Automation Technology, Economic Growth, and Income Distribution in an Economy with Dynasties and Overlapping Generations

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Abstract

This study presents a growth model with automation technology that considers two classes (workers and capitalists) who conduct dynamic optimization in different manners. In addition to two production factors, labor and traditional capital, automation capital is included as the third production factor. Long-run dynamics of input ratios of production factors, income distribution, and per capita output growth are investigated. Regardless of the size of workers' discount factor, workers' own traditional capital has no transitional dynamics and stays constant. When capitalists' discount factor is large, in the long run, the growth rate of per capita output is positive and constant: endogenous growth is obtained. In this case, income gap between workers and capitalists continues to increase through time. When capitalists discount factor is small, two different cases appear. First, when the initial value of traditional capital is large, both capitalists' own traditional capital and automation capital converges to constant values. In this case, income gap between workers and capitalists converges to a constant value. Second, when the initial value of traditional capital is small, capitalists' own traditional capital converges to a constant value while capitalists' own automation capital approaches zero. In this case, income gap between workers and capitalists converges to a constant value. When automation capital becomes zero, after then, the dynamical system switches to a dynamical system without automation capital.

Keywords: automation technology; endogenous growth; income distribution

JEL Classification: E25; O11; O33; O41

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1 Introduction

This study builds a growth model with automation capital (automation technology), and investigates the effect of accumulation of automation capital on economic growth and income distribution. How do advances in automation technology affect our economy?

Automation technology such as artificial intelligence (AI, hereafter) and robots are now widely utilized in our economy. Automation technology are generally thought to substitute labor. Frey and Osborne (2013) point out the possibility that AI and robots drastically substitute human labor. They investigate the effect of automation technology on the U.S. labor markets, and find that about 47% of labor will be substituted in the future. McKinsey Global Institute (2017) analyzes more than 800 jobs and more than 2000 activities in the United States, and finds that about 45% of labor activities can be potentially substituted by AI. Boston Consulting Group (2015) predicts that about 40–50% of jobs in the United States, the United Kingdom, Canada, and Japan will be substituted by AI and robots until 2025.

Recently, attempts to investigate the effect of automation technology on an economy from the viewpoint of macroeconomics have proliferated.[1] These attempts are roughly classified into the task base approach represented by Daron Acemoglu and the automation capital approach represented by Klaus Prettner.[2]

Acemoglu classifies the effects of AI and robots on macro economy into several effects (Acemoglu and Restrepo 2018, 2020). AI and robots substitute human labor and decrease labor demand and wage rate. At the same time, introduction of AI and robots creates new employment in the labor market, which increases labor demand and wage rates. Accordingly, introduction of AI and robots has a counterbalancing effect. Under some assumptions, positive effects dominate negative effects. Acemoglu reaches the conclusion that introduction of AI and robots cannot produce severe unemployment and wage declines.[3]

Prettner (2019) introduces a new production factor, "automation capital" that perfectly substitutes labor such as AI and robots, and differentiates it with "traditional capital" such as machines and factories. He builds an augmented Solow growth model with automation capital, which enters the Cobb–Douglas production function. He assumes that a representative household saves a constant fraction of income, a fraction of the saving is allocated to accumulation of automation capital, and the rest of the saving is allocated to accumulation of traditional capital. The results show that the wage rate and labor share of national income

[1] For analysis of mechanization in a growth model, see the study of Zeira (1998).

[2] For growth models with AI and robots, see also Aghion et. al. (2019).

[3] There are empirical studies that report the effects of introduction of labor substitutable technology on employment and wage rates (Graetz and Michaels 2018; Cords and Prettner 2019; Acemoglu and Restrepo 2020).

decrease with accumulation of automation capital. Moreover, accumulation of automation capital produces endogenous growth of per capita output even though there is no exogenous technological progress.

Heer and Irmen (2019) criticize the study of Prettner (2019). In the Prettner model, two kinds of assets appear—automation capital and traditional capital. However, he does not consider the no-arbitrage condition between the two assets. An economic agent will invest by considering returns of the two assets, and at the equilibrium, returns of the two assets will be equalized. Heer and Irmen introduce the no-arbitrage condition into the Prettner model, and then, show that there is a linear relationship between automation capital and traditional capital. With this linear relationship, the Cobb–Douglas production function leads to the AK production function, which produces endogenous growth as long as traditional capital is accumulated. In Prettner (2019), investment allocation between automation capital and traditional capital is given exogenously. However, they show that due to the no-arbitrage condition, this investment allocation is endogenously determined.

Gasteiger and Prettner (2020) build an overlapping generations model with automation capital, and show that endogenous growth cannot be obtained and an economy becomes stagnant in the long run. In the overlapping generations model, income of working generations is wage income, which decreases through accumulation of automation capital. The decrease in wage income decreases saving of households, which decreases accumulation of traditional capital, leading to the stagnation of the economy.[4]

The present study is an attempt to investigate how advances in automation technology that substitutes labor affect an economy. Based on Prettner (2019), we provide a growth model that includes automation capital as a factor of production.

The main contribution of this study is to investigate the effect of advances in automation technology on income distribution. For this issue, the existing studies with automation capital use a representative household model. Prettner (2019) uses a Solow-type saving function such that a representative household saves a constant fraction of income. In contrast, Gasteiger and Prettner (2020) assume that a representative household solves a two-period overlapping generations model. Both studies assume that a household obtains both labor income and capital income, and hence, they cannot examine the effect of accumulation of automation capital on income distribution and income gap because a change in income distribution is a change in income distribution in a household and not a change between different households.

For this reason, to investigate the effect of accumulation of automation capital on income

[4]Mechanization can decrease wage rates, which consequently stagnates an economy and decreases the social welfare of future generations. For this issue, see Benzell et al.(2015) and Sachs *et al.*(2015).

distribution between households, we introduce two classes, workers and capitalists who show different saving behaviors. To our knowledge, this study is a first attempt that considers automation technology.

For saving behaviors of workers and capitalists, the debate between Pasinetti (1962) and Modigliani and Samuelson (1996) is widely known. Pasinetti (1962) argues that if workers save, workers obtain interest income by holding capital through savings. Accordingly, the total capital of the whole economy is composed of workers' own capital and capitalists' own capital. In addition, he reveals that at the steady state where workers and capitalists coexist (i.e., the Pasinetti steady state), the profit rate (rate of return of capital) is given by the natural growth rate divided by capitalists' saving rate, which is called the Pasinetti theorem.

On the contrary, Samuelson and Modigliani (1966) reveal that the derivation of the Pasinetti theorem critically hinges on the assumption that the capitalists' propensity to save is much higher than the workers' propensity to save. Then, they show that unless the assumption is satisfied, a Dual steady state is obtained.

Almost all previous studies assume that both the workers' propensity to save and capitalists' propensity to save are constant over time: both classes are agents that do not make future consumption plans.

In contrast, this study assumes that workers and capitalists are rational agents that make future consumption plans given the lifetime budget constraints. Specifically, workers solve a two-period overlapping generations (OLG) problem while capitalists solve an infinite-horizon Ramsey problem. Such an attempt was also made by Michl and Foley (2004).[5] They build a growth model in which workers solve a two-period OLG model, and capitalists solve an infinite-horizon dynamic optimization model. They use a fixed-coefficient Leontief production function and the real wage rate that is exogenously given according to the Classical economics assumption: the real wage rate is not determined in order to clear the labor market, but is institutionally determined.

From our analysis, we obtain the following results. (1) Regardless of the size of workers' discount factor, workers' own traditional capital has no transitional dynamics and stays constant. (2) When capitalists' discount factor is large, in the long run, the growth rate of per capita output is positive and constant: endogenous growth is obtained. In this case, income gap between workers and capitalists continues to increase through time. (3) When capitalists discount factor is small and the initial value of traditional capital is large, both capitalists'

[5] According to Caggetti and De Nardi (2008), the mixture of infinitely lived agents and OLG leads to a better performance in the empirical research than the case in which all agents belong to the infinitely lived agents type. For studies that extend Michl and Foley (2004), see Commendatore and Palmisani (2009), Sasaki (2018), and Kurose (2020). Mankiw (2000) also presents a hybrid model such that savers (i.e., capitalist) and spenders (i.e., workers) solve different optimization problems,

own traditional capital and automation capital converges to a constant value. In this case, income gap between workers and capitalists converges to a constant value. (4) When capitalists discount factor is small and the initial value of traditional capital is small, capitalists' own traditional capital converges to a constant value while capitalists' own automation capital approaches zero. In this case, income gap between workers and capitalists converges to a constant value. When automation capital becomes zero, after then, the dynamical system switches to a dynamical system without automation capital.[6]

The remainder of the paper is organized as follows. Section 2 presents our model. Section 3 investigates the dynamics. Section 4 compares the results with those of existing studies. Section 5 concludes the paper.

2 Model

This section presents building brocks of our growth model. First, we provide firms' behavior and the no-arbitrage condition. Second, we provide a capitalists' dynamic optimization problem. Third, we provide a workers' dynamic optimization problem. Fourth, we obtain our dynamical system.

2.1 Firms and no-arbitrage condition

Firms produce a good available for both consumption and investment by using labor, traditional capital, and automation capital. According to Prettner (2019), suppose that the production function takes the following modified Cobb–Douglas form:

$$Y = F(K, L, P) = K^{\alpha}(L + P)^{1-\alpha}, \quad 0 < \alpha < 1, \tag{1}$$

where Y denotes output, K is traditional capital, L is labor, and P is automation capital. This production function has a remarkable characteristic that output is not zero even if $L = 0$ or $P = 0$. Labor is owned by workers and automation capital is owned by capitalists. Traditional capital is owned by both workers and capitalists, and hence, we have

$$K = K^w + K^c, \tag{2}$$

where K^w and K^c denote workers' own traditional capital and capitalists' own traditional capital, respectively.

[6]For the analysis of the dynamical system without automation capital, see Sasaki (2018).

Let w, R^k, and R^p denote the wage rate, the gross rental price of traditional capital, and the gross rental price of automation capital, respectively. Then, workers' income and capitalists' income are as follows:

$$\text{Workers' income} = wL + R^k K^w, \tag{3}$$

$$\text{Capitalists' income} = R^p P + R^k K^c. \tag{4}$$

From profit maximization, factor prices are equal to their marginal products.

$$w = (1-\alpha)\frac{Y}{L+P}, \tag{5}$$

$$R^k = \alpha\frac{Y}{K}, \tag{6}$$

$$R^p = (1-\alpha)\frac{Y}{L+P}. \tag{7}$$

Labor and automation capital are perfect substitutes, and hence, $w = R^p$ holds. An increase in P decreases w, that is, accumulation of automation capital decreases the wage rate of labor.

The production function does not satisfy the Inada conditions. Accordingly, the following relations hold.

$$\lim_{P\to 0} R^p = (1-\alpha)\left(\frac{K}{L}\right)^{\alpha}, \tag{8}$$

$$\lim_{K\to 0} R^k = \infty. \tag{9}$$

When $P \to 0$ and $K \to 0$, $R^p < R^k$ holds. In addition, R^k is decreasing in K while R^p is increasing in K. From these, we find that only after traditional capital K is sufficiently accumulated, automation capital begins to be accumulated.

Gasteiger and Prettner (2020) impose a no-arbitrage condition between two assets K and P. They state that at the equilibrium, the relation $R^k = R^p$ holds. From this we obtain

$$P = \left(\frac{1-\alpha}{\alpha}\right)K - L. \tag{10}$$

This no-arbitrage condition states that there is a linear relationship among the three production factors, P, K, and L. As stated above, accumulation of P starts after K is sufficiently accumulated. From this, we obtain

$$\Rightarrow P = \max\left\{0, \left(\frac{1-\alpha}{\alpha}\right)K - L\right\}. \tag{11}$$

Let $\bar{K}$ be $\bar{K} \equiv \frac{\alpha}{1-\alpha}L$. When $K > \bar{K}$, the accumulation of P starts. Therefore, when $0 < K < \bar{K}$, we have $P = 0$, and when $\bar{K} < K$, we have $P > 0$.

Substituting the no-arbitrage condition into the production function, we obtain

$$Y = \begin{cases} K^{\alpha}L^{1-\alpha} & \text{if } 0 < K < \bar{K} \\ BK & \text{if } \bar{K} \leq K, \ B \equiv \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha} \end{cases} \tag{12}$$

The production function takes the AK form if K is sufficiently large. Accordingly, if traditional capital exceeds its threshold value, perpetual output growth is possible even without exogenous technological progress as long as traditional capital stock is accumulated.

Substituting the no-arbitrage condition into the marginal products, we obtain

$$w = R^k = R^p = \alpha^{\alpha}(1-\alpha)^{1-\alpha} \equiv R. \tag{13}$$

Accordingly, all factor prices are equalized. The wage rate and the rental price of automation are equalized because labor and automation capital are perfect substitutes, and the rental price of automation capital and that of traditional capital are equalizes because of the no-arbitrage condition. For this reason, all factor prices are equalized. The gross rate of return R takes the minimum value $\frac{1}{2}$ when $\alpha = \frac{1}{2}$ while it takes the maximum value unity when $\alpha = 0$ and $\alpha = 1$.

2.2 Capitalists' dynamic optimization

We assume that workers and capitalists are rational agents that make future consumption plans given the life-time budget constraints. Workers solve a two-period over-lapping generations model. Capitalists solve an infinite-horizon Ramsey-Cass-Koopmans model. This kind of hybrid specification is based on the work of Michl and Foley (2004).

Capitalists solve the following infinite horizon dynamic optimization problem:

$$\max \, (1-\beta_c)\sum_{t=0}^{\infty}\beta_c^t \log C_t^c, \quad 0 < \beta_c < 1, \tag{14}$$

$$\text{s.t. } C_t^c + A_{t+1}^c \leq (1+r_t)A_t^c, \tag{15}$$

$$A_0^c, \ r_t : \text{given}, \tag{16}$$

$$\lim_{j\to\infty} \frac{A_{t+j}^c}{\prod_{s=0}^{j}(1+r_{t+j})} = 0, \tag{17}$$

where β_c denotes the discount factor of capitalist, C^c, consumption of capitalists, and equation (17), the transversality condition. $A_t^c = K_t^c + P_t$ denotes total assets of capitalists. The real rate of return of A_t^c is given by $r_t = R_r - \delta$. Note that A^c is composed of two kinds of assets but the rates of return are equalized by the no-arbitrage condition, and hence the common rate r_t is used. In addition, we assume that both capitals have the same depreciation rate.

From the first-order condition, we obtain the Euler equation of consumption as follows:

$$C_{t+1}^c = \beta_c(1 + r_{t+1})C_t^c. \tag{18}$$

From equations (15), (17), and (18), we obtain the following consumption function.

$$C_t^c = (1 - \beta_c)(1 + r_t)K_t^c. \tag{19}$$

From equations (15) and (19), we obtain the dynamic equation of A_t^c as follows:

$$A_{t+1}^c = \beta_c(1 + r_t)A_t^c. \tag{20}$$

2.3 Workers' dynamic optimization

Workers solve the following two-period optimization problem:

$$\max\ (1 - \beta_w)\log c_{1,t}^w + \beta_w \log c_{2,t+1}^w, \quad 0 < \beta_w < 1, \tag{21}$$

$$\text{s.t. } c_{1,t}^w + \frac{c_{2,t+1}^w}{1 + r_{t+1}} \leq w_t, \tag{22}$$

$$w_t,\ r_{t+1} : \text{given}, \tag{23}$$

where β_w denotes the discount factor of workers, c_1^w, workers' consumption in the young period, and c_2^w, workers' consumption in the old period. We assume that $\beta_w < \beta_c$.

From this, we obtain the following consumption and saving functions.

$$c_{1,t}^w = (1 - \beta_w)w_t, \tag{24}$$

$$s_t^w = \beta_w w_t, \tag{25}$$

where s^w denotes the workers' saving. Equation (25) shows that the workers' propensity to save is given by β_w.

Using these equations, the accumulation of workers' aggregate asset is given by

$$A_{t+1}^w = K_{t+1}^w = s_t^w L_t = \beta_w w_t L_t. \tag{26}$$

3 Analysis of dynamics

We obtain the dynamical system. Let $a_t^c = A_t^c/L_t$ and $a_t^w = A_t^w/L_t$. From equations (20) and (26), we obtain

$$a_{t+1}^c = \frac{\beta_c(1+R-\delta)}{1+n} a_t^c = \Theta a_t^c, \tag{27}$$

$$a_{t+1}^w = \frac{\beta_w R}{1+n}. \tag{28}$$

Note that $\Theta > 0$ since $\delta \in [0, 1]$.

In what follows, we call a^c as assets per capitalist. Suppose that the composition of ratio of workers and capitalists is kept constant. Let N_t and N_t^c denote the population and the number of capitalists at t, respectively. Then, N_t and N_t^c grow at the same rate n, and hence, L_t/N_t, N_t^c/N_t, and N_t^c/L_t are kept constant. Capitalists' asset is given by $a_t^c = \frac{N_t^c}{L_t}\frac{A_t^c}{N_c^t}$. Therefore, a_t^c can be regarded as assets per capitalist. In a similar way, we call $k^c = K^c/L$ and $p = P/L$ traditional capital per capitalist and automation capital per capitalist, respectively.

Accordingly, the dynamical equations of traditional capital per capitalist k^c and traditional capital worker k^w are given by

$$k_{t+1}^c + \left(\frac{1-\alpha}{\alpha}\right)(k_{t+1}^c + k_{t+1}^w) - 1 = \Theta\left[k_t^c + \left(\frac{1-\alpha}{\alpha}\right)(k_t^c + k_t^w) - 1\right], \tag{29}$$

$$k_{t+1}^w = k^w = \frac{\beta_w R}{1+n}, \tag{30}$$

$$\text{subject to } k_t^c > \frac{\alpha}{1-\alpha} - \frac{\beta_w R}{1+n} > 0. \tag{31}$$

Equation (31) is a rewritten form of $K > \bar{K}$.

Therefore, we obtain the first-order linear difference equation of k_t^c. The dynamical system can be rewritten as follows:

$$k_{t+1}^c = \Theta k_t^c + (\Theta - 1)[(1-\alpha)k^w - \alpha], \tag{32}$$

$$k_t^c > \frac{\alpha}{1-\alpha} - k^w. \tag{33}$$

Suppose that at $t = 0$, the no-arbitrage condition is satisfied.

The right-hand side of equation (32) is a straight whose slope is Θ and intercept is $(\Theta - 1)[(1-\alpha)k^w - \alpha]$. Depending on whether the slope is more than or less than unity and depending on whether the intercept is positive or negative, different dynamics can be obtained.

3.1 Restrictions on parameters according to four cases

We consider four cases according to the sizes of the parameters. The criteria in classifying the four cases are $\Theta \gtrless 1$ and $(\Theta-1)[(1-\alpha)k^w-\alpha]>0$. From this, we obtain the following relations:

$$\Theta \gtrless 1 \Longrightarrow \frac{\beta_c[1+\alpha^\alpha(1-\alpha)^{1-\alpha}-\delta]}{1+n} \gtrless 1, \tag{34}$$

$$k^w \gtrless \frac{\alpha}{1-\alpha} \Longrightarrow \frac{\beta_w \alpha^\alpha(1-\alpha)^{1-\alpha}}{1+n} \gtrless \frac{\alpha}{1-\alpha}. \tag{35}$$

These conditions can be rewritten as follows:

$$\beta_c \gtrless (1+n)\frac{1}{1+\alpha^\alpha(1-\alpha)^{1-\alpha}-\delta} \equiv \bar{\beta}_c(\alpha), \tag{36}$$

$$\beta_w \gtrless (1+n)\frac{\alpha}{1-\alpha}\frac{1}{\alpha^\alpha(1-\alpha)^{1-\alpha}} \equiv \bar{\beta}_w(\alpha). \tag{37}$$

Note that $\bar{\beta}_c$ and $\bar{\beta}_w$ are functions of α.

The relationship between $\bar{\beta}_c(\alpha)$ and $\bar{\beta}_w(\alpha)$ is given by Figure 1, which states $\bar{\beta}_w(\alpha) \gtrless \bar{\beta}_c(\alpha)$.

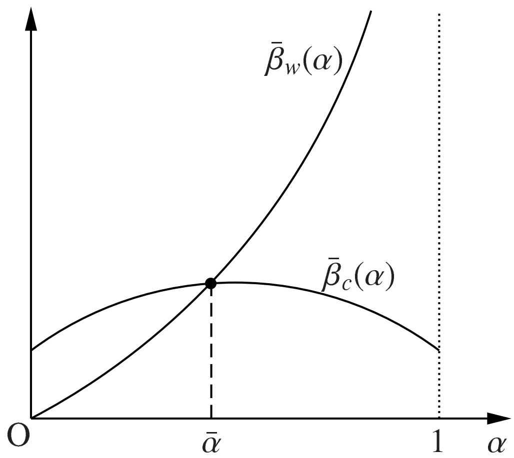

Figure 1: Relation between $\bar{\beta}_c(\alpha)$ and $\bar{\beta}_w(\alpha)$

Considering $\beta_w < \beta_c$, we can consider four cases as follows:

Case 1 : $\beta_w > \bar{\beta}_w$, $\beta_c > \bar{\beta}_c$, and $\bar{\beta}_w \lesseqgtr \bar{\beta}_c$.

Case 2 : $\beta_w > \bar{\beta}_w$, $\beta_c < \bar{\beta}_c$, and $\bar{\beta}_w < \bar{\beta}_c$.

Case 3 : $\beta_w < \bar{\beta}_w$, $\beta_c > \bar{\beta}_c$, and $\bar{\beta}_w \lesseqgtr \bar{\beta}_c$.

Case 4 : $\beta_w < \bar{\beta}_w, \beta_c < \bar{\beta}_c$, and $\bar{\beta}_w \lesseqgtr \bar{\beta}_c$.

The following Figures 2–9 are parameters restrictions on the (β_c, β_w) plane corresponding to Cases 1–4.

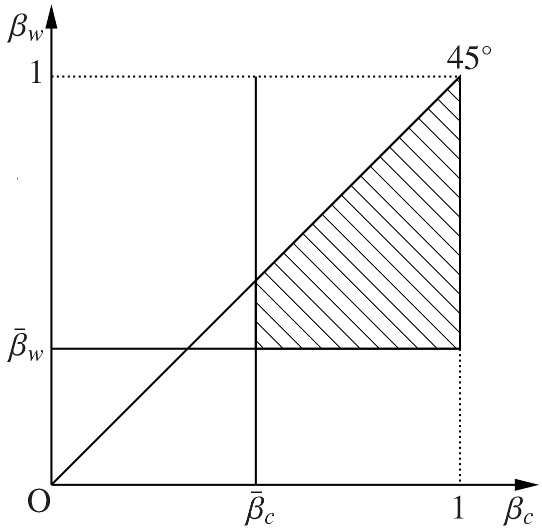

Figure 2: Case 1-1 ($\bar{\beta}_w < \bar{\beta}_c$)

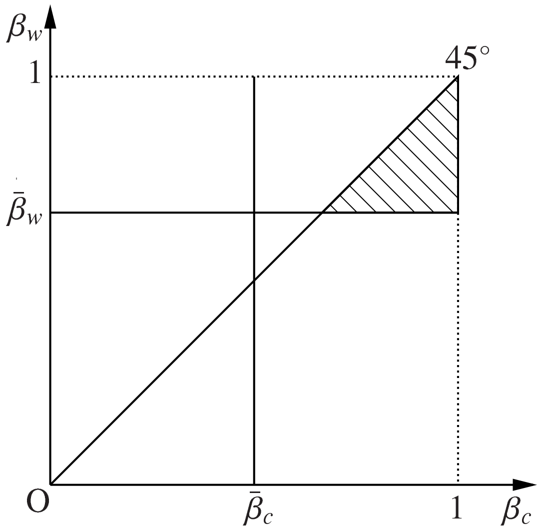

Figure 3: Case 1-2 ($\bar{\beta}_w > \bar{\beta}_c$)

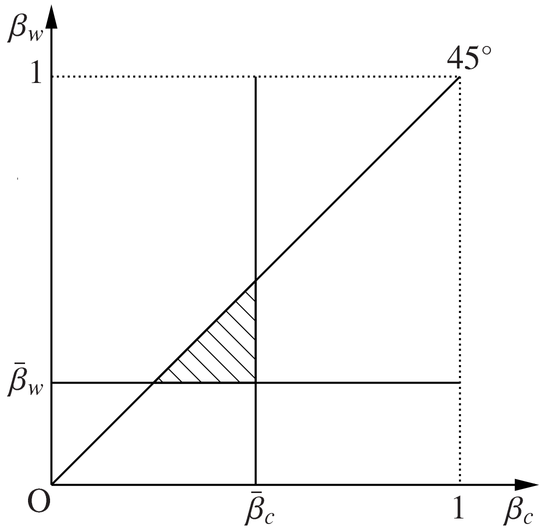

Figure 4: Case 2-1 ($\bar{\beta}_w < \bar{\beta}_c$)

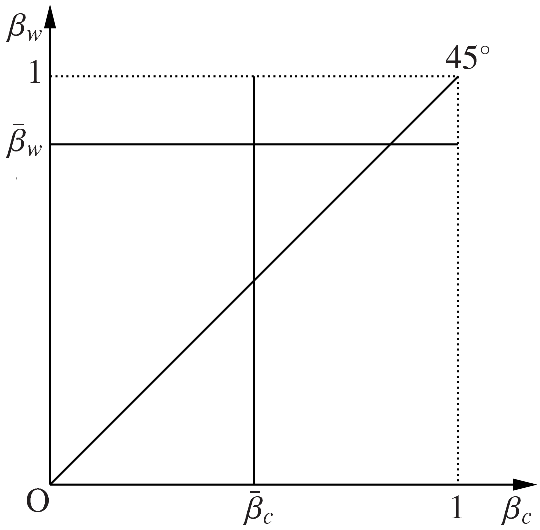

Figure 5: Case 2-2 ($\bar{\beta}_w > \bar{\beta}_c$: empty set)

3.2 Cases 1 and 2

When $k^w > \frac{\alpha}{1-\alpha}$, we classify $\Theta > 1$ and $\Theta < 1$.

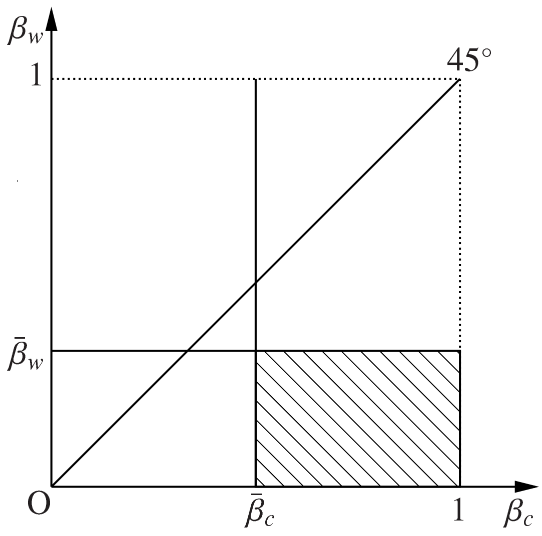

Figure 6: Case 3-1 ($\bar{\beta}_w < \bar{\beta}_c$)

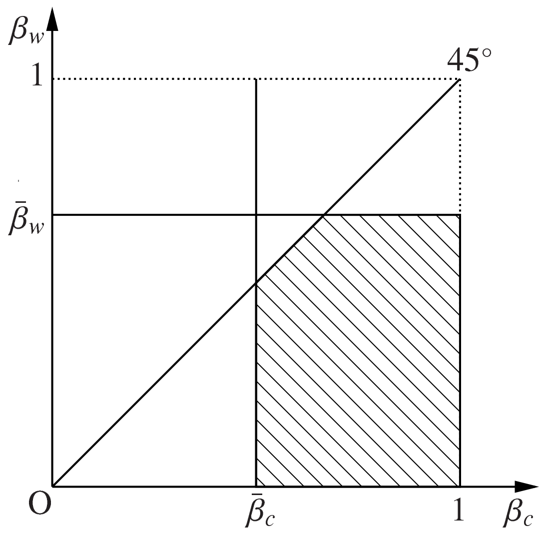

Figure 7: Case 3-2 ($\bar{\beta}_w > \bar{\beta}_c$)

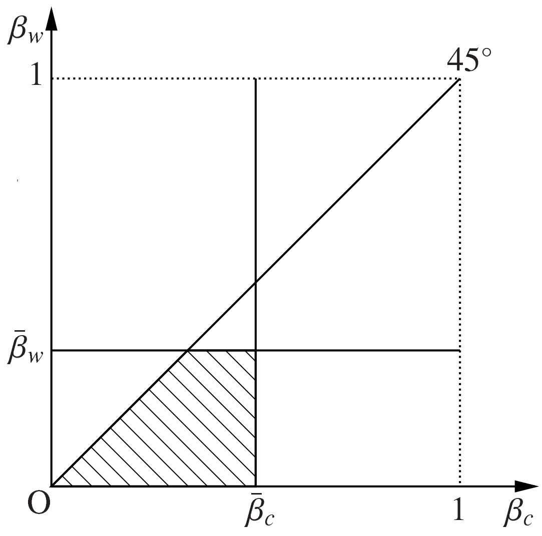

Figure 8: Case 4-1 ($\bar{\beta}_w < \bar{\beta}_c$)

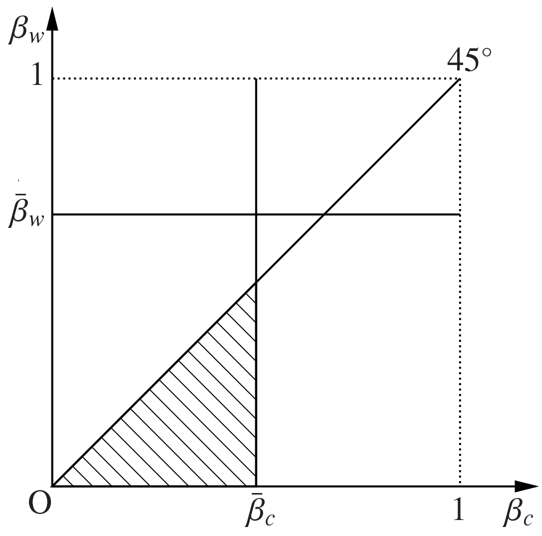

Figure 9: Case 4-2 ($\bar{\beta}_w > \bar{\beta}_c$)

When $\Theta > 1$ and the initial values of k_t^c and k_t^w are relatively large, k_t^w remains constant while k_t^c continues to increase through time, which is shown in Figure 10. In the long run, as stated below, the growth rate of per capita output is positive and constant. The no-arbitrage condition is satisfied through time.

When $\Theta < 1$ and the initial values of k_t^c and k_t^w are relatively large, k_t^w remains constant while k_t^c continues to decrease and approaches a minimum value $\underline{k}^c$, which is given by

$$\underline{k}^c = \frac{\Theta}{(1-\Theta)[(1-\alpha)k^w - \alpha]}. \tag{38}$$

This case is shown in Figure 11. In this case, in the long run, $k^w > 0$, $k^c > 0$, and $p > 0$. The growth rate of per capita output is zero. The no-arbitrage condition is satisfied through time.

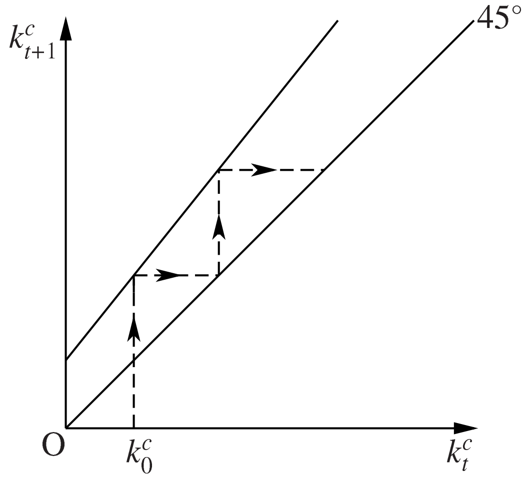

Figure 10: $k^w > \frac{\alpha}{1-\alpha}$ and $\Theta > 1$

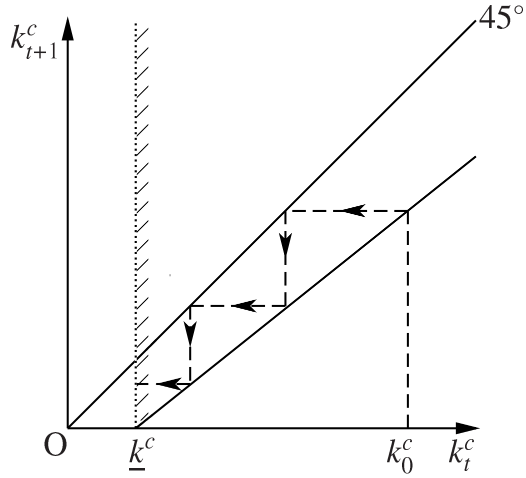

Figure 11: $k^w > \frac{\alpha}{1-\alpha}$ and $\Theta < 1$

3.3 Cases 3 and 4

When $k^w < \frac{\alpha}{1-\alpha}$, we classify $\Theta > 1$ and $\Theta < 1$.

When $\Theta > 1$ and the initial values of k_t^c and k_t^w are relatively small, k_t^w remains constant while k_t^c continues to increase, which is shown in Figure 12. In this case, the growth rate of per capita output is positive and constant. The no-arbitrage condition is satisfied through time.

When $\Theta < 1$ and the initial values of k_t^c and k_t^w are relatively small, k_t^w remains constant while k_t^c continues to decrease and approaches a lower bound value $\hat{k}^c$, which is given by

$$\hat{k}^c \equiv \frac{\alpha - (1-\alpha)k^w}{1-\alpha} > 0, \tag{39}$$

$$k^{c*} \equiv \alpha - (1-\alpha)k^w > 0, \tag{40}$$

$$\Longrightarrow \hat{k}^c > k^{c*}. \tag{41}$$

This case is shown on Figure 13. When k_t^c reaches $\hat{k}^c$, the no-arbitrage condition is violated, and the dynamical system switches to a system that does not include automation capital.

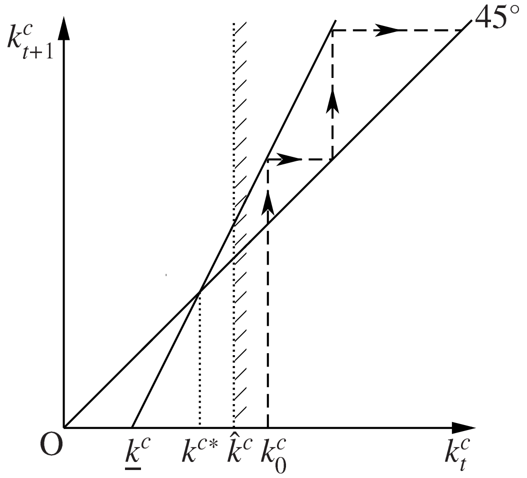

Figure 12: $k^w < \frac{\alpha}{1-\alpha}$ and $\Theta > 1$

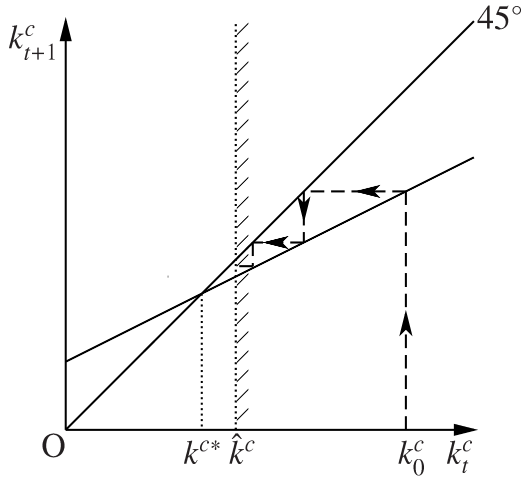

Figure 13: $k^w < \frac{\alpha}{1-\alpha}$ and $\Theta < 1$

Summarizing the analysis of Cases 1–4, we obtain the following results.

Case 1 : $\frac{k_{t+1}^c}{k_t^c} = \frac{p_{t+1}}{p_t} = \frac{a_{t+1}^c}{a_t^c} = \Theta > 1$ and $k^w = a^w = \text{const.}$

Case 2 : $k^c = \underline{k}^c = \text{const.}$, $p = \text{const.}$, $a^c = \text{const.}$, and $k^w = a^w = \text{const.}$

Case 3 : $\frac{k_{t+1}^c}{k_t^c} = \frac{p_{t+1}}{p_t} = \frac{a_{t+1}^c}{a_t^c} = \Theta > 1$ and $k^w = a^w = \text{const.}$

Case 4 : $k^c = \hat{k}^c = \text{const.}$, $p = 0$, $a^c = \text{const.}$, and $k^w = a^w = \text{const.}$

3.4 Ratios of variables and growth rate

To begin with, we investigate the long-run values of input ratios. Depending on k_t^c and hence a_t^c continue to increase or converges to constant values, we obtain different results as follows:

$$\frac{P}{K^c} = \frac{(1-\alpha)(a^c + a^w) - \alpha}{\alpha(1 + a^c + a^w) - a^w} = \begin{cases} \frac{1-\alpha}{\alpha} & \text{in Cases 1 and 3} \\ \text{constant in Cases 2 and 4} \end{cases} \tag{42}$$

$$\frac{P}{L} = (1-\alpha)(a^c + a^w) - \alpha = \begin{cases} +\infty & \text{in Cases 1 and 3} \\ \text{constant in Cases 2 and 4} \end{cases} \tag{43}$$

$$\frac{P}{K} = \frac{(1-\alpha)(a^c + a^w) - \alpha}{\alpha(1 + a^c + a^w)} = \begin{cases} \frac{1-\alpha}{\alpha} & \text{in Cases 1 and 3} \\ \text{constant in Cases 2 and 4} \end{cases} \tag{44}$$

$$\frac{K}{L} = \alpha(1 + a^c + a^w) = \begin{cases} +\infty & \text{in Cases 1 and 3} \\ \text{constant in Cases 2 and 4} \end{cases} \tag{45}$$

Therefore, the ratio of automation capital to traditional capital converges to a constant value.

Next, we investigate the long-run values of income distribution. In Cases 2 and 4, the above variables are constant in the long run. In Cases 1 and 3, k_t^c and hence a_t^c continue to increase through time, and we obtain the following results:

$$\frac{wL}{Y} = \frac{1}{1 + a^c + a^w} = 0, \tag{46}$$

$$\frac{R^k K}{Y} = \alpha, \tag{47}$$

$$\frac{R^p P}{Y} = 1 - \alpha - \frac{1}{1 + a^c + a^w} = 1 - \alpha, \tag{48}$$

$$\frac{R^k K^c + R^p P}{Y} = 1 - \frac{1 + a^w}{1 + a^c + a^w} = 1, \tag{49}$$

$$\frac{wL + R^k K^w}{Y} = \frac{1 + a^w}{1 + a^c + a^w} = 0. \tag{50}$$

Therefore, labor share of income approaches zero while traditional capita share and automation capital share approach constant values. In the long run, workers' income share approaches zero while capitalists' income share approaches unity.

The level of per capita output is given by

$$y_t \equiv \frac{Y_t}{L_t} = B(k_t^c + k^w). \tag{51}$$

When $\Theta < 1$, y becomes constant. On the contrary, when $\Theta > 1$, the growth rate of per capita output is given by

$$1 + g_y = \frac{y_{t+1}}{y_t} = \frac{k^c_{t+1} + k^w}{k^c_t + k^w} \approx \frac{k^c_{t+1}}{k^c_t} \quad \text{when } k^c_t \text{ increases.} \tag{52}$$

From this we obtain

$$1 + g_y \approx \frac{k^c_{t+1}}{k^c_t} = \Theta + \frac{(\Theta - 1)[(1-\alpha)k^w - \alpha]}{k^c_t} \tag{53}$$

$$\approx \Theta \equiv \frac{\beta_c[1 - \delta + \alpha^\alpha(1-\alpha)^{1-\alpha}]}{1+n} > 1 \quad \text{when } k^c_t \text{ increases.} \tag{54}$$

The growth rate of per capital output is endogenously determined by the capitalists' discount factor, the parameter of the production function, the population growth rate, and the depreciation rate. Note that g_y takes the minimum value when $\alpha = \frac{1}{2}$ while it takes the maximum value when $\alpha = 0$ and $\alpha = 1$.

4 Comparisons with existing studies

This section compares our results with the results obtained from the two-class growth models without automation capital mentioned in the Introduction.[7]

- Pasinetti (1962) provides the Pasinetti theorem and the Pasinetti steady state: $r = \frac{n}{s_c}$, where s_c denotes the saving rate of capitalists, $\frac{K^w}{K} > 0$, and $\frac{K^c}{K} > 0$.
- Samuelson and Modigliani (1966) provide the anti-Pasinetti theorem and Dual steady state: $r = \alpha\frac{n}{s_w}$, where s_w denotes the saving rate of workers, $\frac{K^w}{K} = 1$, and $\frac{K^c}{K} = 0$, where $Y = K^\alpha L^{1-\alpha}$ is used.
- Zamparelli (2017) finds the Anti-Dual steady state by using the CES production function with the elasticity of substitution being sufficiently more than unity: $r = \alpha^{\frac{1}{\rho}}$, $\frac{K^w}{K} = 0$, $\frac{K^c}{K} = 1$, and $\frac{rK}{Y} = 1$, where $Y = [\alpha K^\rho + (1-\alpha)L^\rho]^{\frac{1}{\rho}}$ and $s_c > n/\alpha^{\frac{1}{\rho}}$.

In Cases 1 and 3, asset share of capitalists approaches unity, which resembles the Anti-Dual case. In Cases 1 and 3, the no-arbitrage condition is satisfied, and hence, the rate of return of assets is determined only by the parameter of the production function. As Zamparelli (2017) shows, in the neoclassical growth model without automation capital, the

[7] Furuno (1970) presents a Solow type neoclassical growth model with two classes, and shows that according to the sizes of the saving rates of capitalists and workers, an economy converges to either the Pasinetti steady state or the Dual steady state.

rate of return of capital of the Anti-Dual case is determined solely by the parameters of the CES production function.

In Case 2, the interior solutions are obtained: the asset share of capitalists and that of workers are more than zero and less than unity. In this sense, Case 2 is similar to the Pasinetti steady state. However, from the no-arbitrage condition, the rates of return are equalized, that is, $R^k = R$, which states that R is independent of capitalists' discount factor and the growth rate of population. In the Pasinetti steady state, the rate of return of capital is given by the growth rate of population divided by the saving rate of capitalists. Hence, although Case 2 and the Pasinetti steady state are interior solutions, determinants are different.

In Case 4, we obtain $p = 0$, and hence, the dynamical system reduces to a two-class neoclassical growth model without automation capital. This system is investigated by Sasaki (2018) in detail. He shows that depending on the sizes of β_c and β_w, the economy either converges to the Pasinetti steady state or the Dual steady state. In Sasaki (2018), the criteria for the Pasinetti steady state or the Dual steady state are given by

$$\beta_w \lesseqgtr \frac{\alpha}{1-\alpha} \cdot \frac{\beta_c(1+n)}{(1+n)-(1-\delta)\beta_c} \equiv \Gamma. \tag{55}$$

If $\beta_w < \Gamma$, then the Pasinetti steady state exists and is locally stable. On the contrary, if $\beta_w > \Gamma$, then the Dual steady state exists and is locally stable. Comparing the condition given by equation (55) with the condition that produces Case 4, we can confirm that these conditions are compatible. Therefore, in Case 4, when an economy reaches $k_t^c = \hat{k}^c$ and $p = 0$, the economy switches to a dynamical system without automation capital, and converges to either the Pasinetti steady state or the Dual steady state depending on the sizes of the parameters.

5 Conclusions

This study has presented an economic growth model that considers automation capital that is perfectly substitutable for labor, and investigated the effect of accumulation of automation capital on economic growth and income distribution. To conduct analysis of income distribution, we have assumed that workers and capitalists solve the different dynamics optimization problems. The results are summarized as follows.

When the capitalists' discount factor is relatively large, in the long run, per capita output growth rate is positive, and thus, endogenous growth is obtained. Income gap between capitalists and workers continue to increase.

When the capitalists' discount factor is relatively small, endogenous growth is not obtained and per capita output converges to a constant value. This case is further classified into the two cases.

On the one hand, when the initial value of traditional capital is large, both capitalists' own traditional capital and automation capital converges to constant values. Income gap between the two classes also converges to a constant value.

On the other hand, when the initial value of traditional capital is small, capitalists' own traditional capital converges to a constant value whereas capitalists' own automation capital converges to zero. Income gap between the classes converges to a constant value. When automation capital becomes zero, after then, the dynamical system switches to an alternative system that does not include automation capital, and the economy converges to either the Pasinetti steady state or the Dual steady state depending on the sizes of the parameters.

Whether or not the advances in automation technology produces endogenous growth depends on capitalists' attitude toward saving. If the saving rate of capitalists is high, the advances in automation technology produces sustainable growth but income gap between the classes increases. On the contrary, if the saving rate of capitalists is low, the automation technology cannot produce sustainable growth but income gap does not expand in the long run.

Finally, to obtain clear-cut results, this study postulates that automation capital is perfectly substitutable for labor. In reality, however, automation capital is not necessarily perfectly substitutable for labor. In addition, this substitutability differs for types of labor. Consideration of these issues will be left for future research.

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