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Abstract

We analyze the paradox of thrift in a two-sector Kaleckian growth model. We consider an economy with one consumption and one investment good, differential sectoral mark-ups, and profit rates equalization. We show that when the investment function depends on aggregate capacity utilization and on the aggregate profit share (the Bhaduri-Marglin investment function) the paradox of thrift in its growth version may fail if mark-ups are higher in the investment good sector. In this case, an increase in the saving rate produces a reallocation of economic activity towards the investment good sector; the aggregate profit share rises and its positive effect on investment may offset the reduction in average capacity utilization if investment is relatively more sensitive to profitability than to the level of activity.

Keywords: two-sector growth model, paradox of thrift, Bhaduri-Marglin investment function

JEL Classification: D33, E11

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1 Introduction

Dating back to at least Mandeville's popular *Fable of the Bees* (Mandeville, 1714[1988], I, remark Q), the paradox of thrift became a central proposition in Macroeconomics after the publication of Keynes's *General Theory of Employment Interest and Money* (Keynes, 1936[1973]). It is also a distinguishing feature of the Kaleckian model of growth and distribution, where it appears both in a short- and in a long-run version. It states that an increase in the saving rate produces a reduction of production and capacity utilization in the short-run, and of the growth rate in the long-run. A remarkable feature of this result is its robustness to the specification of the investment function, a contentious issue in Kaleckian economics.[1]

We investigate the validity of the paradox of thrift in a two-sector version of the Kaleckian growth model. To the purpose, we consider an economy with one consumption and one investment good, differential sectoral mark-ups, and profit rates equalization. We show that the paradox of thrift is confirmed in both level and growth versions when investment is a function of aggregate (average) capacity utilization or the profit rate. However, when the investment function depends on both aggregate capacity utilization and on the aggregate profit share, that is when the Bhaduri-Marglin investment function (Bhaduri and Marglin, 1990; Marglin and Bhaduri, 1990) is adopted, the paradox of thrift in its growth version may fail if mark-ups are higher in the investment good sector. In this case, an increase in the saving rate produces a reallocation of economic activity towards the investment good sector; the aggregate profit share rises and its positive effect on investment may offset the reduction in average capacity utilization.

Bhaduri and Marglin originally proposed their investment function to analyze the two-sided role that variations in wages play in industrial capitalism (Bhaduri and Marglin, 1990, p. 375). On the one hand, high wages stimulate demand as they increase workers' capacity to spend. This effect is responsible for the Keynesian 'paradox of costs', that is the positive relation between labor costs and economic activity. The paradoxical nature of this result stems from reversing the Marshallian microeconomic prediction that higher marginal costs decrease firms' optimal output. On the other hand, though, high wages may reduce demand as they harm profitability and the incentive to invest. With the aid of the new investment function Bhaduri and Marglin could take both effects into account while studying the relation between income distribution and economic activity. As a result, this interaction became more

[1] The long-run nature of the paradox of thrift is more controversial in models with Harrodian features. Competing views can be found in Shaikh (2009) and Hein et al. (2012) among others.

complex than previously understood. Notably, when the profitability effect dominates, the paradox of cost fails. The economy is in an *exhilarationist* regime, demand is profit-led. Our paper is not concerned with exogenous changes in income distribution, since sectoral mark-ups are given. As such, the adoption of the Bhaduri-Marglin investment function does not serve its original purpose. However, we find it noteworthy that, besides having powerful consequences regarding the paradox of cost, the Bhaduri-Marglin function may also affect the paradox of thrift, admittedly in a specific economic environment.

While Harcourt (1965) produced the first two-sector model of distribution and employment with Keynesian features, the canonical two-sector Keynesian-Kaleckian model of growth and distribution was developed in seminal contributions by Dutt (1988, 1990); Park (1995); Dutt (1997a); Lavoie and Ramirez-Gaston (1997); Franke (2000). We derive our result within a specific version of the model. It is obtained by introducing capital mobility, and the relative implication of profit rates equalization across sectors, in the standard model. This refinement has been proposed by Dutt (1997a) in a discussion with Park (1995) on the risk of over-determination in the Kaleckian two-sector model. The debate clarified that in the canonical model, where the capital stock in each sector grows according to a specific sectoral investment function, profit rates cannot be equalized in the short-run. If we are to add capital mobility and profit rates equalization, we can only specify the aggregate growth rate of the capital stock. This is the reason why the Bhaduri-Marglin investment function we employ depends on the aggregate level of capacity utilization and profit share. The combination of capital mobility and Bhaduri-Marglin investment function is ultimately responsible for our result. If sectoral growth rates could be specified (the no capital mobility case), they would depend on sectoral profit shares and capacity utilization rates. In this scenario, the reallocation of resources that follows a change in the propensity to save would not affect the profitability motive to invest. In fact, the average profit share would change, but investment would depend on the sectoral ones, which would remain constant.

The two crucial assumptions of our paper, capital mobility and the Bhaduri-Marglin investment function, may appear not perfectly compatible. While the profit *rate* determines how capital moves across sectors, the measure of profitability relevant for investment decisions is the profit *share*. But the two variables affect different decisions. Investment plans consist in choosing the *increment* in the capital stock. As discussed above, the Bhaduri-Marglin function may be desirable from this standpoint in that it allows to disentangle the demand from the profitability effect of distributional changes. Capital mobility, on the other hand, concerns the allocation of the *level* of the capital stock. However investment may be determined, it seems reasonable to assume that capital-owners will not overlook profit opportunities. As long as profit rates are not equalized, capital-owners can raise their profits by shifting capital towards the more profitable sector.

The two-sector Kaleckian framework is recently experiencing a revival. Kim and Lavoie (2017) have studied the convergence between the actual and the normal rates of capacity utilization; Fujita (2018) has considered the growth implication of shocks to sectoral mark-ups; Murakami (2018) has analyzed the effect of sectoral interactions on business cycles in a Keynesian model; Beqiraj et al. (2019) have studied how changes in consumers' preferences and the saving rate may affect income distribution through changes in the composition of output; Nishi (2020a) has introduced sectoral endogenous labor productivity growth, and analyzed its effects on cyclical demand, growth and distribution; Nishi (2020b) has shown that the introduction of Kaldor's technical progress function leads to supply-led growth in the long-run; Araujo et al. (2020) revisited the debate on over-determination of the model in an evolutionary dynamics framework. None of these contributions, however, investigate the paradox of thrift in the two-sector Kaleckian model when accumulation is based on the Bhaduri-Marglin investment function.

Our theoretical framework adopts the standard Kaleckian mark-up pricing assumption, where sectoral mark-ups are exogenous and independent of each other. Lavoie and Ramirez-Gaston (1997) argued that this assumption is problematic, since mark-ups in the basic good sector (the investment good in our case) indirectly affect price determination in the non-basic consumption good sector. As a solution, they proposed to replace mark-up pricing with target-return pricing, which means that firms set prices by targeting a specific return rate when capacity utilization is at its normal level. We generalize our analysis by adopting this alternative assumption. From a qualitative point of view, our results hold even while taking into account the dependence of the consumption good sector mark-up on the one in the investment good sector.

The rest of the paper is organized as follows. Section 2 develops the model and states the theoretical results. Section 3 generalizes the main result of the paper when firms adopt target-return, rather than mark-up, pricing. Section 4 offers some concluding remarks while the most tedious proofs can be found in Section 5.

2 The Model

2.1 Production and technology

The economy consists of a consumption (C) and an investment (I) good. Output in both sectors (X_i) is produced through a sector-specific Leontief production function:

$$X_i = min[u_i B_i K_i, A_i L_i], i = C, I \tag{1}$$

where B and A are capital and labor productivities, K is the capital stock, L is employment, and $u \leq 1$ is the degree of capacity utilization. When $u = 1$, output is at its full capacity level (X^p). Capital does not depreciate. Profit maximization ensures:

$$X_i = u_i B_i K_i = A_i L_i. \tag{2}$$

We normalize capital productivities $B_i = 1$.

2.2 Society and saving assumptions

There are two classes in society. Capitalists earn profits on the capital stock they own. They save the share $s > 0$ of their income. Workers earn the wage rate w and do not save. Since labor is homogeneous and workers can move freely without costs, the wage rate is uniform across sectors. Labor supply is infinitely elastic and never constrains growth or production.

2.3 Mark-up prices

In standard Kaleckian fashion, firms set prices by charging an exogenous sector-specific constant mark-up (z_i) over unit labor cost. If we let p_i be the price of good i, and we choose the consumption good as the numeraire we have $p_C = 1 = (1 + z_C)w/A_C$ and $p_I = (1 + z_I)w/A_I$. Accordingly

$$w = \frac{A_C}{1 + z_C}, \tag{3}$$

$$p_I = \frac{1 + z_I}{1 + z_C}\frac{A_C}{A_I} = \frac{1 + z_I}{1 + z_C}\gamma, \tag{4}$$

where $\gamma \equiv A_C/A_I$, is the relative labor productivity ratio. We define the relative price as $p \equiv p_I/p_C = p_I$.

2.4 Value added distribution

In each sector, value added is distributed as wages and profits to labor and capital employed in production. If we let r_i be the profit rate in sector i we have $p_i X_i = wL_i + r_i p_I K_i$, which, after using (2), (3), (4) and rearranging, yields the sectoral profit rates as functions of utilization rates:

$$r_C = \frac{z_C}{1+z_I}\frac{1}{\gamma}u_C, \tag{5}$$

and

$$r_I = \frac{z_I}{1+z_I}u_I. \tag{6}$$

2.5 Output uses

We distinguish consumption depending on its income source. We denote consumption out of wages as C^w, and consumption out of profits as C^π, so that

$$X_C = C^w + C^\pi. \tag{7}$$

Investment good output is fully absorbed in the accumulation of capital. If we let g be the growth rate of the aggregate capital stock we have

$$X_I = gK. \tag{8}$$

2.6 Balanced growth

Since workers do not save, the whole wage fund is spent as consumption out of wages. Using (2) we have

$$C^w = w(L_C + L_I) = w\left(\frac{u_C K_C}{A_C} + \frac{u_I K_I}{A_I}\right).$$

Hence, substituting for the wage rate from (3) yields

$$C^w = \frac{A_C}{1+z_C}\left(\frac{u_C K_C}{A_C} + \frac{u_I K_I}{A_I}\right) = \frac{1}{1+z_C}\left(u_C K_C + \gamma u_I K_I\right). \tag{9}$$

On the other hand, capitalists' propensity to consume out of profits is $(1-s)$. Accordingly

$$C^{\pi} = (1-s)\left(r_I p K_I + r_C p K_C\right),$$

which, using (4),(5) and (6) implies

$$C^{\pi} = \frac{1-s}{1+z_C}\left(z_C u_C K_C + z_I \gamma u_I K_I\right).$$

Once we know consumption out of wages and profits, we can use equation (7) to find

$$X_C = \frac{1}{1+z_C}\left(u_C K_C (1+(1-s)z_C) + \gamma u_I K_I (1+(1-s)z_I)\right).$$

Define $\delta \equiv K_C/K \in (0,1)$ as the share of the capital stock employed in the consumption good sector; δ is the endogenous variables responsible for the instantaneous equalization of sectoral profit rates. Dividing both sides of the previous equation by K and rearranging yields

$$\delta u_C = (1-\delta) u_I \gamma \frac{(1+(1-s)z_I)}{s z_C} = (1-\delta) u_I \gamma \Gamma(s), \tag{10}$$

where $\Gamma(s) \equiv \frac{(1+(1-s)z_I)}{s z_C}$ and $\Gamma'(s) < 0$. Let us now turn to the equilibrium in the investment sector. Remembering from (2) that $X_I = u_I K_I$, we can divide both sides of equation (8) by K to find

$$u_I (1-\delta) = g. \tag{11}$$

Next, we impose the equalization of profit rates across sectors, so that

$$r_C = r_I = r. \tag{12}$$

Using (5) and (6), the equalization yields a relation between the two sectoral utilization rates:[2]

$$u_C = \gamma \frac{z_I}{z_C} u_I. \tag{13}$$

[2]One may argue that firms could achieve profit rates equalization by adjusting mark-ups rather than shifting capital across sectors. If we think that the Kaleckian framework is typically thought of as representative of markets with few big firms with market power, this is likely a more realistic assumption. However, it would make sectoral distribution endogenous thus changing dramatically the nature of the model. This tension between the assumptions of exogenous mark-ups and profit rates equalization points to a possibly imperfect harmony between the Kaleckian model and capital mobility.

We close the model with three alternative investment functions that generalize the standard assumptions of Kaleckian growth to the two-sector growth model; we assume that investment depends either on the rate of capacity utilization, on the profit rate, on the profit share or on some combination of them. We have already mentioned that the instantaneous profit rates equalization implies that sectoral capital stocks are not state variables, and that only aggregate investment and growth can be defined. This hypothesis also implies that firms will have to look at average, rather than sectoral, utilization rates, profit rates and profit shares when making their investment decision. In fact, given total investment, firms will discover the share of the capital stock and investment employed in either sector only after profit rates are equalized.

If we let the average degree of capacity utilization in the economy be $\bar{u}$, and the aggregate profit share be π, we take into account the following investment functions:

- the first one extends to the two-sector case the early Kaleckian models that had capacity utilization as determinant of investment (Amadeo, 1986a; Dutt, 1997b)

$$g_1 = g(\bar{u}); \tag{14}$$

- the second one assumes investment to depend on the profit rate, the 'stagnationist' investment function (Taylor, 1985; Amadeo, 1986b)

$$g_2 = g(r); \tag{15}$$

- the third one generalizes the Bhaduri-Marglin investment function (Bhaduri and Marglin, 1990; Marglin and Bhaduri, 1990) by positing that growth depends on both aggregate capacity utilization and the profit share

$$g_3 = g(\bar{u}, \pi). \tag{16}$$

Under the first and third specifications, the model consists of four equations, (10),(11), (13), and either (14) or (16), for the four unknowns δ, u_I, u_C, g. When the investment function is (15), the unknowns are δ, u_I, u_C, r, g in the five equations (10), (11), (12), (13), and (15). In all three cases we can plug (13) into (10) to find the equilibrium share of capital employed in the consumption goods sector[3]

[3] In what follows, we will denote with x^* the balanced growth value of variable x.

$$\delta^*(s) = \frac{\Gamma(s)}{\Gamma(s) + z_I/z_C} \in (0, 1). \tag{17}$$

The aggregate profit share, that is the ratio between the value of total profits and value added, is also independent of the investment function adopted. Its balanced growth value is

$$\pi^*(s) = \frac{r_C pK_C + r_I pK_I}{X_C + pX_I} = \frac{rp}{\delta u_C + (1-\delta) p u_I} =$$

$$= \frac{z_I}{(1-\delta)\left((1+z_C)\,\Gamma(s) + 1 + z_I\right)} = \frac{z_C \Gamma(s) + z_I}{(1+z_C)\Gamma(s) + (1+z_I)}, \tag{18}$$

where we used (2), (4), (5), (6), (10), (12) and (17). Inspection of (18) shows that π^* is economically meaningful being bounded between zero and one. It is a function of sectoral mark-ups and the saving rate.

We can state:

Proposition 1. *an increase in the saving rate raises the equilibrium profit share if and only if $z_I > z_C$.*

Proof. see Appendix (section 5.1). □

A rise in the saving rate reduces capitalists' consumption, so that the composition of output changes in favor of the investment sector. If mark-ups in the investment sector are higher than in the consumption sector ($z_I > z_C$), the reallocation generates a rise in the aggregate profit share. This finding is not particularly original. Beqiraj et al. (2019), for example, obtained a similar conclusion; but it is instrumental in developing our main argument.

Our next step is to verify whether the paradox of thrift holds under the alternative investment functions we have proposed. In order to obtain closed-form solutions for the growth rate, and in line with most of the Kaleckian tradition, we assume linear functional forms. Let us start with the accelerator version of investment:

$$g_1 = \beta_0 + \beta_1 \bar{u}, \tag{19}$$

where

$$\bar{u} = \frac{X_C + pX_I}{pK} = \frac{1+z_C}{1+z_I}\frac{u_C \delta}{\gamma} + u_I(1-\delta).$$

We can use (10) to find:

$$\bar{u} = u_I(1-\delta)\left(\frac{1+z_C}{1+z_I}\Gamma(s)+1\right). \tag{20}$$

Hence, using (11),

$$\begin{aligned} u_I(1-\delta) = g = \\ = \beta_0 + \beta_1 u_I(1-\delta)\left(\frac{1+z_C}{1+z_I}\Gamma(s)+1\right), \end{aligned} \tag{21}$$

which, by factoring $u_I(1-\delta)$, solves for the steady state growth rate of the first model as a function of the saving rate

$$g_1^*(s) = \frac{\beta_0}{1-\beta_1\left(\frac{1+z_C}{1+z_I}\Gamma(s)+1\right)}. \tag{22}$$

Since $\Gamma'(s) < 0$, the growth rate is a negative function of the saving rate and the paradox of thrift holds in its growth version. Given that $g_1 = \beta_0 + \beta_1\bar{u}$, a reduction in the equilibrium growth rate necessarily requires a decline in the equilibrium aggregate capacity utilization $\bar{u}^*$: the paradox of thrift in its level form is confirmed.

The second investment function makes investment dependent on the profit rate:

$$g_2 = \lambda_0 + \lambda_1 r. \tag{23}$$

Hence, we can use $r_I = r = \frac{z_I}{1+z_I}u_I$, to find

$$\begin{aligned} u_I(1-\delta) = g = \\ = \lambda_0 + \lambda_1\frac{z_I}{1+z_I}u_I. \end{aligned} \tag{24}$$

Next, by factorizing u_I, using (17) and rearranging

$$g_2^*(s) = \frac{\lambda_0\frac{z_I/z_C}{\Gamma(s)+z_I/z_C}}{\frac{z_I/z_C}{\Gamma(s)+z_I/z_C} - \lambda_1\frac{z_I}{1+z_I}}. \tag{25}$$

We show in the Appendix (section 5.2) that $dg_2^*/ds < 0$. The paradox of thrift in its growth version is confirmed also under the second type of investment function. The paradox of thrift in level form also holds. Since $g_2 = \lambda_0 + \lambda_1 r$, a lower growth rate is accompanied by a lower equilibrium profit rate. Both utilization rates are in a direct relation with the profit rate so that both necessarily decline; the aggregate utilization rate, which is a weighted average of the sectoral utilization rates, will also drop.

We now turn to the main result of our paper, and we investigate how growth responds to changes in the saving rate under a Bhaduri-Marglin investment function. In linearized terms, we can specify the function as

$$g_3 = \mu_0 + \mu_1 \bar{u} + \mu_2 \pi. \tag{26}$$

Hence,

$$\begin{aligned} u_I(1-\delta) = g = \\ = \mu_0 + \mu_1 u_I (1-\delta)\left(\frac{1+z_C}{1+z_I}\Gamma(s)+1\right) + \mu_2 \pi(s), \end{aligned} \tag{27}$$

and

$$g_3^*(s) = \frac{\mu_0 + \mu_2 \pi(s)}{1-\mu_1\left(\frac{1+z_C}{1+z_I}\Gamma(s)+1\right)}. \tag{28}$$

We are now able to state:

Proposition 2. *an increase in the saving rate raises the growth rate if and only if* $z_I > z_C$ *and* $\frac{\mu_2}{\mu_1} > \frac{1}{z_I - z_C}\frac{1+z_C}{1+z_I}\frac{\mu_0+\mu_2\pi(s)}{1-\mu_1\left(\frac{1+z_C}{1+z_I}\Gamma(s)+1\right)}\left((1+z_C)\Gamma(s)+(1+z_I)\right)^2$.

Proof. see Appendix (section 5.3). □

A rise in the saving rate has two opposing effects on growth. On the one hand, there is the standard depressing effect due to the reduction in capitalists' consumption and, in turn, in aggregate demand and capacity utilization. On the other hand, though, the higher propensity to save entails a shift in the composition of output away from consumption goods. When the profit share is higher in the investment goods sector, the aggregate profit share rises thus producing a positive incentive to invest. When investment is sufficiently more sensitive to profitability than to economic activity, that is when μ_2/μ_1 is 'high enough', the paradox of thrift in its growth version fails.

On the other hand, the paradox of thrift in level form applies. From (28), we can see that $dg_3^*/ds < 0$ when $\mu_2 = 0$; but since $g_3 = \mu_0 + \mu_1\bar{u}(s) + \mu_2\pi(s)$ this can happen only if the aggregate utilization rate decreases with the saving rate. We provide a formal proof of this result in the Appendix (section 5.2). This conclusion shows that the demand regime is always *stagnationist*: the increase in the profit share due to the higher saving rate necessarily reduces capacity utilization. As shown in Blecker (2002), this is an implication of the linear functional form assumed for the Bhaduri-Marglin investment function. More general functional forms may be able to produce the strong response of investment to the profit share necessary to generate the *exhilarationist* regime.

2.6.1 Discussion

In order to better understand the mechanism underlying our main result, we can focus on the function $\Gamma(s)$. In fact, the propensity to save only enters the system through $\Gamma(s)$. First, remember that from (2) $L_i = u_i K_i / A_i$, so that $L_C/L_I = u_C K_C A_I / [u_I K_I A_C] = \delta u_C / [(1-\delta) u_I \gamma]$. Next, (10) shows that $\Gamma(s) = \delta u_C / [(1-\delta) u_I \gamma] = L_C / L_I$, so that $\Gamma(s)$ equals the employment ratio in the two sectors. Therefore, a rise in the saving rate raises employment and output in the investment sector relative to the consumption one. The change in the composition of output towards the sector with the highest mark-up raises the aggregate profit share (our Proposition 1). Second, $\Gamma(s)$ enters the definition of aggregate capacity utilization through (20). A higher propensity to save reduces capitalists' consumption, thus depressing the aggregate capacity utilization. The negative shock to capacity utilization tends to depress the equilibrium growth rate; this effect can only be offset if investment reacts to the profit share as assumed in the Bhaduri-Marglin investment function (our Proposition 2).

On a different note, it is important to emphasize the relevance of the capital mobility assumption in producing our result. In the standard two-sector model, sectoral growth rates would depend on sectoral profit shares and capacity utilization rates under the Bhaduri-Marglin assumption. An increase in the propensity to save would change the aggregate profit share, while leaving the sectoral ones unaffected. Therefore, there would be no change in the profitability motive to invest. At the same time, capacity utilization in both sectors would decrease. Sectoral growth rates and, in turn, the steady state growth rate would likely fall. The paradox of thrift would not be compromised.

While capital mobility is essential for our result, *perfect* capital mobility is not. We have assumed throughout the analysis that capital instantaneously adjusts to the specific sectoral allocation that equalizes the two profit rates. As shown by Dutt (1997a, p. 447-8), however, we can think of a version of capital mobility where sectoral capital stocks are fixed at a point in time and the profit rate differential regulates the sectoral allocation of aggregate investment rather than of the total capital stocks. In this version of the model, aggregate investment would still depend on the average profit share and capacity utilization as in (16); the allocation of investment between the two sectors, on the other hand, would be governed by the profit rates differential according to: $g_I - g_C = \xi(r_I - r_C)$, where $\xi > 0$ would measure the degree of capital mobility. In steady state, $r_I = r_C$ and $g_I = g_C$ so that equations (11) and (13) would still hold. The system of equations that solves for δ, u_I, u_C, g would be identical to the perfect mobility case. Our result would necessarily follow.

3 A Generalization: the Model with Target-Return Pricing

We developed our results under the standard mark-up pricing assumption that characterizes one- and two-sector Kaleckian growth models. The assumption, however, is controversial. As pointed out by Lavoie and Ramirez-Gaston (1997), since the investment good is a basic good, the mark-up in the consumption sector should not be taken as exogenous and independent of the mark-up in the investment sector.

We develop a generalization of the model that does not suffer from the critique. To the purpose, we replace mark-up pricing with the target-return pricing assumption first proposed by Lavoie and Ramirez-Gaston (1997) and more recently employed by Kim and Lavoie, 2017. We show that the logic of our main results is confirmed even within a framework that takes the interdependence between mark-ups into account.

Let us start by introducing the sectoral normal degree of capacity utilization, u_i^n. Next, we define the sectoral target rate of return (r_i^n) as the return rate that firms target when output and sales correspond to the normal degree of capacity utilization, that is when $X_i = u_i^n K_i \equiv X_i^n$. Given the target rate and normal output, the normal flow of profits (Π_i^n) can be written both as $\Pi_i^n = z_i(w/A_i)X_i^n$ and $\Pi_i^n = r_i^n p K_i = r_i^n p X_i^n / u_i^n$. The equalization of the two profit flows expressions, while using (3) and (4), yields

$$z_i = r_i^n p A_i/(w u_i^n) = r_i^n (1 + z_I) A_i/(A_I u_i^n). \tag{29}$$

Equation (29) shows that mark-ups in the investment good sector, i.e. the basic good sector, depend only on its own economic features:

$$z_I = \frac{r_I^n/u_I^n}{1 - r_I^n/u_I^n} = \frac{r_I^n}{u_I^n - r_I^n}. \tag{30}$$

On the contrary, mark-ups in the consumption good sector are affected by the fundamentals of both sectors:

$$z_C = \frac{r_C^n}{u_C^n}\gamma(1+z_I) = \gamma\frac{r_C^n}{u_C^n}\frac{u_I^n}{u_I^n - r_I^n}. \tag{31}$$

Now, in our exercise we only consider a saving shock. No changes to mark-ups determinants are analyzed and, in turn, the interdependence between the two sectors' mark-ups never comes into play. We can, however, substitute (30) and (31) into (18) to find the aggregate wage share as

$$\pi^*(s) = \frac{\gamma\frac{r_C^n}{u_C^n}\frac{u_I^n}{u_I^n-r_I^n}\widetilde{\Gamma}(s) + \frac{r_I^n}{u_I^n-r_I^n}}{(1+\gamma\frac{r_C^n}{u_C^n}\frac{u_I^n}{u_I^n-r_I^n})\widetilde{\Gamma}(s) + 1 + \frac{r_I^n}{u_I^n-r_I^n}},$$

where $\widetilde{\Gamma}(s) = \frac{1+(1-s)r_I^n/(u_I^n-r_I^n)}{s\gamma r_C^n u_I^n/[u_C^n(u_I^n-r_I^n)]}$. We can now restate Proposition 1 as

Proposition 3. *an increase in the saving rate raises the equilibrium profit share if and only if* $\frac{r_I^n}{u_I^n} > \gamma\frac{r_C^n}{u_C^n}$.

Proof. See Appendix (section 5.4) □

In order to interpret the emended condition for the positive relation between the saving rate and the profit share, let us define the sectoral normal profit shares as $\pi_i^n \equiv \Pi_i^n/X_i^n = r_i^n p/u_i^n$. We can see that the condition found in Proposition 3 is equivalent to $\pi_I^n > \gamma\pi_C^n$, or $A_I\pi_I^n > A_C\pi_C^n$. We thus see that the necessary condition for the violation of the paradox of thrift requires that the normal profit share (weighted by labor productivity) in the investment good sector be higher than in the consumption sector. In order to produce a rise in the profit share, the rise in the saving rate must be associated to a reallocation of resources towards the relatively more profitable sector. From a qualitative point of view, the result confirms what we found under the mark-up pricing assumption.

Once the possibility that $\pi'(s) > 0$ is established, it will always be possible to find a threshold for the relative weights of the profit share and capacity utilization in the investment function (μ_2/μ_1) such that the paradox of thrift is violated. With respect to the result found in Proposition 2, the threshold will be a function of $\gamma, r_C^n, u_C^n, u_I^n$ and r_I^n rather than z_C and z_I; but the economic content is analogous.

4 Conclusions

Our theoretical note shows that the paradox of thrift may not work in the Kaleckian growth and distribution framework, once it is generalized to a two-sector economy. This possibility arises because the saving rate affects not only the level of aggregate demand, but also its composition. In particular, a rise in the saving rate, besides depressing aggregate demand, shifts the sectoral composition of output towards the investment goods sector. If this sector is characterized by relatively high mark-ups the aggregate profit share rises; and such an increase in profitability may have a positive effect on growth if, as assumed in Bhaduri and Marglin (1990) and Marglin and Bhaduri (1990), investment reacts to the profit share.

5 Appendix

5.1 Proof of proposition 1

$\pi'(s) = \frac{d}{ds}\left(\frac{z_C\Gamma(s)+z_I}{(1+z_C)\Gamma(s)+(1+z_I)}\right) = \frac{\Gamma'(s)[z_C(1+z_I)-(1+z_C)z_I]}{((1+z_C)\Gamma(s)+(1+z_I))^2} = \frac{\Gamma'(s)(z_C-z_I)}{((1+z_C)\Gamma(s)+(1+z_I))^2}$, where we used $z_C(1+z_I)-(1+z_C)z_I = z_C + z_Cz_I - z_Cz_I - z_I$.

Therefore $\pi'(s) > 0 \Leftrightarrow z_I > z_C$.

5.2 The paradox of thrift in the stagnationist version of the model

From (25) we have $dg_2^*/ds = \frac{\lambda_0\lambda_1}{\left(\frac{z_I/z_C}{\Gamma(s)+z_I/z_C} - \lambda_1\frac{z_I}{1+z_I}\right)^2}\frac{z_I}{1+z_I}\frac{(z_I/z_C)\Gamma'(s)}{(\Gamma'(s)+z_I/z_C)^2} < 0$.

5.3 The paradox of thrift in the Bhaduri-Marglin model

Let $1-\mu_1\left(\frac{1+z_C}{1+z_I}\Gamma(s)+1\right) \equiv D_0$, from (28)

$$dg_3^*/ds = \frac{1}{D_0^2}\left[\mu_2\pi'(s)\left(1-\mu_1\left(\frac{1+z_C}{1+z_I}\Gamma(s)+1\right)\right)+\mu_1\frac{1+z_C}{1+z_I}\Gamma'(s)\left(\mu_0+\mu_2\pi(s)\right)\right].$$

Therefore

$$sign\left(dg_3^*/ds\right) = sign\left[\mu_2\pi'(s)\left(1-\mu_1\left(\frac{1+z_C}{1+z_I}\Gamma(s)+1\right)\right)+\mu_1\frac{1+z_C}{1+z_I}\Gamma'(s)\left(\mu_0+\mu_2\pi(s)\right)\right].$$

Then, $dg_3^*/ds > 0 \Leftrightarrow \frac{\mu_2}{\mu_1} > -\frac{\frac{1+z_C}{1+z_I}\Gamma'(s)(\mu_0+\mu_2\pi(s))}{\pi'(s)\left(1-\mu_1\left(\frac{1+z_C}{1+z_I}\Gamma(s)+1\right)\right)} = \frac{1}{z_I-z_C}\frac{1+z_C}{1+z_I}\frac{\mu_0+\mu_2\pi(s)}{1-\mu_1\left(\frac{1+z_C}{1+z_I}\Gamma(s)+1\right)}\left((1+z_C)\Gamma(s)+(1+z_I)\right)^2.$ Let us now turn to the utilization rate. From (11) and (20) we can write aggregate capacity utilization as $\bar{u}(s) = g_3^*(s)\left(\frac{1+z_C}{1+z_I}\Gamma(s)+1\right)$. Substituting for $g_3^*(s)$ we have $\bar{u}_3^*(s) = \frac{\mu_0+\mu_2\pi(s)}{1-\mu_1\left(\frac{1+z_C}{1+z_I}\Gamma(s)+1\right)}\left(\frac{1+z_C}{1+z_I}\Gamma(s)+1\right) =$
$= \frac{\mu_0+\mu_2\frac{z_C\Gamma(s)+z_I}{(1+z_C)\Gamma(s)+(1+z_I)}}{1-\mu_1\left(\frac{1+z_C}{1+z_I}\Gamma(s)+1\right)}\left(\frac{1+z_C}{1+z_I}\Gamma(s)+1\right)$, let $D_1 = (1+z_C)\Gamma(s)+(1+z_I)$. Then, $d\bar{u}_3^*/ds =$
$\frac{\mu_2\Gamma'(s)(z_C-z_I)}{D_0D_1^2}\left(\frac{1+z_C}{1+z_I}\Gamma(s)+1\right)+$
$+\frac{1+z_C}{1+z_I}\Gamma'(s)\frac{\mu_0[(1+z_C)\Gamma(s)+(1+z_I)]+\mu_2(z_C\Gamma(s)+z_I)}{D_0^2D_1} =$
$= -\frac{\Gamma'(s)}{D_0D_1}\left\{\frac{\mu_2(z_I-z_C)}{D_1}\left(\frac{1+z_C}{1+z_I}\Gamma(s)+1\right)+\right.$
$\left.-\frac{1+z_C}{1+z_I}\left[\mu_0\left[(1+z_C)\Gamma(s)+(1+z_I)\right]+\mu_2(z_C\Gamma(s)+z_I)\right]/D_0\right\}$. Hence, $d\bar{u}_3^*/ds < 0 \Leftrightarrow \mu_2(z_I-z_C)\left(\frac{1+z_C}{1+z_I}\Gamma(s)+1\right)D_0 <$
$\frac{1+z_C}{1+z_I}\left[\left[(1+z_C)\Gamma(s)+(1+z_I)\right]\left[(1+\mu_0)+\mu_2(z_C\Gamma(s)+z_I)\right]\right]$. Since $D_0 < 1$ and $\mu_2 z_I\left(\frac{1+z_C}{1+z_I}\Gamma(s)+1\right) <$
$\mu_2(z_C\Gamma(s)+z_I)\frac{1+z_C}{1+z_I}\left[(1+z_C)\Gamma(s)+(1+z_I)\right]$, it follows that $d\bar{u}_3^*/ds < 0$ always.

5.4 Proof of proposition 3

Proof. Notice first that $\widetilde{\Gamma}'(s) < 0$. Then, $\pi'(s) = \frac{\widetilde{\Gamma}'(s)\left\{\gamma\frac{r_C^n}{u_C^n}\frac{u_I^n}{u_I^n-r_I^n}\left(1+\frac{r_I^n}{u_I^n-r_I^n}\right)-\frac{r_I^n}{u_I^n-r_I^n}\left(1+\gamma\frac{r_C^n}{u_C^n}\frac{u_I^n}{u_I^n-r_I^n}\right)\right\}}{\left((1+\gamma\frac{r_C^n}{u_C^n}\frac{u_I^n}{u_I^n-r_I^n})\widetilde{\Gamma}(s)+1+\frac{r_I^n}{u_I^n-r_I^n}\right)^2} =$
$\frac{\widetilde{\Gamma}'(s)(\gamma\frac{r_C^n}{u_C^n}\frac{u_I^n}{u_I^n-r_I^n}-\frac{r_I^n}{u_I^n-r_I^n})}{\left((1+\gamma\frac{r_C^n}{u_C^n}\frac{u_I^n}{u_I^n-r_I^n})\widetilde{\Gamma}(s)+1+\frac{r_I^n}{u_I^n-r_I^n}\right)^2}$,where we used
$\gamma\frac{r_C^n}{u_C^n}\frac{u_I^n}{u_I^n-r_I^n}\left(1+\frac{r_I^n}{u_I^n-r_I^n}\right)-\frac{r_I^n}{u_I^n-r_I^n}\left(1+\gamma\frac{r_C^n}{u_C^n}\frac{u_I^n}{u_I^n-r_I^n}\right) = \gamma\frac{r_C^n}{u_C^n}\frac{u_I^n}{u_I^n-r_I^n}+\gamma\frac{r_C^n}{u_C^n}\frac{u_I^n}{u_I^n-r_I^n}\frac{r_I^n}{u_I^n-r_I^n}-\frac{r_I^n}{u_I^n-r_I^n}-\gamma\frac{r_C^n}{u_C^n}\frac{u_I^n}{u_I^n-r_I^n}\frac{r_I^n}{u_I^n-r_I^n}$.

Therefore $\pi'(s) > 0 \Leftrightarrow \frac{r_I^n}{u_I^n} > \gamma\frac{r_C^n}{u_C^n}$. □

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