



Munich Personal RePEc Archive

**Price-setting mixed duopoly,
subsidization and the order of firms'
moves: substitutive, independent and
complementary goods**

Ohnishi, Kazuhiro

Institute for Basic Economic Science, Japan

August 2020

Online at <https://mpra.ub.uni-muenchen.de/105654/>
MPRA Paper No. 105654, posted 30 Jan 2021 08:41 UTC

Price-setting mixed duopoly, subsidization and the order of firms' moves: substitutive, independent and complementary goods

Kazuhiro Ohnishi*

Institute for Basic Economic Science, Japan

Abstract

This study examines a price-setting mixed duopoly with production subsidies and demonstrates that under the optimal production subsidy of each of substitutive, independent and complementary goods, profits and economic welfare are respectively identical in the three regimes of (i) a public firm and a private firm simultaneously set prices, (ii) the public firm acts as a leader, and (iii) both firms act simultaneously as profit-maximizers.

Keywords: Mixed duopoly model; Price competition; Subsidization; Substitutive goods; Independent goods; Complementary goods

JEL classification: C72; D21; L32

* **Corresponding Author Kazuhiro Ohnishi:** 5-1-1 Onoharahigashi, Minoo, Osaka 562-0031, Japan; Phone: +81-72-727-9573; E-mail: ohnishi@e.people.or.jp

1. Introduction

The theoretical work by White (1996) examines the role of production subsidies in a quantity-setting mixed oligopoly market and presents the following main results. If production subsidies are utilized before and after privatization, profits, consumer surplus and economic welfare are not changed. On the other hand, if production subsidies are utilized only before privatization, there is always a reduction in economic welfare. Poyago-Theotoky (2001) extends White's (1996) Cournot-Nash model to a Stackelberg model and demonstrates that the optimal production subsidy is identical regardless of whether (i) a public firm and private firms choose their quantity levels simultaneously, (ii) the public firm acts as a quantity leader, or (iii) all firms move simultaneously as profit-maximizers. Myles (2002) extends Poyago-Theotoky's (2001) model of linear demand to a mixed oligopoly model of general inverse demand and cost functions and shows that the result is essentially similar to that obtained by Poyago-Theotoky (2001). In addition, Hashimzade, Khodavaisi and Myles (2007) extend the works by White (1996), Poyago-Theotoky (2001) and Myles (2002) and examine both quantity and price competition in an oligopoly with substitutive goods. They show that under the optimal subsidy, profits, prices, outputs and economic welfare are respectively identical regardless of the nature of the competition.

In this present paper, we study the role that production subsidies play in a price-setting duopoly market comprising a public firm and a private firm. We consider substitutive,

independent and complementary goods and examine the following three regimes: (i) the public and private firms move simultaneously, (ii) the public firm acts as a price leader, and (iii) both firms act simultaneously as profit-maximizers. We solve the three games and demonstrate that under the optimal subsidy of each of substitutive, independent and complementary goods, both firms' profits, prices, outputs and economic welfare are respectively identical regardless of the nature of the competition.

The remainder of this paper proceeds as follows. In Section 2, we describe the model. Section 3 presents the results of this study. Section 4 presents examples of the results. Finally, Section 5 concludes the paper.

2. Model

Let us consider a model composed of a welfare-maximizing public firm and a profit-maximizing private firm. In the remainder of this paper, subscripts 0 and 1 represent the public firm and the private firm, respectively. The model is from Bárcena-Ruiz and Sedano (2011).

Each firm's demand as a function of the prices p_i and p_j is given by

$$q_i = \frac{a(1-b) - p_i + bp_j}{1-b^2} \quad (i, j = 0, 1; i \neq j),$$

where $a \in (0, \infty)$ and $b \in (-1, 1)$. If $b \in (-1, 0)$, goods are complementary, if $b = 0$, goods are independent, and if $b \in (0, 1)$, goods are substitutive,

Each firm's profit is given by

$$\pi_i = (p_i - c + s)q_i \quad (i = 0, 1), \quad (1)$$

where $c \in (0, a)$ denotes the marginal cost of production and $s \in (0, \infty)$ is the production subsidy rate.

Economic welfare is given by

$$W = CS + \pi_0 + \pi_1 - s(q_0 + q_1), \quad (2)$$

where $CS = [p_0^2 - 2bp_0p_1 + p_1^2 + 2a(1-b)(a - p_0 - p_1)] / 2(1-b^2)$ denotes consumer surplus.

We use subgame perfection as the equilibrium concept and the three games of the next section are solved by backward induction.

3. Results

In this section, we consider the following three price-setting games: (a) mixed Bertrand duopoly, (b) private Bertrand duopoly, and (c) mixed Stackelberg duopoly.

(a) Mixed Bertrand duopoly

There are two stages: in the first stage the government sets the production subsidy level to maximize economic welfare; in the second stage both firms simultaneously and independently choose their prices conditional on the production subsidy. The game is

solved by backward induction to obtain subgame perfect equilibrium values. Maximizing (1) and (2) simultaneously, we arrive at the second-stage equilibrium prices in terms of s :

$$p_0^{\text{MB}} = \frac{ab(1-b) + c(2-b) - bs}{2-b^2}, \quad p_1^{\text{MB}} = \frac{a(1-b) + c(1+b-b^2) - s}{2-b^2}. \quad (3)$$

We now solve the first stage of the game. In the first stage, taking into account how firms will react to the subsidy, the government determines the welfare-maximizing subsidy:

$$s^{\text{MB}} = (1-b)(a-c). \quad (4)$$

From (3) and (4), we can derive the following subgame perfect equilibrium values:

$$p_0(s^{\text{MB}}) = p_1(s^{\text{MB}}) = c, \quad (5)$$

$$q_0(s^{\text{MB}}) = q_1(s^{\text{MB}}) = \frac{a-c}{1+b}, \quad (6)$$

$$\pi_0(s^{\text{MB}}) = \pi_1(s^{\text{MB}}) = \frac{(1-b)(a-c)^2}{1+b}, \quad (7)$$

$$W(s^{\text{MB}}) = \frac{(a-c)^2}{1+b}. \quad (8)$$

Note that prices, outputs and profits are equalized between the public firm and the private firm. Also note that each firm's profit is equal to the amount of subsidy it receives from the government.

(b) Private Bertrand duopoly

In this game only, we assume that the public firm is privatized and maximizes its own profit. In stage one, the government decides the production subsidy level to maximize

economic welfare; in stage two, both firms simultaneously and independently choose their prices conditional on the production subsidy. Maximizing (1) simultaneously, we obtain the second-stage equilibrium in terms of s :

$$p_i^{\text{PB}}(s) = \frac{a(1-b) + c - s}{2-b}. \quad (9)$$

In stage one, the government decides the welfare-maximizing subsidy level:

$$s^{\text{PB}} = (1-b)(a-c) = s^{\text{MB}}.$$

It happens that the optimal subsidy, prices, outputs, profits and economic welfare in this game are identical with those in the mixed Bertrand duopoly game. Therefore, expressions (5) – (8) also represent the relevant expressions for the private Bertrand duopoly game.

(c) Mixed Stackelberg duopoly

We now consider the following three-stage game. In the first stage, the government chooses the production subsidy level. In the second stage, the public firm chooses its price. In the third stage, the private firm chooses its price. Starting from the third stage, we obtain

$$p(p_0, s) = \frac{a(1-b) - c - s + bp_0}{2}. \quad (10)$$

In the second stage, the public firm decides its price for a given subsidy level anticipating how its choice affects the private firm's price decision. This results in

$$p_0^{\text{MS}}(s) = \frac{ab(1-b) + c(4-b-2b^2) - bs}{4-3b^2}, \quad (11)$$

and further we obtain

$$p_1^{\text{MS}}(s) = \frac{a(1-b)(2-b^2) + c(2+2b-2b^2-b^3) - s(2-b^2)}{4-3b^2}. \quad (12)$$

In the first stage, the government, anticipating how its choice of subsidy affects firms' price choices, maximizes (2). The optimal subsidy is

$$s^{\text{MS}} = (1-b)(a-c) = s^{\text{MB}} = s^{\text{PB}}.$$

Prices, outputs, profits and economic welfare are identical to those obtained in (a) and (b), i.e. given by expressions (5) – (8).

Now we can state the following proposition.

Proposition 1: Under each of substitutive, independent and complementary goods, the optimal subsidy, prices, outputs, profits and economic welfare are respectively identical in the three price-setting regimes of (i) the public firm acts simultaneously with the private firm, (ii) the public firm is a Stackelberg leader, and (iii) both firms act simultaneously as profit-maximizers.

4. Examples

In this section, we present each example of substitutive, independent and complementary goods.

4.1. Substitutive goods

In this subsection, we assume that $b = 0.5$. Then the welfare-maximizing subsidy is determined as follows:

$$s^{\text{Subst}} = \frac{(a - c)}{2}.$$

Substitution reveals the following values:

$$p_0(s^{\text{Subst}}) = p_1(s^{\text{Subst}}) = c,$$

$$q_0(s^{\text{Subst}}) = q_1(s^{\text{Subst}}) = \frac{2(a - c)}{3},$$

$$\pi_0(s^{\text{Subst}}) = \pi_1(s^{\text{Subst}}) = \frac{(a - c)^2}{3},$$

$$W(s^{\text{Subst}}) = \frac{2(a - c)^2}{3}.$$

Since each firm's profit is equal to the amount of subsidy it receives from the government, economic welfare is equal to consumer surplus.

4.2. Independent goods

If $b = 0$, then we obtain the following values:

$$s^{\text{Indep}} = a - c,$$

$$p_0(s^{\text{Indep}}) = p_1(s^{\text{Indep}}) = c,$$

$$q_0(s^{\text{Indep}}) = q_1(s^{\text{Indep}}) = a - c,$$

$$\pi_0(s^{\text{Indep}}) = \pi_1(s^{\text{Indep}}) = (a - c)^2,$$

$$W(s^{\text{Indep}}) = (a - c)^2.$$

Notice that each firm's profit, economic welfare and consumer surplus are equalized.

4.3. Complementary goods

If $b = -0.5$, then we have the following values:

$$s^{\text{Compl}} = \frac{3(a-c)}{2}.$$

$$p_0(s^{\text{Compl}}) = p_1(s^{\text{Compl}}) = c,$$

$$q_0(s^{\text{Compl}}) = q_1(s^{\text{Compl}}) = 2(a-c),$$

$$\pi_0(s^{\text{Compl}}) = \pi_1(s^{\text{Compl}}) = 3(a-c)^2,$$

$$W(s^{\text{Compl}}) = 2(a-c)^2.$$

Notice that each firm's profit exceeds the value of economic welfare in this example.

5. Conclusion

We have investigated the role that production subsidies play in a price-setting duopoly comprising a public firm and a private firm. We have considered the following three regimes: (i) the public and private firms choose their prices simultaneously, (ii) the public firm acts as a price leader, and (iii) both firms act as profit-maximizers. In consequence, we have shown that under the optimal subsidy of each of substitutive, independent and complementary goods, prices, outputs, profits and economic welfare are respectively identical in all the three games.

References

- Barcena-Ruiz, J. C., Garzón, M. B., 2007. Capacity choice in a mixed duopoly under price competition. *Economics Bulletin* 12 (26), 1-7.
- Bárcena-Ruiz, J. C., Sedano, M., 2011. Endogenous timing in a mixed duopoly: weighed welfare and price competition. *Japanese Economic Review* 62 (4), 485-503.
- Fjell, K., Heywood, J. S., 2004. Mixed oligopoly, subsidization and the order of firm's moves: the relevance of privatization. *Economics Letters* 83 (3), 411-416.
- Hashimzade, N., Khodavaisi, H., Myles, G., 2007. An irrelevance result with differentiated goods. *Economics Bulletin* 8 (2), 1-7.
- Myles, G., 2002. Mixed oligopoly, subsidization and the order of firms' moves: an irrelevance result for the general case. *Economics Bulletin* 12 (1), 1-6.
- Ohnishi, K., 2010. Domestic and international mixed models with price competition. *International Review of Economics* 57 (1), 1-7.
- Ohnishi, K., 2012. Price-setting mixed duopoly, privatization and subsidization. *Microeconomics and Macroeconomics* 1 (1), 20-23.
- Ohnishi, K., 2020. Price-setting mixed duopoly, subsidization and the order of firms' moves: the relevance of privatization. *Economics Bulletin* 40 (4), 3071-3076.
- Poyago-Theotoky, J., 2001. Mixed oligopoly, subsidization and the order of firms' moves: an irrelevance result. *Economics Bulletin* 12 (3), 1-5.

Scrimatore, M., 2014. Quantity competition vs. price competition under optimal subsidy in a mixed oligopoly. *Economic Modelling* 42 (C), 166-176.

White, M. D., 1996. Mixed oligopoly, privatization and subsidization. *Economics Letters* 53 (2), 189-195.