Pricing (almost) any used goods: a first step towards a theoretical framework

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Pricing (almost) any used goods: a first step towards a theoretical framework

A cohort-specific model for pricing beta-type goods

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Abstract
From the latest technological gadget to the multi-century collectible item, millions of used goods are exchanged on marketplaces every day. Yet, there have been very few attempts to propose a pricing theory that could help buyers and sellers set a price at any point in time. In this paper, we build a robust framework and introduce a cohort-specific model to explain how the price of used goods evolves over time. Despite its simplicity, we believe that this model could be applied to an extensive range of used goods.

Keywords: used goods, cohort-specific, pricing, second-hand markets, beta-type

1. Introduction
While second-hand markets have been in existence for a long time, they have experienced exponential growth over the past 20 years or so with the advent of the internet. The emergence of structured online global marketplaces first and the more recent focus on the benefits of circular economies have helped overcome longstanding barriers potential buyers and sellers of used goods were facing: lack of supply, limited access to used goods, risks associated with trades on the second-hand markets, social perception, and psychological hindrances to name a few (see [Roux00] for more details). Today, millions of used goods are exchanged freely by economic agents on second-hand markets around the world. Apparels, furniture, cars, books, machinery, electronics, antiques, ..., the list is long. Over the last decades, researchers have made significant contributions in advancing our understanding of how second-hand markets function (see [Cays93], [Hris20]). Yet, very few attempts have been made to propose a model that could help buyers and sellers set a price for any goods at any point in time. Can such a model exist? One that applies to types of goods as different as apparel and electronic devices? And one that can explain the price evolution of a good from the day it becomes a used good to the day it acquires the status of collectible decades if not century later? While the objective seems ambitious, data collected from second-hand marketplaces such as Amazon and eBay and auction houses over the past decade suggests that some patterns exist in supply, demand, and price evolution. This finding has motivated us to formulate a simple model that we present in this paper that could explain the observed patterns and stylized facts. First, we introduce some key concepts and definitions that will allow us to establish a rigorous framework for our model. Then we will review some of the
stylized facts observed on second-hand markets. Finally, we will present our proposed model and discuss its advantages and limitations.

2. Used-goods and second-hand markets: Key concepts and definitions

Before delving into the main topic of this article, we shall try to come up with a more precise definition of the terms used goods and second-hand markets. The common definition of used good (sometimes referred to as second-hand good) is the following: a piece of personal property previously owned or used. This definition is rather vague, and we should elaborate on two underlying key concepts: the ones of ownership and usage. While a large proportion of used goods are being sold or transferred on markets by agents who actually consumed it (i.e., who purchased the good on first-hand markets or second-hand markets intending to utilize it / enjoy its benefits and actually did it), many so-called used goods are also being sold by agents who did not consume it and just kept it for future sale (for example, dealers and resellers) which renders the use of the term "used" in used good questionable from a consumption point of view. Similarly, the adjective "used" suggests that the condition of the good has somehow been physically altered. If for many goods the use implies a physical deterioration as soon as they are consumed (nondurable goods, for example), sometimes deterioration can manifest itself in different forms that do not necessarily affect the good physically but may have an impact on its value: damaged packaging, loss of functionality or obsolescence for example. Passage of time is sometimes the only source of loss of value, and used goods are so-called only in contrast to newly manufactured goods. Finally, while physical deterioration seems ineluctable over time, there are also instances where specific use can increase the value of the good (for example, a good once owned by a celebrity). In short, the term "used" is also ambiguous from a usage point of view, and we begin to feel that alternative terminology and sharper definitions could be beneficial, especially to make a more significant distinction between quality or condition and value. Let us now turn our attention to the concept of second-hand markets. The second-hand goods market is commonly defined as a segment of the general commodity market where used goods (defined as pre-owned or pre-used goods) are

1 The term second-hand good could sound more appropriate because it does not impose a constraint on utilization by the owner, but the adjective "second" implies a far-too specific position in the chain of owners.
exchanged (as opposed to first-hand markets, a segment where newly manufactured goods are exchanged). To avoid potential confusion between market segments (first-hand or second-hand) and marketplaces, we believe that it is appropriate to talk about first-hand and second-hand markets for a specific good. Let us take a further step towards defining those markets. Y. Histrova gives the following complete definition (see [Hris20]): "The second-hand goods retail market is considered to be a segment of the general commodity market which operates on the basis of every product bringing value to its owner regardless if it's first or second-hand. This gave rise to the second-hand goods market which separated itself from the first-hand one due to the fact that the products in questions were pre-owned or used at least once. As a rule, the fact that the purchase/sale of the item wasn't a primary act of the final consumer leads to its depreciation in value as well as a certain decrease in its quality". We believe that a fundamental notion emerges from this definition that is worth highlighting: first-hand and second-hand markets are not disjoint but rather permeable. With a few exceptions over time, they also tend to coexist. Indeed, goods of different qualities and conditions, including brand new ones, tend to be sold simultaneously on the same marketplaces, making the segmentation by type of markets difficult. The author goes further, "the second-hand goods market originates from the first-hand one and at the same time replaces it. The two markets are interdependent and influence one another". The existence of a first-hand market is time-bound: at some point, goods are not produced nor sold by official resellers anymore, and brand-new goods still in circulation are then sold on the second-hand market. Again, we face the same difficulties when we try to define second-hand markets as previously with the definition of used goods. We shall now try to address these. In this section, we introduce some key concepts and definitions. These concepts will help us build a rigorous framework for the model we will introduce in the next section.

**Definition 1: ownership (grouping)**

From the manufacturer to consumers to collectors, including a variety of intermediaries, used goods are passed into the hands of a continuum of owners. For the purposes of our model, we divide this continuum into two specific groups:

- The group of primary owners or *p-group*: this group includes the manufacturer and the official distributors who generally create, buy, or keep products only to sell them shortly after.
• The group of secondary owners or \textit{s-group}: this group contains both private users and resellers who acquired products from agents from the \textit{p-group} or the \textit{s-group} to consume, collect, or resell it. Most consumers would fall into this group and also discounters, liquidation or second-hand retailers, refurbishers, etc.

\textbf{Table 1: owners grouping}

<table>
<thead>
<tr>
<th>\textbf{p-group}</th>
<th>\textbf{s-group}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturer</td>
<td>First-hand, second-hand, \ldots, \textit{n}\textsuperscript{th}-hand user</td>
</tr>
<tr>
<td>Distributors (wholesale &amp; retail)</td>
<td>Discounters, second-hand retailers, etc.</td>
</tr>
</tbody>
</table>

\textbf{Definition 2: contemporality}

A good is said to be \textit{contemporary} if a short period of time has elapsed since it was manufactured and/or if the current version or vintage of the good is the latest released (for example, the latest vintage of a given wine or the newest version of an electronic device). On the contrary, if some time has elapsed since it was manufactured or posterior versions/vintages of the good were released, we say that the good is \textit{non-contemporary}.

\textbf{Figure 1: contemporality illustrated}

\textbf{Definition 3: pricing mechanism (centralized vs. decentralized)}

A pricing mechanism is \textit{centralized} if the price of a good can be directly dictated by agents from the \textit{p-group} and cannot be directly influenced by agents from the \textit{s-group} for small quantities at least. Examples of centralized pricing mechanisms are standardized price list, vertical pricing, pricing policy or imposed requirements, etc. We say the pricing mechanism is \textit{decentralized} if the price of a good cannot be directly dictated by agents from the \textit{p-group} anymore but can be influenced by agents from the \textit{s-group} even for small quantities. Examples of decentralized pricing mechanisms are bidding options available for agents (auctions, markets, etc.) or forms of negotiations (private pricing), a peer-to-peer price system, etc.
Definition 4: alpha and beta-type goods

To overcome the difficulties associated with defining used goods based only on their condition, we introduce a new classification. A good is associated with a particular type alpha or beta according to three non-physical factors (see figure 2):

- who owns it (ownership, see definition 1)
- the time elapsed since it was produced (contemporality, see definition 2)
- and how its price is determined (pricing mechanism, see definition 3)

And one physical factor:
- the state of physical depreciation of the good (condition)

An alpha-type good is a good that meets all of the following four criteria:

- ownership: it is held by agents from the p-group (and was never previously owned by an agent of the s-group)
- contemporality: it is contemporary
- pricing mechanism: its price is determined through a centralized pricing mechanism
- condition: the good is new

A beta-type good is a good that meets at least one of the following four criteria:

- ownership: it is held by agents from the s-group.
- contemporality: it is non-contemporary
- pricing mechanism: its price is determined through a decentralized pricing mechanism
- condition: the good is not new

At inception, all goods are of alpha-type (alpha is to be understood in the sense of what comes first, what is new), with the passage of time and/or if other conditions are met, it becomes a beta-type good (beta is used in the sense what comes second). For a given good, we can have a copy of it that is of alpha-type and another one that is of beta-type at the same time for sale on the markets. Typically, this happens when a good was released recently, and consumers can get it from the manufacturer at a standardized price or buy it on second-hand markets.
Let us take a closer look at the four factors used to classify *alpha-type* and *beta-type* good. If agents from the *p-group* own a good and they can somehow fix the price (centralized pricing system), it is a monopolistic or oligopolistic situation. If, in addition to that, consumers do not have any intrinsic element of comparison (time decay, physical depreciation, etc.), there is no form of negotiation possible: it is a “take it or leave it” situation. The fact that at least one of four criteria is met ensures that the price of the beta-type good can be different (but not necessarily) from the price of its alpha-type equivalent. Indeed, it provides tangible elements of discrimination\(^2\) that allow agents to compare the copy of the *beta-type good* that they wish to buy or sell versus a copy of the equivalent *alpha-type good* - whether this later still exists or not; it could just be present as a reference or an anchoring point in the mind of agents. This in turn, induces some flexibility in the pricing. In short, agents can express and confront their respective valuations of the good, and reallocations can occur.

While the definition of beta-type overlaps with the usual one of *used good*, it is broader (and yet more precise) as it does not rely only on the notion of physical depreciation. The ability for agents to have different perceptions of the decay in utility\(^3\), and to trade freely are the necessary conditions for the existence of *beta-type good markets* (a *beta-type good market* is the segment of the commodity markets where *beta-type goods* are exchanged as opposed to *alpha-type good market*, which is the segment of the commodity markets where *alpha-type goods* are exchanged).

\(^2\)Cross-sectional or backward-looking, physical or non-physical.  
\(^3\)In a sense, the introduction of alpha-type and beta-type good helps circumvent the difficulties associated with finding necessary and sufficient conditions for the existence of second-hand markets, in particular the fact that second-hand markets can coexist with first hand-hand markets (see [Cays93]).
Table 2: Classification of goods as per the three non-physical factors (assuming the condition is new).

<table>
<thead>
<tr>
<th>Ownership</th>
<th>Contemporaneity</th>
<th>Pricing</th>
<th>Category</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>p-group</td>
<td>contemporaneous</td>
<td>centralized</td>
<td>α-type good</td>
<td>A new electronic device just released on the markets</td>
</tr>
<tr>
<td></td>
<td></td>
<td>decentralized</td>
<td>β-type good</td>
<td>Goods produced in limited quantities that have already sold out</td>
</tr>
<tr>
<td>non-</td>
<td>non-contemporaneous</td>
<td>centralized</td>
<td>β-type good</td>
<td>wine old vintages still owned by the producer</td>
</tr>
<tr>
<td>contempora-</td>
<td></td>
<td>decentralized</td>
<td>β-type good</td>
<td>Unsold stocks</td>
</tr>
<tr>
<td>norous</td>
<td></td>
<td>central-</td>
<td></td>
<td>Large stock of given good owned by a discounter which can recreate monopoly</td>
</tr>
<tr>
<td></td>
<td></td>
<td>edized</td>
<td></td>
<td>Most used goods produced in large quantity recently released</td>
</tr>
<tr>
<td></td>
<td></td>
<td>central-</td>
<td></td>
<td>Rare collectibles, collectors with a monopolistic position</td>
</tr>
<tr>
<td></td>
<td></td>
<td>edized</td>
<td></td>
<td>Goods produced in large quantity sold on second-hand markets, collectible items.</td>
</tr>
</tbody>
</table>

Definition 5: Cohort

A cohort is a set of goods that share a certain number of defining characteristics. Used goods can be grouped into finite cohorts based on common features. We say a cohort is specific if all the goods within that cohort are identical or share a great number of characteristics (for example, all the bottle of wines produced in a given year by a particular wine estate or all vehicles of the same line and with the same options produced by a particular car manufacturer). On the contrary, we say a cohort is generic if the items within it share only a few numbers of characteristics (for example, books from the sixteenth century). Our main motivation for introducing the concept of cohort is that, in theory at least, identical goods should have the same value/price. One can also expect that if two copies of the same good are different, their difference in value should be a

While value and price are normally distinct concepts in economics, we will use the two terms interchangeably. Indeed, in the context of our model, price can be seen as the representation of the perceived value.
function (not necessarily linear) of the number of defining characteristics they do not share (and, of course, how consumers perceive these characteristics).

3. Used-goods lifecycle: stylized facts

Because the objective of our model is to determine the price evolution of beta-type goods over time, we shall start with a review of some stylized facts about the lifecycle of used goods. We will focus especially on how supply, demand, and price vary over the lifecycle. Of course, each good has a specific evolution, and many variables will come into play in determining supply, demand, consumption, and price. Yet, there are still some clear common patterns in price evolution and some identifiable trends in supply and demand that emerge across markets, even when comparing two radically different goods.

3.1 Price patterns

Let us first study the lifecycle of a good and focus on its price evolution. For the purpose of our analysis, we will consider a standard durable good released in finite measurable quantity, that we will call good $G$. It can be any type of good (electronic devices, apparels, books, cars, furniture, etc.), the only condition that we impose on this good is to be a physical object that can be legally possessed and freely exchanged (for the sake of the example we can assume the good is affordable and produced in quantity large enough). For such good, we can typically break down its price evolution into three phases:

- **Phase $\phi_0$**: Manufacturer release a finite quantity of good $G$ on the markets at an initial price $P_0$. Good $G$ can be sold directly by the manufacturer or by entitled distributors, be it wholesalers or retailers (agents from the $p$-group in any cases). While during this phase, good $G$ is going through different stages (introductory stage, growth stage, saturation stage, etc.), this phase is most of the time relatively short compared to the life expectancy of good $G$. Usually, it stops either when good $G$ has sold out or when buyers preferences have changed leading to a decline in interest for good $G$ or when a next version $G'$ is about to be released. At the onset, $P_0$ is usually fixed (or evolving in a narrow range in case of private pricing), and it is generally determined by the manufacturer according to a variety of factors (cost of production, profit or margin desired, competition pricing,
projected demand, etc.). As time passes in phase $\varphi_0$, the price can fluctuate, but the influence of the manufacturer on the pricing remains high. As per the definition we introduced in the earlier section, phase $\varphi_0$ corresponds to the release of an *alpha-type good* on its *alpha-type market*.

- **Phase $\varphi_1$:** At some point, a *beta-type market* emerges, and good $G$ becomes a *beta-type good*. This is mainly driven by the rise of the *s-group* (most of the time, first-hand consumers trying to resell good $G$ after having used it and discounters coming into play) and the transfer of *pricing power* from the agents of the *p-group* to the ones of the *s-group*. This emergence generally goes hand in hand with a prolonged price decrease\(^5\). More often than not, private consumers who resell good $G$ on (beta-type) markets are likely to sell it at a price inferior to the one at which they bought it in the first place. The price decline is due to the influence of a variety of factors: usage, condition, trust, adjustment of supply relative to demand, the emergence of alternatives, etc. that can be united under the same banner: a decreased interest from buyers for the *beta-type good* versus the *alpha-type good*. Academics have studied the rapid depreciation of *beta-type good* prices in the first years of their life. Several authors have proposed models to describe the convexity of used goods price structure (see [Acke70], [Kurs91] or [Port99]). Although empirical data shows that these convex exponential functions fit well the observed price evolution in the automobile markets, there have been few attempts to apply them to other sectors, although shreds of evidence suggest that they could (see [Ishi16] and [Lica08]). Some authors have also theorized that a relationship exists between good $G$ quality and its trade volume and price depreciation in phase $\varphi_1$ (see [Hend99] and [Ghos09]). Less reliable goods or goods showing faster deterioration tend to exhibit steeper price decline and lower trade volume than high-quality goods. After a sharp decline in the first years, the price of good $G$ seems to plateau. For some authors (see [Acke70]), this phenomenon corresponds to the price convergence toward the scrap value\(^6\) of the good. Indeed, a broken or defective good could still be disassembled, and the different pieces that constituted it be sold separately. There is a segment of agents (recyclers is an example) that are specialized in this. To

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\(^5\) Price Decline is the ratio of the difference between manufacturer price (price of the alpha-type good) and sale price (price of the beta-type good) over the Manufacturer Price. That is, the price decline measures the extent of the residual value of the used product at any given point in time.

\(^6\) The scrap value is the worth of the asset's individual components when the asset itself is deemed no longer usable.
summarize some of the key observations that we highlighted in this section: the price of good $G$ tends to experience an exponential decline in the years that follow its release on beta-type markets before reaching a plateau. Figure 3 provides an illustration of this convex price structure for six different items during phase $\varphi_1$. If most goods (especially the mass-produced ones) see their price go through the phase $\varphi_1$ pattern we describe, there are notable exceptions. Indeed, some goods do not experience a price decrease at all. We can mention the case, for instance, of prestigious items (fine wines "Grand Crus" for example), goods that come in limited editions (stamps, watches, bags, etc.), or manufactured collectibles (dolls, figurines, cards, etc.). Before turning to the next phase, let us make a final comment on the evolution of the quantity of good $G$ in circulation during phase $\varphi_1$. Each good is designed to last or function for a certain period of time. In the course of phase $\varphi_1$, the quantity of good $G$ in existence starts to decrease as the good is being used or consumed. The rate at which copies of $G$ begin to malfunction or cease to exist depends on the characteristics of good $G$. Reliability analysis is a branch of statistics concerned with the study of the survival function of goods and we will see later how we can establish a link.

- **Phase $\varphi_2$:** After a certain amount of time, the quantity of good $G$ available on the (beta-type) markets diminishes up until the point the good becomes scarce. At this point, a combination of factors such as scarcity, non-reproducibility, collectability, etc., seems to permit downward price flexibility and, in some cases, allow the price of good $G$ to appreciate again after it reaches a bottom (see [Lynn96], [Wons15]). If we take a longer-term perspective and we observe the price at which most goods, regardless of their type, are exchanged years, decades, or even centuries later, it has gone up again more often than not. Every day, the results of auctions also provide examples of this phenomenon. Even trivial goods released in large quantities are susceptible to become collectibles and to attract collectors (see [Dobr18],[Wu20],). As an illustration, Figure 4 shows the price evolution of six manufactured collectibles as observed on Amazon. The study of collectibles is relatively new in economics, but literature is already abundant, and many authors have evidenced the key properties of these markets (see [Stol84], [Burt99], [Sari18] for example). There is a large body in the literature that focuses on the motivations and behaviors of collectors (see [Spai18] for a good overview). This price increase generally goes hand in hand with an increase in pricing volatility as the amount of information available decreases (see [Rost16]). Traditionally, second-hand markets are segmented into two distinct sub-markets: the antique markets on the one hand
and the common goods markets on the other hand. As we alluded to earlier, some goods (prestigious goods, limited editions, manufactured collectibles, etc.) do not experience a phase $\phi_1$ and go directly through phase $\phi_2$, which may justify this segmentation. We argue that this distinction should not exist as it implies collectibles and common goods are radically different, whereas most collectibles were common goods once.

To summarize, if we put aside phase $\phi_0$, which solely concerns alpha-type goods, the life cycle of beta-type goods can thus be characterized by two phases: a first phase $\phi_1$ during which the price tends to decline and a second phase $\phi_2$ in which the price tends to go up again. The duration of the phase $\phi_1$ is relatively short compared to the one of phase $\phi_2$. The duration of the phase $\phi_2$ is potentially unlimited in time as long as the good continue to exist. Between the two, the price reaches an inflection point. Another general observation we can make is that the pace at which the price decreases during phase $\phi_1$ tends to be greater than the pace at which it increases in phase $\phi_2$ (except for manufactured collectibles). We illustrate the three phases in figure 5.

3.2 Supply and demand

Let us consider the facts mentioned above from a different perspective: that of supply and demand. Right after good $G$ is released on (beta-type) markets, demand is usually strong (people are expecting to get goods at a discount on marketplaces) while supply builds up progressively. Sellers are in a favorable position. Then trends inverse: demand starts to decrease under the influence of multiple factors. At the same time, supply continues to grow as more and more goods are becoming available on the (beta-type) market through different channels. At a certain point in time, demand bottoms. The good has become out of fashion, or worse, obsolete. Buyers have more bargaining power. This bottoming of demand coincides with the end of phase $\phi_1$, as described in the previous paragraph. Most durable goods are designed to function for a certain period of time. Under the effect of time, the risk of failure increases, and the survival rate decreases. In other words, fewer goods are available in the markets, which pushes the supply lower. And if the item becomes a collectible, demand picks up again.
Figure 3: Historical prices of various beta-type goods (two smartphones, two video games, two books) presumably in phase $\phi_1$. Prices are falling exponentially but at a different pace.

Source: Amazon

- Video Game, Dishonored, for Playstation 3. ASIN: B005C2D1YI. From 4/10/2013 to 16/8/2018.
- Video Game, The Witcher 3: Wild Hunt, for Xbox One. ASIN: B00WTI2HV6. From 11/5/2015 to 10/11/2018
- Book: Lean In: Women, work and the will to lead, Sheryl Sandberg. ASIN: 0385349947. From 18/7/2013 to 1/3/2019
- Book: The girl on the train, Paula Hawkins. ASIN: 1594634025. From 28/5/2016 to 11/12/2018
Figure 4: Historical prices of various manufactured collectibles, presumably in the second phase of their evolution. Prices are rising more linearly but at a different pace.

Source: Amazon

- **Nintendo Game Boy - Original (Gray)**

- **Tekken 3**, video game for Playstation.

- **Mario Amiibo**, figurine.
  - ASIN: B00UL0BMA6. From 26/3/2015 to 5/11/2018

- **WWE Elite Collection Defining Moments Hulk Hogan Figure**
  - ASIN: B00MK0ZU76. From 31/12/2014 to 10/11/2018

- **Bandai "Saint Seya" Taurus Aldebaran figurine**.
  - ASIN: B00JGW4RVG. From 18/5/2014 to 25/10/2018

- **Jurassic World Chomping Indominus Rex figurine**.
  - ASIN: B00PQ7MXKW. From 24/10/2016 to 11/11/2018
Figure 5: Illustrative evolution of Good G price over time.

4. Proposed model

4.1 Scope

Now that we have established the existence of patterns common to most goods, we will try to build a model that explains and predicts these, and that is consistent with existing economic theories such as supply and demand. Let us first set out the scope of our model. It focuses on the pricing of beta-type goods in beta-type good markets. More specifically, it aims at estimating the average price at a given time of goods within a specific cohort. The concept of cohort introduced earlier takes its importance here. Because goods can be grouped into cohorts based on shared defining characteristics, if the cohort is specific enough, goods that compose it are expected to have more or less the same economic value. Thus, the average price should be a good estimator. If defining specific cohorts can prove challenging, they fit well with how agents perceived good. Indeed, as consumers, we are not all attracted by identical goods, and we may favor specific features over others. An interesting property of cohorts is that they are fungible. Indeed, two specific cohorts can be merged in a more generic one if the items of each cohort share a common defining characteristic. The accuracy of the model will depend on the granularity of the cohort, i.e., the model will produce a more representative price if the cohort is specific than if it is generic. We will elaborate further on how to build cohorts efficiently in section 4.3.
The approach we are taking is a holistic one. We are not concerned with single transactions but more with the average of single transactions for a given good with particular characteristics (in a way, the equilibrium price at time \( t \)). These characteristics can be either preexisting (i.e., designed by the manufacturer) or acquired through time (ownership, condition).

Let us now introduce our proposed model. We shall start with a simplified version, and we will see how it can be enriched thereafter.

### 4.2 A simple version

We first assume a "well-mixed" population of agents (i.e., the agents in the population are likely to come into contact with each other) and a given set of beta-type goods that form a cohort. Let us call a copy of good \( G \) an element of the cohort. Let us subdivide the population into four mutually exclusive classes:

- **buyers (B)**: these are the agents who do not own a copy of good \( G \) but are interested in buying it at time \( t \).
- **users (U)**: these are the agents who own (and potentially consume) a copy of good \( G \) and do not wish to sell it at time \( t \).
- **sellers (S)**: these are the agents who own a copy of good \( G \) and are not interested in owning it anymore at time \( t \) and therefore who are considering selling it).
- **others (O)**: this last class includes the agents who do not own a copy of good \( G \) and are not interested in buying it. Others (O) is the adjustment class.

The sum of Users (U) and Sellers (S) represents the total number of copies of good \( G \) in existence (i.e., the cardinal of the cohort), denoted \( Q \), while the sum of Buyers (B) and Users (U) represents the total interest for copies of good \( G \), denoted \( I \). We assume that the total population size is fixed (and set at \( M \) at the beginning of the period), but the number of agents in each class fluctuates over time. \( Q \) and \( I \) change over time, too, under the influence of different factors. Let us codify permissible behaviors for the agent of the four classes at each time-step. In the simplified version of the model, each agent can shift from one compartment to another as follows:

- a user (U) can:
  - stay a user (U)
• become a seller (S) if he is no longer interested in owning the copy of good G
• become an other (O) if the copy of good G ceased to exist

• a buyer (B) can:
  o stay a buyer (B) as long as his interest for a copy of good G is intact but he is not able to strike a deal with a seller
  o become a user (U) if an agreement is found with a seller
  o become an other (O) if he is not interested anymore in acquiring a copy of good G.

• a seller (S) can:
  o stay a seller (S) if he cannot find a counterparty, i.e., a buyer
  o become an other (O) if a deal is struck with a buyer

• an other (O) can only stay an other (O)

In more elaborated versions of the model, we could include additional permissible behaviors for agents of the group other (O). For now, we will assume the copies of good G cannot be renovated or recycled. The other assumptions we make in the simplified version of the model are:

1) the copies of good G are identical (i.e., the cohort is very specific) and can only be in one of the two following states: either in good condition or they cease to exist
2) each agent can only buy, sell, or own one copy.

We designate by $k_n(t)$ or $k_n$ the rates at which individuals are moving from one compartment to another. We summarize in the compartmental diagram below (figure 6) the various elements we introduced.
The dynamics over time of the compartmental system can be written with the following equations:

$$\frac{dB}{dt} = -(k_1 + k_4) \cdot B$$  \hspace{1cm} (1)

$$\frac{dU}{dt} = k_1 \cdot B - (k_2 + k_5) \cdot U$$  \hspace{1cm} (2)

$$\frac{dS}{dt} = k_2 \cdot U - (k_1 + k_3) \cdot S$$  \hspace{1cm} (3)

$$\frac{dO}{dt} = k_4 \cdot B + k_5 \cdot U + (k_1 + k_3) \cdot S$$  \hspace{1cm} (4)

By definition, we also have the following equations for the evolution of Q and I:

$$\frac{dI}{dt} = \frac{dB}{dt} + \frac{dU}{dt} = -k_4 \cdot B - (k_2 + k_5) \cdot U$$  \hspace{1cm} (5)

$$\frac{dQ}{dt} = \frac{dU}{dt} + \frac{dS}{dt} = k_1 \cdot B - k_5 \cdot U - (k_1 + k_3) \cdot S$$  \hspace{1cm} (6)
Now let us try to define the average price \( P \) of a copy of good \( G \). Consistent with classical economic theories, we will assume that \( P \) is a function of supply and demand, or more precisely of excess demand. Supply and demand are represented by the number of sellers and the number of buyers, and we write:

\[
\frac{dP}{dt} = \lambda \left( \frac{dB}{dt} - \frac{dS}{dt} \right)
\]  

(7)

where \( \lambda \) is a parameter characterizing the liquidity or the market depth (the excess demand needed to move the price by one unit).

Using the equations (5) and (6), we can also write:

\[
\frac{dP}{dt} = \lambda \left( \frac{dI}{dt} - \frac{dQ}{dt} \right)
\]

(8)

In short, the price can also be expressed as well as a function of two broader variables: interest and quantity.

So far, the model introduced is intuitive, and despite its simplicity, its dynamic allows the emergence of the stylized facts we observed earlier. Indeed, phase \( \varphi_1 \) would correspond to a state where the sellers (S) compartment grows faster than the buyers (B) compartment, or when the interest I diminishes proportionally faster than the quantity Q. Conversely, phase \( \varphi_2 \) would correspond to a state where the sellers (S) compartment diminishes faster than the buyers (B) compartment, or when the quantity Q diminishes proportionally faster than the interest I.

Let us elaborate further on equation (8) and the fact that price can be expressed as a function of interest and quantity, as we believe this property is key. The compartmental model that we introduced could be as well applied to alpha-type good markets too. When a new good is released, there are only two active classes (if we put the adjustment class others (O) aside), the buyers (B) and the sellers (S). Sellers (S) comprises mainly of agents from the \textit{p-group} (the manufacturer and
the distributors essentially) whose objective is to sell the good, and buyers (B) are agents from the
s-group mainly (consumers in general) who are looking to acquire a copy of the good. Thus, at
inception, the class Users (U) is non-existent. As a result, because all the copies produced are in
the hand of agents of the p-group and because all agents interested in acquiring a copy have not
bought it yet, we have the following identities: Q = S and I = B. In other words, total quantity
coincides with the numbers of sellers (i.e., the supply) and interest coincides with the number of
buyers (i.e., the demand). An interesting fact that we discussed in section 2 is that the emergence
of the Users (U) class creates a new dynamic. Estimating supply and demand becomes complex as
these fluctuate over time. The introduction of class (U) and equation (8) allow us to reduce
complexity. Indeed, we can estimate the co-evolution of quantity Q and interest I, instead of supply
and demand, to predict the evolution of the price P over time. Let us see how.

As mentioned previously, the evolution of the number of copies of good G in existence, or Q, can
be studied statistically. To do this, one has to estimate the lifetime of good G (i.e., the time for
which the copy of good G exists or carries out its appointed task satisfactorily and passes into
"dead" state thereafter), then deduce its probability of survival over time. The lifetime can be
represented by a continuous, non-negative valued random variable. We can study its probabilistic
properties through its cumulative distribution (or equivalent function such as the survival function
or the hazard function). We can then infer the evolution of the proportion of copies of good G in
existence by multiplying the survival function by the total number of copies at inception.
Estimating the evolution of I is less straightforward. Indeed, the interest I is a far more complex
variable influenced by a significant number of subjective factors. In a sense, it is as difficult to
determine I as it is to determine supply and demand. However, we can make a couple of interesting
observations:

- One can predict the evolution of the interest I by predicting the evolution of the rates k_2,
k_4, and k_5. These can be estimated from marketplaces by looking at indicators such as the
number of copies of good G for sale, the number of "watchers," etc.

7 In theory, as per the model above, we have to be able to determine the four coefficients k_1, k_2, k_3
and k_4 that rule the moves from and to classes S and B
• In phase $\varphi_1$, the evolution of the price of good $G$ is mainly driven by the rate of change of $I$. Indeed, during this phase, or at least initially, the good is not scarce enough for its price to experience upward pressure. And the opposite tends to be true in phase $\varphi_2$: it is the rate of change of quantity $Q$ that tends to drive $P$. As a consequence, we could also estimate $I$ statistically by looking at past price evolutions of goods with similar characteristics than good $G$.

In a nutshell, the evolution of price $P$ can be estimated through the co-evolution of two variables: quantity and interest. Quantity $Q$ is almost entirely an intrinsic variable that mainly depends on how good $G$ was designed and manufactured at the onset, while interest $I$ is almost entirely external, linked to people perception.

We believe this approach has two main advantages versus estimating the evolution of price $P$ through the co-evolution of supply and demand:

• the evolution of $Q$ can be estimated statistically through reliability analysis. Moreover, its evolution over time is more constrained than the evolution of supply. While the supply can increase or decrease over time, $Q$ is monotonically decreasing$^8$.

• $Q$ and $I$ are less interdependent than supply and demand. Quantity $Q$ is mainly driven by the characteristics of good $G$ and its physical depreciation over time. Interest expressed by owners and potential buyers can only delay depreciation but not invert it, and this is more likely to occur at later stages of phase $\varphi_2$. Owners usually take extreme care of their goods only when they already have a substantial value.

One last observation we would like to make is that, because a dynamic system can represent price evolution, some inherent sensitivity to the initial conditions exists: the initial quantity $Q_0$, the initial interest $I_0$, and the initial price $P_0$. In other words, while the price evolution will be a function of the evolution over time of the quantity $Q$ of good $G$ in existence and the interest $I$ for it, the initial quantity released, the initial price set by the manufacturer, and the initial interest for good $G$ will also play an important role. In a way, the approach we propose is consistent with a few key findings

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$^8$ At least under the assumptions of no recycling and no renovation.
from behavioral theories such as price anchoring, social norms, and effect of scarcity (see [Tver74] [Meln10], [Wons15]) that may have longer-term effects and is a natural bridge between the subjective and the objective theory of value. More pragmatically, our approach can provide insights to manufacturers who wish to understand the interrelation between primary markets and second-hand markets. In addition to influencing the second-markets through quality (see [Ande94], [Wald96]), manufacturers also have control over $Q_0$ and $P_0$.

4.3 Model sophistication

As stated earlier, the objective of our proposed model is to estimate the average price of goods pertaining to the same cohort. One of the first challenges that may arise is to define the cohort. Also, the simple model we introduced focuses on one single cohort, while in practice, it may be useful to consider multiple cohorts. For these reasons, we will elaborate further on the notion of cohort and how to construct them efficiently.

We defined a cohort as a group of goods that share a set of defining characteristics. The choice of the characteristics is key, and it will determine the level of granularity of the cohort (i.e., if it is specific or generic). The more stringent the criteria used to cluster goods within cohorts, the more specific the cohorts will be. But of course, there is a tradeoff between having a specific enough cohort (which will help improve the accuracy our model, as the more similar the items within a cohort, the more representative the average price will be) and having enough items within the cohort (if $Q_0$ is too small, the model might be too sensitive). We can illustrate this intuitive fact with a simple example. If we consider the cohort formed by all the copies in circulation of the book "The Little Prince" (written by Saint-Exupéry), estimating an average price for this cohort would not be relevant as it is too generic. Since 1943, more than 140 million copies have been edited and sold by multiple editors in more than 300 languages and different editions. Gathering within the same cohort copies as different as a recent pocket edition in German and illustrated coffee-table version from the past century in French reduces our chances of finding a representative price. On the contrary, if we build a cohort with only the 1943 original editions of "The Little Prince" signed by the author, it will surely be specific, but we may end up with very few items in our cohort and face the same issue with the representativeness of the average price. Instead, we might be interested in finding an average price for only the pocket editions of the novel published by a given editor within a specific range of years. Finding the right threshold might sound
complicated, but specific yet populated enough cohorts are common in real life. Indeed, more often than not, goods come in different easily identifiable versions that can serve as a basis for cohort construction. It is a marketing reality (manufacturers increase their chances to reach more consumers by versioning goods) that can be turned to our advantage. When a good is produced in several versions (or come with different options), similar versions will display similar prices at the onset. Until now, we discussed innate physical characteristics that come by design and that tend to be fixed in time (colors, functionalities, materials, textures, format, etc.) that we can use to group goods, but there are also characteristics that are acquired in time. A typical example of this is the condition of the good. At inception, all goods are new, but their state change over time. It is often a search option on marketplaces: New, open box, used, for parts or not working, etc. condition comes in many flavors. Innate characteristics will make cohorts less permeable by nature. For example, suppose we used color as a criterion to split a population of goods into cohorts. In that case, a red-colored good will never transition into the cohort of blue-colored goods unless we change its color or remove the criteria. In contrast, acquired characteristics allow some permeability between cohorts. We used above the example of the condition: a good will generally transition over time from new to used to not working. What are the practical implications of this, and how can we incorporate it into our model? Suppose we are interested in knowing how the average price of a given good will evolve. In that case, we should first use a set of innate characteristics to define a cohort that corresponds to the exact version of the good we are interested in (for example, a black mobile phone of a given brand released in 2007). Then we can consider how subcohorts could emerge over time (see figure 7), as characteristics that can affect price sensibly are acquired (our mobile can stay new, become used, or stop working). The compartmental diagram and the associated system of equations can be modified to reflect this. If we carry on with the example of our mobile phone and the three subcohorts "new," "used," "for parts," we can draw the compartmental diagram shown in figure 8. At inception, we can only find new copies for sale on the markets, so naturally, the subcohorts "used" and "for parts" are empty. As our phone is being bought by agents and consumed, copies gradually move from the "new" subcohort to the "used" one, and as some copies start to malfunction, they subsequently move to the "for parts" subcohort. When a copy ceases to exist, it is removed, and the total quantity in
circulation $Q$ decreases. As we are considering three subcohorts, we can split each compartment of our model into three subcompartments. For example, instead of having one set of users ($U$), we have now three subsets: the users who own a new copy of the mobile ($U$), the ones who own a used copy ($U'$), and the ones who own a defective copy ($U''$). And the same goes for the other compartments: the buyers ($B$, $B'$, and $B''$), the sellers ($S$, $S'$, and $S''$). There is no need to split the other compartment ($O$). As stated previously, it acts as the adjustment variable. This splitting also implies we have three subcompartments for quantity ($Q$, $Q'$, and $Q''$) and interest ($I$, $I'$ and $I''$) that correspond to the number of copies in circulation for each subcohort and the interest associated. The arrows on the diagram represent the flows between the different subcompartments, but again we are not really interested in determining the associated rates. What we will focus on ultimately is $Q$, $Q'$ and $Q''$ and $I$, $I'$ and $I''$ and try to come up with three prices $P$, $P'$, $P''$ for each subcohort. In other words, what a new, a used, and a defective copy is worth on average. The advantage of our approach is that we only have to estimate at which pace a good moves from new to used to defective and estimate the associated interest from agents for the three conditions to be in a position to estimate the corresponding average prices. The model will also naturally reflect why a used or a defective copy will tend to be worth less than a new copy.

Figure 7: A multi-cohort compartmental diagram

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9 In fact, it is equivalent to the copy being transferred to a fourth subcohort "dead", but because there is no point in pricing a copy that does not exist anymore, we can ignore that particular subcohort.
5. Discussions

In this section, we discuss some of the limitations of our proposed model and areas for further development.

5.1 Limitations

Throughout this article, we already touched on some of the caveats of our model. First, there are the limitations imposed by our set of assumptions. We said that our model is cohort-specific and that having a cohort that is not too generic yet populated enough is a necessary condition to ensure a representative price. This constraint might be an issue for goods that come in a minimal number of copies or when a subcohort has too few copies pertaining to it. By definition, the accuracy of our model will tend to decrease over time as the quantity decreases. However, this is consistent with what is observed in markets: price variance increases as illiquidity and scarcity increase.

There are as well some more practical difficulties that we need to overcome. On the quantity side first. Reliability analysis provides an excellent framework to study how the quantity of a given good in circulation will evolve based on the innate characteristics of the goods. However, it does not incorporate external factors that could drive quantity too. We can think of refurbishment and recycling, for example, or the fact that collectibles receive extra care from collectors. Another
difficulty is that each good evolves differently over time. This implies that for each good, we may need to estimate different parametric curves\textsuperscript{10}. If the model is flexible enough to incorporate this, further work is required in order to factor in these adjustments. On the interest side, we also face the same challenges. One of the methods we suggested to estimate interest – a rather difficult variable to measure – leveraged on the fact that quantity and interest were not simultaneously driving price with the same magnitude. For some goods, especially the ones that come in limited quantity, this property might not hold. In the case of rare collectibles, the quantity may also strongly influence interest.

Finally, we would like to mention that the lack of data available poses some challenges from a modeling point of view. The situation is likely to improve over time as we are collecting more and more data points every day on online marketplaces. Still, it is not easy today to get complete price time series (from inception to today) on secondary markets for goods that were released more than ten years ago. It is easy to collect data on goods that are going through phase $\phi_1$ on online second-hand marketplaces and, we also find data in abundance for collectibles in phase $\phi_2$ in specialized auction databases but establishing a complete dataset that covers the two phases for a given set of goods has proven difficult (hence the segmentation proposed by some authors between collectibles and common goods).

The list we have established here is not exhaustive, and additional limitations may be evidence in practice. At first glance, the limitations mentioned above could limit the scope of application of our model, but there are still many instances where it could prove useful.

5.2 Further developments

In our quest to propose a first viable model to understand the pricing of beta-type goods, we believe future research should focus on two areas. The first is the elaboration of complete datasets. Robust data should help us consolidate our understanding of secondary markets and bring further empirical evidence that we could use to refine our model. Data will also be critical for estimation

\textsuperscript{10} although we suspect goods could be clustered by type.
and testing purposes. The second area is the development of robust methods to estimate quantity and interest and \( \lambda \). For quantity, we need to investigate to which extent reliability analysis is applicable. For interest, the work is even in an earlier stage. Efforts should be put on advancing our preliminary knowledge of this newly introduced variable and ways to estimate it. Last but not least, developing protocols to estimate \( \lambda \) is also critical. We need to understand if this parameter is a constant, a coefficient that fluctuates over time, or even a function of other model variables\(^{11}\).

6. Conclusions

Our objective was to propose a model that could help understand how the price of most used goods evolves over time. With this in mind, we first introduced a set of key concepts that we believe were necessary to build a robust framework. We then presented our model that aims at explaining stylized facts that are observed every day on marketplaces. The attractiveness of our proposition lies in its conceptual simplicity, its potential applicability to a large range of used goods, and the large period of time it could cover. Overall, we believe that our approach brings a new perspective in this nascent field of research that are second-hand markets, building on the classical theory of supply and demand and also incorporating findings from more recent economic theories. Further developments are required to make this model fully implementable, but we have laid out strong foundations for future works.

\(^{11}\) For example, it would not be surprising that \( \lambda \) depends on \( Q \).
References


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