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Provision of Renewable Energy using Green Certificates: Market Power and Price Limits

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Abstract

We formulate an analytic equilibrium model for simultaneously functioning electricity market and a market for Green Certificates. The major focus of the paper is the effect of market power in a Green Certificate system. One of the main results from the analysis is that the certificate system faced with market power basically may collapse into a system of per unit subsidies.

1 Introduction

Many countries have adopted the goal to enhance the role of renewable sources in energy supply. It is, for example, a stated goal by the EU to raise the share of electricity based on renewable generation sources from 14 to 22% of total electricity generation by 2010 (see EU/COM (2000)). Typically, in order to promote the generation of green electricity, the relative cost disadvantage of environmentally friendly electricity generation technologies has been compensated by the national authorities through different kinds of subsidy schemes. One example is the fixed-price system used in e.g. Spain and Germany, in which the producers of green electricity get a specified fixed price per unit of their electricity, independent of the quantity they generate. Other subsidy alternatives are investment subsidies and tax reductions for producers of green electricity. The

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Throughout the article, black and green electricity will refer to electricity generated from non renewable and renewable energy sources, respectively.
liberalization of the electricity markets has, however, induced an extra challenge on the countries in their choice of policy measures to promote the provision of green electricity, as the means must be in accordance with market principles. One idea which has been adopted as a possible alternative in many countries is the introduction of a market for Green Certificates (from this point referred to as GCs). The Netherlands has already, since 1998, used a system of “green labelling”, which is a voluntary version of the GC-system. Sweden and Denmark are examples of countries that are close to actually introducing a market for GCs, as it is the intention of both the Swedish and the Danish authorities to have GC systems fully effective by 2003. It seems likely that in the long run all the Member States will support a common EU standard for the markets for GCs. In addition to the EU Member States, countries like Australia, China, India and the US are also considering the introduction of GCs. Despite its popularity, it seems fair to say that the general functioning of the GC market has not yet been fully understood. However, research contributions have been made by e.g. Voogt et al. (1999), Morthorst (2000 and 2001), Amundsen and Mortensen (2001, 2002), Bye et al. (2002) and Jensen and Skytte (2002).

In short, the GC market consists of sellers and buyers of certificates. The sellers are the producers of electricity using renewable sources. The producers are each allowed to sell an amount of certificates corresponding to the electricity they feed into the electricity network. The purchasers of certificates are consumers/distribution companies that are required by the government to hold a certain percentage of certificates corresponding to their total consumption/end-use deliveries of electricity. The GCs are seen as permits for consuming electricity. Hence, this system implies that the producers using renewable energy sources receive both the wholesale price and a certificate for each kWh fed into the electricity network. In this way the GC system is supposed to induce new investments in green electricity generation. An additional indirect effect of increasing the provision of green electricity will be to reduce CO₂ emission if the development of renewable energy technologies is substituting energy production from fossil fuel fired plants.

In the following we formulate an analytic equilibrium model for a GC system. A basic assumption in the model is that the ”percentage requirement” for the possession of certificates of consumers/distribution companies functions as a check on total electricity consumption, as the total number of certificates available are constrained by the total capacity of renewable technologies. Hence, a requirement of e.g. 20% implies that total consumption can be no larger than five times the electricity produced from renewable sources, unless the price of certificates tends to increase above an upper price bound specified by the authorities. This price bound, which is used in most of the proposed GC systems, functions as a penalty which the consumers must pay if they don’t fulfil the

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2 Italy is an exception in this respect as the Italian system is supposed to put the purchase obligation on the producers.

3 In many countries wind mills constitute a significant part of the green production technology. The electricity production from wind mills will typically vary a lot, giving rise to considerable variations in the total production of green electricity between different years.
percentage requirement, i.e. to be allowed to consume more electricity than five
times the amount of certificates they have bought. In the model we also repre-
sent a lower price bound for the certificate price. This price bound is relevant
in case the production of green electricity exceeds the demand for certificates.
If this is the case, the State will buy the excess supply of certificates at a price
corresponding to the lower price bound. Not all the countries considering the
introduction of a GC system will include a minimum price of the certificates,
but the proposed Danish GC system is using such a lower price bound. We will
concentrate on the analysis of the market itself and do not in this setting ad-
dress the question of whether such a system is economically sound as compared
with other alternatives of stimulating the generation of green electricity. Fur-
thermore, we do not consider any uncertainty or financial markets for forwards
or futures trade in certificates, nor do we consider an international system with
trade of GCs.

The major focus of this paper is the effect of market power in a GC system.
We will use our model to characterize the equilibrium conditions for various
levels of competition in the electricity and the GC markets. We will start by
assuming perfect competition in both the electricity and the GC markets and
thereafter go through the three cases of market power in either one or both of
the markets. In addition we look at a case with a monopolistic company that
controls both generation technologies. Throughout the paper we assume Nash-
Cournot (NC-) behavior. As will be shown, the GC system does not always
produce straight forward results. It turns out that the existence of market
power to a large extent will drive the certificate price to either the lower or
the upper price bound and thereby reduce the GC system to a system of direct
subsidies financed through excise taxes. As market power is likely to exist
in many cases, this is a result that should be given serious consideration in the
discussions and development of alternative GC systems. In our analysis we focus
particularly on the generation of green electricity under different assumptions
of competition in the markets. One interesting result from this analysis is that
market power in the generation of black electricity can actually promote an
increase in the generation of green electricity as compared with the result under
perfect competition. Among the other surprising results are that an increase in
the percentage requirement, and an introduction of a CO$_2$ tax on the producers
of black electricity in combination with a GC system, actually both can have
the effect as to reduce the generation of green electricity.

The first section of the paper presents the model. The next section presents
and analyses the equilibrium in the case of perfect competition in both the
electricity and the GC market. Thereafter follows the cases of market power
in the generation of black electricity, market power in the generation of green
electricity, market power in both generation technologies, and market power in
the joint generation of green and black electricity. In section 8 we introduce

\footnote{Alternative models of describing the behavior in the electricity market exist (e.g. Bertrand
games, supply function games and auction games). For a discussion of why NC-behavior is a
reasonable model, see Borenstain and Bushnell (1999) and Borenstein, Bushnell and Knittel
(1999).}
a CO$_2$ tax on the producers of black electricity and analyze the effects on the generation of green and black electricity. Finally, the last section summarizes and concludes.

2 The model

The following model is designed to capture a long run situation for the simultaneous functioning electricity market and a market for GCs.

We will use the following list of variables:

- $p =$ consumer price of electricity
- $s =$ price of GCs
- $\sigma =$ upper price bound of GCs
- $\sigma =$ lower price bound of GCs
- $q =$ wholesale price of electricity
- $x =$ total consumption of electricity
- $y =$ generation of black electricity
- $z =$ generation of green electricity
- $\alpha =$ percentage of green electricity consumption
- $g_d =$ demand for GCs
- $t =$ CO$_2$ tax

We apply the following general functions:

The inverse demand function is:

$$p(x), \text{ with } \frac{\partial p(x)}{\partial x} = p' < 0.$$

The cost function for the producers of black electricity is assumed given by:

$$c = c(y), \text{ with } c'(y) > 0 \text{ and } c''(y) > 0.$$

The rationale for choosing an increasing long run marginal cost function for this industry, is that the expansion of output may drive up the price of CO$_2$-emission permits or CO$_2$-taxes to comply with national CO$_2$-emission constraints. The cost function for the producers of green electricity is assumed given by:

$$h(z), \text{ with } h'(z) > 0 \text{ and } h''(z) > 0.$$

The rationale for choosing an increasing long run marginal cost function for this industry is that good sites for wind-mills may be in scarce supply by nature.

The two groups of producers deliver electricity to a common wholesale market, from where distribution companies purchase electricity for end-use deliveries. The distribution companies are assumed to act as profit maximizers.

In the model we have two markets, one market for electricity and one market for GCs. As we work our way through the different market structures we will apply the subscripts $c$ and $m$ to the endogenous variables in order to indicate

\footnote{For a short run version of the model, see Amundsen and Mortensen (2001).}
whether the markets are competitive, or if there is market power involved in one or both of the generation technologies. The first subscript will correspond to the market structure among the producers of black electricity, while the second subscript is used to describe the market structure in the generation of green electricity. \( q_{cc} \) will thus refer to the wholesale price of electricity in the case of perfect competition among both kinds of electricity producers. In the case of market power in the generation of black electricity and perfect competition among the producers of green electricity, the variable for the wholesale price will be \( q_{mc} \), and so on. In addition we will use the subscript \( M \) for the case where we consider market power in the joint generation of black and green electricity.

We start by considering a market with perfect competition all around.

3 Perfect competition

For the case of perfect competition in both markets all the profit maximizing market participants are price takers. In order to simplify the presentation we are suppressing the subscripts concerning the market structure at this point. The producers of black electricity act as if they jointly maximize:

\[
Max \Pi (y) = qy - c(y).
\]

The first order condition for an optimum in the competitive market is:

\[
q^* = c'(y).
\]

For each unit of green electricity generated there will be issued one certificate. The producers of green electricity will sell all their certificates and will thus earn the wholesale price plus the GC-price per unit of electricity they generate.\(^6\) Jointly they act as to maximize:

\[
Max \Pi (z) = [q + s]z - h(z).
\]

The first order condition is:

\[
q + s = h'(z).
\]

For each unit of electricity bought and sold to the end users the distribution companies will have to pay the wholesale price plus a proportion \( \alpha \) of the certificate price in accordance with the percentage requirement. The distribution companies are throughout the article assumed not to enjoy any market power. Hence, jointly they act as to maximize:

\[
Max \Pi (x) = px - [q + \alpha s]x.
\]

\(^6\)Given the assumption of perfect competition in the generation of green electricity it is obvious that the generators will always sell all their certificates. However, as we will see later, market power in the generation of green electricity makes it relevant for these generators to consider whether they can utilize their market power to affect the price of GCs in order to increase their profit from the GC and the electricity market.
The first order condition is:
\[ p = q + \alpha s. \]

In the market for GCs the demand is given by:
\[ g_d = \alpha x. \]

3.1 cc-equilibrium

Given the objective functions and the first order conditions we can specify the equilibrium for the competitive markets. The key variables used in the analysis are the equilibrium price, generated quantities of black and green electricity and total consumption of electricity. These will depend on whether the price of GCs in equilibrium, \( s^* \), is within the specified price interval, i.e. \( \underline{s} < s^* < \overline{s} \), or on either the upper or lower price bound. As long as the price of GCs is within the interval, total consumption of electricity is given by \( x = \frac{z^*}{\alpha} \). This quantity constraint implied by the percentage requirement is sometimes referred to as the allowable consumption level. In the case of \( s^* = \underline{s} \), the demand for GCs is less than \( z^* \), and there is an excess supply of GCs. It is assumed (as in the Danish GC-proposal) that the State guarantees to buy the otherwise unsold certificates at a price equal to \( \underline{s} \). This means that total electricity consumption is decreasing compared with the preceding case. If the price of GCs in equilibrium turns out to be equal to the upper price bound, \( \overline{s} \), demand for certificates exceeds the maximum possible supply. In this case the consumers are allowed to buy more black electricity if they pay a "fine" equal to \( \overline{s} \) per unit of extra electricity consumption. Including now the subscripts for market form, and assuming that \( c^* \left( y_{cc}^* \right) \) and \( h^* \left( z_{cc}^* \right) \) are representing the aggregate marginal cost functions, we have the following result for the key variables in equilibrium under perfect competition:

- For the case of \( \underline{s} < s_{cc}^* < \overline{s} \)

\[ p \left( x_{cc}^* \right) = q_{cc}^* + \alpha s_{cc}^* \quad (1) \]

\[ x_{cc}^* = y_{cc}^* + z_{cc}^* = \frac{z_{cc}^*}{\alpha} \quad (2) \]

\[ q_{cc}^* + s_{cc}^* = h^* \left( z_{cc}^* \right) \quad (3) \]

\[ q_{cc}^* = c^* \left( y_{cc}^* \right) \quad (4) \]
• For the case of \( s_{cc}^* = \pi \)

\[
p(x_{cc}^*) = q_{cc}^* + \alpha \pi
\]  

(5)

\[
x_{cc}^* = y_{cc}^* + z_{cc}^* > \frac{z_{cc}^*}{\alpha}
\]  

(6)

\[
q_{cc}^* + \pi = h'(z_{cc}^*)
\]  

(7)

\[
q_{cc}^* = c'(y_{cc}^*)
\]  

(8)

• For the case of \( s_{cc}^* = s \)

\[
p(x_{cc}^*) = q_{cc}^* + \alpha s
\]  

(9)

\[
x_{cc}^* = y_{cc}^* + z_{cc}^* < \frac{z_{cc}^*}{\alpha}
\]  

(10)

\[
q_{cc}^* + s = h'(z_{cc}^*)
\]  

(11)

\[
q_{cc}^* = c'(y_{cc}^*)
\]  

(12)

The cc-equilibrium solution is illustrated for the case of \( s < s_{cc}^* < \pi \) in Figure 1. The quantity constraint implied by the percentage requirement is seen to drive a wedge equal to \( \alpha s_{cc}^* \) between the electricity price and the marginal cost of electricity generation. The system thus involves a transfer of consumer- and producer surplus from black electricity generation to a subsidy of green electricity generation.

*Figure 1 here*
3.2 Analysis

In the proposed GC-systems, the "percentage requirement" is perceived as a policy instrument affecting the level of green electricity in end use consumption. Unlike price-fixation (that leaves quantity an endogenous variable) or quantity fixation (that leaves price an endogenous variable) the "percentage requirement" neither fixes price nor quantity and thus leaves both variables to be endogenously determined. The following proposition shows that it is not in general true that an increase of the "percentage requirement" leads to an increased generation of green electricity in equilibrium. It does, however, lead to a reduced generation of black electricity, and therefore - from (4), (8) or (12) - a reduced wholesale price of electricity. As the effect on green electricity is indeterminate, the effect on total consumption and end user price is also indeterminate. 

Proposition 1 Under perfect competition in the electricity and the certificate markets, the "percentage requirement", $s$, has the following effects: i) if $s < s^*_cc < \bar{s}$, then $\frac{dy^*_c}{d\alpha} < 0$ while $\text{sign} \left( \frac{dz^*_c}{d\alpha} \right)$ and $\text{sign} \left( \frac{dx^*_c}{d\alpha} \right)$ are indeterminate, and ii) if $s^*_cc = \bar{s}$ or $s^*_cc = \underline{s}$, then $\frac{dz^*_c}{d\alpha} < 0$, $\frac{dy^*_c}{d\alpha} < 0$, $\frac{dx^*_c}{d\alpha} < 0$.

Proof. i) Inserting (3) and (4) into (1) yields the electricity price as a linear combination of marginal costs of the two groups of generation technologies in equilibrium, i.e. $p(x^*_cc) = (1-\alpha)c' (y^*_cc) + \alpha h' (z^*_cc)$. Take the implicit derivatives of this expression with respect to $\alpha$ and arrive at:

$$\frac{dp}{d\alpha} = \frac{d}{d\alpha} (1-\alpha)c' (y^*_cc) + \frac{d}{d\alpha} (\alpha h' (z^*_cc)) = (1-\alpha)s^*_cc + x^*_cc \left[ \beta p (y^*_cc) - \alpha h' (z^*_cc) \right].$$

ii) Insert (8) in (7) or (12) in (11). Take the implicit derivative with respect to $\alpha$ and get $h'' (z^*_cc) \frac{dz^*_c}{d\alpha} = c'' (y^*_cc) \frac{dy^*_c}{d\alpha}$ As marginal costs are assumed increasing it follows: sign $\left( \frac{dy^*_c}{d\alpha} \right)$ $\frac{dy^*_c}{d\alpha} = \text{sign} \left( \frac{dz^*_c}{d\alpha} \right)$. The last equality follows as $\frac{dz^*_c}{d\alpha} = \frac{d}{d\alpha} (\alpha h' (z^*_cc))$. But the signs cannot be non-negative. To see this insert (8) in (5) and take the implicit derivative with respect to $\alpha$ to obtain

$$\frac{d}{d\alpha} = \frac{d}{d\alpha} (\alpha h' (z^*_cc)) = \frac{d}{d\alpha} \left( \beta p (y^*_cc) - \alpha h' (z^*_cc) \right),$$

where $\bar{s}$ is the level of green electricity in end use consumption. Hence, $\text{sign} \left( \frac{dz^*_c}{d\alpha} \right) = \text{sign} \left( \frac{dy^*_c}{d\alpha} \right) = \text{sign} \left( \frac{dx^*_c}{d\alpha} \right) < 0$ for this equation to hold. 

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7 This is a generalization of results obtained in Amundsen and Mortensen (2001, 2002). See also Bye et al. (2002) and Jensen and Skytte (2002) that obtain more structure on the results by applying specific functions on basically similar models as in Amundsen and Mortensen, e.g. how total electricity consumption varies as a function of the percentage requirement.
4 Market power in the generation of black electricity

We now look at the case where the producers of black electricity enjoy market power. We assume that these producers act as a Nash Cournot (NC)-playing oligopolist (or a perfectly co-ordinated cartel) facing the producers of green electricity as a competitive fringe. We further assume that the NC-playing producers of black electricity consider both the quantity of green electricity and the number of GCs as given, represented by $\pi$. The producers of black electricity will therefore in their generation decision also consider the simultaneous effect their quantity decision has on the price of GCs, because this price affects the wholesale price of electricity ($q = p - \alpha s$). We are considering a one-shot game so the NC-playing oligopolist will not be able to react on the response from the producers of green electricity, i.e. the producers of black electricity are not Stackelberg leaders. In the NC-equilibrium none of the producers will want to change their quantity decision.

Again we are suppressing the subscripts indicating market form as we go through the model specification. The NC-playing producers of black electricity is faced with the following residual demand function for wholesale electricity:

$$q = q(x) = p(x) - \alpha s(x), \text{ where } x = y + \pi,$$

and the following optimization problem:

$$\text{Max} \Pi (y, \pi) = q(x)y - c(y).$$

In equilibrium the profit maximizing oligopolistic generator of black electricity will therefore equate marginal revenue with marginal cost:

$$\frac{\partial \Pi}{\partial y} = \frac{\partial q(x)}{\partial y}y + q - c'(y) = 0.$$

Or more precisely:

$$\frac{\partial \Pi}{\partial y} = \left( \frac{\partial p}{\partial x} - \alpha \frac{\partial s}{\partial y} \right) y + q - c'(y) = 0.$$

Observe that a marginal change of the generation of black electricity may affect the wholesale price through both the electricity market and the GC market. The effect through the electricity market is an ordinary effect on consumer price, while the effect through the GC market stems from the induced change of demand for certificates following from a marginal change in the generation of black electricity (e.g. an increase in the generation of black electricity by one unit will, in equilibrium, increase the demand of electricity by one unit and the demand for certificates by $\alpha$ units). Considering the effect of a marginal change of demand on the price of GCs, $\alpha \frac{\partial p}{\partial y}$, this is clearly zero as long as the certificate price is at one of the price bounds. For the case where the price of GCs is within the interval the derivative of the certificate price does not exist as
the marginal revenue is discontinuous at this point, see Figure 1. At this point
the total demand is equal to \( z^* \), which is the only level of demand where we
actually get a certificate price within the price interval. A marginal change in
the demand of GCs from the allowable consumption level will drive the price of
GCs either to the lower bound (if the demand decreases) or to the upper bound
(if the demand increases). In equilibrium the consumers will never buy more
certificates than they actually need, they always buy an amount of certificates
equal to \( \alpha x \). So if the demand falls below \( \frac{z^*}{\alpha} \) there is an excess supply of certi-
cficates and the certificate price reaches the lower bound. Similarly there will be
an excess demand for certificates if the demand for electricity increases above
the level \( \frac{z^*}{\alpha} \). An excess demand for certificates implies that the certificate price
reaches the upper bound.

4.1 mc-equilibrium

In the case of market power in the generation of black electricity we have the
following equilibrium solution for the key variables:

\[
p(x^*_{mc}) = q^*_{mc} + \alpha s^*_{mc}
\]  
(13)

\[
x^*_{mc} = y^*_{mc} + \frac{s^*_{mc} - z^*_{mc}}{\alpha}
\]  
(14)

\[
q^*_{mc} + s^*_{mc} = h'(z^*_{mc})
\]  
(15)

\[
\left( \frac{\partial p(x^*_{mc})}{\partial x} - \alpha \frac{\partial s(x^*_{mc})}{\partial y} \right) y^*_{mc} + q^*_{mc} = c'(y^*_{mc})
\]  
(16)

In equation (14), \(<, =\) and \(>\) refer to the cases \( s^*_{mc} = s < s^*_{mc} < \overline{s} \)
and \( s^*_{mc} = \overline{s} \), respectively. Note that (16) reduces to \[ \frac{\partial p(x^*_{mc})}{\partial x} y^*_{mc} + q^*_{mc} = c'(y^*_{mc}) \] in
the cases of \( s^*_{mc} = \frac{z}{\alpha} \) and \( s^*_{mc} = \overline{s} \), because, as argued above, \[ \frac{\partial s(x^*_{mc})}{\partial y} = 0 \] if the
GC-price is at one of the price bounds.

4.2 Analysis

We will now compare the mc-equilibrium with the cc-equilibrium. We will do
this by looking at three different cases. The cases will differ with respect to
whether the price of the GCs in the competitive case is at one of the price
bounds or if it is within the price interval. In each of the cases we identify
the cc-equilibrium and consider the effect of introducing market power in the
generation of black electricity. The main results are highlighted in Proposition
2 and 3.

Proposition 2 shows that market power in the generation of black electricity
may effectively distort the functioning of the GC-system and transform it to a
system of tax based subsidies. The GC-price will never be established at an
intermediate level. It may, however, be established at the upper price bound.
Proposition 2 Assume the producers of black electricity act as a NC-playing oligopolist (or a perfectly co-ordinated cartel) facing green producers as a competitive fringe, then - in equilibrium - there will i) never be established an intermediate certificate price such that \( \hat{s} < s_{mc}^* < \pi \), but ii) there may be an equilibrium certificate price at the lower or the upper price bound, i.e. \( s_{mc}^* = \hat{s} \) or \( s_{mc}^* = \pi \).

Proof. i) To show that we cannot have \( \hat{s} < s_{mc}^* < \pi \), assume \( \hat{y} \) is a solution satisfying the first order conditions for the producers of black electricity and that \( \hat{y} + \pi = \hat{\pi} \) where \( \pi \) is the quantity of green electricity that the producers of black electricity, in accordance with the NC-assumption, consider as given. Clearly, if \( y < \hat{y} \), then \( s_{mc}^* = \hat{\pi} \), due to excess supply of certificates (i.e. \( \pi > \alpha (y + \pi) \)) and if \( y > \hat{y} \), then \( s_{mc}^* = \pi \), due to excess demand for certificates. Denote the marginal revenue function, \( g(y,\pi) \), by \( g(y,\pi) = \frac{\partial y}{\partial \pi} y + q \). Observe that \( g(y,\pi) = \frac{\partial y}{\partial \pi} y + q \) for \( y \neq \hat{y} \) as \( \frac{\partial y}{\partial \pi} = 0 \) for such values. Clearly, \( g(y,\pi) \) is discontinuous at \( \hat{y} \) as \( \lim_{y \rightarrow \hat{y}^-} g(y,\pi) = \frac{\partial y}{\partial \pi} y + \hat{q}^- \) and \( \lim_{y \rightarrow \hat{y}^+} g(y,\pi) = \frac{\partial y}{\partial \pi} y + \hat{q}^+ \) where \( \hat{q}^- = \lim_{y \rightarrow \hat{y}^-} q = p(\hat{y} + \pi) - \alpha \hat{s} \) and \( \hat{q}^+ = \lim_{y \rightarrow \hat{y}^+} q = p(\hat{y} + \pi) - \alpha \pi \).

However, as \( \Pi(\hat{y},\pi) = q \hat{y} - c(\hat{y}) \), profit maximization will lead the producers of black electricity to secure \( \hat{q}^- \) (by an infinitesimal quantity reduction of black electricity) implying the corner solution, i.e. \( s_{mc}^* = \hat{s} \). An example is illustrated in Figure 2.

ii) To show that the GC-price may be at either the upper or the lower price bound, it suffices to give examples satisfying the assumptions of the model. Examples are provided in appendix B and illustrated in Figure 3 and 4.

In Figure 2, 3 and 4 we have illustrated the profit curves of the NC-playing producers of black electricity for three possible equilibrium solutions of the model. Figure 2 helps illustrating the point made in Proposition 2 i), that there will never be established an intermediate GC-price in the mc-case. In Figure 3, the equilibrium certificate price is at the lower price bound, while in Figure 4 the price of GCs in equilibrium is established at the upper price bound. The figures are based on a simple numerical model satisfying the assumptions we have made about the electricity market. The model is specified in Appendix A. The specific parameter values and solutions of the model illustrated in the figures are presented in Appendix B. In the figures, the profit curves are illustrated assuming that the NC-playing producers consider both the quantity of green electricity and the number of GCs as given. Fixing the quantity of green electricity at the equilibrium levels, we then draw the profit curve for different quantities of black electricity. Under the assumption of Cournot behavior, the oligopolistic producers of black electricity choose the quantity of black electricity that maximizes profit. The equilibrium quantity of black electricity is thus found where the profit curve is at its maximum. Looking at the figures, we notice that the profit drops discontinuously at a specific value of \( y \). This is the quantity of black electricity at which total consumption of electricity is at the allowable consumption level, i.e. we get a market based price of the GCs. For lower values of \( y \), there is an excess supply of GCs, i.e. the GC-price is at
the lower price bound. If the generation of black electricity increases above the point where the profit curve drops, total consumption is above the allowable consumption level and the certificate price jumps to the upper price bound.

In the numerical example behind Figure 2, the model generates an intermediate GC-price. However, the NC-playing producers of black electricity will not stay at the allowable consumption level implied by the intermediate GC-price. The profit curve shows that profit maximization will lead to an infinitesimal reduction in the quantity of black electricity from the allowable consumption level, inducing $s^*_mc = \frac{y}{\alpha}$. Hence, Figure 2 illustrates the point made in the formal proof above. In Figure 3 the profit curve has a global maximum at the left of the allowable consumption level. We will therefore get an equilibrium GC-price at the lower price bound. In Figure 4, however, the global maximum is to the right of this point, i.e. the GC-price is at the upper price bound.

Figure 2, 3 and 4 here

The following proposition shows that market power exercised by the producers of black electricity may actually lead to an increase of the generation of green power, as compared with the competitive equilibrium, and that this is definitely true if the competitive equilibrium GC-price is at its lower bound. The generation of black electricity will, however, always decrease compared with the competitive equilibrium.

**Proposition 3** Under the assumptions of the model: i) if $s^*_cc = \frac{y}{\alpha}$ then $z^*_mc > z^*_cc$; ii) if $s^*_cc > \frac{y}{\alpha}$ then sign $(z^*_mc - z^*_cc)$ is indeterminate and iii) $y^*_mc < y^*_cc$.

Proof. i) To obtain a contradiction, assume $z^*_mc \leq z^*_cc$. From (11) and (15) we have: $h^*(z^*_cc) = q^*_cc + \frac{y}{\alpha} + s^*_mc = h^*(z^*_mc)$, implying $q^*_cc \geq y^*_mc$. From (12) and (16): $q^*_cc = c(y^*_cc) \geq c(y^*_mc) - \frac{\partial}{\partial y} y^*_mc = y^*_mc$. Hence, $y^*_cc > y^*_mc$. So that $x^*_cc = y^*_cc + z^*_cc > x^*_mc = z^*_mc + y^*_mc$ and $p(x^*_cc) < p(x^*_mc)$. However, successive substitution from the two sets of first order conditions yields: $p(x^*_cc) = (1 - \alpha)q^*_cc + \frac{\alpha}{\alpha} (z^*_cc)$ and $p(x^*_mc) = (1 - \alpha)q^*_mc + \frac{\alpha}{\alpha} (z^*_mc)$. As $q^*_cc \geq y^*_mc$ and $z^*_cc \geq z^*_mc$, this clearly gives $p(x^*_cc) \geq p(x^*_mc)$; a contradiction. Hence, we must have $z^*_mc > z^*_cc$.

ii) Examples satisfying the assumptions of the model are provided in appendix C.

iii) To obtain a contradiction, assume $y^*_mc \geq y^*_cc$. Then from (4) (or (8) or (12)) and (16), $q^*_mc \geq q^*_cc$. We now consider each possible case with respect to the value of $s^*_cc$ for the cc-equilibrium.

Assume first $s^*_cc = \frac{y}{\alpha}$. Then, as $q^*_mc > q^*_cc$ and $s^*_mc \geq \frac{y}{\alpha}$, we have from (9) and (13) that $p(x^*_mc) = q^*_mc + \alpha s^*_mc > p(x^*_cc) = q^*_cc + \alpha \frac{y}{\alpha}$. However, from (11) and (15) we have $h^*(z^*_cc) = q^*_cc + \frac{y}{\alpha} < q^*_mc + s^*_mc = h^*(z^*_mc)$ so that $z^*_mc > z^*_cc$. Hence, as by assumption $y^*_mc \geq y^*_cc$, we must have $x^*_mc = y^*_mc + z^*_mc > y^*_cc + z^*_cc = x^*_cc$ so that $p(x^*_mc) < p(x^*_cc)$. That contradicts the above assumption that $p(x^*_mc) < p(x^*_cc)$. Hence, $y^*_mc < y^*_cc$ as $s^*_cc = \frac{y}{\alpha}$.

Next, assume $s^*_cc > \frac{y}{\alpha}$. Consider first the possibility that $s^*_mc = \frac{y}{\alpha}$. Then, using the same line of reasoning as above, we get $p(x^*_mc) = q^*_mc + \alpha \frac{y}{\alpha} > p(x^*_cc) = q^*_cc + \alpha \frac{y}{\alpha}$. Hence, $x^*_mc > x^*_cc$ as $s^*_cc > \frac{y}{\alpha}$.
$q_{cc}^* + \alpha s_{cc}^*$ and $h'(z_{cc}^*) = q_{cc}^* + s_{cc}^* < q_{mc}^* + \overline{\pi} = h'(z_{mc}^*)$ so that $z_{mc}^* > z_{cc}^*$,

$x_{mc}^* > x_{cc}^*$ and the contradicting result that $p(x_{mc}^*) < p(x_{cc}^*)$.

It remains to consider the case where $s_{cc}^* > \overline{s}$ and $s_{mc}^* = \overline{s}$. As $s_{cc}^* > \overline{s}$, we must have $\frac{z_{cc}^*}{2s_{cc}^*+y_{cc}^*} \leq \alpha$ or $(1-\alpha)z_{cc}^* \leq \alpha y_{cc}^*$, and, furthermore, as $s_{mc}^* = \overline{s}$ we must have $(1-\alpha)z_{mc}^* > \alpha y_{mc}^*$. Then, as by assumption $y_{mc}^* \geq y_{cc}^*$, we have $(1-\alpha)z_{cc}^* < \alpha y_{cc}^* \leq \alpha y_{mc}^* < (1-\alpha)z_{mc}^*$, so that $z_{mc}^* > z_{cc}^*$. Hence, $x_{cc}^* = y_{mc}^* + x_{cc}^* > y_{cc}^* + x_{cc}^* = x_{cc}^*$, so that $p(x_{mc}^*) < p(x_{cc}^*)$. However, successive substitutions from the two sets of first order conditions yield: $p(x_{mc}^*) = (1-\alpha)q_{mc}^* + \alpha h'(z_{mc}^*)$ and $p(x_{cc}^*) = (1-\alpha)q_{cc}^* + \alpha h'(z_{cc}^*)$. As $q_{mc}^* > q_{cc}^*$ and $z_{mc}^* > z_{cc}^*$, we have $p(x_{mc}^*) > p(x_{cc}^*)$, that contradicts the above conclusion.

Hence, we arrive at the final conclusion that $y_{mc}^* < y_{cc}^*$. ■

Compared to the perfect competition equilibrium, market power in the generation of black electricity will always reduce the generation of black electricity (as shown in proposition 3 iii)) and increase the wholesale price of electricity. Even (as stated in Proposition 3 i) and ii)) if the green electricity producers may increase their generation, it can be shown that this increase will never fully compensate for the reduction in generation of black electricity. Thus, total quantity of electricity generated will decrease. This leads unambiguously to an increased end-user price and lower consumption. The market power obviously makes the producers of black electricity better off as they are able to increase their profit as oligopolists (or a cartel) as compared with the competitive solution, and the consumers will always be the losers in this situation due to lower consumption and higher price. Whether the producers of green electricity will gain or loose compared with a competitive market will, however, depend on the price of GCs that would be generated in a competitive market. As stated by Proposition 3 ii), introduction of market power in the generation of black electricity can actually increase the generation of green electricity also in the cases where the competitive market generates a certificate price at the upper bound, $s_{cc}^* = \overline{s}$, or within the price interval. However, the effect on the green producers’ profits is not certain. For the case of $s_{cc}^* = \overline{s}$, the crucial point is whether the equilibrium quantity reduces below the allowable consumption level, $x = \frac{\overline{s}}{\alpha}$. Above this level, we will not get any change in the certificate price as it will stay at the upper bound. In this case the green producers will benefit from the black electricity generator’s market power. Such a case is illustrated in Figure 4. If, however, the equilibrium quantity reduces below the allowable consumption level, as illustrated in Figure 3, the certificate price will jump to the lower bound. The green producers are then facing a lower certificate price and a higher wholesale price of electricity. The effect on their profit is indeterminate. In the case where the perfect competition equilibrium quantity is at the allowable consumption level the certificate price is decided in the market and is within the price interval. As stated in proposition 2 above and illustrated in Figure 2, the oligopolistic generator of black electricity will never want to stay at this generation level. The producers of green electricity will then face a reduced certificate price and an increased wholesale price of electricity. Again the total effect on the green producers’ profits will be indeterminate.
5 Market power in the generation of green electricity

In this section we consider a NC-playing oligopolistic generator of green electricity (or a perfectly co-ordinated cartel) facing a competitive fringe of producers of black electricity. The NC assumption is here implying that the producers of green electricity consider the quantity of black electricity as given (represented below as $y$) when making decisions as to how much green electricity to generate and how many GC’s to sell in the certificate market. As NC-players, the producers of green electricity will recognize that the certificate price will be affected by the number of certificates they sell in the GC market. Therefore they will consider if it can be profitable to hold back some of the certificates they generate. To be able to separate the number of certificates sold from the number of certificates generated we introduce a variable $w$ representing the former ($z$ is still representing the number of generated certificates). The profit maximizing generator of green electricity then maximizes the following objective function:

$$\text{Max} \Pi (z, w, y) = qz + sw - h(z), \text{ s.t. } w \leq z.$$ 

We consider two cases: a) $w = z$ and b) $w < z$:

a) First order condition:

$$\frac{\partial \Pi}{\partial z} = \frac{\partial (q + s)}{\partial z} z + q + s - h'(z) = 0.$$ 

Or more precisely:

$$\frac{\partial \Pi}{\partial z} = \left[ \frac{\partial p}{\partial x} + (1 - \alpha) \frac{\partial s}{\partial z} \right] z + q + s - h'(z) = 0.$$ 

b) First order conditions:

$$\frac{\partial \Pi}{\partial z} = \frac{\partial q}{\partial z} z + q + \frac{\partial s}{\partial z} w - h'(z) = 0 \text{ and } \frac{\partial \Pi}{\partial w} = \frac{\partial q}{\partial w} z + \frac{\partial s}{\partial w} w + s = 0.$$ 

5.1 cm-equilibrium

We have then the following equilibrium solution for the key variables in case a), i.e. $z = w$:

$$p(x_{cm}^*) = q_{cm}^* + \alpha s_{cm}^*$$ 

(17)

$$x_{cm}^* = y_{cm}^* + z_{cm}^* \leq \frac{z_{cm}^*}{\alpha}$$ 

(18)
We will therefore not go into detail about the equilibrium solution of the case where \( w < z \), but rather concentrate on the main results from case a). These are highlighted in Proposition 4, 5 and 6.

5.2 Analysis

As will be shown in proposition 5, it will never be profitable for the producers of green electricity to hold back some of the generated certificates from the market. We will therefore not go into detail about the equilibrium solution of the case where \( w < z \), but rather concentrate on the main results from case a). These are highlighted in Proposition 4, 5 and 6.

Proposition 4 shows that the result regarding non-existence of intermediate certificate prices carries over from the mc-equilibrium to the case of co-ordinated oligopolistic behavior of the producers of green electricity.

**Proposition 4** Assume the producers of green electricity act as a NC-playing oligopolist (or a perfectly co-ordinated cartel) facing a competitive fringe of producers of black electricity, then - in equilibrium - there will i) never be established an intermediate certificate price such that \( \underline{s} < s^*_{cm} < \overline{s} \), but ii) there may be an equilibrium certificate price at the lower or the upper price bound, i.e. \( s^*_{cm} = \underline{s} \) or \( s^*_{cm} = \overline{s} \).

**Proof.** i) To show that we cannot have \( \underline{s} < s^*_{cm} < \overline{s} \), assume \( \hat{z} \) and \( \hat{w} \), with \( \hat{w} \leq \hat{z} \), satisfy the optimality conditions for the producers of green electricity and that \( \overline{s} + z = \overline{w} \) where \( \overline{s} \) is the quantity of black electricity that the producers of green electricity, in accordance with the NC-assumption, consider as given.

We consider two cases: a) \( \hat{w} < \hat{z} \) and b) \( \hat{w} = \hat{z} \).

a) Clearly, if \( z < \hat{z} \) (for given values of \( \overline{s} \) and \( \hat{w} \)) then \( s^*_{cm} = \underline{s} \), due to excess supply of certificates (i.e. \( \hat{w} > \alpha (\overline{s} + z) \)) and if \( \hat{z} > z \), then \( s^*_{cm} = \overline{s} \), due to excess demand for certificates. Denote the marginal revenue function, \( g(z, \overline{s}, \hat{w}) \), by \( g(z, \overline{s}, \hat{w}) = \frac{\partial p}{\partial z} \hat{z} + \frac{\partial w}{\partial z} \hat{w} + q \). Observe that \( g(z, \overline{s}, \hat{w}) = \frac{\partial p}{\partial z} \hat{z} + q \) for \( z \neq \hat{z} \) as \( \frac{\partial w}{\partial z} = 0 \) for such values. Clearly, \( g(z, \overline{s}, \hat{w}) \) is discontinuous at \( \hat{z} \) as \( \lim_{z \to \hat{z}^-} g(z, \overline{s}, \hat{w}) = \frac{\partial p}{\partial z} \hat{z} + \hat{q}^- \) and \( \lim_{z \to \hat{z}^+} g(z, \overline{s}, \hat{w}) = \frac{\partial p}{\partial z} \hat{z} + \hat{q}^+ \) where \( \hat{q}^- = \lim_{z \to \hat{z}^-} q \) and \( \hat{q}^+ = \lim_{z \to \hat{z}^+} q \). The profit function is \( \Pi(\hat{z}, \overline{s}, \hat{w}) = q \hat{z} + s \hat{w} - h(\hat{z}) \). Rewrite this as \( \Pi(\hat{z}, \overline{s}, \hat{w}) = p(\overline{s} + z) \hat{z} + s(\hat{w} - \alpha z) - h(\hat{z}) \) and observe that the assumption of \( \overline{s} + z = \overline{w} \) implies \( \hat{w} > \alpha \hat{z} \) is the case solution \( s^*_{cm} = \overline{s} \). Hence, profit maximization will lead the producers of green electricity to secure \( \hat{q}^+ \) (by an infinitesimal quantity increase of green electricity) implying the corner solution \( s^*_{cm} = \overline{s} \).
b) In this case \( y + \hat{z} = \frac{1}{\pi} \). Clearly, if \( z < \hat{z} \), then \( s_{\text{cm}}^* = \pi \), due to excess demand for certificates (i.e. \( z < \alpha (\gamma + \hat{z}) \)) and if \( z > \hat{z} \), then \( s_{\text{cm}}^* = \alpha \), due to excess supply of certificates. Denote the marginal revenue function, \( g(z, \gamma) \), by \( g(z, \gamma) = \left( \frac{\partial (q + s)}{\partial z} \right) z + q + s \). Observe that \( g(z, \gamma) = \frac{\partial p}{\partial x} z + q + s \) for \( z \neq \hat{z} \) as \( \frac{\partial p}{\partial y} = 0 \) for such values. Clearly, \( g(z, \gamma) \) is discontinuous at \( \hat{z} \) as \( \lim_{z \to \hat{z}^-} g(z, \gamma) = \frac{\partial p}{\partial x} \hat{z} + \hat{q}^- + \pi \) and \( \lim_{z \to \hat{z}^+} g(z, \gamma) = \frac{\partial p}{\partial x} \hat{z} + \hat{q}^+ + \pi \) where \( \hat{q}^- = \lim_{z \to \hat{z}^-} q = p (\gamma + \hat{z}) - \alpha \pi \) and \( \hat{q}^+ = \lim_{z \to \hat{z}^+} q = p (\gamma + \hat{z}) - \alpha \pi \). However, as \( \Pi (\hat{z}, \gamma) = (q + s) \hat{z} - h (\hat{z}) \), profit maximization will lead the producers of green electricity to secure \( \hat{q}^- \) (by an infinitesimal quantity reduction of green electricity) implying the corner solution \( s_{\text{cm}}^* = \pi \).

ii) To show that the GC-price may be at either the upper or the lower price bound, it suffices to give examples satisfying the assumptions of the model. Examples are provided in appendix D and illustrated in Figure 5 and 6.

As in the mc-case we will illustrate two possible equilibrium solutions for the GC-price, one at the lower price bound and one at the upper price bound. In Figure 5 and 6 we have illustrated the profit curves of the producers of green electricity for two sets of assumed parameter values. The numerical examples behind the figures are presented in Appendix D. In the figures, the profit curves are illustrated assuming that the NC-playing producers of green electricity consider the quantity of black electricity as given. Fixing the quantity of black electricity at the equilibrium level and varying the quantity of green electricity, produces the profit curves illustrated in Figure 5 and 6. Again the profit curves drop discontinuously at the point of the allowable consumption level. As opposed to the mc-case, quantity levels below the allowable consumption level will in the cm-case generate a GC-price at the upper price bound. For such low quantities of green electricity, there will be an excess demand for GCs. For higher quantities of green electricity, the GC-price is at the lower bound. As the profit curve in Figure 5 has its global maximum at the right of the allowable consumption level, the GC-price in equilibrium is at the lower price bound. Figure 6, on the other hand, illustrates an equilibrium GC-price at the upper price bound. As in the mc-case, the drop in the profit curves illustrates that an interior GC-price will not be an equilibrium.

**Figure 5 and 6 here**

Proposition 5 shows that the producers of green electricity will never sell less certificates than they generate. Intuitively, the relationship between the number of certificates sold and the equilibrium solution is disconnected when the generator of green electricity act as a NC-oligopolist. Hence, selling an additional certificate has no influence on the market equilibrium and only adds to the profit.

**Proposition 5** Assume the producers of green electricity act as a NC-playing oligopolist (or a perfectly co-ordinated cartel) facing a competitive fringe of pro-
ducers of black electricity, then - in equilibrium - it will never pay to sell less certificates than the amount generated. Formally, we must have $w_{cm}^* = z_{cm}^*$.  

**Proof.** To obtain a contradiction, assume $w_{cm}^* < z_{cm}^*$ in equilibrium. We know the equilibrium implies either $s_{cm}^* = 0$ or $s_{cm}^* = \infty$ and that the number of certificates is not binding, i.e. $w_{cm}^* \neq \alpha (y_{cm}^* + z_{cm}^*)$. Consider the above first order condition $\frac{\partial}{\partial s_{cm}} = \frac{\partial}{\partial z} + \frac{\partial}{\partial w} + s = 0$. As $s_{cm}^*$ is at either the lower or the upper price bound and the number of certificates is not binding we must have $\frac{\partial}{\partial s_{cm}} = 0$ and $\frac{\partial}{\partial z} = 0$, such that $\frac{\partial}{\partial w} = s > 0$. Hence, profits may be increased by selling all the certificates. 

Market power among the producers of green electricity necessarily leads to a reduction of green electricity generation. However, Proposition 6 shows that the generation of black electricity may increase as compared with the competitive equilibrium GC-price if the market power is strong.

**Proposition 6** Under the assumptions of the model: i) $z_{cm}^* < z_{cc}^*$ ii) if $s_{cm}^* = \infty$ then $y_{cm}^* > y_{cc}^*$ and iii) if $s_{cm}^* < \infty$ then sign $(y_{cm}^* - y_{cc}^*)$ is indeterminate.

**Proof.** i) To obtain a contradiction, assume $z_{cm}^* \geq z_{cc}^*$. We consider each possible case with respect to the value of $s_{cc}^*$ for the ce-equilibrium.

Assume first $s_{cc}^* = \infty$. Then, as by assumption, $z_{cm}^* \geq z_{cc}^*$ and $s_{cm}^* < s_{cc}^*$, we have from (7) and (10) $q_{cm}^* = h'(z_{cm}^*) - s_{cm}^* - \frac{dp}{dx_{cm}} < h'(z_{cc}^*) - s_{cc}^* = q_{cc}^*$. By successive substitutions from the two sets of first order conditions we arrive at $p(x_{cm}^*) = (1 - \alpha)q_{cm}^* + \alpha [h'(z_{cm}^*) - \frac{dp}{dx_{cm}}]$ and $p(x_{cc}^*) = (1 - \alpha)q_{cc}^* + \alpha h'(z_{cc}^*)$. As $z_{cm}^* \geq z_{cc}^*$ and $q_{cm}^* > q_{cc}^*$, we have $p(x_{cm}^*) > p(x_{cc}^*)$. However, using (8) and (20) we get $c'(y_{cm}^*) = q_{cm}^* > q_{cc}^* = c'(y_{cc}^*)$ so that $y_{cm}^* > y_{cc}^*$. Consequently, $x_{cm}^* = y_{cm}^* + z_{cm}^* > y_{cc}^* + z_{cc}^* = x_{cc}^*$, so that $p(x_{cm}^*) < p(x_{cc}^*)$. This contradicts the above conclusion that $p(x_{cm}^*) > p(x_{cc}^*)$. Hence, we must have $z_{cm}^* < z_{cc}^*$ as $s_{cc}^* = \infty$.

Next, assume $s_{cc}^* < \infty$. Consider first the possibility that $s_{cm}^* = \infty$. We follow the same line of reasoning as above to arrive at $q_{cm}^* > q_{cc}^*$, as we have from (3), (11) and (19) that $q_{cm}^* = h'(z_{cm}^*) - z_{cm}^* - \frac{dp}{dx_{cm}} > h'(z_{cc}^*) - s_{cc}^* = q_{cc}^*$. Furthermore, by successive substitutions from the two sets of first order conditions we arrive at $p(x_{cm}^*) = (1 - \alpha)q_{cm}^* + \alpha [h'(z_{cm}^*) - \frac{dp}{dx_{cm}}]$ and $p(x_{cc}^*) = (1 - \alpha)q_{cc}^* + \alpha h'(z_{cc}^*)$. Inspection of signs yields $p(x_{cm}^*) > p(x_{cc}^*)$. However, using (4), (12) and (20) we get $c'(y_{cm}^*) = q_{cm}^* > q_{cc}^* = c'(y_{cc}^*)$ so that $y_{cm}^* > y_{cc}^*$. Consequently, $x_{cm}^* = y_{cm}^* + z_{cm}^* > y_{cc}^* + z_{cc}^* = x_{cc}^*$, so that $p(x_{cm}^*) < p(x_{cc}^*)$. This contradicts the above conclusion that $p(x_{cm}^*) > p(x_{cc}^*)$.

It remains to consider the case where $s_{cc}^* < \infty$ and $s_{cm}^* = \infty$. As $s_{cc}^* < \infty$ we must have $(1 - \alpha)z_{cc}^* \geq \alpha y_{cc}^*$ and, furthermore, as $s_{cm}^* = \infty$ we must have $\alpha y_{cm}^* > (1 - \alpha)z_{cm}^*$. Then, as by assumption, $z_{cm}^* \geq z_{cc}^*$ we have $\alpha y_{cm}^* > (1 - \alpha)z_{cm}^* \geq (1 - \alpha)z_{cc}^* \geq \alpha y_{cc}^*$, so that $y_{cm}^* > y_{cc}^*$. Hence, $x_{cm}^* = y_{cm}^* + z_{cm}^* > y_{cc}^* + z_{cc}^* = x_{cc}^*$, such that $p(x_{cm}^*) < p(x_{cc}^*)$. However, using (4), (12) and (20) we get $q_{cm}^* = c'(y_{cm}^*) > c'(y_{cc}^*) = q_{cc}^*$. From the following expressions (derived above): $p(x_{cm}^*) = (1 - \alpha)q_{cm}^* + \alpha [h'(z_{cm}^*) - \frac{dp}{dx_{cm}}]$ and $p(x_{cc}^*) = (1 - \alpha)q_{cc}^* + \alpha h'(z_{cc}^*)$, respectively, we have $p(x_{cm}^*) > p(x_{cc}^*)$. This contradicts the above conclusion.
\((1 - \alpha) q^*_cc + a h^\prime (z^*_cc), \) we are forced to conclude that \(p(x^*_{cm}) > p(x^*_{cc}),\) that contradicts the above conclusion that \(p(x^*_{cm}) < p(x^*_{cc}).\)

Hence, we arrive at the final conclusion that \(z^*_{cm} < z^*_{cc}\) for all cases.

\[\text{ii) To obtain a contradiction, assume } y^*_{cc} \geq y^*_{cm}. \] From (8) and (20) we have \(q^*_{cc} \geq q^*_{cm},\) and from (5) and (17) we have \(p(x^*_{cc}) \geq p(x^*_{me}),\) which implies \(x^*_{cc} \leq x^*_{cm}.\) Then from (6) and (18) we have \(y^*_{cc} + z^*_{cc} \leq y^*_{cm} + z^*_{cm}.\) As \(y^*_{cm} \leq y^*_{cc},\) by assumption, we have \(z^*_{cm} \geq z^*_{cc}\) that is proven to be wrong in i). Hence, we must have \(y^*_{cm} > y^*_{cc}.\)

\[\text{iii) Examples satisfying the assumptions of the model are provided in appendix E.}\]

Market power in the generation of green electricity always induces reduced generation of green electricity and a higher end user price, as compared with the competitive case. The producers of green electricity will always earn a higher profit in the case where they enjoy market power as compared with the cc-equilibrium. On the other hand, also the producers of black electricity may be better off than in the competitive case, and this will always be so in the case of \(s^*_{cc} = \bar{s}.\) The reason is that the wholesale price of electricity will increase in the cm-equilibrium in the case of \(s^*_{cc} = \bar{s},\) as the generation of black electricity increases compared with the competitive solution. However, the effect on the wholesale price in the two other cases, i.e. \(s^*_{cc} = \bar{g} \leq s^*_{cc} < \bar{s},\) is inconclusive because of the indeterminacy of the change in the generation of black electricity. Therefore the effect on the profit of the producers of black electricity is indeterminate in these cases.

\section{Market power in the generation of both black and green electricity}

In this section we will assume that there is market power in both generation technologies. We will therefore have a market in which both the producers of black and green electricity act as NC-playing oligopolists.

The optimization problems for the producers of black and green electricity are identical to the mc-case and the cm-case, respectively. This gives the following first order condition for the NC-playing producers of black electricity:

\[
\frac{\partial \Pi}{\partial y} = \left[ \frac{\partial p}{\partial x} - \alpha \frac{\partial s}{\partial y} \right] y + q - c^\prime (y) = 0.
\]

For the profit maximizing generator of green electricity we consider two cases: a) \(w = z\) and b) \(w < z:\)

a) First order condition:

\[
\frac{\partial \Pi}{\partial z} = \left[ \frac{\partial p}{\partial x} + (1 - \alpha) \frac{\partial s}{\partial z} \right] z + q + s - h^\prime (z) = 0.
\]

b) First order conditions:
\[
\frac{\partial \Pi}{\partial z} = \frac{\partial q}{\partial z} z + q + \frac{\partial s}{\partial z} w - h'(z) = 0 \quad \text{and} \\
\frac{\partial \Pi}{\partial w} = \frac{\partial q}{\partial w} z + \frac{\partial s}{\partial w} w + s = 0.
\]

6.1 \textbf{mm-equilibrium}

We have then the following equilibrium solution for the key variables in case a), i.e. \( z = w \):

\[
p(x_{mm}^*) = q_{mm}^* + \alpha s_{mm}^*
\]

\[
x_{mm}^* = y_{mm}^* + z_{mm}^* \leq \frac{z_{mm}^*}{\alpha}
\]

\[
\left[ \frac{\partial p(x_{mm}^*)}{\partial x} + (1 - \alpha) \frac{\partial s(x_{mm}^*)}{\partial z} \right] z_{mm}^* + q_{mm}^* + s_{mm}^* = h'(z_{mm}^*)
\]

\[
\left[ \frac{\partial p(x_{mm}^*)}{\partial x} - \alpha \frac{\partial s(x_{mm}^*)}{\partial y} \right] y_{mm}^* + q_{mm}^* = c'(y_{mm}^*)
\]

Again, \(<, = \text{and } > \) in equation (22) refer to the cases \( s_{mm}^* = s, s < s_{mm}^* < \bar{s} \) and \( s_{mm}^* = \bar{s} \), respectively. For the cases of \( s_{mm}^* = \bar{s} \) and \( s_{mm}^* = \bar{\bar{s}} \), we have \( \frac{\partial s(x_{mm}^*)}{\partial x} = 0 \). Thus, (23) (24) are reduced to \( \frac{\partial p(x_{mm}^*)}{\partial x} z_{mm}^* + q_{mm}^* = h'(z_{mm}^*) \) and \( \frac{\partial p(x_{mm}^*)}{\partial x} y_{mm}^* + q_{mm}^* = c'(y_{mm}^*) \), respectively.

6.2 \textbf{Analysis}

As in the preceding section, it will be shown (Proposition 7) that it will never be profitable for the producers of green electricity to hold back some of the generated certificates from the market. We concentrate therefore on the main results from case a). These are highlighted proposition 7 and 8.

Proposition 7 shows that, like in the mc- and cm-equilibrium, the certificate market collapses in the sense that the GC-price will never be established at an intermediate level. It may, however, be established at the upper price bound. We further show that again it will not be profitable for the producers of green electricity to utilize their market power to hold back some of the certificates they generate.

Prosoposition 7 Assume both the producers of black and green electricity act as a NC-playing oligopolists, then - in equilibrium - there will i) never be established an intermediate certificate price such that \( s < s_{mm}^* < \bar{s} \), but ii) there may be an equilibrium certificate price at the lower or the upper price bound, i.e. \( s_{mm}^* = \bar{s} \) or \( s_{mm}^* = \bar{\bar{s}} \). Furthermore, we have iii) \( w_{mm}^* = s_{mm}^* \).
Proof. i) The proof follows the same line of reasoning as in the mc- and cm-equilibrium above.

ii) To show that the GC-price may be at either the upper or the lower price bound, it suffices to give examples satisfying the assumptions of the model. Examples are provided in appendix F. An illustration of a possible equilibrium is provided in Figure 7.

iii) The proof for $w^*_{mm} = z^*_{mm}$ is following the same line of reasoning as in the proof for proposition 5.

In Figure 7 we have illustrated a possible equilibrium solution of the numerical model specified in Appendix A. Figure 7 shows the best response curves for the two types of producers in the mm-case for an equilibrium solution with the GC-price at the lower price bound. The figure is based on the example specified in the first part of Appendix F. The best response curve for the producers of green electricity, $R(y)$, is found by fixing the quantity of black electricity at different levels and then identifying the profit maximizing quantity of green electricity at each of these levels. The best response curve for the producers of black electricity, $R(z)$, is produced in the same way, except that we then fix the quantity of green electricity. The equilibrium quantities of black and green electricity is found at the intersection of the best response curves, which confirms the NC-equilibrium solution presented in Appendix F for the assumed parameter values.

Figure 7 here

Proposition 8 states that in the case of market power in both generation technologies the development of the output levels of both black and green electricity is indeterminate as compared with the competitive solution.

Proposition 8 Under the assumptions of the model we have that

\[
sign(y^*_{mm} - y^*_{cc}) \quad \text{and} \quad sign(z^*_{mm} - z^*_{cc})
\]

are both indeterminate, irrespective of whether $\bar{s} < s^*_{cc} < \bar{s}$, $s^*_{cc} = \bar{s}$ or $s^*_{cc} = \bar{s}$.

Proof. It suffices to give examples satisfying the assumptions of the model. Examples are provided in appendix G.

As stated in proposition 8, market power in both generation technologies implies an uncertain effect on the generation of both black and green electricity as compared with the competitive solution. However, the end user price increases as total generation decreases. Both types of producers will use their market power to increase their profits. The effect on the certificate price is indeterminate, except that, as stated in Proposition 7, it will never be realized an interior certificate price in equilibrium.
7 Market power in the joint generation of green and black electricity

In this section we will assume that there is only one generator of electricity. This generator will be able to generate from both renewable and non-renewable sources. The objective function for this generator is:

$$\max \Pi (z, w, y) = q(z + y) + sw - h(z) - c(y), \text{ s.t. } w \leq z.$$ 

We consider two cases: a) $w = z$ and b) $w < z$:

a) First order conditions:

$$\frac{\partial \Pi}{\partial z} = \frac{\partial (q + s)}{\partial z} x + q + s - h'(z) = 0 \text{ and }$$

$$\frac{\partial \Pi}{\partial y} = \frac{\partial q}{\partial y} x + q - c'(z) = 0.$$ 

Or more precisely:

$$\frac{\partial \Pi}{\partial z} = \left[ \frac{\partial p}{\partial x} + (1 - \alpha) \frac{\partial s}{\partial z} \right] x + q + s - h'(z) = 0 \text{ and }$$

$$\frac{\partial \Pi}{\partial y} = \left[ \frac{\partial p}{\partial x} - \alpha \frac{\partial s}{\partial y} \right] x + q - c'(y) = 0.$$ 

b) First order conditions:

$$\frac{\partial \Pi}{\partial z} = \frac{\partial q}{\partial z} x + q + \frac{\partial s}{\partial z} w - h'(z) = 0,$$

$$\frac{\partial \Pi}{\partial y} = \frac{\partial q}{\partial y} x + q + \frac{\partial s}{\partial y} w - c'(y) = 0 \text{ and }$$

$$\frac{\partial \Pi}{\partial w} = \frac{\partial q}{\partial w} x + \frac{\partial s}{\partial w} w + s = 0.$$ 

7.1 M-equilibrium

We have then the following equilibrium solution for the key variables in case a), i.e. $z = w$:

$$p(x_M^*) = q_M^* + \alpha s_M^*$$  \hspace{1cm} (25)

$$x_M^* = y_M^* + z_M^* \leq \frac{z_M^*}{\alpha}$$  \hspace{1cm} (26)

$$\left[ \frac{\partial p(x_M^*)}{\partial x} + (1 - \alpha) \frac{\partial s(x_M^*)}{\partial z} \right] x_M^* + q_M^* + s_M^* = h'(z_M^*)$$  \hspace{1cm} (27)
Examples are provided in appendix H.

As observed in Proposition 9, the profit function for the monopolist reduces to \( \Pi(z, w, y) = p(x) x - c(y) - h(z) \) as \( s < s_M^* < \bar{s} \). Hence, the first order condition for the monopolist reads \( p + \partial_p x = c'(y) = h'(z) \), i.e. just as for any other multiplant monopolist. In the case of a market based GC-price, there are no subsidies paid by the state and it is interesting to note that the monopolist does not itself subsidize its own expensive green technology.

\[
\left[ \frac{\partial p(x_M^*)}{\partial x} - \alpha \frac{\partial s(x_M^*)}{\partial y} \right] x_M^* + q_M^* = c'(y_M^*) \tag{28}
\]

In equation (26), the operators \( <, = \) and \( > \) refer to the cases \( s_M^* = \underline{s} \), \( \underline{s} < s_M^* < \bar{s} \) and \( s_M^* = \bar{s} \), respectively. Remember that, (27) and (28) are reduced to \( \frac{\partial p(x_M^*)}{\partial x} x_M^* + q_M^* = h'(z_M^*) \) and \( \frac{\partial p(x_M^*)}{\partial x} x_M^* + q_M^* = c'(y_M^*) \), respectively, in the cases of \( s_M^* = \underline{s} \) and \( s_M^* = \bar{s} \).

### 7.2 Analysis

As will be shown below, it will never be profitable for the generator to hold back some of the generated certificates from the market. We will therefore not go into detail about the equilibrium solution of the case where \( w < z \), but rather concentrate on the main results from case a). We highlight these results in Proposition 9 and 10. Proposition 9 shows that although there does not exist intermediate equilibrium certificate prices in any of the preceding cases of market power, there will indeed exist such prices when there is joint (monopolistic) market power for black and green electricity producers.

**Proposition 9** Assume the producers have both green and black technologies at their disposal and act as a monopolist, then - in equilibrium - a certificate price may be established at i) an intermediate level, i.e. \( \underline{s} < s_M^* < \bar{s} \) or ii) either of the price bounds, i.e. \( s_M^* = \underline{s} \) or \( s_M^* = \bar{s} \). Furthermore, we have iii) \( w_M^* = z_M^* \).

**Proof.** i) To show that there may be interior certificate prices, \( \underline{s} < s_M^* < \bar{s} \), it suffices to give an example. This is provided in Appendix H. The essential reason for the existence of such interior prices is that the monopolist is indifferent with respect to securing the high, the low or some intermediate certificate price (and correspondingly for the wholesale price) for the case where the optimal solution satisfies \( \hat{x} = \hat{y} = \hat{z} = \frac{w}{\alpha} \) with \( \hat{w} \leq \hat{z} \). To see this, consider the profit function for the monopolist \( \Pi(\hat{z}, \hat{y}, \hat{w}) = q\hat{x} + s\hat{w} - c(\hat{y}) - h(\hat{z}) \). This may be rewritten: \( \Pi(\hat{z}, \hat{y}, \hat{w}) = p\hat{x} + (\hat{w} - \alpha\hat{x}) s - c(\hat{y}) - h(\hat{z}) \). However, as \( \hat{x} = \frac{w}{\alpha} \) the profit function reduces to \( \Pi(\hat{z}, \hat{y}, \hat{w}) = p\hat{x} - c(\hat{y}) - h(\hat{z}) \). Hence, the value of \( s \) does not matter. Intuitively, a larger certificate price is exactly offset by a smaller wholesale price for this case.

ii) To show that there may be a GC-price at either the upper or the lower price bound, it suffices to give examples satisfying the assumptions of the model. Examples are provided in appendix H.

iii) The proof for \( w_M^* = z_M^* \) is following the same line of reasoning as in the proof for proposition 5. \( \blacksquare \)
Proposition 10 Under the assumptions of the model: \( y_M^* < y_{cc}^* \) and \( z_M^* < z_{cc}^* \).

**Proof.** The proposition is proved by considering the following complete set of cases: a) \( s_{cc}^* = \tilde{s} \), b) \( s_{cc}^* = \tilde{\pi} \), c) \( \tilde{s} < s_{cc}^* < \tilde{\pi} \)

a) To obtain a contradiction, assume \( y_M^* \geq y_{cc}^* \). Substitute (12) into (11) and (28) into (27) to obtain \( \tilde{s} = h'(z_{cc}^*) - c'(y_{cc}^*) \leq h'(z_M^*) - c'(y_M^*) = s_M^* \).

As \( y_M^* \geq y_{cc}^* \), we must have \( z_M^* \geq z_{cc}^* \), so that \( x_M^* \geq x_{cc}^* \) and \( p(x_M^*) \leq p(x_{cc}^*) \).

However, successive substitution of the two sets of equilibrium conditions yields \( p(x_M^*) = (1 - \alpha) c'(y_M^*) + ah'(z_M^*) - \frac{\partial p}{\partial x} x_M^* \) and \( p(x_{cc}^*) = (1 - \alpha) c'(y_{cc}^*) + ah'(z_{cc}^*) \).

To obtain a contradiction, assume \( z_M^* \geq z_{cc}^* \) and \( y_M^* \geq y_{cc}^* \) clearly gives \( p(x_M^*) > p(x_{cc}^*) \) that contradicts the above result. Hence, we have \( y_M^* < y_{cc}^* \). Next, we show \( z_M^* < z_{cc}^* \). As \( s_{cc}^* = \tilde{s} \), we must have \( (1 - \alpha) z_{cc}^* > \alpha y_{cc}^* \). Consider the possibility that \( s_M^* > \tilde{s} \) (we consider \( s_M^* = \tilde{s} \) below). If this is to be the case, then \( \alpha y_{cc}^* \geq (1 - \alpha) z_M^* > \alpha y_{cc}^* \), so that \( (1 - \alpha) z_{cc}^* > \alpha y_{cc}^* \). Hence, we must have \( z_M^* < z_{cc}^* \).

b) To obtain a contradiction, assume \( z_M^* \geq z_{cc}^* \). Substitute (8) into (7) and (28) into (27) to obtain \( \tilde{\pi} = h'(z_{cc}^*) - c'(y_{cc}^*) \geq h'(z_M^*) - c'(y_M^*) = s_M^* \). Clearly, \( z_M^* \geq z_{cc}^* \) implies \( y_M^* \geq y_{cc}^* \) so that \( x_M^* \geq x_{cc}^* \) and \( p(x_M^*) \leq p(x_{cc}^*) \). From here on the proof follows the last part of the proof under a). Hence, we must have \( z_M^* < z_{cc}^* \). Next, we show \( y_M^* < y_{cc}^* \) for this case.

As \( s_{cc}^* = \tilde{\pi} \), we must have \( \alpha y_{cc}^* > (1 - \alpha) z_{cc}^* \). Consider the possibility that \( s_M^* < \tilde{\pi} \) (we consider \( s_M^* = \tilde{\pi} \) below). If this is to be the case, then \( (1 - \alpha) z_M^* \geq \alpha y_{cc}^* > (1 - \alpha) z_{cc}^* \), so that \( (1 - \alpha) z_{cc}^* \geq \alpha y_{cc}^* \). Hence, we must have \( y_M^* < y_{cc}^* \). It remains to show the case of \( s_M^* = \tilde{\pi} \). Substitute (12) into (11) and (28) into (27) to obtain \( s_{cc}^* = \tilde{s} = h'(z_{cc}^*) - c'(y_{cc}^*) = h'(z_M^*) - c'(y_M^*) = \tilde{s} = s_M^* \). Clearly, \( y_M^* < y_{cc}^* \) implies \( z_M^* < z_{cc}^* \). This completes the proof of case c).

c) Observe that if \( \tilde{s} < s_{cc}^* < \tilde{\pi} \), then \( \tilde{\pi} - s_{cc}^* = \frac{\gamma_{cc} - \gamma_M}{1 - \alpha} = x_{cc}^* \). Hence, from the relationship \( p(x_M^*) = (1 - \alpha) c'(y_M^*) + ah'(z_M^*) \) we have \( p(x_M^*) = (1 - \alpha) c'(y_M^*) + ah'(z_{cc}^*) \).

Next, consider the possibility that \( \tilde{s} < s_{cc}^* < \tilde{\pi} \) (the possibilities that \( s_M^* = \tilde{s} \) and \( s_M^* = \tilde{\pi} \) are considered below). We then know that \( p(x_M^*) + \frac{\partial p}{\partial x} x_M^* = c'(y_{cc}^*) = h'(z_{cc}^*) \). This may be rewritten: \( p(x_M^*) = (1 - \alpha) c'(y_M^*) + ah'(z_M^*) - \frac{\partial p}{\partial x} x_M^* \). However, as \( \tilde{s} < s_{cc}^* \) we know that \( \frac{\gamma_{cc} - \gamma_M}{1 - \alpha} = x_{cc}^* \). Inserting these relationships in the above expression we arrive at \( p(x_M^*) = (1 - \alpha) c'(y_M^*) + ah'(z_M^*) \). Hence, \( p(x_M^*) = (1 - \alpha) c'(y_M^*) + ah'(z_M^*) \).

To obtain a contradiction assume \( x_M^* \geq x_{cc}^* \). Observe that the l.h.s. of the above expression is non-positive, while the r.h.s. is strictly positive. Hence, we must have \( x_M^* < x_{cc}^* \) and \( y_M^* = (1 - \alpha) x_M^* < (1 - \alpha) x_{cc}^* = y_{cc}^* \). Next, consider the case of \( s_M^* = \tilde{\pi} \). To obtain a contradiction, assume \( y_M^* < y_{cc}^* \). Substituting (4) into (3) and (28) into (27) we have: \( s_{cc}^* = h'(z_{cc}^*) -
\( c' (y_{cc}^*) < h' (z_M^*) - c' (y_M^*) = \pi. \) Clearly, the assumption that \( y_{cc}^* \geq y_{cc} \) implies \( z_M^* \geq z_{cc}^* \) so that \( x_M^* > x_{cc}^* \) and \( p(x_M^*) < p(x_{cc}^*). \) From here, we follow the part of the proof under a) showing that we cannot have \( p(x_M^*) \leq p(x_{cc}^*). \) Hence, we must have \( y_M^* < y_{cc}^* \). We know that this also implies \( z_M^* < z_{cc}^* \). To see this, observe that \( s < s_{cc}^* < \pi \) implies \( (1 - \alpha) z_{cc}^* = \alpha y_{cc}^* \) and that \( s_{cc}^* = \pi \) implies \( \alpha y_{cc}^* > (1 - \alpha) z_{cc}^*. \) Hence, \( (1 - \alpha) z_{cc}^* = \alpha y_{cc}^* > \alpha y_M^* > (1 - \alpha) z_M^* \), so that \( z_M^* < z_{cc}^* \).

To complete the proof we now consider \( s_{cc}^* = s. \) To obtain a contradiction, assume \( z_M^* = z_{cc}^* \). Substituting (12) into (11) and (28) into (27) we get \( s_{cc}^* = h' (z_{cc}^*) - c' (y_{cc}^*) > h' (z_M^*) - c' (y_M^*) = s. \) Clearly, as by assumption \( z_M^* \geq z_{cc}^* \) we must also have \( y_M^* \geq y_{cc}^* \) and \( x_M^* \geq x_{cc}^* \) so that \( p(x_M^*) \leq p(x_{cc}^*). \) From here we follow the part of the proof under a) showing that we cannot have \( p(x_M^*) \leq p(x_{cc}^*). \) Hence, we must have \( z_M^* < z_{cc}^* \).

Finally, we show that \( z_M^* < z_{cc}^* \) implies \( y_M^* < y_{cc}^* \) for the case of \( s_M^* = s. \) Observe that \( s < s_{cc}^* < \pi \) implies \( \alpha y_{cc}^* > (1 - \alpha) z_{cc}^* \) and that \( s_{cc}^* = \pi \) implies \( (1 - \alpha) z_{cc}^* > \alpha y_M^*. \) Hence, \( \alpha y_{cc}^* = (1 - \alpha) z_{cc}^* > (1 - \alpha) z_{cc}^* > \alpha y_{cc}^* = \alpha y_M^*, \) so that \( y_M^* < y_{cc}^* . \)

As expected, the case of joint market power will reduce the generation of both green and black electricity as compared with the competitive solution. The joint market power will also increase the profit from both generation technologies. As stated in proposition 7 above, the certificate price may go in both directions in this case.

8 Compatibility of GCs and CO2 taxes

Assume the producers of black electricity are subject to a CO2 tax, \( t, \) per unit kWh generated. Under perfect competition this gives rise to the following modification of the first order condition for the producers of black electricity:

\[ q = c' (y) + t. \]  \hfill (4' , 8' , 12')

Otherwise, the conditions remain the same. The next proposition shows that - contrary to what one should expect - an increase of the CO2 tax leads to a reduction of the generation of green electricity, provided that the "percentage requirement" is binding and that there is an intermediate certificate price. Hence, the CO2 tax does not stimulate the generation of CO2-free electricity in this setting. However, if the certificate price is at the price bounds it will stimulate the generation of green electricity. To see this use condition (4') instead of (4), (8') instead of (8) and (12') instead of (12).

Proposition 11 Under the assumptions of the model i) \( \frac{ds_{cc}^*}{dt} < 0 \) and \( \frac{dy_{cc}^*}{dt} < 0, \) provided that \( s < s_{cc}^* < \pi \) and ii) \( \frac{dz_{cc}^*}{dt} < 0 \) and \( \frac{dy_{cc}^*}{dt} < 0, \) provided that \( s_{cc}^* = \pi. \)

Proof. See appendix I. \hfill \blacksquare
The intuition for the somewhat surprising first result (i)) is that a CO$_2$ tax leads to an increase in the wholesale price of electricity. However, for each dollar of increased wholesale price the producers of green electricity suffer a loss of $1/\alpha$ dollars. Hence, the producers of green electricity stands to lose as the sum of the wholesale price and the certificate price, $q + s$, is reduced. This then leads to a reduction of the generation of green electricity.

The next proposition shows that a CO$_2$ tax may stimulate green electricity generation for the cases where market power prevails. This will definitely be so if the demand is linear.

**Proposition 12** Under the assumptions of the model i) $\frac{dz_{cm}}{dt} > 0$ and $\frac{dy_{cm}}{dt} < 0$; ii) $\frac{dz_{mm}}{dt} > 0$ and $\frac{dy_{mm}}{dt} < 0$; iii) $\frac{dz_{M}}{dt} > 0$ and $\frac{dy_{M}}{dt} < 0$, provided that $\frac{\partial^2 p}{\partial x^2} \leq 0$.

Proof. See appendix J. ■

Hence, there is nothing surprising in these results. One should remember that the certificate system faced with market power basically has collapsed into a system of per unit subsidies with well known effects.

### 9 Summary and concluding remarks

In this paper we have focused on the effect of market power in a system of Green Certificates (GCs). We have developed an analytic model for the electricity and the GC markets and derived results under different assumptions of competition.

In particular, we have focused on the generation of green electricity. The main results are:

- Under perfect competition, the effect of changing the percentage requirement, $\alpha$, for the obligatory share of GCs a consumer must hold is most inconclusive. If the GC-price is at the lower or upper price bound, an increase of the percentage requirement will actually lead to a decrease in the generation of green electricity. On the other hand, at intermediate GC-prices the effect on generation of green electricity is indeterminate. Hence, increasing the obligatory share of GCs may in fact lead to lower end-use prices and larger consumption under specific formulations of the demand and cost functions. However, in the cases of a GC-price at one of the price bounds, the effect of increasing $\alpha$ will always be reduced generation of green electricity and also reduced total generation. Thus, in these cases one of the costs associated by increasing the generation of green electricity within a GC system is a reduction of consumer and producer surplus.

- The existence of market power in either the generation of black electricity or in the generation of green electricity, and in the case of market power in both generation technologies, will all induce a GC-price at either the upper or the lower price bound. A market based GC-price within the price
bound will never exist in these three cases. In the case of market power in the joint production of black and green electricity we may, however, get an intermediate GC-price.

- Market power exercised by the producers of black electricity may actually lead to an increase of the generation of green electricity as compared with the competitive equilibrium. This is definitely true if the competitive equilibrium GC-price is at its lower bound. The generation of black electricity will, however, always decrease in this case.

- Market power exercised by the producers of green electricity will provide these producers with the possibility to hold back some of the GCs they generate. However, we show that this will never be profitable for them, i.e., the producers of green electricity will never sell less certificates than they generate. This result is also valid in the case of market power in the joint production of black and green electricity.

- Market power among the producers of green electricity necessarily leads to a reduction of green electricity generation. However, the generation of black electricity may increase as compared with the competitive equilibrium. This is definitely true if the competitive equilibrium GC-price is at its upper bound.

- Market power in both generation technologies, resulted in an indeterminate effect on the generation of both black and green electricity, as compared with the competitive case.

- Market power in the joint generation of black and green electricity by a monopolistic company, is shown to induce a reduction in the generation of both black and green electricity from the competitive equilibrium.

- The introduction of a $CO_2$-tax on the producers of black electricity in a GC system may actually induce a reduction in the generation of green electricity. This happens in the competitive case, given an intermediate GC-price. Finally, it has been shown that a $CO_2$ tax may stimulate green electricity generation for the cases where market power prevails. This will definitely be so if the demand is linear.

Based on the above summary it seems fair to conclude that the introduction of a GC system as a means for promoting green electricity generation includes a number of potential pitfalls. As our analysis has shown, a GC system may in many cases induce unexpected effects on the generation of green electricity. In particular the GC system’s sensitivity to market power should be noticed. As market power in most cases will prevent the realization of a market based GC-price within the specified price interval, the GC system will most likely be reduced to a system of direct subsidies financed through consumer/producer taxes. Considering the large probability that some form of market power will exist and the possible high administration costs associated by a GC system,
this result should be given serious attention. It can hardly be cost efficient to introduce a GC system that in the end will function just like an ordinary subsidy scheme.

The main objective of a GC system is to promote the generation of green electricity. To the extent that the generation of green electricity substitutes black electricity, an additional effect of the GC system may be to reduce \( CO_2 \) emission. However, compared with a system of \( CO_2 \)-taxes or quotas, a GC system is less differentiated with respect to the emission of green house gases. At least under the versions of the system considered in various countries at the moment, a generation technology is either green or black. A generation technology with low emission of \( CO_2 \) (e.g. a natural gas power plant) will, in a GC system, be treated on equal terms as a technology with relatively higher emissions (e.g. a coal power plant). In addition, the GC system considers all green electricity as equal. It pays no attention to the fact that also green generation technologies may have negative external effects, e.g. the noise and negative effects in the landscape caused by wind mill farms.

The GC system will typically induce higher prices and lower consumption of electricity. The decision about the percentage requirement, \( \alpha \), will be of importance for the welfare effects of the GC system. In addition, as shown for the competitive equilibrium, it is not generally true that an increase of \( \alpha \) leads to larger generation of green electricity in equilibrium. When deciding the size of \( \alpha \) one should therefore weigh the negative effects on consumer and producer surplus against the assumed positive effects on the generation of green electricity.

On the basis of the results obtained in this article we recommend a great amount of consideration to be done before implementing a GC system.

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Appendix

A A numerical model

In this appendix we will present a simple numerical model satisfying the as-
sumptions we have made about the electricity market. The model will be used
to provide proofs for the existence of some of the results referred to in the
propositions in this article. We assume the following functions:

The inverse demand function is given by:

\[
p(x) = a - bx, \text{ with } a, b > 0.
\]

This gives:

\[
p'(x) = -b < 0.
\]

The technology for generation of black electricity is summarized in the cost
function:

\[
c(y) = \frac{1}{2} y^2, \text{ with } c'(y) = y > 0 \text{ and } c''(y) = 1 \geq 0.
\]

The producers of green electricity have the following cost function:

\[
h(z) = \frac{c}{2} z^2 + g z, \text{ where } c, g > 0, \text{ with } h'(z) = cz + g > 0 \text{ and } h''(z) = c \geq 0.
\]

Running the model produces three sets of possible equilibrium solutions,
one for each of the cases where the GC-price is at either the upper or the lower
bound, and one for the case of an "interior" solution where the GC-price is
decided in the market. However, for an equilibrium solution to be valid, all
the first order conditions must be satisfied. Focus therefore on the first order
conditions specified in equation (14). This expression state that in the case of an
interior solution for the GC-price, total consumption, \(x^*\), in equilibrium, shall
be equal to \( z^* \), while for GC-prices at either the lower or at the upper bound, total consumption shall be higher or lower than \( z^* \), respectively. A violation of this condition will mean that the composition of green and black electricity is incompatible with the amount of GCs sold in the market, i.e. it is not an equilibrium solution. The GC system states that we shall always have \( z \geq \alpha x \). The only case where this condition can be violated is when there is an excess demand for GCs. In that case, the consumers can increase the share of black electricity in their consumption above \( \alpha \) by paying a fine corresponding to the upper price bound. Thus in such a situation we must have \( x > \frac{z^*}{\alpha} \), or \( z^* x < \alpha \). Otherwise, the share of green electricity, as part of total electricity consumption, must be at least equal to the specified \( \alpha \). This means that in the case where we are at the lower price bound for the GC-price we must have that \( \frac{z^*}{\alpha} \geq \alpha \). If this is not the case, the consumption of green electricity as part of total electricity consumption is too low and the rules of the GC system are violated. The NC-playing producers must then chose one of the two other equilibrium solutions. Therefore, for a solution of the numerical model to be valid, the consistency condition must be fulfilled in equilibrium. We will therefore in our examples below include a consistency check variable for each equilibrium solution.

### B Proposition 2

First, according to proposition 2 i) we will show an example of a case where the numerical model generates an interior GC-price in the mc-case. This is used as background for Figure 2, which shows that this cannot be a NC-equilibrium. Then we will show that if the producers of black electricity act as a NC-playing oligopolist (or a perfectly co-ordinated cartel) facing green producers as a competitive fringe, the price of GCs can be either at the lower bound, \( s \), or at the upper bound, \( \bar{s} \).

The following parameter values are used to generate an interior GC-price:

\[ \alpha = 0.6, \quad a = 100, \quad b = 1, \quad c = 2, \quad g = 5, \quad \bar{s} = 50 \text{ and } \underline{s} = 10. \]

The result is presented in the table below.

<table>
<thead>
<tr>
<th>( s^*_{mc} )</th>
<th>( \underline{s}^*_{mc} )</th>
<th>( \bar{s}^*_{mc} )</th>
<th>( x^*_{mc} )</th>
<th>( y^*_{mc} )</th>
<th>( q^*_{mc} )</th>
<th>( s^*_{mc} )</th>
<th>( \Pi (z^*_{mc}) )</th>
<th>( \Pi (y^*_{mc}) )</th>
<th>( \frac{\underline{s}^<em>_{mc}}{z^</em>_{mc}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interior</td>
<td>31.2</td>
<td>20.8</td>
<td>52.0</td>
<td>48.0</td>
<td>20.8</td>
<td>45.4</td>
<td>936.9</td>
<td>215.9</td>
<td>0.6</td>
</tr>
<tr>
<td>Min.</td>
<td>25.4</td>
<td>22.9</td>
<td>48.3</td>
<td>51.8</td>
<td>45.8</td>
<td>50</td>
<td>643.9</td>
<td>784.9</td>
<td>0.526</td>
</tr>
<tr>
<td>Max.</td>
<td>34.4</td>
<td>11.9</td>
<td>46.3</td>
<td>53.8</td>
<td>23.8</td>
<td>50</td>
<td>1181.6</td>
<td>211.5</td>
<td>0.743</td>
</tr>
</tbody>
</table>

The value in the last column is used as a consistency check. In this case, none of the solutions with a GC-price at one of the price bounds fulfill the consistency condition. Hence, the only possible solution is for \( s^*_{mc} = 45.4 \), which is within the price interval. However, as proved formally in Proposition 2 i), and illustrated in Figure 2, this is not a NC-equilibrium as the producers of black electricity can increase their profit by reducing their generation by \( \epsilon \), implying the corner solution \( s^*_{mc} = \underline{s} \).
Proving Proposition 2 (ii), we are assuming the following values for the exogenous variables:
\( \alpha = 0.5, \ a = 100, \ b = 1, \ c = 2, \ g = 5, \ \bar{s} = 28 \) and \( s = 10. \)

<table>
<thead>
<tr>
<th>( s_{mc}^* )</th>
<th>( y_{mc}^* )</th>
<th>( z_{mc}^* )</th>
<th>( p_{mc}^* )</th>
<th>( q_{mc}^* )</th>
<th>( s_{mc}^* )</th>
<th>( \Pi (z_{mc}^*) )</th>
<th>( \Pi (y_{mc}^*) )</th>
<th>( \frac{z_{mc}^<em>}{p_{mc}^</em>} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interior</td>
<td>28.1</td>
<td>28.1</td>
<td>56.3</td>
<td>43.7</td>
<td>31.1</td>
<td>735.7</td>
<td>396.0</td>
<td>0.5</td>
</tr>
<tr>
<td>Min.</td>
<td>25.6</td>
<td>23.1</td>
<td>48.8</td>
<td>51.3</td>
<td>46.3</td>
<td>10</td>
<td>656.6</td>
<td>802.1</td>
</tr>
<tr>
<td>Max.</td>
<td>30.1</td>
<td>18.6</td>
<td>48.8</td>
<td>51.3</td>
<td>37.3</td>
<td>28</td>
<td>907.5</td>
<td>520.3</td>
</tr>
</tbody>
</table>

The NC playing generator of black electricity will choose the quantity that maximizes its profit. Actually we can already exclude the interior solution as we have shown that this will never be optimal for a generator with market power. We can see from the table above that \( y_{mc}^* = 23.1 \) is the profit maximizing quantity of black electricity. This corresponds to a GC-price at the lower price bound. However, to be sure that this is an attainable equilibrium we must check the consistency condition. From the above discussion we know that in the case of \( s_{mc}^* = s \) we must have \( \frac{z_{mc}^*}{p_{mc}^*} > \alpha \). In the example we have \( \alpha = 0.5 \) so this is a valid solution with \( s_{mc}^* = s \).

Using the same parameter values as above, but with \( \alpha = 0.65 \) and \( \bar{s} = 15 \) provides the following result.

<table>
<thead>
<tr>
<th>( s_{mc}^* )</th>
<th>( z_{mc}^* )</th>
<th>( y_{mc}^* )</th>
<th>( x_{mc}^* )</th>
<th>( p_{mc}^* )</th>
<th>( q_{mc}^* )</th>
<th>( s_{mc}^* )</th>
<th>( \Pi (z_{mc}^*) )</th>
<th>( \Pi (y_{mc}^*) )</th>
<th>( \frac{z_{mc}^<em>}{p_{mc}^</em>} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interior</td>
<td>32.1</td>
<td>17.3</td>
<td>49.4</td>
<td>50.6</td>
<td>17.3</td>
<td>51.1</td>
<td>1006.4</td>
<td>149.8</td>
<td>0.65</td>
</tr>
<tr>
<td>Min.</td>
<td>25.3</td>
<td>22.8</td>
<td>48.0</td>
<td>52.0</td>
<td>45.5</td>
<td>10</td>
<td>637.6</td>
<td>776.3</td>
<td>0.526</td>
</tr>
<tr>
<td>Max.</td>
<td>26.3</td>
<td>21.3</td>
<td>47.6</td>
<td>52.4</td>
<td>42.6</td>
<td>15</td>
<td>692.3</td>
<td>681.3</td>
<td>0.552</td>
</tr>
</tbody>
</table>

Again, the monopolist will want the situation where the GC-price is at the lower price bound. However, this gives \( \frac{z_{mc}^*}{p_{mc}^*} = 0.526 < \alpha \). The solution is therefore not consistent with the "percentage requirement". The second best solution for the monopolist is to choose \( y_{mc}^* = 17.6 \), which corresponds to a GC-price at the upper bound, i.e. we have \( s_{mc}^* = \bar{s} \).

We have thus proved the existence of \( s_{mc}^* = \bar{s} \) and \( s_{mc}^* = s \).

C Proposition 3

We want to show the existence of \( z_{mc}^* < z_{mc}^* \) and \( z_{mc}^* > z_{mc}^* \), given that the competitive GC-price is above the lower price bound, i.e. \( s_{cc}^* > \bar{s} \).

We assume the following parameter values:
\( \alpha = 0.46, \ a = 100, \ b = 1, \ c = 2, \ g = 5, \ \bar{s} = 28 \) and \( s = 10. \)

The result is showed in the tables below.
The quantity of green electricity is OK and we have Thus we have interior solution with takers. We see from the

### Table 4

<table>
<thead>
<tr>
<th>$s_{mc}$</th>
<th>$z_{mc}$</th>
<th>$y_{mc}$</th>
<th>$x_{mc}$</th>
<th>$p_{mc}$</th>
<th>$q_{mc}$</th>
<th>$s_{mc}$</th>
<th>$\Pi (s_{mc})$</th>
<th>$\Pi (y_{mc})$</th>
<th>$z_{mc}$</th>
<th>$\frac{z_{mc}}{s_{mc}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interior</td>
<td>26.5</td>
<td>31.1</td>
<td>57.7</td>
<td>42.3</td>
<td>31.1</td>
<td>24.4</td>
<td>635.8</td>
<td>484.7</td>
<td>0.46</td>
<td></td>
</tr>
<tr>
<td>Min.</td>
<td>25.7</td>
<td>23.2</td>
<td>49.0</td>
<td>51.1</td>
<td>46.5</td>
<td>10</td>
<td>661.8</td>
<td>809.1</td>
<td>0.526</td>
<td></td>
</tr>
<tr>
<td>Max.</td>
<td>30.4</td>
<td>18.9</td>
<td>49.3</td>
<td>50.7</td>
<td>37.8</td>
<td>28</td>
<td>924.5</td>
<td>536.1</td>
<td>0.617</td>
<td></td>
</tr>
</tbody>
</table>

Under perfect competition in both technologies, all the producers are price takers. We see from the first table that the competitive equilibrium is at the interior solution with $s_{mc} = 26.7$. The consistency is OK as $\frac{z_{mc}}{s_{mc}} = 0.46 = \alpha$. Thus we have $z_{mc} = 26.2$. From the second table we observe that the equilibrium quantity of green electricity is $z_{mc} = 25.7$. The consistency check proves to be OK and we have $z_{mc} < z_{mc}^*$. Changing the lower price bound on GCs to $\underline{\alpha} = 15$ and solving the model again we get the following result:

### Table 5

<table>
<thead>
<tr>
<th>$s_{mc}$</th>
<th>$z_{mc}$</th>
<th>$y_{mc}$</th>
<th>$x_{mc}$</th>
<th>$p_{mc}$</th>
<th>$q_{mc}$</th>
<th>$s_{mc}$</th>
<th>$\Pi (s_{mc})$</th>
<th>$\Pi (y_{mc})$</th>
<th>$z_{mc}$</th>
<th>$\frac{z_{mc}}{s_{mc}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interior</td>
<td>26.2</td>
<td>30.8</td>
<td>57.0</td>
<td>43.0</td>
<td>30.8</td>
<td>26.7</td>
<td>686.9</td>
<td>473.3</td>
<td>0.46</td>
<td></td>
</tr>
<tr>
<td>Min.</td>
<td>22.6</td>
<td>35.2</td>
<td>57.9</td>
<td>42.1</td>
<td>35.2</td>
<td>15</td>
<td>511.7</td>
<td>620.9</td>
<td>0.391</td>
<td></td>
</tr>
<tr>
<td>Max.</td>
<td>26.6</td>
<td>30.2</td>
<td>56.9</td>
<td>43.1</td>
<td>30.2</td>
<td>28.0</td>
<td>708.8</td>
<td>457.5</td>
<td>0.468</td>
<td></td>
</tr>
</tbody>
</table>

The competitive equilibrium generates then $z_{mc}^* = 26.2$, while oligopolistic producers of black electricity choose $y_{mc}^* = 22.0$, which implies $z_{mc} < z_{mc}^*$. Thus, we have shown the existence of both $z_{mc} < z_{mc}^*$ and $z_{mc} > z_{mc}^*$.

### Proposition 4

According to proposition 4 ii) we will show that if the producers of green electricity act as a NC-playing oligopolist (or a perfectly co-ordinated cartel) facing green producers as a competitive fringe, the price of GCs can be either at the lower bound, $\underline{\alpha}$, or at the upper bound, $\overline{\alpha}$. 

31
First, we solve the model assuming the same parameter values as in the first part of the example for proposition 2 above. Thereafter we solve the model assuming \(\alpha = 0.2\) and \(s = 10\). The results are presented in the following two tables. As we have proved that a solution with a certificate price within the price interval never will be chosen in the cm-case, we are omitting the interior solution from the results.

<table>
<thead>
<tr>
<th>(s^*_cm)</th>
<th>(z^*_cm)</th>
<th>(y^*_cm)</th>
<th>(\pi^*_cm)</th>
<th>(q^*_cm)</th>
<th>(s^*_cm)</th>
<th>(\Pi(z^*_cm))</th>
<th>(\Pi(y^*_cm))</th>
<th>(\bar{z}_{cm})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min. 15.0</td>
<td>40.0 55.0</td>
<td>45.0 40.0</td>
<td>10 450.0</td>
<td>800.0 0.273</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max. 18.9</td>
<td>33.6 52.4</td>
<td>41.6 33.6</td>
<td>28 711.2</td>
<td>563.5 0.360</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As we can see from the first table, the generator of green electricity will choose the equilibrium solution with \(s^*_cm = \pi\), which generates the highest profit. In the second table, the generator also wants to be where the GC-price is at the upper bound. However, the consistency condition in not fulfilled as \(\frac{s^*_cm}{\bar{z}_{cm}} > \alpha\), so this is not an attainable equilibrium. Therefore, the generator chooses the solution which generates a certificate price at the lower bound, i.e. \(s^*_cm = \bar{\pi}\).

### E Proposition 6

We want to show the existence of \(y^*_cm < y^*_cc\) and \(y^*_cm > y^*_cc\), given that the competitive GC-price is below the upper price bound, i.e. \(s^*_{cc} < \pi\).

We assume the following parameter values:
\(\alpha = 0.4\), \(a = 100\), \(b = 1\), \(c = 2\), \(g = 5\), \(\pi = 25\) and \(s = 15\).

The result is showed in the tables below. As for proposition 4 we are omitting the interior solution for the market power case as this is proved to never be realized.

<table>
<thead>
<tr>
<th>(s^*_{cc})</th>
<th>(z^*_{cc})</th>
<th>(y^*_{cc})</th>
<th>(\pi^*_{cc})</th>
<th>(q^*_{cc})</th>
<th>(s^*_{cc})</th>
<th>(\Pi(z^*_{cc}))</th>
<th>(\Pi(y^*_{cc}))</th>
<th>(\bar{z}_{cc})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interior 23.3</td>
<td>35.0 58.3</td>
<td>41.7 35.0</td>
<td>16.7 544.4</td>
<td>612.5 0.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Min. 22.8</td>
<td>35.6 58.4</td>
<td>41.6 35.6</td>
<td>15 519.8</td>
<td>633.7 0.390</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max. 26.0</td>
<td>32.0 58.0</td>
<td>42.0 32.0</td>
<td>25 676.0</td>
<td>512.0 0.448</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(s^*_{cm})</th>
<th>(z^*_{cm})</th>
<th>(y^*_{cm})</th>
<th>(\pi^*_{cm})</th>
<th>(q^*_{cm})</th>
<th>(s^*_{cm})</th>
<th>(\Pi(z^*_{cm}))</th>
<th>(\Pi(y^*_{cm}))</th>
<th>(\bar{z}_{cm})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min. 16.3</td>
<td>38.9 55.1</td>
<td>44.9 38.9</td>
<td>15 530.4</td>
<td>754.9 0.295</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Max. 18.6</td>
<td>35.7 54.3</td>
<td>45.7 35.7</td>
<td>25 689.8</td>
<td>637.8 0.342</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The competitive equilibrium generates \( s^*_{cc} = 16.7 < \bar{\pi} = 25 \), which implies \( y^*_{cc} = 35.0 \). Market power among the producers of green electricity makes them choose \( s^*_{cm} = \bar{\pi} \), which generates \( y^*_{cm} = 35.7 > y^*_{cc} \).

Increasing the upper bound certificate price to \( \bar{\pi} = 28 \), but otherwise keeping the same parameter values as above, we arrive at the following result:

\[
\begin{array}{cccccccc}
\text{s}^*_c & z^*_c & y^*_c & x^*_c & p^*_c & y^*_c & s^*_c & \Pi (y^*_c) \\
\text{Interior} & 23.3 & 35.0 & 58.3 & 41.7 & 35.0 & 16.7 & 544.4 & 612.5 & 0.4 \\
\text{Min.} & 22.8 & 35.6 & 58.4 & 41.6 & 35.6 & 15 & 519.8 & 633.7 & 0.390 \\
\text{Max.} & 27.0 & 30.9 & 57.9 & 42.1 & 30.9 & 28 & 726.8 & 478.0 & 0.466 \\
\end{array}
\]

We get \( s^*_c = 16.7 < \bar{\pi} = 28 \) and \( y^*_c = 35.0 \). Further we have \( s^*_{cm} = \bar{\pi} \), which generates \( y^*_{cm} = 34.8 < y^*_{cc} \).

\[ F \text{ Proposition 7} \]

We will show the existence of \( s^*_{mm} = \underline{s} \) and \( s^*_{mm} = \bar{s} \). We assume the following parameter values:

\( \alpha = 0.35, \kappa = 100, b = 1, c = 2, g = 5, \bar{\pi} = 13 \) and \( \underline{s} = 10 \).

The results are presented in the tables below. Again we are omitting the interior solution for the market power case as this is proved to never be realized in the mm-case.

\[
\begin{array}{cccccccccc}
\text{s}^*_m & z^*_m & y^*_m & x^*_m & p^*_m & y^*_m & s^*_m & \Pi (y^*_m) & \Pi (y^*_m) & \bar{z}^*_m \\
\text{Min.} & 18.9 & 25.9 & 44.8 & 55.2 & 51.7 & 10 & 715.1 & 1003.4 & 0.422 \\
\text{Max.} & 19.5 & 25.3 & 44.8 & 55.2 & 50.6 & 13 & 763.3 & 960.5 & 0.436 \\
\end{array}
\]

The consistency check shows that only \( s^*_{mm} = \underline{s} \) can be a valid solution in this case.

Increasing \( \alpha \) to 0.6, but otherwise keeping the same parameter values, we get the following result:

\[
\begin{array}{cccccccccc}
\text{s}^*_m & z^*_m & y^*_m & x^*_m & p^*_m & y^*_m & s^*_m & \Pi (y^*_m) & \Pi (y^*_m) & \bar{z}^*_m \\
\text{Min.} & 18.5 & 25.2 & 43.6 & 56.4 & 50.4 & 10 & 681.1 & 951.2 & 0.423 \\
\text{Max.} & 18.9 & 24.4 & 43.4 & 56.6 & 48.8 & 13 & 717.9 & 894.4 & 0.437 \\
\end{array}
\]

Again we have only one consistent equilibrium solution, i.e. \( s^*_{mm} = \bar{s} \).
G  Proposition 8

We will show that under the assumptions of the model we have that \( \text{sign} (y_{mm}^* - y_{cc}^*) \) and \( \text{sign} (z_{mm}^* - y_{cc}^*) \) are both indeterminate, irrespective of whether \( \varpi < s_{cc}^* < \varpi \), \( s_{cc}^* = \varpi \) or \( s_{cc}^* = \bar{s} \).

a) \( y_{mm}^* < y_{cc}^* \) and \( z_{mm}^* < z_{cc}^* \), given \( s_{cc}^* = \bar{s} \):

Parameter values: \( \alpha = 0.35, a = 100, b = 1, c = 2, g = 5, \varpi = 25 \) and \( \bar{s} = 15 \).

<table>
<thead>
<tr>
<th>( s_{cc}^* )</th>
<th>( z_{cc}^* )</th>
<th>( y_{cc}^* )</th>
<th>( x_{cc}^* )</th>
<th>( p_{cc}^* )</th>
<th>( q_{cc}^* )</th>
<th>( s_{cc}^* )</th>
<th>( \Pi (z_{cc}^*) )</th>
<th>( \Pi (y_{cc}^*) )</th>
<th>( \frac{z_{cc}^<em>}{x_{cc}^</em>} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interior</td>
<td>20.6</td>
<td>38.3</td>
<td>58.9</td>
<td>41.1</td>
<td>38.3</td>
<td>7.9</td>
<td>425.3</td>
<td>733.4</td>
<td>0.35</td>
</tr>
<tr>
<td>Min.</td>
<td>23.0</td>
<td>35.9</td>
<td>58.9</td>
<td>41.2</td>
<td>35.9</td>
<td>15</td>
<td>526.7</td>
<td>644.4</td>
<td>0.390</td>
</tr>
<tr>
<td>Max.</td>
<td>26.3</td>
<td>32.5</td>
<td>58.8</td>
<td>41.3</td>
<td>32.5</td>
<td>25</td>
<td>689.1</td>
<td>528.1</td>
<td>0.447</td>
</tr>
</tbody>
</table>

The cc-case produces a solution with \( s_{cc}^* = 7.9 < \bar{s} = 15 \), i.e. we get \( s_{cc}^* = \bar{s} \).

The consistency condition is OK. In the mm-case, only the equilibrium with \( s_{cc}^* = \bar{s} \) fulfills the consistency condition, i.e. we get \( y_{mm}^* = 20.0 < y_{cc}^* = 23.0 \) and \( z_{mm}^* = 24.9 < z_{cc}^* = 35.9 \).

b) \( y_{mm}^* > y_{cc}^* \), given \( s_{cc}^* = \bar{s} \):

Parameter values: \( \alpha = 0.73, a = 100, b = 1, c = 2, g = 5, \varpi = 80 \) and \( \bar{s} = 70 \).

<table>
<thead>
<tr>
<th>( s_{mm}^* )</th>
<th>( z_{mm}^* )</th>
<th>( x_{mm}^* )</th>
<th>( y_{mm}^* )</th>
<th>( x_{cc}^* )</th>
<th>( y_{cc}^* )</th>
<th>( s_{mm}^* )</th>
<th>( \Pi (z_{mm}^*) )</th>
<th>( \Pi (y_{mm}^*) )</th>
<th>( \frac{z_{mm}^<em>}{x_{mm}^</em>} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min.</td>
<td>20.0</td>
<td>24.9</td>
<td>44.9</td>
<td>55.1</td>
<td>49.9</td>
<td>15</td>
<td>796.4</td>
<td>932.4</td>
<td>0.445</td>
</tr>
<tr>
<td>Max.</td>
<td>22.0</td>
<td>23.1</td>
<td>45.1</td>
<td>54.9</td>
<td>46.1</td>
<td>25</td>
<td>972.0</td>
<td>798.2</td>
<td>0.489</td>
</tr>
</tbody>
</table>

The cc-case produces a solution with \( s_{cc}^* = 58.6 < \bar{s} = 70 \), i.e. we get \( s_{cc}^* = \bar{s} \).

The consistency condition is OK. In the mm-case, only the equilibrium with \( s_{cc}^* = \bar{s} \) fulfills the consistency condition, i.e. we get \( y_{mm}^* = 7.4 > y_{cc}^* = 6.6 \).

c) \( z_{mm}^* > z_{cc}^* \), given \( s_{cc}^* = \bar{s} \):

Parameter values: \( \alpha = 0.2, a = 100, b = 1, c = 1.4, g = 47, \varpi = 65 \) and \( \bar{s} = 22 \).
The consistency condition is OK. In the mm-case, only the equilibrium with \( z^* \) and \( s^* \) fulfills the consistency condition, i.e. we get \( z_{mm}^* = 12.6 > z_{cc}^* = 12.0 \).

d) \( y_{mm}^* < y_{cc}^* \) and \( z_{mm}^* < z_{cc}^* \), given \( s_{cc}^* = \pi \):

Parameter values: \( \alpha = 0.5, a = 100, b = 1, c = 2, g = 5, \pi = 25 \) and \( \pi = 15 \).
The cc-case produces a solution with $s_{cc}^* = 71.0 > \pi = 70$, i.e. we get $s_{cc}^* = \pi$. The consistency condition is OK. In the mm-case, only the equilibrium with $s_{cc}^* = \pi$ fulfills the consistency condition, i.e. we get $y_{mm}^* = 7.9 < y_{cc}^* = 7.8$.

f) $z_{mm}^* > z_{cc}^*$, given $s_{cc}^* = \pi$.
Parameter values: $\alpha = 0.7$, $a = 100$, $b = 1$, $c = 5$, $g = 10$, $\pi = 30$ and $s = 15$.

<table>
<thead>
<tr>
<th>$s_{cc}^*$</th>
<th>$z_{cc}^*$</th>
<th>$y_{cc}^*$</th>
<th>$x_{cc}^*$</th>
<th>$p_{cc}^*$</th>
<th>$q_{cc}^*$</th>
<th>$s_{cc}^*$</th>
<th>$\Pi(z_{cc}^*)$</th>
<th>$\Pi(y_{cc}^*)$</th>
<th>$\frac{z_{cc}^<em>}{s_{cc}^</em>}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interior</td>
<td>18.4</td>
<td>7.9</td>
<td>26.3</td>
<td>73.7</td>
<td>7.9</td>
<td>94.1</td>
<td>845.5</td>
<td>31.1</td>
<td>0.7</td>
</tr>
<tr>
<td>Min.</td>
<td>9.0</td>
<td>40.2</td>
<td>49.3</td>
<td>50.7</td>
<td>40.2</td>
<td>15</td>
<td>204.6</td>
<td>809.1</td>
<td>0.184</td>
</tr>
<tr>
<td>Max.</td>
<td>18.4</td>
<td>7.9</td>
<td>26.3</td>
<td>73.7</td>
<td>7.9</td>
<td>94.1</td>
<td>845.5</td>
<td>31.1</td>
<td>0.7</td>
</tr>
</tbody>
</table>

The cc-case produces a solution with $s_{cc}^* = 94.1 > \pi = 30$, i.e. we get $s_{cc}^* = \pi$. The consistency condition is OK. In the mm-case, only the equilibrium with $s_{cc}^* = \pi$ fulfills the consistency condition, i.e. we get $z_{mm}^* = 10.9 > y_{cc}^* = y_{cc}^* = 10.8$.

g) $y_{mm}^* < y_{cc}^*$ and $z_{mm}^* = z_{cc}^*$, given $\pi < s_{cc}^* = \pi$.
Parameter values: $\alpha = 0.35$, $a = 100$, $b = 1$, $c = 2$, $g = 5$, $\pi = 25$ and $s = 5$.

<table>
<thead>
<tr>
<th>$s_{cc}^*$</th>
<th>$z_{cc}^*$</th>
<th>$y_{cc}^*$</th>
<th>$x_{cc}^*$</th>
<th>$p_{cc}^*$</th>
<th>$q_{cc}^*$</th>
<th>$s_{cc}^*$</th>
<th>$\Pi(z_{cc}^*)$</th>
<th>$\Pi(y_{cc}^*)$</th>
<th>$\frac{z_{cc}^<em>}{s_{cc}^</em>}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interior</td>
<td>20.6</td>
<td>38.3</td>
<td>58.9</td>
<td>41.1</td>
<td>38.3</td>
<td>7.9</td>
<td>425.3</td>
<td>733.4</td>
<td>0.35</td>
</tr>
<tr>
<td>Min.</td>
<td>19.7</td>
<td>39.3</td>
<td>59.0</td>
<td>41.1</td>
<td>39.3</td>
<td>5</td>
<td>386.1</td>
<td>772.2</td>
<td>0.333</td>
</tr>
<tr>
<td>Max.</td>
<td>26.3</td>
<td>32.5</td>
<td>58.8</td>
<td>41.3</td>
<td>32.5</td>
<td>25</td>
<td>689.1</td>
<td>528.1</td>
<td>0.447</td>
</tr>
</tbody>
</table>

The cc-case produces a solution with $s_{cc}^* = 26.8 < y_{cc}^* = 38.3$ and $z_{mm}^* = 17.9 < z_{cc}^* = 20.6$.

h) $y_{mm}^* > y_{cc}^*$, given $\pi < s_{cc}^* < \pi$.
Parameter values: $\alpha = 0.8$, $a = 100$, $b = 50$, $c = 2.5$, $g = 5$, $\pi = 10$ and $s = 4$.

<table>
<thead>
<tr>
<th>$s_{cc}^*$</th>
<th>$z_{cc}^*$</th>
<th>$y_{cc}^*$</th>
<th>$x_{cc}^*$</th>
<th>$p_{cc}^*$</th>
<th>$q_{cc}^*$</th>
<th>$s_{cc}^*$</th>
<th>$\Pi(z_{cc}^*)$</th>
<th>$\Pi(y_{cc}^*)$</th>
<th>$\frac{z_{cc}^<em>}{s_{cc}^</em>}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interior</td>
<td>1.5</td>
<td>0.4</td>
<td>1.9</td>
<td>7.0</td>
<td>0.4</td>
<td>8.3</td>
<td>2.8</td>
<td>0.1</td>
<td>0.8</td>
</tr>
<tr>
<td>Min.</td>
<td>0.3</td>
<td>1.6</td>
<td>1.9</td>
<td>4.8</td>
<td>1.6</td>
<td>4</td>
<td>0.1</td>
<td>1.4</td>
<td>0.136</td>
</tr>
<tr>
<td>Max.</td>
<td>2.0</td>
<td>-0.1</td>
<td>1.8</td>
<td>7.9</td>
<td>-0.1</td>
<td>10</td>
<td>4.8</td>
<td>0.0</td>
<td>1.061</td>
</tr>
<tr>
<td>$s_{mm}$</td>
<td>$z_{mm}$</td>
<td>$y_{mm}$</td>
<td>$x_{mm}$</td>
<td>$p_{mm}$</td>
<td>$q_{mm}$</td>
<td>$s_{mm}$</td>
<td>$\Pi(z_{mm})$</td>
<td>$\Pi(y_{mm})$</td>
<td>$\tilde{x}_{mm}$</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Min.</td>
<td>0.6</td>
<td>0.7</td>
<td>1.3</td>
<td>36.5</td>
<td>33.3</td>
<td>4</td>
<td>19.4</td>
<td>21.6</td>
<td>0.485</td>
</tr>
<tr>
<td>Max.</td>
<td>0.7</td>
<td>0.6</td>
<td>1.2</td>
<td>37.7</td>
<td>29.7</td>
<td>10</td>
<td>22.4</td>
<td>17.2</td>
<td>0.532</td>
</tr>
</tbody>
</table>

The cc-case produces a solution with $\underline{s} = 4 < s_{cc}^* = 8.3 < \bar{s} = 10$, i.e. we get an interior solution for $s_{cc}^*$. The consistency condition is OK. In the mm-case, only the equilibrium with $s_{cc}^* = \bar{s}$ fulfills the consistency condition, i.e. we get $y_{mm}^* = 0.6 > y_{cc}^* = 0.4$.

1) $z_{mm}^* > z_{cc}^*$ given $\underline{s} < s_{cc}^* < \bar{s}$.

Parameter values: $\alpha = 0.2$, $a = 100$, $b = 50$, $c = 2.5$, $g = 5$, $\bar{s} = 10$ and $\underline{s} = 4$.

<table>
<thead>
<tr>
<th>$s_{cc}$</th>
<th>$z_{cc}$</th>
<th>$y_{cc}$</th>
<th>$x_{cc}$</th>
<th>$p_{cc}$</th>
<th>$q_{cc}$</th>
<th>$s_{cc}^*$</th>
<th>$\Pi(z_{cc}^*)$</th>
<th>$\Pi(y_{cc}^*)$</th>
<th>$\frac{\tilde{x}<em>{cc}}{\tilde{x}</em>{cc}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interior</td>
<td>0.4</td>
<td>1.6</td>
<td>2.0</td>
<td>2.4</td>
<td>1.6</td>
<td>4.4</td>
<td>0.2</td>
<td>1.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Min.</td>
<td>0.3</td>
<td>1.7</td>
<td>2.0</td>
<td>2.5</td>
<td>1.7</td>
<td>4</td>
<td>0.1</td>
<td>1.4</td>
<td>0.139</td>
</tr>
<tr>
<td>Max.</td>
<td>2.0</td>
<td>0.0</td>
<td>2.0</td>
<td>2.0</td>
<td>0.0</td>
<td>10</td>
<td>4.9</td>
<td>0.0</td>
<td>1.014</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th>$s_{mm}$</th>
<th>$z_{mm}$</th>
<th>$y_{mm}$</th>
<th>$x_{mm}$</th>
<th>$p_{mm}$</th>
<th>$q_{mm}$</th>
<th>$s_{mm}^*$</th>
<th>$\Pi(z_{mm}^*)$</th>
<th>$\Pi(y_{mm}^*)$</th>
<th>$\tilde{x}_{mm}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min.</td>
<td>0.6</td>
<td>0.7</td>
<td>1.3</td>
<td>34.9</td>
<td>34.1</td>
<td>4</td>
<td>20.4</td>
<td>22.6</td>
<td>0.485</td>
</tr>
<tr>
<td>Max.</td>
<td>0.7</td>
<td>0.6</td>
<td>1.3</td>
<td>33.8</td>
<td>31.8</td>
<td>10</td>
<td>25.2</td>
<td>19.6</td>
<td>0.529</td>
</tr>
</tbody>
</table>

The cc-case produces a solution with $\underline{s} = 4 < s_{cc}^* = 4.4 < \bar{s} = 10$, i.e. we get an interior solution for $s_{cc}^*$. The consistency condition is OK. In the mm-case, only the equilibrium with $s_{cc}^* = \bar{s}$ fulfills the consistency condition, i.e. we get $y_{mm}^* = 0.6 > y_{cc}^* = 0.4$.

### Proposition 9

We will show the existence of an interior certificate price, i.e. $\underline{s} < s_M^* < \bar{s}$. We assume the following parameter values:

- $\alpha = 0.65$, $a = 100$, $b = 1$, $c = 2$, $g = 5$, $\bar{s} = 80$ and $\underline{s} = 50$.

The model generates the following result:

<table>
<thead>
<tr>
<th>$s_M^*$</th>
<th>$z_M^*$</th>
<th>$y_M^*$</th>
<th>$x_M^*$</th>
<th>$p_M^*$</th>
<th>$q_M^*$</th>
<th>$s_M^*$</th>
<th>$\Pi(z_M^*)$</th>
<th>$\Pi(y_M^*)$</th>
<th>$\frac{\tilde{x}_M}{\tilde{x}_M}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interior</td>
<td>14.0</td>
<td>7.5</td>
<td>21.5</td>
<td>78.5</td>
<td>29.0</td>
<td>76.2</td>
<td>1203.7</td>
<td>189.6</td>
<td>0.650</td>
</tr>
<tr>
<td>Min.</td>
<td>25.3</td>
<td>5.6</td>
<td>30.9</td>
<td>69.1</td>
<td>11.3</td>
<td>50</td>
<td>783.1</td>
<td>47.5</td>
<td>0.818</td>
</tr>
<tr>
<td>Max.</td>
<td>34.1</td>
<td>6.8</td>
<td>27.4</td>
<td>72.6</td>
<td>-13.5</td>
<td>80</td>
<td>934.2</td>
<td>68.3</td>
<td>1.247</td>
</tr>
</tbody>
</table>

The generator now wants to maximize its total profit from the generation of both black and green electricity. Maximum total profit is where $s_{mm}^* = 76.2$. Thus, we have $\underline{s} = 50 < s_M^* = 76.2 < \bar{s} = 80$.  

37
We will now show that we can have \( s^*_M \) at either of the price bounds, i.e. \( s^*_M = \underline{s} \) or \( s^*_M = \overline{s} \). We assume the following parameter values:

\[
\alpha = 0.4, \ a = 100, \ b = 1, \ c = 2, \ g = 5, \ \overline{s} = 25 \text{ and } \underline{s} = 10.
\]

We get the following result:

\[
\begin{array}{ccccccccc}
\text{Interior} & 12.8 & 8.5 & 21.3 & 78.7 & 29.9 & 219.5 & 1195.3 & 279.2 & 0.400 \\
\text{Min.} & 16.6 & 18.3 & 34.9 & 65.1 & 36.5 & 20 & 622.9 & 555.8 & 0.471 \\
\text{Max.} & 18.1 & 16.3 & 34.4 & 65.6 & 32.5 & 25 & 679.7 & 459.4 & 0.517
\end{array}
\]

Maximum total profit is where \( s^*_M = 127.0 \), however this is far above the upper price bound, \( \overline{s} \). The best attainable equilibrium solution in this case is therefore \( s^*_M = \underline{s} \).

Keeping the same parameter values, except for changing the "percentage requirement" to \( \alpha = 0.6 \), the model produces this result:

\[
\begin{array}{ccccccccc}
\text{Interior} & 12.8 & 8.5 & 21.3 & 78.7 & 29.9 & 219.5 & 1195.3 & 279.2 & 0.400 \\
\text{Min.} & 16.6 & 18.3 & 34.9 & 65.1 & 36.5 & 20 & 622.9 & 555.8 & 0.471 \\
\text{Max.} & 18.1 & 16.3 & 34.4 & 65.6 & 32.5 & 25 & 679.7 & 459.4 & 0.517
\end{array}
\]

Here, maximum total profit is at \( s^*_M = 81.3 \). However, again this is far above the upper price bound, \( \overline{s} \). The second best solution is where \( s^*_M = \underline{s} \), but this is incompatible with the consistency condition. The only possible equilibrium is therefore where \( s^*_M = \overline{s} \).

I Proposition 11

In order to simplify the presentation we are omitting the subscripts \( cc \) referring to the case of perfect competition as we go through the proof for proposition 11.

**Proof.** i) The case of \( \underline{s} < s^*_{cc} < \overline{s} \)

The equilibrium is characterized by:

\[
\begin{align*}
p(x) &= q + \alpha s \\
q &= c'(y) + t \\
q + s &= h'(z) \\
x &= y + z \\
x &= \frac{z}{\alpha} \\
z &= ox, \ y = x - z = \frac{z}{\alpha} - z = (1 - \alpha) z, \ y = (1 - \alpha) x \\
\end{align*}
\]

Substitution gives:

\[
\begin{align*}
p(x) &= c'(y) + t + \alpha \left( h'(z) - \left( c'(y) + t \right) \right) \\
p(x) &= (1 - \alpha) \left( c'(y) + t \right) + ah'(z)
\end{align*}
\]
Calculating $\frac{dz}{dt}$:

$$p \left( \frac{z}{\alpha} \right) = (1 - \alpha) \left( c' \left( \frac{1}{1-\alpha} z + t \right) + \alpha h' (z) \right)$$

$$\frac{\partial p}{\partial z} \frac{dz}{dt} + \frac{\partial p}{\partial t} = (1 - \alpha) c'' (y) \frac{1}{1-\alpha} \frac{dz}{dt} + (1 - \alpha) + \alpha h'' (z) \frac{dz}{dt}$$

$$\left[ \frac{\partial p}{\partial z} - \frac{c''}{c} \right] \left[ \frac{dz}{dt} \right] = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Let $A = \begin{bmatrix} \frac{\partial p}{\partial z} - \frac{c''}{c} \\ -h'' \end{bmatrix}$

$$\det A = \begin{vmatrix} 1 & \frac{\partial p}{\partial z} - \frac{c''}{c} \\ -h'' & -h'' \end{vmatrix} < 0$$

$$\frac{dy}{dt} = \left( \frac{\partial p}{\partial z} - \frac{c''}{c} \right) \frac{dz}{dt} - h'' \frac{dz}{dt} > 0$$

$$\frac{dx}{dt} = \left( \frac{\partial p}{\partial z} - \frac{c''}{c} \right) \frac{dz}{dt} - h'' \frac{dz}{dt} > 0$$
J Proposition 12

As in the proof above, we are again omitting the subscripts referring to market structure.

**Proof.** i) The case of \( s^*_{mc} = \bar{s} \) or \( s^*_{mc} = \bar{s} \)

The equilibrium is characterized by:

\[
p(x) = q + \alpha s
\]

\[
q = c'(y) + t
\]

\[
q + s = h'(z)
\]

\[x = y + z\]

We have

\[
\frac{\partial q}{\partial y} = \frac{\partial (q - \alpha s)}{\partial y} = \frac{\partial p}{\partial x} = \frac{\partial p}{\partial x}
\]

Substitution gives:

\[
p(y + z) = h'(z) - s + \alpha s
\]

\[
h'(z) - s + \frac{\partial y}{\partial y} = c'(y) + t
\]

\[
p(y + z) - h'(z) = -(1 - \alpha) s
\]

\[
h'(z) + \frac{\partial p}{\partial y} = c'(y) = s + t
\]

Calculating \( \frac{\partial y}{\partial t} \) and \( \frac{\partial y}{\partial t} \):

\[
\frac{\partial p}{\partial x} + \frac{\partial p}{\partial x} = \frac{\partial h'}{\partial x} (z) \frac{\partial z}{\partial x} = 0
\]

\[
h''(z) \frac{\partial z}{\partial x} + \frac{\partial^2 p}{\partial x^2} \frac{\partial p}{\partial x} + \frac{\partial^2 p}{\partial x^2} \frac{\partial^2 y}{\partial x^2} + \frac{\partial^2 p}{\partial x^2} - c'' \frac{\partial p}{\partial x} = 1
\]

\[
\left[ \frac{\partial^2 p}{\partial x^2} y + \frac{\partial p}{\partial x} - c'' + \frac{\partial^2 p}{\partial x^2} y \right] \left[ \frac{\partial y}{\partial t} \right] = \left[ 0 \right]
\]

Let \( B = \left[ \frac{\partial^2 p}{\partial x^2} y + \frac{\partial p}{\partial x} - c'' + \frac{\partial^2 p}{\partial x^2} y \right] \)

\[
det B = \frac{\partial y}{\partial t} \left( h'' + \frac{\partial^2 p}{\partial x^2} y \right) - \left( \frac{\partial^2 p}{\partial x^2} y + \frac{\partial p}{\partial x} - c'' \right) \left( \frac{\partial y}{\partial t} - h'' \right)
\]

\[
det B = \frac{\partial^2 p}{\partial x^2} h'' + \frac{\partial^2 p}{\partial x^2} y \frac{\partial y}{\partial t} - \left( \frac{\partial^2 p}{\partial x^2} y + \frac{\partial p}{\partial x} - c'' \right)^2 - \frac{\partial p}{\partial x} h'' - c'' \frac{\partial p}{\partial x} + c'' h''
\]

\[
det B = \frac{\partial^2 p}{\partial x^2} h'' + \frac{\partial^2 p}{\partial x^2} y \frac{\partial y}{\partial t} - \left( \frac{\partial^2 p}{\partial x^2} y + \frac{\partial p}{\partial x} - c'' \right)^2 + \frac{\partial p}{\partial x} h'' + c'' \frac{\partial p}{\partial x} + c'' h''
\]

Let \( B < 0 \) given \( \frac{\partial^2 p}{\partial x^2} \leq 0 \)

\[
\frac{\partial y}{\partial t} = \left[ \frac{1}{\det B} h'' + \frac{\partial^2 p}{\partial x^2} y \right] = \left( \frac{\partial y}{\partial t} - h'' \right) < 0 \text{ for } \frac{\partial^2 p}{\partial x^2} \leq 0
\]

\[
\frac{d y}{d t} = \left[ \frac{\partial^2 p}{\partial x^2} y + \frac{\partial p}{\partial x} - c'' \frac{1}{\det B} \right] = \frac{\partial y}{\partial t} > 0 \text{ for } \frac{\partial^2 p}{\partial x^2} \leq 0
\]

ii) The case of \( s^*_{cn} = \bar{s} \) or \( s^*_{cn} = \bar{s} \)

The equilibrium is characterized by:

\[
p(x) = q + \alpha s
\]

\[
q = c'(y) + t
\]

\[
q + \frac{\partial y}{\partial z} + s = h'(z)
\]

\[x = y + z\]

We have

\[
d x = d y + d z
\]
\[ \frac{\partial y}{\partial x} = \frac{\partial (p - \alpha s)}{\partial x} = \frac{\partial p}{\partial x} = \frac{\partial p}{\partial x^2} = \frac{\partial p}{\partial x} \]

Substitution gives:

\[ p(y + z) = c'(y) + t + \alpha s \]
\[ c(y) + t + \frac{\partial}{\partial x} h(z) \]

\[ p(y + z) - c(y) = t + \alpha s \]
\[ c(y) + \frac{\partial}{\partial x} z - h(z) = -(t + s) \]

Calculating \( \frac{dy}{dx} \) and \( \frac{dz}{dx} \):

\[ \frac{\partial p}{\partial x} \frac{dy}{dx} + \frac{\partial p}{\partial x} \frac{dz}{dx} - c'' \frac{dz}{dx} = 1 \]

\[ \left[ \frac{\partial p}{\partial x} - c'' \frac{\partial^2 p}{\partial x^2} \right] \left[ \frac{dy}{dx} + \frac{\partial^2 p}{\partial x^2} \right] = \left[ \frac{1}{-1} \right] \]

Let \( E = \frac{\partial p}{\partial x} - c'' \left( \frac{\partial^2 p}{\partial x^2} - h'' \right) \)

\[ \det E = \left( \frac{\partial p}{\partial x} - c'' \left( \frac{\partial^2 p}{\partial x^2} - h'' \right) \right) \left( \frac{\partial^2 p}{\partial x^2} - h'' \right) = \left( \frac{\partial p}{\partial x} \right)^2 \]

\[ \det E = -\frac{\partial}{\partial x} h'' - c'' \frac{\partial^2 p}{\partial x^2} + c'' h'' - \frac{\partial p}{\partial x} \frac{\partial^2 p}{\partial x^2} \]

\[ \det E > 0 \text{ given } \frac{\partial^2 p}{\partial x^2} \leq 0 \]

\[ \frac{dy}{dx} = \frac{1}{\det E} \left[ \frac{\partial p}{\partial x} \frac{\partial^2 p}{\partial x^2} \right] = \frac{\partial^2 z - h'' - \left( \frac{\partial p}{\partial x} \right)^2}{\det E} < 0 \text{ given } \frac{\partial^2 p}{\partial x^2} \leq 0 \]

\[ -\frac{\partial}{\partial x} - \frac{\partial^2 p}{\partial x^2} > 0 \text{ given } \frac{\partial^2 p}{\partial x^2} \leq 0 \]

(iii) The case of \( s_{mn} = s \) or \( s_{mn} = \Theta \)

The equilibrium is characterized by:

\[ p(x) = q + \alpha s \]
\[ q + \frac{\partial q}{\partial y} = c'(y) + t \]
\[ q + \frac{\partial q}{\partial x} z + s = h'(z) \]
\[ x = y + z \]

We have

\[ \frac{\partial q}{\partial x} = \frac{\partial q}{\partial y} = \frac{\partial q}{\partial x} \]

Substitution gives:

\[ p(y + z) = c'(y) + t - \frac{\partial}{\partial x} y + \alpha s \]
\[ c(y) + t - \frac{\partial}{\partial x} y + \frac{\partial}{\partial x} z + s = h'(z) \]

\[ p(y + z) - c(y) + \frac{\partial}{\partial x} y = t + \alpha s \]
\[ c'(y) - h'(z) - \frac{\partial}{\partial x} y + \frac{\partial}{\partial x} z = -(t + s) \]

Calculating \( \frac{dy}{dx} \) and \( \frac{dz}{dx} \):

\[ \left[ \frac{\partial p}{\partial x} \frac{dy}{dx} + \frac{\partial p}{\partial x} \frac{dz}{dx} - c'' \frac{dz}{dx} \right] \left[ \frac{\partial^2 p}{\partial x^2} \frac{dy}{dx} + \frac{\partial^2 p}{\partial x^2} \frac{dz}{dx} \right] = \left[ \frac{1}{-1} \right] \]

\[ \frac{c''}{\partial x} \frac{dy}{dx} + \frac{\partial^2 p}{\partial x^2} \frac{dy}{dx} + \frac{\partial^2 p}{\partial x^2} \frac{dz}{dx} + \frac{\partial p}{\partial x} \frac{dy}{dx} + \frac{\partial p}{\partial x} \frac{dz}{dx} = 1 \]

\[ c'' \frac{dy}{dx} - h'' \frac{dz}{dx} = -\frac{\partial}{\partial x} - \frac{\partial^2 p}{\partial x^2} - \frac{\partial p}{\partial x} \frac{dy}{dx} + \frac{\partial^2 p}{\partial x^2} \frac{dz}{dx} + \frac{\partial p}{\partial x} \frac{dz}{dx} = -1 \]
\[
\begin{bmatrix}
\frac{\partial p}{\partial t} - c' + \frac{\partial^2 p}{\partial x^2} + \frac{\partial p}{\partial x} \\
\frac{\partial p}{\partial x} - c' + \frac{\partial^2 p}{\partial t^2} y - \frac{\partial p}{\partial x} z - h'' - \frac{\partial^2 p}{\partial x^2} y + \frac{\partial^2 p}{\partial x z^2} + \frac{\partial p}{\partial x} \\
\end{bmatrix}
\begin{bmatrix}
\frac{dp}{dt} \\
\frac{dy}{dt} \\
\end{bmatrix}
= 
\begin{bmatrix}
1 \\
-1 \\
\end{bmatrix}
\]

Let \( F = \begin{bmatrix}
\frac{\partial p}{\partial t} - c' + \frac{\partial^2 p}{\partial x^2} + \frac{\partial p}{\partial x} \\
\frac{\partial p}{\partial x} - c' + \frac{\partial^2 p}{\partial t^2} y - \frac{\partial p}{\partial x} z - h'' - \frac{\partial^2 p}{\partial x^2} y + \frac{\partial^2 p}{\partial x z^2} + \frac{\partial p}{\partial x} \\
\end{bmatrix} \)

Then \( \det F = \begin{bmatrix}
2 \frac{\partial p}{\partial x} + \frac{\partial^2 p}{\partial x^2} x - c' \\
\frac{\partial p}{\partial x} - c' + \frac{\partial^2 p}{\partial x^2} y - \frac{\partial p}{\partial x} z - h'' - \frac{\partial^2 p}{\partial x^2} y + \frac{\partial^2 p}{\partial x z^2} + \frac{\partial p}{\partial x} \\
\end{bmatrix} (\frac{\partial p}{\partial x} + \frac{\partial^2 p}{\partial x^2} y + \frac{\partial p}{\partial x} z) \)

which gives:

\[
\frac{\partial p}{\partial x} + \frac{\partial^2 p}{\partial x^2} x - c' > 0 \text{ given } \frac{\partial^2 p}{\partial x^2} \leq 0
\]

\[
\frac{\partial p}{\partial x} - c' + \frac{\partial^2 p}{\partial x^2} y - \frac{\partial p}{\partial x} z - h'' - \frac{\partial^2 p}{\partial x^2} y + \frac{\partial^2 p}{\partial x z^2} + \frac{\partial p}{\partial x} < 0 \text{ given } \frac{\partial^2 p}{\partial x^2} \leq 0
\]

\[
\frac{\partial p}{\partial x} - \frac{\partial^2 p}{\partial x^2} > 0 \text{ given } \frac{\partial^2 p}{\partial x^2} \leq 0
\]

The case of \( s_M^* = \bar{z} \) or \( s_M^* = \bar{z}^8 \)

The equilibrium is characterized by:

\( p(x) = q + \alpha s \)
\( q + \frac{\partial q}{\partial x} x = c' (y) + t \)
\( q + \frac{\partial q}{\partial x} x + s = h' (z) \)
\( x = y + z \)

We have
\( \frac{\partial q}{\partial x} = \frac{\partial p}{\partial x} = \frac{\partial q}{\partial x} \)

Substitution gives:
\( p(y + z) = c' (y) + t - \frac{\partial p}{\partial x} x + \alpha s \)
\( c' (y) + t - \frac{\partial p}{\partial x} x + \frac{\partial q}{\partial x} x + s = h' (z) \)
\( p(y + z) = c' (y) + t + \alpha s \)
\( c (y) - h' (z) = -(t + s) \)

Calculating \( \frac{\partial p}{\partial y} \) and \( \frac{\partial p}{\partial z} \):

\[
\frac{\partial p}{\partial y} \frac{\partial y}{\partial x} - \frac{\partial p}{\partial x} \frac{\partial y}{\partial x} + \frac{\partial^2 p}{\partial x^2} \frac{\partial y}{\partial y} + \frac{\partial p}{\partial y} \frac{\partial y}{\partial x} + \frac{\partial^2 p}{\partial x^2} \frac{\partial x}{\partial x} + \frac{\partial p}{\partial x} \frac{\partial y}{\partial x} + \frac{\partial^2 p}{\partial x^2} \frac{\partial x}{\partial x} = 1
\]

\[
\frac{\partial p}{\partial x} - c' + \frac{\partial^2 p}{\partial x^2} x - \frac{\partial p}{\partial x} y + \frac{\partial p}{\partial x} x + \frac{\partial^2 p}{\partial x^2} x + \frac{\partial p}{\partial x} y + \frac{\partial^2 p}{\partial x^2} y - h'' - \frac{\partial^2 p}{\partial x^2} y + \frac{\partial^2 p}{\partial x z^2} + \frac{\partial p}{\partial x} = -1
\]

Let \( G = \begin{bmatrix}
\frac{\partial p}{\partial x} - c' + \frac{\partial^2 p}{\partial x^2} x - \frac{\partial p}{\partial x} y + \frac{\partial p}{\partial x} x + \frac{\partial^2 p}{\partial x^2} x + \frac{\partial p}{\partial x} y + \frac{\partial^2 p}{\partial x z^2} + \frac{\partial p}{\partial x} \\
\end{bmatrix} \begin{bmatrix}
\frac{dy}{dt} \\
\end{bmatrix} \)

\[
\det G = \begin{bmatrix}
2 \frac{\partial p}{\partial x} + \frac{\partial^2 p}{\partial x^2} x - c' \\
\frac{\partial p}{\partial x} - c' + \frac{\partial^2 p}{\partial x^2} y - \frac{\partial p}{\partial x} z - h'' - \frac{\partial^2 p}{\partial x^2} y + \frac{\partial^2 p}{\partial x z^2} + \frac{\partial p}{\partial x} \\
\end{bmatrix} \begin{bmatrix}
\frac{dy}{dt} \\
\end{bmatrix} > 0 \text{ given } \frac{\partial^2 p}{\partial x^2} \leq 0
\]

As shown in the proof of proposition 9.1, the monopolist is indifferent with respect to securing the high, the low or some intermediate certificate price (and correspondingly for the wholesale price) for the case the case where the optimal solution satisfies \( \hat{x} - \hat{y} + \hat{z} = \bar{w} \) with \( w \leq \bar{z} \). Therefore, the proof for the case of \( s_M^* = \bar{z} \) or \( s_M^* = \bar{z}^8 \) also work as a proof for the case of \( \bar{x} \leq s_M^* < \bar{z}^8 \).
\[
\frac{dz}{dt} = \frac{1}{\det G} \left| \begin{array}{c}
1 & \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial x^2} \times \\
-1 & -h
\end{array} \right| = -h'' - (-1) \left( \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial x^2} \times \right) \frac{\det G}{\det G} < 0
\]
given \( \frac{\partial^2 p}{\partial x^2} \leq 0 \)

\[
\frac{dx}{dt} = \frac{1}{\det G} \left| \begin{array}{c}
\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial x^2} \times -c'' \\
c'' - c'' - c'' - \frac{\partial^2 p}{\partial x^2} \times
\end{array} \right| = -\left( \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial x^2} \times \right) - c'' \frac{\det G}{\det G} > 0
\]
given \( \frac{\partial^2 p}{\partial x^2} \leq 0 \)

Figure 1: Illustration of the cc-equilibrium for the case of an interior GC-price

Figure 2: Profit curve for the producers of black electricity in the mc-case, illustrating that an intermediate GC-price will never be established in equilibrium.
Figure 3: Profit curve for the producers of black electricity in the mc-case, illustrating an equilibrium GC-price at the lower price bound.

Figure 4: Profit curve for the producers of black electricity in the mc-case, illustrating an equilibrium GC-price at the upper price bound.

Figure 5: Profit curve for the producers of green electricity in the cm-case, illustrating an equilibrium GC-price at the lower price bound.
Figure 6: Profit curve for the producers of green electricity in the cm-case, illustrating an equilibrium GC-price at the upper price bound.

Figure 7: Best response curves for the producers of black electricity, R(z), and the producers of green electricity, R(y), in the mm-case, illustrating an equilibrium with the GC-price at the lower price bound.