Mixed ownership and RD under discriminatory output subsidies

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Abstract

This study considers a (partially privatized) semi-public firm in a mixed duopoly and examines the welfare effects of discriminatory output subsidies under R&D competition. We find that the government grants higher subsidies to the private firm than to the semi-public firm, which induces the private firm to invest more in R&D and to produce a higher output than the semi-public firm. We also show that optimal subsidy rates are higher (lower) than uniform subsidy rates for a sufficiently high (low) degree of privatization, which could decrease (increase) social welfare. This finding sharply contrasts to the case that the committed discriminatory output subsidy always yields the highest welfare compared to non-committed cases.

Keywords: mixed ownership; time-consistency; discriminatory output subsidies; R&D competition

1. Introduction

Many industries are composed of mixed ownership in developed and developing countries, and semi-public firms (partially privatized) are highly concentrated within a few sectors with large portions of the world’s resources. For example, the Chinese government has steadily advanced mixed-ownership reform of state-owned enterprises, which resulted in the number of mixed-owned firms under government control totalling approximately 69% in 2017, while intensively competing with privately-owned firms in certain sectors such as transportation, energy, finance, education, and healthcare.

1 Kowalski et al. (2013, OECD) reported that among the 2,000 largest public companies in the world, more than 10% are (semi-) public and state owned and more than half (in terms of value) of all public firms in OECD countries are significant players in energy-related industries. For some real examples, see Ino and Matsumura (2010), Lee et al. (2018), Xu and Lee (2019), and Liu and Wang (2020).
Many researchers have addressed mixed oligopolies that have been initially defined by De Fraja and Delbono (1989) as the simultaneous presence of private and public firms in a competitive market. Matsumura (1998) introduced the analysis of mixed ownership to the mixed duopoly market in which the government determines the optimal privatization policy of the partially privatized public firm, rather than of fully public or fully private firms. Ever since, recent theoretical literature in the context of applied game theory and industrial economics has focused on the effects of mixed ownership on semi-public firm strategies and social welfare.\(^2\)

Recently, as globalisation and technological improvement have intensified market competition among innovative firms, governments have not only reformed mixed ownership, but significantly enacted various policies to encourage R&D activities. For example, numerous governments in both developed and developing countries offer substantial subsidies or tax credits to firms, to remedy firm-level distortions and for financial resource accessibility to reflect across firm resource reallocation.\(^3\)

Accordingly, some studies examined innovation activities and R&D policies in mixed oligopoly markets.\(^4\) Regarding cost-reducing innovation, Gil-Molto et al. (2011, 2020) and Lee and Muminov (2020) investigated the welfare effects of R&D subsidies and privatization policies, while Kasevayuth and Zikos (2013), Lee et al. (2017), Haruna and Goel (2017), and Atallah (2019) compared the relative superiority between R&D subsidies and output subsidies. They found that R&D subsidies are more socially beneficial when R&D spillovers are high, whereas output subsidies are superior when R&D spillovers are low. Chen et al. (2014), Gelves and Heywood (2016), and Kim et al. (2018) also discussed strategic licensing technology of innovative products in a mixed oligopoly.

\(^2\) Many previous studies examined various topics of mixed oligopolies, such as endogenous timing, product differentiation, free entry competition, foreign penetration, excess burden of taxation, capacity constraints, output subsidy and environmental policies, and so forth. For more detailed descriptions in recent research, see Lee et al. (2013, 2017), Xu et al. (2016b), Lee and Xu (2018), and Kim et al. (2019).

\(^3\) In OECD countries, governments finance between 8–10% of R&D spending by firms (García-Quevedo, 2004). Studwell (2013) shows that subsidies, along with other policies, played an important role in the economic development of Asian countries such as Japan, South Korea, and China.

\(^4\) For example, Delbono and Demicolo (1993), Poyago-Theotoky (1998), Ishibashi and Matsumura (2006), Heywood and Ye (2009), and Nie and Yang (2020) examined R&D activities in a mixed market, however, they did not include the analysis of subsidies or the implications of R&D policies.
In practical policies, while governments decide to subsidise R&D directly, there is some debate on the effectiveness (Kauko, 1996; Rebolledo and Sandonis, 2012). For example, governments might choose to subsidise output rather than R&D either when output enhancement is politically more popular among consumers or when the design and administration of R&D subsidies is relatively more complicated compared to output subsidies. Then, output subsidies can indirectly reduce the production costs of subsidised firms and enhance social welfare through resource redistribution.

From a theoretical viewpoint, several studies examined policy interaction between output subsidies and privatization in mixed oligopolies. White (1996) first showed that welfare is equal irrespective of the privatization policy, if government grants appropriate optimal output subsidies to firms. That is, the so-called privatization neutrality theorem (PNT) states that welfare neutrality existed before and after privatization. Since then, numerous articles have analysed whether this theorem holds in various generalised situations. However, Fjell and Heywood (2004) and Lee and Xu (2018) considered a sequential-move situation in which a public firm acts as a leader after privatization and showed that privatization neutrality failed. That is, firm asymmetry among public and private firms does not result in welfare neutrality.

On the other hand, existing literature has only examined a uniform output subsidy to public and private firms in a mixed oligopoly market. One exception is Hamada (2016, 2017), who recognized firm heterogeneity and demonstrated that even if cost differences (or objective differences) exists between firms and if the public firm acts as a Stackelberg leader before and after privatization, privatization neutrality is satisfied and welfare is maximized under discriminatory output subsidies. Further, if government can credibly commit to a discriminatory subsidy rate from the introduction stage of an

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5 In detail, the PNT states that privatization does not affect welfare, regardless of time structure, competition mode, the number of firms, product differentiation, and the degree of privatization under the optimal output subsidy. Academic debates have been discussed continuously by Pal and White (1998), Poyago-Theotoky (2001), Tomaru and Saito (2010), Matsumura and Ogawa (2012), Matsumura and Okumura (2013, 2017), and Lin and Matsumura (2018).

6 It is also known that the PNT failed under policy considerations of other asymmetric factors such as foreign competitors (Matsumura and Tomaru, 2012), excess burden of taxation (Matsumura and Tomaru, 2013), free entry (Matsumura and Okumura, 2017), and R&D competition (Lee and Tomaru, 2017; Lee et al, 2017).
output subsidy policy, it poses significant implications for supporting superior welfare properties associated with a committed subsidy policy. Therefore, the discriminatory output subsidy always yields the highest welfare compared to the uniform output subsidy, irrespective of the degree of privatization.

In this study, however, we consider a time-consistent output subsidy framework\(^7\) and highlight the interaction between a discriminatory output subsidy policy with ex-ante innovation incentives of the firms. We examine the impact of mixed ownership on the semi-public firm’s activities\(^8\) and show that strategic incentive for innovation among firms do not necessarily result in welfare improvement.

Our main findings are as follows: We show that the government grants a higher subsidy to the private firm than the public firm, resulting in the private firm investing more in R&D and producing a higher output, regardless of the degree of privatization. In the case in which both firms invest in R&D before the government determines the subsidy rate, the private firm intentionally and strategically invests aggressively in R&D to earn a higher subsidy, which also increases the firm’s output and profit. Since R&D investments are strategic substitutes irrespective of the degree of privatization, when the private firm increases R&D and output, the semi-public firm reduces R&D and output. Subsequently, a discriminatory output subsidy redistributes output from the public firm with high marginal costs to the private firm with low marginal costs. Hence, the government grants a lower output subsidy to the public firm than the private firm, while the difference decreases as the degree of privatization increases.

We also compare with the uniform output subsidy and show that optimal discriminatory subsidy rates are higher (lower) than those of uniform subsidy rates for a sufficiently high (low) degree of privatization, which results in a decrease (increase) of social welfare.

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\(^7\) Some research examined the time-consistent tax framework in which the regulator is not able to commit to emission tax and showed that firms undertake increased abatement activities that generate less pollution, which could result in improved welfare. See, for example, Poyago-Theotoky and Teerasuwannajak (2002), Moner-Colonques and Rubio (2015), Leal et al. (2018), Garcia et al. (2018), and Chen et al. (2019). For discussions on the practical evidence of the commitment issue in mixed economies, see Lee et al. (2018) and Ino and Miyaoka (2020).

\(^8\) In the discussion section, we examine a committed regime in which the government can commit a discriminatory output subsidy before firms choose R&D investments and show that the PNT still holds.
On the one hand, if the degree of privatization is sufficiently high, the semi-public firm’s social concern is weak with a much stronger profit incentive, which induces overinvestment between firms as well as welfare loss. However, once the firms choose R&D investments, the distribution of production costs across firms is more symmetric and efficient, and thus, the government grants higher output subsidies to both firms to increase total industry output. That is, although a high degree of privatization can eliminate cost inefficiency, underproduction distortion is serious. By expecting this regulatory flexibility under discriminatory output subsidies, the private firm has stronger incentive to invest more in R&D, which results in overinvestment in total R&D and induces larger welfare loss than under the uniform output subsidy.

On the other hand, if the degree of privatization is sufficiently low, social concern of the semi-public firm is much stronger, which reduces overinvestment between the firms, compared to a higher degree of privatization. Due to the larger asymmetry between the firms, the distribution of production costs across the firms is inefficient, however, the semi-public firm can produce more output to increase total industry output. That is, although a low degree of privatization can increase cost inefficiency, underproduction distortion is less serious. Subsequently, the government grants lower output subsidies to both firms under discriminatory output subsidies and the private firm is less inclined to invest more in R&D. As a result, it reduces overinvestment in total R&D and induces higher welfare than under the uniform output subsidy. This result sharply contrasts to the case of committed output subsidy in which the PNT holds.

The remainder of this paper is organized as follows: In section 2 we provides a cost-reducing innovation model of mixed duopoly with a (semi-) public and private firm. We analyse the discriminatory output subsidy in Section 3 and compare it with uniform output subsidy in Section 4. We also compare the committed discriminatory output subsidy in Section 5. Concluding remarks are presented in Section 6.

2. The Model
We consider a duopoly market which produces homogeneous goods. The inverse demand function is 
\[ P(Q) = a - Q, \] where \( P \) is market price, \( Q(= q_0 + q_1) \) is market output, and \( q_i \) is the output of firm \( i = 0,1 \), respectively. Then, consumer surplus is \( CS = Q^2/2 \).

As assumed in Zikos (2007), Gil-Molto et al. (2011), and Lee et al. (2017), we consider a quadratic cost function of output production where marginal costs increase with respect to output: \( C(q_i, x_i) = (c - x_i)q_i + q_i^2 \), where \( c \) is the initial cost level with \( a > c > 0 \) and \( x_i \) denotes the amount of R&D investment required for firm \( i \) to reduce the cost level. Note that a firm’s R&D shifts the marginal cost function downwards, \( \partial C/\partial q_i = c - x_i + 2q_i \), but does not alter the slope. In addition, R&D is perfectly protected against imitation.\(^9\) We also assume that the firm has to spend \( x_i^2 \) to implement cost-reducing R&D, \( x_i \), which exhibits decreasing returns to scale. Subsequently, the R&D cost function is given as \( \Gamma(x_i) = x_i^2 \) where \( i = 0,1 \).

We consider that each firm receives an output subsidy, \( s_i q_i \), where \( s_i \) denotes the per-unit (discriminatory) subsidy to output quantity, \( q_i \). Then, the profit function of the firm and social welfare, defined as the sum of consumer surplus, industry profits, and net subsidy, are as follows:

\[
\pi_i = (a - q_0 - q_1)q_i - (c - x_i)q_i - q_i^2 - x_i^2 + s_i q_i, \quad (1)
\]

\[
W = CS + \pi_0 + \pi_1 - s_0 q_0 - s_1 q_1. \quad (2)
\]

In general, we assume that subsidies are financed by taxpayers in the form of lump-sum payments, to avoid influencing welfare directly and to be cancelled out when aggregated.

We also assume that firm 1 is a private firm that maximizes profit while firm 0 is a (partially privatized) public firm. That is, we allow the government to sell shares of firm 0 to private investors who are seeking profit maximization. As a result, firm 0 is under mixed ownership of government control and jointly

\(^9\) This assumption is necessary in the analysis of mixed markets for ruling out the uninteresting case of a public monopoly.

\(^{10}\) We ignore the effects of R&D spillovers between the firms even though a part or all R&D outcomes of a firm might spill over to a rival firm in the same market. See Heywood and Ye (2009), Gil-Moltó et al. (2011), Kesavayuth and Zikos (2013), Haruna and Goel (2017), and Lee and Muminov (2020). In this case, Kesavayuth and Zikos (2013) showed that the output subsidy is more socially beneficial than the R&D subsidy.
owned by the government and private investors. Following Matsumura (1998), let $\theta \in [0,1)$ refer to the shares of firm 0 held by private investors, while firm 0 maximizes the convex combination of profit and welfare:\textsuperscript{11}

$$V = (1 - \theta)W + \theta \pi_0.$$  \hspace{1cm} (3)

The timing of the game is as follows. In the first stage, both firms decide to invest in R&D simultaneously. In the second stage, the government sets time-consistent discriminatory output subsidies, $s_i$, $i = 0,1$, after observing both firms’ decisions about R&D. In the last stage, firms compete in quantities in a Cournot fashion. We solve the subgame perfect Nash equilibrium by backward induction.

3. The Analysis

In the third stage of output choice, the first-order conditions provide the following equilibrium output:

$$q_0(x_0, x_1, s_0, s_1) = \frac{3(a - c) + 4x_0 - x_1 + 4\theta s_0 - s_1}{11 + 4\theta},$$

$$q_1(x_0, x_1, s_0, s_1) = \frac{(a - c)(2 + \theta) - x_0 + (3 + \theta)(x_1 + s_1) - \theta s_0}{11 + 4\theta},$$

$$Q(x_0, x_1, s_0, s_1) = \frac{(a - c)(5 + \theta) + 3x_0 + 3\theta s_0 + (2 + \theta)(x_1 + s_1)}{11 + 4\theta}. \hspace{1cm} (4)$$

This shows that an increase in R&D increases the output of the firms, but the increasing rate of a public firm is higher than a private firm, i.e., $\frac{\partial q_0(x_0, x_1, s_0, s_1)}{\partial x_0} > \frac{\partial q_1(x_0, x_1, s_0, s_1)}{\partial x_1} > 0$. However, the firm’s R&D decreases the output of the rival firm, i.e., $\frac{\partial q_1(x_0, x_1, s_0, s_1)}{\partial x_0} = \frac{\partial q_0(x_0, x_1, s_0, s_1)}{\partial x_1} < 0$. In sum, an R&D investment will increase total industry output. It also shows that the output subsidy increases the output of the firm, but the increasing rate of the private firm is higher than that of the public firm.

\textsuperscript{11} Studies on partial privatization are gaining increasing popularity and are used extensively in various contexts since Matsumura (1998). For example, Ino and Matsumura (2010), Lee et al. (2013), and Xu et al. (2016a) reviewed several research topics on partial privatization.
i.e., \( \frac{\partial q_1(x_0,x_1,s_0,s_1)}{\partial s_1} > \frac{\partial q_0(x_0,x_1,s_0,s_1)}{\partial s_0} > 0 \). However, the output subsidy of a firm decreases the output of the rival firm, i.e., \( \frac{\partial q_0(x_0,x_1,s_0,s_1)}{\partial s_1} \leq \frac{\partial q_1(x_0,x_1,s_0,s_1)}{\partial s_0} < 0 \), where the equality holds only if \( \theta = 1 \). Note that the output subsidy of the public firm has no effect on the output of either of the firms if the public firm is fully nationalised, that is, \( \theta = 0 \). Therefore, the output subsidy is insignificant with regards to welfare in the presence of a nationalised public firm.

In the second stage, the government determines the welfare-maximizing output subsidy rate, by considering the firm’s R&D investment. The first order condition yields the following optimal output subsidies:

\[
\begin{align*}
    s^D_0(x_0,x_1) &= \frac{2(a - c) + 3x_0 - x_1}{8} \quad \text{and} \quad s^D_1(x_0,x_1) = \frac{2(a - c) - x_0 + 3x_1}{8}.
\end{align*}
\]

where the superscript \( D \) denotes the equilibrium under the discriminatory output subsidy. This shows that the output subsidy increases in the firm’s R&D, i.e., \( \frac{\partial s^D_i(x_0,x_1)}{\partial x_i} > 0 \), while it decreases in the rival firm’s R&D, i.e., \( \frac{\partial s^D_i(x_0,x_1)}{\partial x_j} < 0 \). Note that given R&D investments, the optimal output subsidy is independent of the degree of privatization. Therefore, without considering R&D investments, the well-known PNT in literature on mixed markets is supported, which states that the optimal output subsidy rate yields similar welfare consequences before and after privatization. However, the PNT does not hold once the R&D setting stage is introduced, since the degree of privatization affects the equilibrium R&D investments, as shown below.

In the first stage of R&D investment, the first-order conditions present the following reaction functions:

\[
\begin{align*}
    x_0(x_1) &= \frac{(2 + \theta)(2(a - c) - x_1)}{26 - 3\theta} \quad \text{and} \quad x_1(x_0) = \frac{3(2(a - c) - x_0)}{23}.
\end{align*}
\]

Then, R&D decisions are strategic substitutes where the slope of the reaction function of each firm is negative, but that of the public firm is higher than the private firm, i.e., \( \frac{\partial x^D_0(x_0)}{\partial x_0} < \frac{\partial x^D_0(x_1)}{\partial x_1} < 0 \).

Solving the reaction function yields the following equilibrium R&D investments:
\[ x_0^D = \frac{5(a-c)(2+\theta)}{74-9\theta} \quad \text{and} \quad x_1^D = \frac{3(a-c)(6-\theta)}{74-9\theta}. \]  

(7)

We can show that \( x_0^D - x_1^D < 0 \) for \( \theta \in [0,1) \). Further, public (private) firm’s R&D increases (decreases) as the degree of privatization increases, i.e., \( \frac{\partial x_0^D}{\partial \theta} > 0 > \frac{\partial x_1^D}{\partial \theta} \).

**Lemma 1.** \( x_0^D < x_1^D \) for \( \theta \in [0,1) \).

Accordingly, the private firm invests more in R&D than the public firm, regardless of the degree of privatization.\(^\text{12}\) That is, the private firm aggressively invests in R&D under the discriminatory production subsidy policy, which can yield higher subsidies and output as per the below analysis.

By substituting (7) with (5), we obtain the following optimal output subsidy rates:

\[ s_0^D = \frac{20(a-c)}{74-9\theta} \quad \text{and} \quad s_1^D = \frac{4(a-c)(6-\theta)}{74-9\theta}. \]  

(8)

We confirm that \( s_0^D - s_1^D < 0 \) for \( \theta \in [0,1) \). Note that output subsidy rates are always positive, but that to the public (private) firm increases according to the degree of privatization, i.e., \( \frac{\partial s_0^D}{\partial \theta} > 0 > \frac{\partial s_1^D}{\partial \theta} \). Thus, a higher degree of privatization can encourage the semi-public firm to invest more in R&D and output production.

**Proposition 1.** \( s_0^D < s_1^D \) for \( \theta \in [0,1) \).

The government grants a higher subsidy to the private firm than the public firm under the discriminatory output subsidy policy. Since R&D investments are strategic substitutes irrespective of the degree of privatization, when the private firm increases R&D and output, the semi-public firm reduces R&D and output. Subsequently, the discriminatory output subsidy redistributes output from the public firm with higher marginal costs to the private firm with lower marginal costs. Hence, the government grants a

\(^{12}\) Note that \( x_0^D = x_1^D \) and \( q_0^D = q_1^D \) when \( \theta = 1 \).
lower output subsidy to the public firm than the private firm, while the difference decreases as the degree of privatization increases.

The equilibrium outputs are as follows:

\[ q_0^p = \frac{20(a - c)}{74 - 9\theta} \quad \text{and} \quad q_1^p = \frac{4(a - c)(6 - \theta)}{74 - 9\theta} \]  

(9)

We demonstrate that \( q_0^p - q_1^p < 0 \) for \( \theta \in [0,1) \). Note that the public (private) firm’s output is always positive and increases (decreases) in the degree of privatization, that is, \( \frac{\partial q_0^p}{\partial \theta} > 0 > \frac{\partial q_1^p}{\partial \theta} \).

**Lemma 2.** \( q_0^p < q_1^p \) for \( \theta \in [0,1) \).

This states that the private firm invests more in R&D and produces more output than the public firm, regardless of the degree of privatization.

Finally, we obtain the resulting profit of the private firm and social welfare as follows:

\[ \pi_1^p = \frac{23(a - c)^2(6 - \theta)^2}{(74 - 9\theta)^2} \quad \text{and} \quad W^p = \frac{10(a - c)^2(152 - \theta(36 + \theta))}{(74 - 9\theta)^2} \]  

(10)

It shows that the profit of the private firm decreases in the degree of privatization, that is, \( \frac{\partial \pi_1^p}{\partial \theta} < 0 \). This is because the (partially privatized) rival firm invests more in R&D and produces more output as the degree of privatization increases. If the degree of privatization is sufficiently high, the profit incentive of the semi-public firm is stronger, which encourages investment by the semi-public firm. Then, the distribution of production costs across the firms is more symmetric and efficient while the firms produce a lower industry output. Hence, although a sufficiently high degree of privatization can remove cost inefficiency, underproduction has distorting effects on welfare. Contrarily, if the degree of privatization is sufficiently low, a high level of asymmetry exists between the firms, which results in inefficient distribution of production costs across the firms, while the semi-public firm can produce more output to increase total industry output. Consequently, although a sufficiently low degree of privatization can increase cost inefficiency, there is a welfare distortion of underproduction distortion. Therefore, social
welfare decreases (increases) when the degree of privatization is high (low), that is, \( \frac{\partial W}{\partial \theta} > 0 \) if \( \theta > 0.153 \).

It implies that the optimal degree of privatization is \( \theta^D = 0.153 \).

**Proposition 2.** Mixed ownership with partial privatization policy is optimal under the discriminatory output subsidy policy.

4. Comparisons with Uniform Output Subsidy

In this section we examine the non-discriminatory uniform output subsidy when the degree of privatization is given exogenously, and then compare it with the discriminatory output subsidy in a time-consistent regulatory framework.

4.1 Non-discriminatory uniform output subsidy

Under the uniform output subsidy, the equilibrium outcomes in the last stage are similar, except \( s_0 = s_1 = s_q \). Then, from (4) we can show the same result of R&D with the discriminatory output subsidy where the firm’s increase in R&D increases the output of the firms, but the increasing rate of the public firm is higher than that of the private firm. Note that even though cost-reducing R&D investment will increase total industry production, the uniform subsidy increases (decreases) the output of the public firm only when the degree of privatization is high (low).

In the second stage of the welfare-maximizing output subsidy, the first order condition yields the following:

\[
q_N(x_0, x_1) = \frac{6(a - c)(1 + 2\theta^2) - (3 + 4\theta(1 - 4\theta))x_0 + (9 + 4(1 - \theta)\theta)x_1}{24 + 48\theta^2}.
\]

where the superscript \( N \) denotes the equilibrium under the non-discriminatory output subsidy. It shows that the output subsidy increases in the private firm’s R&D, i.e., \( \frac{\partial q_N(x_0, x_1)}{\partial x_1} > 0 \), but increases (decreases) the public firm’s R&D only if the degree of privatization is high (low), i.e., \( \frac{\partial q_N(x_0, x_1)}{\partial x_q} < 0 \) if \( \theta > 0.576 \).
Thus, the private firm has greater motivation to increase R&D, while the public firm’s decision depends on the degree of privatization.

In the first stage of R&D investments, the first-order condition yields the following equilibrium R&D investments:

\[
x^N_0 = \frac{(a-c)(360 + 81\theta + 2343\theta^2 + 516\theta^3 + 5076\theta^4 + 1376\theta^5 + 3584\theta^6 + 1280\theta^7)}{2664 + 351\theta + 17529\theta^2 + 1052\theta^3 + 38252\theta^4 - 224\theta^5 + 27776\theta^6 - 1792\theta^7},
\]

\[
x^N_1 = \frac{(a-c)(3 + 4\theta^2)(216 + 27\theta + 909\theta^2 + 8\theta^3 + 992\theta^4 - 64\theta^5)}{2664 + 351\theta + 17529\theta^2 + 1052\theta^3 + 38252\theta^4 - 224\theta^5 + 27776\theta^6 - 1792\theta^7}.
\]  

We confirm that \(x^N_0 - x^N_1 = \frac{96(1+2\theta^2)^2(-3-\theta^2+4\theta^3)}{2664+351\theta+17529\theta^2+1052\theta^3+38252\theta^4-224\theta^5+27776\theta^6-1792\theta^7} < 0 \) for \(\theta \in [0,1)\). It also shows that the private firm’s R&D decreases in the degree of privatization while that of the public firm increases, that is, \(\frac{\partial x^N_0}{\partial \theta} < 0 < \frac{\partial x^N_1}{\partial \theta}\). The total R&D is non-monotone in the degree of privatization, that is, \(\frac{\partial (x^N_0 + x^N_1)}{\partial \theta} < 0\) if \(0.021 < \theta < 0.840\), while \(\frac{\partial (x^N_0 + x^N_1)}{\partial \theta} > 0\) otherwise.

**Lemma 3.** \(x^N_0 < x^N_1\) for \(\theta \in [0,1)\).

It states that the private firm invests more in R&D than the public firm, regardless of the degree of privatization. This is similar to the discriminatory output subsidy in which the private firm invests more aggressively in R&D under the output subsidy policy.\

Substituting equation (12) with equation (11), we obtain the optimal production subsidy level as follows:

\[
s^N_q = \frac{12(a-c)(1 + 2\theta^2)(72 + 13\theta + 287\theta^2 + 4\theta^3 + 320\theta^4)}{2664 + 351\theta + 17529\theta^2 + 1052\theta^3 + 38252\theta^4 - 224\theta^5 + 27776\theta^6 - 1792\theta^7}.
\]

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13 According to numerous theoretical studies, private firm investment in R&D is higher than the welfare-maximizing level due to spillovers. For instance, Haruna and Goel (2017) and Lee and Muminov (2020) indicated a threshold of R&D spillovers, according to which the public firm produces less (more) output than the private firm with higher (lower) R&D spillovers.
Note that the output subsidy rate is positive but non-monotone in the degree of privatization, i.e., \( \frac{\partial s}{\partial \theta} < 0 \) if \( 0.040 < \theta < 0.961 \), while \( \frac{\partial s}{\partial \theta} > 0 \) otherwise. From (4), the equilibrium output of both firms are obtained by:

\[
q_0^N = \frac{12(a - c)(1 + 2\theta^2)(60 + 13\theta + 283\theta^2 + 20\theta^3 + 320\theta^4 - 224\theta^5 + 27776\theta^6 - 1792\theta^7)}{2664 + 351\theta + 17529\theta^2 + 1052\theta^3 + 38252\theta^4 - 224\theta^5 + 27776\theta^6 - 1792\theta^7},
\]

\[
q_1^N = \frac{4(a - c)(1 + 2\theta^2)(216 + 27\theta + 909\theta^2 + 8\theta^3 + 992\theta^4 - 64\theta^5)}{2664 + 351\theta + 17529\theta^2 + 1052\theta^3 + 38252\theta^4 - 224\theta^5 + 27776\theta^6 - 1792\theta^7}. \tag{14}
\]

We illustrate that \( q_0^N - q_1^N = \frac{16(a-c)(-9+3\theta-33\theta^2+19\theta^3-38\theta^4+42\theta^5-16\theta^6+32\theta^7)}{2664+351\theta+17529\theta^2+1052\theta^3+38252\theta^4-224\theta^5+27776\theta^6-1792\theta^7} < 0 \) for \( \theta \in [0,1) \). It also shows that the private firm’s output decreases in the degree of privatization while that of the public firm increases, that is, \( \frac{\partial q_1^N}{\partial \theta} < 0 < \frac{\partial q_0^N}{\partial \theta} \). The total industry output is non-monotone in the degree of privatization, for instance, \( \frac{\partial (q_0^N + q_1^N)}{\partial \theta} < 0 \) if \( 0.116 < \theta < 0.883 \), but \( \frac{\partial (q_0^N + q_1^N)}{\partial \theta} > 0 \) otherwise.

**Lemma 4.** \( q_0^N < q_1^N \) for \( \theta \in [0,1) \).

It states that the private firm produces more output than the public firm, regardless of the degree of privatization. This is similar to the discriminatory output subsidy in which the private firm more aggressively produces output under the output subsidy policy.

The resulting profit of the private firm and social welfare is as follows:

\[
\pi_1^N = \frac{(a - c)^2(23 + 104\theta^2 + 112\theta^4)(216 + 27\theta + 909\theta^2 + 8\theta^3 + 992\theta^4 - 64\theta^5)^2}{(2664 + 351\theta + 17529\theta^2 + 1052\theta^3 + 38252\theta^4 - 224\theta^5 + 27776\theta^6 - 1792\theta^7)^2},
\]

\[
W^N = \frac{2(a-c)^2(984960 + 260496\theta + 13122351\theta^2 + 2537586\theta^3 + 72367623\theta^4 + 9985120\theta^5 + 211711448\theta^6 + 12143008\theta^7 + 346812272\theta^8 - 2339584\theta^9 + 301657344\theta^{10} - 20496384\theta^{11} + 108486656\theta^{12} - 13729792\theta^{13} - 524288\theta^{14})}{2664 + 351\theta + 17529\theta^2 + 1052\theta^3 + 38252\theta^4 - 224\theta^5 + 27776\theta^6 - 1792\theta^7}^2. \tag{15}
\]
Evidently, the profit of the private firm decreases in the degree of privatization, that is, \( \frac{\partial \pi^N}{\partial \theta} < 0 \) while social welfare decreases (increases) if the degree of privatization is high (low), i.e., \( \frac{\partial W^N}{\partial \theta} > 0 \) if \( \theta < 0.933 \). Finally, the optimal degree of privatization is \( \theta^N = 0.933 \). Hence, mixed ownership with the partial privatization policy is optimal under the non-discriminatory output subsidy policy, but the degree is higher than that under the discriminatory output subsidy.

**Proposition 3.** Mixed ownership with partial privatization policy is optimal under the uniform output subsidy policy.

4.2. Comparison and discussion

**Proposition 4.**

\[
 s_0^D \geq s_q^N \text{ if } \theta \geq 0.699 \quad \text{and} \quad s_1^D \geq s_q^N \text{ if } \theta \geq 0.162. \]

This implies that the government is more flexible in granting output subsidies under discriminatory output subsidies, but that the rate depends on the degree of privatization. This is because the firm’s strategic profit incentive for making R&D decisions in a time-consistent framework in which the output subsidy rate is affected by the cost asymmetry between the firms, significantly depends on mixed ownership, that is, the degree of privatization. In particular, the discriminatory output subsidy rate is always higher (lower) than the uniform output subsidy for a sufficiently higher (lower) degree of privatization. That is, when the degree of privatization is high (low) where the semi-public firm is dominantly owned by private investors (government), the government can set a lower (higher) uniform subsidy rate. As a result, welfare could possibly be reduced by aiding a less efficient public firm under the discriminatory output subsidy. However, when the degree of privatization is intermediate, \( s_0^D < s_q^N < s_1^D \) if \( 0.162 < \theta < 0.699 \). For instance, if the semi-public firm is almost equally owned by the government and private investors, the government will set an intermediate subsidy rate under the uniform output subsidy.

14 The proofs of some propositions and lemmas are presented in the Appendix.
**Lemma 5.** Comparing discriminatory and uniform output subsidies, we obtain the following:

(i) $x_0^D > x_0^N$ for any $\theta \in [0,1)$; $x_1^D > x_1^N$ if $\theta > 0.037$ and $X^D > X^N$ for any $\theta \in [0,1)$

(ii) $q_0^D > q_0^N$ for any $\theta \in [0,1)$; $q_1^D > q_1^N$ if $\theta > 0.108$ and $Q^D > Q^N$ if $\theta > 0.028$.

(iii) $\pi_1^D < \pi_1^N$ for any $\theta \in [0,1)$.

Lemma 5 provides the effect of mixed ownership on the equilibrium outcomes under two different output subsidies. Lemma 5 (i) states that the public firm’s R&D and total industry R&D are always higher under the discriminatory output subsidy irrespective of the degree of privatization, while the private firm’s R&D depends on the degree of privatization. That is, the private firm increases (decreases) R&D as the degree of privatization increases (decrease) under the discriminatory output subsidy policy.

Lemma 5 (ii), states that the output of the public firm always increases under the discriminatory output subsidy, while increasing a private firm’s output only in a higher degree of privatization. The total output remains higher in a greater degree of privatization under the discriminatory output subsidy policy.

Lemma 5 (iii) states that the private firm’s profit always increases regardless of the degree of privatization under the uniform output subsidy.

**Proposition 5.** $W^D \geq W^N$ if $\theta \leq 0.033$.

It reveals that in a time-consistent regulatory framework, the discriminatory output subsidy decreases social welfare only when the degree of privatization is not sufficiently low. This implies that when the government determines the output subsidy policy between discriminatory and uniform subsidy rates after observing the firms’ R&D decisions, it crucially depends on the decision of mixed ownership. In most cases, the government prefers the non-discriminatory output subsidy to the discriminatory output subsidy to reduce strategic welfare-distorting R&D decisions of the firms that initiate profit. However, the opposite is true for a sufficiently lower degree of privatization in which the welfare loss from strategic R&D can be minimized. Also, if the government chooses the optimal degree of privatization, as shown in Propositions 2 and 3, the welfare effect of the output subsidy presents the following
relationship: \( W^D(\theta = 0.153) < W^N(\theta = 0.933) \). Therefore, the discriminatory output subsidy could distort welfare under the optimal privatization policy.

Economic explanations of the findings are as follows: the optimal subsidy rates for both firms are higher (lower) than those of the uniform subsidy rate for a high (low) degree of privatization, which results in a decrease (increase) of social welfare. On the one hand, if the degree of privatization is high, social concern of the semi-public firm is weak with a much stronger profit incentive. Subsequently, once the firms choose R&D investments, the distribution of production costs across the firms is more symmetric and efficient. Thus, the government can provide higher output subsidies for both firms to increase total industry output. That is, for a high degree of privatization, underproduction distortion from the private firms becomes more serious and is thus remedied by the higher uniform output subsidy.

On the other hand, if the degree of privatization is sufficiently low, social concern of the semi-public firm is much stronger, which discourages overinvestment between the firms, compared to the higher degree of privatization. Due to the larger asymmetry between the firms, the distribution of production costs across the firms is inefficient, however, the semi-public firm can produce more output to increase total industry output. That is, although the (efficient) private firm receives an output subsidy for a low degree of privatization, so does the (inefficient) public firm under the uniform output subsidy. Therefore, welfare could decrease when less efficient firms are aided. Further, these uniform subsidies distort competition further according to the fact that firm's R&D choices occur before they are set. Thus, the firms’ strategic behaviour that influences subsidies, further reduces the welfare level compared to the discriminatory subsidy. In Section 5, we show that this result sharply contrasts the case of committed output subsidy in which the PNT holds and thus, the discriminatory output subsidy always yields the highest welfare compared to the uniform output subsidy, irrespective of the degree of privatization.

5. Discussion

We compare the committed output subsidy policy in which the government credibly commits to discriminatory output subsidies before the firms choose R&D decisions. The timing of this committed game between the R&D stage and subsidy stage is reversed: in the first stage, the government chooses
either the discriminatory or uniform output subsidy, whereas both firms choose their R&D investment levels in the second stage and their output levels in the third stage.

In the third stage, output choice is similar to the non-committed output subsidy policy, in which the equilibrium output of firms are derived as per (4). In the second stage of R&D investments, the first-order conditions provide the following equilibrium R&D investments:

\[
x_0^C(s_0, s_1) = \frac{(a - c) \left( 275 + \theta (248 + \theta (65 + 4\theta)) \right) + \theta (429 + \theta (337 + 66\theta)) s_0}{1837 + \theta (2159 + 2\theta (425 + 56\theta))}.
\]

\[
x_1^C(s_0, s_1) = \frac{2(3+\theta)((a-c)(33+\theta(33+8\theta)) - 2\theta(11+4\theta)s_0 + 2(3+\theta)(9+4\theta)s_1)}{1837+\theta(2159+2\theta(425+56\theta))}.
\]

(16)

where the superscript \(C\) denotes the equilibrium under the committed discriminatory output subsidy.

Then, we obtain the equilibrium output of the second stage:

\[
q_0^C(s_0, s_1) = \frac{(11 + 4\theta)((a-c)(53 + 21\theta) + 4\theta(19 + 7\theta)s_0) - (215 + \theta (193 + 42\theta)) s_1}{1837 + \theta (2159 + 2\theta (425 + 56\theta))},
\]

\[
q_1^C(s_0, s_1) = \frac{(11+4\theta)((a-c)(33+\theta(33+8\theta)) - 2\theta(11+4\theta)s_0 + 2(3+\theta)(9+4\theta)s_1)}{1837+\theta(2159+2\theta(425+56\theta))}.
\]

(17)

In the first stage, the government chooses the optimal output subsidy for maximizing social welfare as follows:

\[
s_0^C = \frac{4(a - c)(54 + 21131\theta + 30529\theta^2 + 16941\theta^3 + 4277\theta^4 + 414\theta^5)}{\theta (311129 + 443822\theta + 243553\theta^2 + 60996\theta^3 + 5876\theta^4)},
\]

\[
s_1^C = \frac{(a - c)(84381 + 120800\theta + 68317\theta^2 + 18030\theta^3 + 1856\theta^4)}{311129 + 443822\theta + 243553\theta^2 + 60996\theta^3 + 5876\theta^4}.
\]

(18)

It shows that \(s_1^C > 0, \frac{\partial s_1^C}{\partial \theta} > 0 \) and \(\frac{\partial s_0^C}{\partial \theta} < 0\) if \(\theta < 0.408\).

Then, we obtain certain comparisons from the equilibrium outcomes.

**Lemma 6.** \(x_0^C < x_1^C\) for \(\theta \in [0,1)\); \(q_0^C > q_1^C\) for \(\theta \in [0,1)\).

Contrary to the non-committed case, the public firm invests less but produces more than the private firm, irrespective of the degree of privatization.
Proposition 6. \( s_0^C > s_1^C \) for \( \theta \in [0,1) \).

It states that the government grants a higher subsidy to the public firm under the committed discriminatory subsidy policy. Thus, when the government commits output subsidies before firms decide R&D investments, it grants a higher output subsidy to the semi-public firm to increase the total output in the market. This is contrary to the finding in the non-committed case.

Proposition 7. \( s_0^D > s_0^C \) if \( \theta > 0.165 \) and \( s_1^D > s_1^C \) for \( \theta \in [0,1) \).

It states that the government grants a higher (less) output subsidy to the public firm if the degree of privatization is high (low), while that of the private firm is higher for any degree of privatization. That is, the strategic incentive of overinvestment by the private firm can be reduced by the committed output subsidy, while that of the semi-public firm depends on the decision of mixed ownership. If the degree of privatization is high, the profit-incentive increases and thus, the government will reduce the output subsidy of the semi-public firm. However, if the degree of privatization is sufficiently low, the profit incentive decreases and the government will increase the output subsidy of the semi-public firm to increase cost efficiency.

Finally, we obtain the profit of the private firm and welfare in the equilibrium as follows:

\[
\pi_1^C = \frac{2(a - c)^2(103 + 2\theta(38 + 7\theta))(8067 + \theta(8548 + \theta(3231 + 434\theta)))^2}{(311129 + \theta(443822 + \theta(243553 + 52\theta(1173 + 113\theta))))^2},
\]

\[
W^C = \frac{(a - c)^2(177683 + 253550\theta + 139047\theta^2 + 34776\theta^3 + 3344\theta^4)}{2(311129 + 443822\theta + 243553\theta^2 + 60996\theta^3 + 5876\theta^4)}. \tag{19}
\]

It shows that \( \frac{\partial \pi_1^C}{\partial \theta} > 0 \) while \( \frac{\partial W^C}{\partial \theta} < 0 \) if \( \theta < 0.366 \). The optimal mixed ownership is \( \theta^C = 0.366 \), which is higher than that under the non-committed case, that is, \( \theta^D = 0.153 < \theta^C = 0.366 < \theta^N = 0.933 \). Hence, the partial privatization policy is optimal under the committed output subsidy policy. Further, we compare the welfare with that under the non-committed case. Then, it simply shows that

\[
\text{Max}\{W^N, W^D\} < W^C \quad \text{for} \quad \theta \in [0,1] \quad \text{and} \quad W^D(\theta^D) < W^N(\theta^N) < W^C(\theta^C).
\]

18
Proposition 8. Mixed ownership with partial privatization policy is optimal under the committed output subsidy policy and yields the highest welfare.

It reveals that the committed discriminatory output subsidy policy always yields the highest welfare for society, irrespective of the degree of privatization. This is because commitment can eliminate the strategic incentive of the firms to invest more in R&D after observing the output subsidy policy, and thus, reduces the welfare loss. This sharply contrasts the result of the findings in the non-committed subsidy case.

6. Conclusion

We considered R&D competition in a mixed duopoly market with a semi-public firm under discriminatory output subsidies in a time-consistent regulatory framework and examined the strategic incentive of innovation among firms. We highlighted the effect of an output subsidy policy on the innovation strategies of the firms, especially the impact of mixed ownership in a semi-public firm on cost-reduction innovation. We showed that the government grants a higher subsidy to the private firm than the public firm, which yields that the private firm invests more in R&D and produces more output, regardless of the degree of privatization. We also showed that the optimal subsidy rates for both firms are higher (lower) than the uniform subsidy rate for a sufficiently high (low) degree of privatization, which results in a decrease (increase) of social welfare. Finally, we compared the committed discriminatory output subsidy policy according to which our findings under the non-committed case sharply contrast those of the committed case in which the PNT holds, irrespective of the degree of privatization.

Some directions for future research remains. First, theoretical limitations are largely due to model-specific assumptions on the demand and cost functions. Second, different market structures such as Cournot, Bertrand, and Stackelberg competition with the role of product differentiation should be examined to ensure robustness of the findings. Fourth, distinct characteristics of R&D decisions, such as R&D spillovers and joint R&D ventures should be also incorporated for further analysis. Finally,
understanding the effects of endogenous entry in an oligopolistic market and foreign penetration can be promising topics for future research.

Reference


Appendix: The Proofs

A 1. Proof of lemma 5

(i) \( x_0^D - x_0^N = \frac{4(a-c)\theta(3519+1098\theta+20267\theta^2+4206\theta^3+33220\theta^4+5952\theta^5+14624\theta^6+640\theta^7)}{(74-9\theta)(2664+351\theta+17529\theta^2+1052\theta^3+38252\theta^4-224\theta^5+27776\theta^6-1792\theta^7)} \) > 0 for \( \theta \in [0,1) \).

\( x_1^D - x_1^N = \frac{4(a-c)\theta(-459+\theta(12366+\theta(-2775+4\theta(12330+\theta(-2965+16\theta(803-2\theta(119-6\theta)))))))}{(-74+9\theta)(-2664+\theta(-351+\theta(-17529+4\theta(-263+\theta(-9563+56\theta(1-4\theta(31-2\theta)))))))} \)

Figure 1. \( x_1^D - x_1^N \)

\( X^D > X^N = \frac{16(a-c)\theta(765+3366\theta+4373\theta^2+13380\theta^3+5340\theta^4+14336\theta^5-152\theta^6+352\theta^7)}{(74-9\theta)(2664+351\theta+17529\theta^2+1052\theta^3+38252\theta^4-224\theta^5+27776\theta^6-1792\theta^7)} \) > 0 for \( \theta \in [0,1) \).

(ii) \( q_0^D - q_0^N = \frac{4(a-c)\theta(489-1470\theta+5929\theta^2-4190\theta^3+13922\theta^4-2120\theta^5+8320\theta^6)}{(74-9\theta)(2664+351\theta+17529\theta^2+1052\theta^3+38252\theta^4-224\theta^5+27776\theta^6-1792\theta^7)} \) > 0 for \( \theta \in [0,1) \).

\( q_1^D - q_1^N = \frac{8\theta(-306+2916\theta-1868\theta^2+10539\theta^3-5377\theta^4+9816\theta^5-5600\theta^6+320\theta^7)}{(74-9\theta)(2664+351\theta+17529\theta^2+1052\theta^3+38252\theta^4-224\theta^5+27776\theta^6-1792\theta^7)} \)

Figure 2. \( q_1^D - q_1^N \)
\[ Q^D - Q^N = \frac{4\theta (-123+4362\theta+2193\theta^2+16888\theta^3+3160\theta^4+175126\theta^5-2880\theta^6+640\theta^7)}{(74-9\theta)(2664+351\theta+17529\theta^2+1052\theta^3+38252\theta^4+224\theta^6+27776\theta^6-1792\theta^7)} \]

\textbf{Figure 3.} \( Q^D - Q^N \)

\[
\pi_1^D - \pi_1^N = \frac{8(a-c)^2(-56247696+153661860\theta-713018997\theta^2+1540286847\theta^3-3939725871\theta^4+6189922937\theta^5-12116293520\theta^6+21692704400\theta^7+14229001088\theta^8)}{(74-9\theta)^2(2664+351\theta+17529\theta^2+1052\theta^3+38252\theta^4-224\theta^5+27776\theta^6-1792\theta^7)^2} < 0 \text{ for } \theta \in [0,1).
\]

\textbf{A 2. Proof of lemma 6}

From some necessary calculations, we can obtain the following equilibrium outcomes.

\[
x_0^C = \frac{2(a-c)(22142 + \theta(33747 + \theta(19592 + \theta(5129 + 510\theta(1173 + 113\theta)))))}{311129 + \theta(443822 + \theta(243553 + 52\theta(1173 + 113\theta)))},
\]

\[
x_1^C = \frac{2(a-c)(3 + \theta)(8067 + \theta(8548 + \theta(3231 + 434\theta))))}{311129 + \theta(443822 + \theta(243553 + 52\theta(1173 + 113\theta)))},
\]

\[
q_0^C = \frac{4(a-c)(22241 + \theta(31715 + \theta(17355 + \theta(4325 + 414\theta(1173 + 113\theta)))))}{311129 + \theta(443822 + \theta(243553 + 52\theta(1173 + 113\theta)))},
\]

\[
q_1^C = \frac{(a-c)(11 + 4\theta)(8067 + \theta(8548 + \theta(3231 + 434\theta))))}{311129 + \theta(443822 + \theta(243553 + 52\theta(1173 + 113\theta)))}.
\]

Then, using the comparisons, we can show that \( x_0^C - x_1^C < 0 \) for \( \theta \in [0,1) \); \( q_0^C - q_1^C > 0 \) for \( \theta \in [0,1) \).

\textbf{A 3. Proof of proposition 4}

i) \[
s_0^D - s_0^N = \frac{4(a-c)(-2664+813\theta-7686\theta^2+10237\theta^3-6398\theta^4+21242\theta^5-2984\theta^6+8320\theta^7)}{(74-9\theta)(2664+351\theta+17529\theta^2+1052\theta^3+38252\theta^4+224\theta^6+27776\theta^6-1792\theta^7)}
\]
\[ s_1^D - s_q^D = \frac{8(a-c)\theta(-750+4746\theta-3120\theta^2+15401\theta^3-8617\theta^4+12500\theta^5-10624\theta^6+896\theta^7)}{(74-9\theta)(2664+351\theta+17529\theta^2+1052\theta^3+38252\theta^4-224\theta^5+27776\theta^6-1792\theta^7)} \]

Figure 4. \( s_0^D - s_q^N \) and \( s_1^D - s_q^N \)

A 4. Proof of proposition 5

\[ W^D - W^N = \left\{ \frac{8(a-c)^2\theta(7336656-222360201\theta+82856520\theta^2-2170762938\theta^3+575205120\theta^4-8664521629\theta^5+2257209472\theta^6-17909243336\theta^7+4666856160\theta^8-19808166416\theta^9+4716875904\theta^{10}-10635702592\theta^{11}+1855867392\theta^{12}-192658048\theta^{13}+83369984\theta^{14}+6602752\theta^{15})}{(74-9\theta)^2(2664+351\theta+17529\theta^2+1052\theta^3+38252\theta^4-224\theta^5+27776\theta^6-1792\theta^7)^2} \right\} \]

Figure 5. \( W^D - W^N \)

A 5. Proof of proposition 6

It is easy to see that \( s_0^{CD} - s_1^{CD} = \frac{216+143\theta+1316\theta^2-553\theta^3-922\theta^4-200\theta^5}{311129\theta^2+443822\theta^3+243553\theta^4+60996\theta^5+5876\theta^6} > 0 \) for \( \theta \in [0,1) \).

A 6. Proof of proposition 7

\[ s_0^C = s_0^{CD} = \frac{4(a-c)(-3996-7563\theta+150143\theta^2+238892\theta^3+140951\theta^4+37237\theta^5+3726\theta^6)}{\theta(74-9\theta)(311129+443822\theta+243553\theta^2+60996\theta^3+5876\theta^4)} \]
ii) \[ s^D_1 - s^C_1 = \frac{(a-c)(-1222902-1227441\theta-101726\theta^2+229675\theta^3+78034\theta^4+6800\theta^5)}{(-74+9\theta)(311129+443822\theta+243553\theta^2+60996\theta^3+5876\theta^4)} > 0 \text{ for } \theta \in [0,1). \]