# Why Does Productivity Matter?

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**Abstract**

Productivity is a key concept in economics and crucial for economic growth. By using different theoretical models, we show the role of several kinds of productivity, including total factor productivity (TFP) and labor productivity.

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## 1 Introduction

Productivity is a key concept in economics. It is crucial for economic growth. Since the total output, generally measured by gross domestic product (GDP), is produced by different inputs (such as capital, labor, land, raw materials, ...), there are different ways to measure productivity. We can use capital productivity which is defined as output per unit of capital used in the production process during a given time reference period.

$$\text{Capital Productivity } \equiv \frac{\text{Output}}{\text{Capital input use}} \tag{1}$$

or labor productivity defined as output per unit of labor (measured in terms of the number of workers or hours worked):

$$\text{Labor Productivity } \equiv \frac{\text{Output}}{\text{Labor input use}} \tag{2}$$

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There is another measure of productivity: the total factor productivity (TFP) which is the portion of growth not explained by growth in inputs used in the production process. The TFP measures the efficiency with which factor inputs are combined and is often used to proxy technological progress.

Estimating contribution to growth of different factors is not an easy task.[1] Looking back to history, Solow (1957) estimated that TFP growth accounted for 87.5% of growth in output per worker of the US over the period 1909-1949.[2] Zhu (2012) estimated that the contribution of TFP growth to economic growth is 78% percent of the growth in GDP per capita of China during 1978-2007.

The goal of this chapter is to explore the role of different kinds of productivity on economic growth from a theoretical point of view. We will focus on TFP and labor productivity.

# 2 Total factor productivity

## 2.1 TFP and economic growth

Let us start our exposition by investigating the relationship between TFP and economic growth. Solow (1957), using the data of the US economy of the 50 beginning years of the 20th century, ran a regression

$$\Delta \ln(Y_t) = B + \alpha \Delta \ln(K_t) + \beta \Delta \ln(N_t) \tag{3}$$

where $Y_t, K_t, N_t$ are respectively the GDP, physical capital and number of workers. Solow (1957) found that TFP growth accounted for 87.5% of growth in output per worker over that period.

This regression is derived from a production function (Cobb-Douglas function)

$$Y_t = A_t K_t^{\alpha} N_t^{\beta} \tag{4}$$

Obviously, $B = \Delta \ln(A_t)$ in the regression. $B$ is called *Solow residual* while $A_t$ is called technical progress or Total Factor Productivity (TFP). TFP is the portion of growth in output not explained by growth in traditionally measured inputs of labor and capital used in production. TFP is measured as the ratio of aggregate output (e.g., GDP) to aggregate inputs (here, this is the quantity $k^{\alpha} N^{\beta}$ when $\beta = 1 - \alpha$). The rate of TFP growth is calculated by subtracting average growth rates of labor and capital inputs from the growth rate of output

In the following, we explore the role of TFP by using different growth models. Let us start with the Harrod model. Consider an infinite horizon closed economy starting

[1] See Hulten (2009) for a great treatment of growth accounting.

[2] Solow was awarded the Nobel Memorial Prize in Economic Sciences in 1987 for his contributions to the theory of economic growth. The paper Solow (1957) is an important part of these contributions.

with an initial capital stock $k_0 > 0$:

$$\textbf{Harrod Model:} \quad \begin{aligned} & c_t + S_t = Y_t \\ & I_t = S_t \\ & k_{t+1} = k_t(1-\delta) + I_t \\ & S_t = sY_t \\ & Y_t = A_t k_t \end{aligned}$$

where $c_t, S_t, I_t$ are consumption, saving, investment at date $t$ ($t = 0, 1, \ldots, +\infty$), $s \in (0,1)$ is the exogenous saving rate, $k_t$ is the physical capital at date $t$ ($k_0 > 0$ is given), $\delta \in [0,1]$ is the capital depreciation rate, $Y_t$ is the output.

The production function in this model ($Y_t = A_t k_t$) can be interpreted in several ways: (i) It is a special case of the general form of Cobb-Douglas function with $\beta = 0$, (2) the labor $N_t$ has an exogenous rate of growth $N_t = N_0(1+n)^t$. In this case the TFP becomes $A_t N_0^\beta (1+n)^{\beta t}$, (3) if $\beta = 1 - \alpha$, the function can be written as:

$$\frac{Y_t}{N_t} = A_t \left(\frac{k_t}{N_t}\right)^\alpha$$

i.e., we consider the output per capita as function of capital per capita.

From the above system, we obtain that, for any $t \geq 0$,

$$\begin{aligned} Y_t \quad &= A_t((1-\delta)k_{t-1} + sY_{t-1}) \\ \text{and } \frac{\Delta Y_t}{Y_t} \quad &= \tfrac{A_{t+1}}{A_t}(1-\delta) + sA_{t+1} - 1 \\ \text{where } \Delta Y_t \quad &\equiv Y_{t+1} - Y_t \end{aligned}$$

. Therefore, we have the following result.

**Proposition 1.** *Consider the above Harrod model. Suppose $A_t \to A > 0$ when $t$ tends to infinity. We have that:*

- $\frac{\Delta Y_t}{Y_t} \to sA - \delta$
- *If $sA - \delta > 0$ then $Y_t \to +\infty$*
- *If $sA - \delta < 0$ then $Y_t \to 0$*

According to this result, the economy may grow or collapse, depending to the TFP $A$: if $A$ is high enough ($A > \delta/s$), then we have economic growth without bounds.

We now consider a model à la Solow. This model is quite similar to the Harrod Model, excepted the production function,

$$\textbf{Solow Model:} \quad \begin{aligned} & c_t + S_t = Y_t \\ & I_t = S_t \\ & k_{t+1} = k_t(1-\delta) + I_t \\ & S_t = sY_t \\ & Y_t = A_t k_t^\alpha L_t^{1-\alpha}, \alpha \in (0,1) \\ & A_t = a(1+\gamma)^t \\ & L_t = L_0(1+n)^t \end{aligned}$$

Here $\gamma > -1$ is the rate of growth of the TFP $A_t$, $n > -1$ is the rate of growth of the labor force. Both of them are assumed to be exogenous.

From the above system, we obtain that, for any $t \geq 0$,

$$\begin{aligned} Y_t &= a(1+\gamma)^t k_t^\alpha L_t^{1-\alpha} \\ \frac{Y_{t+1}}{Y_t} &= (1+\gamma)(1+n)^{1-\alpha}\Big(\frac{k_{t+1}}{k_t}\Big)^\alpha \\ k_{t+1} &= k_t(1-\delta) + sa(1+\gamma)^t k_t^\alpha L_t^{1-\alpha} \end{aligned}$$

Therefore, we obtain the following result:

**Proposition 2.** *Consider the above Solow model. We have that:* $\frac{\Delta Y_t}{Y_t} \to g$ *where* $g$ *satisfies*

$$1 + g = (1+n)(1+\gamma)^{\frac{1}{1-\alpha}}$$

The long-term rate of growth $g$ of the output depends strongly on the rate of growth of the TFP $A_t$. The higher $A$, the higher the rate of growth $g$.

Although the Harrod and Solow models help us to explain the role of TFP, they have two limits: (1) the rate of saving is exogenous and (2) the rate of growth of the output is also exogenous. With the Ramsey model, we can endogenize the rate of saving but we do not resolve the question of the exogeneity of the rate of growth of the output. This question will be resolved with endogenous growth models.

We now present a Ramsey model. We assume there exists a representative consumer who lives for an infinite number of periods. She/he maximizes her/his intertemporal utility under sequential constraints

$$\textbf{Ramsey model:} \qquad \begin{aligned} &\max_{(c_t,k_t,I_t)} \sum_{t=0}^{+\infty} \beta^t u(c_t) \\ &\text{subject to: } c_t + I_t \leq F_t(k_t) \\ &\qquad\qquad k_{t+1} = k_t(1-\delta) + I_t \end{aligned}$$

where $k_0 > 0$ is given, $\beta \in (0,1)$ represents the rate of time preference, $u$ is the utility function. $u$ is strictly increasing, strictly concave, differentiable, $u'(0) = +\infty$. The production function $F_t$ is concave, strictly increasing, differentiable, and $F_t(0) = 0$. Note that this function is time-dependent.[3]

**Remark 1.** *As in the Harrod Model, in the Ramsey Model, implicitly, either we consider the number of workers is exogenous and has an exogenous rate of growth, or we consider in fact output per capita and capital per capita.*

As usual, if $(c_t^*, k_{t+1}^*, I_t^*)_{t\geq 0}$ is the list of the optimal solutions of the above Ramsey problem, the optimal rates of saving are $s_t^* = \frac{I_t^*}{F_t(k_t^*)}$ and we have Euler equations:

$$u'(c_t^*) = \beta u'(c_{t+1}^*)\big(1 - \delta + F'_{t+1}(k_{t+1})\big). \tag{5}$$

In general, finding solutions of the Ramsey problem is not easy. To explore the importance of the TFP, we consider two examples where we can explicitly compute the optimal paths and rate of growth.

[3]See Le Van and Dana (2003) for a detailed presentation of optimal growth models.

**Example 1** (AK model)**.** *Suppose $u(c) = \ln(c)$, $F_t(k) = A_t k$. Let us denote $A'_t = A_t + 1 - \delta$. We can prove that the optimal path $(k_t)$ is given by $k_{t+1} = \beta(1 - \delta + A_t)k_t$ $\forall t$. Then the optimal output $Y^*_t$ satisfies*

$$Y^*_t = \beta^t (A'_0 A'_1 \dots A'_t) Y_0$$

*with $Y_0 = A'_0 k_0$. The optimal rate of saving is*

$$s^*_t = \frac{\beta A_t + (1-\delta)(\beta - 1)}{A_t} \leq \beta < 1$$

*which is increasing in $A_t$. If $A_t \leq A_{t+1}$ then $s^*_t \leq s^*_{t+1}$.*

*We can also compute the rate of growth by $\frac{Y^*_{t+1}}{Y^*_t} = \beta(A_t + 1 - \delta)$.*

*Now suppose $A_t \to A > 0$ as $t \to +\infty$. In this case $\frac{Y^*_{t+1}}{Y^*_t} \to \beta(A + 1 - \delta)$ and $s^*_t \to s = \frac{\beta A + (1-\delta)(\beta-1)}{A}$. Let us look at two cases:*

- *If $\beta(A + 1 - \delta) > 1 \Leftrightarrow sA - \delta > 0$ then $\frac{Y^*_{t+1}}{Y^*_t} \to +\infty$*
- *If $\beta(A + 1 - \delta) < 1 \Leftrightarrow sA - \delta < 0$ then $\frac{Y^*_{t+1}}{Y^*_t} \to 0$*

*We get the same results as in the Harrod model: the TFP plays a crucial role on the economic growth.*

**Example 2.** *Assume that $u(c) = \ln(c), F_t(k) = Ak^\alpha, \alpha \in (0,1)$, and $\delta = 1$. In this case, we can prove that the optimal path is given by $k_{t+1} = \beta\alpha A k_t^\alpha$ $\forall t \geq 0$, and the saving rate is $\alpha\beta$. Therefore, the optimal output is*

$$y^*_{t+1} = A^{\frac{1-\alpha^{t+2}}{1-\alpha}} (\alpha\beta)^{\frac{\alpha - \alpha^{t+2}}{1-\alpha}} k_0^{\alpha^{t+2}}$$

*When $t$ goes to infinity, the output $y^*_{t+1}$ converges to a steady state*

$$y^s = A^{\frac{1}{1-\alpha}} (\alpha\beta)^{\frac{\alpha}{1-\alpha}}$$

*There is no growth. It is due to the fact the production function is of strictly decreasing returns to scale. However, observe when $A$ increases, the steady state becomes higher.*

## 2.2 How to increase TFP and obtain economic growth?

So far the TFP $A_t$ seems to be a blackbox in a production function of the type

$$y_t = A_t k_t^\alpha N_t^\beta$$

where $k_t, N_t$ are the number of machines and the number of workers. In this modeling, we do not take into account the quality of the machines, nor the skill of the workers. Actually, the production function should be written as

$$y_t = am_t(\mathcal{K}_t)^\alpha (\mathcal{N}_t)^\beta$$

where $m_t$ is the quality of the management, the macroeconomic environment (stability, law rule), $\mathcal{K}_t$ is the effective capital stock, $\mathcal{N}_t$ is the effective labor. Let $\zeta_t$ denote the technology embedded in the machines, $\theta_t$ denote the working time, $h_t$ the human capital (education, training, health) of the workers. We then have

$$\mathcal{K}_t = \zeta_t k_t \text{ and } \mathcal{N}_t = \theta_t h_t N_t.$$

The production function now is $y_t = A_t k_t^\alpha N_t^\beta$ where the TFP is $A_t \equiv \left[a m_t \zeta_t^\alpha (\theta_t h_t)^\beta\right]$. If we assume $\theta_t$ depends positively on wages or bonus (incentive mechanism) then

$$y_t = A_t k_t^\alpha N_t^\beta \tag{6}$$

$$\text{where the TFP} \qquad A_t = \left[a m_t \zeta_t^\alpha \left(\theta(w_t) h_t\right)^\beta\right] \tag{7}$$

The TFP is not anymore a black box. If we invest in the quality of management,[4] in technology, in training, education, health and if the salaries of the workers are sufficiently incentive, we will have a high TFP. Using endogenous growth models (Lucas, 1988; Romer, 1990), we can prove that there may be economic growth even with strictly decreasing returns to scale production function.

In the following, we present a simple endogenous growth model. The representative household maximizes her intertemporal utility $\sum_{t=0}^{\infty} \beta^t u(c_t)$ subject to sequential constraints: $c_t + S_{t+1} = G_t F(k_t)$ $\forall t \geq 0$, where $c_t, S_{t+1}$ are consumption, saving.

We now assume that the saving $S_{t+1}$ is shared in investment in physical capital $k_{t+1}$ and in investment $T_{t+1}$ in TFP, i.e., $k_{t+1} + T_{t+1} = S_{t+1}$. $G_{t+1}$ is a function of $T_{t+1}$ and we write $G(T_{t+1})$. We rewrite the model as follows

$$\begin{aligned}
& \max \sum_{t=0}^{\infty} \beta^t u(c_t) \\
\text{for } t \geq 1 \qquad & c_t + S_{t+1} = H(S_t) \equiv \max\{G(T_t)F(k_t) : T_t + k_t = S_t, \text{ and } T_t, k_t \geq 0\} \\
& \text{where } k_{t+1} + T_{t+1} = S_{t+1}.
\end{aligned}$$

For the sake of tractability, we assume that $F(k) = k^\alpha, \alpha \in (0,1), G(T) = (\lambda T + 1)^\xi, \xi > 0$, and $\lambda > 0$.[5] The parameter $\xi$ measures the quality of the TFP investment technology. The higher $\xi$ the more efficient the TPF investment. The parameter $\lambda$ measures the utilization of $T_t$. For instance $\lambda$ is small because of diversion of $T_t$.

We firstly look at the static problem and the properties of the function $H$. Under our specifications, we have $H(S_t) \equiv \max\{(\lambda T_t + 1)^\xi k_t^\alpha : T_t + k_t = S_t, \text{ and } T_t, k_t \geq 0\}$. Solving this problem is equivalent to solving the following problem whose objective function is strictly concave

$$\max\{\xi ln(\lambda T_t + 1) + \alpha ln(k_t) : T_t + k_t = S_t, \text{ and } T_t, k_t \geq 0\}$$

[4] Bloom et al. (2013) ran a management field experiment on large Indian textile firms and provided free consulting on management practices to randomly chosen treating plants. By comparing the performance of these plants to a set of control plants, they found that adopting these management practices raised the TFP by 17% in the first year.

[5] Here, we implicitly assume that $u$ is continuously differentiable, strictly increasing, concave, $u'(0) = \infty$ and $\sum_{t=0}^{\infty} \beta^t u(D_t) < \infty$ where the sequence $(D_t)$ is defined by $D_0 = H(S_0)$, $D_{t+1} = H(D_t)$.

$(T_t, k_t)$ is an optimum point if and only if there are non-negative values $\mu_1, \mu_2$ such that

$$\frac{\alpha}{k_t} = \mu_1, \quad \xi\frac{\lambda}{\lambda T_t + 1} + \mu_2 = \mu_1, \quad \mu_2 T_t = 0$$

If $T_t = 0$ at optimal, then we have $\lambda\xi = \mu_1 - \mu_2 \leq \mu_1 = \alpha/k_t = \alpha/S_t$. Thus, we have $S_t \leq \alpha/(\lambda\xi)$.

If $T_t > 0$ at optimal, the FOC implies that $\frac{\alpha}{k_t} = \xi\frac{\lambda}{\lambda T_t+1}$, i.e., $(\lambda T_t + 1)\alpha = \xi\lambda k_t = \xi\lambda(S_t - T_t)$. So, we can compute that

$$T_t = \frac{\xi\lambda S_t - \alpha}{\lambda(\alpha+\xi)}, \quad k_t = \frac{\alpha(\lambda S_t + 1)}{\lambda(\alpha+\xi)}$$
$$H(S_t) = \Big(\frac{\xi(\lambda S_t+1)}{\alpha+\xi}\Big)^{\xi}\Big(\frac{\alpha(\lambda S_t+1)}{\lambda(\alpha+\xi)}\Big)^{\alpha} = \frac{\xi^{\xi}\alpha^{\alpha}}{(\alpha+\xi)^{\alpha+\xi}}\frac{(\lambda S_t+1)^{\alpha+\xi}}{\lambda^{\alpha}}$$

Of course, $T_t > 0$ is equivalent to $\xi\lambda S_t - \alpha > 0$.

Summing up, we obtain the following result:

**Lemma 1.** • *If $S_t \leq \frac{\alpha}{\xi\lambda}$ then $T_t = 0$. It is not optimal to invest in TFP, when $S_t$ is small. In this case $S_t = k_t$ and $H(S_t) = S_t^{\alpha}$.*

- *If $S_t > \frac{\alpha}{\xi\lambda}$ then $T_t > 0$. (If $S_t$ is high enough then it is worthwhile to invest in TFP.) In this case*

$$H(S_t) = a_h\frac{(\lambda S_t+1)^{\alpha+\xi}}{\lambda^{\alpha}}$$

*where $a_h \equiv \frac{\xi^{\xi}\alpha^{\alpha}}{(\alpha+\xi)^{\alpha+\xi}}$ depending on $(\alpha, \xi)$.*

*The function $H$ is increasing in $\lambda$ when $S > \frac{\alpha}{\xi\lambda}$. The lower the level of diversion, the higher the total output.*

Notice that the function $H(S)$ is increasing return to scale and convex for any $S > \alpha/(\xi\lambda)$. This is one way to introduce increasing return to scale technology is growth models (see Romer (1986) for more detailed discussions).

We now show the dynamics of the optimal path. It is easy to see that the optimal path $(S_t)$ is monotonic. We then have the convergence of optimal paths.[6]

**Proposition 3.** *Assume that $\beta\alpha^{\alpha}\xi^{1-\alpha}\lambda^{1-\alpha} > 1$ and $\alpha + \xi \geq 1$. Then any optimal sequence $\{S_t^*\}_t$, and hence any optimal sequence of outputs $\{y_t^* = H(S_t^*)\}$ converge to infinity.[7] By consequence, there is a date $\tau$ such that the country invests in TFP from date $\tau$ on (i.e., $T_t > 0\ \forall t \geq \tau$).*

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[6] We do not provide a full analysis in this paper. However, more dynamic properties may be obtained by adopting the method in Kamihigashi and Roy (2007), Bruno et al. (2009).

[7] Proof: If $S < \frac{\alpha}{\xi\lambda}$, then we have $H'(S) = \alpha S^{\alpha-1} > \alpha\big(\frac{\alpha}{\xi\lambda}\big)^{\alpha-1} = \alpha^{\alpha}\xi^{1-\alpha}\lambda^{1-\alpha}$.

If $S > \frac{\alpha}{\xi\lambda}$, then we have

$$H'(S) = a_h(\alpha+\xi)\lambda\frac{(\lambda S_t+1)^{\alpha+\xi-1}}{\lambda^{\alpha}} > a_h(\alpha+\xi)\lambda\frac{(\lambda\frac{\alpha}{\xi\lambda}+1)^{\alpha+\xi-1}}{\lambda^{\alpha}} = \alpha^{\alpha}\xi^{1-\alpha}\lambda^{1-\alpha}. \tag{8}$$

Since $\beta\alpha^{\alpha}\xi^{1-\alpha}\lambda^{1-\alpha} > 1$, by applying Proposition 4.6. in Kamihigashi and Roy (2007), we have that every optimal path increasingly converges to infinity.

According to our result, if the utilization of investment in technology (parameter $\lambda$) and the quality of the TFP investment technology (parameter $\xi$) are high, and we have increasing return to scale ($\alpha + \xi \geq 1$) technology, we get growth without bounds.

The rate of growth ($\frac{y^*_{t+1}}{y^*_t} - 1$) is now endogenous. It is obtained by an optimal share between investing in physical capital and investing in HC, Technology, Management Quality, incentive mechanisms. For that reason, we call these types of models Endogenous Growth Models.

The above results (Lemma 1 and Proposition 3) above deserve some comments.

- The country will wait until some date $\tau$, when the optimal output generates enough saving $S_\tau > \frac{\alpha}{\xi\lambda}$, before investing in TFP.
- If the diversion of the $T_t$ is high (i.e. $\lambda$ is low), the country may never invest in TFP and will not have growth.
- If $\lambda$ is lower (the diversion exists), the date $\tau$ becomes larger. The country has to wait longer before starting to invest in TFP.

## 2.3 TFP and Competitiveness

### 2.3.1 Competition between physical capital and financial asset

The financial market has been considered as one of the main causes of recession or/and fluctuation. But does the financial market always cause an recession in the productive sector? To address this question, let us consider a two-period economy with one consumer, one producer. In period 0, the consumer has a revenue $R_0$ and consumes $c_0$ and saves $s_0$. She wants to invest $k_1$ in capital stock, $\xi_0$ in financial asset. We suppose the numeraire is the consumption good. Let $r_1$ denote of the return of asset in period 1. The consumer wants to maximize the revenue $R_1$ in period 1. We have $R_1 = Ak_1^\alpha + r_1\xi_0$, $\alpha \in (0,1)$. She solves the problem

$$\max_{k_1 \geq 0, \xi_0 \in \mathbb{R}} \{Ak_1^\alpha + r_1\xi_0 : k_1 + \xi_0 = s_0\}$$

The optimal value $\xi_0^*$ solves the equation

$$A\alpha(s_0 - \xi_0^*)^{\alpha-1} = r_1$$

It is easy to see that $\xi_0^*$ is a decreasing function of the TFP $A$ (crowding out effect). In particular, when $A$ is very small, the optimal value $k_1^*$ is also very small.

If the consumer anticipates a high value of asset return $r_1$ (speculation), then she invests in the financial asset ($\xi_0^*$ is close to $s_0$) and reduces the physical capital ($k_1^*$ is small). Whether people invest more in physical capital or financial assets strongly depends on the TFP of the production sector and the asset return. See Le Van and Pham (2016) for the interaction between the financial market and the production section in an infinite-horizon general equilibrium model.

### 2.3.2 Competition between two countries

We now investigate the role of TFP in the context of globalization. Assume we have two countries $a, b$. Country $a$ has the production function $A_a k^\alpha$ and its saving in period 0 is $s_0$. Country $b$ has the production function $A_b k^\alpha$ and its saving in period 0 is $s_0$.

Each country maximizes its revenue in period 1: $A_a k_{a,1}^\alpha + r_1 \xi_{a,0}$, $A_b k_{b,1}^\alpha + r_1 \xi_{b,0}$.

The two countries exchange consumption good and financial assets. We investigate the equilibrium, i.e., a list of allocations and price $(k_{a,1}^*, \xi_{a,0}^*, k_{b,1}^*, \xi_{b,0}^*, r_1^*)$ such that

1. For each $i = a, b$, given $r_1^*$, the pair $(k_{i,1}^*, \xi_{i,0}^*)$ solves the following problem
$$\max_{k_1 \geq 0, \xi_0 \in \mathbb{R}} \{A_i k_1^\alpha + r_1^* \xi_0 : k_1 + \xi_0 = s_0\}$$
2. The financial market clears: $\xi_{a,0}^* + \xi_{b,0}^* = 0$.

We then obtain the equilibrium return $r_1^*$ from these equilibrium relations. The following result shows the impact of TFP on the equilibrium outcomes.

**Proposition 4.** *If the TFP $A_a$ of country $a$ is smaller than the one of country $b$, $A_b$, then at equilibrium we have*

- $\xi_{a,0}^* > 0$ *(Country a buys financial asset),* $\xi_{b,0}^* = -\xi_{a,0}^* < 0$ *(Country b sells financial asset)*
- $k_{a,1}^* < k_{b,1}^*$
- $c_{a,1}^* = A_a (k_{a,1}^*)^\alpha + r_1^* \xi_{a,0}^* < c_{b,1}^* = A_b (k_{b,1}^*)^\alpha + r_1^* \xi_{b,0}^*$, *i.e. the consumption of country a is lower than the consumption of country b.*

Our result suggests that the TFP matters in the context of globalization: The higher the TFP, the higher the input quantity used for production and hence the higher the income of the country.

# 3 Labor Productivity

We consider the following definition of labor productivity:

$$\pi^L \equiv \frac{y}{N}$$

where $y$ is the output and $N$ is the number of workers.

In general, the number of workers can be calculated as follows

$$N = (1-u) r_p w_a P \tag{9}$$

where $P$ is the total population, $w_a$ is the proportion of the working-age population (often defined as 15-64 year old) to the total population, $r_p$ is the participation rate (in other words, $1 - r_p$ is the fraction of the working age population does not participate in the labor market), and $u$ is the unemployment rate.

We have the following relationship between labor productivity and per capita GDP

$$\underbrace{\frac{y}{P}}_{\text{GDP per capita}} = (1-u)r_p w_a \underbrace{\frac{y}{N}}_{\text{Labor productivity}} \tag{10}$$

If $w_a$, $r_p, u$ are unchanged, then, *by definition*, increasing labor productivity is equivalent to increasing GDP per capita. In reality, however, all factors, including labor productivity and per capita GDP are inter-dependent. Hence, the very issue is to understand determinants of labor productivity and GDP as well as TFP. To do so, go back to relations (6), (7) with $\beta = 1-\alpha$. We obtain

$$y_t = A_t k_t^{\alpha} N_t^{1-\alpha} \tag{11}$$

$$\text{where the TFP is} \qquad A_t = \left[a m_t \zeta_t^{\alpha} \left(\theta(w_t)h_t\right)^{1-\alpha}\right] \tag{12}$$

From these relations we obtain

$$\pi_t^L = \frac{y_t}{N_t} = \left[a m_t \zeta_t^{\alpha} \left(\theta(w_t)h_t\right)^{1-\alpha}\right] \left(\frac{k_t}{N_t}\right)^{\alpha} \tag{13}$$

According to this expression, the labor productivity depends on (1) physical capital per worker $k_t/N_t$, (2) quality of machines $\zeta_t$, (3) human capital (education, training, health) $h_t$ of workers, (3) quality of management $m_t$ and (4) incentive mechanism (wages, bonus, for instance).

This observation is consistent with that in Baumol et al. (1989): "*Historically, labor productivity growth has been driven by innovation, better education, and investment in physical capital. Innovation and investment by private sector require a growth-friendly environment, with supportive institutions and policies, including policies that promote macroeconomic stability and the rule of law*". Here, we contribute by mentioning the role of incentive mechanisms ($w$) for the short term.

We now look at the connection between labor productivity and economic growth by using a supply side view. Assume that the total output is produced by a Cobb-Douglas production function: $y_t = A_t k_t^{\alpha} N_t^{\beta}$ where $0 < \alpha, \beta < 1$. We then obtain

$$y_t^{1-\beta} = A_t k_t^{\alpha} \left(\frac{N_t}{y_t}\right)^{\beta} = A_t k_t^{\alpha} \left(\pi_t^L\right)^{-\beta}$$

Hence

$$\left(\frac{y_{t+1}}{y_t}\right)^{1-\beta} = \frac{A_{t+1}}{A_t} \left(\frac{k_{t+1}}{k_t}\right)^{\alpha} \left(\frac{\pi_{t+1}^L}{\pi_t^L}\right)^{-\beta}.$$

This equation leads to an interesting observation: If the TFP and physical capital remain unchanged ($A_{t+1} = A_t$, $k_{t+1} = k_t$), an increasing of labor productivity ($\pi_{t+1}^L > \pi_t^L$) does decrease the GDP ($y_{t+1} < y_t$). Therefore, we should focus not only on labor productivity but also on TFP and physical capital.

# 4 Conclusion

We have presented several models showing how productivity matters for economic growth. However, economic growth is not an outcome of a single factor but several factors. Moreover, many factors, for instance, TFP and labor productivity, are not separable. Focusing only on one indicator may be misleading. It is important to find an optimal share between the purchases of machines with new technology, the expenditures for training, education, and the wage policy (in the labor market). In this regard, we cannot say that labor productivity (respectively, capital productivity) is crucial for growth. A good combination of both of them is crucial for economic growth.

# References


Baumol, W. J., Blackman, S. A. B., Edward N. Wolff. (1989). Productivity and American leadership: the long view. MIT press.

Bloom, N., Eifert, B., Mahajan, A., McKenzie, D., Roberts, J., (2013). Does management matter? Evidence from Inida. *Quarterly Journal of Economics*, 128.

Barro, R. (1990). Government Spending in a Simple Model of Endogeneous Growth. *Journal of Political Economy*, 98, S103-S125.

Bruno O., Le Van, C., & Masquin, B. (2009). When does a developing country use new technologies?. *Economic Theory*, 40, 275-300.

Hulten, C. R. (2009). Growth Accounting. NBER Working Paper No. 15341.

Jones, L., & Manuelli, R. (1990). A convex model of equilibrium growth: Theory and policy implications. *Journal of Political Economy*, 98(5), 1008-1038.

Kamihigashi, T., & Roy, S. (2007). A nonsmooth, nonconvex model of optimal growth. *Journal of Economic Theory*, 132, 435-460.

Le Van C., & Dana, R-A. (2003). *Dynamic Programming in Economics*, Kluwer Academic Publishers.

Le Van, C., Pham, N.-S., 2016. Intertemporal equilibrium with financial asset and physical capital. *Economic Theory* 62, p. 155-199.

Lucas, R. E. (1988). On the mechanics of economic development. *Journal of Monetary Economics*, 22, 3-42.

Romer, P.M. (1986). Increasing returns and long-run growth. *Journal of Political Economy*, 94, 1002-1037.

Romer, P.M. (1990). Endogenous technological change. *Journal of Political Economy*, 98, pp. S71-S102.

Solow, R. (1957). Technical change and the aggregate production function. *Review of Economics and Statistics*, 39 (3): 312–320.

Zhu, X. (2012). Understanding China's Growth: Past, Present, and Future. *The Journal of Economic Perspectives*, Vol. 26, No. 4 (Fall 2012), pp. 103-124.