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Mavrotas, George and Florios, Kostas and Vlachou, Dimitra

National Technical University of Athens

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George Mavrotas*, Kostas Florios, Dimitra Vlachou

Laboratory of Industrial and Energy Economics, School of Chemical Engineering, National Technical University of Athens,, Zographou Campus, 15780, Athens, Greece

* corresponding author: mavrotas@chemeng.ntua.gr

Abstract

For more than 40 years, Mathematical Programming is the traditional tool for energy planning at the national or regional level aiming at cost minimization subject to specific technological, political and demand satisfaction constraints. The liberalization of the energy market along with the ongoing technical progress increased the level of competition and forced energy consumers, even at the unit level, to make their choices among a large number of alternative or complementary energy technologies, fuels and/or suppliers. In the present work we develop a modelling framework for energy planning in units of the tertiary sector giving special emphasis to model reduction and to the uncertainty of the economic parameters. In the given case study, the energy rehabilitation of a hospital in Athens is examined and the installation of a cogeneration, absorption and compression unit is examined for the supply of the electricity, heating and cooling load. The basic innovation of the given energy model lies in the uncertainty modelling through the combined use of Mathematical Programming (namely, Mixed Integer Linear Programming, MILP) and Monte Carlo simulation that permits the risk management for the most volatile parameters of the objective function such as the fuel costs and the interest rate. The results come in the form of probability distributions that provide fruitful information to the decision maker. The effect of model reduction through appropriate data compression of the load data is also addressed.

Key words: Energy Planning, Mathematical Programming, MILP, Uncertainty, Monte Carlo

1. INTRODUCTION

Mathematical Programming (MP), and in particular, Linear Programming (LP) models have been the traditional tool for energy planning. The main objective is usually the minimization of cost subject to specific technological, political and demand satisfaction constraints [1]. In many cases it is advisable to use, apart from the continuous, integer variables in order to describe inherently discrete phenomena (economies of scale, technology choices, logical conditions etc.), which may be present in the decision situation. The resultant Mixed Integer Linear Programming (MILP) models are harder to solve than the corresponding LP models, but they represent a more realistic view of the decision situation, whereas the increasing computer capacities make their implementation all the more easier [2].

However, the liberalization of the energy market along with the ongoing technical progress increased the level of competition and forced energy consumers to make their own choices among a large number of alternative or complementary energy technologies and/or suppliers. Examples of such energy planning applications of multi-objective models in units of the industrial or tertiary sector can be found in [3-5].

The existence of numerous alternative energy options, which may differ in technical, economic and or environmental performance, has caused a growing need in implementing energy planning models in smaller systems. This need becomes even more imperative nowadays, as the increasing interest towards environmental issues has led to a growing public awareness. Moreover, the intrinsic uncertainty in some of the key energy planning parameters (e.g. the load demand or energy prices) should be properly addressed. As a result, decision makers, at the level of individual units, are often confronted to a complicated decision problem, which becomes even more difficult to solve because of the underlying uncertainties.

The purpose of this paper is to develop an energy planning framework combining Mathematical Programming and Monte Carlo simulation that can be properly used in buildings of the Services' sector (hospitals, hotels, sport centers, universities etc) taking into account the uncertainty in cost parameters that are expressed by probability distributions. The application of the method is performed in a case study refereed to the energy rehabilitation of a hospital in Athens. Since hospitals are among the largest energy consumers in the Services' sector, it is highly recommended to upgrade the existing energy supply system by using more efficient energy technologies. Besides the traditional energy supply options, it is possible to implement Combined Heat and Power (CHP) systems, in combination with an absorption chiller for air conditioning.

More specifically, the aim is to meet the hospital's energy demand as determined by typical daily load profiles provided for each month, in a more efficient way at the lowest possible cost. CHP units, absorption and/or compression units comprise the candidate technologies, to be combined with the necessary back-up systems. The appropriate capacity of the new units as well as their operational characteristics have to be determined.

Moreover, due to the large size of the full model, a model reduction technique is proposed through compression of the relevant load data. This issue is extremely important in similar problems as the model reduction may hopefully drive in much faster solution times, without affecting much the accuracy of the results obtained. In case that multiple instants of the energy planning model have to be solved (e.g. multi-objective formulations or multiple scenarios), the savings in computational time are crucial. Consequently, the effect of the model reduction on the final results is also investigated.

The remainder of the paper is organized as follows: After this introductory section, the methodological framework is presented, describing briefly the Mathematical programming and the Monte Carlo simulation. A short description of the case study follows in section 3 while the section 4 is devoted to the data compression techniques. Section 5 deals with the model building describing in detail the corresponding mathematical relations. Section 6 contains a detailed analysis of the obtained results and discussion while the last section includes concluding remarks.

2. METHODOLOGICAL FRAMEWORK

2.1 Mathematical Programming in the optimization of energy systems

Mathematical Programming is the most popular method for the optimization of various systems. Usually MP models in their various forms (Linear Programming, Integer Programming, Non Linear Programming etc) have a number of decision variables (the unknowns of the problem) and parameters (the data of the problem). The relations of the decision variables and the parameters that describe the system are the constraints of the problem and the objective function expresses the optimization criterion. The MP models are usually written as follows:

$$
\begin{aligned}
&\max\ z = f(\boldsymbol{x}) \\
&\text{st} \\
&g_i(\boldsymbol{x}) \leq b_i \quad i=1 \ldots m
\end{aligned}
\qquad (1)
$$

where $\boldsymbol{x}$ is the vector of the decision variables, $f(\boldsymbol{x})$ is the objective function, $g_i(\boldsymbol{x})$ are the constraint functions and b_i the parameters expressing the Right Hand Side of the constraints.

MP and mainly LP models are for many years the most widely applied tools in energy planning, and usually aim at minimizing the discounted cost of meeting energy demand (investment and operational) over the entire planning horizon (see e.g. [2, 6-11]. The constraints of the problem typically represent the demand of various energy consuming sectors or activities, as well as the imposed technological (energy balances, capacities etc) and possibly political limitations (e.g. independence from imported fuels). The decision variables usually refer to the amounts of energy forms and the units' capacities. When the problem includes discrete elements like new facilities, economies of scale, logical conditions etc. that cannot be properly represented by continuous variables, Mixed Integer Linear Programming (MILP) models are used. MILP problems require much more computational effort than the corresponding LP problems due to the discontinuities in the decision variable space. However, the improvements in relevant software in combination with the enormous increase in computer capacities are greatly facilitating their practical implementation (see e.g. [12]).

In the building level, we can express the energy planning problem borrowing ideas from the field of process synthesis in chemical engineering. Specifically, the problem can be formulated as a multi-period **structure**, **design** and **operational optimization** problem [13, 14]. All the available energy options and their interdependencies can be considered in the **superstructure** of the system (topology of all the available energy options) and the Mathematical Programming model proposes the best solution in terms of structure (which units are selected), design (what are their sizes) and operation (how they will operate).

2.2. Monte Carlo simulation for dealing with uncertainty

Monte Carlo simulation is a widely used tool whenever the uncertainty is expressed in the form of probability distributions [15]. Usually, the uncertain parameters are given in the form of specific probability distributions (e.g. normal, uniform, triangular). Mathematical Programming and Monte Carlo can be combined whenever the nature of uncertainty is stochastic, i.e. the uncertain parameters are given in the form of probability distributions [16-18]. The combination of the Monte Carlo method and the Mathematical Programming model solution is working as follows: first, the user determines the number of iterations (usually 500-1000) of the Monte Carlo simulation. Subsequently for each sampling of the uncertain parameters, the MP model is solved and the results for the objective function and the main decision variables are recorded. The combination of Monte Carlo with Mathematical Programming is illustrated in the flowchart of Figure 1.

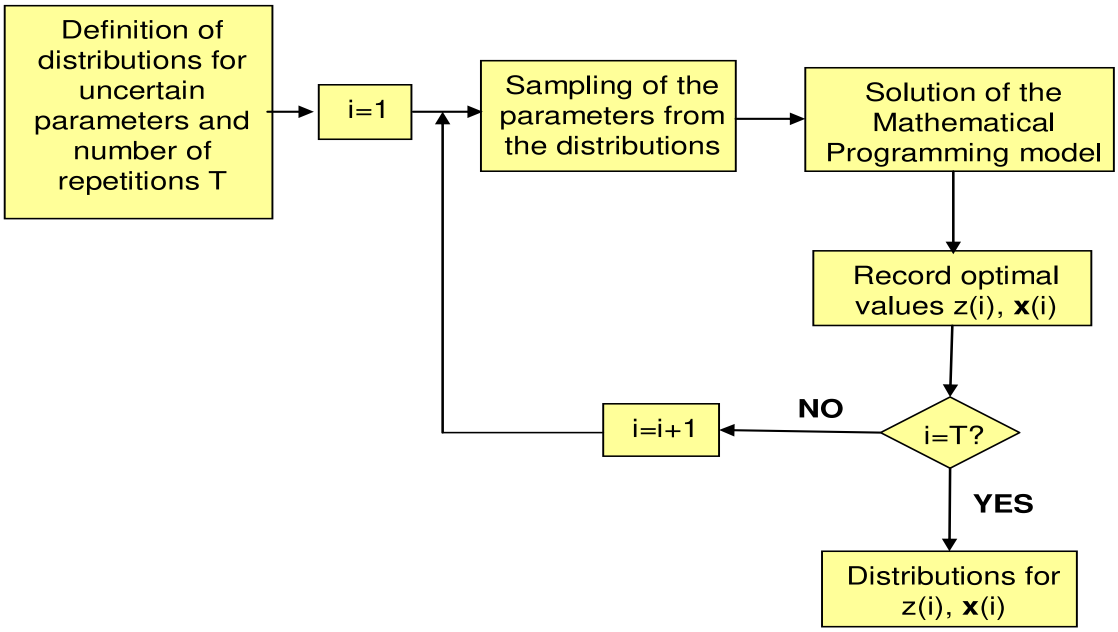

Figure 1. Flowchart of the combination of Monte Carlo simulation and Mathematical Programming.

The information obtained by the decision maker as the optimal values of the objective function and the key variables is given in the form of probability distributions. Therefore, the decision maker can see how the objective function and the decision variables can vary, given the specific uncertainty on the model's parameters.

3. CASE STUDY

The examined hospital is located in the greater Athens area and has a capacity of 400 beds. Actually, energy requirements are covered by the electricity from the electric power grid, from the water heating and space heating boilers, as well as from boilers for providing heat to various medical and non-medical uses. Cooling demand is only partially satisfied with split units, but a more integrated solution is sought. The development of the natural gas grid in the area offers the opportunity to proceed to a radical restructuring of the hospital's energy supply system by simultaneously upgrading the overall energy efficiency. The technologies under consideration are a CHP unit (driven by an internal combustion engine) for providing power and heat, an absorption unit and/or a compression unit for providing cooling load. Electricity from the grid and the already existent boilers can be optionally used to cover any excess demand would occur during system operation. The superstructure of the proposed new system is presented in Figure 2.

The results of the energy supply optimization model will determine the candidate units that should be installed as well as their optimal size. The load demand is given in the form of hourly data (in kW) for each load category (heating, cooling and electricity) for one typical day of each month. Consequently, there are in total 12×24 = 288 data for each one of the three loads. The monthly 24-hour profiles for each one of the three loads are shown graphically in Figure 3.

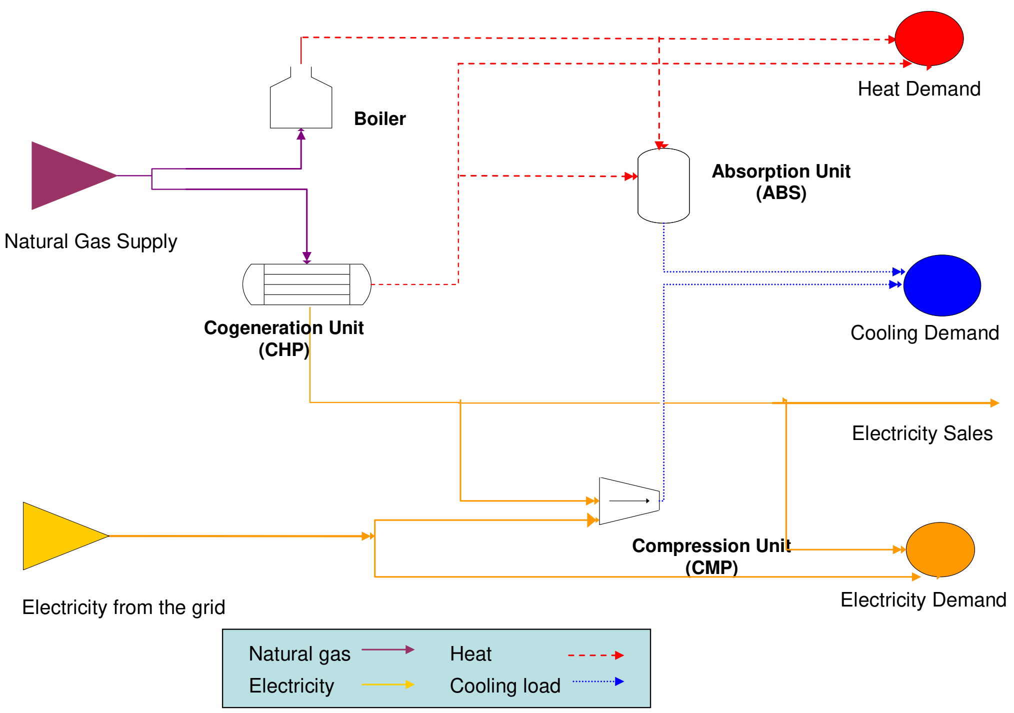

Figure 2. The superstructure of the hospital's energy system.

It must be noted that the planned demand side management measures have been taken into account during the calculation of these loads. One of the issues emphasized in this study is the possibility of reducing the size of the model by appropriately applying data compression techniques (i.e. grouping of load data) and how these affect the accuracy of the obtained results. The candidate unit characteristics (investment cost, O&M cost, efficiency, operating limits etc) are specified according to the information supplied by a relevant study [19].

4. MODEL REDUCTION THROUGH DATA COMPRESSION

The demand data for the three types of load, namely heating, cooling and electricity, are given on an hourly basis for a typical day of each month. The full 12×24 model (1 typical day for each month is subdivide in the 24-hour intervals) is developed using the hourly energy balances for meting the load demand for every hour and for every load (electricity, heating, cooling). Besides the full 12×24 model, we developed two additional models obtained through the data compression process in order to explore the effect of the model compression on the accuracy of the obtained results. The first one depicts a 6×12 representation (i.e. 6 seasons with 12 intraday periods) and the second one, an even more compact model, depicts a 3 x 6 representation (i.e 3 seasons with 6 intraday periods). In order to achieve this representation, a technique grouping the months of similar characteristics to seasons was applied and subsequently the hours of the day to intraday periods were grouped. The challenge is to perform efficient grouping, so that the obtained reduced models will not present significant divergence from the initial full model. The whole data compression procedure is performed in Microsoft Excel associated with coding performed in Visual Basic.

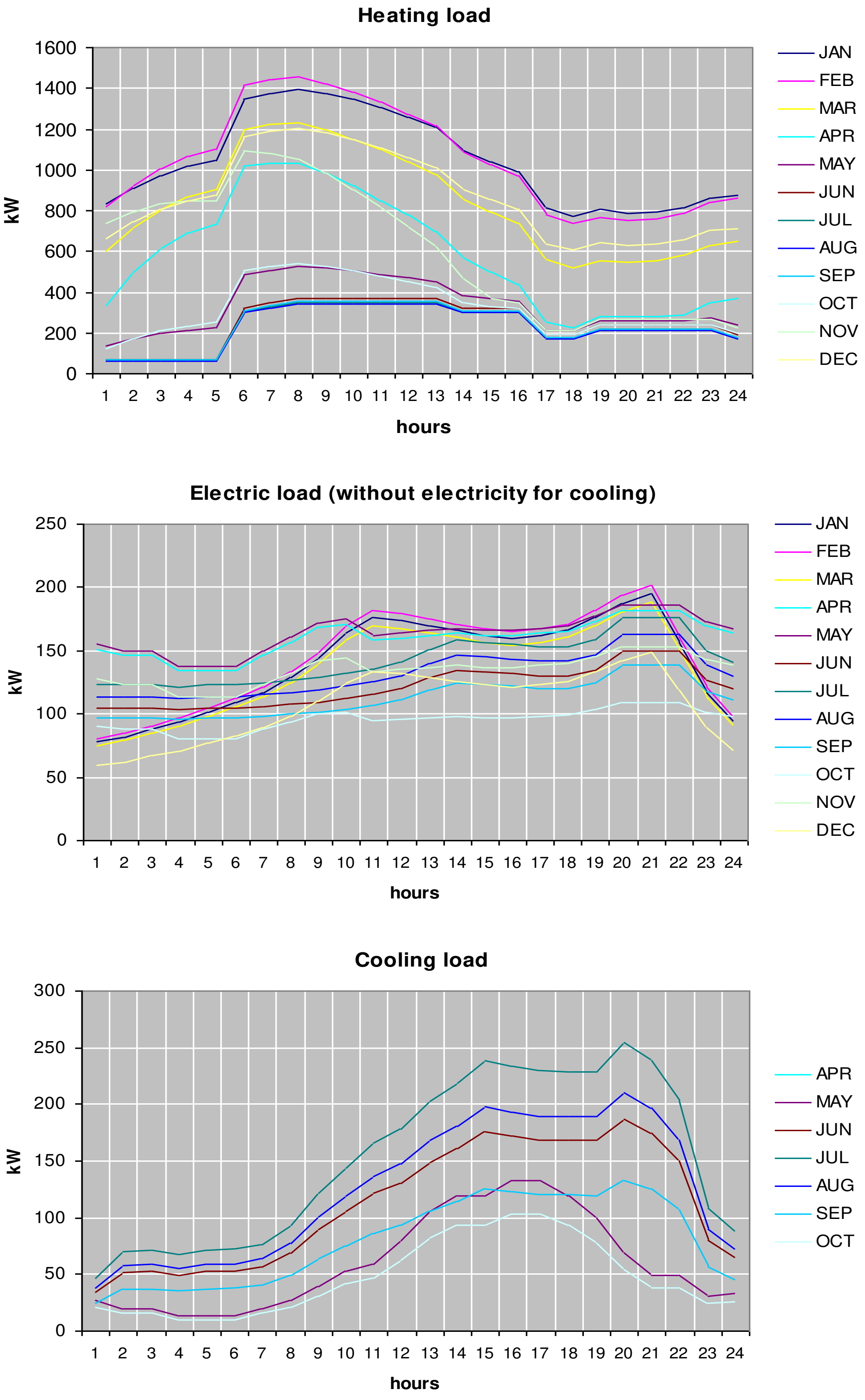

Figure 3. Load profiles for a typical day for the three loads

4.1 Grouping of months to seasons

For each month, a 24-hour profile for the three loads (electricity, heating and cooling) of a representative (or typical) day is available. The aim of the monthly grouping to seasons is to create a representative day for a season, keeping (as much as possible) the characteristics of the corresponding months. In doing so, we create the 24-hour profile for each one of the three loads as an aggregation of the months' load profiles. The rationale of the calculation is twofold: to assign, as the maximum power for the seasonal load profile, the maximum power found in the represented months and, on the other hand, to assign, as daily energy consumption, the average daily energy consumption of the represented months (the daily energy consumption is calculated as the area under the load profile). In order to achieve this, the months are grouped to a predetermined number of seasons according to the similarity of their load profiles. The algorithm for grouping the months is illustrated using the following example: Assume that we would like to group the 12 months into three seasons. Then, using a simple clustering method based on their load profiles, the following months, January, February, March, April, November and December will form the cluster "winter season ", May and October will form the cluster "intermediate season" and June, July, August and September will form the cluster "summer season". In Figure 4, the heating load of the winter season is depicted.

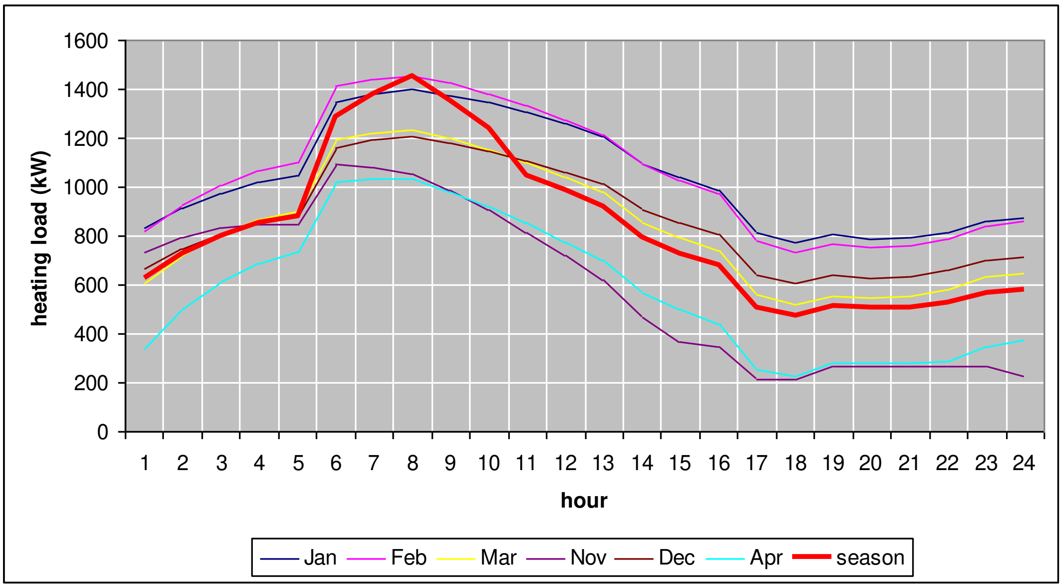

Figure 4. Example of grouping the monthly profiles into seasonal profiles

The calculation of the seasonal heat load profile is performed as follows: The maximum load value in the 24-hour profiles of the grouped months is located (8 am, February with value 1456 kW) and attributed as the maximum load value of the season's typical day. For the remainder of the hours, the seasonal load is calculated as the average of the 6 months. In order to smooth the seasonal profile around the maximum load, the load, two hours before and after the maximum load, is calculated as the weighted sum, with more weight given on the month where the maximum load is recorded (e.g. February in the present case). Namely, at 7am and 9 am, the load for the seasonal profile is calculated as 0.75×FEB + 0.25×average(JAN, MAR, APR, NOV, DEC), while at 6am and 10am the load is calculated as 0.5×FEB + 0.5×average(JAN, MAR, APR, NOV, DEC). However, in doing so, the daily energy consumption of the typical season will be somewhat higher than the average daily energy consumption of the respective months. In order to restore the required equivalence, the calculated loads for the remainder 19 (= 24-5) hours are accordingly and evenly reduced. Following this procedure, the 24-hour load profiles for a typical day of a season are calculated, corresponding to the 24-hour load profiles of the respective months.

4.2 Grouping of hours to intraday periods

The next step in the data compression procedure is to group the hourly load data, as obtained from the daily load profiles, into intraday periods. The aim is to appropriately linearize the daily profiles in order to obtain a surrogate profile with flat (constant) loads assigned to each one of the intraday periods. In this way, we actually divide the day to 6 or 12 intraday periods of constant load instead of the 24-hours partition. Due to the formulation of the subsequent mathematical programming model, the grouping must be the same for the three load profiles (electricity, heating and cooling load). The challenge is to apply a grouping procedure so that the surrogate, linearized profile will retain the maximum load while keeping the daily energy consumption (the area under the load profile) of the original daily profile. In Figure 5, the three load profiles for a typical day of the summer season along with their surrogate profiles are shown. In the surrogate profiles, the day is divided into 12 intraday periods.

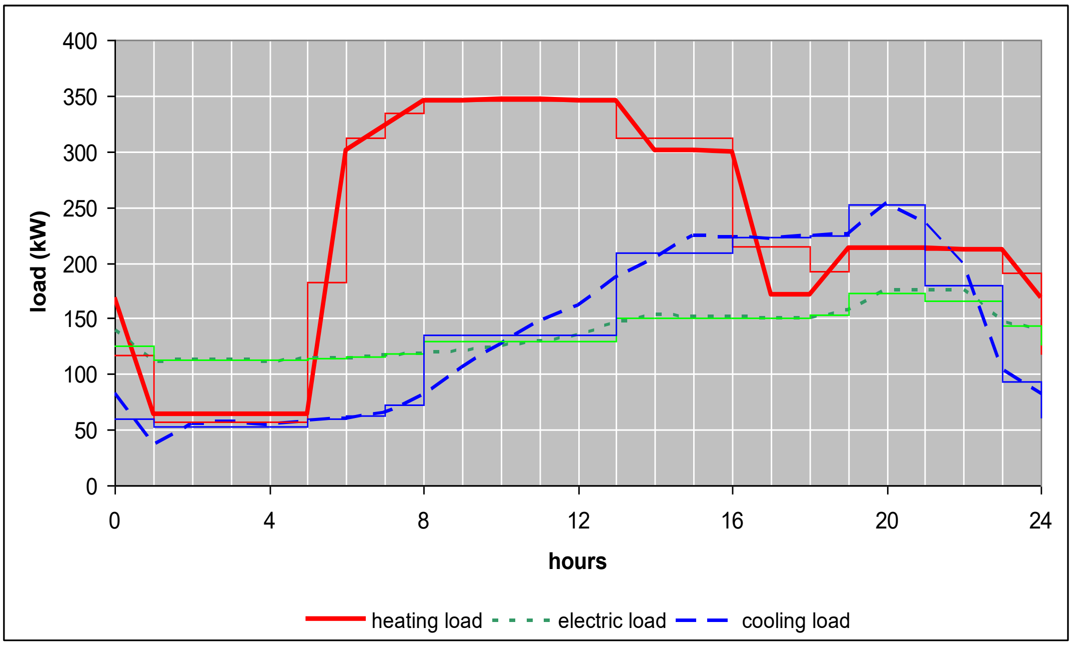

Figure 5. Example of linearization of load profiles for a typical summer-season day

The cut points in the surrogate profiles can be either set by the user (manual setting) or calculated automatically (automatic setting). The automatic process calculates the cut points according to the slopes of the original load profiles. Namely, it calculates the slope in every point of the three load profiles and ranks them in decreasing order. Then, it assigns the cut points to the *n* first points, where *n* is the number of required intraday periods plus one. If the user has already set some cut points, the number of those automatically calculated is appropriately reduced. A useful rule of thumb is that the cut points around the maximum load for each one of the three load profiles are set manually in order to guarantee the maximum load bracketing (as it is done for hours 19 and 21 in Figure 4 in order to capture the maximum cooling load). These cut points define the corresponding intraday periods. The load for each one of them is the average of the corresponding hourly loads. Finally, a fine-tuning may be needed in order to assure that the area under the linearized profile is equal to the area under the corresponding hourly profile. The fine- tuning is performed by appropriately reducing the base loads or/and increasing the peak loads.

5. MODEL BUILDING

A MILP model is formulated for the cost optimization of the energy superstructure shown in Fig. 1. Continuous decision variables indicate energy flows and equipment capacities, while binary decision variables refer to the adoption or rejection of the considered types of units, as well as to the operation of a unit during a time period. In order to maintain the linear characteristics of the model, we use piecewise linear approximations for the characteristics of the equipment as a function of its capacity. The model is treated and formulated as a multi-period synthesis and operational problem according to the guidelines proposed in [13].

Economic objective function (annualized cost)

As already mentioned, the objective function of the developed MILP model is to minimize the annualized cost, namely, the sum of the annualized investment cost (assuming a discount rate i= 8% and lifetime N= 15 years) and the annual operational and maintenance cost.

The annualized investment cost is obtained from the investment cost by multiplying with the Capital Recovery Factor (CRF) given by:

$$CRF = \frac{i(1+i)^N}{(1+i)^N - 1} \quad (2)$$

where i is the discount rate and N is the equipment's lifetime. The objective function that expresses the minimization of the annualized energy cost is given by the following equation:

$$\min Z_1 = \sum_{k=el,ng} c^k ENERGY^k - p^{els}\ ELCHPS + \sum_{m=1}^{4} (icpt^m B^m + slop^m CAP^m) \quad (3)$$

where, $ENERGY^k$ is the purchased amount of energy of k-th type (k=electricity from the grid and natural gas), c^k is the corresponding cost, p^{els} is the selling price of the electricity from the CHP to the grid, *ELCHPS* the electric energy from the CHP that is sold to the grid and m is the index for the different units. The parameters $icpt^m$ and $slop^m$ are the intercept and the slope respectively of the line expressing the annualized cost of the unit (investment and O&M) as a function of unit capacity drawn from available cost data of various units (depicted in Figure 6).

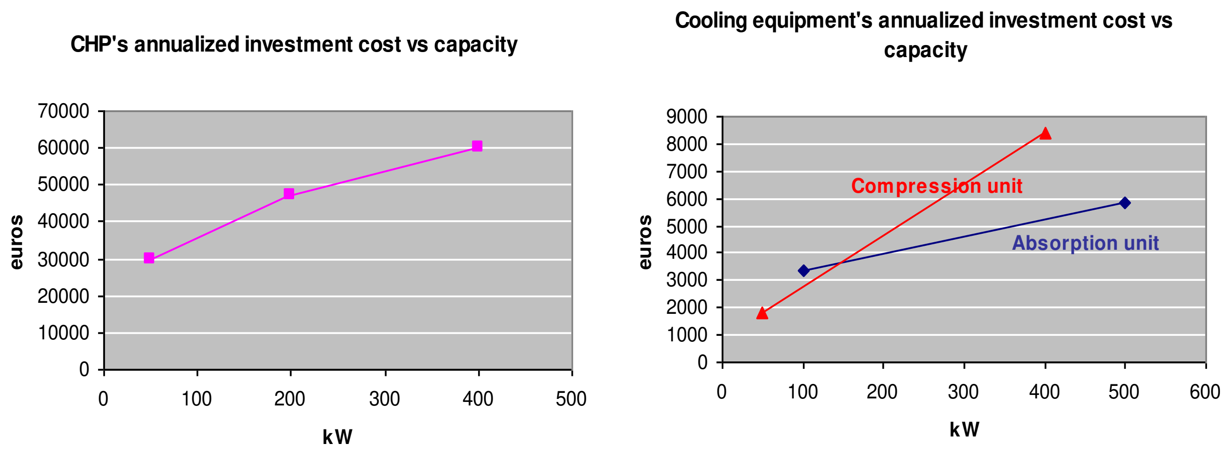

Figure 6. Annualized investment cost as a function of nominal capacity

The decision variables B^m and CAP^m express the existence or not of m-th unit (binary variable) and the capacity of m-th unit respectively (we consider four cases as the CHP's modeling requires two line segments – see also modeling of CHP unit below). The electricity and natural gas prices are drawn from the Regulatory Authority of Energy [20]. The electricity price is 102 €/MWh (34€/MWh for off-peak electricity i.e. from 11pm-6am), the natural gas price is 30 €/MWh and the selling price of electricity from the CHP to the grid is 56 €/MWh. It must be mentioned that these values are used as reference values for creating the probability distributions that are going to be used in the Monte Carlo simulation.

The constraints of the model are grouped as follows:

Demand satisfaction: The heat, electricity and cooling load produced in the network must meet the corresponding demands in each period. In the case of electricity, exchanges with the grid are possible:

$$POWER^k_{ij} \geq ld^k{}_{ij} \qquad i=1..s, \quad j=1..p, \quad k=elec, heat, cool \qquad (4)$$

where s,p denote the number of seasons and periods of the day respectively, $POWER^k_{ij}$ is the decision variable denoting the required output for serving the *k*-th load in season *i* and period *j,* and ld^k_{ij} is the parameter that expresses the required load of k-th type in the i-th season and the j-th period of day.

Energy Balances: they refer to energy inputs and outputs from the CHP, absorption and compression units and the boiler for each period. Therefore, for the *m*-th unit the energy balance for season *i* (*i=1…s*) and period of day *j* (*j=1…p*) becomes:

$$eff^m INPUT^m_{ij} - OUTPUT^m_{ij} = 0 \qquad i=1\ldots s,\ j=1\ldots p,\ m=1\ldots 4 \qquad (5)$$

where eff^m is the efficiency of the m-th unit, $INPUT^m_{ij}$ is the variable indicating the power input for unit *m* (in terms of power) referring to season i and period of day j, while $OUTPUT^m_{ij}$ is the corresponding output.

Equipment Capacity: they set the corresponding upper bound to the output of each unit in each period, whether the latter is a parameter (i.e. for existing units such as the boiler) or a decision variable (i.e. for new units such as CHP, absorption and compression units).

$$CAP^m - lo^m B^m \geq 0 \quad \& \quad CAP^m - up^m B^m \leq 0 \qquad m=1\ldots 4 \qquad (6)$$

$$OUTPUT^m_{ij} - CAP^m \leq 0 \quad i=1\ldots s,\ j=1\ldots p,\ m=1\ldots 4 \qquad (7)$$

where CAP^m is the decision variable indicating the capacity of unit *m*, B^m is the binary decision variable indicating the existence or not of the m-th unit, while lo^m and up^m are the parameters indicating the lower and upper bound for these capacities.

Technical Minimum: these constraints set the corresponding lower bound to the output of each unit in each period (usually 30% of its nominal capacity). There are binary variables for each unit *m*, indicating if the unit is operating or not in the i-th season and the j-th period of day (Y^m_{ij}). These variables are necessary for modeling the technical minimum requirement.

$$OUTPUT^m_{ij} - up^m Y^m_{ij} \leq 0 \qquad i=1\ldots s,\ j=1\ldots p,\ m=1\ldots 4 \qquad (8)$$

$$-OUTPUT^m_{ij} + tmin^m CAP^m + (tmin^m\ up^m)\ Y^m_{ij} \leq (\ tmin^m\ up^m) \quad i=1 \ldots s, j=1 \ldots p, m=1 \ldots 4$$

(9)

where $tmin^m$ is the technical minimum of the m-th unit as the percentage of the unit's capacity. If Y^m_{ij}=1 the constraint is activated while if Y^m_{ij}=0 it becomes inactive.

At this point, it must be noted that the constraint of the technical minimum in the operation of the units is the main cause for the introduction of the binary variables Y^m_{ij}. These binary variables increase significantly the computational effort of solving the MILP problem (e.g. in the 12x24 model there are 3x12x24 = 864 such variables). Omitting the technical minimum constraints the computational performance is considerably improved (the problem is solved in a few seconds). Moreover, omitting the technical minimum constraints in the present case, the obtained solution underestimates the annual cost by almost 2%. However, although this underestimation is relatively small, a considerable divergence in the optimal capacity of the installed units is observed, rendering its omission questionable. It becomes obvious, the problem arising comprises another example characterized by the trade-off between realistic modeling and computational effectiveness.

Reserve margin for cooling load: the sum of capacities of the compression and the absorption unit should be 20% greater than the annual hourly peak in cooling load (*maxcldm*).

$$CAP^{abs} + CAP^{cmp} \geq 1.2\ maxcldm \tag{10}$$

CHP modeling: the CHP unit is modeled by using two size domains: from 50 to 200 kW and from 201 to 400 kW with different investment cost and technical characteristics (i.e. efficiency and power to heat ratio).

$$CAP_v - locap_v\ S_v \geq 0 \quad v=1,2 \tag{11}$$

$$CAP_v - upcap_v\ S_v \leq 0 \quad v=1,2 \tag{12}$$

$$CAP^{chp} - \Sigma_v\ CAP_v = 0 \tag{13}$$

$$S_1 + S_2 \leq 1 \tag{14}$$

S_v is the binary variable indicating the size domain where the selected CHP unit belongs (v=1, 2). The binary variables S_v are also used to link electricity and heat production through the appropriate power to heat ratio:

$$EL_{ij} - \Sigma_v ph_v HT_{vij}=0 \qquad i=1\ldots s, j=1\ldots p \tag{15}$$

where ph_v is the power to heat ratio for the v-th type, EL_{ij} is the electricity output of the CHP unit in season i and period j, HT_{vij} is the heat output of the CHP unit of v-th type in season i and period j. In order to deactivate the redundant HT_{vij} variables (corresponding to the v-th type of CHP unit that is not selected), we add the two following constraints:

$$\Sigma_{ij} HT_{vij} - (2\ \ s\ \ p\ \ up_v)\, S_v \leq 0 \qquad v=1,2 \tag{16}$$

where up_v is the upper bound on the capacity for the CHP unit of v-th type (the factor 2 is added because capacity is defined in terms of electric output and the thermal output is at least 1 ½ times higher).

Conversion of power to energy: Annual energy figures are obtained by summing the products of the number of hours in each day-period of a season with the respective load (heating, cooling or electricity).

$$ENERGY^k - \Sigma_{ij} h_{ij} POWER^k{}_{ij} = 0 \qquad k=1\ldots K \tag{17}$$

where $ENERGY^k$ is the total annual energy for the k-th load, h_{ij} is the parameter indicating the number of hours in the i-th season and the j-th period of day, $POWER^k{}_{ij}$ is the respective power of the k-th load and K is the number of different energy forms that we need to calculate. In this set of constraints, we further discriminate the energy required in off-peak periods (with reduced electricity cost) and we calculate the value of the objective function Z_1 accordingly.

The problem is solved by using the GAMS (General Algebric Modeling System) modeling language [21] and the CPLEX 10.0 solver for MILP. The full 12x24 model comprises 4925 constraints, 5209 continuous and 868 binary variables, while data compression reduces these numbers, for both 6x12 and 3x6 models, to 1254, 1321, 220, and to 335, 349, 58 respectively.

6. RESULTS AND DISCUSSION

6.1 Effect of model reduction

First, the effect of model reduction is studied by solving the three models (3x6, 6x12 and 12x24) without the application of the Monte Carlo simulation on the uncertain parameters (e.g. no uncertainty of the given parameters is regarded, and only the expected values have been considered). The results concerning the objective function and some key variables are shown in Table 1.

Table 1 : Objective function values and main variables of the MILP model without Monte Carlo

Model size	Annual cost (€)	CHP (kW)	ABS (kW)	CMP (kW)	Electricity from grid (MWh)	Electricity to grid (MWh)	Nat. gas supply (MWh)	Solution Time (sec)
3x6	289,271	193	204	214	7	572	7478	0.11
6x12	287,477	197	207	227	14	552	7338	0.67
12x24	286,031	198	276	203	11	604	7315	20.64

Comparing the models, it is observed that small differences in the values of the objective function are present with remarkable increase in the solution time (for an Intel Core 2 Duo 2.0 GHz). Regarding the size of the equipment, the greatest difference is observed in the equipment for cooling, where a significant switch to the absorption is observed for the full model. Furthermore, it is also observed that as the size of the model increases reflecting more detailed modelling the annual cost expressed by the objective function is reduced. The conclusions regarding the size of the model is that we may experience significant variations in the sizes of the equipment although it may not be reflected in significant variations in the annual cost.

6.2 Monte – Carlo simulation and the effect of various probability distributions

The combination of Monte Carlo simulation – Optimization is performed developing appropriate code in GAMS. The MILP model is solved for 1000 replications of the Monte Carlo method for every model as depicted in Figure 7. The produced results comprise the optimal solutions for every state of nature and the corresponding optimal solutions are described in histograms for the design variables of the hospital energy system and the cost.

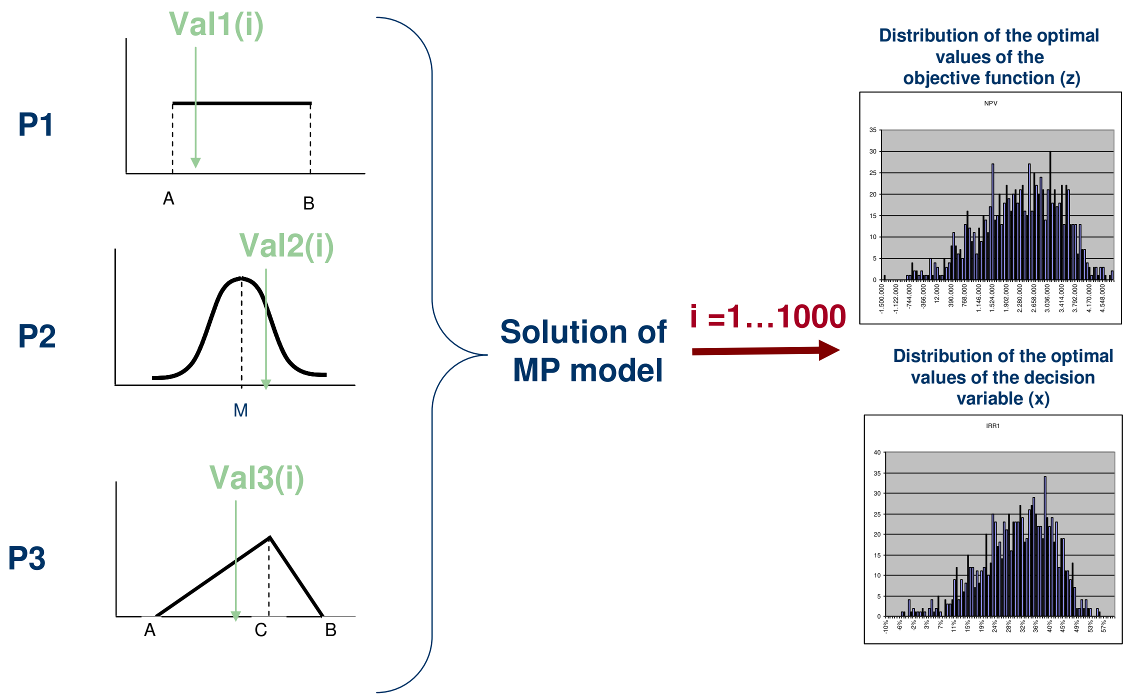

Figure 7. Monte Carlo simulation and Mathematical Programming.

As uncertain parameters we consider the natural gas price, the electricity price and the discount rate used to annualize the investment costs. The selling electricity price and the off-peak electricity price are linked to the electricity price at a constant percentage of 55% and 34% respectively. Three probability distributions were used for the uncertain parameters: the uniform, the normal and the triangular. The characteristics of the distributions for the uncertain parameters are given in Table 5. These are the minimum and the maximum value for the uniform distribution, the mean and the standard deviation for the normal distribution and the minimum, the most likely and the maximum value for the triangular distribution.

Table 2: Characteristic of the applied probability distributions

	Natural gas cost (ngcost)	Electricity cost (elcost)	Discount rate (ir)
uniform	(25, 35)	(90, 125)	(5, 12)

normal	(30, 1.67)	(107.5, 5.83)	(8.5, 1.17)
triangular	(25, 27, 35)	(90, 116.5, 125)	(5, 8, 12)

The uniform distribution expresses the maximum uncertainty situation, where the value of the parameter can vary within an interval with equal probabilities. The normal distribution is used when the value of the parameter can vary around a mean value, while the triangular distribution is useful whenever we want to express a non-symmetrical variation around an average value. The Monte Carlo method produces the results in the form of probability distributions for the objective function as well as for the fundamental decision variables. Sampling from the defined distributions and is performed inside the GAMS model. The computational time is 183 seconds for the 3x6 model, 720 seconds for the 6x12 model and 25311 seconds for the full 12x24 model.

The output of the model is in the form of histograms with the probability distributions of the annual cost and the key variables. Except from the obtained range of values these outputs provide fruitful information to the decision maker as he is able to see how the objective function and the optimal values of the key decision variables vary along this range. As it was expected for different type of probability distributions for the input parameters different probability for the output variables are obtained. For example, in Figure 8 one can see the histograms of the probability distribution of the annual cost for the three types of distributions (uniform, normal, triangular) of the input variables for the 6x12 model. It is observed that more or less the distribution of the annual cost follows the distribution of the input parameters.

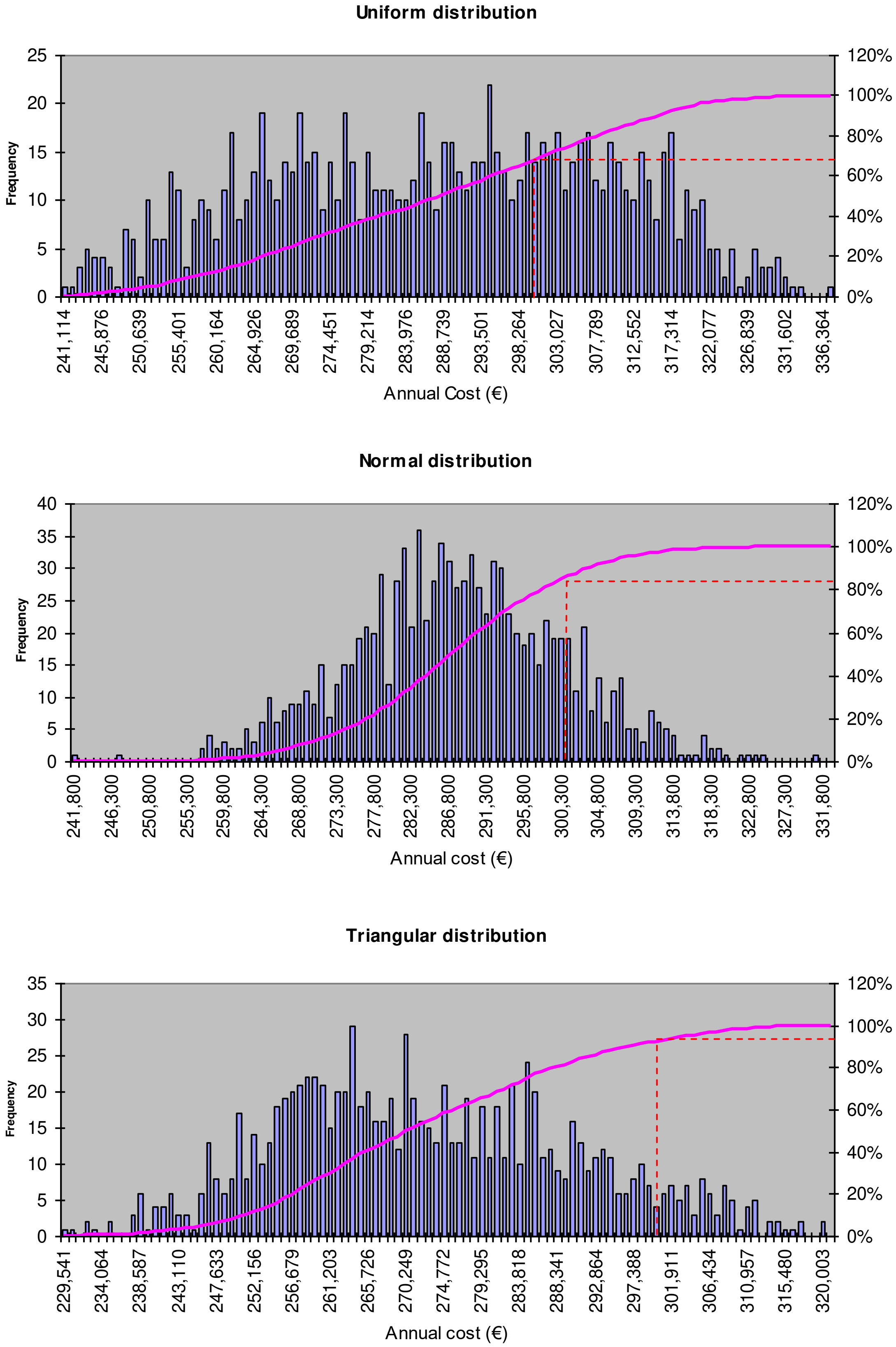

Figure 8: The probability distributions of the annualized cost (output of the model)

From these charts one can also extract information in the terms of probabilities. For example the probability that the annual cost is under 300,000€ is about 67% for the uniform distribution, 85%

for the normal distribution and 92% for the triangular distribution.

Likewise we can observe the probability distribution of the optimal values of some key variables. For example the probability distribution of the CHP size if the input variables are sampled from the normal distribution of Table 2 is shown in Figure 9. From this chart one can see which is the most likely optimal size of the CHP unit. From Figure 9 it is also noticed that there is a small, but not negligible probability (5% which is 50 out of 1000 replications) that the installation of the CHP unit is not an optimal choice. This happens when the sampled price of the natural gas is high and simultaneously the price of the electric power is low, so it is more beneficial to leave out the CHP unit and satisfy the demand in heating and cooling load by electricity from the grid. Similar conclusion can be drawn for the other distribution and the other decision variables. These pieces of information are very useful for the decision maker because they show not only the optimal decisions (at which one may arrive without Monte Carlo analysis) but also the sensitivity of these decisions with regard to some critical parameters.

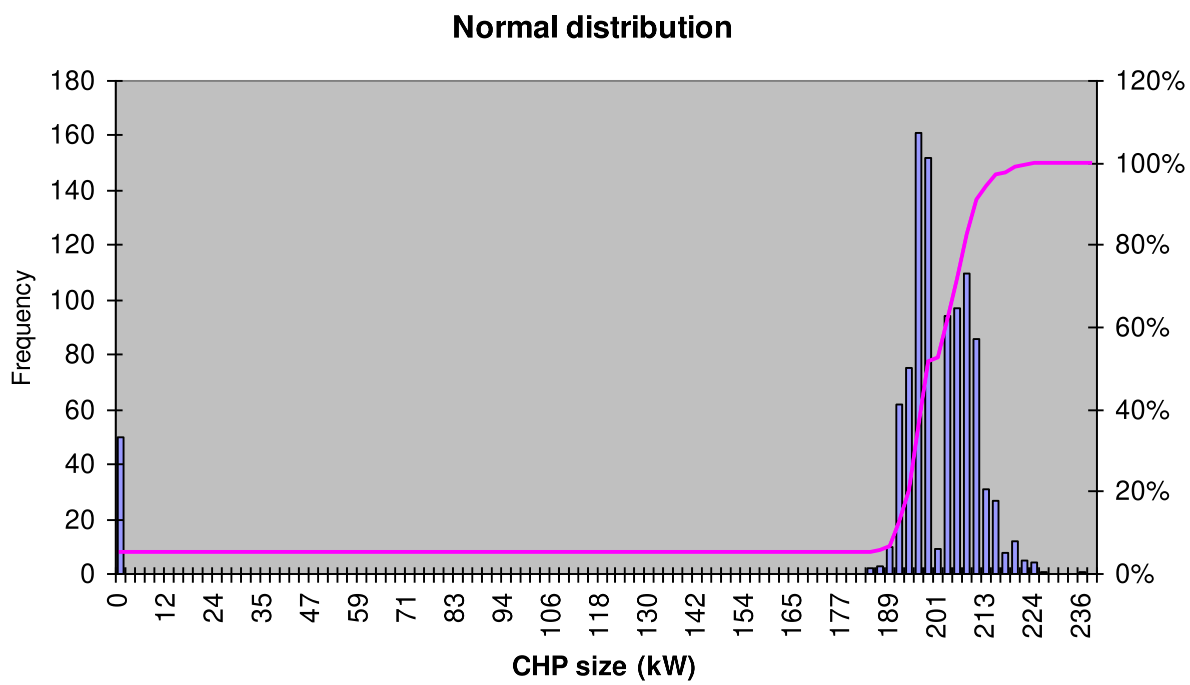

Figure 9: Probability distribution of CHP size (model 12x24)

7. CONCLUSIONS

In the present paper an integrated energy planning framework based on the combined use of Monte Carlo simulation and Mixed Integer Linear Programming is proposed for buildings of the tertiary sector, taking into account the uncertainty in the fuel costs and the discount rate. The application of the method is demonstrated in a case study regarding the energy restructuring of a hospital. A key concept is the modelling of the superstructure of the hospital's energy system and the subsequent optimization through Mathematical Programming. The output of this process is the optimal structure (which equipment), the optimal size of the equipment and the optimal operational conditions (energy flows for each time interval). The above approach is enhanced with the Monte Carlo simulation in order to take into account the stochastic uncertainty on some key input parameters like the required energy prices and the discount rate that are use as objective function coefficients.

The combined use of Monte Carlo simulation and Mixed Integer Programming gives us the capability to study the variation of the values for the examined variables and the robustness of the solution with regard to the volatile parameters. The Monte Carlo technique permits the risk management for the most uncertain parameters of the objective function and produces results that are the optimal solutions for every state of nature and come in the form of probability distributions.

The scale reduction through data compression proved to be successful as it leads to satisfactorily convergent results, with remarkable simultaneous reduction of the run time. However, relevant effects should be further investigated in other systems of different size and other structural characteristics. The intermediate model (6x12) seems to perform better in terms of convergence and computational time as it can give a good approximation to the full model with a small increase in computational time regarding the simplified model. For a single run, the full model is not computationally prohibitive but for a series of runs (like e.g. in the Monte Carlo simulation) the solution time increases dramatically.

Regarding the specific case study, it was found that the introduction of new energy technologies is beneficial for the hospital under the vast majority of the examined scenarios. The prevailing option is the installation of a CHP unit combined with an absorption unit.

Future research may focus on a similar uncertainty analysis taking into account the uncertainty in load demand or the combination of both (uncertainty on load demand and energy prices).

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