Stabilizing Inflation under Heterogeneity: a welfare-based measure on what to target

Sinigaglia, Daniel

PUC-Rio

16 September 2008

Stabilizing Inflation under Heterogeneity: a welfare-based measure on what to target

Daniel L. Sinigaglia
PUC-Rio

September 16, 2008

Abstract

What measure of inflation a Central Bank should respond to? This paper characterizes the optimal targeting index in a multisectorial economy with Calvo-pricing, defined as a composition of sectorial inflations that maximizes a selected welfare criterion. This is a purely quadratic approximation to the representative agent's utility in an environment of distorted steady state and sectorial heterogeneity of price stickiness. The Central Bank is modeled as following a historical Taylor Rule. For most parameter values, weights of sectorial inflations are increasing functions of the degrees of nominal rigidity and productivity volatility and decreasing functions of sectorial wage markup volatilities, resembling most of the conclusions from related literature. Bayesian estimation for the structural model using sectorial quantum and price indexes for Personal Consumption Expenditure (PCE) provides the parameter values that allow constructing the optimal index for the US economy. The result points out towards a price index with similar properties than the PCE, with more weight on services and less weight of inflation from durable goods. I find no evidence that a core index based on the exclusion of food and energy goods is welfare improving.

1 Introduction

In the last couple of decades, inflation targeting has attracted much of academic and practical interest in monetary policy. Since it was firstly adopted in 1990 by New Zealand, several industrial and emerging economies have chosen this line of approach while attempting to stabilize inflation. In the United States a vivid debate has been taken place on whether the Federal Reserve should adopt it as a strategy for monetary policy, being Ben Bernanke one of the most enthusiastic advocators. Arguments pushing forth the implementation of this regime stress the role of accountability and transparency in molding the private sector's expectation while reducing uncertainty about the future paths of key macroeconomic variables. In addition, many authors believe that, in opposition to money-growth rules, inflation targeting can prevent dramatic swings in monetary policy that could be held partially responsible for the macroeconomic mistakes of the past, such as the Great Depression of the 1930's and the accelerating inflation of the 1970's. Arguments against usually evoke the limited flexibility or discretion to adjust policy objectives in the face of unexpected circumstances.

Among several practical aspects regarding the implementation of Inflation Targeting regimes, one has drawn particular attention: what measure of inflation should the monetary authority focus on? Consumer price indexes are often used because they provide an accurate description of the cost of living and, therefore, a direct measure of the costs of inflation. However, as pointed out by Mankiw and Reis (2003), such price index is not necessarily the best one to serve as a target for counter-cyclical policies. These authors propose that central banks should use an alternative index that gives substantial weight to the level of nominal wages. Similarly, some economists have argued that "core inflation", that is, inflation measure with the exclusion of certain components of high price volatility such as food and energy goods, may provide a better assessment of inflationary welfare losses. The intuition is that, under the assumption of a positive long-term trade-off between inflation and output growth, focusing on prices subject to a smaller degree of nominal rigidity would imply a higher frequency of policy interventions and, possibly, a higher volatility in real variables. In a seminal paper, Aoki (2001) stresses that optimal policy consists of stabilizing a sticky-price inflation measure, rather than a broader measure encompassing flexible prices. In a related two-region model, Benigno (2003) shows that inflation targeting policy in which higher weight is given to the inflation in the region with higher degree of nominal rigidity is nearly optimal.

Recent empirical research on price stickiness, such as Nakamura and Steinsson (2008) and Bill and Klenow (2004), has underlined the fact that prices are not only sticky, but there is a substantial amount of heterogeneity across sectors. This conclusions call for a reassessment of previous works, under a more realistic multi-sectorial environment. The objective of this paper is to establish a measure of inflation that a Central Bank committed to stabilizing inflation through a simple policy rule should target. This measure is defined as a composition of sectorial inflations that maximizes a selected welfare criterion, which ranks policies according to Ramsey rational expectation equilibrium with commitment. I depart from a multi-sector version of the New Keynesian framework with Calvo (1983) pricing and derive a purely quadratic approximation to the representative consumer's utility function by following Benigno and Woodford's (2003). The behavior of the Central Bank is modeled by a simplistic Taylor rule: interest rate is set taking into account the current level of inflation, output gap and the past level of interest rate. Although this is not a free of controversy assumption, as stressed in Svensson (2003), I believe it provides a simple way to characterize some of the central features of an inflation targeting regime: absence of other nominal anchors, such as money growth target or fixed exchange-rate systems, absence of fiscal dominance and policy instrument independency and, of course, institutional commitment to price stability by increasing interest rate in response to deviations from some measure of inflation to an established target.

The next Section presents the model economy. In particular, it establishes a welfare criterion that rank suboptimal paths for endogenous variables. Section 3 defines the ideal Targeting Index (TI) and presents some of its theoretical properties. For the great range of values considered, the weight of sectorial inflation on the Targeting Index is an increasing function of the degree of nominal rigidity and of the variance of productivity shocks, and a decreasing function of the variance of sectorial wage markup shocks. In Section 4, an empirical attempt using Bayesian methodology determines the parameter values for the US economy using quantum and price indexes for thirteen categories of consumption products from Personal Consumption Expenditure (PCE), obtained at the Bureau of Economic Analysis. Sectorial inflation weights on the Targeting Index are, then, optimally established using the mean from posteriori distributions from parameter estimations. I find no support that the exclusion of food or energy goods from the Targeting Index is welfare improving. In fact, optimal weights are close to the sector weights from PCE price index, with more emphasis going from durable goods to service goods. The last Section concludes.

2 Model

The model is a multi-sector version of the standard New Keynesian setup, detailed in Woodford (2003). I depart from that framework by allowing for heterogeneity in price stickiness *a la* Calvo (1983): firms in different productive sectors may have different probabilities of updating their nominal prices. There is a set Z of measure one of differentiated goods and respective suppliers working under monopolistic competition. These suppliers can be aggregated into a finite number of intervals or K productive sectors. Each good as well as each supplier is indexed by $z \in [0,1]$ and $k \in [1,2,...,K]$. We denote as m_k the measure obtained from the aggregation of all suppliers working under sector k, which can be understood as the relative weight of sector k, since $\sum_{k=1}^{K} m_k = 1$, where $0 < m_k < 1$.

2.1 Agents

A representative household chooses a Dixit-Stiglitz (1977) composite of differentiated consumption goods and supplies labor hours to a continuum of different types to monopolistically competitive firms (i.e., respectively, C_t and $h_{k,t}(z)$)

$$U_t \equiv E_t \sum_{j=t}^{\infty} \beta^{j-t} \left[u\left(C_j\right) - \sum_{k=1}^{K} \int_{m_k} v\left(h_{k,t}(z)\right) dz \right], \tag{1}$$

where β is the discount factor and the utility is isoelastic for simplicity,

$$u\left(C_t\right) \equiv \frac{C_t^{1-\sigma}}{1-\sigma}, \tag{2}$$

$$\sum_{k=1}^{K}\int_{m_k} v\left(h_{k,t}(z)\right)dz \equiv \sum_{k=1}^{K}\int_{m_k}\frac{\lambda}{1+\nu}h_{k,t}(z)^{1+\nu}dz, \tag{3}$$

where σ, ν and λ are all greater than zero and represent, respectively, the inverse of the intertemporal elasticity of substitution of consumption, the inverse of the Frisch elasticity of labor supply, and λ is a normalizing constant.

The CES aggregate good C_t is a weighted sum of sector aggregates $C_{k,t}$:

$$C_t \equiv \left[\sum_{k=1}^{K} m_k^{1/\eta} C_{k,t}^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)}, \tag{4}$$

where $\eta > 0$ is the elasticity of substitution across sectors. The sector composite consumption good $C_{k,t}$ is:

$$C_{k,t} \equiv \left[m_k^{-1/\theta}\int_{m_k} c_{k,t}\left(z\right)^{(\theta-1)/\theta} dz\right]^{\theta/(\theta-1)}, \tag{5}$$

where $c_{k,t}\left(z\right)$ is the quantity purchased of produced good z in sector k and θ the elasticity of substitution among goods produced within each sector. For simplicity, there is no capital, investment or liquidity services provided by money. The aggregate price index of composite consumption good produced in sector k is defined as:

$$P_{k,t} \equiv \left[m_k^{-1}\int_{m_k} p_{k,t}\left(z\right)^{1-\theta} dz\right]^{1/(1-\theta)} \tag{6}$$

and the aggregate consumer price-level is:

$$P_t \equiv \left[\sum_{k=1}^{K} m_k P_{k,t}^{1-\eta}\right]^{1/(1-\eta)}. \tag{7}$$

At the beginning of each period t, the representative household receives a nominal tax-free gross interest rate R_{t-1} over the nominal stock of risk free bonds acquired in the previous period, B_{t-1}. The flow budget constraint faced by the household is:

$$P_tC_t + B_t - R_{t-1}B_{t-1} = \sum_{k=1}^{K}\int_{m_k} W_{k,t}(z)h_{k,t}(z)dz + \int_0^1 \Psi_t(z)dz - P_t\tau_t, \tag{8}$$

where $\Psi_t(z)$ are dividends transferred from firm z and τ_t are (real) lump-sum taxes adjusted by the government in every date t.

Firms operate a constant-returns to scale technology and are subject to a sector-specific technology factor $a_{k,t}$, that is exogenously determined and independent across sectors for simplicity[1]

[1] It is assumed that the productivity factor has the same steady state level across sectors, that is $\bar{a}_k = 1$, all k:

$$y_{k,t}(z) = a_{k,t}h_{k,t}(z), \tag{9}$$

where $y_{k,t}(z)$ denotes the quantity produced by firm z in sector k. Within the same sector, firms are identical: they all have the same degree of market power, they face the same productive shocks and employ the same amount of differentiated labor hours. Across sectors, firms differ in terms of their productivity and are subject to different degrees of price stickiness.

In each date, an independent monetary authority determines the nominal interest rate R_t while the government issues new debt and adjusts taxation. Government expenses are represented by an exogenous process G_t, taken as the model's fiscal shock. By hypothesis, aggregate government expenses follow the same CES characterization of household consumption:

$$G_t \equiv \left[\sum_{k=1}^{K} m_k^{1/\eta} G_{k,t}^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)}, \tag{10}$$

where $G_{k,t}$ is the government consumption of sector composite good k. Government consumption of sector composite good is defined in terms of differentiated goods produces by firms within that sector, analogous to (5), where $g_{k,t}(z)$ is government consumption of good z:

$$G_{k,t} \equiv \left[m_k^{-1/\theta} \int_{m_k} g_{k,t}\left(z\right)^{(\theta-1)/\theta} dz\right]^{\theta/(\theta-1)}. \tag{11}$$

For simplicity, all government revenue come from lump sum taxes, which are adjusted in order to ensure government's solvency. In a date t perspective, it is given according to

$$R_{t-1}B_{t-1}^G = B_t^G + S_t, \tag{12}$$

where B_t^G denotes the end-of-period nominal liabilities of the government in terms of the one period risk-free bond, S_t the government nominal primary surplus defined in terms of sectorial aggregates according to:

$$S_t \equiv [\tau_t - G_t]P_t, \tag{13}$$

where τ_t are (real) the lump sum taxes.

2.2 Competitive Equilibrium

The first-order conditions on consumer's problem imply the following demand for good z in terms of sector aggregate and for the sector aggregate in terms of aggregate consumption and relative price:

$$c_{k,t}\left(z\right) = m_k^{-1} C_{k,t} \left[\frac{P_{k,t}}{p_{k,t}\left(z\right)}\right]^{\theta}, \tag{14}$$

$$C_{k,t} = m_k C_t \left[\frac{P_t}{P_{k,t}}\right]^{\eta}. \tag{15}$$

Following the definition of overall and sector consumption, government's demand for differentiated goods or sector aggregates can be derived in a similar fashion as household's demands, leading to demands analogous to (14) and (15):

$$g_{k,t}(z) = m_k^{-1} G_{k,t} \left[\frac{P_{k,t}}{p_{k,t}(z)}\right]^{\theta}, \tag{16}$$

$$G_{k,t} = m_k G_t \left[\frac{P_t}{P_{k,t}}\right]^{\eta}. \tag{17}$$

The economy is closed and all markets clear at all dates:

$$y_{k,t}(z) = g_{k,t}(z) + c_{k,t}(z),$$

all t, k and z. Expressions for overall sector and differenced good demands are, then, given by:

$$Y_{k,t} = m_k Y_t \left[\frac{P_t}{P_{k,t}}\right]^{\eta} \tag{18}$$

and

$$y_{k,t}(z) = m_k^{-1} Y_{k,t} \left[\frac{P_{k,t}}{p_{k,t}(z)}\right]^{\theta}. \tag{19}$$

From the representative consumer's maximization problem, sectorial real wages must satisfy:

$$\mu_{k,t} \frac{\lambda h_{k,t}(z)^{\nu}}{C_t^{-\sigma}} = w_{k,t}(z), \tag{20}$$

where $\mu_{k,t} \geq 1$ is an *ah hoc* exogenous sector-specific markup factor in the labor market, which is allowed to vary over time.[2] The consumer's intertemporal problem define an Euler equation

$$C_t^{-\sigma} = \beta R_t E_t \left[\frac{C_{t+1}^{-\sigma}}{\Pi_{t+1}}\right], \tag{21}$$

as well as a unique stochastic discount factor and the transversality condition:

$$\Theta_{t,j} = \beta^{j-t} E_t \left[\frac{C_j^{-\sigma}}{C_t^{-\sigma}} \frac{P_t}{P_j}\right], \tag{22}$$

[2] Benigno and Woodford (2003) introduce the same labor market disturbance in the aggregate economy. An alternative approach is undertaken by Steinsson (2003), who motivate the cost-push shock by considering the elasticity of substitution between goods stochastic. Both approaches reach the same log-linearized system.

$$\lim_{j\to\infty} \beta^j E_t \left[C_j^{-\sigma}\right] = 0. \tag{23}$$

As usual, α_k^{j-t} defines the probability that the price defined by firm z at period t, $p_{k,t}(z)$, will remain valid until period $t+j$. Firm z chooses a price $p_{k,t}(z)$ that maximizes the present discounted value of expected future profits:

$$\max_{\{p_{k,t}(z)\}} E_t \sum_{j=t}^{\infty} \alpha_k^{j-t} \Theta_{t,j} \left[y_{k,j}(z) p_{k,t}(z) - h_{k,j}(z) W_{k,j}(z)\right]. \tag{24}$$

The term Θ is the stochastic discount factor, common throughout firms. Solving the optimization problem for the firm yields the following rule for price setting in terms of sectorial and overall aggregate variables (similarly to Benigno and Woodford (2003) and detailed in the Appendix A):

$$\frac{p_{k,t}^*(z)}{P_{k,t}} = \left[\frac{K_{k,t}}{F_{k,t}}\right]^{1/(1+\theta\nu)}, \tag{25}$$

$$K_{k,t} \equiv \frac{\theta\lambda}{\theta-1} m_k^{-\nu} E_t \sum_{j=t}^{\infty} (\alpha_k \beta)^{j-t} \mu_{k,t} \Pi_{k,j}^{\theta(\nu+1)} \left[\frac{Y_{k,j}}{a_{k,j}}\right]^{\nu+1}, \tag{26}$$

$$F_{k,t} \equiv E_t \sum_{j=t}^{\infty} (\alpha_k \beta)^{j-t} C_j^{-\sigma} \Pi_{k,j}^{\theta-1} p_{k,j} Y_{k,j}, \tag{27}$$

where $p_{k,t}$ stands for the relative price of sector k , or $p_{k,t} = P_{k,t}/P_t$ and $\Pi_{k,j}$ is the gross inflation rate from period t to $t+j$ in sector k, or $\Pi_{k,j} = P_{k,j}/P_{k,t}$. $K_{k,t}$ is the discounted sum of (constant) markups over present and future marginal costs and $F_{k,t}$ represent the discounted sum of present and future net revenues. In equilibrium, all prices set within the same sector at a given date are equivalent. The relevant difference from the homogeneous stickiness case is the presence of sectorial aggregates and the sectorial relative price level term.

Define the measure for sectorial price dispersion $\Delta_{k,t}$ as

$$\Delta_{k.t} \equiv m_k^{-1} \int_{m_k} \left[\frac{p_{k,t}(z)}{P_{k,t}}\right]^{-\theta(1+\nu)} dz. \tag{28}$$

Given the Calvo price setting, one can show that $\Delta_{k,t}$ evolves according to the following law of motion:

$$\Delta_{k.t} = \alpha_k \Pi_{k,t}^{\theta(1+\nu)} \Delta_{k.t-1} + (1-\alpha_k) \left(\frac{1-\alpha_k \Pi_{k,t}^{\theta-1}}{1-\alpha_k}\right)^{\theta(1+\nu)/(\theta-1)}, \tag{29}$$

where $\Pi_{k,t}$ is the sectorial gross inflation between periods $t-1$ and t. Also from Calvo pricing, one can show that expression (25) can be rewritten in terms of a sectorial non-linear Phillips Curve which relates sectorial inflation to the expected discounted sum of sectorial markups over marginal costs and inversely to the discounted sum of future net revenues.

$$\frac{F_{k,t}}{K_{k,t}} = \left(\frac{1 - \alpha_k \Pi_{k,t}^{\theta-1}}{1 - \alpha_k} \right)^{\theta(1+\nu)/(\theta-1)} . \tag{30}$$

Iterating forward expression in (12) allow us to write the government budget constraint as:

$$W_t = E_t \sum_{j=t}^{\infty} \beta^{j-t} C_j^{-\sigma} s_j, \tag{31}$$

where s_t is the real value of (13) and W_t is defined as

$$W_t \equiv \frac{C_t^{-\sigma}}{\Pi_t} R_{t-1} b_{t-1} \tag{32}$$

and b_t the real value of debt at date t, or $b_t = B_t^G / P_t$. Financial markets clear at all dates

$$B_t^G = B_t, \tag{33}$$

and debt sustainability is subject to the transversality condition on the value of consolidated debt

$$\lim_{T \to \infty} E_t \left[\beta^T W_T \right] = 0. \tag{34}$$

Finally, it is worth noting that the relative price of sector k evolves according the difference between sectorial and aggregate gross inflation rates

$$\frac{p_{k,t}}{p_{k,t-1}} = \frac{\Pi_{k,t}}{\Pi_t}. \tag{35}$$

From the definition of aggregate price level, one can establish the following relation between sectorial and aggregate gross inflation:

$$\Pi_t^{1-\eta} = \sum_{k=1}^{K} m_k \left(\Pi_{k,t} p_{k,t-1} \right)^{1-\eta} . \tag{36}$$

Definition 1 *A competitive equilibrium is a sequence of endogenous variables* $\mathcal{X}_t^{En} = \{\Pi_t,\, \Pi_{k,t},\, Y_t,\, Y_{k,t},\, F_{k,t},\, K_{k,t},\, W_t,\, C_t,\, C_{k,t},\, b_t,\, \Delta_{k.t},\, p_{k,t}\}$, *policy variables* $\mathcal{X}_t^{P} = \{\tau_t,\, R_t\}$ *and initial conditions* $\mathcal{X}_{t_0-1}^{In} = \{\Delta_{k.t_0-1},\, p_{k,t_0-1},\, R_{t_0-1},\, b_{t_0-1}\}$ *for all k and $t \geq t_0$, that satisfy (14)-(19), (21), (23), (29), (30), (31), (34), (35), (36) and the market clearing conditions plus relevant definitions, given the exogenous processes* $\mathcal{X}_t^{Ex} = \{G_t,\, e_t,\, a_{k,t},\, \mu_{k,t}\}$ *, all k.*

2.3 Policy Rule

In order to fully specify the model, it is assumed the Central Bank is committed to stabilize the economy by following a Taylor rule while determining R_t in each date, such that:

$$\frac{R_t}{\bar{R}} = \left[\frac{R_{t-1}}{\bar{R}}\right]^{\rho_R} \left[\left(\frac{\check{P}_t}{\check{P}_{t-1}}\right)^{\phi_\pi} \left(\frac{Y_t}{\bar{Y}}\right)^{\phi_y}\right]^{(1-\rho_R)} \exp(e_t), \tag{37}$$

where ϕ_π and ϕ_π are the parameters that measure the intensity of the reaction of interest rates to, respectively, some measure of inflation and output fluctuation, ρ_R is the coefficient of interest rate smooth, $\bar{R}$ is the steady state interest rate (equals to β^{-1}), e_t is an exogenous monetary policy shock and $\check{P}_t$ is the aggregate targeting price index defined by

$$\check{P}_t \equiv \left[\sum_{k=1}^{K} \omega_k P_{k,t}^{1-\eta}\right]^{1/(1-\eta)}, \tag{38}$$

which can be different from (7)[3]. The term ω_k refer to the relative weight of sector k's price level defined in (6) over the aggregate measure $\check{P}_t$, to be optimally determined by the Central Bank. The fiscal regime is completely Ricardian. In each date, the government issue new debt taking as given the choice of R_t and G_t and adjusts lump sum taxes τ_t according to (31) in a way to ensure solvency of public debt. Thus, fiscal policy is passive in Sargent and Wallace's (1982) sense. Such set of policy rules determine a subset of (suboptimal) competitive equilibria.

2.4 Welfare Criterion

Any path of variables satisfying the definition of a competitive equilibrium is a solution for the model described above. Different paths, however, can be ranked according to some convenient welfare criterion. Consider the utility function for the representative consumer given in (1). After some manipulation and using the market-clearing conditions, one can rewrite the consumer's utility function as:

$$U_t = E_t \sum_{j=t}^{\infty} \beta^{j-t} \left[\frac{(Y_t - G_t)^{1-\sigma}}{1-\sigma} - \frac{\lambda}{1+\nu} \sum_{k=1}^{K} m_k \left[\frac{Y_{k,t}}{m_k a_{k,t}}\right]^{1+\nu} \Delta_{k.j}\right]. \tag{39}$$

Expression (39) can be used to categorize policies that lead to different paths for variables characterizing any competitive equilibrium:

[3] In a cashless economy, it is well known that an interest rate smooth term is clearly suboptimal. Nontheless, many authors have considered the present shape of the Taylor rule more conected with the empirical regularities for the behaviour of central banks throughout the world. Adding or subtracting an interest rate smooth term does not change the results presented. What is important is the modeling hypothesis. In other words, what kind of equation best describes the Central Bank policy rule.

Definition 2 *In a Ramsey rational expectation equilibrium with commitment, the social planner selects a competitive equilibrium by choosing policy instruments $\mathcal{X}_t^P$, all t, in order to maximize (39).*

It is well known that, in the absence of further constraints, the solution to the Ramsey problem above implies time-inconsistency for the optimal plan.[4] One possibility for obtaining a time invariant solution follows Woodford (1999), where the optimal solution with commitment is characterized from a *timeless perspective*. This approach imposes restrictions on the problem to prevent the social planner from internalizing the gains from private expectations on the evolution of inflation under commitment in the first period. In other words, consider a vector of quantities $X_t = \{F_{k,t},\ K_{k,t},\ W_t\}$, all k and t. A restricted Ramsey equilibrium from a timeless perspective imposes a set of preconditions on quantities so that optimization takes place also subject to the fact that X_{t_0} must take certain values.[5]

Definition 3 *In a restricted Ramsey rational expectation equilibrium with commitment, the social planner uses policy instruments in order to select a competitive equilibrium that maximizes (39) subject to the additional constraint of timeless perspective $X_{t_0} = \{\bar{F}_k,\ \bar{K}_k,\ \bar{W}\}$, all k.*

Hereafter, the restricted Ramsey equilibrium defined above establishes the metric through which different choices of policy instruments can be compared. Of particular interest are those choices of instruments based on explicit policy rules, such as the Taylor rule used to establish the level of interest rates in the model. The following lemma establishes the characteristics of the deterministic steady state around which a second order approximation of the welfare criterion is obtained. It provides the grounds for the log-approximations for the whole set of equations considered.[6]

Lemma 4 *There is a deterministic symmetric steady state, characterized by zero inflation rate, constant lump sum taxation and positive level of public debt.*

Proof. Appendix B. ■

In order to express (39) purely in quadratic terms around a steady state with positive government expenses and taxation[7] I follow Benigno and Woodford (2003). The procedure consists of deriving second-order approximations for the whole set of restrictions and use the second order terms of such restrictions in order to express the discounted sum of the linear term for aggregate and sectorial outputs only in terms of quadratic endogenous variables. The following

[4] Stokey and Lucas (1983).

[5] In particular, quantities X_{t_0} are chosen such as the first order conditions for the policy problem applied over t_0 are exactly the same as those applied in any date t.

[6] As standard in the related literature, it is assumed that the random disturbances that characterize the model are small enough so that shocks are unable to drive the economy away from its approximation point to the extend that equations become miss-specified.

[7] Rotemberg and Woodford (1999) eliminate this term by assuming a distortive subsidy on firms' production level ($\bar{\tau} < 0$) financed by lump-sum taxes.

proposition presents a second-order approximation to the utility function in an environment of heterogeneity of Calvo pricing:

Proposition 5 *The representative consumer's utility function can be approximated up to second-order by*

$$U_{t_0} = -\frac{\Omega}{2} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\lambda_{y_k} \sum_{k=1}^{K} m_k y_{k,t}^2 + \sum_{k=1}^{K} m_k \lambda_{k,\pi} \pi_{k,t}^2 \right] + T_0 + tips, \quad (40)$$

where the relative weights of sectorial inflations and sectorial output gaps depend on structural parameters of the economy, T_0 is a set of predetermined variables and tips stands for "terms independent of policy".

Proof. Appendix C. ■

The loss function written only in quadratic terms of endogenous variables resembles the usual definition for the loss function of a single-sector economy.[8] However, Berriel and Sinigaglia (2008) show it presents some different features from the standard case. In particular, policies closer to optimum should display a fixed sectorial inflation dispersion and strong commovements of sectorial output gaps.

3 What to Target?

3.1 Approximated Policy Problem

As for the approximate model equations, first-order Taylor expansion over the sectorial supply equation yields:

$$\pi_{k,t} = \kappa_k \left[\varsigma y_t + \chi y_{k,t} \right] + \beta E_t \pi_{k,t+1} + u_{k,t}. \quad (41)$$

This sectorial Phillips Curve is similar to homogeneous price stickiness case, in the sense that contemporaneous inflation depends on output gap[9] and expected future inflation. These are sectorial rather than aggregate relations. Moreover, there is an additional term that relates sectorial inflation to aggregate output. If the elasticity of substitution among different sectors is high (η^{-1} close to zero), a higher aggregate output leads to higher sectorial inflation. The term $u_{k,t}$ is a cost-push shock, defined in terms of the model's primitive shocks in the Appendix D. The same appendix also define the coefficients in terms of the structural parameters of the model economy.

Euler equation and market clearing condition on goods yield a standard IS equation of the form

$$\hat{R}_t = \tilde{\sigma} E_t \Delta y_{t+1} + E_t \pi_{t+1} - E_t \Delta r_{t+1}, \quad (42)$$

[8] Appendix C presents the details of derivation as well as the definitions of relevant terms.

[9] Given the hypothesis of an inefficient steady state, the output gap is defined as the difference between output and a target output level, defined in the Appendix D.

where r_t can be interpreted as a aggregate demand shock, defined in terms aggregate government expenses, productivity and wage markup shocks (Appendix D).

By log-linearizing the Taylor rule in (37), one gets:

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1-\rho_R)[\phi_\pi \breve{\pi}_t + \phi_y y_t] + e_t, \tag{43}$$

where e_t is the monetary policy shock. From (35), (36) and the definition of sectorial relative prices, it is possible to obtain the definition for aggregate output gap in terms of it sectorial counterparts:

$$y_t = \sum_{k=1}^{K} m_k y_{k,t}. \tag{44}$$

Definitions for consumer price inflation and inflation targeting index are, respectively, given by:

$$\pi_t = \sum_{k=1}^{K} m_k \pi_{k,t}, \tag{45}$$

$$\breve{\pi}_t = \sum_{k=1}^{K} \omega_k \pi_{k,t}, \tag{46}$$

while both m_k is the (given) relative size of sector k in the economy and ω_k is the weight of inflation in sector k over the targeting index, such that

$$0 \leq \omega_k \leq 1, \tag{47}$$

all k, and

$$\sum_{k=0}^{K} \omega_k = 1. \tag{48}$$

Finally, from (35) and the definition of sectorial relative prices, one gets[10]:

$$y_t - y_{k,t} = \eta[\pi_{k,t} - \pi_t] + y_{t-1} - y_{k,t-1} + \Delta\zeta_{k,t}, \tag{49}$$

where $\zeta_{k,t}$ is a reduced form shock that depends of productivity and wage markup shocks, defined in the Appendix D, and can be interpreted as a relative demand shock.

Definition 6 *A welfare-based Targeting Index (TI) is defined by selecting relative weights ω_k on sectorial inflations, all k, in order to maximize the welfare criterion in (40) taking as given the economy's restrictions (41)-(49), definitions of reduced form shocks, $u_{k,t}$, r_t and $\zeta_{k,t}$, all k, and stochastic processes governing monetary policy shocks, government expenses, sectorial productivity and wage markup shocks.*

[10] For $K-1$ sectors, once the definition for aggregate output in terms of sectorial output gaps define the remaining sector's output level.

3.2 Comparative statics in a two-sector economy

According to the previous Section, the Central Bank takes as given the parameters of the economy and selects optimally the weights of sectorial inflations over an aggregate inflation index to be targeted seeking to maximize the welfare criterion subject to the constraints of sectorial supplies, aggregate and relative demand as well as policy rule that characterize the economy. In particular, it is assumed the Central Bank is committed to follow a linear policy rule with all relevant parameters are taken as given, as in the case of the Taylor rule detailed in (43). In this Section, I depart from a simplistic symmetric two-sector economy calibrated at usual parameter values and present numerical results for weights of sectorial inflations on the Targeting Index defined in the previous Section as some key structural parameters of the economy are allowed to vary. I refer to Schmitt-Grohé and Uribe (2004) for computing the welfare under alternative compositions of sectorial inflations.

The Appendix E reports a detailed description of the values assigned to each parameter of the model. For simplicity, there are only two sectors with the same size and general characteristics. Under this special circumstances, it is no surprise that $\omega_k^* = .5$. Shocks are assumed to follow independent AR(1) processes[11]. Standard deviation of innovations and inertia parameters are set to the same value in both sectors, for simplicity [12]. Figure 1 displays the main results for the weight of the inflation in sector A in the Targeting Index, under the calibration considered and under the assumption that all parameters in sector B (reference sector) are constant.

As pointed out in the first panel in the upper left, the relative importance of sectorial inflation in the Targeting Index is an increasing function of the degree of price stickiness in that sector under a great range of values considered. In Calvo pricing, higher stickiness leads to higher real distortions and, therefore, higher concerns for cyclical stabilization. This result is by no means surprising and coincides with the conventional wisdom as pointed out by many authors, including Mankiw and Reis (2003) in a different framework. Evidently, the great majority of values assigned for differences in price stickiness across sectors under the calibration used lead to corner solutions, resembling Aoki's (2001) and Benigno's (2001) results for a situation in which the degrees of stickiness are not polar cases. In this sense, even if a sector has not completely flexible prices, it can have zero weight in the Targeting Index under the present calibration.

[11] Let $\rho_e, \rho_G, \rho_{a_k}$, and ρ_{μ_k} denote the inertia coefficients for, respectively, monetary shocks, government expenses, sectorial productivity and sectorial wage markup in sector k. The standard deviations are given by $\sigma_e, \sigma_G, \sigma_{a_k}$, and σ_{μ_k}.

[12] Respectively, .2 and .5.

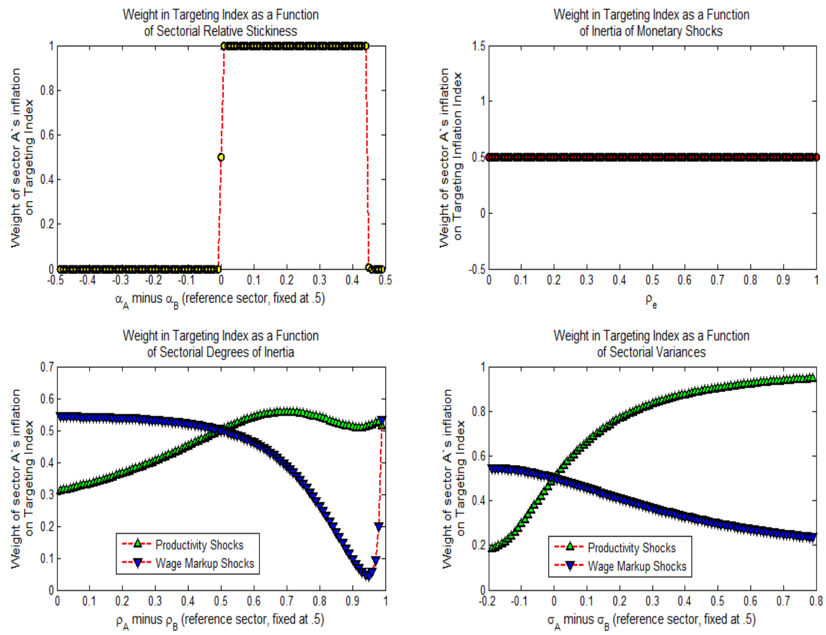

Figure 1: Weight of sectorial inflation on Targeting Index as a function of varing structural parameters

However, one of the distinct features of the analysis carried out stresses the fact that the weight of sectorial inflation over the Targeting Index implied by the degree of nominal stickiness is not a monotone function. In other words, for some (high) values of nominal rigidity, it is welfare improving to target the inflation in the sector with more flexible prices, in the present case, the reference sector where the probability of nominal rigidity equals .5. This feature contrasts with most results reported by the current literature. Berriel and Sinigaglia (2008) report that under sectorial heterogeneity of price stickiness, optimal policy prescribes a fixed distribution of sectorial inflation rates, which is given by the different degrees of nominal rigidity, and a strict output gap alignment. In other words, square deviations of aggregate inflation might not be as relevant as how sectorial inflation rates commove. Hence, some aggregate inflation can be desirable, provided it leads to commovements of sectorial inflations close to optimal.

In the case considered, a close to one Clavo parameter implies a degree of real distortion that approaches infinity: firms respond almost exclusively through output while prices remain unchanged. Therefore, as α_A increases, the degree of real distortions in sector A increases exponentially to the point in which such sector presents a very small inflation and great output variability. As the optimal policy requires alignment of output gaps, that implies an increasing distance from first best as α_A increases for a fixed α_B. In the absence of sectorial specific instruments, the Taylor rule and the IS equation jointly determine the contemporary aggregate output as a sum of future expected outputs and future aggregate inflations. A recognition by the private sector of a higher reaction of interest rates to inflation or output gap by the Central Bank imply an even smaller output gap today. Once aggregate output is determined, sectorial outputs and inflations are determined according to the structural conditions of the model, given by sectorial demands and relative prices. But because one of the sectors displays an increasingly higher output variability as α increases, the other sector has to display a smaller output to conform to a fixed aggregate output.

For sufficiently higher degrees of nominal rigidity and sufficiently lower degrees of aversion to inflationary risk (as in the case of the present calibration), output misalignment can only be mitigated with an increasingly strong contraction of aggregate demand, which means higher real interest rates. One possible way is to produce an stronger reaction from nominal interest rates to inflation by targeting, instead, more flexible prices. In other words, by targeting sectors that, according to the values assigned for the structural parameters, present a smaller degree of nominal rigidity.

Another predictable result is exemplified in the second panel to the upper right, where relative weights are presented as functions of the inertia coefficient of AR(1) monetary disturbances. The same pattern extends to other aggregate disturbance parameters (not reported), that is: in the benchmark calibration, aggregate shock have no influence over the relative weight of sectorial inflations. Only sector-specific asymmetries seems to be relevant.

This last result is exemplified in the last two boxes, where relative weight of sectorial inflation in the Targeting Index is reported as a function of the parameters that characterize the AR(1) disturbances in sectorial wage markups and productivity shocks. Weights in the Targeting Index are negative (positive) functions of the variance of sectorial wage markup (productivity) shocks, a result that aligns with Mankiw and Reis (2003), reported in the bottom right. It also support the conventional wisdom that central banks should target core indexes based on the exclusion of certain product categories, such as energy goods or food, which usually display comparably highly volatile cost structures. Taking into account prices from such sectors imply more frequent interest rate adjustments and, therefore, more output contractions required to stabilize inflation.

In addition, the weight of a particular sector in the Targeting Index is a non-monotone function of the degree of inertia in the wage markup or productivity shocks in that sector, as reporter in the bottom left panel. This is direct result from the hypothesis that exogenous variables considered in the model follow AR(1) processes. Under such circumstances, the variance of an exogenous variable is a quadratic function of the inertia coefficient. Therefore, the persistence of sectorial shocks has an ambiguous effect upon the relative weight of sectorial inflations in the Targeting Index.

4 An Empirical Attempt

In this Section, I present and empirical attempt of a Targeting Index for US economy. In the first stage, I use Bayesian estimation to establish the values for the parameters of the log-linearized model. In the second stage, the maximization problem described in the last Section is carried out by using the means from posterior distributions of relevant parameters and then determining the optimal weights of sectorial inflation on the Targeting Index.

Data set consists of sectorial price and quantum indexes for Personal Consumption Expenditure (PCE), obtained at the Bureau of Economic Analysis (BEA), which are used to construct quarterly measures for sectorial inflations and output gaps for 13 different categories of products. Series extent from the last quarter of 1954 until the first quarter of 2008, comprising 214 observations. Effective Federal Funds Rate are also used as measure for nominal interest rate. Sectorial measures of inflation are demeaned from a common (linear) trend. Measures for output gaps consist of percent change of per capita quantum index, also demeaned form a common linear trend. Per capita measures are obtained dividing the quantum indexes by the Civilian Labor Force[13], from the Bureau of Labor Statistics (BLS). I employ MCMC methodology in order to estimate the main parameters of interest[14]. Sectorial productivity as well as fiscal shocks are assumed to follow AR(1) processes. Wage markup shocks are modeled as i.i.d disturbances, following Smets and Wouters (2005). Monetary shocks are also modeled as i.i.d. disturbances, as the Taylor rule parameters are estimated. Other aggregate parameters and steady state level variables are calibrated at their benchmark values. Appendix F reports the prior distributions and posterior means along with 95% confidence intervals for all estimated parameters.

Of particular interest are the estimations for the degrees of nominal rigidity. Table 1 presents the duration of price spells implied by the estimated Calvo (1983) parameters[15] for the PCE categories and compares this values with results from microdata, extracted from Nakamura and Steinsson (2008) and Bills and Klenow (2004), respectively, NS and BK[16]. Categories are not directly comparable, since both NS (2008) and BK (2004) report frequencies of price adjustments for CPI instead of PCE groups. Nonetheless, comparisons point out roughly similar degrees of nominal rigidity. One important exception is Medical Care.

[13] Sixteen years and over.

[14] Four chains, with 120,000 replications, while the first 25% are dropped. Acceptance rate from jumping distribuion is around 20%.

[15] Given by $-1/\ln(\alpha_k)$; times 3 for monthly durations.

[16] These include temporary sales.

Table 1: Sectors of PCE and implied duration of price spells (in months)

k	Categories	P. Mean	NS (2008)	BK (2004)
1	Motor vehicles and parts	9.1	-	-
2	Furniture and household equipment	6.1	4.6[1]	3.8[1]
3	Other durable goods	5.2	-	-
4	Food	3.0	2.8[2]	3.9
5	Clothing and shoes	2.9	2.7	3.4
6	Gasoline, fuel oil, and other energy goods	0.8	0.5[3]	-
7	Other nondurable goods	3.9	-	-
8	Housing	4.7	-	-
9	Household operation	3.0	2.1[4]	-
10	Transportation	1.6	2.7	2.5
11	Medical care	3.7	-	10.6
12	Recreation	5.0	7.9	8.8
13	Other services	2.0	-	-

[1] Home Furnishing.

[2] Weighted average of processed and unprocessed food.

[3] Vehicle Fuel.

[4] Utilities.

Table 2 compares the weights of PCE categories and the weights obtained from the optimization of the welfare criterion subject to equations that characterize the economy, taking all parameters as given. Optimal weights are roughly similar to the weights of PCE sectors in the representative consumer's consumption bundle. Considering broader categories, there is a small increase in the participation of Services, sponsored by a decrease in the weights from durables. Interestingly, as observed in the previous Table, this category displays the highest degree of nominal rigidity among all categories, which should account for an increase in the participation of durable goods in the Targeting Index as seen in the previous section. The observed decrease, however, is based on the fact that this group also displays the highest degree of wage markup volatility. Non-durables, which encompasses, among others, food and energy goods, have roughly the same weight. This result provides a word of caution against the use of core inflation indexes based on the exclusion of such categories of consumption goods, provided information concerns and estimation accuracy of sectorial variables are not relevant issues.

How relevant is to target the optimal index instead of the PCE price index? In order to answer that question, I compare the welfare losses generated by monetary policy conducted under the estimated Taylor rule for both price indexes. Other parameter values are given by means from posterior distributions, as detailed above. Schmitt-Grohé and Uribe (2004) provide a detailed description on comparing suboptimal policy rules by using a second order approximation of the consumption equivalent of two alternative policies, as in Lucas (1987). Using as reference the average consumption expenditure in 2006, provided by the BLS, the welfare losses per US household per year amount for US$25,86. In other words, each American household would be willing to forego twenty five dollars of its annual consumption in order to observe a shift from the PCE to the ideal index, whose weights in terms of sectorial inflation are given by Table 2.

Table 2: Sectors weights on PCE (m_k) and weights on Targeting Index (ω_k^*)

k	Categories	m_k (%)	ω_k^* (%)
1	Motor vehicles and parts	4.91	0.00
2	Furniture and household equipment	2.52	2.80
3	Other durable goods	1.71	0.00
4	Food	18.94	23.11
5	Clothing and shoes	3.69	0.79
6	Gasoline, fuel oil, and other energy goods	4.21	4.51
7	Other nondurable goods	7.96	4.78
8	Housing	16.18	19.78
9	Household operation	5.63	2.24
10	Transportation	4.19	5.21
11	Medical care	14.37	18.95
12	Recreation	2.91	0.00
13	Other services	12.77	17.83
	Durable goods	9.15	2.80
	Non-durable goods	34.80	33.19
	Services	56.05	64.01

5 Conclusion

This paper establishes an optimal inflation measure which a Central Bank operating under a historical Taylor rule should target. This measure is obtained by weighting sectorial inflations in a way to maximize the representative consumer's utility function subject to the set of equations that characterize the economy. In theoretical grounds, weights of sectorial inflations on the Targeting Index as function of some key parameter values reflect the results established by the related literature. For some extreme values of Calvo parameters, however, it is possible to show that the reported increasing relation is non-monotone, a result credited to the use of a Taylor rule sensitive to aggregate output gap in an environment of heterogeneity of price stickiness. In more concrete grounds, the Targeting Index derived for the US economy using Bayesian estimation to calibrate the values for sectorial parameters points towards two main observations. First, the optimal Targeting Index is considerably similar to inflation measured by the Personal Consumption Expenditure, with less weight given to durable goods and more to inflation in the sector of service goods. Second, there is no evidence that a core inflation index based on the exclusion of food and energy goods is welfare improving. This result holds as long as considerations regarding information costs from more volatile sectorial variables are left aside. Hence, there is evidence on the importance of price stickiness, but other sectorial parameters as the variance and inertia of cost-push shocks are also decisive. Durable goods received small weight in the Targeting Index in spite the fact of displaying the highest degree of nominal rigidity, a fact attributable to the high variances of wage markup in those sectors.

References

[1] Aoki, Kosuke. (2001). "Optimal monetary policy responses to relative price changes." Journal of Monetary Economics, 48, 55–80.

[2] Benigno, Pierpaolo. (2004). "Optimal monetary policy in a currency area." Journal of International Economics, 63, 293-320.

[3] Benigno, Pierpaolo and Michael Woodford . (2003). "Optimal Monetary and Fiscal Policy: A Linear-Quadratic Approach." NBER Working Papers 9905, July 2003.

[4] Bils, Mark and Pete Klenow. (2004). "Some Evidence on the Importance of Sticky Prices." Journal of Political Economy, 112, 947-985.

[5] Calvo, Guillermo. (1983). "Staggered prices in a utility-maximizing framework." Journal of Monetary Economics, 12, 382-398.

[6] Dixit, Avinash K., and Joseph E. Stiglitz. (1977). "Monopolistic Competition and Optimum Product Diversity." American Economic Review, 67, 297-308.

[7] Lucas, Robert E., and Nancy Stokey. (1983). "Optimal Fiscal and Monetary Policy in an Economy without Capital." Journal of Monetary Economics 12, 55-93.

[8] Lucas, Robert E. (1987). "Models of Business Cycles." Basil Blackwell, New York, NY.

[9] Mankiw, N. Gregory and Ricardo Reis. (2003). "What measure of inflation should a central bank target?" Journal of European Economic Association 1, 1058 - 1086.

[10] Nakamura, Emi, and Jón Steinsson. (2008). "Five Facts about Prices: a Reevaluation of Menu Cost Models." Quarterly Journal of Economics, forthcoming.

[11] Rotemberg, Julio J., and Michael Woodford. (1999). "The cyclical behavior of prices and costs". In Handbook of Macroeconomics, edition 1, vol. 1, edited by John B. Taylor and Michael Woodford, pp. 1051-1135. Elsevier.

[12] Sargent, Thomas, and Neil Wallace. (1981). "Some Unpleasant Monetarist Arithmetic." Federal Reserve Bank of Minneapolis Quarterly Review, 5(Fall), 1-17.

[13] Smets, Frank, and Rafael Wouters. (2005). "Comparing Shocks and Frictions in US and Euro Area Business Cycles: A Bayesian DSGE Approach." Journal of Applied Econometrics, 20, 161-185.

[14] Schmitt-Grohé, Stephanie, and Martin Uribe. (2004). "Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function." Journal of Economic Dynamics and Control, 28, 755-775.

[15] Svensson, Lars (2003). "What Is Wrong with Taylor Rules? Using Judgment in Monetary Policy through Targeting Rules." Journal of Economic Literature. American Economic Association, 41(2), 426-477, June.

[16] Steinsson, Jón. (2003). "Optimal monetary policy in an economy with inflation persistence." Journal of Monetary Economics, 50, 1425 - 1456.

[17] Woodford, Michael. (1999). "Commentary: How Should Monetary Policy Be Conducted in an Era of Price Stability?". In: New Challenges for Monetary Policy, Federal Reserve Bank of Kansas City.

[18] Woodford, Michael. (2003). Interest and Prices: Foundations of a Theory of Monetary Policy. Princeton, NJ: Princeton University Press.

6 Appendix A - The Firms' Problem

Noting that $\theta > 1$, FOC from firms' optimization problem is given by:

$$E_t \sum_{j=t}^{\infty} \alpha_k^{j-t} \Theta_{t,j} \frac{\partial \Psi_j \left(p_{k,t}\left(z\right),\ .\right)}{\partial p_{k,t}\left(z\right)} = 0; \tag{50}$$

taking derivatives and dividing resulting expression by $1-\theta$

$$E_t \sum_{j=t}^{\infty} \alpha_k^{j-t} \Theta_{t,j} \frac{p_{k,t}(z)}{P_{k,t+j}}^{-\theta} Y_{k,j} \{1 + \frac{\theta}{1-\theta} \frac{w_{k,j}(z)}{a_{k,j}} \frac{P_j}{P_{k,j}} \frac{P_{k,j}}{p_{k,t}(z)}\} = 0;$$

using expression in the main text for labor supply, production function and discount factor:

$$E_t \sum_{j=t}^{\infty} (\alpha_k \beta)^{j-t} \frac{C_j^{-\sigma}}{P_j} \frac{p_{k,t}(z)}{P_{k,j}}^{-\theta} Y_{k,j} \{1 + \\ + \mu_{k,j} \frac{\theta\lambda}{1-\theta} \frac{y_{k,j}(z)^{\nu}}{C_j^{-\sigma}} \frac{1}{a_{k,j}^{\nu+1}} \frac{P_j}{P_{k,j}} \frac{P_{k,j}}{p_{k,t}(z)}\} = 0;$$

using expression for demand for good z in terms of sectorial aggregates and isolating terms $p_{k,t}(z)/P_{k,t}$.

$$\frac{p_{k,t}(z)}{P_{k,t}}^{1+\theta\nu} E_t \sum_{j=t}^{\infty} (\alpha_k \beta)^{j-t} \frac{C_j^{-\sigma}}{P_j} \frac{P_{k,t}}{P_{k,j}}^{-\theta} Y_{k,j} = \\ \mu_{k,j} \frac{\theta\lambda}{\theta-1} m_k^{-\nu} E_t \sum_{j=t}^{\infty} (\alpha_k \beta)^{j-t} \frac{P_{k,j}}{P_{k,t}}^{1+\theta(\nu+1)} \frac{Y_{k,j}}{a_{k,j}}^{\nu+1} \frac{1}{P_{k,j}}$$

$$\frac{p_{k,t}(z)}{P_{k,t}}^{1+\theta\nu} = \frac{\frac{\theta\lambda}{\theta-1} m_k^{-\nu} E_t \sum_{j=t}^{\infty} (\alpha_k \beta)^{j-t} \mu_{k,j} \frac{P_{k,j}}{P_{k,t}}^{\theta(\nu+1)} \frac{Y_{k,j}}{a_{k,j}}^{\nu+1}}{E_t \sum_{j=t}^{\infty} (\alpha_k \beta)^{j-t} C_j^{-\sigma} \frac{P_{k,j}}{P_{k,t}}^{\theta-1} p_{k,j} Y_{k,j}} \tag{51}$$

7 Appendix B - Steady State

There is a steady state characterized by zero inflation and constant values for all variables, where exogenous disturbances also assume constant values, that is: $\bar{\xi} = \{\bar{G}, \bar{a}_k, \bar{\mu}_k, \bar{e}_t\}$, where $\bar{a}_k = 1$ and $\bar{\mu}_k = \bar{\mu} > 1$, all k. We focus particular attention to a steady state with positive real debt, that is $\bar{b}_{-1} = \bar{b} > 0$, price dispersion equals one, $\Delta_{k,-1} = \bar{\Delta}_k = 1$ and relative price also equals one, $p_{k,-1} = \bar{p}_k = 1$, all k. Consider the government budget constraint, which in steady state is given by:

$$(1-\beta)\bar{b} = \bar{\tau} - \bar{G}. \tag{52}$$

Assuming government expenses are non-zero in steady state (i.e.: $\bar{G} > 0$), imply, according to the hypothesis of a Ricardian regime, that $\bar{\tau}$ is determined directly from (52) and proportional to both $\bar{G}$ and $\bar{b}$. From the firms' maximization problem, considering $\bar{\Pi}_k = 0$, all k:

$$\bar{K}_k = \bar{F}_k.$$

Using definitions for both terms:

$$\frac{\theta\lambda}{\theta-1}\bar{\mu}m_k^{-\nu}\bar{Y}_k^{\nu} = \bar{C}^{-\sigma}, \tag{53}$$

From (18) in the text, $\bar{Y}_k = m_k\bar{Y}$, which implies that

$$\bar{Y} = \left[\frac{\theta\lambda}{\theta-1}\bar{\mu}{s_c}^{\sigma}\right]^{-1/\nu+\sigma}, \tag{54}$$

while s_c is defined as

$$s_c = \bar{C}/\bar{Y},$$

which is determined through the market clearing and the fact that $\bar{G}$ is positive and exogenously given. Therefore, $\bar{Y}$ and $\bar{C}$ are defined by the equations above in terms of the parameters of the economy.[Is this part really important???]

8 Appendix C - Approximation to Welfare Criterion

8.1 Second Order Approximation of Utility Function

I start with a second order Taylor expansion of the representative consumer's welfare function where ξ_t refers to the full vector of random disturbances, as in Benigno and Woodford (2003). Define hereafter, for any variable X_t,

$$\tilde{X}_t \equiv \frac{X_t - \bar{X}}{\bar{X}},$$

$$\hat{X}_t \equiv \log\frac{X_t}{\bar{X}}.$$

It is know that the following relation holds up to second order:

$$\tilde{X}_t \simeq \hat{X}_t + \frac{1}{2}\hat{X}_t^2. \tag{55}$$

Given the functional form assumed in the main text for the utility function, define

$$u\left(Y_t,\xi_t\right) \equiv \frac{\left(Y_t - G_t\right)^{1-\sigma}}{1-\sigma}.$$

A second order Taylor expansion yields:

$$u\left(Y_t,\xi_t\right) = \bar{C}^{-\sigma}\bar{Y}[\tilde{Y}_t - \frac{\sigma}{2}\frac{\bar{Y}}{\bar{C}}\tilde{Y}_t^2 + \sigma\frac{\bar{Y}}{\bar{C}}\tilde{Y}_t\tilde{G}_t] + tips + O_p^3, \tag{56}$$

where $\tilde{G}_t$ represents the absolute deviation *over GDP*. Defining $s_C = \bar{C}/\bar{Y}$, yields

$$u\left(Y_t,\xi_t\right) = \bar{C}^{-\sigma}\bar{Y}[\hat{Y}_t + \frac{1}{2}\hat{Y}_t^2(1-\sigma s_C^{-1}) + \sigma s_C^{-1}\hat{Y}_t\hat{G}_t] + tips + O_p^3. \tag{57}$$

Define also:

$$v\left(Y_{k,t},\xi_t\right)\Delta_{k.t} \equiv \frac{\lambda}{1+\nu}\left[\frac{Y_{k,t}}{m_k a_{k,t}}\right]^{1+\nu}\Delta_{k.j}$$

A second order Taylor expansion around steady state values yield

$$\begin{aligned} v\left(Y_{k,t},\xi_t\right)\Delta_{k.t} \quad = \quad & v\left(\bar{Y}_k,\bar{\xi}\right)\tilde{\Delta}_{k.t} + v_{Y_k}\left(\bar{Y}_k,\bar{\xi}\right)\bar{Y}_k(\hat{Y}_{k,t} + \frac{1}{2}\hat{Y}_{k,t}^2) + \\ & + \frac{1}{2}v_{Y_kY_k}\left(\bar{Y}_k,\bar{\xi}\right)\bar{Y}_k^2(\hat{Y}_{k,t}^2) + v_{Y_k}\left(\bar{Y}_k,\bar{\xi}\right)\bar{Y}_k(\hat{Y}_{k,t})\tilde{\Delta}_{k.t} + \\ & + v_{Y_k\xi}\left(\bar{Y}_k,\bar{\xi}\right)\bar{Y}_k(\hat{Y}_{k,t}\hat{a}_{k,t}) + v_{\xi}\left(\bar{Y}_k,\bar{\xi}\right)\tilde{\Delta}_{k.t}(\hat{a}_{k,t}) + \\ & + tips + O_p^3. \end{aligned} \tag{58}$$

Using the definition for $\Delta_{k,t}$ one can show that $\tilde{\Delta}_{k,t}$ is a term of second order. In this sense, interactions between $\tilde{\Delta}_{k,t}$ and $\hat{a}_{k,t}$ or $\tilde{\Delta}_{k,t}$ and $\hat{Y}_{k,t}$ can be ignored up to second order. Hence, expression (58) simplifies to

$$v\left(Y_{k,t},\xi_t\right)\Delta_{k.t} = \lambda\left[\frac{\bar{Y}_k}{m_k}\right]^{1+\nu}\{\frac{\hat{\Delta}_{k.t}}{1+\nu} + \hat{Y}_{k,t} + \frac{1+\nu}{2}\hat{Y}_{k,t}^2 - (1+\nu)\hat{Y}_{k,t}\hat{a}_{k,t}\} + tips + O_p^3, \tag{59}$$

once one notice that $\hat{\Delta}_{k,t}^2$ is of higher order than O_p^2. Using a second order Taylor expansion over the law of motion for sectorial price dispersion given by (29) in the main text yields:

$$\hat{\Delta}_{k.t} = \alpha_k\hat{\Delta}_{k.t-1} + \frac{1}{2}\frac{\alpha_k}{(1-\alpha_k)}\theta(1+\nu)(1+\theta\nu)\pi_{k,t}^2 + O_p^3,$$

once one uses the relation $\hat{\Pi}_{k,t} = \pi_{k,t} + (1/2)\pi_{k,t}^2$, where $\pi_{k,t}$ is the percent variation of sectorial price level $\pi_{k,t} = \log P_{k,t}/P_{k,t-1}$. Iterating backwards yields

$$\hat{\Delta}_{k.t} = \alpha_k^{t-1}\hat{\Delta}_{k.-1} + \frac{1}{2}\frac{\alpha_k}{(1-\alpha_k)}\theta(1+\nu)(1+\theta\nu)\sum_{j=0}^{t}\alpha_k^{t-j}\pi_{k,j}^2 + O_p^3.$$

Here it is convenient to consider the sectorial price dispersion in the remote past as a "term independent of policy". Further considering that it is possible to change positions of sums over t and k on (59), and re-ordering the terms:

$$\sum_{t=0}^{\infty}\beta^t\hat{\Delta}_{k.t} = \frac{1}{2}\frac{\alpha_k}{(1-\alpha_k)(1-\alpha_k\beta)}\theta(1+\nu)(1+\theta\nu)\sum_{t=0}^{\infty}\beta^t\pi_{k,t}^2 + tips + O_p^3. \quad (60)$$

Substituting (60) over (59) yields

$$v(Y_{k,t},\xi_t)\,\Delta_{k.t} = \lambda\left[\frac{\bar{Y}_{k,t}}{m_k}\right]^{1+\nu}\{\frac{1}{2}\frac{\alpha_k\theta(1+\theta\nu)}{(1-\alpha_k)(1-\alpha_k\beta)}\pi_{k,t}^2 + \hat{Y}_{k,t} + \\ + \frac{1+\nu}{2}\hat{Y}_{k,t}^2 - (1+\nu)\hat{Y}_{k,t}\hat{a}_{k,t}\} + tips + O_p^3.$$

Considering expressions for $u(Y_t,\xi_t)$ and $v(Y_{k,t},\xi_t)\,\Delta_{k.t}$, we can approximate the representative consumer utility up to second order by the following expression:

$$\begin{aligned} U_{t_0} &= \Omega E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\{\hat{Y}_t + \frac{(1-\tilde{\sigma})}{2}\hat{Y}_t^2 + \tilde{\sigma}\hat{Y}_t\hat{G}_t + \\ &\quad -\sum_{k=1}^{K} m_k(1-\Phi)[\frac{\theta}{\kappa_k}\frac{\pi_{k,t}^2}{2} + \hat{Y}_{k,t} + \frac{1+\nu}{2}\hat{Y}_{k,t}^2 + \\ &\quad -(1+\nu)\hat{Y}_{k,t}\hat{a}_{k,t}]\} + tips + O_p^3, \end{aligned} \quad (61)$$

where

$$\Omega \equiv \bar{C}^{-\sigma}\bar{Y}, \quad (62)$$

$$\kappa_k \equiv \frac{(1-\alpha_k)(1-\alpha_k\beta)}{(1+\theta\nu)\alpha_k}, \quad (63)$$

$$\tilde{\sigma} \equiv \sigma s_C^{-1} \quad (64)$$

and

$$(1-\Phi) \equiv \frac{\theta-1}{\theta}\frac{1}{\bar{\mu}}, \quad (65)$$

where feasibility constraint in (53) was used to eliminate inconvenient terms in $v(Y_{k,t},\xi_t)\,\Delta_{k.t}$. Following Benigno and Woodford (2003), we seek to eliminate linear terms by obtaining second order approximations to all equations that describe the economy.

8.2 Second Order Approximation to AS Equation

The starting point is the expression for the sectorial non-linear Phillips Curve, given by:

$$\left(\frac{1-\alpha_k \Pi_{k,t}^{\theta-1}}{1-\alpha_k}\right)^{\frac{1+\theta\nu}{\theta-1}} = \frac{F_{k,t}}{K_{k,t}}. \tag{66}$$

We define $V_{k,t}$ as

$$V_{k,t} = \frac{1-\alpha_k \Pi_{k,t}^{\theta-1}}{(1-\alpha_k)}. \tag{67}$$

Using a second order Taylor expansion on $\hat{V}_{k,t}$:

$$\hat{V}_{k.t} = -\frac{\alpha_k(\theta-1)}{(1-\alpha_k)}\left[\pi_{k,t} + \frac{1}{2}\frac{(\theta-1)}{(1-\alpha_k)}\pi_{k,t}^2\right] + O_p^3. \tag{68}$$

Considering the expression for $K_{k,t}$ define $\Pi_{k,t,s} = P_{k,s}/P_{k,t}$, where $s \geq t$ is some date in the future and $P_{k,t}$ the aggregate price level in sector k in period t. We use a second order Taylor expansion:

$$\tilde{K}_{k,t} = (1-\beta\alpha_k)E_t\sum_{j=t}^{\infty}(\alpha_k\beta)^{j-t}\{\hat{k}_{k,j} + \frac{1}{2}\hat{k}_{k,j}^2\} + O_p^3, \tag{69}$$

where the term $\hat{k}_{k,t}$ can be defined as

$$\hat{k}_{k,j} = \theta(1+\nu)\pi_{k,t,j} + (1+\nu)\hat{Y}_{k,j} - (1+\nu)\hat{a}_{k,j}.$$

Taking a second order Taylor expansion of (27) in the text:

$$\tilde{F}_{k,t} = (1-\beta\alpha_k)E_t\sum_{j=t}^{\infty}(\alpha_k\beta)^{j-t}\{\hat{f}_{k,j} + \frac{1}{2}\hat{f}_{k,j}^2\} + O_p^3, \tag{70}$$

where we define

$$\hat{f}_{k,j} = -\sigma\hat{C}_j + \hat{Y}_{k,j} + \hat{p}_{k,j} + (\theta-1)\,\pi_{k,t,j}.$$

Using $\tilde{F}_{k,t}$, $\tilde{K}_{k,t}$, as well as $\hat{V}_{k.t}$, $\hat{F}_{k,t}$ and $\hat{K}_{k,t}$,after some algebra, we get:

$$\left[\frac{1+\theta\nu}{\theta-1}\right]\hat{V}_{k,t} = (1-\beta\alpha_k)E_t\sum_{j=t}^{\infty}(\alpha_k\beta)^{j-t}\{[z_{k,j}-(1+\theta\nu)\pi_{k,t,j}]+$$
$$+\frac{1}{2}[z_{k,j}-(1+\theta\nu)\pi_{k,t,j}][\hat{X}_{k,j}+[(\theta-1)+\theta(1+\nu)]\pi_{k,t,j}]\}$$
$$-\frac{1}{2}\left[\frac{1+\theta\nu}{\theta-1}\right]\hat{V}_{k,t}(1-\beta\alpha_k)E_t\sum_{j=t}^{\infty}(\alpha_k\beta)^{j-t}\{\hat{X}_{k,j}+[(\theta-1)+\theta(1+\nu)]\pi_{k,t,j}\}+O_p^3,$$

where

$$\hat{X}_{k,j} \equiv -\sigma\hat{C}_j + (2+\nu)\hat{Y}_{k,j} + \hat{p}_{k,j} - (1+\nu)\hat{a}_{k,j} + \hat{\mu}_{k,t}, \tag{71}$$

$$\hat{f}_{k,j} - \hat{k}_{k,j} = z_{k,j} - (1+\theta\nu)\pi_{k,t,j}$$

and

$$z_{k,j} = -\sigma\hat{C}_j - \nu\hat{Y}_{k,j} + \hat{p}_{k,j} + (1+\nu)\hat{a}_{k,j} - \hat{\mu}_{k,t}. \tag{72}$$

Define

$$Z_{k,t} \equiv E_t\sum_{j=t}^{\infty}(\alpha_k\beta)^{j-t}\left\{\left[\hat{X}_{k,j}+[(\theta-1)+\theta(1+\nu)]\pi_{k,t,j}\right]\right\} \tag{73}$$

We can replace in the expression above and after some algebra we get:

$$\frac{(1+\theta\nu)}{(\theta-1)(1-\beta\alpha_k)}\hat{V}_{k,t}(\pi_{k,t+1}) = (\pi_{k,t+1})E_t\sum_{j=t+1}^{\infty}(\alpha_k\beta)^{j-t-1}\{z_{k,j}-(1+\theta\nu)(\pi_{k,t,j})\}+O_p^3 \tag{74}$$

We can use the definition for $\hat{V}_{k,t}$ and replace above, also ignoring the terms O_p^3 or of higher order:

$$-\kappa_k^{-1}\left[\pi_{k,t}+\frac{1}{2}\frac{(\theta-1)}{(1-\alpha_k)}\pi_{k,t}^2-\alpha_k\beta E_t\pi_{k,t+1}-\frac{1}{2}\frac{(\theta-1)}{(1-\alpha_k)}\alpha_k\beta E_t\pi_{k,t+1}^2\right]=$$
$$z_{k,t}+\frac{1}{2}z_{k,t}\hat{X}_{k,t}-(1+\theta\nu)\frac{\alpha_k\beta}{(1-\alpha_k\beta)}E_t\pi_{k,t+1}+$$
$$-\frac{1}{2}[(\theta-1)+\theta(1+\nu)]\frac{\beta}{\kappa_k}E_t\pi_{k,t+1}^2+$$
$$-\frac{1}{2}(1+\theta\nu)(\alpha_k\beta)E_t[\pi_{k,t+1}Z_{k,t+1}]+$$
$$+\frac{1}{2}\frac{(1+\theta\nu)\alpha_k}{(1-\alpha_k)}[\pi_{k,t}Z_{k,t}-\alpha_k\beta E_t[\pi_{k,t+1}Z_{k,t+1}]]+O_p^3,$$

where we have defined κ_k elsewhere.

Further simplification yields

$$
\begin{aligned}
-\kappa_k^{-1}\pi_{k,t} - \frac{1}{2}\kappa_k^{-1}\frac{(\theta-1)}{(1-\alpha_k)}\pi_{k,t}^2 - \frac{1}{2}\frac{(1+\theta\nu)\alpha_k}{(1-\alpha_k)}\pi_{k,t}Z_{k,t} & \\
= z_{k,t} + \frac{1}{2}z_{k,t}\hat{X}_{k,t} - \kappa_k^{-1}\beta E_t\pi_{k,t+1} & \\
- \frac{1}{2}\kappa_k^{-1}\{\frac{(\theta-1)}{(1-\alpha_k)} + \theta(1+\nu)\}\beta E_t\pi_{k,t+1}^2 & \\
- \frac{1}{2}\frac{(1+\theta\nu)\alpha_k}{(1-\alpha_k)}\beta E_t[\pi_{k,t+1}Z_{k,t+1}] + O_p^3. &
\end{aligned}
$$

Multiplying both sides for $-\kappa_k$ allow us to write above expression as

$$
\mathcal{V}_{k,t} = -\kappa_k\{z_{k,t} + \frac{1}{2}z_{k,t}\hat{X}_{k,t}\} + \frac{\theta(1+\nu)}{2}\pi_{k,t}^2 + \beta E_t\mathcal{V}_{k,t+1} + O_p^3, \qquad (75)
$$

where:

$$
\mathcal{V}_{k,t} = \pi_{k,t} + \frac{1}{2}\{\frac{(\theta-1)}{(1-\alpha_k)} + \theta(1+\nu)\}\pi_{k,t}^2 + \frac{1}{2}\frac{\kappa_k\alpha_k}{(1-\alpha_k)}[\pi_{k,t}Z_{k,t}]. \qquad (76)
$$

Log-approximation on consumption as a function of aggregate output and government expenses yields:

$$
\hat{C}_t = s_C^{-1}\hat{Y}_t - s_C^{-1}\hat{G}_t + \frac{1}{2}s_C^{-1}(1-s_C^{-1})\hat{Y}_t^2 - \frac{1}{2}s_C^{-1}(1+s_C^{-1})\hat{G}_t^2 + s_C^{-2}\hat{Y}_t\hat{G}_t + O_p^3. \quad (77)
$$

Using this result, one can be generally express (75) as

$$
\mathcal{V}_{k,t} = E_{t_0}\sum_{j=t}^{\infty}\beta^{j-t}\{-\kappa_k[z_{k,t} + \frac{1}{2}z_{k,t}\hat{X}_{k,t}] + \frac{\theta(1+\nu)}{2}\pi_{k,t}^2\} + tips + O_p^3. \quad (78)
$$

One could finally note that a first order approximation to (78) yields the known Phillips Curve of the form:

$$
\pi_{k,t} = \kappa_k\{(\tilde{\sigma}-\eta^{-1})\hat{Y}_t + (\nu+\eta^{-1})\hat{Y}_{k,t} - \tilde{\sigma}\hat{G}_t - (1+\nu)\hat{a}_{k,t} + \hat{\mu}_{k,t}\} + \beta E_t\pi_{k,t+1}.
$$

8.3 Aggregate and Sectorial Output Relation

Sectorial demand expressed is (18) can be log-linearized as

$$\hat{p}_{k,t} = \eta^{-1}(\hat{Y}_t - \hat{Y}_{k,t}), \tag{79}$$

which establishes an exact (inverse) relation between sector relative price and sector relative product. It is used to eliminate references to relative prices in all equations. Also, using (35) in the text and (79), one gets:

$$\eta^{-1}(\hat{Y}_t - \hat{Y}_{k,t}) = \pi_{k,t} - \pi_t + \eta^{-1}(\hat{Y}_{t-1} - \hat{Y}_{k,t-1}), \tag{80}$$

all k, which is also an exact relation. Also using (35) and (79) over (36) in the main text yields:

$$Y_t^{(\eta-1)/\eta} = \sum_{k=1}^{K} m_k^{1/\eta} Y_{k,t}^{(\eta-1)/\eta}, \tag{81}$$

which relates aggregate and sectorial outputs. Log linearization of (81) yields

$$\hat{Y}_t + \frac{1}{2}(1-\eta^{-1})\hat{Y}_t^2 = \sum_{k=1}^{K} m_k \hat{Y}_{k,t} + \frac{1}{2}(1-\eta^{-1})\sum_{k=1}^{K} m_k \hat{Y}_{k,t}^2 + O_p^3. \tag{82}$$

whose first order approximation in simply the definition of aggregate output in terms of sectorial outputs:

$$\hat{Y}_t = \sum_{k=1}^{K} m_k \hat{Y}_{k,t}. \tag{83}$$

8.4 Matrix Notation

We start by defining

$$x_t' = \left[\begin{array}{cccccccc} \hat{Y}_t & \hat{Y}_{1,t} & ... & \hat{Y}_{K,t} & \pi_{1,t} & ... & \pi_{K,t} \end{array}\right] \tag{84}$$

and

$$\xi_t' = \left[\begin{array}{cccccccc} \hat{G}_t & \hat{a}_{1,t} & ... & \hat{a}_{K,t} & \hat{\mu}_{k,t} & ... & \hat{\mu}_{K,t} \end{array}\right]. \tag{85}$$

For notational convenience, we also define the following terms:

$$\upsilon \equiv 1 + \nu, \tag{86}$$

$$\omega_\eta \equiv 1 - \eta^{-1}, \tag{87}$$

$$\chi \equiv \nu + \eta^{-1}, \tag{88}$$

$$\tilde{\sigma} \equiv \sigma s_C^{-1}, \tag{89}$$

$$\varsigma \equiv \tilde{\sigma} - \eta^{-1}, \tag{90}$$

and

$$\omega_C \equiv \frac{\bar{Y} - \bar{C}}{\bar{C}}, \tag{91}$$

in addition to:

$$s_C \equiv \bar{C}/\bar{Y}. \tag{92}$$

Using the definitions above, expression in (61) can be written in matrix notation as

$$U_{t_0} \equiv \Omega E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \{A_x' x_t - \frac{1}{2} x_t' A_{xx} x_t - x_t' A_\xi \xi_t\} + tips + O_p^3, \tag{93}$$

where A_x, A_{xx}, and A_ξ are, respectively, $(2K+1) \times 1$, $(2K+1) \times (2K+1)$ and $(2K+1) \times (2K+1)$ matrices, such as:

$$A_x' = \left[\begin{array}{cccccccc} 1 & -m_1(1-\Phi) & \ldots & -m_K(1-\Phi) & 0 & \ldots & 0 \end{array} \right], \tag{94}$$

$$A_{xx} = \left[\begin{array}{ccc} A_{xx}^{11} & 0 & 0 \\ 0 & A_{xx}^{22} & 0 \\ 0 & 0 & A_{xx}^{33} \end{array} \right], \tag{95}$$

where A_{xx}^{11} is a 1×1 matrix such as

$$A_{xx}^{11} = -(1 - \tilde{\sigma}),$$

A_{xx}^{22} is a $K \times K$ diagonal matrix such as its typical k^{th} element is

$$\left(A_{xx}^{22}\right)_{kk} = m_k(1-\Phi)\upsilon,$$

A_{xx}^{33} is a $K \times K$ diagonal matrix such as its typical k^{th} element is

$$\left(A_{xx}^{33}\right)_{kk} = \frac{m_k(1-\Phi)}{\kappa_k}\theta,$$

and

$$A_\xi = \left[\begin{array}{ccc} A_\xi^{11} & 0 & 0 \\ 0 & A_\xi^{22} & 0 \\ 0 & 0 & 0 \end{array} \right], \tag{96}$$

where

$$A_\xi^{11} = -\tilde{\sigma}$$

and A_ξ^{22} is a $K \times K$ diagonal matrix such as its typical k^{th} element is

$$\left(A_\xi^{22}\right)_{kk} = -m_k(1-\Phi)\upsilon,$$

and where we have observed the definitions in (62)-(65).

The Sectorial Phillips Curve expressed in (78) can also be written in matrix notation. We start by substituting expressions for $\hat{p}_{k,t}$ into definitions for $z_{k,t}$ and $\hat{X}_{k,t}$, underlined in (72) and (71). Quadratic and linear terms of random disturbances are placed into *tips*. After some manipulation one obtains:

$$\mathcal{V}_{k,t_0} = E_{t_0} \sum_{j=t_0}^{\infty} \beta^{j-t_0} \{C'_{x,k} x_t + \frac{1}{2} x'_t C_{xx,k} x_t + x'_t C_{\xi,k} \xi_t\} + tips + O_p^3, \qquad (97)$$

for a generic sector k. As in (93), matrices $C_{x,k}$, $C_{xx,k}$, and $C_{\xi,k}$ have, respectively, dimension $(2K+1) \times 1$, $(2K+1) \times (2K+1)$ and $(2K+1) \times (2K+1)$, such as:

$$C'_{x,k} = \left[\begin{array}{ccc} C_{x,k}^{11\prime} & C_{x,k}^{12\prime} & 0 \end{array}\right], \qquad (98)$$

where $C_{x,k}^{11\prime}$ is 1×1 matrix such as

$$C_{x,k}^{11\prime} = \kappa_k \varsigma$$

every k, $C_{x,k}^{12\prime}$ is $1 \times K$ matrix such as

$$\left(C_{x,k}^{12\prime}\right)_{1k} = \kappa_k \chi$$

and zeros elsewhere; and

$$C_{xx.k} = \left[\begin{array}{ccc} C_{xx.k}^{11} & C_{xx.k}^{12} & 0 \\ C_{xx.k}^{21} & C_{xx.k}^{22} & 0 \\ 0 & 0 & C_{xx.k}^{33} \end{array}\right] \qquad (99)$$

such that $C_{xx,k}^{11}$ is 1×1 matrix

$$C_{xx,k}^{11} = -\kappa_k[\tilde{\sigma}\omega_C + \varsigma^2]$$

for every k, $C_{xx,k}^{12}$ is $1 \times K$ matrix such that

$$\left(C_{xx,k}^{12}\right)_{1k} = \kappa_k \varsigma \omega_\eta$$

and zeros elsewhere, all k, and $C_{xx,k}^{12\prime} = C_{xx,k}^{21}$; $C_{xx,k}^{22}$ is $K \times K$ diagonal matrix such that, all k,

$$\left(C^{22}_{xx,k}\right)_{kk} = \chi\kappa_k(\upsilon + \omega_\eta)$$

$C^{33}_{xx,k}$ is $K \times K$ diagonal matrix such that, for all k,

$$\left(C^{33}_{xx,k}\right)_{kk} = \theta\upsilon$$

Also, matrix $C_{\xi,k}$ can be defined as

$$C_{\xi,k} = \begin{bmatrix} C^{11}_{\xi,k} & 0 & 0 \\ C^{21}_{\xi,k} & C^{22}_{\xi,k} & C^{23}_{\xi,k} \\ 0 & 0 & 0 \end{bmatrix} \tag{100}$$

where $C^{11}_{\xi,k}$ is 1×1 matrix, such that

$$C^{11}_{\xi,k} = \kappa_k[\omega_C + \tilde{\sigma} + \omega_\eta]\tilde{\sigma}$$

for every k; $C^{21}_{\xi,k}$ is a $K \times 1$ matrix, such as

$$\left(C^{21}_{\xi,k}\right)_{1k} = -\kappa_k\omega_\eta\tilde{\sigma}$$

and zero elsewhere, $C^{22}_{\xi,k}$ is $K \times K$ diagonal matrix such that

$$\left(C^{22}_{\xi,k}\right)_{kk} = -\kappa_k\upsilon^2$$

and zero elsewhere, $C^{23}_{\xi,k}$ is $K \times K$ diagonal matrix such that

$$\left(C^{23}_{\xi,k}\right)_{kk} = \kappa_k\upsilon$$

and zero elsewhere.

Equation (82) can be expressed in matrix notation as

$$0 = \sum_{j=t}^{\infty} \beta^{j-t}\{H'_x x_t + \frac{1}{2}x'_t H_{xx} x_t\} + O^3_p \tag{101}$$

where we have used the fact that the definition for aggregate output in terms of its sectorial counterparts expressed in (82) is valid at all dates. Matrices H_x and H_{xx} have, respectively, dimension $(2K+1) \times 1$ and $(2K+1) \times (2K+1)$, such as:

$$H'_x = \begin{bmatrix} 1 & -m_1 & ... & -m_K & 0 & ... & 0 \end{bmatrix}, \tag{102}$$

$$H_{xx} = \omega_\eta \begin{bmatrix} 1 & 0 & 0 \\ 0 & H^{22}_{xx} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{103}$$

where H^{22}_{xx} is a $K \times K$ diagonal matrix such as

$$\left(H^{22}_{xx}\right)_{kk} = -m_k,$$

for every k.

8.5 Elimination of Linear Terms

In order to eliminate linear terms in (93), we need to find a set a multipliers $\vartheta_C^1, ..., \vartheta_C^K, \vartheta_H$, such as

$$\vartheta_C^1 C_x^{1\prime} + ... + \vartheta_C^K C_x^{K\prime} + \vartheta_H H_x' = A_x' \tag{104}$$

By solving the linear system of equations, one gets the following set of solution:

$$\vartheta_H = \frac{\chi + \varsigma(1 - \Phi)}{\varsigma + \chi} \tag{105}$$

and, for every k,

$$\vartheta_C^k = \frac{m_k}{\kappa_k} \frac{\Phi}{\varsigma + \chi} \tag{106}$$

where we have used the definitions in (63)-(65).

Hence, using relations (93), (97), (101) and (104) one can write:

$$\begin{aligned} E_{t_0} \sum_{j=t_0}^{\infty} \beta^{j-t_0} A_x' x_t &= E_{t_0} \sum_{j=t_0}^{\infty} \beta^{j-t_0} [\sum_{k=1}^{K} \vartheta_C^k C_x^{k\prime} + \vartheta_H H_x'] x_t \\ &= -E_{t_0} \sum_{j=t_0}^{\infty} \beta^{j-t_0} \{\frac{1}{2} x_t' D_{xx} x_t + x_t' D_\xi \xi_t\} + \sum_{k=1}^{K} \vartheta_C^k \mathcal{V}_{k,t_0}, \end{aligned} \tag{107}$$

where

$$D_{xx} = \sum_{k=1}^{K} \vartheta_C^k C_{xx,k} + \vartheta_H H_{xx}$$

and

$$D_\xi = \sum_{k=1}^{K} \vartheta_C^k C_\xi^k.$$

We use this last relations in order to rewrite (93) as

$$U_{t_0} \equiv -\Omega E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \{\frac{1}{2} x_t' Q_{xx} x_t + x_t' Q_\xi \xi_t\} + T_{t_0} + tips + O_p^3 \tag{108}$$

where

$$T_{t_0} = \Omega\{\sum_{k=1}^{K} \vartheta_C^k \mathcal{V}_{k,t_0}\} \tag{109}$$

is a vector of predetermined variables and where Q_{xx} and Q_ξ can be defined, respectively, as

$$Q_{xx} = \begin{bmatrix} Q_{xx}^{11} & Q_{xx}^{12} & 0 \\ Q_{xx}^{21} & Q_{xx}^{22} & 0 \\ 0 & 0 & Q_{xx}^{33} \end{bmatrix}, \tag{110}$$

where Q_{xx}^{11} is a 1×1 matrix such as

$$Q_{xx}^{11} = -(1-\tilde{\sigma}) - [\tilde{\sigma}\omega_C + \varsigma^2]\frac{\Phi}{\varsigma + \chi} + \omega_\eta \frac{\chi + \varsigma(1-\Phi)}{\varsigma + \chi},$$

Q_{xx}^{22} is a $K \times K$ diagonal matrix such as, for a generic k diagonal element,

$$\left(Q_{xx}^{22}\right)_{kk} = m_k\{(1-\Phi)\upsilon + \frac{\omega_\eta}{\varsigma + \chi}[2\varsigma\Phi - \chi - \varsigma]\},$$

Q_{xx}^{33} is a $K \times K$ diagonal matrix such as, for a generic k diagonal element,

$$\left(Q_{xx}^{33}\right)_{kk} = \frac{m_k}{\kappa_k}\theta[1 - \Phi + \frac{\Phi}{\varsigma + \chi}\upsilon],$$

Q_{xx}^{12} a $1 \times K$ such as its typical k^{th}-column element is

$$\left(Q_{xx}^{12}\right)_{1k} = m_k \varsigma \omega_\eta \frac{\Phi}{\varsigma + \chi},$$

and $Q_{xx}^{21} = Q_{xx}^{12\prime}$. In the same fashion, we define the matrix Q_ξ as

$$Q_\xi = \begin{bmatrix} Q_\xi^{11} & 0 & 0 \\ Q_\xi^{21} & Q_\xi^{22} & Q_\xi^{23} \\ 0 & 0 & 0 \end{bmatrix}, \tag{111}$$

where Q_ξ^{11} is a 1×1 matrix such as

$$Q_\xi^{11} = -\tilde{\sigma} + [\omega_C + \tilde{\sigma} + \omega_\eta]\tilde{\sigma}\frac{\Phi}{\varsigma + \chi},$$

Q_ξ^{22} is a $K \times K$ diagonal matrix such as, for a generic k diagonal element,

$$\left(Q_\xi^{22}\right)_{kk} = -m_k \upsilon[1 - \Phi + \frac{\Phi}{\varsigma + \chi}\upsilon],$$

Q_ξ^{21} a $K \times 1$ dimension matrix such as its typical k^{th}-line element is

$$\left(Q_\xi^{21}\right)_{k1} = -m_k \omega_\eta \tilde{\sigma} \frac{\Phi}{\varsigma + \chi},$$

Q_ξ^{23} a $K \times K$ diagonal matrix such as its typical k^{th}-line element is

$$\left(Q_\xi^{23}\right)_{k1} = m_k \upsilon \frac{\Phi}{\varsigma + \chi}.$$

Simplifying (108) further by getting rid-off tax rates references and by separating terms referring to sectorial and overall outputs from references to sectorial inflation. Proceeding in such fashion yields

$$U_{t_0} = -\frac{\Omega}{2} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \{x'_{y,t} \tilde{Q}_y x_{y,t} + 2x'_{y,t} \tilde{Q}_\xi \xi_t + x'_{\pi,t} \tilde{Q}_\pi x_{\pi,t}\} + T_{t_0} + tips + O_p^3, \tag{112}$$

where $x_{y,t}$ is a $K + 1 \times 1$ vector containing only references to aggregate and sectorial outputs measures, or

$$x'_{y,t} = \left[\begin{array}{cccc} \hat{Y}_t & \hat{Y}_{1,t} & ... & \hat{Y}_{K,t} \end{array} \right],$$

$x_{\pi,t}$ is a $K \times 1$ vector containing only sectorial inflation measures, or

$$x'_{\pi,t} = \left[\begin{array}{ccc} \pi_{1,t} & ... & \pi_{K,t} \end{array} \right],$$

and $\tilde{Q}_y$, $\tilde{Q}_\xi$ and $\tilde{Q}_\pi$ are given, respectively, by:

$$\tilde{Q}_y = \left[\begin{array}{cc} Q_{xx}^{11} & Q_{xx}^{12} \\ Q_{xx}^{21} & Q_{xx}^{22} \end{array} \right],$$

$$\tilde{Q}_\pi = \left[\begin{array}{c} Q_{xx}^{33} \end{array} \right],$$

$$\tilde{Q}_\xi = \left[\begin{array}{ccc} Q_\xi^{11} & 0 & 0 \\ Q_\xi^{21} & Q_\xi^{22} & Q_\xi^{23} \end{array} \right],$$

where accurate specifications for submatrices Q_{xx}^{ij} and Q_ξ^{ij} are given in (110) and (111). From (112), we now focus on the term

$$x'_{y,t} \tilde{Q}_y x_{y,t} = q_y Y_t^2 + \sum_{k=1}^{K} m_k q_{y_k} Y_{k,t}^2 + 2 \sum_{k=1}^{K} m_k q_{y,y_k} Y_t Y_{k,t}, \tag{113}$$

where q terms are defined according to

$$q_y = -(1 - \tilde{\sigma}) - [\tilde{\sigma} \omega_C + \varsigma^2] \frac{\Phi}{\varsigma + \chi} + \omega_\eta \frac{\chi + \varsigma(1 - \Phi)}{\varsigma + \chi}, \tag{114}$$

$$q_{y_k} = (1-\Phi)\upsilon + \frac{\omega_\eta}{\varsigma+\chi}[2\varsigma\Phi - \chi - \varsigma], \tag{115}$$

$$q_{y,y_k} = \varsigma\omega_\eta \frac{\Phi}{\varsigma+\chi}. \tag{116}$$

Under the assumption that wage markups is steady state as well as markups over marginal costs are the same across sectors ($\bar{\mu}_k = \bar{\mu}$ and $\theta_k = \theta$) , q coefficients are all independent of k. We use the following proposition in order to simplify (113) further:

Proposition 7 *The following expression relating sum of sectorial output variances and covariances of sectorial outputs and aggregate output is of third order:*

$$\hat{Y}_t \sum_{k=1}^{K} m_k \hat{Y}_{k,t} - \sum_{k=1}^{K} m_k \hat{Y}_{k,t}^2 = O_p^3.$$

Proof. On one hand, from (82)

$$\hat{Y}_t - \sum_{k=1}^{K} m_k \hat{Y}_{k,t} = \frac{(1-\eta^{-1})}{2}(\sum_{k=1}^{K} m_k \hat{Y}_{k,t}^2 - \hat{Y}_t^2) + O_p^3. \tag{117}$$

On the other hand, from the definition of sectorial demand it is possible to establish the following exact relation:

$$\hat{p}_{k,t} = \eta^{-1}(\hat{Y}_t - \hat{Y}_{k,t}). \tag{118}$$

Summing across sectors yields:

$$\sum_{k=1}^{K} m_k \hat{p}_{k,t} = \eta^{-1}(\hat{Y}_t - \sum_{k=1}^{K} m_k \hat{Y}_{k,t}). \tag{119}$$

From the definition of aggregate price level in terms of sectorial prices:

$$1 = \sum_{k=1}^{K} m_k p_{k,t}^{1-\eta}. \tag{120}$$

Log-approximation on (120) yields:

$$\sum_{k=1}^{K} m_k \hat{p}_{k,t} = \frac{1}{2}(1-\eta) \sum_{k=1}^{K} m_k \hat{p}_{k,t}^2 + O_p^3.$$

One can use (118) and (119) in order to replace for $\hat{p}_{k,t}$, which yields:

$$\hat{Y}_t - \sum_{k=1}^{K} m_k \hat{Y}_{k,t} = -\frac{(1-\eta^{-1})}{2}(\hat{Y}_t^2 - 2\hat{Y}_t \sum_{k=1}^{K} m_k \hat{Y}_{k,t} + \sum_{k=1}^{K} m_k \hat{Y}_{k,t}^2) + O_p^3. \tag{121}$$

Comparing (117) and (121) yields the result. ■

Given proposition above, (113) is equivalent to:

$$x'_{y,t}\tilde{Q}_y x_{y,t} = q_y Y_t^2 + q'_{y_k}\sum_{k=1}^{K} m_k Y_{k,t}^2 + O_p^3, \tag{122}$$

where:

$$q'_{y_k} = q_{y_k} + 2q_{y,y_k}.$$

We now focus on the second term of (112), containing the interactions between endogenous variables and exogenous processes:

$$x'_{y,t}\tilde{Q}_\xi \xi_t = q_{yG}\hat{Y}_t\hat{G}_t + q_{y_k G}\sum_{k=1}^{K} m_k Y_{k,t}\hat{G}_t + \sum_{k=1}^{K} m_k\hat{Y}_{k,t}[q_{y_k a_k}\hat{a}_{k,t} + q_{y_k\mu_k}\hat{\mu}_{k,t}]. \tag{123}$$

where coefficients defined as

$$q_{yG} = -\tilde{\sigma} + \tilde{\sigma}[\omega_C + \tilde{\sigma} + \omega_\eta]\frac{\Phi}{\varsigma + \chi}, \tag{124}$$

$$q_{y_k a_k} = -\upsilon[1 - \Phi + \frac{\Phi}{\varsigma + \chi}\upsilon], \tag{125}$$

$$q_{y_k G} = -\omega_\eta\tilde{\sigma}\frac{\Phi}{\varsigma + \chi}, \tag{126}$$

$$q_{y_k\mu_k} = \frac{\Phi}{\varsigma + \chi}\upsilon \tag{127}$$

are all independent of sector-specific characteristics.

Proposition 8 *The following expression is, at least, of second order:*

$$\hat{Y}_t - \sum_{k=1}^{K} m_k\hat{Y}_{k,t} = O_p^2.$$

Proof. Follows directly from (82). ■

From above, the following holds:

Proposition 9 *The following expression holds:*

$$[\hat{Y}_t - \sum_{k=1}^{K} m_k Y_{k,t}]\hat{G}_t = O_p^3.$$

Proof. From proposition above plus the fact that all exogenous processes are O_p^1. ■

From (123), one can use above to get:

$$x'_{y,t}\tilde{Q}_\xi \xi_t = \sum_{k=1}^{K} m_k Y_{k,t}[q'_{y_k G}\hat{G}_t + q_{y_k a_k}\hat{a}_{k,t} + q_{y_k \mu_k}\hat{\mu}_{k,t}] + O_p^3, \tag{128}$$

where

$$q'_{y_k G} = q_{yG} + q_{y_k G}.$$

We now focus our attention on (122). The following lemma can help us simplify the expression even further.

Proposition 10 *The following expression is of third order:*

$$\hat{Y}_t^2 - \sum_{k=1}^{K} m_k \hat{Y}_{k,t}^2 = O_p^3.$$

Proof. From the first proposition:

$$\hat{Y}_t \sum_{k=1}^{K} m_k \hat{Y}_{k,t} - \sum_{k=1}^{K} m_k \hat{Y}_{k,t}^2 = O_p^3. \tag{129}$$

From the second proposition:

$$\hat{Y}_t - \sum_{k=1}^{K} m_k \hat{Y}_{k,t} = O_p^2. \tag{130}$$

Replacing (130) over (129) yields:

$$\hat{Y}_t^2 - \sum_{k=1}^{K} m_k \hat{Y}_{k,t}^2 = O_p^3,$$

once we notice that $\hat{Y}_t O_p^2$ is O_p^3. ■

From (122):

$$x'_{y,t}\tilde{Q}_y x_{y,t} = q_y[Y_t^2 - \sum_{k=1}^{K} m_k Y_{k,t}^2] + [q'_{y_k} + q_y]\sum_{k=1}^{K} m_k Y_{k,t}^2 \tag{131}$$

Applying the last Proposition above:

$$x'_{y,t}\tilde{Q}_y x_{y,t} = q''_{y_k}\sum_{k=1}^{K} m_k Y_{k,t}^2 + O_p^3, \tag{132}$$

where

$$q''_{y_k} = q'_{y_k} + q_y.$$

Replacing (128) and (132) over (112) yields the expression for the second order approximation for the utility function:

$$U_{t_0} = -\frac{\Omega}{2}E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\{\lambda_{y_k}\sum_{k=1}^{K} m_k y_{k,t}^2 + \sum_{k=1}^{K} m_k\lambda_{k,\pi}\pi_{k,t}^2\} + T_{t_0} + tips + O_p^3,$$

where

$$y_{k,t} = \hat{Y}_{k,t} - \hat{Y}^*_{k,t}$$

and

$$-\hat{Y}^*_{k,t} = \lambda_{y_k}^{-1}[(q_{yG} + q_{y_k G})\hat{G}_t + q_{y_k a_k}\hat{a}_{k,t} + q_{y_k \mu_k}\hat{\mu}_{k,t}], \tag{133}$$

all k, and, most importantly,

$$\lambda_{y_k} \equiv q_{y_k} + 2q_{y,y_k} + q_y, \tag{134}$$

$$\lambda_{k,\pi} \equiv \frac{\theta}{\kappa_k}[1 - \Phi + \frac{\Phi}{\varsigma + \chi}\upsilon], \tag{135}$$

while terms such as q_{y_k}, q_y , and q_{y,y_k} are defined from (114) to (116) and terms such as q_{yG}, $q_{y_k G}$, $q_{y_k a_k}$ and $q_{y_k \mu_k}$ are defined from (124) to (127).

9 Appendix D - Log-linear Model

9.1 Definition of Target Variables

Explicitly using the assumption that sector specific tax rates as well as wage markups in steady state are the same across sectors, we can define the target level of aggregate output using (133):

$$-\hat{Y}^*_t = \lambda_{y_k}^{-1}[(q_{yG} + q_{y_k G})\hat{G}_t + q_{y_k a_k}\hat{a}_t + q_{y_k \mu_k}\hat{\mu}_t], \tag{136}$$

where coefficients q are defined elsewhere and $\hat{a}_t$ and $\hat{\mu}_t$ are respectively defined as:

$$\hat{a}_t = \sum_{k=1}^{K} m_k \hat{a}_{k,t} \tag{137}$$

and

$$\hat{\mu}_t = \sum_{k=1}^{K} m_k \bar{\mu}_{k,t}. \tag{138}$$

9.2 Aggregate supply and cost-push disturbance term

We take the first order terms of AS equation in (78), valid for all k. Adding and subtracting, respectively, the terms referring to overall and sectorial output targets with the appropriate coefficients yield

$$\pi_{k,t} = \kappa_k\{\varsigma y_t + \chi y_{k,t}\} + \beta E_t \pi_{k,t+1} + u_{k,t}, \tag{139}$$

for all k, where the definition for the cost-push $u_{k,t}$ is given in terms of primitive shocks as

$$\begin{aligned} u_{k,t} = -\kappa_k\{[(\varsigma+\chi)\lambda_{y_k}^{-1}(q_{yG}+q_{y_kG})+\tilde{\sigma}]\hat{G}_t \quad & +\varsigma\lambda_{y_k}^{-1} q_{y_k a_k}\hat{a}_t + \\ +\varsigma\lambda_{y_k}^{-1} q_{y_k\mu_k}\hat{\mu}_t \quad & +[\chi\lambda_{y_k}^{-1} q_{y_k a_k}+\upsilon]\hat{a}_{k,t} + [\chi\lambda_{y_k}^{-1} q_{y_k\mu_k}-1]\hat{\mu}_{k,t}\}. \end{aligned} \tag{140}$$

9.3 Aggregate and Sectorial Output Relations

First order approximation to (82) can be redefined in terms of deviation from aggregate and sectorial output targets, yielding

$$y_t = \sum_{k=1}^{K} m_k y_{k,t}. \tag{141}$$

First order approximation to aggregate inflation measured by consumer prices is:

$$\pi_t = \sum_{k=1}^{K} m_k \pi_{k,t}. \tag{142}$$

In the same way, targeting inflation measure is given by

$$\breve{\pi}_t = \sum_{k=1}^{K} \omega_k \pi_{k,t}. \tag{143}$$

Finally, from (80)

$$y_t - y_{k,t} = \eta[\pi_{k,t} - \pi_t] + y_{t-1} - y_{k,t-1} + \Delta\zeta_{k,t}, \tag{144}$$

where

$$\zeta_{k,t} = \lambda_{y_k}^{-1} q_{y_k a_k}[\hat{a}_t - \hat{a}_{k,t}] + \lambda_{y_k}^{-1} q_{y_k \mu_k}[\hat{\mu}_t - \hat{\mu}_{k,t}]. \tag{145}$$

9.4 Euler Equation

Taking the first order approximation of the Euler equation in the main text yields

$$\hat{R}_t = \tilde{\sigma} E_t \Delta \hat{Y}_{t+1} - \tilde{\sigma} E_t \Delta \hat{G}_{t+1} + E_t \pi_{t+1} + O_p^2,$$

where we have used the relation in (77) to substitute for $\hat{C}_t$ in terms of $\hat{Y}_t$ and $\hat{G}_t$. Expressing equilibrium interest rates in terms of aggregate output gap by using definition in (133), which yields

$$\hat{R}_t = \tilde{\sigma} E_t \Delta y_{t+1} + E_t \pi_{t+1} - E_t \Delta r_{t+1}, \tag{146}$$

where

$$r_t = \tilde{\sigma}[\lambda_{y_k}^{-1}(q_{yG} + q_{y_k G}) + 1]\hat{G}_t + \tilde{\sigma}\lambda_{y_k}^{-1} q_{y_k a_k} \hat{a}_t + \tilde{\sigma}\lambda_{y_k}^{-1} q_{y_k \mu_k} \hat{\mu}_t. \tag{147}$$

9.5 Taylor Rule

Taking the first order approximation of the Taylor rule yields

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R)[\phi_\pi \breve{\pi}_t + \phi_y y_t] + e_t. \tag{148}$$

10 Appendix E - Benchmark Calibration

The following table presents the parameter values for the benchmark calibration, along with its definitions.

Table 3: Benchmark Calibration

Symbol	Parameter Definition	Assigned Value
K	Number of Sectors	2
σ	Coeff. of risk aversion	1.1
ν	Inv. of the Frisch elasticity of labor supply	.47
β	Discount parameter	.99
α_k	Calvo prob. of price stickiness	.5
m_k	Sector size	$1/K$
η	Cross-sector elasticity of substitution	1.5
θ	Within-sector elasticity of substitution	11
λ	Desutility of sectorial labor	.98
ρ_g	AR(1) coeff. of fiscal shock	.5
ρ_e	AR(1) coeff. of monetary shock	.5
ρ_{μ_k}	AR(1) coeff. of wage markup shock	.5
ρ_{a_k}	AR(1) coeff. of productivity shock	.5
σ_g	Standard deviation of fiscal shock	.2
σ_e	Standard deviation of monetary shock	.2
σ_{μ_k}	Standard deviation of wage markup shock	.2
σ_{a_k}	Standard deviation of productivity shock	.2
s_C	Steady state consumption over GDP	78%
$\overline{\tau}$	Steady state lump sum tax level over GDP	22%
$\overline{G}$	Steady state gov. expenses over GDP	19.5%
$\overline{\mu}$	Steady state wage markup	5%
$\overline{b}$	Steady state public debt level over GDP	50% (annual)
$\overline{R}$	Steady state interest rate level	4.05% (annual)
ϕ_π	Taylor rule reaction parameter to inflation	1.5
ϕ_y	Taylor rule reaction parameter to output gap	.25
ρ_R	Taylor rule interest rate smooth parameter	.85

For simplicity, only two sectors are considered. Shocks follow an AR(1) defined for any variable x as:

$$x_{t+1} = \rho_x x_t + \varepsilon_{t+1},$$

where ε_t follows a Normal Distribution, with mean zero and variance σ_x^2. Parameters ρ_x are calibrated at .5 for reasons of symmetry. σ_x parameters are calibrated at .2. All other parameters have approximated values of those used in the literature.

11 Appendix F - Bayesian Estimation

11.1 Sector Weights and Prior Distributions

Table below present the PCE sectors with respective weights. These are averages on the sample period of 1954, last quarter, to the first quarter of 2008. The following table presents the prior distributions of the estimated parameters.

Table 4: Sectors of PCE and respective weights

k	Categories	Weight (m_k), in %.
1	Motor vehicles and parts	4.91
2	Furniture and household equipment	2.52
3	Other durable goods	1.71
4	Food	18.94
5	Clothing and shoes	3.69
6	Gasoline, fuel oil, and other energy goods	4.21
7	Other nondurable goods	7.96
8	Housing	16.18
9	Household operation	5.63
10	Transportation	4.19
11	Medical care	14.37
12	Recreation	2.91
13	Other services	12.77

Table 5: Prior Distributions

Parameter	Distribution	Prior Mean	Prior Std.
α_k	Uniform (0,1)	.5	.28
ρ_{a_k}	Beta	.5	.2
σ_{a_k}	Inverse Gamma	.01	1
σ_{μ_k}	Inverse Gamma	.01	1
ρ_g	Beta	.8	.1
σ_g	Inverse Gamma	.01	1
σ_e	Inverse Gamma	.01	1
ρ_R	Beta	.85	.1
ϕ_π	Gamma, truncated at 1	1.5	.1
ϕ_y	Gamma, truncated at 0	.25	.05

11.2 Estimation Results

This section presents the posterior distributions for the estimated parameters. Other aggregate parameters not displayed are calibrated according to the benchmark values, presented in Appendix E.

11.2.1 Degrees of Nominal Rigidity

Table 6: Posterior Distribution - Degrees of Price Stickiness

Categories	Symbol	Posterior Mean	95% C.I.
1	α_1	0.7190	[0.6966, 0.7412]
2	α_2	0.6129	[0.4921, 0.7158]
3	α_3	0.5648	[0.5201, 0.6071]
4	α_4	0.3626	[0.3140, 0.4127]
5	α_5	0.3606	[0.2992, 0.4173]
6	α_6	0.0291	[0.0033, 0.0525]
7	α_7	0.4628	[0.4097, 0.5146]
8	α_8	0.5306	[0.4829, 0.5793]
9	α_9	0.3680	[0.3089, 0.4266]
10	α_{10}	0.1547	[0.1230, 0.1846]
11	α_{11}	0.4404	[0.3754, 0.5041]
12	α_{12}	0.5528	[0.5075, 0.5986]
13	α_{13}	0.2304	[0.1592, 0.3006]

11.2.2 Productivity Shock Parameters

Table 7: Posterior Distribution - Productivity Shocks: AR(1) Coeffs.

Categories	Symbol	Posterior Mean	95% C.I.
1	ρ_{a_1}	0.0261	[0.0031, 0.0494]
2	ρ_{a_2}	0.6394	[0.4717, 0.7777]
3	ρ_{a_3}	0.0672	[0.0066, 0.1236]
4	ρ_{a_4}	0.4116	[0.3364, 0.4854]
5	ρ_{a_5}	0.1735	[0.0648, 0.2754]
6	ρ_{a_6}	0.0206	[0.0024, 0.0384]
7	ρ_{a_7}	0.3067	[0.1931, 0.4242]
8	ρ_{a_8}	0.5690	[0.5175, 0.6217]
9	ρ_{a_9}	0.0433	[0.0066, 0.0795]
10	$\rho_{a_{10}}$	0.0202	[0.0024, 0.0376]
11	$\rho_{a_{11}}$	0.5170	[0.4520, 0.5823]
12	$\rho_{a_{12}}$	0.0692	[0.0093, 0.1266]
13	$\rho_{a_{13}}$	0.1602	[0.0402, 0.2788]

Table 8: Posterior Distribution - Productivity Shocks: Std. Deviations

Categories	Symbol	Posterior Mean	95% C.I.
1	σ_{a_1}	0.1074	[0.0979, 0.1167]
2	σ_{a_2}	0.0878	[0.0514, 0.1173]
3	σ_{a_3}	0.0395	[0.0348, 0.0439]
4	σ_{a_4}	0.0257	[0.0222, 0.0294]
5	σ_{a_5}	0.0291	[0.0244, 0.0337]
6	σ_{a_6}	0.0906	[0.0833, 0.0980]
7	σ_{a_7}	0.0241	[0.0194, 0.0288]
8	σ_{a_8}	0.0250	[0.0218, 0.0282]
9	σ_{a_9}	0.0298	[0.0269, 0.0326]
10	$\sigma_{a_{10}}$	0.0433	[0.0396, 0.0471]
11	$\sigma_{a_{11}}$	0.0345	[0.0293, 0.0393]
12	$\sigma_{a_{12}}$	0.0315	[0.0278, 0.0350]
13	$\sigma_{a_{13}}$	0.0290	[0.0245, 0.0334]

11.2.3 Wage Markup Shock Parameters

Table 9: Posterior Distribution - Wage Markup Shocks: Std. Deviations

Categories	Symbol	Posterior Mean	95% C.I.
1	σ_{μ_1}	0.7232	[0.6071, 0.8460]
2	σ_{μ_2}	0.3604	[0.1711, 0.5155]
3	σ_{μ_3}	0.2020	[0.1565, 0.2445]
4	σ_{μ_4}	0.0588	[0.0469, 0.0713]
5	σ_{μ_5}	0.0846	[0.0636, 0.1047]
6	σ_{μ_6}	0.0713	[0.0592, 0.0829]
7	σ_{μ_7}	0.0680	[0.0527, 0.0830]
8	σ_{μ_8}	0.0535	[0.0415, 0.0656]
9	σ_{μ_9}	0.0667	[0.0490, 0.0833]
10	$\sigma_{\mu_{10}}$	0.0410	[0.0347, 0.0471]
11	$\sigma_{\mu_{11}}$	0.0553	[0.0394, 0.0711]
12	$\sigma_{\mu_{12}}$	0.1441	[0.1113, 0.1768]
13	$\sigma_{\mu_{13}}$	0.0123	[0.0026, 0.0238]

11.2.4 Other Estimated Parameters

Table 10: Prior Distributions - Other Parameters

Parameter Definition	Posterior Mean	95% C.I.
ρ_g	0.9874	[0.9828, 0.9921]
σ_g	0.0814	[0.0609, 0.1013]
σ_e	0.0026	[0.0024, 0.0029]
ρ_R	0.7329	[0.7000, 0.7683]
ϕ_π	1.5197	[1.3990, 1.6422]
ϕ_y	0.5372	[0.4300, 0.6439]