigma\eta_{t+1} \tag{3}$$

The time-varying uncertainty of stockholder consumption growth is defined as $\frac{1}{w^s} g^k_t \xi_t + \sigma\eta_{t+1}$, which is a function of capital share growth g^k_t. Also relevant to our model is the consumption volatility risk (CVR) factor derived by Boguth and Kuehn (2013). In the theoretical motivation of their volatility risk factor, the consumption growth is assumed to switch between high and low volatility states according to agent's beliefs. In our model, instead of assuming a Markov switching process based upon changing beliefs, volatility is explicitly modeled using a capital share factor.

[3]The wealth-weighted participation rate is lower than the aggregate participation rate, regardless which quantile of wealth distribution is selected as a benchmark (Lettau et al., 2019).

[4]The DGP of stockholder consumption growth is consistent with the assumption of Bansal and Yaron's (2004) long-run risk framework.

2.2 Capital Return Uncertainty and Long-run Risks

In this subsection, I combine the stockholder consumption growth in equation (3) with the long-run risk model proposed by Bansal and Yaron (2004). In line with the assumption by Lettau et al. (2019), labour workers do not influence equity prices and, consequently, they are independent from the stock market and their participation is not modeled.

To solve the relationship between equity returns and capital share growth, this paper extends the model of Bansal and Yaron (2004) to derive the equity premium explicitly. I develop a hybrid model of the constant volatility case (Case I) and the time-varying volatility case (Case II) of Bansal and Yaron (2004). The uncertainty in the stockholder consumption growth contains both σ that captures economic fundamental uncertainties and $\frac{1}{w^s} g_t^k \xi_t$ that captures time-varying uncertainty associated with capital returns.

Consider a representative stockholder with Epstein and Zin (1989) type recursive preferences. Based upon the recursive preference utility function, the asset pricing restrictions for gross return $R_{i,t+1}$ satisfy

$$E_t[\delta^\theta (G^s_{t+1})^{-\frac{\theta}{\psi}} R_{a,t+1}^{-(1-\theta)} R_{i,t+1}] = 1 \tag{4}$$

where $\theta = (1-\gamma)/(1-\frac{1}{\psi})$. In the following parts of this paper, expectations are conditional on the stockholders' information set, which is omitted in equations for simplifying the model. In equation (4), G^s_{t+1} is the stockholder consumption growth. $R_{a,t+1}$ is the gross return on an asset that generates dividends that cover the aggregate stockholder consumption. $0 < \delta < 1$ is the time discount factor, $\gamma \geq 0$ is the risk-aversion parameter, and $\psi \geq 0$ is the intertemporal elasticity of substitution (IES). $logG^s_{t+1} = g^s_{t+1}$ holds by a standard Taylor approximation.[5]

Given the asset pricing constraint in equation (4), the intertemporal marginal rate of substitution (IMRS) is:

$$m_{t+1} = \theta log\delta - \frac{\theta}{\psi} g^s_{t+1} + (\theta - 1) r_{a,t+1} \tag{5}$$

where g^s_{t+1} and $r_{a,t+1}$ are the natural logarithm of G^s_{t+1} and $R_{a,t+1}$, respectively.

[5] $G^s_{t+1} = \frac{C^s_{t+1}}{C^s_t}$ where C^s_t is the stockholder time-t consumption.

We also adopt the standard approximation proposed by Campbell and Shiller (1988b) to derive the functional form of the equity premium. The innovation of log gross consumption $r_{a,t+1}$ and log market return $r_{m,t+1}$ are assumed to follow:

$$r_{a,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + g_{t+1} \tag{6}$$

$$r_{m,t+1} = \kappa_{0,m} + \kappa_{1,m} z_{m,t+1} - z_{m,t} + g_{d,t+1} \tag{7}$$

where z_t is the log price-consumption ratio ($log(\frac{P_t}{C_t})$) and $z_{m,t}$ is the log price-dividend ratio ($log(\frac{P_t}{D_t})$).[6]

I incorporate the stockholder consumption growth in equation (3) into the long-run risk framework and set $g_{t+1} = g^s_{t+1}$ for simplifying notations. In my model, the stock market is driven by a persistent growth component (x_{t+1}), economic fundamental uncertainty (σ), capital share growth (g^k_t) and elevated stockholder consumption volatility (ξ_{t+1}). The system is as follows:

$$\begin{aligned}
x_{t+1} &= \rho x_t + \phi_e \sigma e_{t+1} \\
g_{t+1} &= \mu + x_t + \frac{1}{w^s} g^k_{t+1} \xi_{t+1} + \sigma \eta_{t+1} \\
g_{d,t+1} &= \mu_d + \phi x_t + \phi_d \sigma_{d,t+1} u_{t+1} \\
& e_{t+1}, u_{t+1}, \eta_{t+1} \sim N_{i.i.d.}(0,1) \qquad \xi_t \sim N_{i.i.d.}(0, \Sigma)
\end{aligned} \tag{8}$$

where $g_{d,t+1}$ is the dividend growth rate, and ρ is the persistence of the expected growth rate process. Parameters μ and μ_d are the constant component of g_{t+1} and $g_{d,t+1}$, respectively. Following Bansal and Yaron (2004), I set $\phi_e > 1$ and $\phi_d > 1$. The parameter ϕ can be interpreted as the leverage ratio on expected consumption growth, see Bansal and Yaron (2004) and Abel (1999). The stochastic error terms e_{t+1}, u_{t+1}, and η_{t+1} are independent from each other (Bansal and Yaron, 2004). σ is a constant which captures the volatility of x_{t+1} and g_{t+1}.[7]

[6] D_t denotes the dividend.

[7] Bansal and Yaron (2004) Case II adds time varying volatility and fluctuating economic uncertainty into their model through a general error term. Our model does not assume a stochastic innovation of σ in order to isolate the volatility effect generalized by the introduction of the capital share factor.

The dividend growth volatility is correlated with consumption growth volatility, as suggested by Bansal and Yaron (2004). Therefore, $\sigma_{d,t+1}$ is assumed to be partially correlated with both $g^k_{t+1}\xi_{t+1}$ and σ in our model.[8] Intuitively, the level change of capital share represents the capital return fluctuations. As important sources of capital returns, dividends should be correlated with the magnitude of capital share when the labour supply is constant. Therefore, the uncertainty of dividend returns should be correlated with capital share growth.

The innovation of $g_{d,t+1}$ which is found to be more volatile than g_{t+1} (Campbell, 1999) is tackled by ϕ_d. Prices are partially myopic to future fundamentals but very sensitive to capital flows in inelastic markets (Gabaix and Koijen, 2020). Our model therefore assumes the capital share growth operates in the volatility of the dividend growth $g_{d,t+1}$. We formalises uncertainty in terms of the impact of high income consumption variability, rather than a generic uncertainty as set out by Bansal and Yaron (2004).

In addition, this paper defines the dynamics of capital share growth to solve the equity premium. According to Lettau et al. (2019), the capital share growth follows an AR(1) process:[9]

$$g^k_{t+1} = \rho^k g^k_t + e^k_{t+1} \tag{9}$$

where e^k_{t+1} captures unexpected shocks and ρ^k captures the persistence of capital share growth. Since the consumption growth g_t, dividends growth $g_{d,t}$, and the capital share growth are exogenous processes in my model, the functional form of the innovation of consumption return, the pricing kernel, and equity returns in this economy can be derived explicitly using equations (5)-(9).[10]

Following Bansal and Yaron (2004), I derive solutions for the log price–consumption ratio z, t and the log price–dividend ratio $z_{m,t}$ to characterize the returns $r_{a,t+1}$ and $r_{m,t+1}$. z_t and

[8]The specification of $\sigma_{d,t+1}$ also relaxes the setting by Bansal and Yaron (2004) Case II, in which g_{t+1} and $g_{d,t+1}$ are cointergated, to be consistent with empirical literature (Campbell and Cochrane, 1999).

[9]The constant is not significant according to our AR(1) estimation.

[10]Detailed proofs are provided in the Appendix.

$z_{m,t}$ are assumed to satisfy $z_t = A_0 + A_1 x_t + A_2 \xi_t$ and $z_{m,t} = A_{0,m} + A_{1,m} x_t + A_{2,m} \xi_t$.[11] The relevant state variables in solving for the equilibrium are x_t and ξ_t. We modify the functional form of the log price-consumption and log price-dividend ratios assumed by Bansal and Yaron (2004) to include the time-varying part of stockholder consumption growth volatility.[12]

I first solve the parameters of the persistent consumption growth x_t and excess volatility ξ_t on price-consumption and price-dividend ratios, which track expected risk prices (Campbell and Cochrane, 2000). In my model, the resulting A_1 and $A_{1,m}$ are identical to Bansal and Yaron (2004). The sensitivity of the price-consumption (and price-dividend) ratio to the excess volatility ξ_t is constant over time. A_2 (and $A_{2,m}$) are constants when holding w^s, ρ^k and g_t^k constant:[13]

$$A_2 = \frac{1 - \frac{1}{\psi}}{w^s(1-\kappa_1)} \rho^k g_t^k \tag{10}$$

$$A_{2,m} = -\frac{\rho^k}{w^s \psi (1-\kappa_{1,m})} g_t^k \tag{11}$$

$A_{2,m}$ is always negative in my model. This finding is consistent with Bansal and Yaron (2004) that a rise in economic uncertainty lowers the price–consumption ratio and increases risk prices. Notice that, in this paper, the IES of stockholders is not necessarily to be greater than 1 to ensure a positive effect of rising capital return uncertainty on equity premiums.[14]

A non-zero $A_{2,m}$ is based upon the assumption that the capital share growth is sufficiently persistent over time. Lettau et al. (2019) find that the long-term capital share growth has strong pricing power when ρ^k is statistically insignificant from 1. This paper investigates the importance of capital share growth persistence in the asset pricing performance. I conduct the Lettau et al.'s (2019) F-MB test on the 2, 4, 6, 8, 10 and 12-month capital share growth and plot the results in Figure 1. Also, I estimate an AR(1) model of the 2, 4, 6, 8, 10 and 12-month capital share growth. As shown in Figure 1, the R^2 of capital share growth increases

[11] A_0 and $A_{0,m}$ are constants; A_1 and $A_{1,m}$ are parameters of the persistent consumption growth component x_t; $A_{2,t}$ and $A_{2,m}$ are parameters of the excess volatility ξ_t

[12] See Bansal and Yaron (2004) Case II.

[13] A_2 and $A_{2,m}$ are derived in Online Appendix.

[14] The sign of $A_{2,m}$ is independent from IES and is always to be negative.

as we increase the growth horizon. The AR(1) coefficients of the 2, 4, 6, 8, 10 and 12-month capital share growth are 0.702, 0.833, 0.903, 0.927, 0.938 and 0.954, respectively. Therefore, the pricing power increases as the persistence of capital share growth raises. This empirical finding implies that the persistence of capital share growth is important for a constant $A_{2,m}$. When capital share growth is not sufficiently persistent, its impact on price-consumption ratio will not be priced, which explains the poor performance of short-term capital share growth as in Lettau et al. (2019).

According to the parameters of excess volatility in equations (10) and (11), given the stochastic nature of ξ_t, the capital share growth does not have an impact on the magnitude but affects the uncertainty of the price-consumption and price-dividend ratios.

2.3 Short-run versus Long-run Expectations

In this subsection, I derive equity premiums under short-run and long-run expectations, respectively.[15] With Epstein-Zin type recursive utilities, equity premiums are affected by both the covariance between asset returns and instantaneous consumption growth, and that between asset returns and the long-run consumption growth (Gârleanu and Panageas, 2020). Therefore, I now set out equity returns in the short-run and the long-run. The difference between these two settings is due to the difference between short-run and long-run expectations of ξ_{t+1}. Conditioning on information at t, the short-run (conditional) expectation $E_t(\xi_{t+1}) = \xi_t$ due to smoothed consumption and the locally deterministic instantaneous consumption growth as in Gârleanu and Panageas (2020), while the long-run (unconditional) expectation of ξ_{t+1} is zero.

I start from the short-run case. Conditional on information at time t, the stockholder consumption is expected to be locally deterministic. Therefore, the short-run innovation of the pricing kernel m_{t+1} is:

$$m_{t+1} - E_t(m_{t+1}) = \lambda_\eta \sigma \eta_{t+1} + \lambda_e \sigma e_{t+1} + \lambda_{\xi,t+1} \xi_{t+1} \tag{12}$$

[15] Full details are in the Online Appendix.

The short-run innovation of market return $r_{m,t+1}$ is:

$$r_{m,t+1} - E_t(r_{m,t+1}) = \phi_d \sigma_{d,t+1} u_{t+1} + \lambda_{m,e} \sigma e_{t+1} + \lambda_{m,\xi} \xi_{t+1} \tag{13}$$

In equations (12) and (13), $\lambda_{m,e}$, λ_η and λ_e are constants, while $\lambda_{m,\xi}$ and $\lambda_{\xi,t+1}$ are functions of e^k_{t+1}.[16] Therefore, the conditional pricing kernel innovation in equation (12) is only correlated to unexpected capital share growth e^k_t, but the conditional market return innovation is correlated with capital share growth through $\sigma_{d,t+1}$.

The equity premium is determined by the conditional covariance between the return $r_{m,t+1}$ and the SDF m_{t+1} (Bansal and Yaron, 2004). Following Bansal and Yaron (2004), the time-varying equity premium in the presence of short-run consumption uncertainty is:

$$\begin{aligned} E_t(r_{m,t+1} - r_{f,t}) &= -(\lambda_{m,e}\lambda_e - 0.5\lambda^2_{m,e})\sigma^2 + 0.5\phi^2_d \sigma^2_{d,t+1} + E_t(\lambda_{m,\xi}\lambda_{\xi,t+1} - 0.5\lambda^2_{m,\xi}) \\ &= -(\lambda_{m,e}\lambda_e - 0.5\lambda^2_{m,e})\sigma^2 + 0.5\phi^2_d \sigma^2_{d,t+1} \end{aligned} \tag{14}$$

where $r_{m,t+1}$ is the market return rate and $r_{f,t}$ is the risk free rate. At time t, the short-run expectation $E_t(\xi_{t+1}) = \xi_t$, so the effect of predictable capital share growth is omitted in equation (14). Therefore, $E_t(\lambda_{m,\xi}\lambda_{\xi,t+1} - 0.5\lambda^2_{m,\xi}) = 0$ holds. As shown by equation (14), the conditional equity premium is constant in the short-run and has one source of systematic risk that relates to the expected consumption growth volatility σ^2. However, the capital share factor enters the innovation of market return in equation (13). Hence, the elevated consumption volatility of stockholders, through capital share growth, is linked to the equity return volatility. The unexpected stockholder consumption volatility is perceived as a fraction of the systematic uncertainty σ under short-run expectations.

Under long-run expectations, the consumption growth uncertainty of stockholders is identical to the fundamental uncertainty of the economy. Therefore, $E(\xi_t) = 0$ in this case. The long-run innovation of the pricing kernel is as follows:

$$m_{t+1} - E(m_{t+1}) = \lambda_\eta \sigma \eta_{t+1} + \lambda_e \sigma e_{t+1} + \lambda^u_{\xi,t+1} \xi_{t+1} \tag{15}$$

[16] Details of $\lambda_{m,e}$, λ_η, λ_e, $\lambda_{m,\xi}$ and $\lambda_{\xi,t+1}$ are in Appendix.

The long-run innovation of market return is:

$$r_{m,t+1} - E(r_{m,t+1}) = \phi_d \sigma u_{t+1} + \lambda_{m,e} \sigma e_{t+1} + \lambda^u_{m,\xi} \xi_{t+1} \tag{16}$$

Detailed functional forms of the parameters in equations (15) and (16) are in the Appendix. Using equations (15) and (16), the long-run equity premium is calculated as:

$$E(r_{m,t+1} - r_{f,t}) = -(\lambda_{m,e}\lambda_e - 0.5\lambda^2_{m,e} - 0.5\phi^2_d)\sigma^2 + E\left[\lambda^u_{m,\xi}\lambda^u_{\xi,t+1} - 0.5(\lambda^u_{m,\xi})^2\right] \tag{17}$$

where $E\left[\lambda^u_{m,\xi}\lambda^u_{\xi,t+1} - 0.5(\lambda^u_{m,\xi})^2\right]$ is positively correlated with $E\left[(g^k_{t+1})^2\right]$.[17] Under long-run expectations, the equity premium is a function of fluctuations in expected consumption growth σ^2 and capital return variability $E\left[(g^k_{t+1})^2\right]$.

Overall, due to constant excess volatility between two adjacent periods, capital share growth does not shift the expected rate of return under short-run (conditional) expectations. However, in the long-run, volatility shocks fail to feature in expectations and the increased uncertainty of returns generate redistribution risks between stockholders and labour workers.

2.4 The Capital Return Variability Factor

In this subsection, I propose a new asset pricing factor, namely the capital return variability factor (CRV), as a long-run proxy for the capital return impact on the equity return volatility. As shown by equation (17), the CRV ($E\left[(g^k_{t+1})^2\right]$) enters the equity premium under long-run expectations. The unexpected shocks of capital return associated stockholder consumption uncertainty, captured by e^k_t in equation (17), is very small and gets magnified under long-run expectations because of the long-lasting nature of the volatility shock (Bansal and Yaron, 2004). Therefore, only the predictable component of capital share growth has asset pricing powers.

Intuitively, the ratio of the conditional risk premium to the conditional volatility of the market portfolio fluctuates with consumption volatility (Bansal and Yaron, 2004). The maximal

[17] Details of $\lambda_{m,e}$, λ_η, λ_e, $\lambda^u_{m,\xi}$ and $\lambda_{\xi,t+1}$ are in Appendix. The functional form of $E\left[\lambda^u_{m,\xi}\lambda^u_{\xi,t+1} - 0.5(\lambda^u_{m,\xi})^2\right]$ is in equation (B.48).

Sharpe ratio approximated by volatility of the pricing kernel innovation also varies with consumption volatility. In my model, the stockholder consumption volatility operates through capital share growth. Therefore, risk prices will rise as economic uncertainty rises.

The model calibration of Bansal and Yaron (2004) indicates that the stockholder IES is greater than 1. Also, Ogaki and Atkeson (1997) and Andreasen and Jørgensen (2020) find stockholders that are wealthy and relatively less risk averse tend to have higher IES than labour workers. Therefore, the coefficients of capital share growth in parameters $A_{2,t}$ in equation (10) and $A_{2,m}$ in equation (11) are both negative, which ensures that capital share growth is negatively correlated with the uncertainty in the price-consumption and the price-dividend ratio. In response to lower expected equity return uncertainty, asset demand rises to generate positive risk prices of CRV. The utility study of Colacito et al. (2018) also highlights that increased macroeconomic volatility increases the stochastic discount factor under the recursive utility framework, thus rises expected returns and generates a positive volatility risk price.

The model specification of Boguth and Kuehn's (2013) consumption volatility risk factor is a potential explanation of the nonlinear relationship between the equity premium and capital share growth in equation (17). Changes in beliefs about consumption growth volatility are found important in explaining unconditional equity returns by Boguth and Kuehn (2013), which indicates that the assumption of two volatility states is reasonable. My model can be alternatively explained by assuming infinite states of stockholder consumption growth volatility. At each time t, consumption growth volatility has only two latent states, but ξ_t is an unknown stochastic variable and, hence, this is a setup that is consistent with the quadratic relationship between equity returns and capital share growth in my model.

3 Empirical Analysis

In this section, I present empirical justifications of the theoritical model in Section 2. My theoretical model provides following asset pricing implications. Under short-run expectations, capital share growth captures the market volatility and is not priced. Under long-run expectations, capital return variability (CRV) proxies the impact of elevated stockholder consumption growth uncertainty on equity returns. Therefore, the capital return variability (CRV) is a long-run risk factor.

The empirical analysis by Lettau et al. (2019) does not allow for time variation of the capital share parameters. However, empirically risk factor loadings may vary over time, see Jensen (1968), Jagannathan and Wang (1996) and Lewellen and Nagel (2006). For instance, the static CCAPM fails to capture the effect of time-varying investment opportunities (Lettau and Ludvigson, 2001b). The non-zero unconditional price anomalies do not necessarily indicate non-zero conditional alphas, given time-varying factor loadings that are correlated with the equity premium or market volatility (Lewellen and Nagel, 2006).

Therefore, I use both conditional (parameters are time-varying) and unconditional (parameters are time-invariant) estimations to measure the short-run and long-run factor risk exposures, respectively. For testing the short-run case, I use a rolling-window multiplicative GARCH approach to test if the capital share growth is significant in equity return variance, and the Bayesian time-varying beta with stochastic volatility (B-TVB-SV) estimation from Bianchi et al. (2017) to estimate risk price of the capital share growth. For testing the short-run case, I adopt the bootstrapped Fama-MacBeth (FMB) procedure that is identical to Lettau et al. (2019).

3.1 Data

In this subsections, I specify the data used for empirical tests. Capital share is calculated as one minus labour share. Labour share data used in this paper is the nonfarm sector labor share, which is identical to that used by Lettau et al. (2019) and Gomme and Rupert (2004). Lettau et al. (2019) use quarterly capital share and quarterly portfolio returns converted from monthly data. In this paper, instead of modifying monthly to quarterly returns in a relatively ad-hoc manner, I interpolate the capital share using a reasonable indicator to reduce information loss.[18] The theoretical model addresses the importance of the capital share growth persistence in pricing asset returns. Therefore, I first test capital share growth over different horizons and select the one with highest pricing power. The construction of capital return variability (CRV) factor is based on the selected long-term capital share growth.

3.1.1 Long-term Capital Share Growth and Capital Return Variability

This part reports the construction of long-term capital share growth and capital return variability. Long-term capital share growth is adopted to partial out the measurement error effect, as measurement error leads to biased estimation of CAPMs (Lettau et al., 2019).[19]

Following Lettau et al. (2019), the H-period capital share growth tested in this paper is:

$$g_t^k = \frac{k_t}{k_{t-H}} \tag{18}$$

where k_t denotes time-t capital share. In the test of the capital share growth, Lettau et al. (2019) compare H = 1, 4, 8, 12 and 16-quarter cases and select the one with highest asset pricing powers. The 4-quarter capital share growth is found to have the highest pricing power. Under the monthly frequency setting, I conduct similar test as in Lettau et al. (2019). [20] The results are plotted in Figure 3. As shown by the Figure 3, the 12 and 24-month capital share growth both have the highest R^2. Following Lettau et al. (2019), I adopt 12-month capital share growth as the risk factor in this paper.

[18]The monthly capital share is obtained by the Chow-Lin interpolation. Detailed capital share interpolation is in Appendix. Data for constructing capital share and the Chow-Lin indicator is from FRED.

[19]During the data collection process, the filtering approach introduces measurement error problem.

[20]In the first stage of F-MB regression, I regress the 3, 12, 24, and 36-month capital share growth on 3, 12, 24, and 36-month size/BM sorted portfolio returns, respectively. The second stage is identical to a standard F-MB regression.

According to the long-run case of equity premium in equation (17), CRV, also denoted as $(E(g_k)^2)$, enters the unconditional mean equation of equity returns. CRV is constructed based on the AR(1) innovation process of g_k as in Lettau et al. (2019):

$$g^k_{t+1} = \rho^k g^k_t + e^k_{t+1} \tag{19}$$

where e^k_{t+1} captures unexpected shock in capital share growth. The magnitude of the estimate of ρ^k is 0.947, which is statistically indifferent from 1. The innovations of capital share growth and CRV are plotted in Figure 2. The summary statistics of the capital share factor and CRV are reported in Table A10 in the Appendix.

3.1.2 Portfolio Returns

In this paper, capital share growth and CRV are tested on different groups of portfolio returns. The portfolio groups used include 25 size/BM, 10 long-term reverse (REV), 25 size/INV, and 25 size/OP sorted portfolio returns. Descriptive statistics of benchmark portfolio returns are reported in Appendix. For the multiplicative GARCH estimation, this paper takes cross-sectional averages of size/BM, REV, size/INV, and size/OP sorted portfolio returns respectively to mimic different market portfolios. All portfolio returns are monthly from the Kenneth R. French Data Library. Sample spans January 1964 to August 2018.

3.2 The Short-run Case

Following structure of Section 2, I first test the short-run case. This subsection reports the rolling-window multiplicative GARCH and the B-TVB-SV results. The B-TVB-SV estimates the risk prices of the capital share growth when capital share betas are allowed to move freely. The rolling-window multiplicative GARCH tests the significance of capital share growth in the time-varying variance of equity premium.

3.2.1 Rolling-window Multiplicative GARCH

The rolling-window multiplicative GARCH assumes that capital share growth influences the equity premium volatility only. The innovation of market return in equation (13) is correlated with capital share growth, while the innovation of equity premium in equation (14) is independent from capital share growth. Therefore, the rolling-window multiplicative GARCH is used to test equations (13) and (14) jointly.

Within each regression window, the asset pricing model estimated by the multiplicative GARCH is as follows:

$$r_{i,t} = \beta_{i0,t} + \epsilon_{i,t} \qquad \epsilon_{i,t} \sim N(0, \sigma^2_{i,t}) \tag{20}$$

where $Var(\epsilon_{i,t}) = \sigma^2_{i,t}$, and $\sigma^2_{i,t}$ is consistent with the form in equation (22) below.

In the short-run expectation case, the conditional variance of equity premium is correlated with the capital share growth. To test this theoretical prediction, I employ a general form for the variance equation:

$$\sigma^2_{i,t} = \gamma^k (g^k_t)^2 \tag{21}$$

where γ^k is the coefficient of squared capital share growth. The functional form in equation (21) is motivated by the market return innovation in equation (13), as capital share growth is an $O(n^2)$ addend in the variance equation $\sigma^2_{i,t}$.

Following Judge et al. (1988), I rewrite equation (21) as follows to ensure $\sigma^2 \geq 0$ holds:

$$\sigma^2_{i,t} = exp\left[\lambda_0 + \lambda_1 log\left((g^k_t)^2\right)\right] \tag{22}$$

Due to the limitation of maximum likelihood convergence, 60 and 90-month window lengths are selected by this paper for the GARCH estimates. The magnitude of the GARCH estimates are not of interest. However, significant GARCH coefficients can justify that capital share growth has variance effects. $log\left((g^k_t)^2\right)$ in the variance equation (22) is expected to be significant over time if equations (13) and (14) hold. Also, the significance of GARCH coefficients is expected to decrease as the window length increases.

The 60 and 90-month rolling-window GARCH estimates for capital share growth are plotted in Figures 4 and 5, respectively. In Figure 4, 60-month GARCH estimates show that capital share growth is always significant in the equity premium variance. However, in Figure 5, the 90-month GARCH estimates are less significant, which is expected given that the capital return variability fells within stockholders' expectations over horizons that is sufficiently long. The magnitude of 60-month GARCH estimates are significant and stable. Therefore, the capital share growth has a strong impact on the variance equation of equity premium in the short-run. This variance effect dominates the mean effect under conditional estimations, which empirically justifies the short-run innovation of market return in equations (13) and equity premium in equation (14).

3.2.2 Time-Varying Beta With Stochastic Volatility

I now test the short-run equity premium in equation (14) independently. Following Bianchi et al. (2017), I adopt the Bayesian time-varying beta with stochastic volatility (B-TVB-SV) model to estimate capital share growth in the mean equation of the short-run equity premium. This method allows the factor loadings varying over time and partials out the volatility effects of capital share growth. Capital share growth is expected to be insignificant when the econometric model specification fully captures time variation of factor loadings and the stochastic volatility.

The B-TVB-SV model for asset return $r_{i,t}$ as a function of risk factors $F_{j,t}$ is:

$$r_{i,t} = \beta_{i0,t} + \sum_{j=1}^{K} \beta_{ij,t} F_{j,t} + \sigma_{i,t}\epsilon_{i,t} \qquad \epsilon_{i,t} \sim N(0,1) \tag{23}$$

Factor risk prices $\lambda_{j,t}$ are estimated by:

$$r_{i,t} = \lambda_{0,t} + \sum_{j=1}^{K} \lambda_{j,t}\beta_{ij,t} + e_{i,t} \qquad e_{i,t} \sim N(0,\tau^2) \tag{24}$$

The B-TVB-SV framework assumes the time-varying betas $\beta_{ij,t}$ and residuals in equation (23) take the following forms:

$$\beta_{ij,t} = \beta_{ij,t-1} + \kappa_{ij,t}\eta_{ij,t} \qquad j = 0, ..., K \tag{25}$$

$$ln(\sigma_{i,t}^2) = ln(\sigma_{i,t-1}^2) + \kappa_{iv,t}\upsilon_{i,t} \qquad i = 0, ..., N \tag{26}$$

where $\kappa_{ij,t}$ is the structural break of factor loading $\beta_{ij,t}$, and $\kappa_{iv,t}$ is the structural break of idiosyncratic variance $ln(\sigma_{i,t}^2)$.[21] The stochastic terms $\eta_{ij,t}$ and $\upsilon_{i,t}$ follow normal distributions with zero mean and variances q_{ij}^2 and q_{iv}^2, respectively. A $\kappa_{ij,t}$ equal to one indicates that structural breaks are present in the factor loadings, and $\kappa_{iv,t}$ equal to one indicates that structural breaks are present in the idiosyncratic variance. The advantage of including structural breaks is that the model captures discrete movements of the factor loadings. [22] Other detailed break and risk price prior specifications and sampling approaches are discussed in the Appendix.

As shown in equations (26) and (23), the B-TVB-SV estimation allows volatility change to have structural breaks and autocorrelations, incorporating variance effects of the risk factors that are assumed to enter the mean equation.[23] The B-TVB-SV approach is a robustness check for the true data dynamics of the short-run equity premium in equation (14), as a risk factor will generate significant factor loadings and risk price estimates if this factor enters the mean equation. Therefore, the distribution of capital share risk prices is expected to be centered at zero as indicated by equation (14), and the variance of this distribution should change over time, as in equation (13).

Following Bianchi et al. (2017), I use 2,000 burn-ins and 10,000 iterations of the Markov chain Monte Carlo (MCMC) to estimate a parsimonious capital share growth model. [24] All Bayesian estimates in this paper passed the Geweke (1991) convergence diagnostic.

Figure 6 plots the time-average break probabilities calculated by averaging all estimated $\kappa_{ij,t}$ in equation (25). The average break probabilities of capital share growth are around 0.427 among the four equity portfolio classes. I also plot the capital share growth loadings in Figure

[21] As specified by the B-TVB-SV model, $\kappa_{ij,t}$ is a binary variable that equals 0 or 1. Therefore, the estimated time-average break probabilities can be viewed as a structural break test (structural breaks exist when the break probability estimates are non-zero).

[22] In equation (25), the innovation of factor loading maintains the random walk properties to retain the shrinkage power of the selected prior to the largest extent. Therefore, the B-TVB-SV approach tackles factor selection automatically.Weak priors are used for the distributions of $\beta_{ij,t}$ and $ln(\sigma_{i,t}^2)$. Evidence indicates when the number of

Table 1: B-TVB-SV Risk Price Estimates

	Average	*Std.err*	*t-stat*	*p-value*	*2.5%*	*50%*	*97.5%*
				Panel A: size/BM sorted portfolios			
β_0	0.832**	0.214	5.610	0.000	−9.512	1.126	9.403
g^k	−0.017	0.197	−0.085	0.932	−7.296	−0.019	7.784
				Panel B: REV sorted portfolios			
β_0	0.652**	0.201	3.249	0.001	−9.620	0.950	8.717
g^k	0.104	0.242	0.431	0.667	−7.932	0.154	8.632
				Panel C: size/INV sorted portfolios			
β_0	0.839**	0.215	3.909	0.000	−9.404	1.176	9.406
g^k	−0.054	0.157	−0.344	0.731	−6.505	−0.015	8.066
				Panel D: size/OP sorted portfolio			
β_0	0.801**	0.216	3.707	0.000	−9.579	1.166	9.302
g^k	0.085	0.149	0.568	0.570	−6.338	0.053	8.119

Note: This table reports B-TVB-SV risk price estimates. The short-run equity premium in equation (14) is tested by including capital share growth g^k in the mean equation. Estimates in this table are robust to time variation and volatility clustering of factor loadings. Risk prices (%) in panels A, B, C and D are estimated by the capital share growth model using size/BM, REV, size/INV, and size/OP sorted portfolios, respectively. The 2.5%, 50%, and 97.5% quantiles of estimated risk price distribution are included in this table. ** and * denote significance at the 5% and 10% levels, respectively. Data used are monthly from January 1964 to August 2018.The first 10-year data in the sample is used for hyperparameter estimation. Sample estimates cover January 1974 to August 2018.

7. In this figure, the width of factor loadings is very volatile over time. Overall, loadings of capital share growth $\beta_{ij,t}$ follow a jump process with frequent structural breaks over time. Also, the capital share growth loadings for all portfolios are around zero. Factor loadings are shrunk toward zero by the weak prior when the risk factor has little effect on the level of the

variable is small (K=5), flat prior works quite well with the sparse specification and performs modest with the dense specification (Huber et al., 2020). The weak prior adopted by V-TVB-SV approach also has shrinkage effects.

[23]The prior specification of factor loading allows volatility clustering and frequent structural breaks.

[24]Following Bianchi et al. (2017), to robustify structural break estimates, this paper demeans all risk factors within both the training and the estimation samples to cancel out all potential bias caused by multicollinearity between the constant and the risk factors. The demeaned factors will not affect the results estimated by the B-TVB-SV since all level movements and moment conditions are retained in the sample.

true equity premium. These findings indicate that capital share growth performs poorly in capturing equity premium dynamics when factor loadings are allowed to change over time.

Figure 8 plots risk prices of capital share growth, and Table 1 reports numerical results. In Figure 8, the distribution of risk prices of capital share growth is centered at zero. The risk prices in Table 1 are insignificant for all portfolios. Therefore, risk prices of capital share growth are insignificant in the short-run equity premium, after ruling out the potential influence of outliers and stochastic volatility. Therefore, I conclude that capital share growth does not enter the mean equation of the short-run equity premium, which is consistent with the theoretical model in equation (14).

3.3 The Long-run Case and Capital Return Variability Factor

For testing the long-run case, I adopt the Fama-Macbeth (FMB) approaches following Lettau et al. (2019). I conduct identical FMB test as in Lettau et al. (2019), and estimate long-run risk prices of capital share growth and capital return variability (CRV). [25] Significant risk price estimates indicate that there is a risk-reward relationship between the factor tested and the equity premium. Therefore, I estimate risk prices of capital share growth proposed by Lettau et al. (2019) and CRV in equation (17) in different settings.

As shown in Section 2, the capital share growth explains the variance of the short-run equity premium, see equation (13). According to equation (17), CRV enters the long-run equity premium dynamics. The static FMB makes a strong assumption that factor loadings are time-invariant. When a risk factor enters the variance equation of the true DGP, the ordinary least squares (OLS) regression in FMB first stage will be biased due to heteroskedasticity problems. In second stage, risk price estimates of this factor will be significant due to width

[25] The FMB bootstrap is based upon the static FMB procedure, and can be used to correct both cross-sectional correlations and firm effects (Lettau et al., 2019). Lettau et al. (2019) adopt the non-overlapping block residual bootstrap for both stages of the FMB procedure. Although it is argued that utilizing the overlapping bootstrap is a more robust method, Andrews (2004) compares overlapping and non-overlapping block bootstraps, and reaches the conclusion that although the former is often favored in applications, the latter generates similar numerical results.

changes of the factor loading distribution. Therefore, in this paper, FMB tests/estimates will generate significant capital share growth prices as capital share growth has variance effects.

I first plot FMB test results for capital share growth and CRV in Figures 9 and 10, respectively. In Figure 9, due to the higher variation in monthly data, the R^2 estimates are generally lower for each portfolio class compared to the quarterly data estimates by Lettau et al. (2019). In addition, the R^2 estimated by REV sorted portfolios is 0.26 in this figure, while other R^2 estimated from monthly data deviate modestly from their quarterly counterparts. Overall, the monthly capital share growth has substantial explanatory power for expected returns. However, slope of regression lines in Figure 9 significantly deviates from 1, which implies a presence of heteroskedasticity or non-linearity. In Figure 10, although the average of R^2 estimated by CRV is lower, the R^2 estimates across equity portfolios are more stable than those of Figure 9. Also, slopes of regression lines estimated by the CRV is closer to 1 than those estimated by capital share growth. The FMB results of the CRV are more robust to heteroskedasticity or nonlinearity problems than that of capital share growth. The empirical evidences are in line with the long-run equity premium innovation, see equation 17.

I report FMB bootstrap estimates of risk prices in Table 2. In this table, Panel A reports the single capital share growth model estimates, Panel B reports the single CRV model estimates, and panel C reports a 2-factor model that includes capital share growth and the CRV for comparison.

Panel A tests the asset pricing performance of monthly capital share growth. In Panel A, all of the risk prices estimates of capital share growth (g_{KS}) are statistically significant at 5% level. For the bootstrap interval of R^2 estimates, the lower bound of R^2 for REV portfolios is 0.000, while for other portfolios are all above 0.300. Overall, the monthly data estimates for capital share growth are significant and capital share growth captures equity premium dynamics. However, the pricing power of capital share growth diminishes in higher frequency data. This finding indicates that capital share growth might be correlated with the volatility of the long-run equity premium. According to the theoretical model. capital share growth generates significant risk prices due to its non-linear relationship with the equity premium dynamics. The multicollinearity between capital share growth and CRV also explains the significance of capital share growth prices.[26]

Panel B justifies the asset pricing power of CRV. In Panel B, CRV risk prices are significant for all equity returns. The $\bar{R}^2$ estimates are stable across different portfolios and, overall, are higher than those estimated by the capital share growth model in Panel A. For REV sorted portfolios, the $\bar{R}^2$ estimate is insignificant since its lower bound of confident interval is zero. All $\bar{R}^2$ estimates are significant in Panel B. Therefore, for REV portfolios, the low R^2 in Panel B explains the insignificant capital share growth risk price in panel A. The correlation relationship between CRV and the long-run equity premium dominates that between capital share growth and the long-run equity premium.

Panel C compares capital share growth and CRV. In Panel C, the $\bar{R}^2$ estimates are of similar magnitude as those in Panel B, which is due to the multicolinearity between CRV and capital share growth. In panel C, capital share growth is strongly dominated by CRV. Following the inclusion of CRV, risk prices of capital share growth decrease significantly for all portfolios. Also, for REV sorted portfolios, capital share growth turns insignificant. Although, CRV risk prices also decrease following the inclusion of the capital share factor due to the colinearity between the capital share factor growth, the partial effect of CRV remains significant in the 2-factor model.

This paper also estimates the risk price of CRV using Generalized Method of Moments (GMM) following Hansen (1982).[27] The GMM estimate of CRV risk price is 5.657 (%) and factor loading is 4.832 using the monthly sample. The factor loading of CRV is also a measure of risk aversion in the long-run. Therefore, the magnitude of CRV factor loading is consistent with literature, see Bansal and Yaron (2004) and Andreasen and Jørgensen (2020).

[26] $g_{KS,t}$ and CRV all contains the mean of $g_{KS,t}$ plus terms containing deviation from the mean. Therefore, g_{KS} and CRV are correlated.

[27] Detailed GMM settings are in Appendix.

According to empirical evidences, I conclude that CRV captures the true long-run equity premium dynamics. Therefore, the theoretical model in Section 2 is justified by the empirical analysis in this section. In the long-run setting, CRV explains high cross-sectional equity return variations and dominates the capital share growth.

Table 2: FMB Bootstrap Risk Price Estimates

	Size/BM	*REV*	*Size/INV*	*Size/OP*
		Panel A: capital share growth		
α	1.213**	1.256**	1.170**	1.189**
	[1.068, 1.362]	[0.769, 1.731]	[1.055, 1.288]	[1.085, 1.291]
g^k	2.405**	2.560**	2.010**	2.124**
	[1.755, 3.073]	[0.756, 4.262]	[1.517, 2.554]	[1.858, 2.708]
$\bar{R}^2$	0.697	0.511	0.721	0.832
	[0.372, 0.898]	[0.000, 0.898]	[0.429, 0.903]	[0.618, 0.944]
		Panel B: capital return variability (CRV)		
α	1.139**	1.054**	1.092**	1.181**
	[0.992, 1.280]	[0.735, 1.411]	[0.952, 1.227]	[0.959, 1.409]
CRV	8.488**	7.611**	6.966**	9.230**
	[6.277, 10.730]	[3.462, 11.79]	[4.943, 9.081]	[6.109, 12.460]
$\bar{R}^2$	0.705	0.623	0.659	0.612
	[0.425, 0.888]	[0.083, 0.935]	[0.366, 0.866]	[0.256, 0.854]
		Panel C: 2-factor model		
α	1.220**	1.099**	1.170**	1.197**
	[1.054, 1.384]	[0.657, 1.544]	[1.021, 1.315]	[1.066, 1.327]
g^k	1.769**	0.980	1.768**	2.237**
	[0.969, 2.540]	[-0.615, 2.562]	[1.001, 2.539]	[1.787, 2.707]
CRV	6.811**	6.475**	4.423**	4.464**
	[4.707, 8.967]	[2.093, 10.83]	[2.125, 6.787]	[2.786, 6.203]
$\bar{R}^2$	0.777	0.561	0.752	0.849
	[0.498, 0.930]	[0.000, 0.913]	[0.468, 0.922]	[0.641, 0.952]

Note: This table reports FMB bootstrap estimations of factor risk prices (%). Factor estimated are capital share growth and CRV. CRV captures the true DGP of long-run equity premium in equation (17). For maintaining the consistency of moments and distribution functions (Horowitz, 1997), I set the optimal block-length as $536^{(\frac{1}{5})} \approx 4$ following Hall et al. (1995). The optimal block-length for FMB second stage is identical to Lettau et al. (2019). I use 10,000 simulations for bootstraps. In this table, Panel A reports the capital share growth model estimates, Panel B reports the CRV model estimates, and Panel C reports estimates of a 2-factor model including both capital share growth and CRV. Portfolio returns used for estimation are size/BM, REV, size/INV, and size/OP sorted portfolios. Bootstrapped 95% confidence intervals are reported in square brackets. ** denotes the estimate is significant at 5% level. * denotes the estimate is significant at 10% level. Sample spans January 1974 to August 2018.

4 Conclusion

Inspired by Lettau et al. (2019), I further investigate the role of capital share growth theoretically in pricing equity returns. My paper develops a theoretical model of capital share growth and proposes capital return variability (CRV) as an long-run risk factor.

Based on the long-run risk model by Bansal and Yaron (2004), this paper develops a heterogeneous asset pricing model that separates stockholders from labour workers. My theory finds that the elevated stockholder consumption growth volatility operates through capital share growth. Under short-run expectations, capital share growth is found to affect the innovation of market returns. However, the short-run equity premium is not determined by capital share growth. Under long-run expectations, CRV is a priced risk factor as a proxy for accumulated volatility effect of capital share growth.

Empirical evidences are in line with the theoretical predictions. Capital share growth is significant in the variance but insignificant in the mean equation of short-run equity return. For the long-run equity premium, both capital share growth and CRV generate significant risk prices. However, capital share growth has non-zero long-run risk prices is due to the accumulation of its volatility effect. After the inclusion of CRV, the volatility effect of capital share growth is captured by CRV. Therefore, CRV strongly dominates capital share growth in pricing long-run equity returns.

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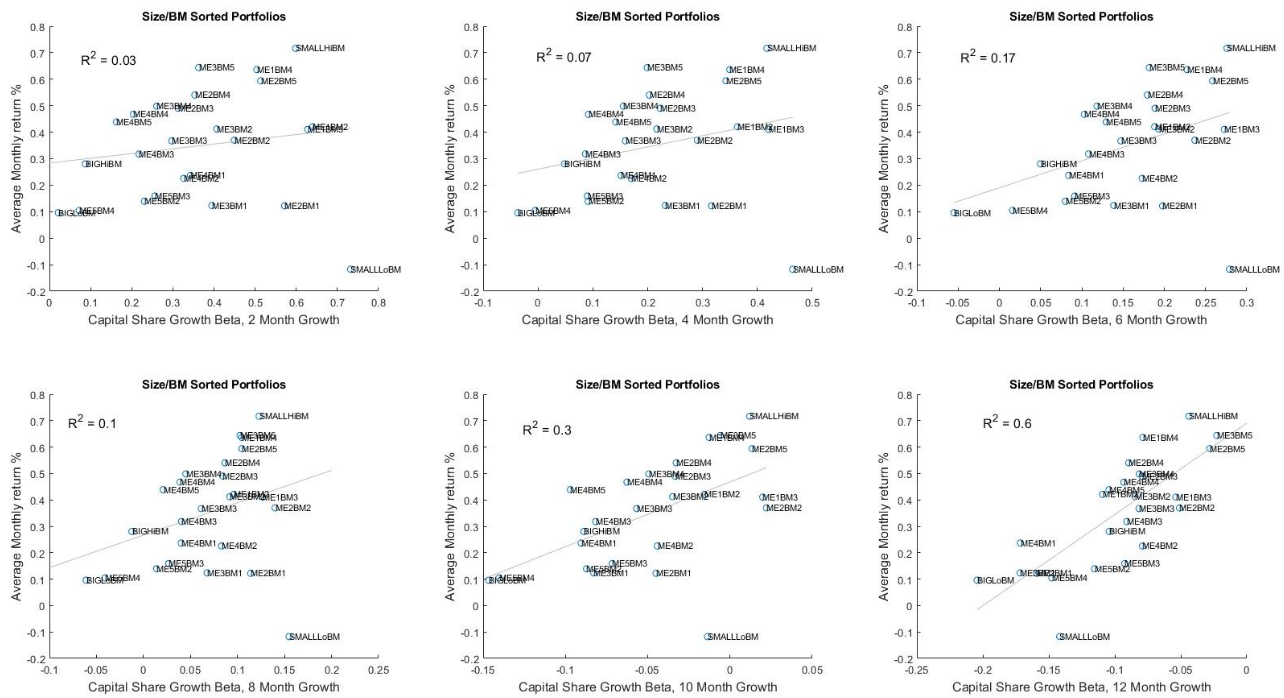

Figure 1: Pricing powers of capital share growth over different horizon. This figure plots the Fama-MacBeth results of capital share growth over 2, 4, 6, 8, 10 and 12-month. The Fama-MacBeth procedure used for this figure is identical to Lettau et al. (2019). Portfolio used are 25 size/BM sorted portfolios. The sample spans January 1964 to August 2016 due to the limitation of data.

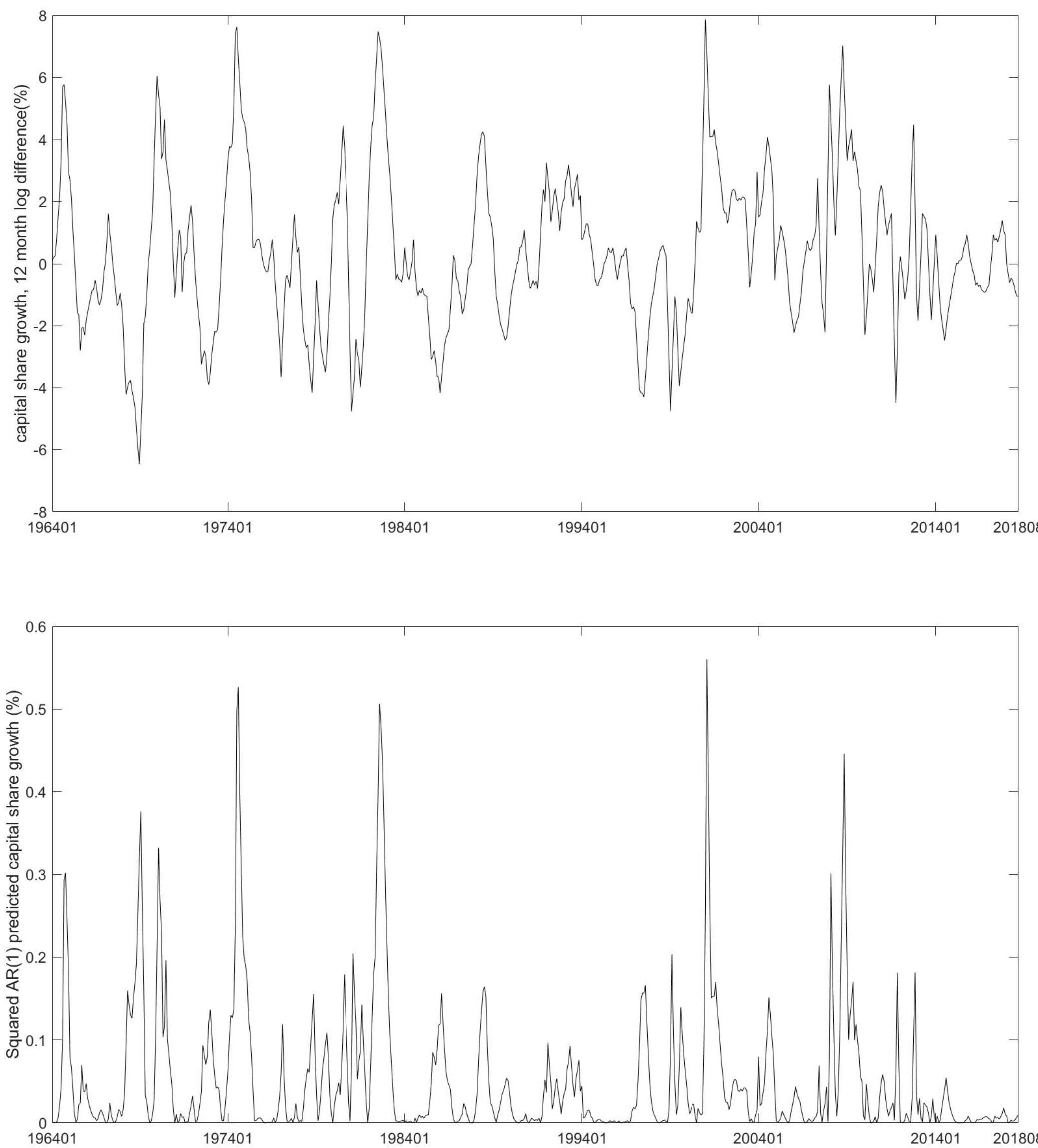

Figure 2: Capital share growth and CRV (%). Sample spans January 1964 to August 2018.

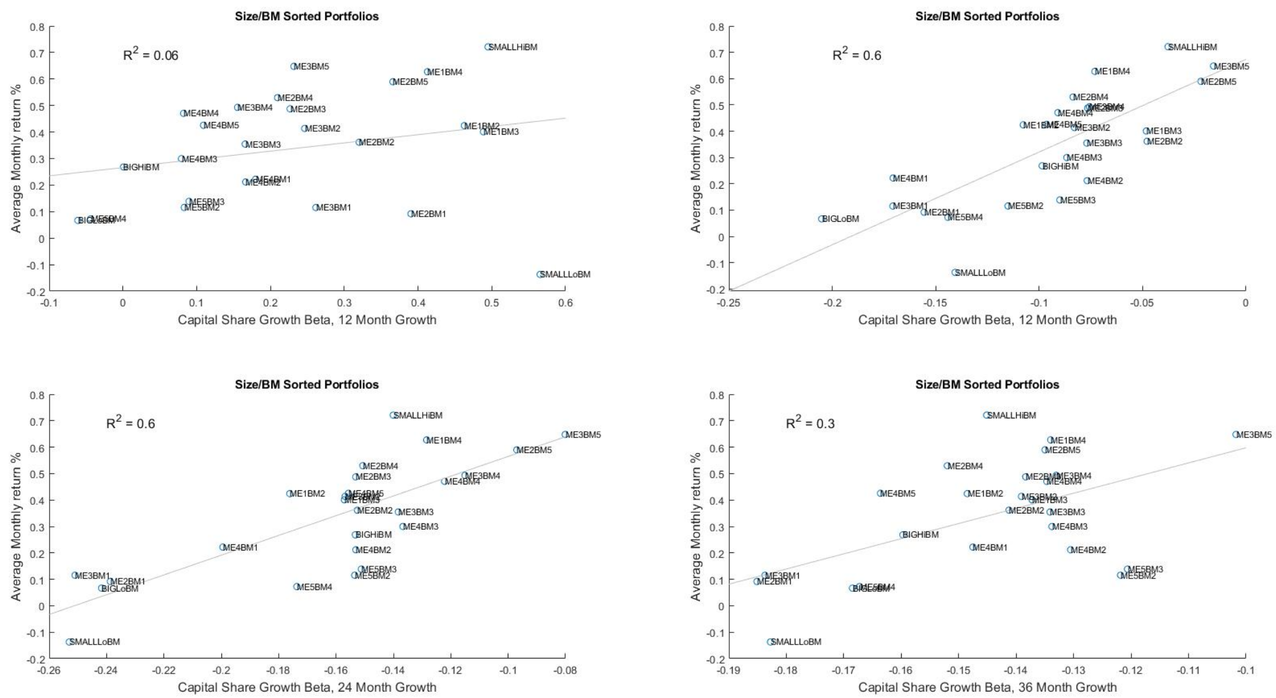

Figure 3: Pricing powers of capital share growth over different horizons. This figure plots the FMB results of capital share growth over 3, 12, 24 and 36-months. The FMB test used for this figure is identical to Lettau et al. (2019). Portfolio used are 25 size/BM sorted portfolios. Sample spans January 1964 to August 2016.

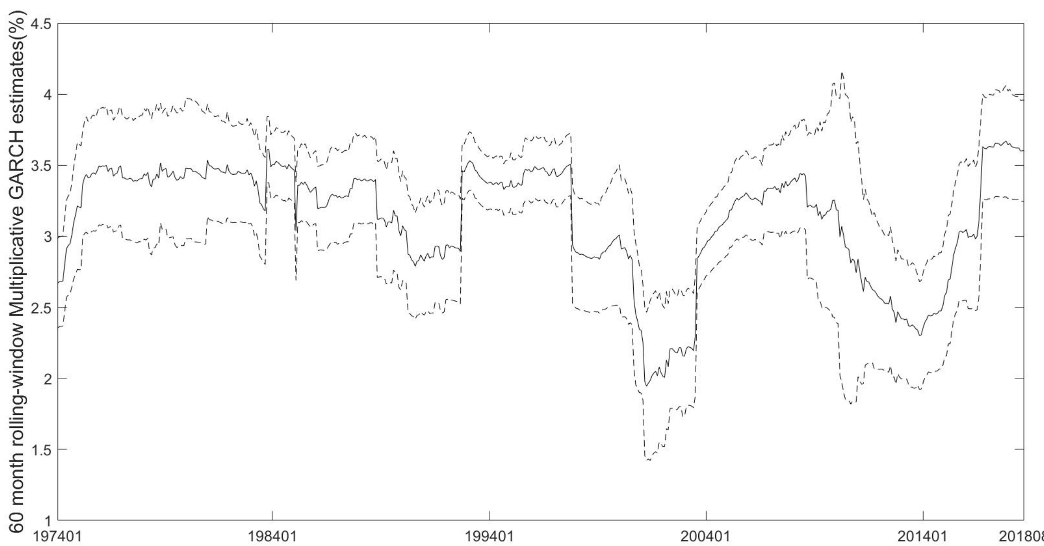

Figure 4: 60-month rolling-window multiplicative GARCH estimates (%). This figure shows estimates for testing short-run market return in equation (13) and equity premium in (14). Capital share growth enters the variance equation of the short-run equity premium. The coefficient of capital share growth is estimated using monthly average returns of size/BM sorted portfolios. The 95% confidence intervals are plotted using dashed lines. Sample spans July 1975 to August 2018.

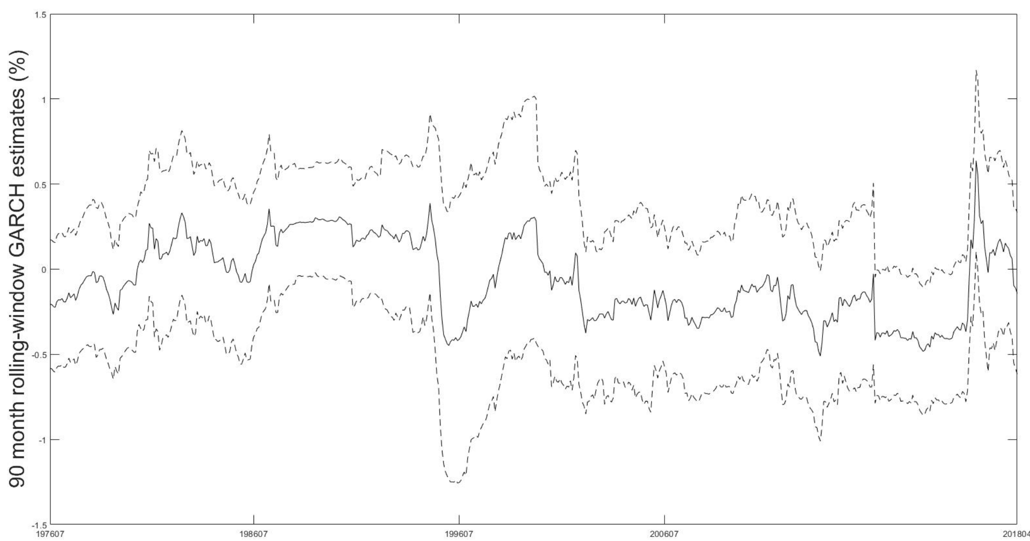

Figure 5: 90-month rolling-window multiplicative GARCH estimates (%). This figure shows estimates for testing short-run market return in equation (13) and equity premium in (14). Capital share growth enters the variance equation of the short-run equity premium. The coefficient of capital share growth is estimated using monthly average returns of size/BM sorted portfolios. The 95% confidence intervals are plotted using dashed lines. Sample spans June 1980 to August 2018.

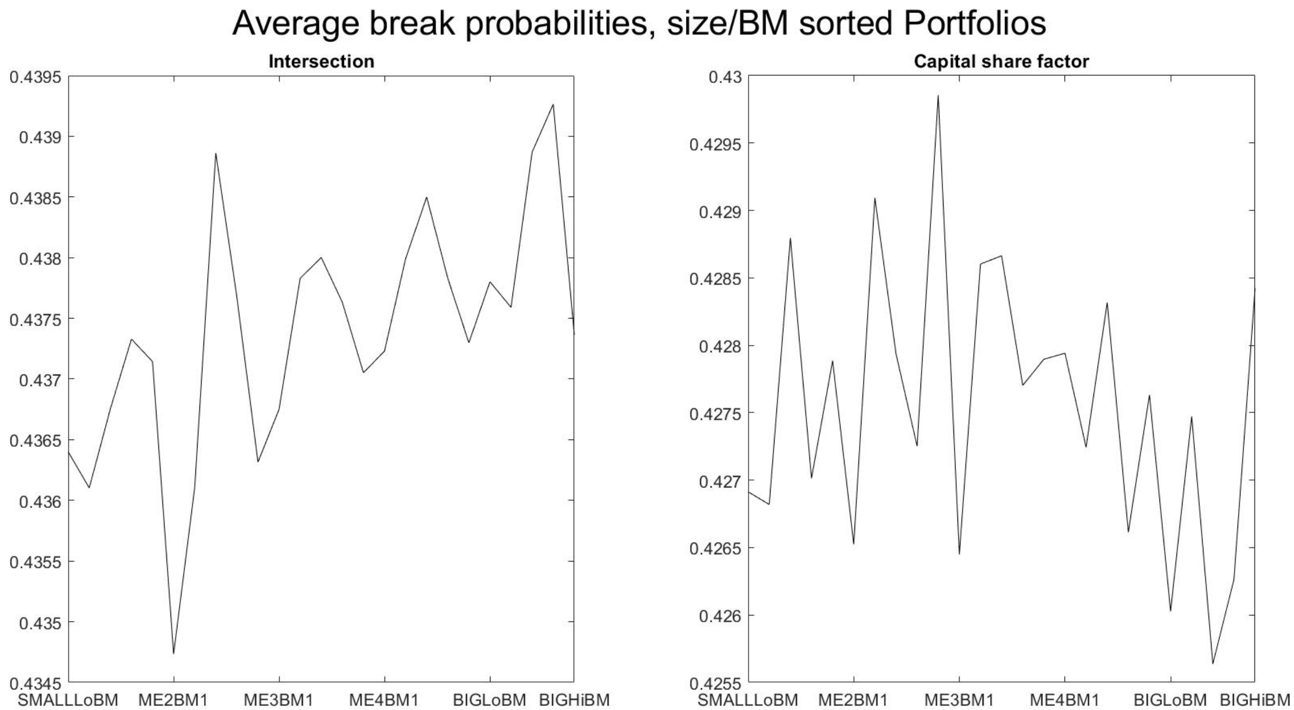

Figure 6: B-TVB-SV average break probabilities of factor loadings, the capital share growth model. The break probabilities are estimated using 25 size/BM sorted portfolios. Average probabilities reported are the time-average for each portfolios. Sample spans January 1964 to August 2018. The first 10-year data in the sample is used for hyperparameter estimation. Sample estimates cover January 1974 to August 2018.

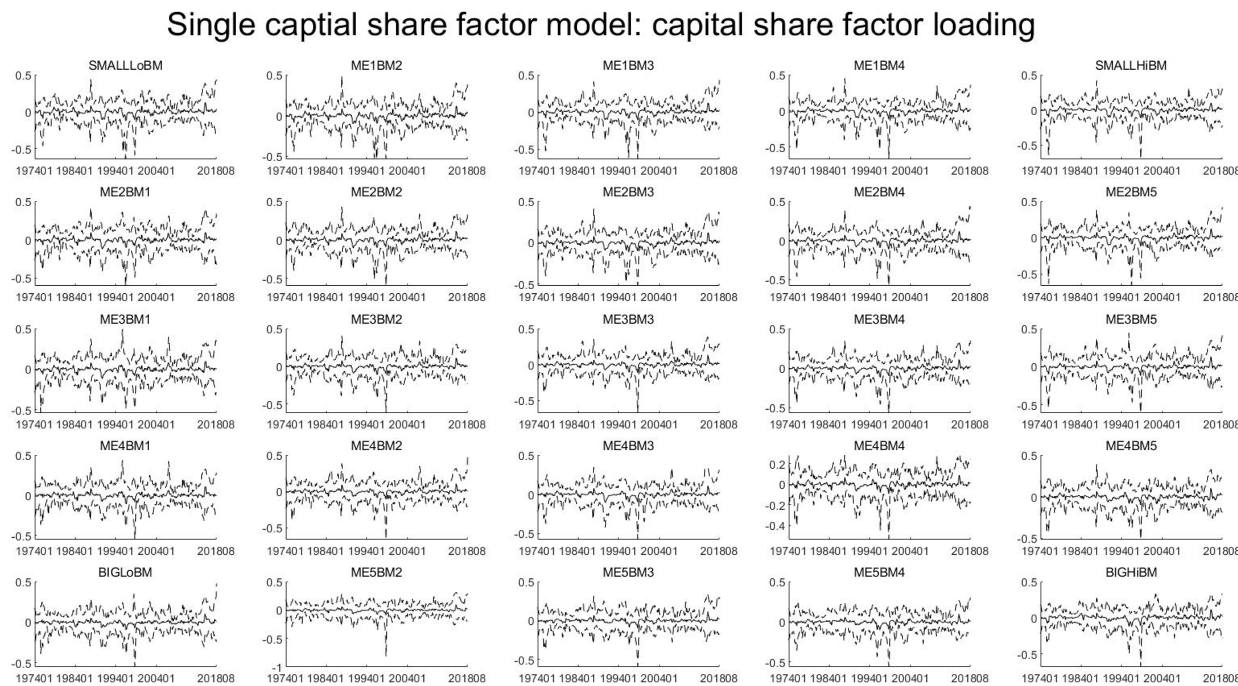

Figure 7: B-TVB-SV factor loadings, the capital share growth model. Factor loadings are estimated by the capital share growth model using monthly size/BM sorted portfolio returns. The 95% confidence intervals are plotted using dashed line. Sample spans January 1964 to August 2018. The first 10-year data in the sample is used for hyperparameter estimation. Sample estimates cover January 1974 to August 2018.

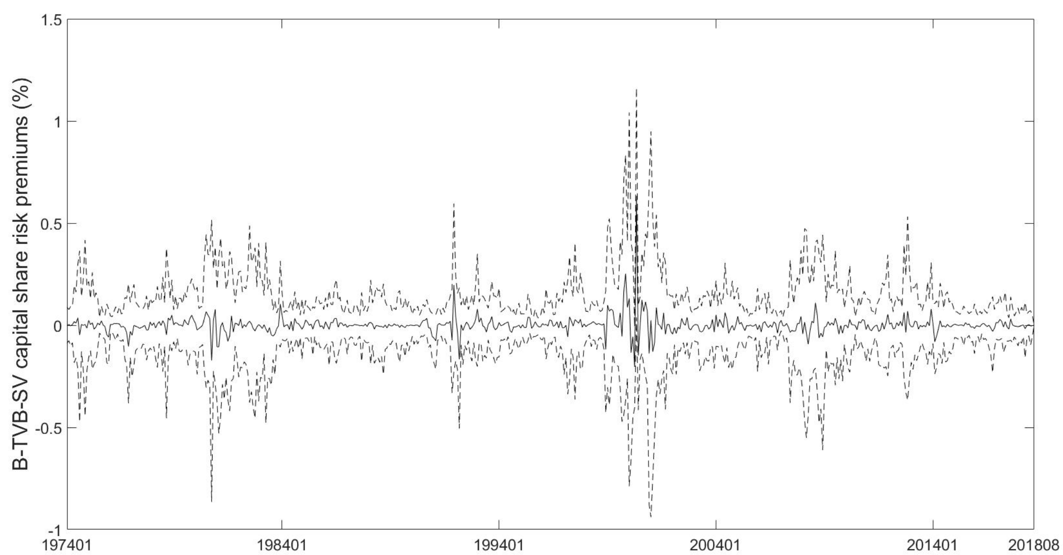

Figure 8: B-TVB-SV risk prices (%), the capital share growth model. This figure plots risk prices estimated by the capital share growth using monthly size/BM sorted portfolio returns. The 95% confidence intervals are plotted using dashed lines. Sample spans January 1964 to August 2018. The first 10-year data in the sample is used for hyperparameter estimation. Sample estimates cover January 1974 to August 2018.

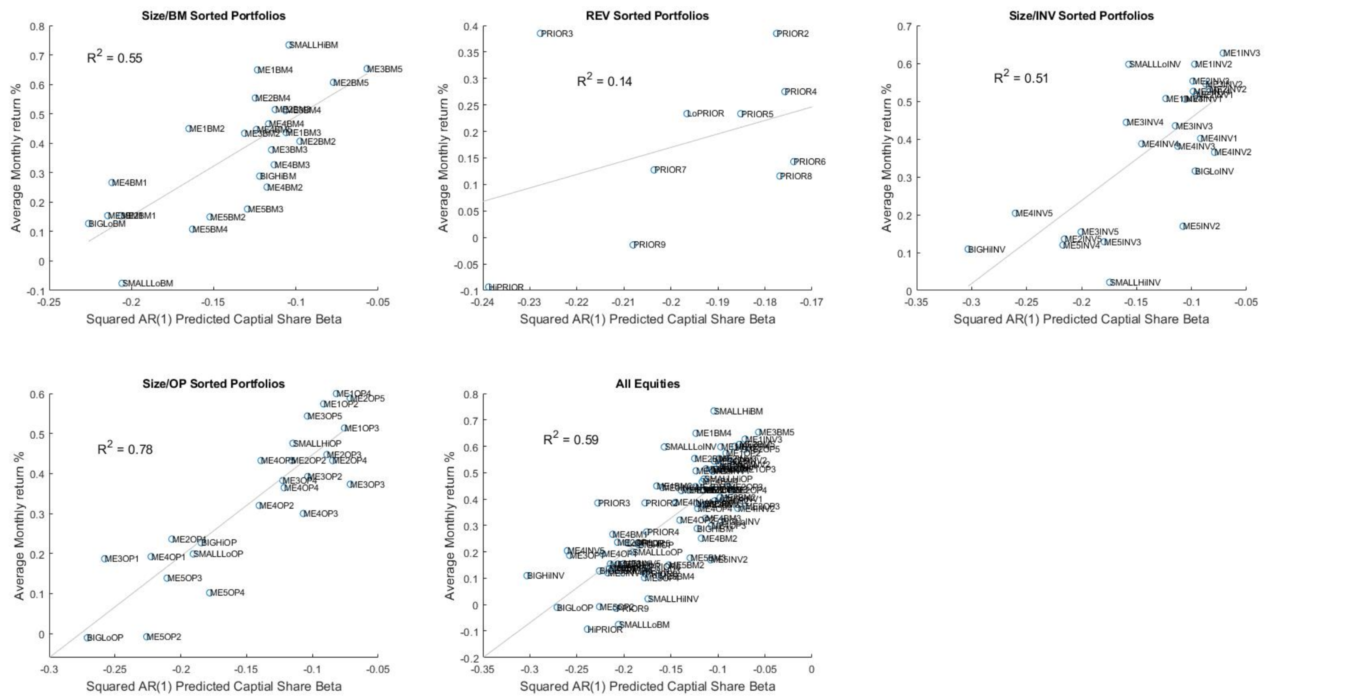

Figure 9: Capital share growth betas (%). This plot depicts the betas constructed by the F-MB regression of average portfolio returns on capital share beta. The monthly average returns are on the y-axis and the portfolio factor betas are on the x-axis. The portfolios estimated include size/BM, REV, size/INV and size/OP sorted portfolios or using all equities together. R^2 estimates of each regression are reported in the graph. Sample spans January 1974 to August 2018.

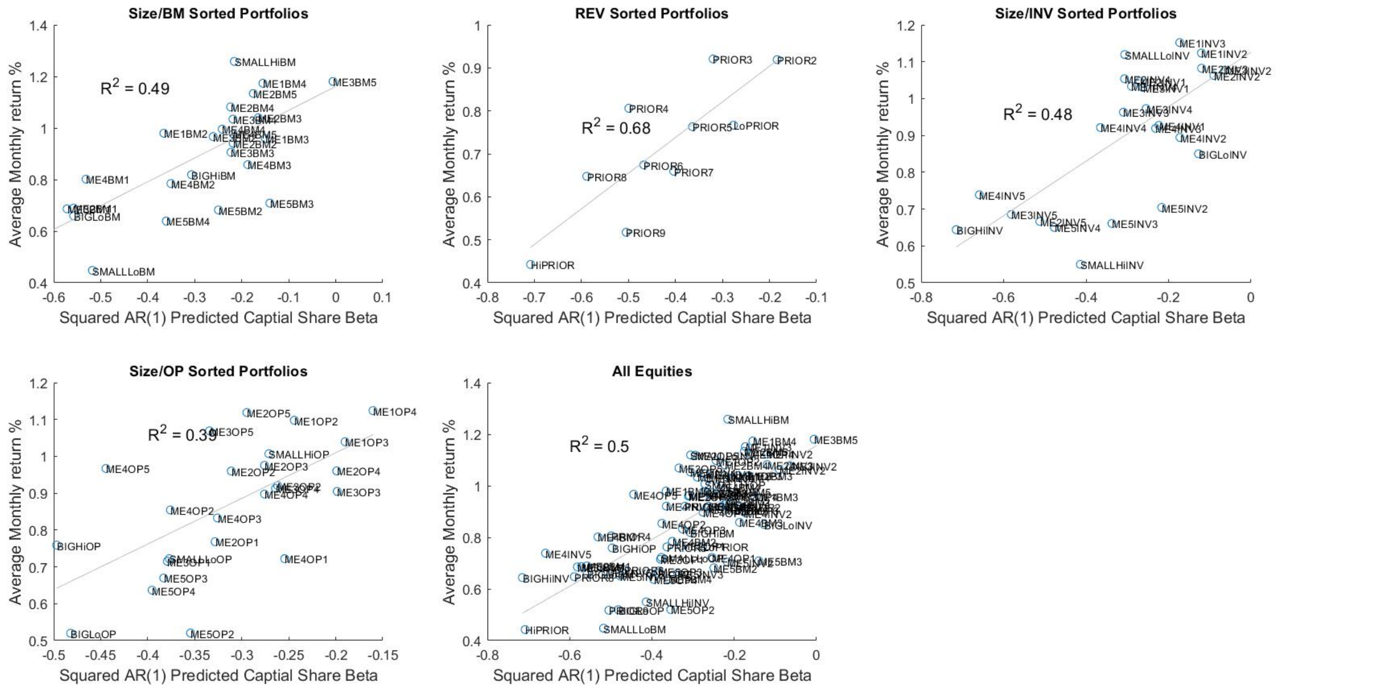

Figure 10: Capital return variability (CRV) betas (%). This plot depicts the betas constructed by the F-MB regression of average portfolio returns on CRV beta. The monthly average returns are on the y-axis and the portfolio factor betas are on the x-axis. The portfolios estimated include REV, size/BM, size/INV and size/OP sorted portfolios or using all equities together. R^2 estimates of each regression are reported in the graph. Sample spans January 1974 to August 2018.

Appendix - Online Supplement

This appendix is not for publication and describes the Bayesian time varying beta with stochastic volatility (B-TVB-SV) specification, the detailed theoretical induction of my model, the construction of the dataset, and basic statistics and estimations.

A The B-TVB-SV model specification

Bianchi et al. (2017) assumes the structural breaks are independent both across portfolio returns and over time. Equation (A.1) defines the structural break probabilities:

$$\begin{aligned} Pr[\kappa_{ij,t} = 1] &= \pi_{ij} \qquad && i = 1, ..., N \\ Pr[\kappa_{iv,t} = 1] &= \pi_{iv} \qquad && j = 0, ..., K \end{aligned} \tag{A.1}$$

The probabilities π_{ij} and π_{iv} are sampled using a uninformative prior to retain the robustness of estimations. The priors are assumed to follow beta distributions:

$$\begin{aligned} \pi_{ij} &\sim Beta(a_{ij}, b_{ij}) \qquad && i = 1, ..., N \\ \pi_{iv} &\sim Beta(a_{iv}, b_{iv}) \qquad && j = 0, ..., K \end{aligned} \tag{A.2}$$

The structural break estimation in Bianchi et al. (2017) uses an efficient generation of mixing variables developed by Gerlach et al. (2000). In modeling intervention in dynamic mixture models, this sampling approach allows the state matrix to be singular and, hence, estimations are allowed to depend on unknown parameters. The breaks innovations $\kappa_{ij,t}$ in equation (25) are assumed to be conditional on the residual variance matrix (Σ), the break probability matrix of σ (K_σ), the simulated model parameter θ, excess returns R, and factors F. In equation (26), $\kappa_{iv,t}$ is assumed to follow a similar innovation process to $\kappa_{ij,t}$. The conditional variance parameters of the size of the structural breaks are assumed to follow an inverted Gamma-2 distribution, of which the shape parameter is linked to the scale parameter (Bianchi et al., 2017).

The prior of the second step risk prices is a mixture of 10 random normal distributions. Priors of these normal distributions are proposed by Omori et al. (2007). The risk price prior is as follows:

$$\lambda \sim MN(\underline{\lambda}, \underline{V}) \tag{A.3}$$

The prior of τ^2 in equation (24) follows a inverse Gamma-2 distribution with shape parameter $\bar{\psi}_0$ and scale parameter Ψ, where

$$\Psi = \Psi_0 + (r - \beta\lambda)'(r - \beta\lambda) \tag{A.4}$$

The risk prices are sampled conditional on the price error matrix $r - \beta\lambda$ linking the time-series regression in equation (23) and the second-step cross-sectional regression in equation (24). Therefore, although the risk prices are estimated in a similar manner to the F-MB procedure within each iteration, the estimated standard deviations of risk prices are robust when a firm effect is present in portfolio returns.

B Theoretical Framework

B.1 Stockholder Consumption Growth

In this subsection, I provide empirical evidences for the functional form of stockholder consumption growth in equation (2). I first approximate the stockholder consumption using the aggregate consumption deduct the product of labour share and the personal disposable income. Following Lettau et al. (2019), I assume that the labour workers consume all their income. This assumption is also justified by their low IES by Gârleanu and Panageas (2015) and low wealth accumulation by Saez and Zucman (2016). Empirically, the stockholder consumption growth is approximated as follows:

$$g^s_{t+1} = \frac{\bar{C}_{t+1} - PI_{t+1} l_{t+1}}{\bar{C}_t - PI_t l_t} - 1 \tag{B.1}$$

where $\bar{C}_t$ is aggregate consumption, PI_t is the personal disposable income and l_t is the labour share. The correlation between monthly stockholder consumption growth and the aggregate consumption growth is 0.726. I plot the two time series in Figure A1.

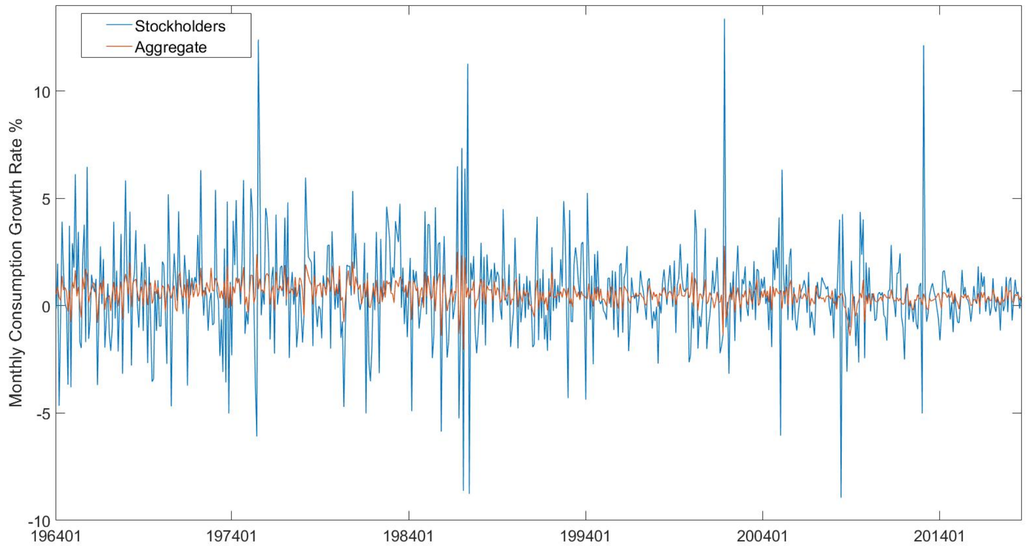

Figure A1: Stockholder consumption growth and aggregate consumption growth (%). This figure plots stockholder consumption growth constructed in equation (B.1) and aggregate consumption growth. Sample spans January 1964 to August 2018.

Figure A1 is consistent with Lettau et al. (2019) that the consumption growth of the top wealth distribution is more volatile than that of the rest of the population.

I also derive the elevated stockholder consumption growth volatility using the stockholder consumption growth constructed in equation (B.1) minus the aggregate consumption growth. The correlations between capital share growth and aggregate consumption growth, capital share growth and the stockholder consumption growth, and the capital share growth and the elevated stockholder consumption growth volatility are -0.035, 0.120, and 0.145, respectively. Therefore, capital share growth is weakly correlated with aggregate consumption growth. Also, the correlation between capital share growth and the stockholder growth comes from the elevated stockholder consumption growth volatility. Empirical evidences imply that the magnitude of elevated stockholder consumption volatility operates trough capital share growth.

I further carry out a test of the relationship between the elevated stockholder consumption growth volatility and capital share growth. I test different modifications of g_t^k in Table A1. This table reports correlations between the the excess stockholder consumption growth and different functional forms of capital share growth.

Table A1: Functional Forms of $g_{KS,t}$

Functional Forms	Corr.
g_t^k	0.1451*
$exp(g_t^k)$	0.1449
$log(1 + g_t^k)$	0.1454**
$(g_t^k)^2$	-0.0176
$\|g_t^k\|^{\frac{1}{2}}$	0.0000

Note: This table reports the correlation between the excess stockholder consumption volatility and functional forms of capital share growth. The excess stockholder consumption volatility is constructed as difference between the stockholder consumption in equation (B.1) and aggregate consumption. Sample spans January 1964 to August 2018.

Given Table A1, I can conclude that the standard form for the capital share growth better captures the elevated stockholder consumption growth volatility. Therefore, the stockholder consumption growth in equation (2) is robust to empirical evidences.

B.2 Baseline Model

In this subsection, I derive the impact of elevated shareholder consumption volatility on the price-consumption ratio (see equation (10) in the main text) and the price-dividend ratio (see equation (11) in the main text). Also, the short- and long-run innovation of the pricing kernel (equations (12) and (15)), the short- and long-run innovation of equity returns (equations (13) and (16)). The equity premium with the short- and long-run expectations (equations (14) and (17)) in the main text are derived in this section.

With Epstein and Zin (1989) recursive preferences, the asset pricing restrictions for gross return $R_{i,t+1}$ satisfy

$$E_t[\delta^{\theta}(G^s_{t+1})^{-\frac{\theta}{\psi}} R_{a,t+1}^{-(1-\theta)} R_{i,t+1}] = 1 \tag{B.2}$$

where $\theta = (1-\gamma)/(1-\frac{1}{\psi})$. In equation (4), G^s_{t+1} denotes the stockholder consumption growth, and $R_{a,t+1}$ denotes the gross return on an asset that generates dividends that cover the aggregate stockholder consumption. $0 < \delta < 1$ is the time discount factor, $\gamma \geq 0$ is the risk-aversion parameter, and $\psi \geq 0$ is the intertemporal elasticity of substitution (IES).

Our system equation is:

$$\begin{aligned} x_{t+1} &= \rho x_t + \phi_e \sigma e_{t+1} \\ g_{t+1} &= \mu + x_t + \frac{1}{w^s} g^k_{t+1} \xi_{t+1} + \sigma \eta_{t+1} \\ g_{d,t+1} &= \mu_d + \phi x_t + \phi_d \sigma_{d,t+1} u_{t+1} \\ & e_{t+1}, u_{t+1}, \eta_{t+1} \sim N_{i.i.d.}(0,1) \qquad \xi_t \sim N(0,\Sigma) \end{aligned} \tag{B.3}$$

According to Bansal and Yaron (2004), dividend growth volatility is correlated with consumption growth volatility. Thus, $\sigma_{d,t+1}$ is partially correlated with $g^k_{t+1}\xi_{t+1}$.

The IMRS is

$$m_{t+1} = \theta log\delta - \frac{\theta}{\psi} g_{t+1} + (\theta - 1) r_{a,t+1} \tag{B.4}$$

Consumption return follows:

$$r_{a,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + g_{t+1} \tag{B.5}$$

where

$$z_t = A_0 + A_1 x_t + A_{2,t} \xi_t \tag{B.6}$$

where $A_{2,t}$ is assumed to vary over time. However, this paper will prove $A_{2,t}$ is a constant next and uses A_2 in the main text. Following Bansal and Yaron (2004), assuming $r_{a,t+1} = r_{i,t+1}$, IMRS in equation (B.4) indicates:

$$log\delta - \frac{1}{\psi} g_{t+1} + r_{a,t+1} = 0 \tag{B.7}$$

Substituting equations (B.3), (B.5) and (B.6) into equation (B.7), I get:

$$
\begin{aligned}
&log\delta + (1 - \frac{1}{\psi})(\mu + x_t + \frac{1}{w^s} E_t(g^k_{t+1}) E_t(\xi_{t+1}) + \sigma\eta_{t+1}) \\
&+ \kappa_0 + \kappa_1(A_0 + A_1\rho x_t + A_1\phi_e\sigma + E_t(A_{2,t+1})E_t(\xi_{t+1})) - (A_0 + A_1 x_t + A_{2,t}\xi_t) = 0 \qquad \text{(B.8)}
\end{aligned}
$$

To ensure equation (B.8) holds, the following must hold:

$$
(1 - \frac{1}{\psi})x_t + \kappa_1\rho A_1 x_t - A_1 x_t = 0 \qquad \text{(B.9)}
$$

$$
(1 - \frac{1}{\psi})\frac{1}{w^s} E_t(g^k_{t+1}) E_t(\xi_{t+1}) + \kappa_1 E_t(A_{2,t+1}) E_t(\xi_{t+1}) - A_{2,t}\xi_t = 0 \qquad \text{(B.10)}
$$

Notice that although the long-run expectation of ξ_{t+1} is zero, this term is relatively stable between t and $t+1$. My model assumes the existence of an r_ξ such that $\xi_t - r_\xi < \xi_{t+1} < \xi_t + r_\xi$ due to smoothed consumption of each income group. r_ξ is a very small number which allows ξ_{t+1} to deviate from ξ_t while ruling out explosive growth. Therefore, $E_t(\xi_{t+1}) \approx \xi_t$. My model also assumes $E_t(A_{2,t+1}) = A_{2,t}$ due to the following relationship derived from equation (B.10):

$$
E_t(A_{2,t+1}) = (1 - \frac{1}{\psi})\frac{1}{w^s}\frac{\rho^k g^k_t}{\kappa_1} + A_{2,t} \qquad \text{(B.11)}
$$

Assume that the value of $A_{2,t}$ equals $A_{2,0}$ at $t = 0$, it is easy to solve that:

$$
A_{2,t} = (1 - \frac{1}{\psi})\frac{1}{w^s}[\frac{\kappa_1}{\rho^k(\kappa_1 - \rho^k)}(g^k_0 - \frac{g^k_t}{\kappa^t_1}) + \frac{A_{2,0}}{\kappa^t_1}] \qquad \text{(B.12)}
$$

According to Bansal and Yaron (2004) and Campbell and Shiller (1988a), the magnitude of κ_1 is very close to 1. The value of $A_{2,t}$ is bounded by definition, thus the true κ_1 and $A_{2,0}$ are not concerns. As shown by equation (B.12),

$$
\begin{aligned}
E_t(A_{2,t+1}) &= (1 - \frac{1}{\psi})\frac{1}{w^s}[\frac{\kappa_1}{\rho^k(\kappa_1 - \rho^k)}(g^k_0 - \rho^k\frac{g^k_t}{\kappa^{t+1}_1}) + \frac{A_{2,0}}{\kappa^{t+1}_1}] \\
&\approx (1 - \frac{1}{\psi})\frac{1}{w^s}[\frac{\kappa_1}{\rho^k(\kappa_1 - \rho^k)}(g^k_0 - \frac{g^k_t}{\kappa^t_1}) + \frac{A_{2,0}}{\kappa^t_1}] \\
&= A_{2,t} \qquad \text{(B.13)}
\end{aligned}
$$

when $\kappa_1 \approx \rho^k \approx 1$. Therefore, assuming $E_t(A_{2,t+1}) = A_{2,t} = E(A_{2,t+1})$ is reasonable, and I use A_2 in the main text.

Following Lettau et al. (2019), this paper assumes that the capital share growth rate follows an AR(1) process[28]:

$$g^k_{t+1} = \rho^k g^k_t + e^k_t \tag{B.14}$$

The functional form of A_1 and A_2 have been solved:

$$A_1 = \frac{1-\frac{1}{\psi}}{1-\kappa_1\rho} \tag{B.15}$$

$$A_2 = \frac{1-\frac{1}{\psi}}{w^s(1-\kappa_1)}\rho^k g^k_t \tag{B.16}$$

Following the same steps used in deriving the consumption premium, this paper further derives the equity premium. Equity returns have the following functional form:

$$r_{m,t+1} = \kappa_{0,m} + \kappa_{1,m} z_{t+1} - z_t + g_{d,t+1} \tag{B.17}$$

where

$$z_t = A_{0,m} + A_{1,m}x_t + A_{2,m,t}\xi_t \tag{B.18}$$

where $A_{2,m,t}$ is assumed to vary over time. However, this paper will prove that $A_{2,m,t}$ is a constant when holding g^k_t constant, and uses $A_{2,m}$ in the main text.

To further derive the equity premium $r_{m,t}$, this paper invokes the Euler condition $E[exp(m_{t+1}+r_{m,t+1})] = 1$. The following condition holds:

$$\theta log\delta - \frac{\theta}{\psi}g_{t+1} + (\theta-1)r_{a,t+1} + r_{m,t+1} = 0 \tag{B.19}$$

To solve $A_{1,m}$ and $A_{2,m,t}$, substitute equations (B.4), (B.5), (B.17) and (B.18) into equation (B.19), collecting all terms containing x_t and ξ_t respectively:

$$\begin{aligned} &(\theta - 1 - \frac{\theta}{\psi})x_t + (\theta-1)(\kappa_1\rho - 1)A_1 x_t + \kappa_{1,m}A_{1,m}\rho x_t - A_{1,m}x_t\phi x_t \\ &= -\frac{1}{\psi}x_t + \kappa_{1,m}A_{1,m}\rho x_t - A_{1,m}x_t + \phi x_t = 0 \end{aligned} \tag{B.20}$$

$$\begin{aligned} &(\theta - 1 - \frac{\theta}{\psi})\frac{1}{w^s}E_t(g^k_{t+1})E_t(\xi_{t+1}) + (\theta-1)(\kappa_1 E_t(A_{2,t+1})E_t(\xi_{t+1}) - A_{2,t}\xi_t) \\ &+ \kappa_{1,m}E_t(A_{2,m,t+1})E_t(\xi_{t+1}) - A_{2,m,t}\xi_t \\ &= -\frac{1}{\psi}\rho^k g^k_t \frac{1}{w^s} + \kappa_{1,m}A_{2,m,t} - A_{2,m,t} = 0 \end{aligned} \tag{B.21}$$

[28]The constant is not significant due to the AR(1) estimation. The magnitude of ρ^k is 0.947.

The functional form of $A_{1,m}$ and $A_{2,m,t}$ can now be solved as:

$$A_{1,m} = \frac{\phi - \frac{1}{\psi}}{1 - \kappa_1 \rho} \tag{B.22}$$

$$A_{2,m,t} = -\frac{\rho^k}{w^s \psi (1 - \kappa_{1,m})} g_t^k \tag{B.23}$$

Therefore, $A_{2,m}$ is a constant and I use $A_{2,m}$ in the main text.

B.3 Short-Run Case: Conditional on information set at time t

The short-run innovation of consumption return is:

$$\begin{aligned} r_{a,t+1} - E_t(r_{a,t+1}) =& \sigma \eta_{t+1} + \kappa_1 A_1 \phi_e \sigma e_{t+1} + (\frac{1}{w^s} g_{t+1}^k + \kappa_1 A_{2,t+1}) \xi_{t+1} \\ & - E_t(\frac{1}{w^s} g_{t+1}^k + \kappa_1 A_{2,t+1}) \xi_{t+1} \\ =& \sigma \eta_{t+1} + \lambda_{r,e} \sigma e_{t+1} + \lambda_{r,\xi,t+1} \xi_{t+1} \end{aligned} \tag{B.24}$$

The short-run innovation of the pricing kernel is:

$$\begin{aligned} m_{t+1} - E_t(m_{t+1}) =& (\theta - 1 - \frac{\theta}{\psi}) \sigma \eta_{t+1} + (\theta - 1)(\kappa_1 A_1 \phi_e) \sigma e_{t+1} \\ & + (\theta - 1)[\kappa_1 (A_{2,t+1} - E_t(A_{2,t+1}))] \xi_{t+1} \\ =& \lambda_\eta \sigma \eta_{t+1} + \lambda_e \sigma e_{t+1} + \lambda_{\xi,t+1} \xi_{t+1} \end{aligned} \tag{B.25}$$

In equations (B.24) and (B.25), the parameters are as follows:

$$\lambda_{r,e} = \kappa_1 \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 \rho} \phi_e \tag{B.26}$$

$$\lambda_{r,\xi,t+1} = (\frac{1}{w^s} + \kappa_1 \frac{1 - \frac{1}{\psi}}{w^s (1 - \kappa_1)} \rho_k) e_{t+1}^k \tag{B.27}$$

$$\lambda_\eta = \theta - 1 - \frac{\theta}{\psi} \tag{B.28}$$

$$\lambda_e = (\theta - 1)(\kappa_1 \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 \rho} \phi_e) \tag{B.29}$$

$$\lambda_{\xi,t+1} = (\theta - 1)(\kappa_1 \frac{1 - \frac{1}{\psi}}{w^s (1 - \kappa_1)} \rho^k) e_{t+1}^k \tag{B.30}$$

The short-run consumption premium in the presence of time-varying economic uncertainty is

$$\begin{aligned} E_t(r_{a,t+1} - r_{f,t}) =& cov_t((m_{t+1} - E_t(m_{t+1}))(r_{a,t+1} - E_t(r_{a,t+1})) + 0.5Var_t(r_{a,t+1}) \\ =& -(\lambda_\eta + \lambda_{r,e}\lambda_e - 0.5\lambda_{r,e}^2 - 0.5)\sigma^2 \\ &+ E_t(\lambda_{r,\xi,t+1}\lambda_{\xi,t+1} - 0.5\lambda_{\xi,t+1}^2) \end{aligned} \tag{B.31}$$

The short-run innovation of equity return is:

$$\begin{aligned} r_{m,t+1} - E_t(r_{m,t+1}) =& \phi_d\sigma_{d,t+1}u_{t+1} + \kappa_{1,m}A_{1,m}\phi_e\sigma e_{t+1} + \kappa_{1,m}A_{2,m,t+1}\xi_{t+1} \\ =& \phi_d\sigma_{d,t+1}u_{t+1} + \lambda_{m,e}\sigma e_{t+1} + \lambda_{m,\xi,t+1}\xi_{t+1} \end{aligned} \tag{B.32}$$

In equation (B.32), the parameters are as follows:

$$\lambda_{m,e} = \kappa_{1,m}\frac{\phi - \frac{1}{\psi}}{1 - \kappa_1\rho} \tag{B.33}$$

$$\lambda_{m,\xi,t+1} = \kappa_{1,m}\frac{\frac{1}{w^s}\rho^k}{\psi(1 - \kappa_{1,m})}e_{t+1}^k \tag{B.34}$$

where $\lambda_{m,\xi,t+1}$ is a constant when holding e_{t+1}^k constant. The short-run equity premium in the presence of time-varying economic uncertainty is

$$\begin{aligned} E_t(r_{m,t+1} - r_{f,t}) =& cov_t((m_{t+1} - E_t(m_{t+1}))(r_{m,t+1} - E(r_{m,t+1})) + 0.5Var(r_{m,t+1}) \\ =& -(\lambda_{m,e}\lambda_e - 0.5\lambda_{m,e}^2)\sigma^2 + 0.5\phi_d^2\sigma_{d,t+1}^2 \\ &+ E_t(\lambda_{m,\xi,t+1}\lambda_{\xi,t+1} - 0.5\lambda_{m,\xi,t+1}^2) \end{aligned} \tag{B.35}$$

where $E_t(\lambda_{m,\xi,t+1}\lambda_{\xi,t+1} - 0.5\lambda_{m,\xi,t+1}^2) = 0$ due to $E_t(e_{t+1}^k) = 0$; σ_g^2 is close to σ^2 due to very small ξ^2. Therefore, the expected equity premium can be viewed as a constant when the model only contains capital share growth as the independent variable. The deviation of equity returns is correlated with $\sigma_{d,t+1}$ which is a function of g_{t+1}^k and ξ_{r+1}. In the short-run case, g_{t+1}^k is a variable that enters the variance equation.

B.4 Long-run Case: Unconditional Expectations

Under long-run expectations, $E(\xi_t) = 0$. Therefore, the long-run innovation of consumption return is:

$$\begin{aligned} r_{a,t+1} - E(r_{a,t+1}) =& \sigma\eta_{t+1} + \kappa_1 A_1 \phi_e \sigma e_{t+1} + (\frac{1}{w^s} g_{t+1}^k + \kappa_1 A_{2,t+1})\xi_{t+1} \\ =& \sigma\eta_{t+1} + \lambda_{r,e}\sigma e_{t+1} + \lambda_{r,\xi,t+1}^u \xi_{t+1} \end{aligned} \tag{B.36}$$

The long-run innovation of the pricing kernel is:

$$\begin{aligned} m_{t+1} - E(m_{t+1}) =& (\theta - 1 - \frac{\theta}{\psi})\sigma\eta_{t+1} + (\theta-1)(\kappa_1 A_1 \phi_e)\sigma e_{t+1} + (\theta-1)(\kappa_1 A_{2,t+1})\xi_{t+1} \\ =& \lambda_\eta \sigma\eta_{t+1} + \lambda_e \sigma e_{t+1} + \lambda_{\xi,t+1}^u \xi_{t+1} \end{aligned} \tag{B.37}$$

The long-run consumption premium in the presence of time-varying economic uncertainty is

$$\begin{aligned} E(r_{a,t+1} - r_{f,t}) =& cov((m_{t+1} - E(m_{t+1}))(r_{a,t+1} - E(r_{a,t+1})) + 0.5Var(r_{a,t+1}) \\ =& -(\lambda_\eta + \lambda_{r,e}\lambda_e - 0.5\lambda_{r,e}^2 - 0.5)\sigma^2 \\ &+ E[\lambda_{r,\xi,t+1}^u \lambda_{\xi,t+1}^u - 0.5(\lambda_{r,\xi,t+1}^u)^2] \end{aligned} \tag{B.38}$$

In equations (B.36), (B.37) and (B.38), the parameters are as follows:

$$\lambda_{r,e} = \kappa_1 \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 \rho} \phi_e \tag{B.39}$$

$$\lambda_{r,\xi,t+1}^u = \frac{1}{w^s} g_{t+1}^k + \kappa_1 \frac{1 - \frac{1}{\psi}}{w^s(1 - \kappa_1)} \rho^k g_{t+1}^k \tag{B.40}$$

$$\lambda_\eta = \theta - 1 - \frac{\theta}{\psi} \tag{B.41}$$

$$\lambda_e = (\theta - 1)(\kappa_1 \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 \rho} \phi_e) \tag{B.42}$$

$$\lambda_{\xi,t+1}^u = (\theta - 1)(\kappa_1 \frac{1 - \frac{1}{\psi}}{w^s \psi (1 - \kappa_1)} \rho^k g_{t+1}^k) \tag{B.43}$$

where $\lambda_{r,\xi,t+1}^u$ and $\lambda_{\xi,t+1}^u$ are functions of g_{t+1}^k. Therefore, I set $\lambda_{r,\xi,t+1}^u = \lambda_{r,\xi}^u(g_{t+1}^k)$ and $\lambda_{\xi,t+1}^u = \lambda_\xi^u(g_{t+1}^k)$. The long-run innovation of equity returns is:

$$\begin{aligned} r_{m,t+1} - E(r_{m,t+1}) =& \phi_d E(\sigma_{d,t+1})u_{t+1} + \kappa_{1,m} A_{1,m} \phi_e \sigma e_{t+1} + \kappa_{1,m} A_{2,m,t+1} \xi_{t+1} \\ =& \phi_d \sigma u_{t+1} + \lambda_{m,e}\sigma e_{t+1} + \lambda_{m,\xi,t+1}^u \xi_{t+1} \end{aligned} \tag{B.44}$$

The long-run expectation of $E(\sigma_{d,t+1})$ equals to σ due to $E(\xi_t) = 0$. In equations (B.44) and (B.47), the parameters are as follows:

$$\lambda^u_{m,e} = \kappa_{1,m} \frac{\phi - \frac{1}{\psi}}{1 - \kappa_1 \rho} \tag{B.45}$$

$$\lambda^u_{m,\xi,t+1} = -\kappa_{1,m} \frac{\rho^k}{w^s \psi (1 - \kappa_{1,m})} g^k_{t+1} \tag{B.46}$$

where $\lambda^u_{m,\xi,t+1}$ is a function of g^k_{t+1} and I set $\lambda^u_{m,\xi,t+1} = \lambda^u_{m,\xi}(g^k_{t+1})$ Therefore, the long-run equity premium in the presence of time-varying economic uncertainty is

$$\begin{aligned} E(r_{m,t+1} - r_{f,t}) =& cov((m_{t+1} - E_t(m_{t+1}))(r_{m,t+1} - E(r_{m,t+1})) + 0.5Var(r_{m,t+1}) \\ =& -(\lambda_{m,e}\lambda_e - 0.5\lambda^2_{m,e} - 0.5\phi^2_d)\sigma^2 \\ &+ E[\lambda^u_{m,\xi,t+1}\lambda^u_{\xi,t+1} - 0.5(\lambda^u_{m,\xi,t+1})^2] \end{aligned} \tag{B.47}$$

where $E\left[\lambda^u_{m,\xi,t+1}\lambda^u_{\xi,t+1} - 0.5(\lambda^u_{m,\xi,t+1})^2\right]$ is a function of $E\left[(g^k_{t+1})^2\right]$:

$$\begin{aligned} & E\left[\lambda_{m,\xi,t+1}\lambda_{\xi,t+1} - 0.5(\lambda^u_{m,\xi,t+1})^2\right] \\ &= (1-\theta)\kappa_1\kappa_{1,m} \frac{(\rho^k)^2(1 - \frac{1}{\psi})}{(w^s\psi)^2(1-\kappa_1)(1-\kappa_{1,m})} E\left[(g^k_{t+1})^2\right] \\ &- 0.5(\kappa_{1,m}\frac{\rho^k}{w^s\psi(1-\kappa_{1,m})})^2 E\left[(g^k_{t+1})^2\right] \\ &= \kappa_{1,m}(\rho^k)^2 \frac{(1-\theta)\kappa_1(1-\frac{1}{\psi})(1-\kappa_{1,m}) - 0.5\kappa_{1,m}(1-\kappa_1)}{(w^s\psi)^2(1-\kappa_1)(1-\kappa_{1,m})^2} E\left[(g^k_{t+1})^2\right] \end{aligned} \tag{B.48}$$

Given the DGP of capital share growth in equation (B.14), the $E\left[(g^k_{t+1})^2\right]$ is a predicted value derived by an AR(1) model. In the long-run case, $E\left[(g^k_{t+1})^2\right]$ is a risk factor that enters the mean equation.

B.5 Factor Interpolation

This paper estimates the risk exposure and risk premium of the capital share factor in a monthly setting. However, the highest frequency of capital share data is quarterly. I interpolate capital share into monthly data due to the following reasons: 1) to avoid likely information loss when converting monthly portfolio returns into quarterly data; 2) to maintain a high degree of freedom in the training set in Bayesian estimations; 3) to avoid projection errors: in the projection process of the capital share factor, the quarterly horizon is more sensitive than the monthly horizon in terms of model missimplification (Lamont, 2001). To convert the factor into monthly frequency, this paper adopts the Chow-Lin interpolation approach, which is a linear regression based model with autocorrelation in the error term (Chow and Lin, 1971).

B.5.1 Indicator calculation

The commonly used Chow-Lin interpolation (Chow and Lin, 1971) and other alternative interpolation approaches (see Fernandez (1981), Litterman (1983), etc.) are all based upon the assumption that the monthly observations of interest satisfy a multiple regression relationship with some related series. Accordingly, regression based interpolation methods require related series as indicators to capture the latent monthly movement out of a quarterly time series.

The capital share at time t, denoted by k_t, can be calculated as

$$k_t = 1 - l_t \tag{B.49}$$

under the assumption that all risk sharing across workers and stockholders is imperfect (Lettau et al., 2019). l_t denotes labour share at time t.

Table A2 shows the personal income and its disposition. The personal income and the compensation of employees are selected by this paper for indicator construction. An additional assumption is made to increase the robustness of the indicator, as shown in equation (B.50), which is that the share of compensation of employees is constantly proportional to the labour share.

$$ES_t = \gamma_m l_t \tag{B.50}$$

In equation (B.50), ES_t denotes the compensation of employees share over personal income, and γ_m is a constant.

Table A2: Personal income and its disposition (FRED, 2019b)

Unit: Bil. of %	*2011:12*	*Percentage*	*1972:01*	*Percentage*
Personal income	13,572.40	100%	898.8	100%
Compensation of employees	**8,283.50**	**61%**	**644.5**	**72%**
Proprietors' income with inventory valuation and capital consumption adjustments	1,286.10	9%	80.2	9%
Rental income of persons with capital consumption adjustment	508.30	4%	21.1	2%
Personal income receipts on assets	2,049.30	15%	122.4	14%
Personal current transfer receipts	2,367.10	17%	81	9%
Less: Contributions for government social insurance, domestic	922.00	7%	50.3	6%
Less: Personal current taxes	1,478.80	11%	97.5	11%
Equals: Disposable personal income	12,093.60	89%	801.3	89%
Less: Personal outlays	11,153.00	82%	694.5	77%
Equals: Personal saving	940.50	7%	106.8	12%

Note: Personal income is the income obtained from provision of labour, land, and capital used in current production and the net current transfer payments received from business and government. Percentage denotes the proportion of each element in personal income. Data selected are monthly, and covers the period from January 1972 to December 2011.

The intuition behind the indicator selection is simple. Labour share is calculated by labour compensation divided by national income.[29] Lettau et al. (2019) uses the labour share of national income in the nonfarm business sector to compute capital share. However, national income is only available quarterly. Therefore, personal income is the most appropriate proxy for monthly interpolation due to its relevantly stable relationship with national income. In Table A2, personal income refers to the broad measure of household income, and the compensation of employees denotes the gross wages paid to employees within a certain period.[30] Personal income is calculated by national income minus indirect business taxes,

[29] Labour compensation: compensation of employees in national currency.

[30] Here the period is one year.

corporate income taxes and undistributed corporate profits, then adds transfer payments.[31] Gomme and Rupert (2004) show that indirect taxes and subsidies are stable over time. Hence, when studying the movement of data, the difference between national income and personal income can be ignored, because the difference is mainly caused by indirect tax and subsidies.

The calculation method for ES_t is as follows:

$$ES_t = \frac{Com_t}{PI_t} \tag{B.51}$$

where Com_t denotes the compensation of employees and PI_t denotes personal income.

To roughly estimate γ_m, this paper assumes that γ_m and γ_q share the same data generation process (DGP). Quarterly compensation share ES_q and labour share l_q can be used to calculate quarterly γ_q using the following function:

$$\gamma_q = \frac{ES_q}{l_q} \tag{B.52}$$

Table A3: Descriptive Statistics of γ_q

Min.	*1st Qu.*	*Median*	*Mean*	*3rd Qu.*	*Max.*	*Std.dev*
1.048	1.087	1.097	1.099	1.110	1.154	0.020

Note: γ_q is estimated by compensation of employee share in personal income over labour share (equation B.52). Data is quarterly and covers the sample period 1972:Q1−2011:Q4. γ_q is assumed to share the same DGP as γ_m.

Table A3 shows the descriptive statistics of γ_q calculated using equation (B.52). The standard deviation of γ_q is 0.020, and the mean and median are close to each other. The dispersion of γ_q is low according to the descriptive statistics. Therefore, monthly γ_m can be treated as a constant according to properties of quarterly γ_q.

[31] Personal income equals to national income minus corporate profits with inventory valuation and capital consumption adjustments, taxes on production and imports less subsidies, contributions for government social insurance, net interest and miscellaneous payments on assets, business current transfer payments (net), current surplus of government enterprises, and wage accruals less disbursements, plus personal income receipts on assets and personal current transfer receipts (FRED, 2019a)

The movement of labour share can be represented by the share of compensation of employees. In the Chow-Lin Interpolation, the constant multiplier of the indicator is unimportant due to the regression nature of the approach. Therefore, the monthly indicator, denoted by Ind_t, is calculated as follows:

$$Ind_t = 1 - ES_t \tag{B.53}$$

Figures A2 and A3 show the patterns of quarterly capital share factor and indicator, respectively. Although the capital share factor is overall more volatile compared to the indicator, comovements between them can still be found easily by eyeballing the two figures.

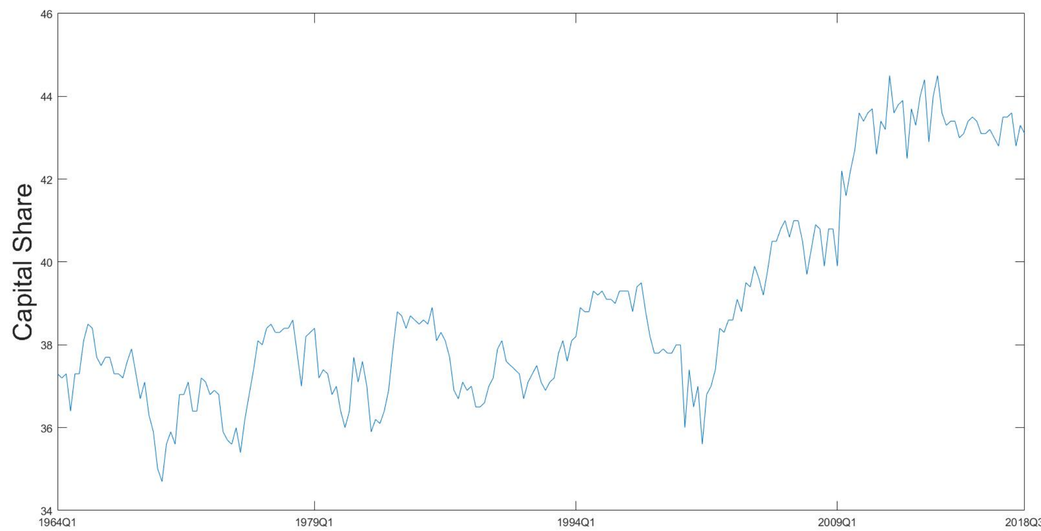

Figure A2: Capital share (quarterly).

B.5.2 Interpolation of Capital Share

Chow and Lin (1971) proposes an interpolation approach based upon the assumption of a regression relationship between the latent monthly time series of interest and indicators. Based upon Chow-Lin method, Fernandez (1981) and Litterman (1983) approaches introduce unit roots in the error term. This paper adopts the Chow-Lin approach for interpolation

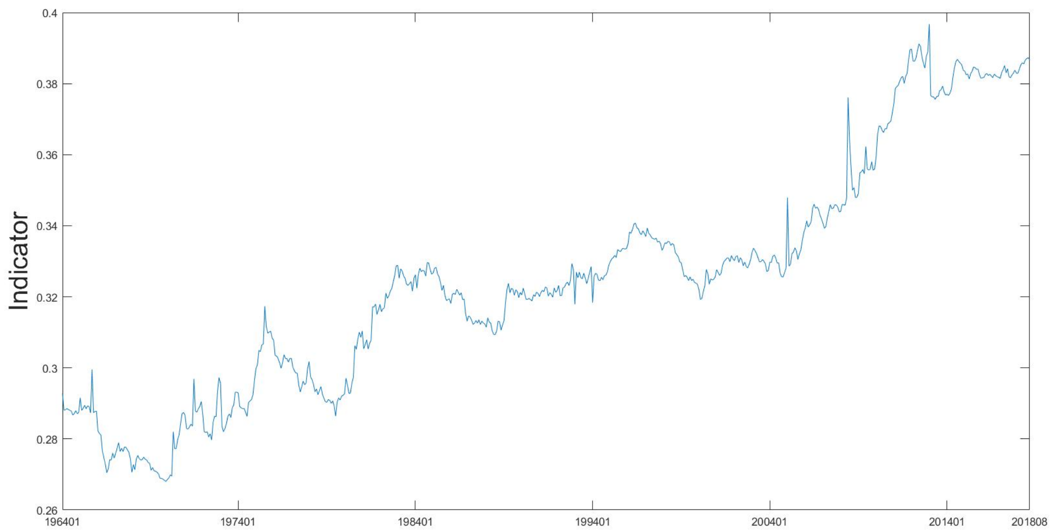

Figure A3: Indicator Dynamics

and also takes potential autocorrelations in the error term of the target time series into consideration.

Therefore, this paper assumes the following relationship holds:

$$k_{monthly} = \beta_0 + \beta_{ind} Ind + \mu \tag{B.54}$$

The error term μ has the following form to avoid spurious discontinuities between the last month of the previous year and the first month of the next year:

$$\mu_t = \rho \ \mu_{t-1} + \epsilon_t \tag{B.55}$$

where $KS_{monthly}$ denotes the target time series data matrix after interpolation. Ind is the monthly indicator. μ_t is assumed to be an autocorrelated variable as shown in equation (B.55). The covariance matrix of μ is denoted by V. β_0 and β_{Ind} denote the constant and the coefficient of the indicator, respectively. ρ is the coefficient of μ_{t-1} and captures the autocorrelation is present in the error term. ϵ_t is *i.i.d.* and follows a standard normal distribution.

The generalized least squares estimators are defined as follows in this paper:

$$\beta_{Ind} = (Ind' \ V^{-1} \ Ind)^{-1} \ Ind' \ V^{-1} \ k_{monthly} \tag{B.56}$$

where

$$V = C(A'A)^{-1}C' \tag{B.57}$$

In equation (B.57), A is an auxiliary matrix with the following form (n equals to the quarterly data length) to factor in the autocorrelation of the error term:

$$A = \begin{bmatrix} (1-\rho^2)^{\frac{1}{2}} & 0 & 0 & 0 & \dots & & & \\ -\rho & 1 & 0 & 0 & \dots & & & \\ 0 & -\rho & 1 & 0 & \dots & & & \\ \vdots & \vdots & \ddots & \ddots & & \vdots & \vdots & \vdots \\ & & & & & \ddots & \ddots & \\ & & & & & & -\rho & 1 \\ & & & & & & & -\rho \end{bmatrix}_{3n \times 3n} \tag{B.58}$$

C is an $n \times 3n$ matrix with the following form:

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots & & & & & \\ 0 & 0 & 0 & 1 & 0 & 0 & \dots & & & \\ & & & & & & \dots & & & \\ & & & & & & & & & \\ 0 & \dots & & & & & & 1 & 0 & 0 \end{bmatrix}_{n \times 3n} \tag{B.59}$$

Grid search is used in the estimation process of the autocorrelation coefficient ρ. The objective function of grid searches could be the Weighted Least Square or the Log Likelihood Function. The formats of the objective functions are as follows (Bournay and Laroque, 1979):

$$WLS = \mu' V^{-1} \mu \tag{B.60}$$

$$LL = -\frac{n}{2} ln(2\pi \frac{\mu' V^{-1} \mu}{n-1}) - \frac{1}{2} log(|V|) - \frac{n}{2} \tag{B.61}$$

To select proper options of the Chow-Lin interpolation, Table A4 shows the information criteria values under different settings. According to this table, the first element Chow-Lin interpolation with constant and WLS as an objective function has the lowest AIC and BIC. Hence, this paper chooses this Chow-Lin setting to generate artificial monthly capital share data.

Table A4: Information Criteria of Different Chow-Lin Settings

Chow-Lin Settings (N=160, n=480, Quarterly to Monthly)				
	Last Element			
(opc, rl)	**WLS**		**LL**	
	AIC	**BIC**	**AIC**	**BIC**
(0, [ ])	−11.222	−11.183	−11.201	−11.162
(1, [ ])	−11.384	−11.327	−11.349	−11.291
	First Element			
(opc, rl)	**WLS**		**LL**	
	AIC	**BIC**	**AIC**	**BIC**
(0, [ ])	−11.349	−11.310	−11.329	−11.291
(1, [ ])	**-11.404**	**-11.346**	−11.373	−11.315

Note: *opc* denotes the option related to the constant. When *opc* equals zero or one, the regression includes zero or one constant respectively. *rl* denotes the innovational parameter. $rl = [\ \ ]$ indicates the autocorrelation parameter ρ is generated by grid search, and the calculation process adopts 100 grids of $\rho \in [0.050, 0.999]$. WLS and LL denotes the objective function for the grid search: Weighted Least Square and Log Likelihood Function respectively.

The coefficients calculated by the Chow-Lin interpolation are shown in Table A5. The estimated constant and the indicator coefficient are both larger than two standard deviations. Although the estimated ρ is close to the upper bound (0.999) of the grid search, since ρ does not go beyond 1, the conditions of partition of residuals still hold (Bournay and Laroque, 1979). Figure A4 plots the interpolated monthly capital share data.

Table A5: Chow-Lin coefficients under selected model specification

	Values	*Std.dev*	*t-stat*
Constant (β_0)	**0.192**	0.055	3.515
β_{Ind}	**0.449**	0.121	3.704
ρ	**0.989**		

Notes: Bold denotes significant or feasible autocorrelation coefficients. β_0 and β_{Ind} are both significant at 95% confident level. The estimated autocorrelation coefficient ρ is within the range of $\rho \in [0.050, 0.999]$ for grid search, indicating no unit roots present in the error term.

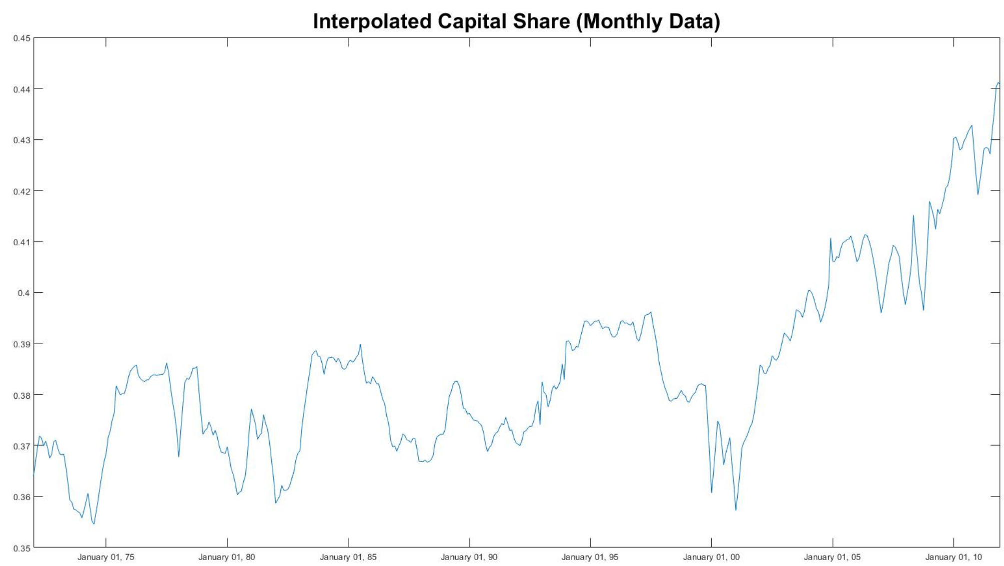

Figure A4: Interpolated Capital Share

C Descriptive Statistics

The descriptive statistics of all portfolio returns and control factors are in Tables (A6) to (A9) below:

Table A6: 10 REV sorted portfolio returns (%)

	10 Size/REV sorted portfolios, value-weighted			
Portfolio/Factor	*Mean*	*Median*	*Std.dev*	*Sharp ratio*
LoPRIOR	1.000	1.135	7.184	0.139
PRIOR2	1.152	1.140	5.674	0.203
PRIOR3	1.154	1.375	5.040	0.229
PRIOR4	1.039	1.335	4.656	0.223
PRIOR5	0.996	1.165	4.422	0.225
PRIOR6	0.907	1.240	4.270	0.213
PRIOR7	0.892	1.105	4.227	0.211
PRIOR8	0.881	1.155	4.375	0.201
PRIOR9	0.750	0.855	4.676	0.161
HiPRIOR	0.676	0.795	5.403	0.125

Notes: Data frequency is monthly. Time span of data is from January 1964 to August 2018.

D Robustness Check

This section presents the robustness check of the empirical works in the main text. The monthly rolling-window estimation and GMM settings are included in this section. Also, I test the pricing power of quarterly capital share growth and CRV.

D.1 Rolling-Window Fama-MacBeth

This subsection presents the rolling-window FMB setting and estimation results. The rolling-window FMB estimates are supplementary evidence of the theoretical model in the main text.

As shown by equation (14), the mean equation of the equity premium is independent from the capital share factor. I compare the performance of capital share factor loadings under different window length to infer the latent DGP of capital risks. To visualize the volatility pattern, I first adopt rolling-window regressions to estimate factor loadings in a time-varying manner as suggested by Lewellen and Nagel (2006). This paper estimates the FMB first step regression following Lewellen and Nagel (2006) with 12, 30, 60, 90-month window lengths. The second step estimation of risk prices is identical to the original cross-sectional regression of the FMB approach. The results of the rolling-window regression serves as a benchmark for the true DGP of factor loadings under the assumption of a modest level of temporal variation.

I start with a very short window for estimation is adopted for the following reasons. Within each window, the regression using short horizon data can be viewed as an estimation that is robust to firm effects, especially since the autocorrelation of stock returns is weaker over a relatively short regression window (Fama and French, 1988). Another function of the rolling-window regression is to serve as a volatility estimator. Volatility is constant within each window, but varies across windows.

I only investigate the volatility patterns using the rolling-window approach due its widely known limitations. The rolling-window FMB is an appropriate approximation for time-varying factor loadings, only conditional on the assumption that there are no structural breaks present within each window. The time variations are still not fully captured due to the ad-hoc window length selection: robustness of the rolling-window approach is diminished when extreme outliers are present in the sample. Therefore, the assumption of rolling-window FMB is still too strong and vulnerable. Further, the rolling-window FMB is subject to a common problem of 2-step estimations, which is that the second step estimation is dependant on the first step results. This approach cannot pass the variability of factor loadings into the second step estimation and, therefore, is insufficient to ensure unbiased estimation of risk prices. The rolling-window approach also views the factor loading as a constant at each time point, causing information carried by the change of factor loading volatilities to be retained within the first step estimation. The time variation of risk prices are thus inflated compared to the true underlying DGP by the rolling window FMB approach when stochastic volatility is present in factor loadings.

As shown by the innovation of market premium in equation (14), the loading of the capital share factor is expected to be centered at zero, and a strong volatility clustering is expected to be present under rolling-window estimation. Due to heteroskedasticity and the model misspecification problem highlighted by Jagannathan and Wang (1996), risk price estimates should be insignificant but vary dramatically over time for short window lengths, and the significance should raise as the window length increase.[32]

I estimate the factor loadings using a rolling-window regression in the first step of the FMB procedure. Risk prices are estimated in the same manner as the static FMB but within each window. Table A11 reports the rolling-window estimates of the parsimonious capital share factor model. As shown in this table, the capital share risk prices are insignificant for under all window lengths. Statically insignificant F_{KS} rules out the possibility that the capital share factor is priced under short-run expectations.

Figure A5-A8 plots the 12, 30, 60, 90-month rolling-window estimated factor loadings of the parsimonious capital share model, and the portfolio returns estimated are size/BM sorted portfolios. As the Figure A5 shows, the factor loadings have small jumps in levels but big structural breaks in volatilities under conditional estimation. The overall level of factor loadings is centered at zero. Figure A5 also shows a strong volatility clustering pattern in factor loadings. As I increase the window length, the volatility clustering diminishes and the factor loading's volatility increase. Also, the capital share factor loading distribution narrows as I increase the window length. Rolling-window results further enhance the possibility that the factor model estimated might be misspecified, in the sense that the capital share factor does not enter the mean equation if I account for the time evolution of risk prices.

[32] An insignificant risk factor in the true equity dynamic might be significant under FMB estimations.

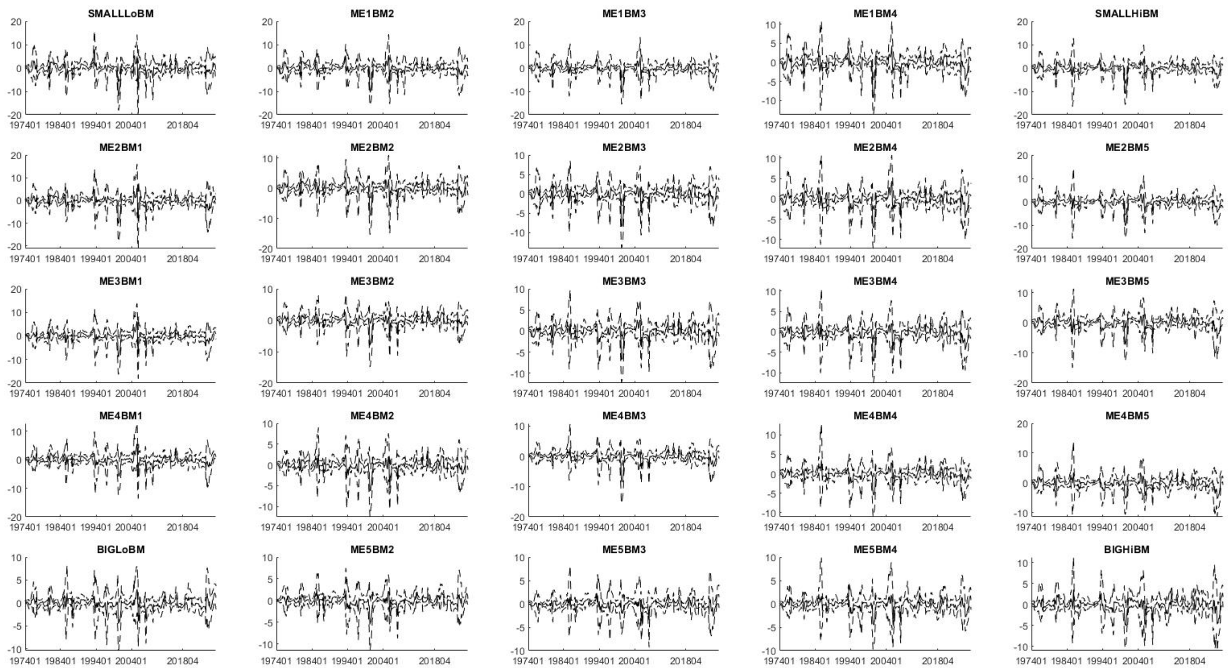

Figure A5: 12-month rolling-window estimation of capital share factor loadings, single factor model. The factor loadings are estimated using monthly size/BM sorted portfolio returns and 12-month window length. The 95% confidence intervals are plotted using dashed line. Sample spans January 1974 to August 2018.

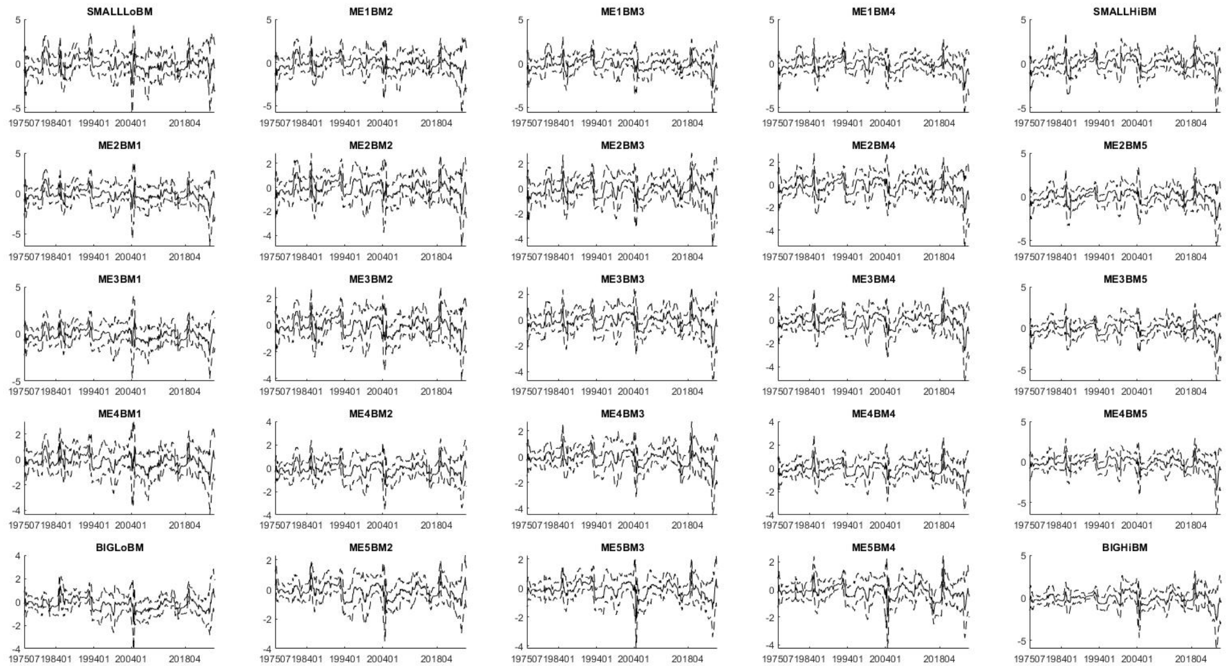

Figure A6: 30-month rolling-window estimation of capital share factor loadings, single factor model. The factor loadings are estimated using monthly size/BM sorted portfolio returns and 12-month window length. The 95% confidence intervals are plotted using dashed line. Sample spans July 1975 to August 2018.

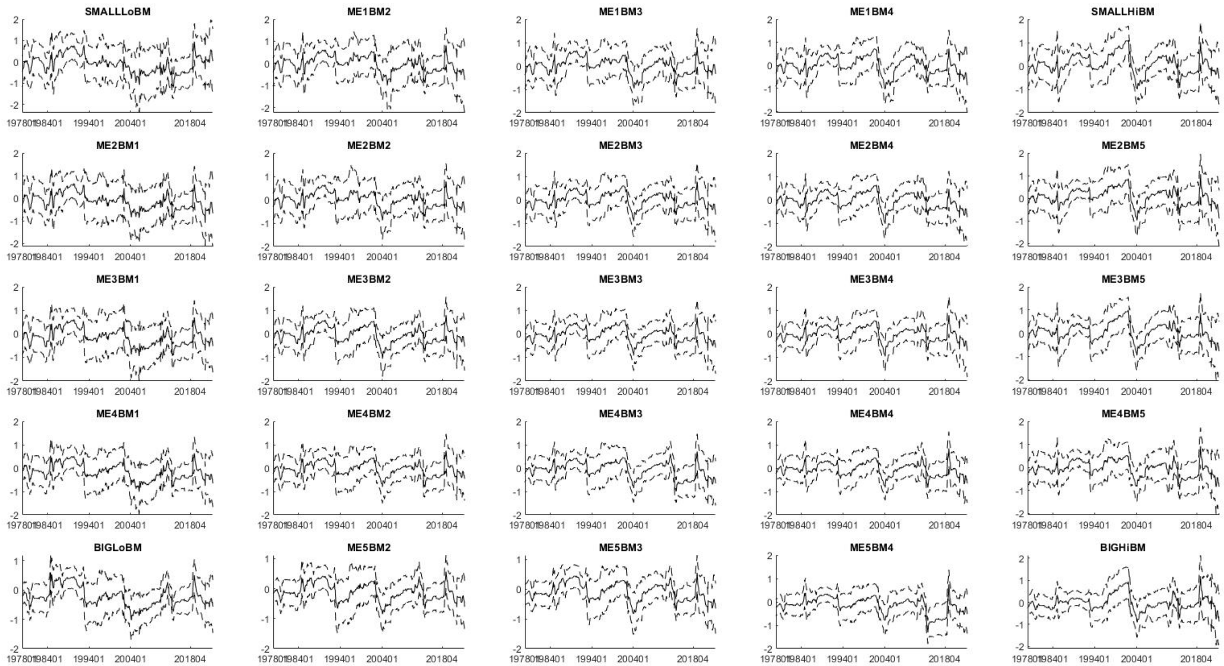

Figure A7: 60-month rolling-window estimation of capital share factor loadings, single factor model. The factor loadings are estimated using monthly size/BM sorted portfolio returns and 12-month window length. The 95% confidence intervals are plotted using dashed line. Sample spans January 1978 to August 2014.

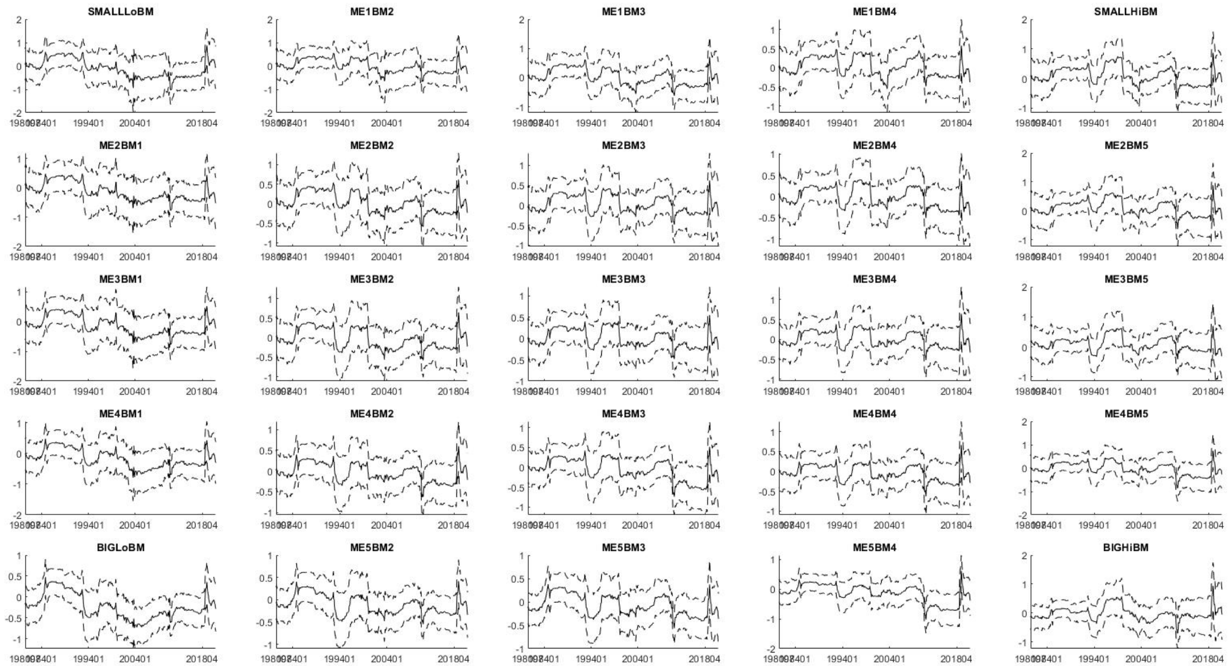

Figure A8: 90-month rolling-window estimation of capital share factor loadings, single factor model. The factor loadings are estimated using monthly size/BM sorted portfolio returns and 90-month window length. The 95% confidence intervals are plotted using dashed line. Sample spans July 1980 to August 2018.

Figure A9 plots the capital share risk prices estimated by the single factor model using 12-month window length. This figure shows that, the time variation of risk prices is very high across the sample, and the level of risk prices witnesses frequent structural breaks. In the first step of the rolling-window FMB estimation, the factor loadings only capture the effects caused by level changes and not the effects caused by volatility changes. In the second step estimation, the factor loadings at each time are treated as a constant, leading to a more volatile risk price series over the time when volatility varies across windows.

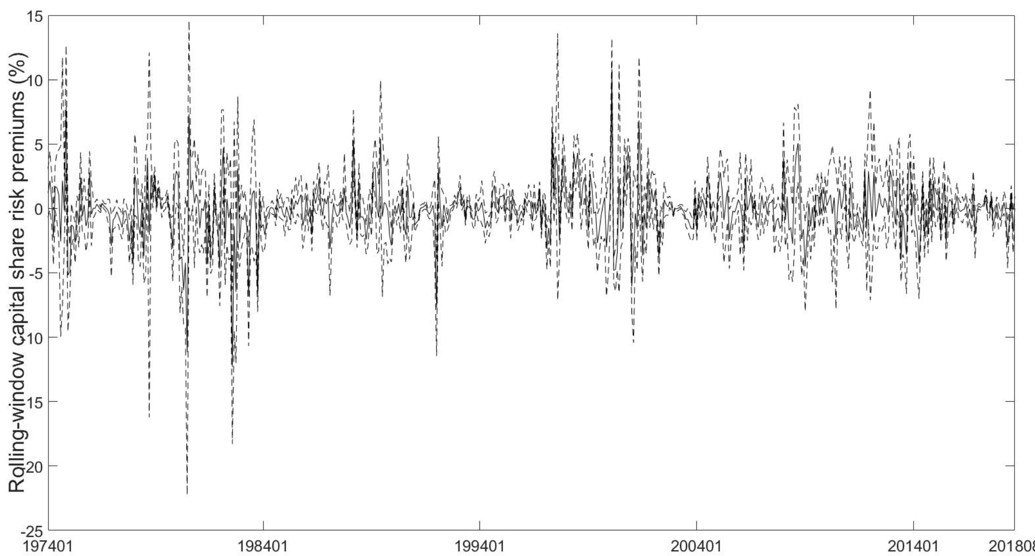

Figure A9: Rolling-window capital share factor risk price (%). Following Fama and MacBeth (1973) and Lewellen and Nagel (2006), the factor loadings are estimated using monthly size/BM sorted portfolio returns and 12-month window length. The 95% confidence intervals are plotted using dashed line. The sample spans January 1974 to August 2018.

Overall, the rolling-window FMB estimates are consistent with the theoretical model in equations (13) and equation (14) in that the capital share factor loadings are centered at zero with strong volatility clustering. However, this analysis cannot rule out the potential impact of large outliers on risk price estimates due to the very short window length used. The results derived by the rolling-window FMB procedure also support accounting for structural breaks and stochastic volatility for further robustness in the main text using Bayesian analysis.

D.2 GMM estimation

This subsection includes details of GMM estimation of monthly CRV risk prices. In this paper, I use two-pass regression GMM estimation following Lettau et al. (2019).

The moment conditions for the expected return-beta representations are:

$$g_T(b) = \begin{bmatrix} E(\mathbf{R_t^e} - \lambda_0 - \beta\lambda) \\ E(\mathbf{R_t^e} - \alpha - \beta CRV_t) \\ E\left((\mathbf{R_t^e} - \alpha - \beta CRV_t) \otimes CRV_t\right) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{D.1}$$

where $\alpha = [\alpha_1 ... \alpha_{25}]'$ is the vector of constants, and $\beta = [\beta_1 ... \beta_{25}]$ is the vector of factor loadings. R_t^e is the vector of expected portfolio returns.[33] The parameter vector $b' = [\alpha\ \beta\ \lambda_0\ \lambda]$. λ_0 is the constant and λ is the CRV risk price. The point estimates from GMM are identical to those from FMB regressions (Lettau et al., 2019). Therefore, α and β are obtained in the first-stage time-series regression of the standard FMB procedure. λ_0 and λ are obtained in the second-stage cross-sectional regression of the standard FMB procedure. As in Lettau et al. (2019), I choose parameters b to set the following linear combination of moments to zero:

$$a_T g_T(b) = 0 \tag{D.2}$$

where

$$a_T = \begin{bmatrix} \mathbf{I} & 0 \\ 0 & [\mathbf{1}_{25} \quad \beta]' \end{bmatrix} \tag{D.3}$$

[33] $\otimes$ is the Kronecker tensor product.

Following Hansen (1982), I compute the d matrix of derivatives for computing standard errors:

$$d = \frac{\partial g_T(b)}{\partial b'}$$
$$= \begin{bmatrix} -\mathbf{I}_{25\times25} & -\mathbf{I}_{25\times25} \otimes E(CRV_t) & \mathbf{0}_{25\times2} \\ -\mathbf{I}_{25\times25} & -\mathbf{I}_{25\times25} \otimes E(CRV_t^2) & \mathbf{0}_{25\times2} \\ -\mathbf{0}_{25\times25} & -\mathbf{I}_{25\times25} \otimes \lambda & -[\mathbf{1}_{25} \quad \beta] \end{bmatrix} \tag{D.4}$$

The the spectral density matrix S at frequency zero in this paper is:

$$S = \sum_{-\infty}^{\infty} E\left(\begin{bmatrix} E(\mathbf{R^e_t} - \lambda_0 - \beta\lambda) \\ E(\mathbf{R^e_t} - \alpha - \beta CRV_t) \\ E\left((\mathbf{R^e_t} - \alpha - \beta CRV_t) \otimes CRV_t\right) \end{bmatrix} \begin{bmatrix} E(\mathbf{R^e_{t-j}} - \lambda_0 - \beta\lambda) \\ E(\mathbf{R^e_{t-j}} - \alpha - \beta CRV_{t-j}) \\ E\left((\mathbf{R^e_{t-j}} - \alpha - \beta CRV_{t-j}) \otimes CRV_{t-j}\right) \end{bmatrix}\right) \tag{D.5}$$

Following Lettau et al. (2019), denote:

$$h_t(b) = \begin{bmatrix} E(\mathbf{R^e_t} - \lambda_0 - \beta\lambda) \\ E(\mathbf{R^e_t} - \alpha - \beta CRV_t) \\ E\left((\mathbf{R^e_t} - \alpha - \beta CRV_t) \otimes CRV_t\right) \end{bmatrix} \tag{D.6}$$

and use the Newey-West (1986) correction to the standard errors with lag L:

$$S_T = \sum_{j=-L}^{L} (\frac{L-|j|}{L}) \frac{1}{T} \sum_{t=1}^{T} h_t(\hat{b}) h_{t-j}(\hat{b})' \tag{D.7}$$

The optimal lag length in this paper is $L = 656^{\frac{1}{3}} \approx 9$. Therefore, for λ estimation, I use the asymptotic standard variance as:

$$Var(\hat{b}) = \frac{1}{T}(\alpha_T d)^{-1} \alpha_T S_T \alpha_T' (\alpha_T d)'^{-1} \tag{D.8}$$

The GMM estimate of CRV risk price is 5.657 (%) and factor loading is 4.832.

D.3 Quarterly Estimations

This subsection presents quarterly data estimates using size/BM sorted portfolio returns. The quarterly size/BM returns converted from monthly data as in Lettau et al. (2019).

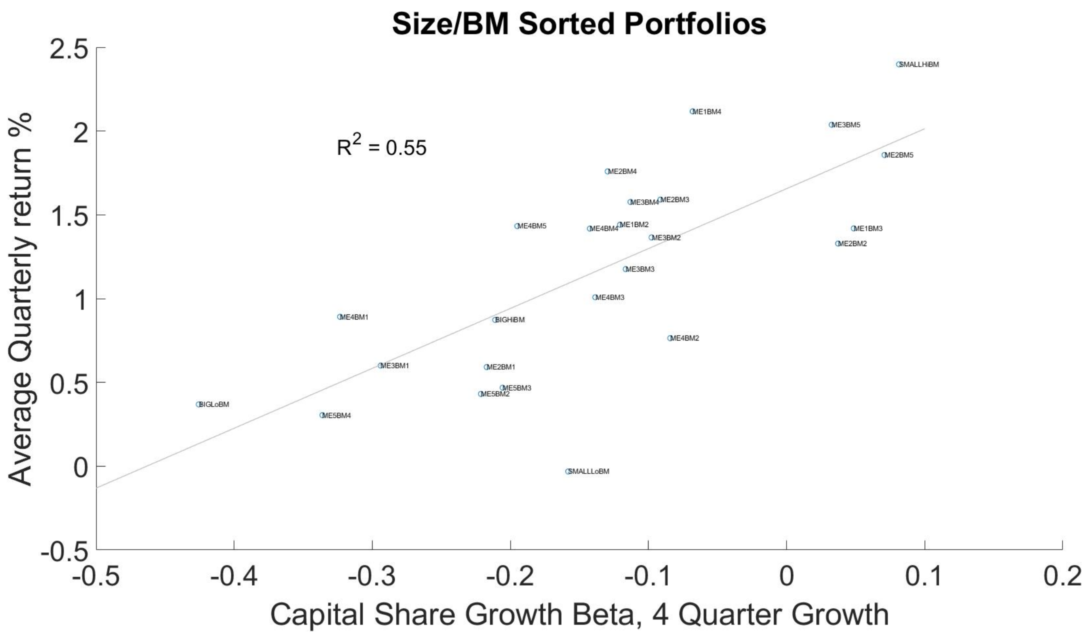

Figure A10: Capital share growth betas (%), quarterly. This plot depicts the betas constructed by the F-MB regression of average portfolio returns on capital share beta. The quarterly average returns are on the y-axis and the portfolio factor betas are on the x-axis. R^2 estimates of each regression are reported in the graph. Sample spans 1974Q1 to 2018Q3.

To obtain the quarterly excess return, I use the following method:

$$R^p_q = (1 + R^p_{m1})(1 + R^p_{m2})(1 + R^p_{m3}) - (1 + R^f_{m1})(1 + R^f_{m2})(1 + R^f_{m3}) \tag{D.9}$$

where R^p_q is the quarterly excess return. R^p_{m1}, R^p_{m2}, and R^p_{m3} are monthly return of the first, second and third month in this quarter, respectively. R^f_{m1}, R^f_{m2}, and R^f_{m3} are monthly risk free rate of the first, second and third month in this quarter, respectively.

The FMB tests of 4-quarter capital share growth and quarterly CRV are plotted in Figures A10 and A11. In quarterly data, both capital share growth and CRV explains high cross-sectional return variations, as estimated R^2 for these two factors are both 0.55. The slop of regression line is closer to 1 in Figure A11 than that in Figure A10. Therefore, the monthly conclusion that CRV captures the true DGP also holds in quarterly data.

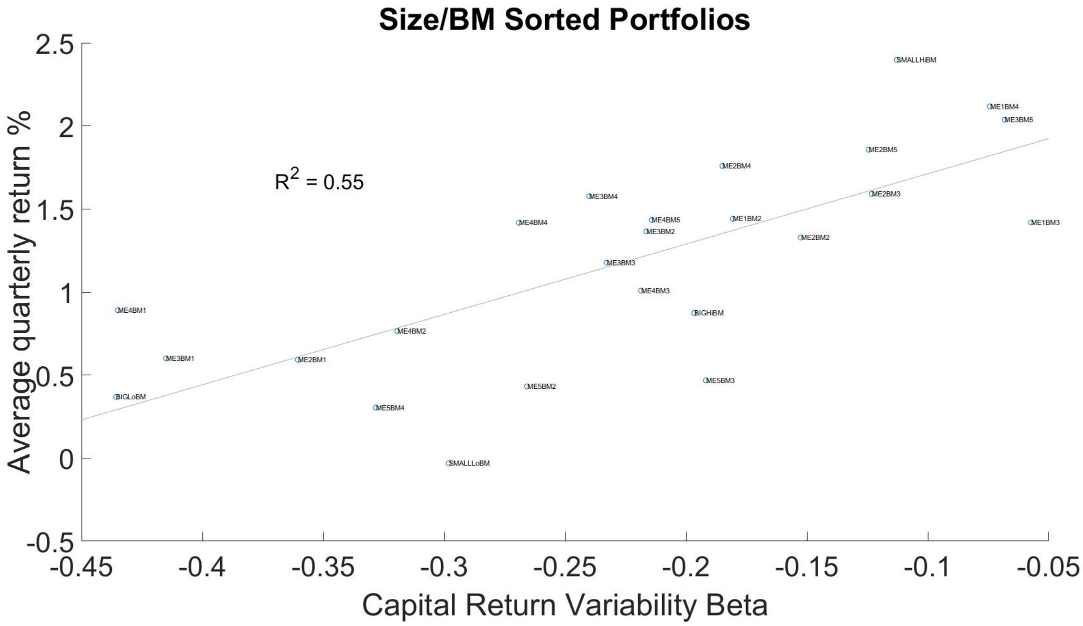

Figure A11: Capital return variability betas (%), quarterly. This plot depicts the betas constructed by the F-MB regression of average portfolio returns on capital share beta. The quarterly average returns are on the y-axis and the portfolio factor betas are on the x-axis. R^2 estimates of each regression are reported in the graph. Sample spans 1974Q1 to 2018Q3.

I also reports the quarterly FMB bootstrap estimates of risk prices in Table A12. In this table, Column A reports a single capital share growth model estimates, Column B reports a single CRV model estimates, Column B reports estimates of a 2-factor model including capital share growth and CRV. In Column A, capital share growth generates significant a risk price, but the $\bar{R}^2$ is low. In Column B, CRV is significant and generates higher $\bar{R}^2$ than that estimated by capital share growth. In Column C, capital share growth is strongly dominated by CRV, and both CRV risk price and $\bar{R}^2$ are of similar magnitudes as those in Column B. Therefore, in quarterly data, CRV is a long-run risk factor that outperforms capital share growth and captures the true DGP.

Table A7: 25 Size/BM sorted portfolio returns (%)

	25 Size/BM sorted portfolios, value-weighted			
Portfolio/Factor	*Mean*	*Median*	*Std.dev*	*Sharp ratio*
SMALLLoBM	0.681	1.060	7.854	0.087
ME1BM2	1.213	1.523	6.849	0.177
ME1BM3	1.192	1.254	5.934	0.201
ME1BM4	1.407	1.450	5.644	0.249
SMALLHiBM	1.491	1.485	5.946	0.251
ME2BM1	0.923	1.376	7.094	0.130
ME2BM2	1.174	1.456	5.924	0.198
ME2BM3	1.273	1.530	5.374	0.237
ME2BM4	1.315	1.528	5.197	0.253
ME2BM5	1.367	1.788	5.964	0.229
ME3BM1	0.920	1.546	6.515	0.141
ME3BM2	1.20	1.505	5.383	0.223
ME3BM3	1.19	1.486	4.943	0.230
ME3BM4	1.268	1.442	4.855	0.261
ME3BM5	1.414	1.524	5.587	0.253
ME4BM1	1.035	1.157	5.823	0.178
ME4BM2	1.018	1.215	5.052	0.201
ME4BM3	1.091	1.354	4.906	0.222
ME4BM4	1.229	1.420	4.720	0.260
ME4BM5	1.210	1.423	5.626	0.215
BIGLoBM	0.893	0.998	4.569	0.195
ME5BM2	0.915	1.073	4.375	0.209
ME5BM3	0.942	1.215	4.231	0.223
ME5BM4	0.872	0.995	4.581	0.190
BIGHiBM	1.052	1.319	5.326	0.198

Notes: Data frequency is monthly. Time span of data is from January 1964 to August 2018.

Table A8: 25 Size/INV sorted portfolio returns (%)

	25 Size/INV sorted portfolios, value-weighted			
Portfolio/Factor	*Mean*	*Median*	*Std.dev*	*Sharp ratio*
SMALLLoINV	1.353	1.376	7.183	0.188
ME1INV2	1.357	1.413	5.599	0.242
ME1INV3	1.385	1.631	5.603	0.247
ME1INV4	1.266	1.561	5.926	0.214
SMALLHiINV	0.783	1.028	7.049	0.111
ME2INV1	1.280	1.636	6.298	0.203
ME2INV2	1.296	1.586	5.209	0.249
ME2INV3	1.315	1.490	5.197	0.253
ME2INV4	1.287	1.595	5.633	0.228
ME2INV5	0.900	1.235	6.893	0.131
ME3INV1	1.263	1.460	5.673	0.223
ME3INV2	1.310	1.475	4.775	0.274
ME3INV3	1.196	1.383	4.761	0.251
ME3INV4	1.206	1.495	5.273	0.229
ME3INV5	0.919	1.307	6.441	0.143
ME4INV1	1.160	1.455	5.318	0.218
ME4INV2	1.127	1.388	4.709	0.239
ME4INV3	1.152	1.402	4.620	0.249
ME4INV4	1.154	1.269	4.867	0.237
ME4INV5	0.972	1.224	6.240	0.156
BIGLoINV	1.083	1.125	4.554	0.238
ME5INV2	0.937	0.920	3.957	0.237
ME5INV3	0.894	1.000	4.066	0.220
ME5INV4	0.883	1.045	4.379	0.202
BIGHiINV	0.877	1.113	5.390	0.163

Notes: Data frequency is monthly. Time span of data is from January 1964 to August 2018.

Table A9: 25 Size/OP sorted portfolio returns (%).

	25 Size/OP sorted portfolios, value-weighted			
Portfolio/Factor	*Mean*	*Median*	*Std.dev*	*Sharp ratio*
SMALLLoOP	0.955	0.980	7.218	0.132
ME1OP2	1.331	1.471	5.791	0.230
ME1OP3	1.273	1.581	5.583	0.228
ME1OP4	1.357	1.505	5.739	0.237
SMALLHiOP	1.240	1.477	6.546	0.190
ME2OP1	1.001	1.522	6.944	0.144
ME2OP2	1.193	1.633	5.640	0.212
ME2OP3	1.209	1.510	5.244	0.230
ME2OP4	1.194	1.298	5.509	0.217
ME2OP5	1.352	1.698	6.143	0.220
ME3OP1	0.948	1.208	6.535	0.145
ME3OP2	1.154	1.485	5.091	0.227
ME3OP3	1.138	1.384	4.866	0.234
ME3OP4	1.146	1.286	5.106	0.225
ME3OP5	1.302	1.554	5.753	0.226
ME4OP1	0.955	1.077	6.044	0.158
ME4OP2	1.087	1.391	5.057	0.215
ME4OP3	1.066	1.250	4.720	0.226
ME4OP4	1.131	1.293	4.833	0.234
ME4OP5	1.200	1.558	5.307	0.226
BIGLoOP	0.753	1.051	5.444	0.138
ME5OP2	0.753	0.926	4.412	0.171
ME5OP3	0.903	1.033	4.325	0.209
ME5OP4	0.870	1.127	4.357	0.200
BIGHiOP	0.992	1.106	4.273	0.232

Notes: Data frequency is monthly. Time span of data is from January 1964 to August 2018.

Table A10: Descriptive Statistics of Risk Factors(%)

Portfolio/Factor	*Mean*	*Median*	*Std.dev*	*Sharp ratio*
g^k				
January 1964 - January 1974	-0.245	-0.502	2.690	-0.091
January 1974 - August 2018	0.435	0.195	2.336	0.186
January 1964 - August 2018	0.310	0.074	2.416	0.129
$E\left[(g^k)^2\right]$				
January 1964 - January 1974	0.065	0.024	0.085	0.764
January 1974 - August 2018	0.051	0.017	0.083	0.615
January 1964 - August 2018	0.054	0.018	0.084	0.643

Notes: g^k denotes the capital share growth and $E\left[(g^k)^2\right]$ denotes CRV. The training sample spans January 1964 to January 1974. The sample used for estimation spans January 1974 to August 2018. The full sample spans January 1964 to August 2018.

Table A11: Capital Share Beta Rolling-window Estimations

	12-month	*30-month*	*60-month*	*90-month*
β_0	0.674**	0.857**	0.861**	0.923**
	(0.000)	(0.000)	(0.000)	(0.000)
F_{KS}	-0.623*	0.325	0.361	-0.188
	(0.530)	(0.596)	(0.551)	(0.689)
R^2	0.380	0.392	0.477	0.487

Note: This table reports risk prices (%) of the capital share factor. Conditional equity premium in equation (14) is tested by including capital share factor F_{KS}, which is the 12, 30, 60, or 90-month capital share growth, in the mean equation. Portfolio returns used for estimation is size/BM sorted portfolios. The model estimated is a single capital share factor model, where β_0 is the constant and $F_{KS,t}$ is the capital share factor. the P-values are reported in parentheses below estimates. ** and * denote significance at the 5% and 10% levels, respectively. Sample spans the period January 1974 to August 2018.

Table A12: FMB Bootstrap Risk Price Estimates

	Column A	*Column B*	*Column C*
α	3.505**	3.615**	3.612**
	[2.664, 4.358]	[2.702 4.475]	[2.533, 4.705]
g^k	1.397**		0.481
	[1.755, 3.073]		[-0.860, 1.842]
CRV		2.951**	2.978**
		[1.067 4.813]	[1.517, 2.554]
$\bar{R}^2$	0.192	0.335	0.340
	[0.000 0.530]	[0.000, 0.675]	[0.000, 0.691]

Note: This table reports FMB bootstrap estimations of factor risk prices (%). Factor estimated are capital share growth and CRV. CRV captures the true DGP of long-run equity premium in equation (17). For maintaining the consistency of moments and distribution functions (Horowitz, 1997), I set the optimal block-length as $536^{(\frac{1}{5})} \approx 4$ following Hall et al. (1995). The optimal block-length for FMB second stage is identical to Lettau et al. (2019). I use 10,000 simulations for bootstraps. This table reports the capital share growth model estimates, the CRV model estimates, and estimates of a 2-factor model including both capital share growth and CRV. Portfolio returns used for estimation are size/BM sorted portfolios. Bootstrapped 95% confidence intervals are reported in square brackets. ** denotes the estimate is significant at 5% level. * denotes the estimate is significant at 10% level. Sample spans 1974Q1 to 2018Q3.