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GENERAL EQUILIBRIUM MODELLING: THE STATE OF THE ART

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Content Outline

Abstract

Introduction

Historical Evidence

Pros, Cons, and Underlying Gap

General Equilibrium Modelling: The State of the Art

Conclusion

References
Abstract

The general equilibrium approach's theoretical superiority has always been accepted in economic literature, as a robust, complete, and detailed general temporal equilibrium model that shows how money, production, saving level, capital goods and services prices, and the interest rate are jointly determined. This article is a time-path exploration of the literature on propositions related to general equilibrium modelling, and a highlight of its contemporary relevance. Over time and till now, the Walras’ proposition at best takes primacy, amidst others, and improvements in the development of sophisticated sub-models of asset markets within the general equilibrium framework had meant substantial progress since this permit analysis under more realistic conditions. However, limitations are clear, and steady advances in the model can better improve its application.
1.0 Introduction

The theoretical superiority of the general equilibrium approach has always been accepted in economic literature. This robust model is credited to the timeless economic foundation of Smith (1776), Walras (1874) and Edgeworth contract curve, Arrow-Debreu proof of existence, and Leontief input and output model (Manne 1983).

Walras (1874) must be credited to have opined an existing relationship between market mechanism and factor prices, which can be determined by Smith’s invisible hand. More so, Walras (1874) conventional applied general equilibrium model also describes the allocation of resources in the market economy due to the interaction of supply and demand, leading to an equilibrium price.

However, the building block of this model are equations representing the behaviour of the relevant economic agents (consumer, producer, and government), each of these agents’ demand and supply of goods and services, and factors of production as a function of their prices, assuming that market forces will lead to equilibrium between demand and supply. The general equilibrium thereby computes the prices that all markets clear and determines the allocation of resources and income distribution that result from the equilibrium (Borges, 1986).

To this end, certain question(s) therein include: What are the possible pros, and inherent limitations, echoed over time in the literature relative to the general equilibrium modelling? What is the possible way forward established over time in the literature that could be re-echoed, in respect to asset markets’ examination beyond the place for a physical asset (or capital), given the recurring instance of government deficit financing, especially in developing economies? Thus, the purpose of this article is to undertake a time-path exploration of the literature on propositions related to general equilibrium modelling (tracing the historical antecedents, as well as future
possibilities in general equilibrium modelling), and to highlight still the contemporary relevance of general equilibrium modelling.

Nevertheless, the possible non-awareness of general equilibrium modelling as an analytical framework that could accommodate financial asset and financial intermediation in examining asset markets may render intended empirical investigations respective to issues relating to government deficit financing to be thought non-feasible.

The rest of this article as follows: Section 2 discusses the historical evidence. Section 3 reiterates the pros, cons, and underlying gap. Section 4 presents state of the art in general equilibrium modelling. Section 5 concludes.

**2.0 Historical Evidence**

The general equilibrium theory's historical origin is to be found in the marginal utility or neoclassical school (the school of economics active in the mid to late nineteenth century). Based on the theory developed by this school, Gossen (1854), Jevons (1871) and Walras (1874) – who used mathematical notations – and Menger (1871) – who did not – took the first steps to develop general equilibrium theory. The most effective and outstanding researcher in this group is L. Walras. Walras can be considered the protagonist of the theory.

General equilibrium’s simplest problem lies in the analysis of exchange economies. In this type of economy, the consumers' budget constraint is determined by their initial resource endowment and the price vector. The individual demand function is the optimal response of the individual consumer to the given price system. Aggregating individual functions obtain the market demand
function, and market equilibrium emerges when we find a price for which the addition of net demands equal zero.

This idea was already present in classical economic theory expressed as ‘supply should match demand’. Although Cournot (1838) in his discussion of international money flow, and Mill (1848) in his arguments on international trade, had already sensed this point, we owe its expression as a set of mathematical equations to Walras (1874).

Some years later, Pareto (1909) defined the property of market equilibrium. Under the assumptions that goods were perfectly divisible and utility functions were differentiable if every consumer made an equilibrium allocation of goods, an infinitesimal change in this allocation would not affect the utility levels if it did not affect the budget restriction levels. The so-called Pareto optimum could occur in competitive equilibrium, but it would require more severe conditions. Arrow (1951) set out the first theorem for developing this question.

The following step in developing general equilibrium theory was the introduction of production into a static framework. Producers were assumed to minimize production costs given the market prices. Market equilibrium was defined as a situation in which, given a price vector, supply matched demand. Walras considered a productive sector with a single good, and Hicks (1939) generalized this model to include more than just one output.

Earlier on Cassel (1918) had already developed a productive sector model, understood as a set of potential linear activities. He applied a simplified Walrasian model that preserved demand functions and production coefficients but did not deduce the utility functions or preferences' demand functions. The model was generalized by von Neumann (1937) to allow for production in a spatial context.
A little later, Koopmans (1951) made a more complete and sophisticated analysis, creating a model explicitly introducing intermediate products. But the general linear model of production was not enough to deal with the choice of activities as a cost-minimizing process, given the price vector and the quantities. Cost minimization had to be replaced by a condition according to which no activity could provide profits, and no activity could suffer any losses in competitive equilibrium. This was exactly the condition used by Walras to initially define production equilibrium in a general production model by von Neumann (1937) and was called the von Neumann law for production activities models.

Meanwhile, an alternative productive sector model was being developed. This model emphasized producer organizations or firms, rather than activities or technology. The equilibrium condition in the productive sector was that each firm maximizes its profits, calculated as the value of the input-output combination over its production potential, given the input and output prices. This version of the production sector, specified in a partial equilibrium context by Cournot (1838), was implicit in the work of Marshall (1890), and Pareto (1909). It was further specified in a general equilibrium context by Hicks (1939), and especially in the Arrow and Debreu (1954) model.

The Arrow and Debreu’s (1954) model is the one we can identify as the ‘first complete general equilibrium model’. It formally demonstrated the existence of equilibrium with a productive sector formed by enterprises. Each of the enterprises had a set of production possibilities based on the resources it owned. The productive sector reached equilibrium when each enterprise chose the input-output combination of its technical possibilities that maximized profits at market prices. This was also the first model to include Walras-style preferences through demand-side hypotheses directly.
More or less simultaneously, McKenzie (1959) built another formal general equilibrium model. It formalized Walras’ theory and used a linear production model. McKenzie (1959) proved equilibrium in this model through hypotheses made on-demand functions rather than directly on preferences. It is considered a linear technology instead of a set of enterprises. It was a generalized form of Wald's (1936) model, omitting the structure of production, and the key hypothesis stated that demand functions satisfied the so-called ‘weak axiom of revealed preferences’.

The rationale of the static equilibrium analysis was to choose a short enough period to avoid a big distortion of reality, and suppose that all transactions would conclude within that period. This type of analysis had been developed by Walras (1874), Hicks (1939), and Arrow and Debreu (1954), although Arrow and Debreu (1954) explicitly dealt with inter-temporal planning, of both consumers and producers.

Walras’ (1874) approach to static equilibrium was suitable only when everything remained constant: technology, tastes, resources, and maybe even capital and population growth rates. Therefore, static comparisons had to be made as comparisons between the different stages. On the other hand, Hicks (1939) considered the possibility of analysing equilibrium not from a static perspective, but over time assuming agents’ present price expectations remained unchanged in the future.

Several authors have tried to solve this problem. One was Radner (1972) whom solution was to assume perfect forecasting, considering that all the agents had unchanged price expectations. Only a finite number of events could happen each time. From the point of view of the given market, the key events were the sequences of states of nature that could occur over time. For each sequence, the agents correctly anticipated their corresponding price sequence. Rational expectations were
implicit in this equilibrium model, where all agents had the same available information. The trouble with this model is that the agents may behave differently from how they are expected to, and this have served other authors to demonstrate the non-existence of equilibrium; see Green (1977), and Kreps (1977).

Hick’s (1939) model was naturally developed by Grandmont (1977) amongst others. Grandmont (1977) assigned each agent an expectation function that provided a distribution of probabilities on future prices and possibly other relevant variables as well. Therefore, assuming that each consumer had a criterion for choosing the optimum plan according to his or her expectations, the model would determine the excess demand as a function of current prices. Equilibrium is reached if the market were cleared at the given prices.

Theorems on static equilibrium have been developed and demonstrated for many special cases, particularly for perfectly competitive economies where production is not taken into account, and the number of periods is finite. The application of a fixed-point theorem like the one developed by Brouwer (1910) completes the proof that a price system causes market-clearing if every excess demand function equal to zero. Despite these achievements, there are also some problems with his theory.

The most remarkable oversights in Walras’ static equilibrium still are the analysis of the demand for assets, and saving for future consumption. For this reason, one of the main lines for general equilibrium theory development is the introduction of money. Money performs several economic functions, being a means of exchange, an asset, or a numeraire. Authors such as Grandmont and Younes (1972), and Grandmont (1977) proved equilibrium in monetary models.
To prove monetary equilibrium, a hypothesis, similar to previous assumptions for the same purpose of limiting price expectations, like Green's (1973) conjecture was needed to prove the existence of a temporal equilibrium in non-monetary economies. The hypothesis was that, on a finite temporal horizon, the expected set of prices that resulted from all possible choices between current prices was assumed to be positive. Then, if all consumers had expectations that satisfied this and the previous model’s hypotheses, a temporal equilibrium would also exist in this case.

This review of the main contributions to general equilibrium theory would not be complete without a reference to temporal equilibrium with infinite horizon. As noted earlier, the Arrow and Debreu (1954) general equilibrium model had a finite number of periods, events, and goods. The main objection to the finite number of goods' constraint was that it required a finite horizon and there was no natural way to choose the end of the period.

Two types of models were developed to solve this problem, leading to an infinite number of goods. One model has an infinite number of living consumers. Each consumer could be considered a descendent of a series in an undefined future. This way, consumers living in the present period have an interest in the goods of all periods. This model is called an overlapping generation model. It was first proposed and analyzed by Samuelson (1958). Later on, it was rigorously developed by Balasko et al. (1980), and by Wilson (1981).

The second model, introduced by Peleg and Yaari (1970), was a competitive general equilibrium model with a finite number of consumers, and an infinite number of goods. Peleg and Yaari (1970) presented an exchange model without production. Bewley (1972) produced a competitive general equilibrium model that included production with an infinite number of goods. It represented a
generalized form of the existence theorem developed by McKenzie (1959) in the case of many goods, retaining the hypothesis of a finite number of goods.

In sum, Walras’ theory had been the most complete and detailed general temporal equilibrium model ever developed. It is remarkable in that it is also the first formal general equilibrium model. Walras was able to build a model that jointly determined money, production, saving level, capital goods and services prices, and the interest rate.

3.0 Pros, Cons, and Underlying Gaps

The general equilibrium model has several strong advantages and expectedly, some drawbacks. It is agreed in the economic literature that, general equilibrium provides conceptual consistency for model analysis. Iwayemi and Adenikinju (2009) argued that general equilibrium is based on well-established axioms and the microeconomic foundation principle such as profit and utility maximization, and rational behaviour of economic agents. For instance, under Walras law, households are presumed to be on a budget constraint. There are no zero profit conditions of firms, and demand and supply are equal for all commodities and production factors. In addition to conceptual consistency, the social accounting matrix, which provides the GE models database, ensures consistency. A suggestion that expenditure cannot exceed incomes and consistent factor allocation makes sure market clears.

Another benefit GE model is the analysis of multi-sector backward and forward linkages. Hence, they permit analysis of resource allocation and how policy impacts or permeate through the various sector of the economy. A major strength here is that it allows for welfare analysis, particularly in a gain or loss situation which helps for compensatory and economic reforms.
Admittedly, the general equilibrium approach has, of course, some setbacks which reduce its interest and limit its applicability to carefully chosen area or issues. The most frequently mentioned drawback is the lack of empirical validation of the model, in the sense that usually there is no measure of the degree to which the model fits the data or track historical facts. More worrisome is the fact that the model is usually large, including a substantial number of parameters, and often embedded rather complex structure, making it difficult to estimate econometrically. Such, therefore suggests that the model does not pretend to make any reasonable forecast as it's far from reality.

Perhaps more fundamental is the fact that the general equilibrium assumption rules out the use of those models for important policy decisions. It is assumed that all markets must clear and that nothing happens until equilibrium is reached. The GE approach is not very relevant to discuss macroeconomic issues related to stabilization policy. The approach may provide insight in a case where the market does not clear, but cannot solve unemployment and deficit budget.

There also is a reference to the case of data issues. Researchers in the use of general equilibrium modelling as an analytical framework are often faced with problems of the unreliability of data, model bias from data for the selected year, and limited model structure. Uncertainty of data is a very big problem with general equilibrium models, especially because the parameters' value is very important for later determination of results from different simulations. Here, the main problem is that most of the data is derived from databases, but other data are derived from external sources (mostly data on the elasticity of substitution). The biggest problem for those data is they are estimated under various assumptions. This explains precisely the problem of unreliability of such data (Skare and Stjepanovic, 2013).
In the same vein on the issue of data, another drawback is related to the quality of the data selected that directly affects the quality of the whole model. In econometric models – which typically lack sufficient structure for complex policy analysis, stochastic distribution tries to reduce errors in measuring endogenous and exogenous variables in the model. However, in the calibration process, it is assumed that stochastic distribution is zero, which leads to that calibration parameters must absorb all the errors occurring in the data for the selected base year. Furthermore, a social accounting matrix – a less suitable, limited alternative analytical framework, and similar to the input-output framework - is not always in equilibrium, or the sum of rows is not equal to the total sum of the column, which also leads to some error that occurs in the process of bringing matrix into balance (Partridge and Rickman, 2010; Skare and Stjepanovic, 2013).

Lastly, the limited applicability of general equilibrium modelling to developing economies due to their peculiarities is also of concern. In the literature, the limited use of general equilibrium models for analysis in small economies (in general terms) is attributable to their formulation, implementation, and description when typically patterned after those models used in large (or developed) economies. Thence, GE models applied in small (or developing) economies are more often than not non-consistent with the small economy's location theory as well as non-reflective of the dynamics of the developing economy's adjustment processes (to policy shocks), i.e. not completely representative of the economy of application's economic settings and be such with the absence of a time element, which likely causes inaccurate policy assessments (Partridge and Rickman, 2010). Wendner (1999) had argued that an applied model is useful only to the extent that its structure is appropriate to study the problem in question.
4.0 General Equilibrium Modelling: The State of the Art

Extensive research is underway to overcome the weaknesses noticed in the general equilibrium modelling and enhance its application's further extension to reflect current economic reality. Though, it has been argued that present research gives considerable effort to a physical asset, and no direction on applying the general equilibrium model to a financial asset, financial intermediation, and portfolio choice.

Perhaps, the most substantial efforts thus far are concentrated on developing sophisticated sub-models of asset markets within the general equilibrium framework. In the first versions of these models, the only asset is considered as physical capital. No financial asset was taken into account, no financial intermediation existed, and therefore there was no modelling of portfolio choice.

However, some models developed had included a stronger emphasis on asset markets along two lines: either dealing with financial intermediation issue or incorporating some costs of adjustment in the analysis of certain policy decisions. The possibility of incorporating financial asset and financial intermediation in the general equilibrium model represent substantial progress since this permits analysis under the more realistic condition when some agents will like to borrow and lend. This, of course, particularly is relevant to government deficit, and the way deficit is financed. It is relevant to the issue of development aid, and the financial crisis of developing economies (on the first issue, see Borges (1984); on the second, see Chichilnisky, Heal and McLeod (1983)).

Still, the existence of different financial assets with different return rates and the modelling of portfolio choice only makes sense in the context of uncertainty, that is if the difference among these assets and their rate of return is related to the degree of risk associated with each of them. Thus, introducing a systematic way of financial assets in the general equilibrium models involves
a major new step in modelling. Such requires that the first generation of the model's deterministic simplicity be abandoned in favour of a much more complex specification, including a substantial stochastic element. Significant research efforts are underway in this direction, but no major results have been reported thus far apart from steps made.

The second motivation for consideration of asset markets is related to capturing the adjusted cost related to some policy changes through asset prices. In this area, some results have been presented dealing with the consequence of tax policy (Summer, 1982, 1983). It is argued that as tax change, the rate of return to a different factor of production may be modified. Still, eventually, in the long run, the mobility of factors will equalize all rate of return, and the effect of the policy will lead to a new allocation of resources. During the transition, however, the change in return rate translated into a change in asset prices, which describes the path towards the new long run, and allocates the cost-benefit of changes. This approach combines the capital asset principles with general equilibrium assumptions, leading to new tax policy studies that are very promising since it focuses on issues close to policymakers' concern.

A second research area that attempts to improve the dynamic aspect of general equilibrium models is research efforts devoted to a better specification of the production sector to deal with the problem related to capital mobility (Fullerton, 1983). In energy policy, it is clear that the main consequence of policy changes is the impact that the policy may have on the economic viability of a certain type of physical capital, which may be economically obsolete. This issue thereof can only be studied, if specific vintages of capital are defined, with limited or alternative use once the capital is installed.
Finally, another important area of research is an attempt to model consistently market structure, representing departures from the competitive standard to analyze many important issues for which this feature is relevant. The tradition of general equilibrium models and general equilibrium theory is based on the paradigm of a perfectly competitive economy, with prices determined by market forces and agents' economic behaviour as price takers. Industries are models without economies of scale and product differentiation. For many applications of this model, the assumption of a perfectly competitive market is acceptable. In contrast, some studies have incorporated economic of scale and product differentiation to enhance a better explanation of trade and trade flow.

5.0 Conclusion

In all, the general equilibrium models' power and potential are recognized and well understood today. Limitations are clear, although steady improvements in the model can better improve its application. The inclusion of the financial market in the modelling of the general equilibrium permits the study of financial policy. The breakthrough in resolving complex issues relating to the market structure would permit us to include economies of scale, and product differentiation in our modelling. Lastly, efforts related to capital asset pricing modelling have helped us to resolve various criticisms in a financial market study.
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