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Abstract

We investigate a finite-horizon dynamic pricing problem of a seller under limited commitment. Even when the buyers are *ex-ante symmetric* to the seller, the seller can charge *different* prices to different buyers. We show that under the class of posted-price mechanisms this asymmetric treatment of symmetric buyers *strictly revenue-dominates* symmetric treatment. The seller implements this by using a priority-based deterministic tie-breaking rule instead of using a random tie-breaking rule. The effect of asymmetric treatment on revenue increment increases monotonically as we increase the time horizon of the game.

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1 Introduction

Dynamic pricing is a pricing strategy in which firms set flexible prices. These prices are adjusted according to market conditions to achieve specific sales objectives like profit maximization. This is increasingly being used in different revenue-management industries like airlines, hospitality, retail etc. At its core, a dynamic pricing strategy is a price-posting mechanism that involves selling the same product at different prices either to different groups of people or in different time-periods.

In this paper we develop a model of revenue-management pricing for a group of homogeneous buyers. We show that charging different prices to different buyers is indeed a good policy for the firms. Although standard theories of third-degree price discrimination deal with differential pricing, but the underlying assumption in such cases is buyer heterogeneity which serves as the driving force for the monopolist to price discriminate. On the contrary, we show that for revenue-management pricing even if the buyers are *ex-ante* homogeneous to the seller, the seller can charge personalized prices and this will improve the seller's revenue.

We know that for sale of a single item, when the buyers are *ex-ante* symmetric, the revenue maximizing mechanism is a standard Vickrey auction with a reserve price (Myerson, ′81). The underlying philosophy for such a mechanism is '*equal treatment of equals*'. It is only with *ex-ante* heterogeneous buyers that the optimal mechanism is discriminatory. This is true in a general Myerson environment when we can take all possible mechanisms into consideration. But in our case of revenue-management pricing, we are restricted only to the feasible set of price-posting mechanisms. In such cases, we show that the 'equal treatment of equals' rule breaks down. Our result is stark: even if buyers are *ex-ante* symmetric, the seller might choose to treat the symmetric buyers differently as this differential treatment *strictly revenue-dominates* symmetric treatment. Secondly, this asymmetric treatment becomes even stronger in a dynamic environment. As the time-horizon of the game increases, the revenue difference between asymmetric and symmetric treatment increases further.[1]

Revenue-management pricing is basically a series of posted-price offers in a finite horizon setting with more than one buyer. We assume that there is only a single unit of good for sale. This implies that there is an inherent competition among the buyers to acquire the good. In each period the seller sets prices which the buyers can either accept and end the game, or reject in which case the game moves to the next period for possible price revisions. The seller cannot pre-commit to any fixed price paths, so each price has to be *sequentially rational.* The seller has to sell the good within the deadline after which the good perishes.

The standard tie-breaking rule in a symmetric environment is a random tie-breaking rule. We consider

[1] Our model can also be applied to situations where the seller himself prefers to use posted-prices than running the optimal auction, maybe due to the simplicity of running the former. Einav *et al* (2013) uses data from *eBay.com* to show that online sellers are increasingly preferring posted-prices than auctions. Hammond (2010), Hammond (2013) etc. also show similar trends of favoring posted-prices over auctions.

this as our benchmark case and compare it with an asymmetric tie-breaking rule which we propose. In case of asymmetric treatment we change the rule to a deterministc tie-breaking allocation rule. Here the seller pre-specifies a buyer arbitrarily for each set of accepting buyers such that in case of a tie, he would allocate the good *deterministically* to that buyer. This deterministic allocation rule results in an equilibrium where the seller sets buyer-specific prices in each period. In case of two buyers, if an allocation rule is biased in favor of buyer 1, in equilibrium buyer 1 is charged the higher price.

For a single period game, each buyer has a dominant strategy to accept if the price is lower than his value. So the intuition for asymmetric treatment is relatively simple: the seller treats buyer 2, the lower price one, as an outside option. If buyer 1 does not accept the higher price option, then there is always buyer 2 charged with a lower price and thus having higher chance of accepting. But for finite horizon multi-period setting, there are more economic forces in play. If buyer 2 knows that the allocation rule always favors buyer 1 in the future periods in case of tie, this increases his incentive to buy earlier. Also, since the buyers' game is that of strategic complementarity, this in turn increases incentives for buyer 1 to accept an earlier price. Thus the overall competition among the buyers increases. The asymmetric treatment intensifies the competition among the buyers and this increases the expected revenue.

Our contribution is two-fold: First, it shows the importance of asymmetric treatment of symmetric buyers in increasing the revenue of the seller under posted-price mechanisms. Our second important contribution is to show that our revenue-increment result becomes even stronger in a dynamic setting. While the first result can hold even in a one-shot game, the multi-period set-up adds more insights. We show that the asymmetric treatment has even more impact in a dynamic setting. As the time-horizon of the game increases, the revenue difference between the symmetric and asymmetric mechanism increases monotonically. Since the deadline is a commitment from the seller, longer time-horizon implies less commitment from the seller not to decrease the price in future. Thus for a longer time-horizon, buyers are less willing to accept a higher price at an early stage which lowers the competition. In that case the asymmetric treatment becomes more important tool for the seller as it increases the buyer-competition.

For a special case of uniform distribution the equilibrium is unique. If we compare the asymmetric treatment with the symmetric treatment, we find that for asymmetric treatment, one price path is above and the other price path is below the symmetric treatment equilibrium price path. Also, the lower price path is steeper than the rest. This means that as we approach the deadline, the seller tries to keep the fallback option much lower in order to increase the probability of trade.

Apart from the theoretical insights, our result also contributes to the public policy debate on affirmative actions. Sometimes organizations favor a subset of agents relative to others with a distributional or affirmative action objective. He does this solely on the basis of some observed characteristics of the buyers, e.g. race, gender etc. If a differential treatment, or affirmative action, can increase the revenue of the seller, then such policies can have a two-fold justification. Apart from achieving a distributional goal, the revenue increment will mean that the social cost of such a policy is definitely less than the standard belief.

In terms of literature, Kotowski (2018) has a similar result. They restrict the feasible set of mechanisms to first price auctions and show that for single-period case setting different reserve prices to different *ex-ante* symmetric bidders increases the seller's revenue. But their result holds only when the value distributions are non-regular. On the contrary our result is more general in the sense that for posted-price mechanisms even for regular distributions, differential treatment to symmetric buyers increases revenue. Secondly, we establish the importance of dynamic environment in strengthening the effect of asymmetric treatment.

We set up our model following Hörner and Samuelson (2011) which, under the feasible set of posted-prices, derives the revenue-maximizing price path for symmetric buyers in the context of revenue-management industries, like airlines, packaged tours etc. Their mechanism is a symmetric mechanism which is the benchmark case in our paper. We show that such a symmetric mechanism of posting a common price in each period to all buyers acts as a *binding constraint* to the seller. If we relax the constraint, it *strictly increases* the seller's revenue.

The paper also fits into the literature on dynamic mechanism design problems. Although there is a relatively longer literature on dynamic mechanisms *with commitment*, (see Pai and Vohra (2009), Gershkov and Moldovanu (2010), Said (2008, 2009), Gallien (2006), Board and Skrzypacz (2010), Courty and Li (2000), Eso and Szentes (2000), Pavan, Segal and Toikka (2009a), Battaglini (2005) etc.), our paper contributes to the growing literature on dynamic mechanism design *without commitment* where the mechanism designer cannot pre-commit to the mechanisms in the future periods. (For e.g. see Hörner and Samuelson, 2011, Skreta, 2006, Skreta, 2015 etc.). Unlike most of the literature we restrict ourselves to only those situations where posted price is the only feasible mechanism. Thus we take the indirect mechanisms approach in order to portray a specific type of interaction between buyer and seller.

2 The Model

2.1 Anonymous Price Posting Mechanisms

We consider a T-period game where a seller posts take-it-or-leave-it prices to sell an indivisible good. There are 2 buyers. The good is consumed at the end of the T periods. The seller has to sell the good within the T periods, after which the good becomes valueless. We denote the T periods as $\{\Delta, 2\Delta, \ldots, T\Delta \equiv 1)$, where $\frac{1}{\Delta} \equiv T$. Thus the first period is denoted by T and likewise, $t = T-1$ denotes the next period while $t = 1$ is the last period.

The **timeline** for the game is as follows: In each period t, the seller announces a price $p_t \in \mathbb{R}$, and the buyers upon observing the price simultaneously decide whether to accept or to reject the price. If only one of the buyers accepts the price, the game ends and the good is sold to the accepting buyer at price p_t. If both buyers accept, then the good is randomly allocated to one of the accepting buyers at the announced price. If no one accepts the good, the game moves to the next period $t-1$.

Each buyer draws his private valuation v independently and identically from a known distribution $F : [0,1] \to [0,1]$ such that F is strictly increasing and continuously differentiable. A buyer with valuation v who obtains the good at price p derives a payoff of $(v - p)$. The seller having no intrinsic valuation over the good has a payoff equal to the price p at which the good is sold. The payment takes place at the deadline.

We denote this game as Γ^T. A non-trivial **history** $h_t \in H_t$ is the history at period t where the game does not end effectively. The history h_t at period $t \in \{1, \ldots, T\}$, consists of prices till period $(t+1)$: $\{p_T, \ldots, p_{t+1}\}$ that were rejected by the buyers. The history is public history as all the buyers can see the previous prices and acceptance decisions. The set of all possible histories at period t is H_t, and we assume $H_T \equiv \emptyset$. A *behavior strategy* of the seller $\{\sigma_S^t\}_{t=1}^T$ is a sequence of prices p_t which maps from the history to a probability distribution of prices. A behavior strategy of a buyer i, $\{\sigma_i^t\}_{t=1}^T$, is a collection of maps from his type, history of prices, and current price to a probability of acceptance, i.e.,

$$\sigma_i^t : [0, 1] \times H_t \times \mathbb{R} \to \{0, 1\}.$$

The solution concept we adopt is *perfect Bayesian equilibrium*.[2] We assume that the seller does not have any commitment power and each price is *sequentially rational* given the previous history and the belief about the optimal continuation payoff. In anonymous price posting mechanism the seller posts a single price in each period to all buyers and the buyers use symmetric strategies, $\sigma_i^t = \sigma_j^t$ for $i \neq j$. That is buyers of same type base their strategies on the same conditional distribution. The strategy of a buyer depends only on his valuation but not on his identity.

In an anonymous price-posting mechanism, in equilibrium the seller posts a single price to all the buyers in each period. Each individual buyer chooses a particular time period (if any) to accept the corresponding price and end the game. Thus the buyers' problem is an optimal stopping problem. The incentive from waiting one extra period increases for a buyer, the more likely he believes that his opponents will also wait. Consequently, the buyers' game is one with *strategic complementarity* where in general there is a possibility of *multiple equilibria*.[3] We shall show that for *uniform distribution* of buyer valuations, however, there is a *unique* solution to the problem.

Buyers with higher valuations are more eager to accept earlier. If a buyer with valuation v weakly prefers to accept a price p_t in period t, then a buyer with valuation v', where $v < v' \leq 1$, will strictly prefer to accept p_t at period t. Given our focus on symmetric perfect Bayesian equilibrium, the buyers who accept at time period t are those whose valuations exceed a critical threshold v_t. **Lemma** 1 illustrates the seller's posterior beliefs after a history of no sales up to a particular time period.

Lemma 1 *(Hörner and Samuelson (2011)) Fix an equilibrium, and suppose period t has been reached without a price having been accepted. Then the seller's posterior belief is that the buyers' valuations are identically and independently drawn from the distribution $F(v)/F(v_{t+1})$, with support $[0, v_{t+1}]$, for some $v_{t+1} \in (0, 1]$.*

[2]Existence of such an equilibrium in our setting is similar to that in Horner and Samuelson (2011), and follows from standard arguments (see Chen (2012)). We generalize Fudenberg and Tirole definition (Definition 8.2) to our infinte game. The only off-path histories are triggered by the uninformed player, so no additional difficulty arises.

[3]For a particular example of a case where multiple equilibria can arise, see Hörner and Samuelson (2011).

In the last period, *i.e.* $t = 1$, it is a dominant strategy for a buyer to accept the price p_1 if $v > p_1$ and reject if $v < p_1$. In the earlier periods each buyer faces a trade-off whether to accept at the posted price, or to wait till the next period .

Consider an arbitrary time period t and a buyer i with valuation v. Given a critical threshold v_t, buyer i's expected payoff from accepting the price p_t is :

$$\begin{aligned} &\sum_{j=0}^{1} \frac{1}{j+1} \left(1 - \frac{F(v_t)}{F(v_{t+1})}\right)^j \left(\frac{F(v_t)}{F(v_{t+1})}\right)^{1-j} (v - p_t) \\ = \; &\frac{1 - \left(F(v_t)/F(v_{t+1})\right)^2}{1 - F(v_t)/F(v_{t+1})} \frac{(v - p_t)}{2}. \end{aligned} \tag{1}$$

In the above expression $\left(1 - \frac{F(v_t)}{F(v_{t+1})}\right)^j$ is the probability that j other buyers ($j = \{0, 1\}$) have valuations above the threshold v_t and thus accept the price p_t. Under this event, the good will be allocated with probability $\frac{1}{j+1}$.

On the other hand, if buyer i with type v waits for another period to accept price p_{t-1}, his expected payoff is

$$\begin{aligned} &\left(\frac{F(v_t)}{F(v_{t+1})}\right) \sum_{j=0}^{1} \frac{1}{j+1} \left(1 - \frac{F(v_{t-1})}{F(v_t)}\right)^j \left(\frac{F(v_{t-1})}{F(v_t)}\right)^{1-j} (v - p_{t-1}) \\ = \; &\left(\frac{F(v_t)}{F(v_{t+1})}\right) \frac{1 - \left(F(v_{t-1})/F(v_t)\right)^2}{1 - F(v_{t-1})/F(v_t)} \frac{(v - p_{t-1})}{2}. \end{aligned} \tag{2}$$

If this critical threshold v_t is interior, then a v_t-type buyer is indifferent between accepting at price p_t in this period and waiting for the next period to accept p_{t-1}. In other words, we have

$$(IND_A): \frac{1 - \left(F(v_t)/F(v_{t+1})\right)^2}{1 - F(v_t)/F(v_{t+1})} \frac{(v - p_t)}{2} = \left(\frac{F(v_t)}{F(v_{t+1})}\right) \frac{1 - \left(F(v_{t-1})/F(v_t)\right)^2}{1 - F(v_{t-1})/F(v_t)} \frac{(v - p_{t-1})}{2}. \tag{3}$$

The above equation recursively defines a set of thresholds v_t such that for a buyer with valuation v if the optimal time period to accept is t, then $v \in [v_t, v_{t+1})$. The seller's optimization problem is to choose a sequence of prices $\{p_t\}_{t=T}^{1}$ so as to maximize his expected payoff. To solve the problem using backward induction, it is convenient to write the seller's expected payoff in t recursively as follows:

$$\pi_t(v_{t+1}) = \left(1 - \left(\frac{F(v_t)}{F(v_{t+1})}\right)^2\right) p_t + \left(\frac{F(v_t)}{F(v_{t+1})}\right)^2 \pi_{t-1}(v_t),$$

where $\pi_{t-1}(v_t)$ is the continuation expected payoff. The buyers' incentive constraints fix v_t in period t, $\pi_{t-1}(v_{t-1})$, v_{t-1}, and p_{t-1}, are fixed by the continuation payoff. The seller's t^{th} period problem can be stated as follows:

$$Max_{p_t}\pi_t(v_{t+1}) \quad s.t. \quad (IND_A)$$

Since this is a finite horizon problem, we apply backward induction to recursively solve for the seller's optimal sequence of prices. But the problem is complicated by the possibility of multiple equilibria in the continuation game, i.e., multiple sequences of critical thresholds $\{v_t\}$ can be consistent with a sequence of equilibrium prices.

2.1.1 Symmetric Equilibrium

Suppose the buyers' valuations are drawn from $U[0,1]$. We show that with uniform distribution, we can uniquely pin down the equilibrium solution of the problem. The buyers' indifference condition (IND_A) is given by the following equation:

$$\frac{1-\gamma_t^2}{1-\gamma_t}(v_t - p_t) = \gamma_t \frac{1-\gamma_{t-1}^2}{1-\gamma_{t-1}}(v_t - p_{t-1}) \tag{4}$$

where $\gamma_t = \frac{v_t}{v_{t+1}}$. By recursive substitution, and writing $\frac{p_t}{v_t}$ as $\frac{p_t}{v_{t+1}}\frac{1}{\gamma_t}$, the equation can be rewritten as

$$\frac{1-\gamma_t^2}{1-\gamma_t}(1 - \frac{p_t}{v_{t+1}}\frac{1}{\gamma_t}) = \gamma_t(1 - \Pi_{\tau=1}^{t-1}\gamma_\tau^2) \tag{5}$$

The equation pins down γ_t as a function of $\frac{p_t}{v_{t+1}}$ and the solution is unique. The seller's problem is then to maximize his expected payoff subject to the buyers' indifference condition.

$$Max_{p_t}\pi_t(v_{t+1}) = Max_{p_t}[(1-(\frac{v_t}{v_{t+1}})^2)p_t + (\frac{v_t}{v_{t+1}})^2\pi_{t-1}(v_{t-1})].$$

This along with the indifference condition on the buyers gives $\pi_t(v_{t+1})$ as a linear function of v_{t+1} which suggests that the solution is unique.[4] The seller's problem shows that the solution is interior, i.e. there exists some buyer valuations in each period who accept the good.

2.2 Non-Anonymous Price Posting Mechanisms

The key difference between the two mechanism is that a non-anonymous mechanism is *identity dependent*, *i.e.* the seller can identify the different buyers. [5] Apart from that, the buyers are otherwise *ex-ante* symmetric in their valuations. Since the seller treats the buyers differently, the equilibrium notion is an *asymmetric perfect Bayesian equilibrium* where the buyers use asymmetric strategies, $\sigma_i^t \neq \sigma_j^t$, for $i \neq j$. In such an equilibrium, the strategy of a buyer depends not only on the type but also on the buyer's

[4] See Horner and Samuelson (2011) for a detailed description of the equilibrium.

[5] While this is not a particularly strong assumption (i.e., the seller can simply assign each buyer a particular number that will be fixed throughout the T periods), such mechanisms will be typically feasible in settings where the number of buyers is not too large.

identity.

In our mechanism, the differential treatment appears in the *tie-breaking rule*. In the anonymous mechanism the tie is broken *randomly*. In the non-anonymous case the seller can modify this random tie-breaking rule to a *deterministic* tie-breaking rule in order to achieve a higher expected payoff. The rule is as follows:

Deterministic Tie-breaking Rule:

1. First, the two buyers are arranged in any *arbitrary permutation*, and are ranked in an increasing order.

2. Then the tie is broken in favor of that accepting buyer who has the highest pre-assigned rank.

It should be noted that our result will be robust to any change in the permutation. Regarding implementing the rule, the seller simply pre-specifies that in case of a tie, he will allocate the good to one of the buyers, chosen arbitrarily (say buyer 1, for example).[6]

In equilibrium the seller chooses different prices to different buyers, and if there is a tie, he allocates the good to the accepting buyer with the highest price in each period. As a result, in a non-anonymous price posting mechanism, we shall have 2 different price paths, each designed for a particular buyer, instead of a single one as in the case of anonymous buyers.

The **timeline** for the game is as follows: In each period t, the seller announces two (possibly) different prices p_t (for buyer 1) and q_t (for buyer 2) to the two different buyers, $p_t, q_t \in \mathbb{R}$, and the buyers upon observing their corresponding prices simultaneously decide whether to accept or to reject the prices.[7] If only one buyer accepts, the game ends and the good is sold to the accepting buyer at the corresponding price. If both accept, then the good is allocated arbitrarily to buyer 1 (say). If no one accepts the good, the game moves to the next period $t-1$.

We denote this game as G^T. A non-trivial **history** $h_t \in H_t$ at period $t \in \{1, \ldots, T\}$, consists of prices till period $(t+1)$: $\{(p_T, q_T), \ldots, (p_{t+1}, q_{t+1})\}$ that were rejected by the buyers. The history is public as both buyers can see the past prices offered not only to him but also to his rival. The set of all possible histories at period t is H_t, and we assume $H_T \equiv \emptyset$. A *behavior strategy* of the seller $\{\sigma_S^t\}_{t=1}^T$ is a sequence of prices (p_t, q_t) which maps from the history to a probability distribution of prices. A behavior strategy of a buyer i, $\{\sigma_i^t\}_{t=1}^T$, is a collection of maps from his type, history of prices, and current price to a probability of acceptance, i.e.,

$$\sigma_i^t : [0,1] \times H_t \times \mathbb{R} \to \{0,1\}.$$

Suppose the critical valuation thresholds are $u_t \in [0, u_{t+1})$ (buyer 1) and $v_t \in [0, v_{t+1})$ (buyer 2) at time t for the two buyers respectively. In a non-anonymous price posting mechanism, since the allocation rules for the two buyers are different, the indifference conditions that pin down the corresponding threshold

[6] Of course there are many other tie-breaking rules that the seller can adopt and the above tie-breaking rule is not necessarily the revenue-maximizing rule in the entire dynamic game. We shall however restrict the strategy-space of the seller to such an intuitive tie-breaking rule.

[7] Our mechanism does not exclude the case of $p_t = q_t$.

types are different for the two buyers.

In time period t, the incentives for a u_t-type of buyer 1 is given by the indifference condition

$$(IND^1_{NA}) : u_t - p_t = \frac{F(v_t)}{F(v_{t+1})}(u_t - p_{t-1}). \tag{6}$$

Notice that in period t, buyer 1 can get the good with certainty if he accepts the offer. On the other hand, if he rejects the offer, the game goes to the next period $(t-1)$ only in the event that buyer 2 has also rejected his own price offer in period t.

Similarly, in time period t, the incentives for a v_t-type of buyer 2 is given by the indifference condition

$$\begin{aligned}(IND^2_{NA}) \quad &: \quad \frac{F(u_t)}{F(u_{t+1})}(v_t - q_t) = \frac{F(u_t)}{F(u_{t+1})}\frac{F(u_{t-1})}{F(u_t)}(v_t - q_{t-1}) \\ &\Rightarrow \quad v_t - q_t = \frac{F(u_{t-1})}{F(u_t)}(v_t - q_{t-1}).\end{aligned} \tag{7}$$

The seller's period-t payoff is as follows:

$$\pi_t(u_t, v_t) = \\ \left(1 - \frac{F(u_t)}{F(u_{t+1})}\right) p_t + \frac{F(u_t)}{F(u_{t+1})}\left(1 - \frac{F(v_t)}{F(v_{t+1})}\right) q_t + \frac{F(v_t)}{F(v_{t+1})}\frac{F(u_t)}{F(u_{t+1})}\pi_{t-1}(v_{t-1})$$

The seller's optimization problem in period t is to choose p_t and q_t to maximize $\pi_t(u_t, v_t)$ given the continuation payoff $\pi_{t-1}(v_{t-1})$, and subject to the buyers' indifference conditions.

$$\max_{p_t, q_t} \pi_t(u_t, v_t) \ s.t. \ (IND^1_{NA}) \text{ and } (IND^2_{NA}).$$

In the next subsection we characterize the asymmetric equilibrium.

2.2.1 Asymmetric Equilibrium

We assume the distribution of buyers' valuation to be $U[0,1]$. Buyer 2's indifference condition gives

$$\begin{aligned} v_t - q_t &= \frac{u_{t-1}}{u_t}(v_t - q_{t-1}) \\ &= \beta_{t-1}(1-\gamma_t)v_t + \beta_t(v_{t-1} - q_{t-1}), \text{ where } \beta_t = \frac{u_t}{u_{t+1}} \text{ and } \gamma_t = \frac{v_t}{v_{t+1}}.\end{aligned} \tag{8}$$

Proceeding recursively, and writing $v_t - q_t$ as $v_t(1 - \frac{q_t}{v_{t+1}}\frac{1}{\gamma_t})$, we can write the indifference condition as

$$(1 - \frac{q_t}{v_{t+1}}\frac{1}{\gamma_t}) = \beta_{t-1} + \sum_{\tau=1}^{t} \beta_{t-\tau}\Pi^t_{\tau=1}\gamma^2_{t-\tau} - \sum_{\tau=2}^{t}\beta_{t-\tau}\gamma_{t-\tau}\Pi^t_{\tau=2}\gamma^2_{t-\tau+1} - \beta_{t-1}\gamma_{t-1}. \tag{9}$$

The left hand side of the equation is monotonic in γ_t while the right hand side is independent of γ_t. Thus the equation can pin down γ_t as a function of $\frac{q_t}{v_{t+1}}$. Thus in the continuation game with t periods to go, given the price offered by the monopolist, there can be only one threshold type of Buyer 2 who is indifferent between accepting the price and waiting for the next period.

Similarly we can write down buyer 1's indifference condition and substitute recursively as

$$u_t - p_t = \frac{v_t}{v_{t+1}}(u_t - p_{t-1})$$

Similarly writing $u_t - p_t$ as $u_t(1 - \frac{p_t}{u_{t+1}}\frac{1}{\beta_t})$, the above equation can be rewritten as

$$\frac{1}{\gamma_t}(1 - \frac{p_t}{u_{t+1}}\frac{1}{\beta_t}) = \sum_{\tau=1}^{t} \Pi_{\tau=1}^{t}\gamma_{t-\tau}\beta_{t-\tau} - \sum_{\tau=1}^{t} \Pi_{\tau=1}^{t}\beta_{t-\tau}\gamma_{t-\tau+1}\gamma_{t-\tau} - \beta_{t-1}. \quad (10)$$

The left side of the equation is a function of β_t and γ_t, and since γ_t is pinned down from buyer 2's indifferent condition, thus the left side becomes monotonic in only β_t, while the right hand side is independent of it, thus pinning down β_t.Thus we can claim that the equilibrium of the monopolist's problem is unique.

The seller's problem is to maximize his expected payoff

$$\pi_t(u_{t+1}, v_{t+1}) = (1 - \frac{u_t}{u_{t+1}})p_t + \frac{u_t}{u_{t+1}}(1 - \frac{v_t}{v_{t+1}})q_t + \frac{u_t}{u_{t+1}}\frac{v_t}{v_{t+1}}\pi_{t-1}(u_t, v_t).$$

This along with the indifference conditions of the buyers gives $\pi_t(u_{t+1}, v_{t+1})$ as a linear function of u_{t+1} and v_{t+1}. This again suggests that the solution is unique. This can be stated formally in the following Lemma:

Lemma 2 *In the continuation game with t periods remaining, the prices for the two buyers p_t and q_t, and the seller's payoff function are linear functions of u_{t+1} and v_{t+1} for every t.*

The first order conditions from the seller's maximization problem characterize the price path of the monopolist in any general t^{th} period, and the second order condition shows that the solution is interior. The interior solution implies that in each period there exist some buyer valuations that do accept the prices in that period. The corresponding t^{th} period first order conditions that define the price paths are

$$\beta_t : -2(1-\beta_t)\beta_t[\Sigma_{\tau=1}^{t-1}(1-\beta_\tau)\Pi_{l=\tau+1}^{t-1}\beta_l^2]u_{t+1} - \gamma_t(1-\gamma_t)\Sigma_{\tau=1}^{t-1}(1-\gamma_\tau)\Pi_{l=\tau+1}^{t-1}\gamma_l^2\gamma_\tau v_{t+1}$$

$$+ [\Sigma_{\tau=1}^{t-1}(1-\beta_\tau)\Pi_{l=\tau+1}^{t-1}\beta_l^2]u_{t+1} + (1-2\beta_t)u_{t+1} + (1-\gamma_\tau)\gamma_t v_{t+1} + \gamma_t\pi_{t-1} = 0 \quad (11)$$

and

$$\gamma_t : -\beta_t(1-2\gamma_t)[\Sigma_{\tau=1}^{t-1}(1-\gamma_\tau)\Pi_{l=\tau+1}^{t-1}\gamma_l^2\gamma_\tau v_{t+1}] + \beta_t(1-2\gamma_t)v_{t+1} + \beta_t\pi_{t-1} = 0 \quad (12)$$

The seller thus sets prices in each period such that the corresponding cut-off types of buyers are indifferent between accepting the price and waiting for the next period. The buyers on the other hand follow the strategy in any period to accept the price if their valuations (or types) are strictly greater than the respective cutoff valuations in that period, otherwise they wait for the next period. This gives the unique perfect Bayesian equilibrium of the continuation game. The exact closed form solution and its derivation are given in **Proposition** $A.1$ in the **Appendix**. The equilibrium threshold valuations can be ranked among themselves. In each period, the threshold valuation u_t for buyer 1 is higher than v_t, the threshold valuation for buyer 2. The following Proposition states this:

Proposition 1 *In each period t, the equilibrium threshold valuations of the two buyers, u_t and v_t, can be ranked in the following way:*

$$u_t > v_t, \ \forall t.$$

Proof. Detailed Proof is in the Appendix. ■

2.3 Examples with $T = 1$ and $T = 2$

For the one period model, Table 1 shows the prices and revenues under both the mechanisms.

	Anonymous Mechanism	Non-Anonymous Mechanism
Price	0.57	$0.625\ (p^1), 0.5\ (p^2)$
Expected revenue	0.38	0.39

Table 1: Comparison of prices and revenues between the Anonymous and Non-anonymous cases for $T = 1$

Thus the seller's revenue increases by 0.01 under the non-anonymous mechanism.

Thus the non-anonymous mechanism performs better *even in the static case* (without any effect of dynamics).The intuition is that the seller wants to diversify his portfolio of prices. the seller offers a higher price to buyer 1 with lower probability of acceptance. In case buyer 1 does not accept, he has a fall back option on buyer 2, charged with a lower price with a higher probability of acceptance. This portfolio of prices is better for the seller than posting a single price.

For the $T = 2$, the expected revenues are the following: $\pi_2^A = 0.404 > \pi_2^N = 0.4$. Table 2 compares the performance of the optimal anonymous mechanism with the optimal non-anonymous mechanism along with the prices posted in two periods.

	Anonymous Mechanism	Non-Anonymous Mechanism
Price in period 2	$0.58\ (p_2)$	$0.62\ (p_2^1), 0.56\ (p_2^2)$
Price in period 1	$0.48\ (p_1)$	$0.55\ (p_1^1), 0.40\ (p_1^2)$
Expected revenue	0.4	0.404

Table 2: Comparison of prices and revenues between the Anonymous and Non-anonymous cases for $T = 2$

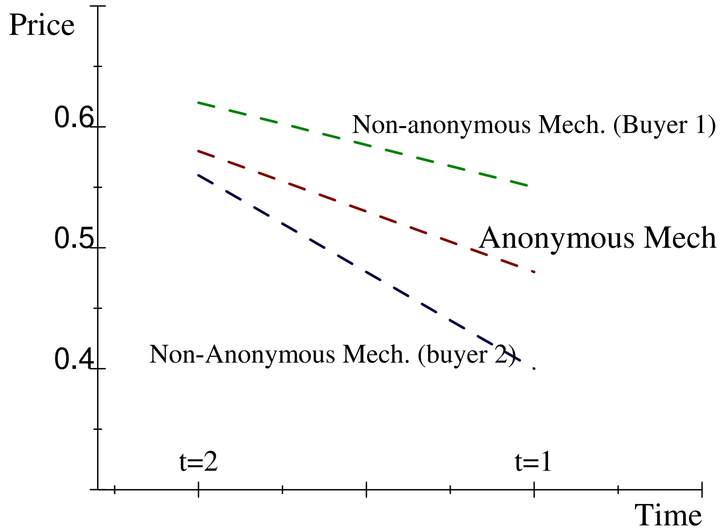

Figure 1: Single Price Path for Anonymous Mechanism and Two Price Paths for Non-anonymous Mechanism for buyers 1 and 2 respectively

Denote p_t as the optimal price in period t under the anonymous mechanism, $t \in \{1, 2\}$ and denote p_t^i as the optimal price in period t for buyer $i \in \{1, 2\}$, $t \in \{1, 2\}$ under the non-anonymous mechanism. A first observation from the table is that the two prices in each period under the non-anonymous mechanism are a "spread" from the corresponding price under the anonymous mechanism, i.e., $p_t^1 > p_t > p_t^2$ for each t. Hence buyer 1 is charged with a price higher than the anonymous mechanism price while buyer2 is charged a price lower than the anonymous mechanism price.

A second useful observation is that $|p_t^i - p_t|$ is decreasing in t for each $t = 1, 2$ and $i = 1, 2$. In other words, in the earlier period the spread of the prices is less than that in the final period. The line of the price path for buyer 2 in the non-anonymous mechanism is steeper than that of buyer 1 in the non-anonymous mechanism, while the slope of the line of the price path for the anonymous mechanism lies in the mid-way. In the final period, the seller tends to diversify even more to make it more likely for at least one of the buyers accepts the good in the final period. In addition, we can see that the price difference (between the two mechanisms) for buyer 1 is relatively higher in the first period than that of buyer 2, i.e., $|p_2^1 - p_2| > |p_2^2 - p_2|$, while in the final period the price difference for buyer 2 is higher, i.e. $|p_1^1 - p_1| < |p_1^2 - p_1|$.

Finally, we can see that the revenue difference between the two mechanisms is higher (0.04) for $T = 2$ than it is for $T = 1$ (0.01). Thus as we have increased the time-horizon of the game, the revenue difference has increased. We will formalize the observation in the next section.

2.4 Revenue

This section discusses our primary result of higher revenue that a non-anonymous mechanism generates. We argue through a local perturbation method. Suppose we perturb the tie-breaking allocation rule from $(\frac{1}{2}, \frac{1}{2})$ under anonymous mechanism to some discriminatory allocation rule. We know that under $(\frac{1}{2}, \frac{1}{2})$

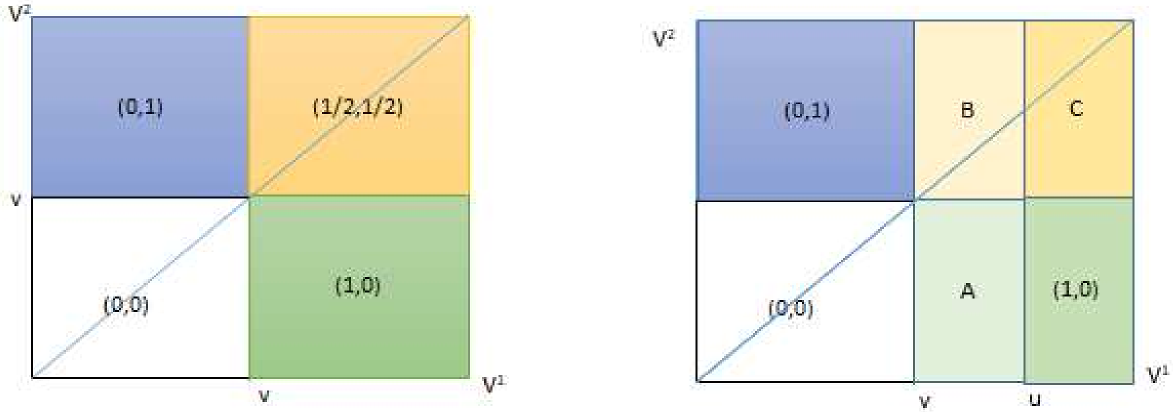

Figure 1: Figure 2 : Allocation Rules for Anonymous and Non-anonymous Mechanisms

allocation rule, buyers are treated symmetrically, and in equilibrium the cutoff valuation for both players are the same in each period. Hence the prices are also the same for both. But under discriminatory allocation rule under non-anonymous mechanism, buyers are treated differently. The cutoff valuations are different for the two buyers in each period. In Section 2.4.1 we first prove the results for a special case when the buyer valuations follow uniform distribution. This is for expositional simplicity and in no way our results are restricted only to uniform distribution. In Section 2.4.2 we generalize our result to a general class of distributions. We show that the revenue increment result still holds with some sufficiency conditions on the distribution types.

2.4.1 Uniform Distribution

Here we will start with the static case and illustrate the argument for revenue increment. Then we will extend the argument to the dynamic case. Let v be the equilibrium cutoff valuation (and also price) under the anonymous mechanism for both the buyers. Let us assume a general tie-breaking allocation rule that is perturbed in favor of buyer 1. Due to some such perturbation, the equilibrium cutoff valuation for buyer 1 is changed to u, where $u = v + k$, $k > 0$. Figure 2 shows the different allocation rules in different regions under this perturbation.

Following Figure 2, the regions A, B and C are the regions where the allocation rule changes. In Region A, the perturbation implies no-trade compared to the anonymous case where buyer 1 was allocated the good. Thus in Region A, the non-anonymous mechanism decreases revenue for the seller. In Regions B and C, for the anonymous mechanism, each buyer is allocated the good with probability $\frac{1}{2}$. Under perturbation, the allocation probability shifts in favor of buyers 2 and 1 respectively for Regions B and C.

The potential gain in revenue by shifting the allocation probabilities to either buyer in Regions B and C should dominate the certain revenue loss under Region A. Thus the non-anonymous mechanism generates higher revenue if the following condition holds:

$$((1-(v+k))+v)(1-v)(v+k) > 2(1-v)^2v+(1-v)v^2 \tag{13}$$

We can conclude that there exists $\underline{k}, \overline{k} > 0$, such that for all perturbations of allocation rule which results in difference in cutoff valuations $k \in (\underline{k}, \overline{k})$, the expected revenue of the non-anonymous mechanism dominates that of the anonymous mechanism. Here there is a trade-off with two opposing forces working together. A higher cutoff for buyer 1 implies a higher equilibrium price charged by the seller in case he sells to buyer 1, but there is also a higher probability that the good remains unsold. The idea is that the positive effect should outweigh the negative one. If k is too high, the price the seller gets is greater, but the negative impact of no-trade probability outweighs the positive impact. If k is too low, the no-trade probability is of course low, but it is still higher than that in the anonymous mechanism. Thus the lower positive impact it gets with lower increase in price (as the perturbation k is low) cannot outweigh the negative impact. This rationalizes the range of values for perturbation amount.

The logic can be applied to the $T-$period game as well. In each period t, we perturb the allocation rule such that the equilibrium cut-off value of buyer 1 is increased by an amount $k_t > 0$. Of course, for a dynamic game, the trade-off is more complicated. We show that *for each continuation game*, under discriminatory allocation rule, the expected revenue of the seller at each continuation game strictly increases with perturbation. This is formalized in the following Proposition.

Proposition 2 *Let v_t be the equilibrium cutoff valuation in period t under anonymous mechanism. Under non-anonymous mechanism, the tie-breaking allocation rule is perturbed in favor of buyer* 1. *The non-anonymous mechanism generates strictly higher revenue than the anonymous mechanism and in equilibrium the cutoff valuation for buyer 1 in period t is $u_t = v_t + k_t$ where $k_t \in (\underline{k_t}, \overline{k_t})$ for $\overline{k_t}$, $\underline{k_t} > 0$ for each t.*

Proof. Detailed Proof is in the Appendix. ■

Thus the revenue-dominance result of the static model ($T = 1$) can be translated qualitatively to any T. But surprisingly, the magnitude of this revenue difference is not the same for any T and most importantly there is a monotonic relationship between the revenue difference and the time-horizon of the game. The following Proposition shows that as we increase the time-horizon of the game the revenue increment from non-anonymous mechanism also increases. As T increases, the deadline becomes less stringent. So the commitment from the seller not to decrease future prices is lost to an extent as T increases. This in turn decreases the force of competition among the buyers. The buyers are less willing to accept a higher earlier price and want to wait for future. The symmetric treatment thus loses some value. Herein lies the importance of the asymmetric treatment in the non-anonymous mechanism. The asymmetric treatment intensifies the buyer-competition. The force works in opposite direction to that of increase in T. Thus with higher T, non-anonymous mechanism has more bite.

Proposition 3 *Let v_t be the equilibrium cutoff valuation for the anonymous mechanism in period t. Under non-anonymous mechanism, the equilibrium cutoff valuation for buyer 1 is $u_t = v_t + k_t$, for $k_t \in (\underline{k_t}, \overline{k_t})$ for $\overline{k_t}$, $\underline{k_t} > 0$ for each t. Let Π_T^A and Π_T^{NA} are the expected revenues under the anonymous and non-anonymous mechanisms respectively. Then the difference in revenue in the two mechanisms, $D_T = \Pi_T^{NA} - \Pi_T^A$ increases as T increases.*

Proof. Detailed Proof is in the Appendix. ■

2.4.2 Non-Uniform Distributions

We have shown how the non-anonymous mechanism increases revenue for the seller for an uniform distribution. For non-uniform distributions, we will shed light on some distribution properties for generation of higher revenues. We will also restrict ourselves to the static case comparison.

Monotone hazard rate (MHR) condition implies that for a random variable x following a probability distribution $F(x)$, the following term

$$Q(x) = \frac{1 - F(x)}{f(x)}.$$

is monotone in x. If $Q(x)$ is non-decreasing in x, we call it **decreasing hazard rate (DHR)** condition. If $Q(x)$ is non-increasing in x, we call it **increasing hazard rate (IHR)** condition.[8]

Let v be the equilibrium cutoff value for the anonymous mechanism. We perturb the tie-breaking allocation rule so that the cutoff value for buyer 1 is increased by an amount k. Referring to the the allocation rules in Figure 2, the following table shows the revenues of the seller in different regions.

Region	Anonymous Mechanism	Non-Anonymous Mechanism
A	$(1, 0)$; $Q(v)F(v)f(v)v$	$(0, 0)$; 0
B	$(\frac{1}{2}, \frac{1}{2})$; $Q(v)(1 - F(v))f(v)v$	$(0, 1)$; $Q(v)f(v)v(F(u) + F(v) - 1)$
C	$(\frac{1}{2}, \frac{1}{2})$; $Q(v)(1 - F(v))f(v)v$	$(1, 0)$; $Q(u)Q(v)(1 - F(v))f(v)f(u)u$

Table 2: Allocation Rules and Seller's Revenue in Different Regions

The non-anonymous mechanism generates a higher revenue than the anonymous mechanism if the following condition holds:

$$I = Q(v)f(v)v(Q(u)f(u)u - 2Q(v)f(v) + F(u) - \frac{F(v)}{f(v)}) > 0. \tag{14}$$

We can verify that $Q(v) > 0$, and $u = v + k, k > 0$. If $F(u) > \frac{F(v)}{f(v)}$ and $Q(u)$ is very high compared to $Q(v)$, I can be positive. This means if the distribution of valuations follows a sharp **DHR** condition, it is

[8] The inverse of $Q(x)$ is defined as the Hazard Rate.

possible to generate higher revenue by a sufficient level of perturbation k. One example of a distribution with DHR condition is a Pareto distribution.

For a distribution that follows **IHR** condition, $Q(u) < Q(v)$. For such distributions, for the condition $I > 0$ to hold, the gap between u and v should be sufficiently high. Thus for a high enough perturbation k, the non-anonymous mechanism can generate higher revenue. However, there should also be an upper bound on k such that $Q(u)$ does not become too high compared to $Q(v)$. Thus for a given closed and bounded set of perturbation values k, the non-anonymous mechanism generates higher revenue when the distribution follows the IHR condition. Earlier we have already shown that for Uniform distribution, which follows IHR condition, the non-anonymous mechanism creates higher revenue for the seller. This is depicted in the following Proposition.

Proposition 4 *Let v_t be the equilibrium cutoff valuation that maximizes revenue under anonymous mechanism. Under non-anonymous mechanism favoring buyer 1, the cutoff valuation for buyer 1 is $u = v + k$.*

*i) When $F(.)$ follows **DHR** condition: Let $Q(x) = \frac{1-F(x)}{f(x)}$.* Let $|Q(u) - Q(v)| > K > 0$. *Then the non-anonymous mechanism generates strictly higher revenue than the anonymous mechanism and in equilibrium $k > \widetilde{k} > 0$.*

*ii) When $F(.)$ follows **IHR** condition: The non-anonymous mechanism generates strictly higher revenue than the anonymous mechanism and in equilibrium $k \in (\overline{k}, \underline{k})$,* where $\overline{k}, \underline{k} > 0$.

Proof. Detailed Proof is in the Appendix. ■

Similar intuitions can be derived for dynamic games as well. If the time-horizon of the game is T periods, then for each continuation game starting at period t, there exists an optimal range of perturbations such that for any perturbations within this range, the non-anonymous mechanism generates strictly higher revenue. Different optimal ranges correspond to whether $F(.)$ follows DHR or IHR condition.

Proposition 5 *Let v be the equilibrium cutoff valuation under anonymous mechanism in period t. Under non-anonymous mechanism favoring buyer 1, the cutoff valuation for buyer 1 is $u_t = v_t + k_t$.*

*i) When $F(.)$ follows **DHR** condition: Let $Q(x) = \frac{1-F(x)}{f(x)}$.* Let $|Q(u) - Q(v)| > K > 0$. *Then the non-anonymous mechanism generates strictly higher revenue than the anonymous mechanism and in equilibrium $k_t > \widetilde{k_t} > 0$.*

*ii) When $F(.)$ follows **IHR** condition: The non-anonymous mechanism generates strictly higher revenue than the anonymous mechanism and in equilibrium $k_t \in (\overline{k_t}, \underline{k_t})$,* where $\overline{k_t}, \underline{k_t} > 0$.

While Proposition 5 states that the asymmetric mechanism revenue-dominates the symmetric mechanism, Proposition 6 mentions the result that with increase in the time-horizon, the revenue increment due to asymmetric treatment is even higher.

Proposition 6 *From Proposition* 5, *for cases (i) and (ii), the respective sufficiency conditions hold. Let v_t be the equilibrium cutoff valuation for the anonymous mechanism in period t. Suppose under non-anonymous mechanism, the equilibrium cutoff valuation for buyer* 1 *is $u_t = v_t + k_t$, for $k_t \in (\underline{k_t}, \overline{k_t})$ for*

$\overline{k_t}$, $\underline{k_t} > 0$ *for each* t. *Then the difference in expected revenues in the two mechanisms,* $D_T = \Pi_T^{NA} - \Pi_T^A$ *increases as* T *increases.*

3 Conclusion

Under a non-anonymous mechanism, the seller can asymmetrically treat symmetric buyers and this revenue-dominates the symmetric mechanism. The result holds true for a one-shot game as well as in the dynamic version of the game under non-commitment. More importantly, as the time-horizon increases, the revenue-difference between the non-anonymous and anonymous mechanisms increases monotonically.

It would be interesting to identify the 'optimal' or the revenue-maximizing tie-breaking rule under the current feasible set of posted-price mechanism, when we relax the constraint of 'treating equals equally'. Of course our deterministic tie-breaking rule is also a constraint to the seller very much like the random tie-breaking rule. In complete absence of constraints, we can get the mechanism that maximizes revenue even for more than two buyers. This should be an interesting extension for a future research.

Appendix

Proposition A.1: *When the buyers are non-anonymous, at any period t, if the seller's posterior beliefs are such that the buyers' valuations are drawn uniformly from $[0, u_{t+1}]$ and $[0, v_{t+1}]$ respectively, then in the unique perfect Bayesian equilibrium, the t^{th} period prices are given by*

$$p_t = u_t - \gamma_t \sum_{\tau=1}^{t-1} (1-\beta_\tau)(\Pi_{l=\tau}^{t-1} \gamma_l) u_{\tau+1}$$

and

$$q_t = v_t - \sum_{\tau=1}^{t-1} (1-\gamma_\tau)(\Pi_{l=\tau}^{t-1} \beta_l) v_{\tau+1},$$

and given prices $\widetilde{p}_t$ and $\widetilde{q}_t$, buyers 1 and 2 with their respective valuations $u > u_t(\widetilde{p}_t, u_{t+1})$ and $v > v_t(\widetilde{p}_t, v_{t+1})$, the threshold types at time period t, accept their prices, and buyers 1 and 2 with their respective valuations $u < u_t(\widetilde{p}_t, u_{t+1})$ and $v < v_t(\widetilde{p}_t, v_{t+1})$ reject the prices, where $u_t(\widetilde{p}_t, u_{t+1})$ and $v_t(\widetilde{p}_t, v_{t+1})$ are given by

$$(1 - \frac{\widetilde{q}_t}{v}) = \beta_{t-1} + \sum_{\tau=1}^{t} \beta_{t-\tau} \Pi_{\tau=1}^{t} \gamma_{t-\tau}^2 - \sum_{\tau=2}^{t} \beta_{t-\tau}\gamma_{t-\tau} \Pi_{\tau=2}^{t} \gamma_{t-\tau+1}^2 - \beta_{t-1}\gamma_{t-1}.$$

and

$$\frac{1}{\gamma_t}(1 - \frac{\widetilde{p}_t}{u}) = \sum_{\tau=1}^{t} \Pi_{\tau=1}^{t} \gamma_{t-\tau}\beta_{t-\tau} - \sum_{\tau=1}^{t} \Pi_{\tau=1}^{t} \beta_{t-\tau}\gamma_{t-\tau+1}\gamma_{t-\tau} - \beta_{t-1}.$$

Proof: We have assumed without loss of generality that the two buyers face t^{th} period prices p_t and q_t respectively with $p_t \geq q_t$. We start from the last period, *i.e.* $t = 1$. In the last period, the buyers accept the price if and only if their valuations are at least the prices they face in that period *i.e.* $u_1 \geq p_1$ and $v_1 \geq q_1$ respectively for buyers 1 and 2. The seller updates his posterior belief that the buyers' valuations are drawn from uniform distributions in the range $[0, u_2]$ and $[0, v_2]$ respectively.

The seller sets $u_1 = p_1$ and $v_1 = q_1$. The objective function of the seller is:

$$\begin{aligned} & (1 - \frac{u_1}{u_2})u_1 + \frac{u_1}{u_2}(1 - \frac{v_1}{v_2})v_1 \\ = \;& (1-\beta_1)\beta_1 u_2 + \beta_1(1-\gamma_1)\gamma_1 v_2 \end{aligned}$$

where $\beta_1 = \frac{u_1}{u_2}$ and $\gamma_1 = \frac{v_1}{v_2}$.

From the first order conditions we get:

$$\beta_1 \; : \; (1-2\beta_1)u_2 + ((1-\gamma_1)\gamma_1)v_2 = 0 \tag{15}$$

$$\gamma_1 \; : \; (1-2\gamma_1)\beta_1 v_2 = 0 \tag{16}$$

Solving the first order conditions,

$$\begin{aligned} u_1 &= \frac{4u_2 + v_2}{8} \\ v_1 &= \frac{v_2}{2} \end{aligned}$$

As we can see, in the last period, u_1 and v_1 can be expressed as linear functions of u_2 and v_2. The value of the problem is

$$\pi_1 = \mu_1 u_2 + \upsilon_1 v_2$$

where $\mu_1 = (1 - \beta_1)\beta_1$ and $\upsilon_1 = \frac{\beta_1}{4}$. Thus in the last period the solution is linear in u_2 and v_2.

Now we use the logic of induction on the number of time periods to show that the solution is unique for any general t^{th} period problem. Let us first fix t and assume that for all periods up to $t-1$, the solution is unique and is characterized by μ_{t-1}, β_{t-1} and γ_{t-1}. Now let us consider the t^{th} period problem where the posterior beliefs are that the valuations of the two buyers are drawn from uniform distributions in $[0, u_{t+1}]$ and $[0, v_{t+1}]$ respectively.

The indifference conditions of the two buyers in the t^{th} period are:

$$u_t - p_t = \frac{v_t}{v_{t+1}}(u_t - p_{t-1}) \tag{17}$$

and

$$v_t - q_t = \frac{u_{t-1}}{u_t}(v_t - q_{t-1}). \tag{18}$$

Writing $\frac{v_t}{v_{t+1}} = \gamma_t$ and $\frac{u_t}{u_{t+1}} = \beta_t$, we can write for buyer 1,

$$\begin{aligned} u_t - p_t &= \gamma_t(u_t - p_{t-1}) \\ &= \gamma_t(u_t - u_{t-1}) + \gamma_t(u_{t-1} - p_{t-1}) \\ &= \gamma_t(1 - \beta_{t-1})u_t + \gamma_t(\gamma_{t-1}(1 - \beta_{t-2})u_{t-1} + \gamma_{t-1}(u_{t-2} - p_{t-2})) \\ &= \sum_{\tau=1}^{t-1}(1 - \beta_\tau)(\Pi_{l=\tau+1}^{t}\gamma_l)u_{\tau+1}. \end{aligned} \tag{19}$$

Similarly, for buyer 2,

$$\begin{aligned} v_t - q_t &= \gamma_{t-1}(v_t - q_{t-1}) \\ &= \gamma_{t-1}(v_t - v_{t-1}) + \gamma_{t-1}(v_{t-1} - q_{t-1}) \\ &= \gamma_{t-1}(1 - \gamma_{t-1})v_t + \gamma_{t-1}(\gamma_{t-2}(1 - \gamma_{t-2})v_{t-1} + \gamma_{t-2}(v_{t-2} - q_{t-2})) \\ &= \sum_{\tau=1}^{t-1}(1 - \gamma_\tau)(\Pi_{l=\tau}^{t-1}\gamma_l)v_{\tau+1}. \end{aligned} \tag{20}$$

Now again let us consider buyer 1. Buyer 1's indifference condition can also be written as:

$$(1 - \frac{p_t}{u_{t+1}}\frac{1}{\beta_t}) = (\sum_{\tau=1}^{t-1} \Pi_{l=t-1}^{t-\tau+1}\gamma_l^2(1-\beta_{t-\tau}) \tag{21}$$

We can write a similar expression for buyer 2 as well. Thus we have characterized the buyers' behavior completely and uniquely. Given the sequences $\{\beta_t\}_{t=t-1}^{T}$ and $\{\gamma_t\}_{t=t-1}^{T}$ in each period t, we can pin down β_t and γ_t uniquely as functions of $\frac{p_t}{u_{t+1}}$ and $\frac{p_t}{v_{t+1}}$.

In the above equation, the left hand side is monotonic in β_t while the right hand side is independent of it. Thus β_t can be pinned down uniquely given u_{t+1} and the values in the continuation game.

Next we come to the seller's problem. The seller's expected payoff is:

$$\pi_t(u_{t+1}, v_{t+1}) = (1 - \frac{u_t}{u_{t+1}})p_t + \frac{u_t}{u_{t+1}}(1 - \frac{v_t}{v_{t+1}})q_t + \frac{u_t}{u_{t+1}}\frac{v_t}{v_{t+1}}\pi_{t-1}(u_t, v_t).$$

The seller maximizes the objective function subject to the indifference conditions of the buyers.

The first order conditions from the seller's maximization problem characterize the price path of the monopolist in the t^{th} period.

$$\beta_t : -2(1-\beta_t)\beta_t[\Sigma_{\tau=1}^{t-1}(1-\beta_\tau)\Pi_{l=\tau+1}^{t-1}\beta_l^2]u_{t+1} - \gamma_t(1-\gamma_t)\Sigma_{\tau=1}^{t-1}(1-\gamma_\tau)\Pi_{l=\tau+1}^{t-1}\gamma_l^2\gamma_\tau v_{t+1} \tag{22}$$

$$+ [\Sigma_{\tau=1}^{t-1}(1-\beta_\tau)\Pi_{l=\tau+1}^{t-1}\beta_l^2]u_{t+1} + (1-2\beta_t)u_{t+1} + (1-\gamma_\tau)\gamma_t v_{t+1} + \gamma_t\pi_{t-1} = 0$$

and

$$\gamma_t : -\beta_t(1-2\gamma_t)[\Sigma_{\tau=1}^{t-1}(1-\gamma_\tau)\Pi_{l=\tau+1}^{t-1}\gamma_l^2\gamma_\tau v_{t+1}] + \beta_t(1-2\gamma_t)v_{t+1} + \beta_t\pi_{t-1} = 0 \tag{23}$$

From the second order condition it can be shown that the solution is also interior. Thus the solution to the t^{th} period problem is unique and interior given the continuation game.

Proof of Proposition 1

Suppose $n = 2$. Under anonymous mechanism, suppose a buyer with valuation v weakly prefers to accept at a price p in period t. Then the following inequality must hold:

$$(q(1)\frac{1}{2} + q(0))(v-p) \geq q(0)\sum_{p_l \in P/p_t} \rho(p_l)(v-p_l)$$

$q(k)$ is the probability that k other buyers accept at the price p. P is the finite set of prices the seller sets in equilibrium. $\rho(p_l)$ is the probability that the buyer will accept such a price. Let $\rho^A(p_t)$ $= q(1)\frac{1}{2} + q(0)$. Under non-anonymous mechanism, for buyer 1, $\rho^{NA}(p_t) = q(1) + q(0)$, which is greater than $\rho^A(p_t)$. Thus the derivative with respect to $\rho(p_t)$ on both sides of the inequality maintains the weak inequality. Also, if $q(0) < 1$ or $\rho(p_l)$ attaches a non-unitary probability to prices equal to p, the inequality becomes strict. In the same logic, the inequality reverses for buyer 2.

Thus under non-anonymous mechanism, buyer 1 has lower incentive to accept earlier while buyer 2 has higher incentive to accept earlier. Thus, $u_t > v_t$.

Proof of Proposition 2

Let v_t be the equilibrium cutoff valuation for each buyer under the anonymous mechanism. Suppose that the cutoff valuation for buyer 1 in each period t is higher by an amount k_t. We apply backward induction and start from the last period. Let us first assume that only the last period cutoff valuation for buyer 1 is higher by an amount k_1. In the last period, the perturbed cutoff value for buyer 1 is

$$u_1 = v_1 + k_1.$$

Therefore if $\kappa_1 = \frac{k_1}{v_2}$ and $\alpha_2 = \frac{v_2}{u_2}$, then

$$\beta_1 = (\gamma_1 + \kappa_1)\alpha_2$$

Corresponding to Figure 2, the anonymous mechanism revenue in regions A, B and C together is

$$\mu_1^A = (1-\gamma_1)\gamma_1^2 + 2(1-\gamma_1)^2\gamma_1$$

The corresponding non-anonymous mechanism revenue for the three regions is

$$\begin{aligned} \mu_1^{NA} &= (\gamma_1+\kappa_1)(1-\gamma_1)\gamma_1\alpha_2 + (1-(\gamma_1+\kappa_1)\alpha_2)(1-\gamma_1)(\gamma_1+\kappa_1)\alpha_2 \\ &= \alpha_2\left(1-\gamma_1\right)\left(\kappa_1+\gamma_1\right)\left(\gamma_1-\kappa_1\alpha_2-\alpha_2\gamma_1+1\right) \end{aligned}$$

We define $\Delta_t = \mu_t^{NA} - \mu_t^A, \forall t$. Thus

$$\Delta_1 = \mu_1^{NA} - \mu_1^A = (1-\gamma_1)\left(-\kappa_1^2\alpha_2^2 - 2\kappa_1\alpha_2^2\gamma_1 + \kappa_1\alpha_2\gamma_1 + \kappa_1\alpha_2 - \alpha_2^2\gamma_1^2 + \alpha_2\gamma_1^2 + \alpha_2\gamma_1 + \gamma_1^2 - 2\gamma_1\right) \quad (24)$$

For $\Delta_1 > 0$ there must exist $\overline{\kappa_1}$, $\underline{\kappa_1} > 0$, such that $\kappa_1 \in (\underline{\kappa_1}, \overline{\kappa_1})$. Thus we show that if only the last period cutoff value for buyer 1 is perturbed, given the equilibrium history, the revenue increases.

Next we assume that for the continuation game starting from period $(t-1)$, $\Delta_{t-1} > 0$. Also, let Λ_{t-1} be the difference in total flow of revenue (as a sum of current revenue and the continuation revenue). For $t = 1, \Lambda_t = \Delta_t$. We also assume that $\Lambda_{t-1} > 0$. Then we are required to show that $\Lambda_t > 0$.

Next we determine how the p_t changes with the perturbations. Let $\rho_t = \frac{p_t}{v_{t+1}}$. We pin down perturbed model $\rho_t^P(\gamma_t + \kappa_t) = \rho_t(\gamma_t) + \theta_t$, for $\theta_t > 0, \rho_t'(\gamma_t) > 0, \rho_t^{P\prime}(\gamma_t + \kappa_t) > 0$. So the t^{th} period expected revenues are the following:

$$\mu_t^A = (1-\gamma_t)\gamma_t\rho_t(\gamma_t) + 2(1-\gamma_t)^2\rho_t(\gamma_t)$$

$$\mu_t^{NA} = (\gamma_t+\kappa_t)(1-\gamma_t)\rho_t(\gamma_t)\alpha_{t+1} + (1-(\gamma_t+\kappa_t)\alpha_{t+1})(1-\gamma_t)\rho_t^P(\gamma_t+\kappa_t)\alpha_{t+1}$$

The difference between the two revenues is

$$\Delta_t = \mu_t^{NA} - \mu_t^A = (1-\gamma_t)((\gamma_t+\kappa_t)\rho_t(\gamma_t)\alpha_{t+1} + (1-(\gamma_t+\kappa_t)\alpha_{t+1})\rho_t^P(\gamma_t+\kappa_t)\alpha_{t+1} - \gamma_t\rho_t(\gamma_t) - 2(1-\gamma_t)\rho_t(\gamma_t)) \tag{25}$$

Analytical solution to the range of κ_t is non-trivial. But since $\rho_t'(\gamma_t) > 0, \rho_t^{P\prime}(\gamma_t+\kappa_t) > 0$, we can apply the same logic for the last period revenue. The value of κ_t should be sufficiently high so that the positive impact of higher price outweighs the negative impact higher probability of no-trade event. Also, there is an upper bound of κ_t beyond which the negative impact probability of no-trade becomes too high to outweigh the positive effect. Thus there should exist, in each period t, $\overline{\kappa_t}$, $\underline{\kappa_t} > 0$, such that $\kappa_t \in (\underline{\kappa_t}, \overline{\kappa_t})$, so that $\Delta_t > 0$.

Δ_t is the difference in revenues for only for the combined Regions A, B and C. The total difference in revenues is

$$\Lambda_t = \Delta_t + (\gamma_t+\kappa_t)\alpha_{t+1}\Lambda_{t-1} \tag{26}$$

Since $\Lambda_{t-1} > 0$ and $\Delta_t > 0$, we have $\Lambda_t > 0$. Then we can apply the logic of induction to claim that for any Γ_T, $T > 2$, we have $\Lambda_T > 0$. This completes the proof.

Proof of Proposition 3

Let us first establish the following Lemma:

Lemma 3 *There exists $\overline{p_1} \in (p_1, 1)$, such that if the seller increases p_1 from the equilibrium to any $p_1' > \overline{p_1}$, then for each time period t, equilibrium p_t increases.*

Proof. Under anonymous mechanism, suppose a buyer with valuation v weakly prefers to accept at a price p in period t. Then the following inequality must hold:

$$(q(1)\frac{1}{2} + q(0))(v-p) \geq q(0) \sum_{p_l \in P/p_t} \rho(p_l)(v-p_l)$$

$q(k)$ is the probability that k other buyers accept at the price p. P is the finite set of prices the seller sets in equilibrium. $\rho(p_l)$ is the probability that the buyer will accept such a price. Let $\rho^A(p_t)$ $= q(1)\frac{1}{2} + q(0)$. Under non-anonymous mechanism, for buyer 1, $\rho_1^{NA}(p_t) = q(1) + q(0)$, and for buyer 2, $\rho_2^{NA}(p_t) = q(0)$. If for any l, $p_l \in P/p_t$ increases to any $p_l' > \overline{p_l}^A$, then to maintain the inequality, for given v, p increases. Similarly for non-anonymous mechanism, we can find $\overline{p_l}^{NA,1}$ and $\overline{p_l}^{NA,2}$ for buyers 1 and 2 respectively, such that for any l, if p_l increases to p_l' which are beyond the corresponding thresholds, p increases. Since this holds for all $p_l \in P/p_t$, this is true for p_1. ■

Suppose the cutoff valuation of buyer 1, v_t is perturbed by an equilibrium amount k_t in any period

t, as in **Proposition** 2. In the last period, the difference in revenue of the two mechanisms is given by

$$\begin{aligned}\Delta_1 &= \mu_1^{NA} - \mu_1^{A} = (\gamma_1 + \kappa_1)(1-\gamma_1)\gamma_1\alpha_2 + (1-(\gamma_1+\kappa_1)\alpha_2)(1-\gamma_1)(\gamma_1+\kappa_1)\alpha_2 \\ &\quad -(1-\gamma_1)\gamma_1^2 + 2(1-\gamma_1)^2\gamma_1\end{aligned}$$

From **Proposition** 2, we know that $\Delta_1 > 0$. Also Δ_1 is decreasing in γ_1. Combining with **Lemma 3,** if p_1 is increased to $p_1' > \overline{p_l}^{NA,1}$, the expected revenue in the last period is decreasing given the history. Also if p_1 is increased to p_1', in any period t, p_t increases.

Next we assume that for the continuation game starting from period $(t-1)$, Δ_{t-1} is decreasing in γ_{t-1}. Also, let Λ_{t-1} be the difference in total flow of revenue (as a sum of current revenue and the continuation revenue). For $t = 1, \Lambda_t = \Delta_t$. We also assume that Λ_{t-1} is decreasing in γ_{t-1}. Then we can similarly show that Λ_t is decreasing in γ_t where

$$\begin{aligned}\Lambda_t &= \mu_t^{NA} - \mu_t^{A} = (1-\gamma_t)((\gamma_t+\kappa_t)\rho_t(\gamma_t)\alpha_{t+1} + (1-(\gamma_t+\kappa_t)\alpha_{t+1})\rho_t^P(\gamma_t+\kappa_t)\alpha_{t+1} \\ &\quad -\gamma_t\rho_t(\gamma_t) - 2(1-\gamma_t)\rho_t(\gamma_t)) + (\gamma_t+\kappa_t)\alpha_{t+1}\Lambda_{t-1}\end{aligned} \tag{27}$$

Since Λ_t decreases monotonically with γ_t, and therefore with p_1, for $p_1 \geq p_1' > \overline{p_l}^{NA,1}$. Thus compared to equilibrium p_1, when $p_1 = 1$, Λ_t decreases. Also, if Λ_t decreases, by induction the overall expected profit of the seller decreases. Again, since $p_1 = 1$ implies that this is effectively $(T-1)$ period game. This completes the proof.

Proof of Proposition 4

The difference between the revenues of non-anonymous and anonymous mechanisms is given by the following expression:

$$I = Q(v)f(v)v(Q(u)f(u)u - 2Q(v)f(v) + F(u) - \frac{F(v)}{f(v)}) \tag{28}$$

The non-anonymous mechanism generates higher revenue if $I > 0$.

We know that $Q(v) = \frac{1-F(v)}{f(v)} > 0$ for $v > 0$.

Suppose, for any perturbation k, let us define $K = \frac{Q(v)(2-f(v+k)(v+k))}{f(v+k)(v+k)} > 0$.

If $Q(v+k) - Q(v) > K$, then it implies that $Q(u)f(u)u - 2Q(v)f(v) > 0$, $(u = v+k)$.

For any $F(.)$, this happens only if $k > \overline{k_1} > 0$.

Again, for any $F(.)$, $F(u) - \frac{F(v)}{f(v)}) > 0$ if $k > \overline{k_2} > 0$. Thus for a perturbation $k > \widetilde{k} = \max(\overline{k_1}, \overline{k_2})$, we have $I > 0$.

If $F(.)$ follows IHR, then $Q(u) < Q(v)$, since $u > v$. Thus there must exist an optimal range $(\overleftrightarrow{k}, \underleftrightarrow{k})$ such that the equilibrium perturbation $k \in (\overleftrightarrow{k}, \underleftrightarrow{k})$ and $I > 0$.

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