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Abstract

We develop a New Keynesian Phillips curve based on a combination of staggered price contracts and indexation to past inflation. This Phillips curve links current inflation dynamics to past inflation with a positive weight, as well as current and lagged expectations of inflation and output, giving a possible alternative explanation for recent empirical findings on the role of expectations in the determination of inflation.

JEL Classification: E31, E3, E5.

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1 Introduction

There is a debate on the role of expectations in inflation dynamics. The standard forward-looking New Keynesian model is often considered as empirically inaccurate (Estrella and Fuhrer, 2002). A part of the literature has then claimed that introducing non rational price setters along the rational ones helped to reproduce the role of past inflation in the determination of current inflation that is observed in the data (see Woodford, 2003 for an early presentation). Another part of the literature, from Roberts (1997) to Coibion, Gorodnichenko, and Kamdar (2018), has argued that introducing "real world" expectations in the structural form of the model helped to improve its empirical plausibility. Fuhrer (2017) has shown recently that the inertia of aggregate variables was indeed well captured by the inertia of expectations, both for inflation and output, that is found in expectations surveys, leaving a small role only for lagged inflation.

By combining staggered prices with indexation, we propose a framework that allows both elements to play a role in inflation dynamics. Even when price setters form rational expectations, the structure of price rigidity is such that lagged expectations of macroeconomic variables play a role in the Phillips curve. Past inflation also has a direct impact, either with a negative or a positive sign, depending on the degree of indexation of current prices to past inflation.

To do that, we develop a New Keynesian Phillips curve that is based on a staggered pricing structure proposed by Taylor (1980), in which we introduce a partial indexation of prices to lagged inflation, as in Woodford (2003). The staggered prices structure offers a role for inertia of expectations in the determination of current inflation. While it has been shown that in a purely forward-looking models, this lagged inflation has always a negative coefficient (Yao, 2009, Whelan, 2007), we show that introducing price indexation transforms this influence into a positive one. We also show that if we remove expectational error terms, this Phillips curve is similar to the inflation equation proposed by Fuhrer and Moore (1995), which contained several lags and leads of inflation to explain the current inflation rate.

2 A New Keynesian Model with Staggered Contracts and Indexation

We develop a staggered prices model in the sense of Taylor (1980). Prices are set in advance for a length of N periods. N different cohorts of price setters of equal size coexist at any given time. Each price contract has a duration of N periods. When a new contract is set, the firms chooses rationally the price to set initially, and then during the rest of the life of the contract, this initial price is updated to the last known inflation rate (see Woodford, 2003). Compared to Taylor (1980), prices are no longer fully predetermined and fixed since they can be modified each period to adjust to last period inflation.

2.1 Price setting under indexed staggered contracts

We consider a standard linearized model of price setting in which the profit-maximizing flexible price at time t, noted p_t^* is equal to[1]:

$$p_t^* = p_t + \phi y_t \tag{1}$$

where p_t is the general price level, $\phi > 0$ a parameter measuring real rigidities and strategic complementarities, and y_t the real marginal costs of firms[2]. Each cohort of firms can set a new price at a different period. When a firm can set a new price at period t, it does it in order to minimize the distance between the price effectively set and the profit maximizing price p^* during the N periods (including the current one) of the duration of the price contract. $x_{t,t+i}$, $i \in [0, ..., N-1]$ is the value of the price effective during period $t+i$ for a contract set in period t. The initial value of the contract is $x_{t,t} = x_t$. In the model of Taylor (1980), prices are fixed all over the duration of the contract and then we have $x_{t,t+i} = x_t$, $\forall i \in [0, ..., N-1]$. However, we propose to modify the model by introducing a mechanism of partial indexation to past inflation similar to the one proposed by Woodford (2003) for the model of Calvo (1983). In Woodford (see p.214), a fraction of price setters sets its optimal price, and the remaining prices are partially indexed on the past inflation rate. We adapt this assumption to the structure of Taylor. At the beginning of each price contract, a firm sets its price optimally. This price remains unchanged during the remaining period of the contract, excepted that during each period of the contract consecutive to the initial period, a fraction γ of the value of the previous period inflation is added to the current value of the price. For example, the value in $t+1$ of a price optimally modified in t is $x_{t,t+1} = x_t + \gamma\pi_t$, with $\pi_t = p_t - p_{t-1}$ the inflation rate in t. More generally, for $0 < i < N$, we have:

[1] See Woodford (2003) or Sheedy (2010)

[2] The model is linear. All variables are in log. Under restrictive assumptions, the real marginal cost can be linearly related to the output gap.

$$x_{t,t+i} = x_t + \gamma \sum_{l=0}^{i-1} \pi_{t+l} \tag{2}$$

A firm deciding in t to set the initial value of its new price contract has the following program:

$$\min_{x_t} \sum_{i=0}^{N-1} \beta^i \mathbb{E}_t (x_{t,t+i} - p^*_{t+i})^2 \tag{3}$$

where β is the discount factor, $x_{t,t+i}$ is given by (2) for $i \in]0, N]$, and $x_{t,t} = x_t$. The first order condition of this programs leads to:

$$x_t = \frac{\sum_{i=0}^{N-1} \beta^i \mathbb{E}_t p^*_{t+i} - \gamma \sum_{i=0}^{N-2} \beta^{i+1} \sum_{l=0}^{i} \mathbb{E}_t \pi_{t+l}}{\sum_{i=0}^{N-1} \beta^i} \tag{4}$$

Given the short-run horizon of the model and the finite duration of contracts, to simplify the exposition, without altering the results, we assume that $\beta = 1$. Using equation (1) and given that for $i \geqslant 1$, we have $p_{t+i} = p_t + \sum_{k=1}^{i} \pi_{t+k}$. it is possible to write the previous equation as:

$$x_t = p_t + \frac{1}{N} \left[\sum_{i=1}^{N-1} (N-i) \mathbb{E}_t (\pi_{t+i} - \gamma \pi_{t+i-1}) \right] + \frac{1}{N} \phi \sum_{i=0}^{N-1} \mathbb{E}_t y_{t+i} \tag{5}$$

A price set in t responds to the current price level, to expected inflation and to the expected trajectory of the driving variable y. Without indexation ($\gamma = 0$), this responds to the expected path of inflation over the duration of the contract. With full indexation ($\gamma = 1$), it only responds to expected variations of inflation.

Since all cohorts of prices have the same size, the price level in the economy is an average of the price contracts at a given date: $p_t = \frac{1}{N} \sum_{j=0}^{N-1} x_{t-j,t}$. Given the value of prices given in (2), we can rewrite the price level in t such as:

$$p_t = \frac{1}{N} \left[\sum_{j=0}^{N-1} x_{t-j} + \gamma \sum_{j=1}^{N-1} (N-j) \pi_{t-j} \right] \tag{6}$$

2.2 The Phillips curve

Combining equation (5) and equation (6), we have:

$$Np_t = \sum_{i=0}^{N-1} p_{t-i} + \frac{1}{N}\sum_{j=0}^{N-1}\mathbb{E}_{t-j}\left(\sum_{i=1}^{N-1}(N-i)(\pi_{t+i-j} - \gamma\pi_{t+i-j-1}) + \phi\sum_{i=0}^{N-1} y_{t+i-j}\right) + \gamma\sum_{j=1}^{N-1}(N-j)\pi_{t-j} \tag{7}$$

Noting that $Np_t - \sum_{i=0}^{N-1} p_{t-i} = (N-1)\pi_t + \sum_{i=1}^{N-1}(N-i-1)\pi_{t-i}$, we obtain the following Taylor hybrid Phillips curve:

$$\pi_t = \eta\sum_{i=1}^{N-1} a_i\pi_{t-i} + \eta\sum_{j=0}^{N-1}\sum_{i=1}^{N-1} b_{ji}\mathbb{E}_{t-j}\pi_{t+i-j} + \eta\phi\sum_{j=0}^{N-1}\sum_{i=0}^{N-1}\mathbb{E}_{t-j}y_{t+i-j} \tag{8}$$

with $\eta = \frac{1}{(N-1)(N+\gamma)}$, $a_i = [\gamma + N(1-\gamma)(1-N+i)]$ and $b_{ji} = [\gamma + (1-\gamma)(N-i)]$. The first term represents the role of lagged inflation. The second term represents the role of current and lagged expectations of inflation. The third term represents the role of current and past expectations of output gap. We then obtain for Taylor staggered contracts a similar equation to the equation [2.6] proposed by Sheedy (2010) for different hazard functions of price adjustment[3]. Here, the number of lags and leads of inflation is finite, and the value of the coefficients on them have an explicit value depending on the length of contracts and the level of indexation to past inflation. Compared to Whelan (2007) and Yao (2009), the presence of indexation can give a positive weight to lagged inflation, which is excluded in the purely forward-looking model. Indeed, when $\gamma = 0$, the weight on past inflation is strictly negative for any term of past inflation. However, when there is full indexation to past prices ($\gamma = 1$), the weight on past inflation is always positive. For partial indexation, $\gamma \in]0,1[$, there exists a mix of positive and negative weights on past inflation. For example, if $N = 4$, the weight on π_{t-3} is positive for any value $\gamma > 0$, the weight on π_{t-2} is positive for $\gamma > 4/5$ and the weight on π_{t-1} is positive for $\gamma > 8/9$.

3 The staggered Phillips curve without lagged expectations

Lagged expectations of inflation represent an interesting feature of staggered prices models since they allow a sluggish response of inflation to shocks, even in the case of a disinflation policy with forward-looking agents (see Musy, 2006). However, it is frequent to remove

[3] The analysis of Sheedy (2010) does not include Taylor contracts as a specific case (see the discussion p. 1052). The output gap terms are different. He also does not consider indexation at all.

lagged expectations under the argument that they represent expectational errors that can be neglected (see Roberts, 1995 or Walsh, 2010)[4]. In this section, we follow this approach and we rewrite lagged expectational of current and past terms as expectational errors terms, equal ot 0 on average. We then assume $\mathbb{E}_{t-j}\pi_{t-j+i} - \pi_{t-j+i} = 0$, for $j > i \geq 0$. We also assume that for $i > 0$, $j > 0$, $\mathbb{E}_{t-j}\pi_{t+i} - \mathbb{E}_t\pi_{t+i} = 0$. Under such assumptions, the New Keynesian Phillips curve becomes:

$$\pi_t = \eta_E \sum_{i=1}^{N-1} c_{i,j}\pi_{t-i} + \eta_E \sum_{i=1}^{N-1} d_{i,j}\mathbb{E}_t\pi_{t+i} + \eta_E\phi\left(Ny_t + \sum_{i=1}^{N-1}(N-i)(y_{t-i} + \mathbb{E}_t y_{t+i})\right) \tag{9}$$

with $\eta_E = \frac{2}{N(N-1)(1+\gamma)}$, $c_{i,j} = [\gamma(N-i) - (1-\gamma)\sum_{j=0}^{N-1-i} j]$ and $d_{i,j} = [(N-i)\gamma + (1-\gamma)\sum_{j=1}^{N-i} j]$. To illustrate, the Phillips curve obtained when $N = 2$ is :

$$\pi_t = \frac{\gamma}{1+\gamma}\pi_{t-1} + \frac{1}{1+\gamma}\mathbb{E}_t\pi_{t+1} + \phi\hat{y}_t \tag{10}$$

where $\hat{y}_t$ is a moving average of output gaps. We observe that when there is no indexation ($\gamma = 0$), this reduces to the standard New Keynesian Phillips curve: $\pi_t = \mathbb{E}_t\pi_{t+1} + \phi\hat{y}_t$. When there is full indexation ($\gamma = 1$), this reduces to the famous hybrid Phillips curve proposed by Fuhrer and Moore (1995) (eq. 6 p. 130): $\pi_t = \frac{1}{2}(\pi_{t-1} + \mathbb{E}_t\pi_{t+1}) + \phi\hat{y}_t$.

For higher values of N, when $\gamma = 0$, the weight on past inflation remains strictly negative. When $\gamma = 1$, the Phillips curve is:

$$\pi_t = \frac{1}{N(N-1)}\left[\sum_{i=1}^{N-1}(N-i)\left(\pi_{t-i} + \mathbb{E}_t\pi_{t+i}\right)\right] + \frac{\phi}{N(N-1)}\left[Ny_t + \sum_{i=1}^{N-1}(N-i)(y_{t-i} + \mathbb{E}_t y_{t+i})\right] \tag{11}$$

This Phillips curve with full indexation is also close to the one proposed by Fuhrer and Moore (1995), equation (25) p. 147, who have different foundations. Considering expectational errors as equal to 0, they display a two-sided inflation equation of the form:

$$\pi_t = f(L)f(L^{-1})[\pi_t + \phi g^{-1}(L)y_t] \tag{12}$$

where L stands for the lag operator. The number of lags and leads of inflation depends on the length of contracts N. For intermediate values of indexation, the weights on past and

[4] Sheedy (2010) proposes a formal discussion of removing these terms.

lead inflation are no longer symmetric. Coefficient on past inflation can even be negative for low values of γ. The assumptions of Fuhrer and Moore (1995) have been strongly criticized by Holden and Driscoll (2003), but we show that it is possible to obtain a similar Phillips curve with a standard assumption of price indexation.

4 Conclusion

We have proposed a New Keynesian Phillips curve that combines the staggered price structure of Taylor (1980) with indexation to past inflation in the sense of Woodford (2003). This results in a hybrid form in which are present lagged terms of inflation and output as well as lagged and current expectations of those variables. When expectational errors are ruled out we show that the case with full indexation leads to the Phillips curve proposed by Fuhrer and Moore (1995).

When expectational errors are not ruled out, we obtain a Phillips curve containing past expectations of inflation and output, as well as possible positive weights on past inflation, which was not the case in the purely forward-looking model. Such a form, built with the standard assumptions used in the New Keynesian literature, can be an alternative way to understand why some recent studies find an influence of past expectations in inflation dynamics (see Fuhrer, 2017).

Assuming Taylor contracts with indexation can then offer different dynamics than the models built on the Calvo assumption, while keeping a tractable and intuitive structure. Ben Aıssa and Musy (2011) compare the standard forms of inflation dynamics of the literature for two period contracts. It could be interesting to extend this comparative work for longer price durations and variate degrees of price indexation.

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