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Abstract

This paper offers a rationale for production subcontracting by a market power firm from smaller firms despite the latter's ability to sell the good for themselves. Particularly, in a dominant firm (DF) model in which the good can be sold through linear pricing or through nonlinear two-part tariff (2PT) contracts, we demonstrate that the DF finds it optimal, whenever it sells its own production plus outsourced production, to subcontract production from fringe firms by setting nonlinear 2PT contracts.

Keywords: Dominant firm model, linear prices, nonlinear 2PT contracts, horizontal subcontracting, welfare

JEL Classification: L11, L14

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1. Introduction

The traditional dominant firm-competitive fringe textbook model of price leadership can be sketched out as follows (Shepherd, 1997; Carlton and Perloff, 2000). The dominant firm (DF), which knows the market demand, sets a price that takes into account the response of a set of fringe (price-taking) firms at any price. This allows the DF to determine residual demand, defined as the difference between market demand and the collective supply function of fringe firms at any price that the DF chooses. The DF ultimately decides the optimal price. This model fits well to a number of industries, and especially those emerging from restructuring processes, where the incumbent is obliged to sell a portion of its capacity to different firms and newly independent producers (Kahai et al., 1996; Rassenti and Wilson, 2003; Gowrisankan and Holmes, 2004; Bonacina and Gulli, 2007).

Subcontracting production among firms when each is capable of producing and marketing independently is quite common in industries in which there is a large (market-power) firm competing with a set of smaller firms (Spiegel, 1993; Baake et al., 1999). However, this strategy consisting of the large firm acquiring production, rather than capacity, from its rivals in the same stage of the industry, cannot be rationalized in the standard dominant firm-competitive fringe model in which it is assumed that all firms sell their production through a uniform or linear price. Particularly, Newbery (1984) showed that forward contracts are not utilized by the DF if all firms are risk-neutral, since firms on the fringe would freeride on the DF, making it unprofitable for the DF to subcontract production from the fringe.[1]

In this paper, we demonstrate the usefulness of the dominant firm-competitive fringe model to rationalize the practice of production subcontracting if we depart from its basic formulation. This allows us to simulate and reconcile the potential and merits of this model to industries better adjusted to that model than to the oligopoly model. This model has recently been applied to landmark antitrust cases referring to Standard Oil and Alcoa and, more recently, to the analysis of deregulated markets for electric power (Wilson and Rassenti, 2004). We particularly extend the basic DF model by assuming that firms are not restricted to selling the good through linear or uniform price contracts, but can use nonlinear two-part tariff (2PT) contracts. We also assume that there is some intra-consumer heterogeneity regarding the marginal value of each unit of consumption, such that a firm cannot extract all consumer surplus if selling the good through linear pricing contracts, but only through nonlinear 2PT contracts.

[1] Allaz and Vila (1993) show that firms would use forward contracts in a Cournot setting as a way to strategically increase market share, leading to fiercer competition that would benefit consumers.

In a previous paper (Antelo and Bru, 2020), we demonstrate that if nonlinear contracts are feasible, the DF, but not the fringe firms, will use them to sell its production: in other words, the DF practices price discrimination. In this current paper, we show that this encourages the DF to “monopolize” sales by purchasing capacity from fringe firms and becoming their distributor in the market. We consider two kinds of contracts for purchasing production capacity, one equivalent to a merger and the other equivalent to horizontal subcontracting. The DF’s incentive to set capacity contracts equivalent to a merger emerge when it is restricted to selling its production to end users through linear pricing, as was pointed out by Gowrisankaran and Holmes (2004). A market-power firm restricted to using linear prices with consumers finds it optimal to acquire production capacity from the competitive fringe. Our novel finding in this paper is that the DF’s incentive to acquire capacity from the fringe persists when the output is sold through nonlinear 2PT contracts, and is even reinforced with respect to the case in which this selling method is not allowed.

Regarding subcontracting practices (forward contracts), two findings emerge from our model. First, unlike what Newbery (1984) argued, we show that selling the good through nonlinear 2PT contracts leads the DF to set forward contracts with fringe firms. Second, and contrary to Allaz and Vila (1993), this yields a less competitive outcome. Hence, our results on forward contracting parallel previous findings that cast doubt on the pro-competitive effects of such contracts (Antelo and Bru, 2002; Mahenc and Salanie, 2004). Moreover, subcontracting with the fringe does not seek to foreclose the market, since fringe firms are already established and there is nothing to prevent them from selling directly to consumers. Indeed, fringe firms’ profits increase when such contracts are allowed, since the outside option for consumers becomes less attractive. Fringe firms can thus charge higher prices if they choose to sell in the product market. This ultimately leads the DF to offer an attractive contract for the fringe firms not to sell directly to consumers. In sum, it is the consumers who, on seeing their surplus reduced, lose out.

The remainder of this paper consists of four sections. In Section 2 we set out the model. In Section 3 we discuss the impact of horizontal subcontracting by the DF. In Section 4 we make some final remarks and discuss directions of future research. An appendix contains the proofs of the results.

2. The model

Consider an industry selling a homogeneous good comprised of a dominant firm (DF) and a set of price-taking firms, collectively known as the fringe. We start by detailing preferences, technology and market interaction in this industry.

Preferences. There is a continuum of symmetric and homogeneous consumers of size one, with preferences given by the same quasi-linear utility function $u(q,m) = U(q) + m$, where $U(q)$ represents utility derived from consumption of the good and m stands for the numeraire. As usual, $U(0) = 0$, $U'(q) > 0$, $U''(q) < 0$, and $\rho \equiv -\frac{U'''(q)q}{U''(q)} < 2$.[2]

Technology. The industry comprises K production plants, which we normalize to $K = 1$. Both the DF and fringe firms have the same cost function per plant, $c(Q)$,[3] which is assumed to satisfy $c(0) = 0$, $c'(Q) > 0$, and $c''(Q) > 0$. The number of plants belonging to the fringe as a whole is $1 - k$, so k plants are in hands of the DF.

Market interaction. The interaction between the DF, fringe firms and consumers follows the standard treatment described in textbooks.[4] The difference is that all firms are not restricted to sell the good through linear pricing contracts, but can also do it through nonlinear two-part tariff (2PT) contracts. Moreover, horizontal contracts can be agreed among them.

The timing of the game we consider is as follows:

1. The DF can sell production q for a payment $T(q) = F + pq$, where $F = 0$ if prices are restricted to be linear or $F \geq 0$ if nonlinear 2PT contracts are allowed. The DF also offers exclusive or nonexclusive contracts to fringe firms to acquire production from them. All contracts are offered simultaneously.

2. Consumers accept or reject the DF's contract. Those who accept it can consume quantity q that maximizes consumer surplus, i.e., that verifies $U'(q) = p$.

3. Fringe firms observe the type of contract the DF chooses to sell the good and simultaneously decide whether to accept or reject the DF's production contracts. The fringe firms not receiving a DF offer and those rejecting the DF's offer simultaneously decide the number of consumers they will supply and the type of contract (linear pricing contract or nonlinear 2PT contract).

4. The DF delivers the good to (residual) consumers not served by the fringe.

To evaluate inefficiencies, we consider, as the first-best scenario, the aggregate welfare achieved when the quantity produced and consumed in the industry is that which solves the

[2] Condition $\rho < 2$ is the usual restriction on the convexity of the demand function to ensure that the second order condition of the monopolist's problem is satisfied. See, for instance, Mrázová and Neary (2017).
[3] Most of our results hold if we allow the DF to have a different technology in its plants provided this still leads to an increasing and strictly convex cost function. However, our assumption of the same technology across firms allows us to simplify the discussion of comparative statics when we change how production capacity is distributed between the DF and fringe firms.
[4] See, for instance, Carlton and Perloff (1994) and Gowrisankaran and Holmes (2004).

problem $\max_{q,Q}\{U(q)-c(Q)\}, q \leq Q$. It is straightforward to see that: (i) $a^{fb}=k$, i.e., the number of consumers served by the DF in the first-best scenario is proportional to its share of plants in the industry;[5] (ii) each buyer consumes quantity q^{fb} satisfying $U'(q^{fb})=c'(q^{fb})$; and (iii) the consumer surplus amounts to $U(q^{fb})-U'(q^{fb})q^{fb}$. Finally, note that a competitive market would implement this outcome.

3. Does the dominant firm subcontract production from fringe firms?

A well-known result when the DF sells the good through uniform pricing is that it never subcontracts production from the fringe unless it enjoys some cost advantage (Newbery, 1984). In our set-up, the DF has no cost advantage over fringe firms, but it does end up selling the good through nonlinear 2PT contracts – unlike the fringe firms, which resort to linear pricing (Antelo and Bru, 2020). Can this difference in selling procedure incentivize the DF to contract production from the fringe? And if so, what is the welfare impact of this horizontal subcontracting coupled with price discrimination?

To investigate these issues, we assume that the DF can acquire production from fringe firms on an exclusive or nonexclusive basis. Nonexclusive contracts are equivalent to forward contracts or horizontal subcontracting, whereas exclusive contracts are strategically equivalent to a merger. Let k_c denote the number of horizontal contracts chosen by the DF, i.e., the number of fringe firms with which a contract is established. If the DF acquires the entire production of those firms, the customers of the remaining fringe firms consume quantity q_f that satisfies

$$U'(q_f)=c'(Q_f), \tag{1}$$

where $Q_f=\frac{(1-a)q_f}{1-k-k_c}$ is the quantity these fringe firms are willing to produce, with a the number of customers of the DF. From Eq. (1) it can be shown that, in equilibrium, q_f is a function of a and k_c, in such a way that

$$\frac{\partial q_f(a,k_c)}{\partial a}=\frac{c''q_f}{(1-a)c''-(1-k-k_c)U''}>0 \tag{2}$$

and

$$\frac{\partial q_f(a,k_c)}{\partial k_c}=-\frac{1-a}{1-k-k_c}\frac{\partial q_f(a,k_c)}{\partial a}<0. \tag{3}$$

[5] The superscript fb denotes first-best.

A nonexclusive contract between the DF and a fringe firm (horizontal subcontracting) is a contract that allows the fringe firm to participate in the final market with the amount of noncontracted production. Therefore, for each subcontracted unit of product the DF must pay the same price as the fringe firms charge their consumers, $p^s = U'(q_f)$, and the DF must purchase the quantity that such firms are willing to produce at that price, Q_f. Thus, if the DF signs nonexclusive contracts with k_c fringe firms, its profit is

$$\pi_D^s(a,q,k_c) = a\left[U(q) - \left(U(q_f) - U'(q_f)q_f\right)\right] - kc\left(\frac{aq - k_c Q_f}{k}\right) - k_c U'(q_f)Q_f. \quad (4)$$

Contrariwise, an exclusive contract between the DF and a fringe firm (a merger) prevents that fringe firm from selling directly to consumers. Hence, if the DF signs exclusive contracts with k_c fringe firms, the remaining fringe firms have profits

$$\pi_f(a,k_c) = U'(q_f)Q_f - c(Q_f), \quad (5)$$

where Q_f follows from Eq. (1). For fringe firms to accept an exclusive contract $\{b, Q_c\}$ paying b for production Q_c, that contract must satisfy

$$b - c(Q_c) \geq \pi_f(a,k_c). \quad (6)$$

Hence, the DF pays $b = c(Q_c) + \pi_f(a,k_c)$ and its profit amounts to

$$\pi_D^c(a,q,k_c) = a\left(U(q) - \left(U(q_f) - U'(q_f)q_f\right)\right) - kc\left(\frac{aq - k_c Q_c}{k}\right) - k_c\left(c(Q_c) + \pi_f(a,k_c)\right). \quad (7)$$

Since the DF minimizes production costs $kc\left(\frac{aq - k_c Q_c}{k}\right) + k_c c(Q_c)$ when production is the same in the DF's and the subcontracted fringe firms' plants, $\frac{aq - k_c Q_c}{k} = Q_c$, then the profit stated in Eq. (7) can be rewritten as

$$\pi_D^c(a,q,k_c) = a\left[U(q) - \left(U(q_f) - U'(q_f)q_f\right)\right] - (k + k_c)c\left(\frac{aq}{k + k_c}\right) - k_c \pi_f(a,k_c). \quad (8)$$

We next investigate the industry outcome in different scenarios depending on the type of contract that the DF sets with consumers and with fringe firms.

4.1 The DF offers nonexclusive contracts to fringe firms and sells its own and outsourced production through linear pricing

In this set-up, the DF acquires Q^s units of product from the fringe at price p^s and sells its total output (own plus outsourced production) to a customers through linear contracts. Customers of the fringe consume quantity q_f that satisfies Eq. (1), and the DF's customers receive the same quantity q_f . Therefore, the DF maximizes profits given in Eq. (4) with $q = q_f$, namely,

$$\max_{\{a,k_c\}} \pi_D^s(a, q_f, k_c) = aU'(q_f)q_f - kc\left(\frac{aq_f - k_cQ_f}{k}\right) - k_cU'(q_f)Q_f. \tag{9}$$

We define the DF's total production as $Q_D \equiv aq_f - k_cQ_f$. There are many different equivalent combinations $\{a, k_c\}$ with the same impact on the fringe firms' offer to consumers, provided that Eq. (1) can be written as $U'(q_f) = c'\left(\frac{q_f - Q_D}{1-k}\right)$; hence, q_f becomes a function of Q_D, and the DF's profits as given in Eq. (9) can be written as $\pi_D^s(Q_D) = U'(q_f)Q_D - kc\left(\frac{Q_D}{k}\right)$. Therefore, the same industry outcome under no subcontracting ($a = a^l$ and $Q^s = 0$) yields the levels of horizontal subcontracting given by $Q^s = (a - a^l)q_f$, with a ranging from $a = 0$ to $a = 1$. When $a = 0$, the DF sells all its production to the competitive fringe, $Q^s = -a^lq_f < 0$, and so does not interact with consumers; and when $a = 1$, $Q^s = (1 - a^l)q_f > 0$ and the DF is the only firm serving consumers. Horizontal subcontracting is thus innocuous in terms of both consumer surplus and industry profits.

Summarizing, if the DF is restricted to using linear pricing to deal with consumers, then the use of horizontal subcontracting either cannot be rationalized or is a dull instrument in terms of its impact on industry performance.[6]

We now analyze what happens in three further scenarios: when the DF sells the good through linear pricing but contracts with fringe firms can be settled on an exclusive basis, and when the DF sells the good through nonlinear 2PT contracts and simultaneously offers either exclusive or nonexclusive production contracts to the fringe.

4.2. The DF sells through linear pricing and signs exclusive contracts with fringe firms

In this case, the DF is restricted to offering $q = q_f$ and production per plant amounts to $\frac{aq_f}{k+k_c}$. The profits stated in Eq. (8) thus become

[6] If contracts with fringe firms were set before contracts with consumers, rather than simultaneously, the DF would never outsource production from the fringe under linear pricing. This was first noted for forward contracts (strategically similar to subcontracting) by Newbery (1984). To subcontract under linear pricing, the DF must have some advantage over fringe firms, as described in Antelo and Bru (2002).

$$\pi_D^c(a, q_f, k_c) = aU'(q_f)q_f - (k + k_c)c\left(\frac{aq_f}{k+k_c}\right) - k_c\pi_f(a, k_c) \tag{10}$$

and the DF chooses $\{a, k_c\}$ to maximize Eq. (10), with q_f defined by Eq. (1).

4.3. The DF sells through nonlinear 2PT contracts and signs nonexclusive contracts with fringe firms

In this case, the DF chooses $\{a, k_c\}$ to maximize Eq. (4), where we consider that fringe firms offer their buyers a quantity q_f that satisfies Eq. (1), with the DF then having to pay the same price $p = U'(q_f)$ to fringe firms.

4.4. The DF sells through nonlinear 2PT contracts and signs exclusive contracts with fringe firms

In this case, the DF offers customers the quantity q that maximizes their joint profits, i.e., that satisfy $U'(q) = c'\left(\frac{aq}{k+k_c}\right)$. It then chooses $\{a, k_c\}$ to solve Eq. (8), with q_f defined by Eq. (1).

The following result emerges.

Proposition 1. *Under scenarios (4.2), (4.3) and (4.4), the DF subcontracts production with all firms on the fringe, $k_c^* = 1 - k$, and becomes the only supplier in the industry, $a^* = 1$.*

Proof. See the Appendix.

Perhaps the most striking result from Proposition 1 is that, for (effective) horizontal subcontracting to emerge, the DF must sell the good to consumers through nonlinear 2PT contracts. The explanation is as follows. The convexity of the cost function of all the firms implies that, when the DF subcontracts a given amount of production to fringe firms, the marginal costs of these firms increase, leading them to reduce the quantity offered to their customers. This, in turn, reduces the consumer surplus of customers supplied by fringe firms, which allows the DF to charge higher prices to those customers. Since fringe firms charge the DF the same price as obtained by selling directly to consumers, there is now a wedge in the marginal price charged by the DF and the fringe firms, $U'(q^c) - U'(q_f)$. In sum, the DF finds it profitable to subcontract production.

Once the DF subcontracts production to fringe firms and sells the good through 2PT contracts in the product market, it purchases the total quantity $(1-k)Q_f$ from the fringe at unit price $p^s = U'(q_f)$. Therefore, it offers a deal to consumers that yields consumption q and consumer surplus $U(q_f) - U'(q_f)q_f$, and that solves:

$$\max_{\{q,q_f\}} \left\{U(q) - [U(q_f) - U'(q_f)q_f] - kc\left(\frac{q-(1-k)Q_f}{k}\right) - (1-k)U'(q_f)Q_f\right\},$$

$$\text{s.t: } Q_f \text{ defined in Eq. (1)} \tag{11}$$

If the DF acquires production from fringe firms through exclusive contracts, these firms will accept the contracts if they obtain the same profit as would be obtained from being active in the final market, $\pi_f^c = U'(q_f)Q_f - c(Q_f)$, with q_f and Q_f according to Eq. (1). If the DF is restricted to selling the good through linear pricing, the price chosen is that which induces consumption q_f and that solves

$$\max_{q_f} \ \{U'(q_f)q_f - c(q_f) - (1-k)\pi_f^c\}. \tag{12}$$

Finally, if the DF delivers the good through nonlinear 2PT contracts, it induces consumption q and consumer surplus $U(q_f) - U'(q_f)q_f$ that solves

$$\max_{\{q,q_f\}} \{U(q) - [U(q_f) - U'(q_f)q_f] - c(q) - (1-k)\pi_f^c\}. \tag{13}$$

From here, we can summarize the impact of allowing the DF to sign contracts with fringe firms on the DF's profits.

Proposition 2. *The DF's profit increases:*

i) *When the DF sells the good through nonlinear contracts, if it moves from nonexclusive to exclusive contracts with the fringe.*

ii) *When the DF subcontracts production through exclusive contracts, if it moves from selling the good through linear to nonlinear contracts.*

Proof. See the Appendix.

We can also evaluate how the consumer surplus evolves with changes in the contracts available to the DF.

Proposition 3. *Regarding consumers, the following hold:*

a) *Save for the case in which DF sells the good through linear contracts and acquires production from fringe through nonexclusive contracts, horizontal subcontracting harms consumers.*
b) *The consumer surplus decreases:*
   b.1) when the DF sells the good through nonlinear 2PT contracts, if it can move from nonexclusive to exclusive contracts with fringe firms.
   b.2) when the DF signs exclusive contracts with fringe firms, if it can move from selling the good through linear to nonlinear 2PT contracts.

Proof. See the Appendix.

What part (a) of Proposition 3 states is that, except when horizontal subcontracting is coupled with selling through linear pricing in the final market (when the consumer surplus is not affected), any contract between the DF and fringe firms is harmful to the end consumer. The equilibrium outcome, however, is not the monopoly outcome unless the DF already owns all production capacity: compared to monopolistic behavior, when solving Eqs. (11), (12) and (13), the DF chooses a larger value of q_f in order to reduce payment to the fringe.

Part (b) of Proposition 3 shows that the DF, when selling the good through a 2PT contract and signing exclusive contracts to acquire production from the fringe, chooses the efficient quantity q^{fb}. However, with nonexclusive contracts with the fringe (horizontal subcontracting), and even though in equilibrium, the DF becomes the only active supplier in the industry, and the outcome is not equivalent to a monopoly operating under nonlinear 2PT contracting. A major inefficiency arises from the fact that the DF subcontracts too much production from the fringe, $Q_f > q^{fb}$, and end users are therefore better off than in a monopoly. The explanation is that, since the fringe firms have a marginal cost $c'(Q_f)$ that is below $U'(0)$, they potentially constitute an alternative to the DF for end users and, hence, the DF must leave them some consumer surplus.

We can also see that q_f decreases in k. If $k = 1$, then Eqs. (11), (12) and (13) indeed represent the monopoly situation. However, the interests of the DF and the fringe firms are aligned, and so fringe firms are better off if exclusive contracts are allowed and the DF becomes the only active supplier in the industry.

5. A numerical example

To obtain closed-form results when the DF can subcontract production from the fringe, we consider a numerical example consisting of customers that have preferences given by the utility function $U(q) = \left(1 - \frac{q}{2}\right) q$, and firms with a technology described by the per-plan cost function $c(Q) = Q^2/2$. In this case, if the DF signs exclusive contracts with fringe firms and uses linear prices with customers, the problem stated in Eq. (12) becomes

$$\max_{q_f} \left\{ (1 - q_f) q_f - \tfrac{1}{2} q_f^2 - (1-k)\frac{1}{2}\left(1 - q_f\right)^2 \right\} \tag{14}$$

and each customer consumes the efficient quantity $q_f^S = \frac{2-k}{4-k}$, which, whenever $0 < k < 1$, is below the quantity consumed if exclusive contracts between firms were prohibited, $q_f = \frac{2-k^2}{4-k^2}$.

Contrariwise, if the DF sells the good through nonlinear 2PT contracts, the problem defined in Eq. (13) becomes

$$\max_{\{q, q_f\}} \left\{ (1 - \tfrac{q}{2}) q - \tfrac{1}{2} q_f^2 - \frac{1}{2} q^2 - (1-k)\frac{1}{2}\left(1 - q_f\right)^2 \right\} \ , \tag{15}$$

the DF's customers buy quantity $q = 1/2$ and their consumer surplus, inferred from the consumption of buyers supplied by the fringe, $q_f = \frac{1-k}{2-k}$, amounts to $CS = q_f^2/2$.

If horizontal contracts are nonexclusive, then the DF only subcontracts production from the fringe if it can sell the good through nonlinear 2PT contracts. In this case, the DF subcontracts $\frac{Q^S}{1-k} = \frac{2}{4-k-k^2}$ units from each fringe firm. Since the DF asks fringe firms to produce above the efficient quantity, $\frac{Q^S}{1-k} > \frac{1}{2}$, those firms have little interest in making deals with consumers. A fringe firm with marginal costs $c'\left(\frac{Q^S}{1-k}\right)$ can, in any case, offer the quantity q_f satisfying

$U'(q_f) = c'\left(\frac{Q^s}{1-k}\right)$ to buyers, i.e., $q_f = \frac{2-k-k^2}{4-k-k^2}$. This quantity is decreasing in k, but strictly positive unless $k = 1$; hence, the DF must offer a contract that guarantees a strictly positive consumer surplus to its buyers, i.e., $U(q_f) - U'(q_f)q_f = \frac{1}{2}q_f^2 > 0$. The DF offers end users the consumption level $q = \frac{2-k^2}{4-k-k^2}$, which becomes the efficient level, $q = q^{fb} = 1/2$, when $k \in \{0, 1\}$, but otherwise larger, $q > q^{fb}$. Finally, the DF can charge a positive fee F slightly below $U(q) - U'(q)q - (U(q_f) - U'(q_f)q_f) = \frac{1}{2}(q^2 - q_f^2)$.

In sum, when the DF sells the good through nonlinear 2PT contracts, consumers are better off (worse off) when production contracts between the DF and fringe firms are exclusive (nonexclusive). Horizontal subcontracting therefore harms end users.

From Table 1 referring to the consumer surplus, it follows that at $k = 0$ (competitive industry), $CS^A = CS^B = CS^C = CS^D = 1/8$, whereas at $k = 1$ (monopoly), $CS^A = CS^B = 1/18 > CS^C = CS^D = 0$ (the monopolist appropriates all rents). Finally, at $k \in (0, 1)$, the consumer surplus is decreasing in k in all scenarios, and, for a given value of k, $CS^D < CS^C < CS^B < CS^A$.

Table 1. Consumer surplus in the quadratic example.

		DF's contracts with consumers	
		Linear contracts	Nonlinear 2PT contracts
DF's contracts with fringe firms	Nonexclusive contracts (subcontracting)	$CS^A = \frac{1}{2}\left(\frac{2-k^2}{4-k^2}\right)^2$	$CS^C = \frac{1}{2}\left(\frac{2-k-k^2}{4-k-k^2}\right)^2$
	Exclusive contracts (capacity acquisition)	$CS^B = \frac{1}{2}\left(\frac{2-k}{4-k}\right)^2$	$CS^D = \frac{1}{2}\left(\frac{1-k}{2-k}\right)^2$

Likewise, from Table 2 referring to aggregate welfare, it follows that at $k = 0$ (competitive industry), $W^A = W^B = W^C = W^D = 1/4$, whereas at $k = 1$ (monopoly), $W^A = W^B = 2/9 < W^C = W^D = 1/4$ (under 2PT, the monopolist chooses the efficient quantity). Finally, at $k \in (0, 1)$, aggregate welfare is decreasing in k under linear prices, and efficiency is always achieved with a 2PT and exclusive contracts. For a given value of k, it holds that $W^A <$

min $\{W^B, W^C\} < \max \{W^B, W^C\} < W^D$. Finally, $W^B < W^C$ if the DF's size is such that $0.642074 < k \leq 1$.

Table 2. Aggregate welfare in the quadratic example.

		DF's contracts with consumers	
		Linear contracts	Nonlinear 2PT contracts
DF's contracts with fringe firms	Nonexclusive contracts (subcontracting)	$W^A = \dfrac{4+2k-k^2-k^3}{2(2-k)(2+k)^2}$	$W^C = \dfrac{8-4k-4k^2+k^3+k^4}{2(4-k-k^2)^2}$
	Exclusive contracts (capacity acquisition)	$W^B = \dfrac{2(2-k)}{(4-k)^2}$	$W^D = \dfrac{1}{4}$

5. Final remarks

We have provided a rationale for the emergence of horizontal subcontracting — even when cost asymmetries among firms do not exist — as a stylized fact commonly observed in real-life industries, where large firms subcontract part of their production to smaller firms and these smaller firms produce for the large firm rather than for consumers. To that end, we built a dominant firm-competitive fringe model in which all firms can either sell their production to consumers through uniform prices or through (more sophisticated) nonlinear 2PT contracts.

In a previous paper (Antelo and Bru, 2020), we showed that while firms with no market power to affect overall market performance use linear pricing to sell the good to their customers, the DF prefers to utilize nonlinear 2PT contracts; hence, intrapersonal price discrimination emerges in equilibrium. In contrast, therefore, with the market foreclosure literature that rationalizes 2PT as a barrier to entry (see Aghion and Bolton, 1987), we provide another rationale for why a large firm signs nonlinear 2PT contracts with smaller firms, which is their use as a collusive tool that favors all firms in the industry as compared to the context in which the large firm would use linear prices with consumers and yield allocative inefficiencies.

In this paper, we show that, in sharp contrast with the standard DF model where all firms are assumed to use uniform prices to sell the good and the DF has no advantage over fringe firms, there is room for the DF to subcontract production from the fringe. Indeed, we find that the DF purchases all the fringe's production and thus becomes the only supplier in the industry. Therefore, the possibility of setting nonlinear 2PT contracts with customers, and the subsequent different selling methods used by the DF and fringe firms, explain the emergence of horizontal subcontracting practices whereby the DF acquires production from the fringe, exacerbating all the effects caused by intrapersonal price discrimination.

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Appendix

Proof of Proposition 1. When the DF sells the good through a nonlinear 2PT contract and subcontract s units of product from fringe firms, it serves a customers, and all customers that purchase from the fringe pay the unit price $p(a,k,s)=\frac{1-a-s}{2-k-a}$ and buy the quantity $q_f(a,k,s)=\frac{1-k-s}{2-k-a}$. On the other hand, the DF's customers purchase the quantity $q(a,k,s)=\frac{k+s}{k+a}$ and the DF's profits in the second stage (in which payments to the fringe are sunk), $\big(U(q(a,k,s))-CS(q(a,k,s))\big)a-kc\left(\frac{aq(a,k,s)-s}{k}\right)$, amount to

$$\pi_D(a,k,s)=\frac{1}{2}\left(\left(\frac{a(k+s)(k-s+2a+ka^2)}{(k+a)^2}-\left(\frac{1-k-s}{2-k-a}\right)^2\right)a-k\left(\frac{a-s}{k+a}\right)^2\right). \tag{A1}$$

Thus, the DF chooses to supply the number of buyers $a^c(k,s)$ that satisfies the FOC $\frac{\partial\pi_D(a,k,s)}{\partial a}=0$. In the first stage, the DF chooses to subcontract from fringe firms the quantity s of product that maximizes

$$\pi_D(a(k,s),k,s)=\big(U(q(a(k,s),k,s))-CS((a(k,s),k,s))\big)a(k,s)-$$
$$-kc\left(\frac{a(k,s)q(a(k,s),k,s)-s}{k}\right)-p(a(k,s),k,s)\,s, \tag{A2}$$

where it is assumed that, in exchange of production s purchased from the fringe, the DF must pay the equilibrium final price $q(a(k,s),k,s)=U'\big(q(a(k,s),k,s)\big)$. Lastly, the DF acquires from fringe the quantity s of product that satisfies the first-order condition (FOC)

$$\frac{\partial\pi_D(a(k,s),k,s)}{\partial s}+\frac{\partial\pi_D(a(k,s),k,s)}{\partial a}\frac{\partial a(k,s)}{\partial a}=0. \tag{A3}$$

If the DF offers nonlinear 2PT contracts to consumers and horizontally subcontracts from the fringe, then it chooses $\{a,q,k_c\}$ to maximize the profit given in Eq. (4). The DF's customers consume the quantity q that solves the FOC

$$0=\frac{\partial\pi_D^s}{\partial q}=a\left(U'(q)-c'\left(\frac{aq-k_cQ_f}{k}\right)\right) \tag{A4}$$

and, given the number of fringe firms that the DF deals with, k_c, the DF's optimal number of customers, a, is that which satisfies the FOC

$$0 = \frac{\partial \pi_D^S}{\partial a} = \left(U(q)\, q - U'(q)\, q\right) - \left(U\left(q_f\right) - U'^{(q_f)\, q_f}\right) + \frac{\partial \pi_D^S}{\partial q_f} \frac{\partial q_f}{\partial a}, \tag{A5}$$

where $\frac{\partial \pi_D^S}{\partial q_f} = U''\left(q_f\right)\left(a q_f - k_c Q_f\right)$ and we use (A4) to replace $c'\left(\frac{aq - k_c Q_f}{k}\right)$ with $U'(q)$. The marginal impact of setting contracts with k_c fringe firms on the DF's profits is

$$\frac{\partial \pi_D^S}{\partial k_c} = \left(U'(q) - U'\left(q_f\right)\right) Q_f + \frac{\partial \pi_D^S}{\partial q_f} \frac{\partial q_f}{\partial k_c}, \tag{A6}$$

where we again use (A4) to replace $c'\left(\frac{aq - k_c Q_f}{k}\right)$ with $U'(q)$. Using (A5) and bearing in mind that $\frac{\partial q_f}{\partial k_c} = -\frac{1-a}{1-k-k_c} \frac{\partial q_f}{\partial a}$, the derivative given in (A6) can be rewritten as

$$\frac{\partial \pi_D^S}{\partial k_c} = \left(U'(q) - U'\left(q_f\right)\right) Q_f + \frac{1-a}{1-k-k_c}\left(U(q) - U'(q) q - \left(U\left(q_f\right) - U'\left(q_f\right) q_f\right)\right), \tag{A7}$$

which, in turn, can be rewritten, using $Q_f = \frac{1-a}{1-k-k_c} q_f$, as

$$\begin{aligned} \frac{\partial \pi_D^S}{\partial k_c} &= \frac{1-a}{1-k-k_c}\left(U(q) - U\left(q_f\right) - U'(q)\left(q - q_f\right)\right) \\ &= \frac{1-a}{1-k-k_c} \int_{q_f}^{q} \left(U'(s) - U'(q)\right) \mathrm{d}s, \end{aligned} \tag{A8}$$

with (A8) strictly positive, since $q > q_f$ and $U'(s) > U'(q)$ for $q_f < s < q$. Therefore, in equilibrium, the DF becomes the only active seller in the industry, $k_c^* = 1 - k$.

If the DF sells the good through linear pricing and sets exclusive horizontal contracts with fringe firms, it chooses the pair $\{a, k_c\}$ that maximizes the profit stated in Eq. (10). Given k_c, the DF chooses a that solves the FOC

$$0 = \frac{\partial \pi_D^C}{\partial a} = \left(U'\left(q_f\right) - c'(Q_D)\right) q_f + \frac{\partial \pi_D^C}{\partial q_f} \frac{\partial q_f}{\partial a}, \tag{A9}$$

where $Q_D = \frac{a q_f}{k + k_c}$ and $\frac{\partial \pi_D^C}{\partial q_f} = a\left(U''\left(q_f\right) q_f + U'\left(q_f\right) - c'(Q_D)\right) - k_c\, U'\left(q_f\right) Q_f$. The marginal impact of contracts k_c on the DF's profits is therefore

$$\frac{\partial \pi_D^C}{\partial k_c} = c'(Q_D) Q_D - c(Q_D) - \pi_f + \frac{\partial \pi_D^C}{\partial q_f} \frac{\partial q_f}{\partial k_c}, \tag{A10}$$

which, using (A9) and bearing in mind that $\frac{\partial q_f}{\partial k_c} = -\frac{1-a}{1-k-k_c}\frac{\partial q_f}{\partial a}$, can be written as

$$\frac{\partial \pi_D^c}{\partial k_c} = c'(Q_D)Q_D - c(Q_D) - \pi_f + \left(U'(q_f) - c'(Q_D)\right)Q_f, \tag{A11}$$

i.e., as

$$\begin{aligned}\frac{\partial \pi_D^c}{\partial k_c} &= c'(Q_D)Q_D - c(Q_D) - \left(c'(Q_D)Q_f - c(Q_f)\right)\\ &= \int_{Q_D}^{Q_f}\left(c'(s) - c'(Q_D)\right)\mathrm{d}s\end{aligned} \tag{A12}$$

with (A12) strictly positive, since $Q_f > Q_D$. Therefore, in equilibrium, the DF again becomes the only active seller in the industry, $k_c^* = 1 - k$.

Finally, if the DF uses nonlinear 2PT contracts to sell the good and sets exclusive horizontal contracts with fringe firms, it chooses $\{a, q, k_c\}$ to maximize the profit given in Eq. (8). The DF's customers consume q that solves the FOC

$$0 = \frac{\partial \pi_D^c}{\partial q} = a\left(U'(q) - c'\left(\frac{aq}{k+k_c}\right)\right) \tag{A13}$$

and, given the number of fringe firms that the DF deals with, k_c, the DF's optimal number of consumers a is that which satisfies the FOC

$$0 = \frac{\partial \pi_D^c}{\partial a} = \left(U(q) - U'(q)q\right) - \left(U(q_f) - U'(q_f)q_f\right) + \frac{\partial \pi_D^s}{\partial q_f}\frac{\partial q_f}{\partial a}, \tag{A14}$$

where $\frac{\partial \pi_D^s}{\partial q_f} = U''(q_f)\left(aq_f - k_c Q_f\right)$ and we use (A4) to replace $c'\left(\frac{aq - k_c Q_f}{k}\right)$ with $U'(q)$. The marginal impact of the set of contracts with k_c fringe firms on the DF's profits is

$$\frac{\partial \pi_D^s}{\partial k_c} = \left(U'(q)Q_f - c(Q_D)\right) - \pi_f + \frac{\partial \pi_D^c}{\partial q_f}\frac{\partial q_f}{\partial k_c}, \tag{A15}$$

where $Q_D = \frac{aq}{k+k_c}$. Using (A14) and bearing in mind that $\frac{\partial q_f}{\partial k_c} = -\frac{1-a}{1-k-k_c}\frac{\partial q_f}{\partial a}$, then (A15) can be rewritten as

$$\frac{\partial \pi_D^c}{\partial k_c} = \left(U'(q)Q_f - c(Q_D)\right) - \pi_f + \frac{1-a}{1-k-k_c}\left((U(q) - U'(q)q) - \left(U(q_f) - U'(q_f)q_f\right)\right). \tag{A16}$$

If we define $Q(s) \equiv \text{argmax}_Q \, (U'(s)Q - c(Q))$, where $Q'(s) = \frac{U''(s)}{c''(Q)} < 0$, then (A16) becomes

$$\frac{\partial \pi_D^c}{\partial k_c} = \int_{q_f}^{q} U''(s)\left(Q(s) - \frac{1-a}{1-k-k_c}s\right) \mathrm{d}s \tag{A17}$$

and (A17) is strictly positive since $Q(q_f) = \frac{1-a}{1-k-k_c}q_f$ and $\frac{\partial}{\partial s}\left(Q(s) - \frac{1-a}{1-k-k_c}s\right) = Q'(s) - \frac{1-a}{1-k-k_c} < 0$ implies that $Q(s) - \frac{1-a}{1-k-k_c}s < 0$ for $q_f < s < q$. Therefore, in equilibrium, the DF becomes the only supplier in the industry and, as result, $k_c^* = 1 - k$. ■

Proof of Proposition 2. It is immediate that, for any given interaction with fringe firms (no contracts, nonexclusive contracts or exclusive contracts), the DF's profits increase when contracts with customers move from linear pricing to nonlinear 2PT contracts, because a 2PT can always replicate a linear price, and the DF always chooses a contract that leads to $q > q_f$, which means that there is an increase in the joint surplus for the relationship between the DF and buyers, and the DF can appropriate this surplus increase.

If the DF is restricted to offering linear prices to consumers, then it solves[7]

$$\max_{q_f} \pi^{l,ne}(q_f) = U'(q_f)q_f - kc\left(\frac{q_f-(1-k)Q_f}{k}\right) - (1-k)U'(q_f)Q_f, \tag{A18}$$

under nonexclusive horizontal contracts and

$$\max_{q_f} \pi^{l,e}(q_f) = U'(q_f)q_f - c(q_f) - (1-k)\pi_f^c\,, \tag{A19}$$

under exclusive horizontal contracts, where $\pi_f^c = U'(q_f)Q_f - c(Q_f)$ and Q_f satisfies Eq. (1). If we evaluate $\pi^{l,e}(q_f)$ at the quantity q_f that solves (A18), we have

[7] Recall that the DF has the same profits as those achieved in the absence of horizontal contracts with fringe firms.

$$\pi^{l,e}(q_f) = \pi^{l,ne}(q_f) + \left(kc\left(\frac{q_f-(1-k)Q_f}{k}\right) + (1-k)c(Q_f) - c(q_f)\right), \quad \text{(A20)}$$

which is larger than $\pi^{l,ne}(q_f)$, because $kc\left(\frac{q_f-(1-k)Q_f}{k}\right) + (1-k)c(Q_f) > c(q_f)$. Therefore, under linear pricing to sell the good, the DF can achieve larger profits when horizontal subcontracting moves from nonexclusive to exclusive contracts.

If the DF can offer nonlinear 2PT contracts to consumers, profits under nonexclusive horizontal contracts are those given in Eq. (11), which, according to Proposition 1, are larger than those achieved when no horizontal contracts exist, while profits under exclusive horizontal contracts are those in Eq. (13). If we evaluate the profits given in Eq. (13) at the quantities q and q_f that solve Eq. (11), we have

$$\pi^{c,e}(q, q_f) = \pi^{c,ne}(q, q_f) + \left(kc\left(\frac{q-(1-k)Q_f}{k}\right) + (1-k)c(Q_f) - c(q)\right), \quad \text{(A21)}$$

which is larger than $\pi^{c,ne}(q, q_f)$, because $kc\left(\frac{q-(1-k)Q_f}{k}\right) + (1-k)c(Q_f) > c(q)$. Therefore, under nonlinear 2PT pricing, the DF can obtain larger profits when subcontracting exists if it moves from nonexclusive to exclusive contracts. ■

Proof of Proposition 3.

Assume that the DF sells through linear pricing. According to Proposition 1, the DF monopolizes sales, a=1, when there is subcontracting from fringe firms, in which case customers have the consumer surplus $CS(q_f^s) = U(q_f^s) - U'(q_f^s)q_f^s$, where q_f^s is the quantity that solves Eqs. (11), (12) and (13). On the other hand, with no subcontracting, customers obtain the same consumer surplus if the DF supplies a number of buyers $\hat{a}$ that leads fringe firms to offer the quantity q_f^s. Thus, $\hat{a}$ must satisfy

$$\frac{1-\hat{a}}{1-k} q_f^s = Q_f^s. \quad \text{(A22)}$$

Customers are better off if the DF choose to serve a number of buyers a such that $a > \hat{a}$, since according to Eq. (7), q_f is increasing in a. We see below that this is indeed the case.

If the DF sells the good through linear pricing, customers are worse off if subcontracting is by means of exclusive contracts than in the absence of horizontal subcontracting. In fact, when there is no subcontracting, the DF maximizes the profit given in Eq. (10) and chooses quantity q_f that solves the problem stated in Eq. (11). On the other hand, under subcontracting with exclusive contracts, the DF solves the problem defined in Eq. (12), whose FOC is

$$U''(q_f^s)\, q_f^s + U'(q_f^s) - c'(q_f^s) - (1-k)\, U''(q_f^s)\, Q_f^s = 0, \tag{A23}$$

where q_f^s and Q_f^s denote the quantities q_f and Q_f that solve Eq. (1) and (A23). Using (A22), we can rewrite (A23) as

$$\hat{a}\, U''(q_f^s)\, q_f^s + U'(q_f^s) - c'(q_f^s) = 0. \tag{A24}$$

If we evaluate the FOC at $\hat{a}$, we have

$$\begin{aligned} \left.\frac{\partial \pi_D}{\partial a}\right|_{a=\hat{a}} &= \left(U'(q_f^s) - c'\left(\frac{\hat{a} q_f^s}{k}\right)\right)\left(q_f^s + \hat{a}\,\frac{\partial q_f}{\partial a}\right) + U''(q_f^s)\, q_f^s\, \hat{a}\, \frac{\partial q_f}{\partial a} \\ &= -U''(q_f^s)\, q_f^s\, \hat{a}\left(q_f^s - (1-\hat{a})\frac{\partial q_f}{\partial a}\right) = -\hat{a}\,\frac{\left(U''(q_f^s)\, q_f^s\right)^2}{U''(q_f^s) - \frac{1-\hat{a}}{1-k} c'(Q_f^s)} > 0, \end{aligned} \tag{A25}$$

where we use Eq. (7) and (A24). Thus, the DF chooses $a^c > \hat{a}$ when there is no subcontracting and customers are worse off than if there is horizontal subcontracting.

Assume now that the DF offers nonlinear 2PT contracts to consumers. In this case, consumers are worse off if, in addition, there is nonexclusive subcontracting. With 2PT contracts and no subcontracting, the DF chooses the pair $\{a^c, q^c\}$ that solves the problems given in Eqs. (13) and (14), and q_f^c that satisfies Eq. (6). With nonlinear 2PT contracts with customers and nonexclusive subcontracting, the DF chooses the pair $\{q, q_f\}$ that solves the FOCs

$$0 = \frac{\partial \pi_D^{c+s}}{\partial q} = U'(q) - c'\left(\frac{q-(1-k)Q_f}{k}\right) \tag{A25}$$

and

$$0 = \frac{\partial \pi_D^{c+s}}{\partial q_f} = U''(q_f)\left(\left(q_f - (1-k)Q_f\right) + \frac{1-k}{c''(Q_f)}\left(c'\left(\frac{q-(1-k)Q_f}{k}\right) - U'(q_f)\right)\right), \tag{A26}$$

where, from Eq. (17), $\frac{\partial Q_f}{\partial q_f} = \frac{U''(q_f)}{c''(Q_f)}$. Let us denote by q^s, q_f^s and Q_f^s the values of the quantities q, q_f and Q_f that solve Eqs. (1), (A25) and (A26), respectively. With horizontal subcontracting, customers obtain the consumer surplus $CS = U(q_f^s) - U'(q_f^s)q_f^s$. Using (A22), we have $q_f^s - (1-k)\, Q_f^s = \hat{a}\, q_f^s$, and this equality and (A25) allow us to rewrite FOC given in (A26) as

$$U''(q_f^s)\left(\hat{a}\, q_f^s + \frac{1-k}{c''(Q_f^s)}\left(U'(q^s) - U'(q_f^s)\right)\right) = 0\,, \tag{A27}$$

which implies

$$\hat{a}\, q_f^s\, \frac{c''(Q_f^s)}{1-k} = U'(q_f^s) - U'(q^s). \tag{A28}$$

Therefore, using Eq. (7) and (A28), the derivative of the profit stated in Eq. (10) evaluated at $\hat{a}$ becomes

$$\begin{aligned}\left.\frac{\partial \pi_D}{\partial a}\right|_{a=\hat{a}} &= \left(U(q^c) - U'(q^c)\, q^c - \left(U(q_f^s) - U'(q_f^s)\, q_f^s\right)\right) - \\ &\quad - \left(U'(q_f^s) - U'(q^s)\right) q_f^s \frac{U''(q_f^s)}{U''(q_f^s) - \frac{1-\hat{a}}{1-k} c''(Q_f^s)},\end{aligned} \tag{A29}$$

where q^c is the value that solves the problem given in Eq. (13) evaluated at $a = \hat{a}$. We thus have

$$\begin{aligned}\left.\frac{\partial \pi_D}{\partial a}\right|_{a=\hat{a}} &> \left(U(q^c) - U'(q^c)q^c - \left(U(q_f^s) - U'(q_f^s)q_f^s\right)\right) - \left(U'(q_f^s) - U'(q^s)\right) q_f^s \\ &> \left(U(q^c) - U'(q^c)q^c - \left(U(q_f^s) - U'(q_f^s)q_f^s\right)\right) - \left(U'(q_f^s) - U'(q^c)\right) q_f^s \\ &= \left(U(q^c) - U(q_f^s)\right) - U'(q^c)\left(q^c - q_f^s\right) \\ &= \int_{q_f^s}^{q^c} \left(U'(s) - U'(q^c)\right) ds \\ &> 0,\end{aligned} \tag{A30}$$

where the first inequality comes from the fact that $U'(q_f^s) - U'(q^s) > 0$ and $\frac{U''(q_f^s)}{U''(q_f^s) - \frac{1-\hat{a}}{1-k} c''(Q_f^s)} \in (0,1)$. To prove this, first note that $q_f^s < q^s$. Assume otherwise that

$q^s < q_f^s$, in which case $U'' < 0$ implies $U'(q^s) > U'\left(q_f^s\right)$. From Eq. (1) and (A25) we have $c'\left(\frac{q-(1-k)Q_f}{k}\right) > c'\left(Q_f\right)$. We can therefore see that $c'' > 0$ means that $\frac{q-(1-k)Q_f}{k} > Q_f \Rightarrow q > Q_f$; however, in (A20) we must have $q < (1-k)\, Q_f$, which contradicts the above. Therefore, we must have $q_f^s < q^s$ and since $U'' < 0$, we then have $U'\left(q_f^s\right) > U'(q^s)$ as stated. The second inequality, on the other hand, holds from $q^c > q^s$ at $a = \hat{a}$. In fact, we know that $U'(q^c) = c'\left(\frac{\hat{a}q^c}{k}\right)$, whereas from (A19) and (A21), we can see that q^s satisfies $U'(q^s) = c'\left(\frac{q^s-(1-k)Q_f^s}{k}\right) = c'\left(\frac{q^s-(1-\hat{a})q_f^s}{k}\right) = c'\left(\frac{\hat{a}q^s}{k} + X\right)$, where $X = \frac{(1-\hat{a})(q^s-q_f^s)}{k} > 0$. Since q^s is decreasing in X, $\frac{\partial q^s}{\partial X} = \frac{c"}{U"-\frac{a}{k}c"} < 0$, we have $q^c > q^s$ as stated. Finally, the fact that $\left.\frac{\partial \pi_D}{\partial a}\right|_{a=\hat{a}} > 0$ implies that the DF chooses $a^c > \hat{a}$ when there is no horizontal subcontracting with the fringe and that customers are worse off if there is horizontal subcontracting.

That consumers are worse off if subcontracting moves from nonexclusive to exclusive contracts can be proved as follows. If the DF sells the good through nonlinear 2PT contracts and it horizontally sets exclusive contracts with fringe firms, it chooses the quantities q^{es} and q_f^{es} that maximize the problem given in Eq. (13), namely the quantities that solve the FOCs

$$U'(q^{es}) - c'(q^{es}) = 0 \tag{A31}$$

and

$$U''\left(q_f^{es}\right)\left(q_f^{es} - (1-k)Q_f^{es}\right) = 0, \tag{A32}$$

where, in addition, Q_f^{es} satisfies Eq. (1), which can be written as

$$U'\left(q_f^{es}\right) - c'\left(\frac{q_f^{es}}{1-k}\right) = 0 \tag{A33}$$

if we use (A32). If, on the other hand, there is nonexclusive horizontal subcontracting, the DF chooses, according to (A26), $Q_f^s > \frac{q_f^s}{1-k}$ and thus Eq. (1) can be written as

$$U'\left(q_f^s\right) - c'\left(\frac{q_f^s}{1-k} - S\right) = 0 \tag{A34}$$

for $S > 0$. From (A33) and (A34) it follows that $q_f^{es} < q_f^s$ and, therefore, customers end up with a smaller consumer surplus when subcontracting moves from nonexclusive to exclusive contracts.

Assume now that the DF sets nonexclusive contracts with fringe firms, as well as nonlinear 2PT contracts instead of linear prices with its customers. With linear prices, the DF's optimal quantity q_f is the quantity that makes the derivative of the DF's profit with respect to q_f zero, i.e.,

$$U''(q_f)(q_f - (1-k)Q_f) - \left((1-k)\frac{U''(q_f)}{c''(Q_f)} - 1\right)\left(U'(q_f) - c'\left(\frac{q_f-(1-k)Q_f}{k}\right)\right) = 0, \quad \text{(A35)}$$

where, from Eq. (1), $\frac{\partial Q_f}{\partial q_f} = \frac{U''(q_f)}{c''(Q_f)}$. On the other hand, with nonlinear 2PT contracts with customers, the optimal quantities $\{q, q_f, Q_f\}$ simultaneously satisfy Eqs. (1), (A25) and (A26). If we evaluate (A35) at these quantities, it follows that

$$-(1-k)\frac{U''(q_f)}{c''(Q_f)}\left(U'(q) - c'\left(\frac{q_f-(1-k)Q_f}{k}\right)\right) + \left(U'(q_f) - c'\left(\frac{q_f-(1-k)Q_f}{k}\right)\right) > 0, \quad \text{(A36)}$$

since both $U'(q) - c'\left(\frac{q_f-(1-k)Q_f}{k}\right)$ and $U'(q_f) - c'\left(\frac{q_f-(1-k)Q_f}{k}\right)$ are positive. Hence, under linear prices the DF chooses a larger quantity q_f and buyers are better off than under 2PT.

Finally, assume a context in which the DF sets exclusive contracts with fringe firms and is allowed to offer nonlinear 2PT contracts instead of linear pricing to buyers. With linear prices, the FOC of problem given in Eq. (12) is

$$U''(q_f)(q_f - (1-k)Q_f) + U'(q_f) - c'(q_f) = 0, \quad \text{(A37)}$$

where $U'(q_f) - c'(q_f) > 0$ and thus $Q_f < \frac{q_f}{1-k}$. With nonlinear 2PT contracts to sell the good, the FOCs of problem stated in Eq. (13) are given by (A31) and (A32), with $Q_f^{es} = \frac{q_f^{es}}{1-k}$. A similar reasoning to that used in (A33) and (A34) leads the DF to choose a lower quantity q_f with nonlinear 2PT pricing to sell the good, and customers are therefore worse off. ■

Supporting Information

The Mathematica file shows that, for values of k in the interval $[0.698878\,,1]$, in equilibrium, the DF becomes the only seller in the market, i.e., $a^c = 1$, where superscript c indicates that the DF sets a nonlinear 2PT contract with customers, and thus subcontracts all the fringe's production. Note that despite the DF becoming a de facto monopolist in the market, consumers have a strictly positive surplus whenever $k < 1$, although they are worse off than when the DF cannot subcontract production from the fringe.

Appendix S1

Further proofs (pdf file). ■