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# Inventory Holding and a Mixed Duopoly with a Foreign Joint-Stock Firm

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## Abstract

This paper investigates a mixed duopoly model in which there is a state-owned firm competing with a foreign joint-stock firm. The following situation is considered. In the first period, each firm non-cooperatively decides how many it sells in the current market. In addition, each firm can hold inventories for the second-period market. By holding large inventories, a firm may be able to commit to large sales in the next period. In the second period, each firm non-cooperatively chooses its second-period output. At the end of the second period, each firm sells its first-period inventory stocked and its second-period output. The paper discusses the firms' reaction functions in the mixed duopoly model.

**Keywords:** Inventory holding, state-owned firm, foreign joint-stock firm, reaction curves

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## 1. Introduction

Rotemberg and Saloner (1989) examine a two-period model in which inventories are used by duopolists to deter deviations from an implicitly collusive arrangement, and establish that higher inventories allow duopolists to punish cheaters more strongly and can thus help to maintain collusion. Matsumura (1999) considers a Cournot duopoly model with finitely repeated competition, and establishes that two-period competition is insufficient to make private firms collusive. These studies investigate private duopoly models with inventories as a strategic device. Ohnishi (2011) considers a mixed duopoly model in which a welfare-maximizing public firm and a profit-maximizing private firm can use inventory holding as a strategic device, and demonstrates that the equilibrium coincides with the Stackelberg solution where the private firm is the leader.

The analysis of mixed oligopoly models that incorporate state-owned public firms has been performed by many researchers (e.g., see Delbono and Rossini, 1992; Nett, 1994; Willner, 1994; Fjell and Pal, 1996; George and La Manna, 1996; White, 1996; Mujumdar and Pal, 1998; Pal, 1998; Pal and White, 1998; Poyago-Theotoky, 1998; Nishimori and Ogawa, 2002; Bárcena-Ruiz and Garzón, 2003; Ohnishi, 2006, 2009; Bárcena-Ruiz, 2007; Fernández-Ruiz, 2009; Heywood and Ye, 2010; Wang and Lee, 2010; Pal and Saha, 2014). However, these studies consider mixed market models in which state-owned firms compete with capitalist or labor-managed firms, and do not include joint-stock firms.

Only a few studies consider joint-stock firms. For example, Meade (1972) examines the differences in incentives, short-run adjustment and so forth among labor-managed, joint-stock and capitalist firms. Hey (1981) restricts attention to the case of a perfectly competitive firm producing a homogeneous final good with inputs of capital and labor, and examines the behavior of labor-managed, joint-stock and capitalist firms. Ohnishi (2010b) presents the equilibrium solution of a quantity-setting model comprising a joint-stock firm and a capitalist firm, and shows that introducing lifetime employment into the model of quantity-setting duopoly is beneficial only for the joint-stock firm. In addition, Ohnishi (2015b) investigates a mixed duopoly model in which a joint-stock firm and a state-owned firm are allowed to offer lifetime employment as a strategic commitment, and presents the equilibrium solution of the mixed duopoly model.

We consider a two-period mixed market model in which a state-owned firm and a foreign joint-stock firm can hold inventories as a strategic device. The game runs as follows. In period one, each firm non-cooperatively decides how many it sells in the current market. In addition, each firm non-cooperatively decides the inventory level it holds for the second-period market. In period two, each firm non-cooperatively chooses its output. At the end of period two, each firm sells its first-period inventory stocked and its second-period output. This paper traces out the firms' reaction functions in the mixed duopoly model.

The remainder of this paper is organized as follows. In the second section, we describe the model. The third section characterizes best replies for firms in the model. The fourth section presents the results of this study. The final section concludes the paper.

## 2. Basic Setup

We consider a mixed duopoly model in which there is a domestic state-owned firm (firm D) competing with a foreign joint-stock firm (firm F). Throughout this paper, subscripts D and F refer to firms D and F, respectively, and superscripts 1 and 2 refer to periods 1 and 2, respectively. In addition, when  $i$  and  $j$  are used to refer to firms in an expression, they should be understood to denote D and F with  $i \neq j$ . The duopolists produce perfectly substitutable goods. The price of each period is determined by  $P(S^t)$ , where  $S^t = s_D^t + s_F^t$  is the aggregate sales of each period. We assume that  $P' < 0$  and  $P'' \leq 0$ . The two periods of the game are as follows. In the first period, each firm non-cooperatively and simultaneously decides its first-period production  $q_i^1 \in [0, \infty)$  and its first-period sales  $s_i^1 \in [0, q_i^1]$ . Firm  $i$ 's inventory level  $I_i^1$  becomes  $q_i^1 - s_i^1$ . In the second period, each firm non-cooperatively and simultaneously decides its second-period production  $q_i^2 \in [0, \infty)$ . At the end of the second period, each firm sells  $s_i^2 = I_i^1 + q_i^2$ . For simplicity, we consider the game with no discounting.

Since  $\sum_{t=1}^2 q_i^t = \sum_{t=1}^2 s_i^t$ , domestic economic welfare is

$$W = \sum_{t=1}^2 \left[ \int_0^{S^t} P(x) dx - m_D q_D^t - P(S^t) q_F^t \right] = \sum_{t=1}^2 \left[ \int_0^{S^t} P(x) dx - m_D s_D^t - P(S^t) s_F^t \right] \quad (1)$$

We define

$$w^t \equiv \int_0^{S^t} P(x) dx - m_D s_D^t - P(S^t) s_F^t \quad (2)$$

where  $m_D \in (0, \infty)$  represents firm D's constant marginal cost of production. Firm D seeks to maximize (1). The demand and cost conditions that firms face remain unchanged over time.

In addition, since  $\sum_{t=1}^2 q_F^t = \sum_{t=1}^2 s_F^t$ , firm F's profit per capital is

$$\Phi_F = \sum_{t=1}^2 \left[ \frac{P(S^t) s_F^t - m_F q_F^t - f_F^t}{k_F(s_F^t)} \right] = \sum_{t=1}^2 \left[ \frac{P(S^t) s_F^t - m_F s_F^t - f_F^t}{k_F(s_F^t)} \right] \quad (3)$$

We define

$$\phi_F^t \equiv \frac{P(S^t) s_F^t - m_F s_F^t - f_F^t}{k_F(s_F^t)} \quad (4)$$

where  $m_F \in (0, \infty)$  represents firm F's constant marginal cost of production,  $f_F \in (0, \infty)$  is firm F's fixed cost, and  $k_F(s_F^t)$  is firm F's capital input function. Firm F aims to maximize (3).

We assume that  $k_F(s_F^t)$  is the function of  $s_F^t$  with  $k_F' > 0$  and  $k_F'' > 0$ . This assumption means that the marginal quantity of capital used is increasing.

We also assume that firm D is less efficient than firm F, i.e.  $m_D > m_F$ . This assumption is justified in Gunderson (1979) and Nett (1993, 1994), and is often used in literature studying mixed oligopoly markets (e.g., see George and La Manna, 1996; Mujumdar and Pal, 1998; Pal, 1998; Nishimori and Ogawa, 2002; Matsumura, 2003; Ohnishi, 2006, 2015a; Fernández-Ruiz, 2009). If  $m_D \leq m_F$ , then firm D chooses  $q_D^t$  and  $s_D^t$  such that price equals marginal cost of production. Therefore, firm F does not operate in the market, and firm D can act as a monopolist.

### 3. Supplementary Explanations

First, we derive firm D's reaction functions from (2). In period one, since there is no inventory holding, firm D's reaction function is defined by

$$R_D^1(s_D^1) = \arg \max_{\{s_D^1 \geq 0\}} \left[ \int_0^{S^1} P(x) dx - m_D s_D^1 - P(S^1) s_F^1 \right] \quad (5)$$

In period two, firm D's reaction function without inventory holding is defined by

$$R_D^2(s_D^2) = \arg \max_{\{s_D^2 \geq 0\}} \left[ \int_0^{S^2} P(x) dx - m_D s_D^2 - P(S^2) s_F^2 \right] \quad (6)$$

and therefore its best response is given by

$$\bar{R}_D^2(s_F^2) = \begin{cases} R_D^2(s_F^2) & \text{if } s_D^2 > I_D^1 \\ I_D^1 & \text{if } s_D^2 = I_D^1 \end{cases} \quad (7)$$

Firm D maximizes  $W$  with respect to  $s_D^t$ , given  $s_F^t$ . The equilibrium solution needs to satisfy the following conditions: If the inventory level is zero, the first-order condition for firm D is

$$P - m_D - P' s_F^t = 0 \quad (8)$$

and the second-order condition is

$$P' - P'' s_F^t < 0 \quad (9)$$

Therefore, we obtain

$$R_D^{t'}(s_F^t) = \frac{P'' s_F^t}{P' - P'' s_F^t} \quad (10)$$

In period one, firm D's reaction function is upward sloping. In period two, firm D's best response also slopes upward for  $s_D^2 > I_D^1$ . This indicates that firm D treats  $s_D^t$  as strategic complements. The concept of strategic complements is due to Bulow, Geanakoplos, and Klemperer (1985).

Next, we derive firm F's reaction functions from (4). In period one, since there is no

inventory holding, firm F's reaction function is defined by

$$R_F^1(s_D^1) = \arg \max_{\{s_F^1 \geq 0\}} \left[ \frac{P(S^1)s_F^1 - m_F s_F^1 - f_F^1}{k_F(s_F^1)} \right] \quad (11)$$

In period two, firm F's reaction function without inventory holding is defined by

$$R_F^2(s_D^2) = \arg \max_{\{s_F^2 \geq 0\}} \left[ \frac{P(S^2)s_F^2 - m_F s_F^2 - f_F^2}{k_F(s_F^2)} \right] \quad (12)$$

and therefore its best response is given by

$$\bar{R}_F^2(s_D^2) = \begin{cases} R_F^2(s_D^2) & \text{if } s_F^2 > I_F^1 \\ I_F^1 & \text{if } s_F^2 = I_F^1 \end{cases} \quad (13)$$

Firm F maximizes  $\Phi_F$  with respect to  $s_F^t$ , given  $s_D^t$ . The first-order condition without inventory holding is

$$(P's_F^t + P - m_F)k_F - (Ps_F^t - m_F s_F^t - f_F^t)k_F' = 0 \quad (14)$$

and the second-order condition is

$$(P''s_F^t + 2P')k_F - (Ps_F^t - m_F s_F^t - f_F^t)k_F'' < 0 \quad (15)$$

In addition, we obtain

$$R_F^t'(s_D^t) = - \frac{P''s_F^t k_F + P'(k_F - s_F^t k_F')}{(P''s_F^t + 2P')k_F - (Ps_F^t - m_F s_F^t - f_F^t)k_F''} \quad (16)$$

Since  $k_F'' > 0$ ,  $k_F - s_F^t k_F' < 0$ , and hence  $P''s_F^t k_F + P'(k_F - s_F^t k_F')$  is positive. This means that firm F treats  $s_F^t$  as strategic complements.

#### 4. Reaction Curves

In this section, we draw the firms' reaction curves in the model described in section 2. There is no inventory holding in period one, and firm D's and firm F's reaction curves in period two can be determined by the level of  $I_i^1$ . Hence, we consider the second period. In the remainder of this paper, we delete the superscript 2 for brevity's sake.

We illustrate both firms' reaction curves by using figures 1-8. We consider the following three cases.

Case 1: Only firm D uses inventory holding.

Case 2: Only firm F uses inventory holding.

Case 3: Both firms use inventory holding.

We discuss these cases in orders.

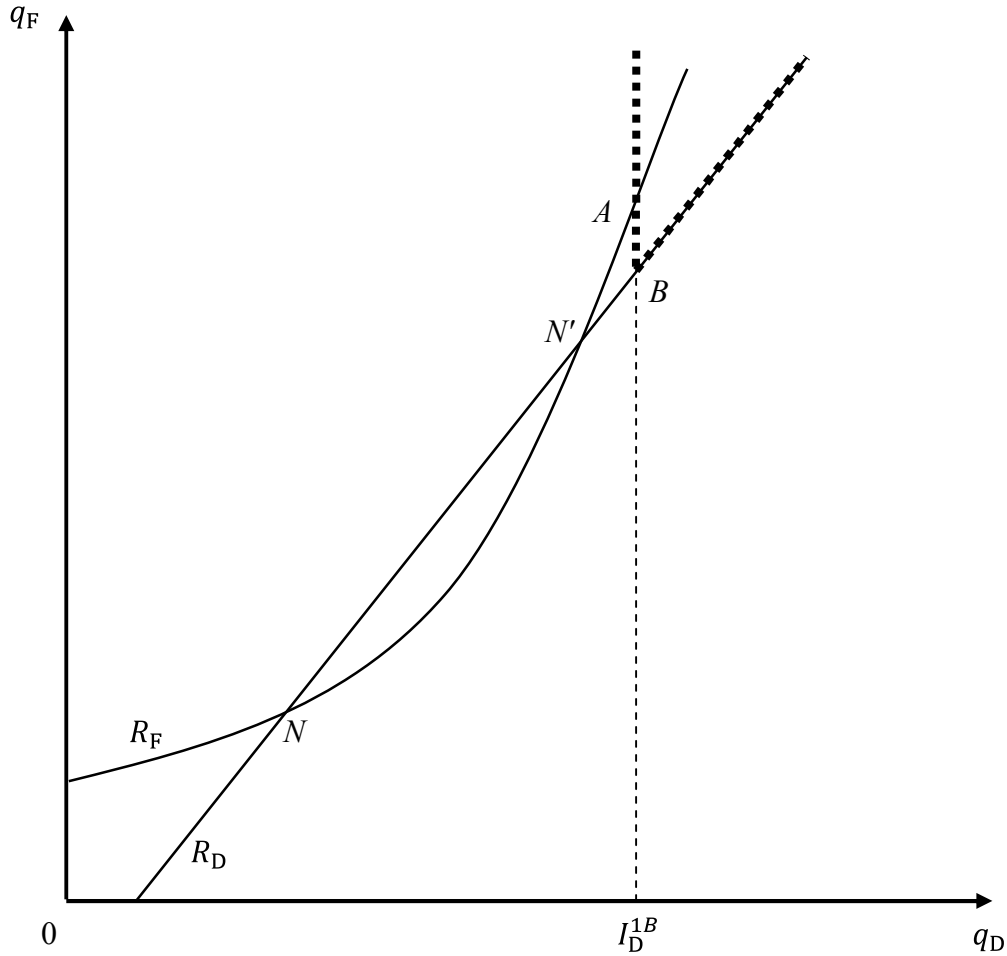


Figure 1: Firm D's best response is kinked at the level of  $I_D^{1B}$ .

### Case 1

This is the case in which only firm D uses inventory holding as a strategic commitment device, and is illustrated in figures 1-2. Here,  $R_i$  represents firm  $i$ 's second-period reaction curve with no inventory holding. Both firms' reaction curves are upward sloping. The equilibrium solution is determined in Cournot fashion, i.e., the intersection of firm D's and firm F's second-period reaction curves gives us the equilibrium of the game.  $R_D$  and  $R_F$  cross twice as depicted in figure 1. Only point  $N$  is a stable Cournot equilibrium, since in point  $N'$ ,  $R_F$  crosses  $R_D$  from above.

We first consider the change of firm D's best response curve, which is drawn in figure 1. We suppose that firm D maintains the inventory level of  $I_D^{1B}$  in period two. By holding inventories, firm D's best response changes to (7). Firm D's inventory holding creates a kink in its reaction curve at the level of  $I_D^{1B}$ . Therefore, firm D's reaction curve becomes the kinked bold broken lines. From figure 2, we see that the inventory level of  $I_D^{1B}$  changes the solution of the game. The intersection of the reaction curves is the equilibrium solution in period two. That is, if firm D holds  $I_D^{1B}$ , the solution occurs at  $A$ .

Domestic economic welfare is higher at  $A$  than at  $N'$ , and  $A$  is a stable solution.

Next, we examine the situation drawn in figure 2. If firm D maintains the inventory level of  $I_D^{1D}$  in period two, its inventory holding creates a kink in its reaction curve at the level of  $I_D^{1D}$ . In this figure, the reaction curves cross at two points. Both  $C$  and  $N$  are stable solutions. We see that domestic economic welfare is higher at  $N$  than at  $C$ .

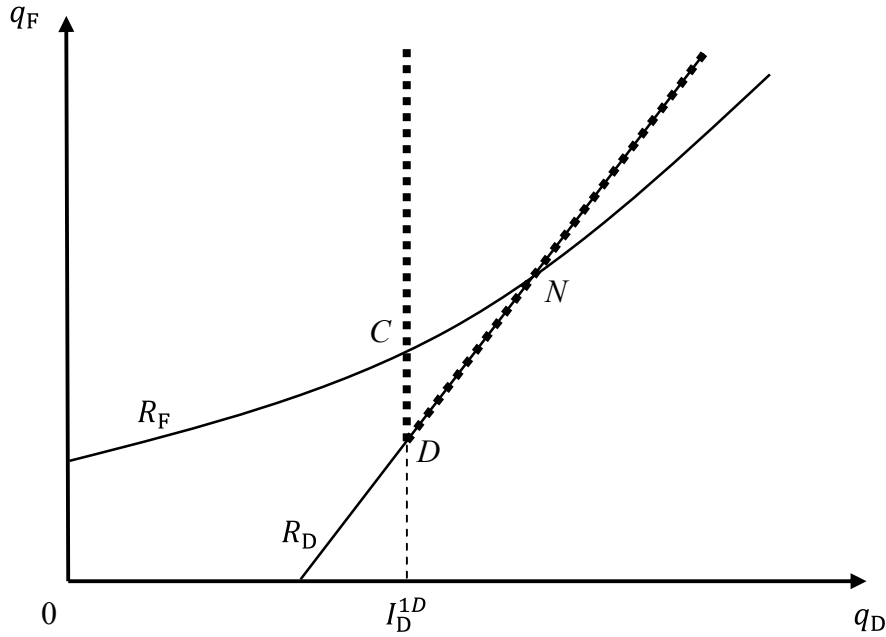


Figure 2: There are two stable solutions.

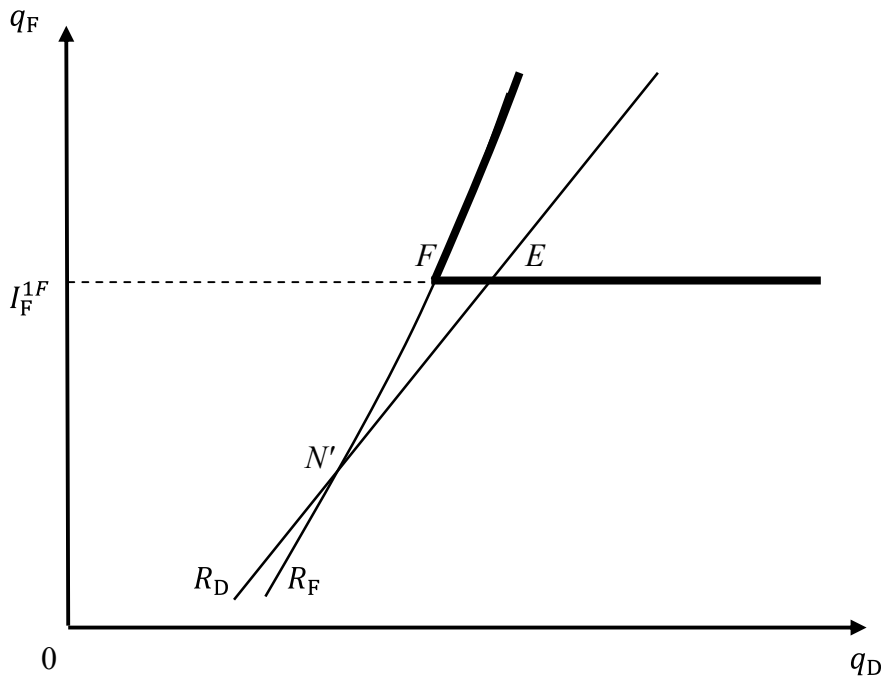


Figure 3: Firm F's best response is kinked at the level of  $I_F^{1E}$ .



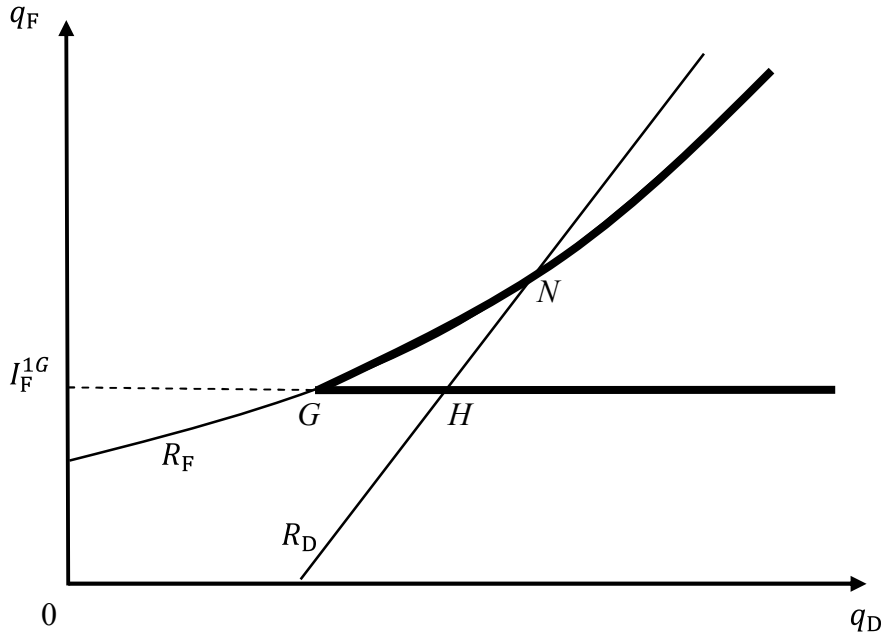


Figure 4: There are two stable solutions.

### Case 2

This case is illustrated in figures 3-4. We first consider the situation in figure 3. We suppose that the firm F holds  $I_F^{1F}$  in the second period. By holding inventories, firm F's best response changes to (13). Firm F's inventory holding creates a kink in its reaction curve at the level of  $I_F^{1F}$ . Therefore, firm F's reaction curve becomes the kinked bold lines. From figure 3, we see that the inventory level of  $I_F^{1F}$  changes the solution of the game. If firm F holds  $I_F^{1F}$ , then the solution is at  $E$ . However, we see that firm F's profit per capital is lower at  $E$  than at  $N'$ .

Next, we examine the situation drawn in figure 4. If firm F holds  $I_F^{1G}$  in period two, then its inventory holding creates a kink in its reaction curve at the level of  $I_F^{1G}$ . The intersection of the reaction curves is the equilibrium solution in period two. That is, if firm F maintains the inventory level of  $I_F^{1G}$ , then the best response curves cross at two points as in this figure.

### Case 3

This case is illustrated in figures 5-8. In this case, both firms use inventory holding as a strategic commitment device. First, we consider the situation in figure 5. We suppose that firm D maintains the inventory level of  $I_D^{1M}$  in period two. By holding inventories, firm D's best response changes to (7). Therefore, firm D's reaction curve becomes the kinked bold broken lines. In addition, we suppose that firm F holds  $I_F^{1J}$  in period two. By holding inventories, firm F's best response changes to (13). Therefore, firm F's reaction curve becomes the kinked bold lines. The solution is determined in Cournot fashion. From figure 5, we see that inventory holding by each firm changes the solution of the game. Figure 5 says that there are three stable solutions.

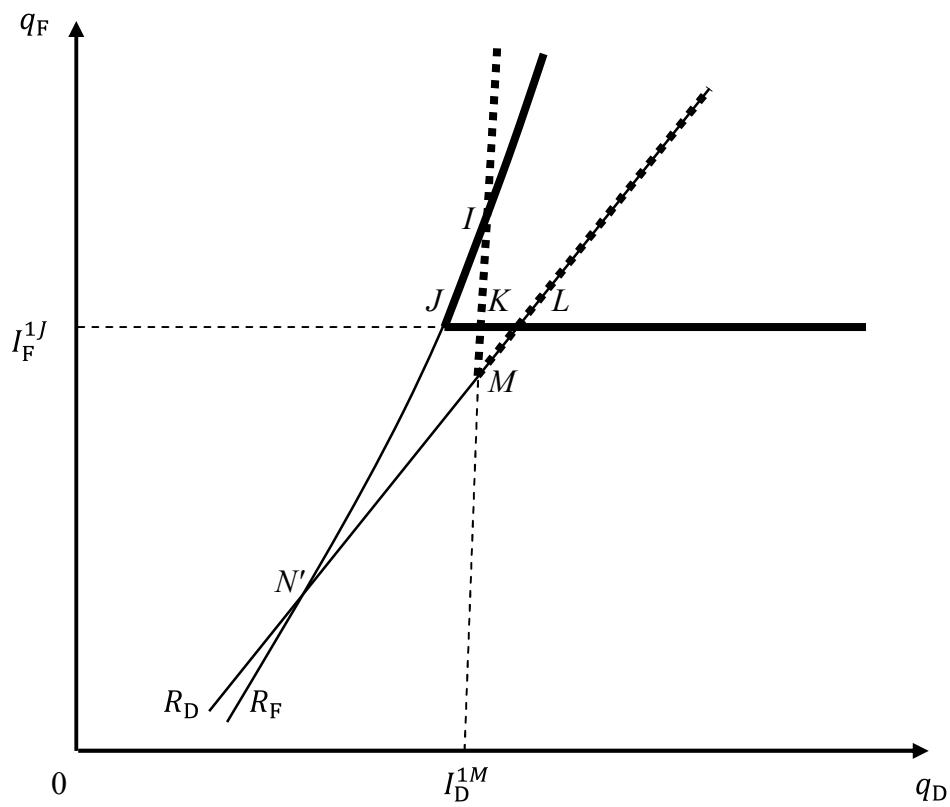


Figure 5: There are three stable solutions.

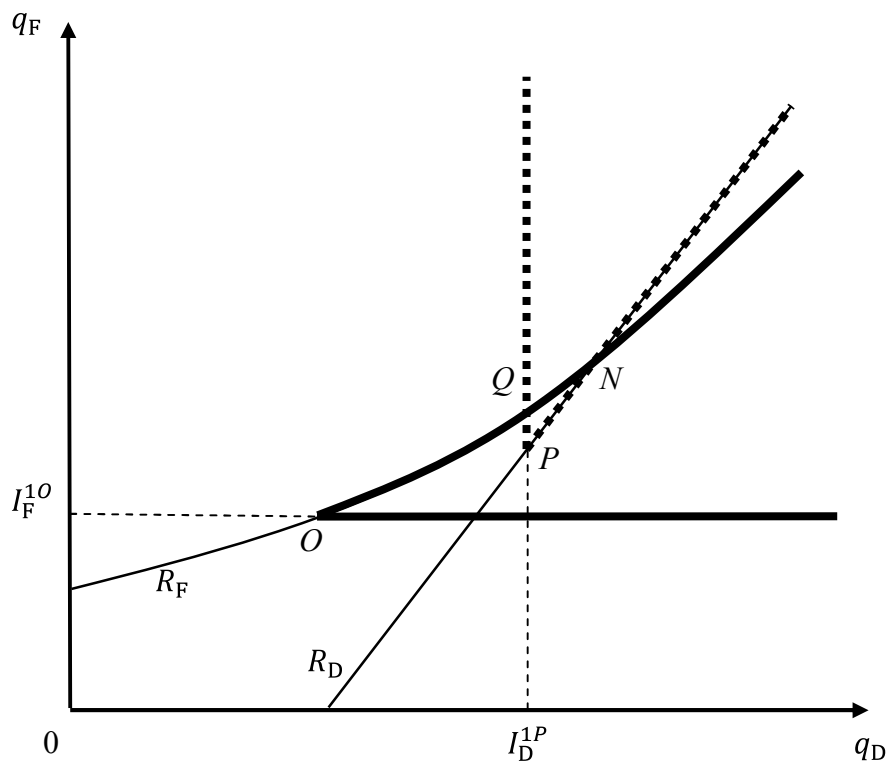


Figure 6: Both  $N$  and  $Q$  are stable solutions.

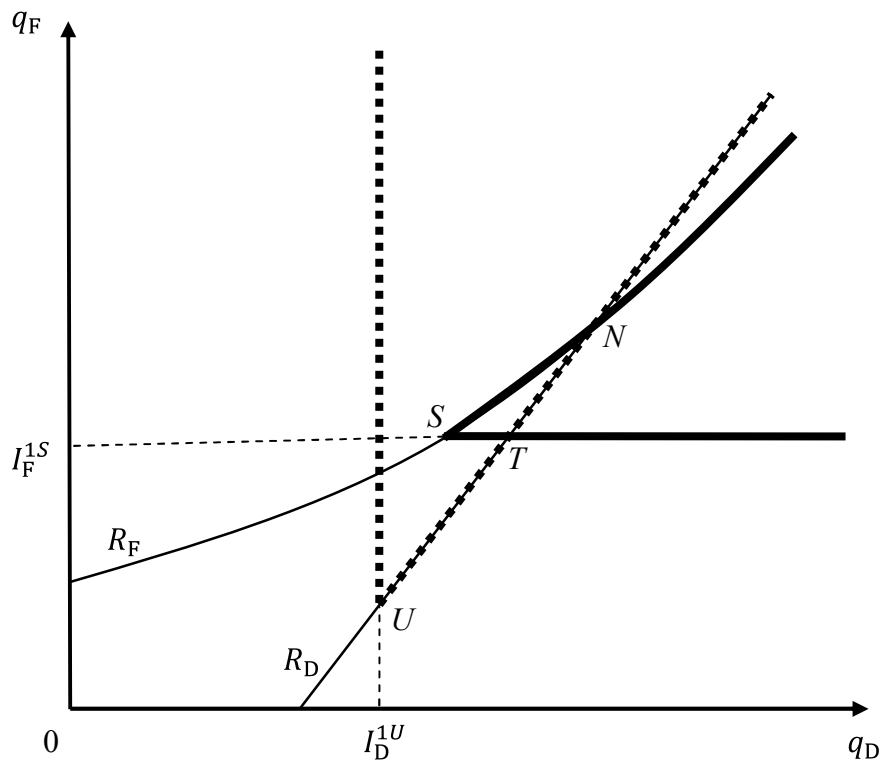


Figure 7: Both  $N$  and  $T$  are stable solutions.

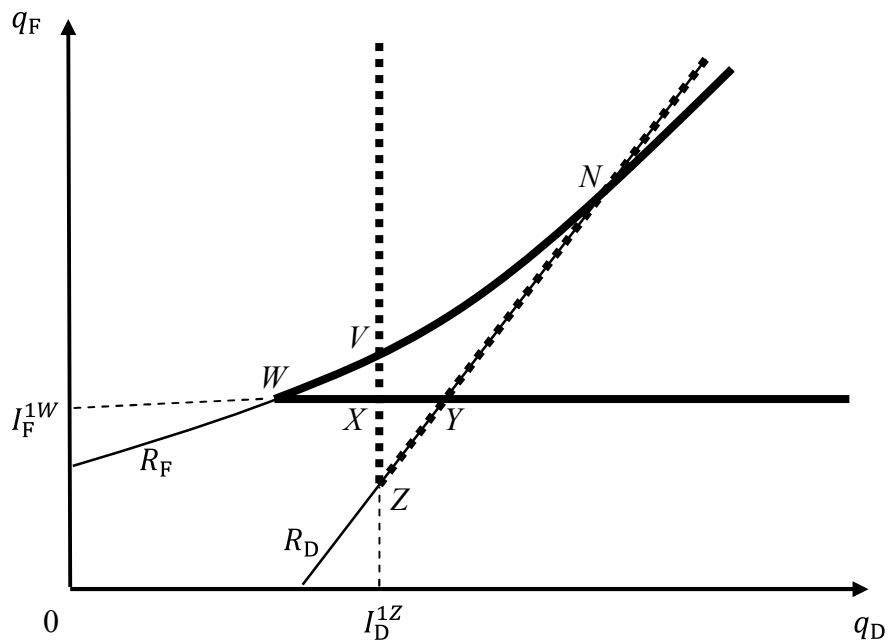


Figure 8: There are four stable solutions.

Secondly, we discuss the situation in figure 6. If firm D maintains the inventory level of  $I_D^{1P}$ , then its quantity best response curve is kinked at the level of  $I_D^{1P}$ . In addition, inventory holding by firm F kinks its quantity best response curve. Therefore, firm D's best response is depicted as the thick broken lines, while firm F's best response is

the thick lines. The firms' best response curves cross twice as in figure 6. It is obvious that both  $N$  and  $Q$  are stable.

Thirdly, we examine the situation drawn in figure 7. If firms D and F hold  $I_D^{1U}$  and  $I_F^{1S}$  respectively, then firm D's best response is depicted as the thick broken lines, and firm F's best response curve is the thick lines. In this figure, the firms' best response curves cross at two points. It is obvious that both  $N$  and  $T$  are stable solutions.

Fourthly, we consider the case drawn in figure 8. Firm D's best response is depicted as the thick broken lines, while firm F's best response curve is the thick lines. The firms' best response curves cross four times as in figure 8. We see that all these points are stable solutions.

## 5. Conclusion

We have considered a two-period mixed duopoly model in which there is a state-owned firm competing with a foreign joint-stock firm. Each firm is allowed to hold inventories as a strategic device. As a result, we have shown that there may be multiple stable Cournot solutions in the international mixed duopoly model. In the near future, we will extend our analysis by considering a mixed oligopoly model where a state-owned firm competes with both domestic and foreign joint-stock firms.

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