Optimal interventions on strategic fails in repo markets

Fukai, Hiroki

Chukyo University

30 January 2021

Optimal Interventions on Strategic Fails in Repo Markets*

Hiroki Fukai†

January 30, 2021

Abstract

A search theoretic model of repurchase agreements is constructed wherein the sellers' incentives to fail to deliver securities are explicitly incorporated. In equilibrium, too many sellers choose to fail relative to the social optimum. Two types of interventions are studied: a fails charge and an interest reset. These interventions improve efficiency by lowering the fraction of sellers who fail and making it easier for buyers to find their counterparties. In extensions of the model, the two types of optimal interventions are differently affected by fundamental variables. Thus, a policymaker needs to carefully distinguish between the workings of the two.

Keywords: fails charge; over-the-counter market; repo interest; repo market

JEL Classification: D53, G01, G18

*This research was supported by grants-in-aid from the Japan Center for Economic Research, Nihon Hoseigakkai Foundation, Zengin Foundation for Studies on Economics and Finance, and JSPS KAKENHI (Grant Number JP19K13648).

†Chukyo University. Address: 101-2 Yagoto-Honmachi, Syowa-ku, Nagoya-shi, Aichi 466-8666, Japan. E-mail: fukai.hiroki@gmail.com

1 Introduction

Repurchase agreements (*repos*) are one of the important sources of short-term funding for major financial institutions (see Garbade *et al.* [10]). A repo is a promise by a seller (denoted as "she") to sell a security to a buyer (denoted as "he") for an agreed price on a purchase date (*starting leg*) and to repurchase the security from the buyer for a different price on a repurchase date (*closing leg*), as illustrated in Figure 1. However, even if the seller agrees to, she may strategically choose to fail to deliver the security on a timely basis, which Fleming and Garbade [7] call a *strategic fail.* Because there was an extraordinary volume of fails during the 2007-2009 financial crisis, novel interventions were introduced to mitigate dysfunctionality in repo markets since then (see Garbade *et al.* [10]). The purpose of this study is to analyze how these interventions reduce sellers' incentives to fail and to characterize the optimal levels of such interventions.

An incentive for a strategic fail arises as follows. Through a repo, a seller pays a buyer the difference between the repurchase price and the purchase price, called the *repo interest.* The seller remains obliged to pay the full amount of the repo interest to the buyer regardless of whether she delivers the promised security late or not at all, as illustrated in Figure 2. This convention provides an incentive for the seller to deliver the security on the scheduled starting date for a sufficiently high repo interest rate (see Fleming and Garbade [8]). In the absence of any ancillary costs or penalties, the seller has little incentive to deliver the security at a repo interest rate of zero. She may even strictly prefer failing to lending money if the repo interest rate is negative (see Fleming and Garbade [8]).

When the repo interest rate is so low, fails even become *chronic* (see Garbade *et al.* [10]). When fails become chronic, a buyer who bought (but did not receive) a security is nailed to a relationship with a failing seller, wherein he must bargain with the seller in order to liquidate his position. The failed buyer faces the risk that the failing seller

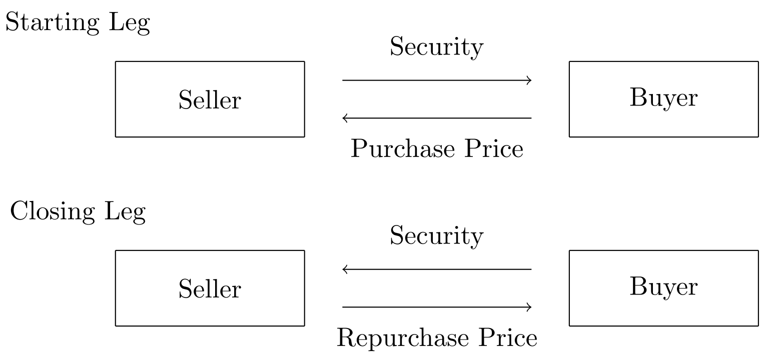

Figure 1: The figure describes a repo when no fails occur. The seller sells a security for a purchase price at the starting leg and repurchases it for a repurchase price at the closing leg.

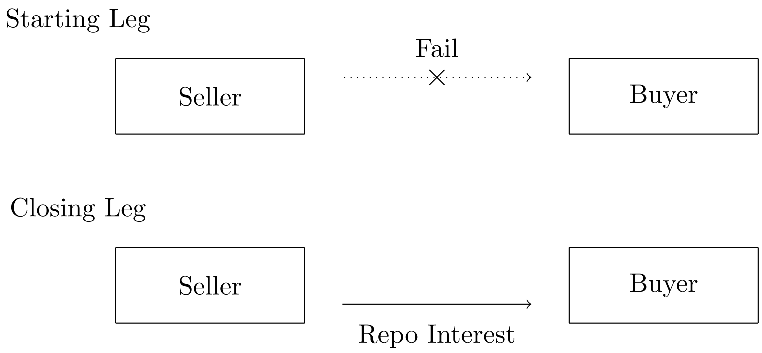

Figure 2: The figure describes a fail at the starting leg. When the seller fails to deliver the security at the starting leg, she is still obliged to pay the repo interest at the closing leg.

becomes insolvent during this period and that, to replace the security, the buyer may have to pay more than the original price negotiated with the insolvent seller. The fear of failing to receive securities on a timely basis can harm market liquidity and function. Thus, fails have been a matter of primary concern to the Federal Reserve Bank of New York for decades (see Garbade and Keane [11]).

Even since before the financial crisis, there have been instances wherein particular securities have exhibited repo interest rates near zero and high volumes of fails. Such examples include the squeeze in the 30-year Treasury bond in May 1986, the chronic fails following the terrorist attacks in September 2001, and the chronic fails in the 10-year Treasury note in June 2003 (see Garbade and Keane [11]). During the financial crisis, primary dealer fails in Treasury securities rose sharply to an average of $379 billion per day during the week of October 9, 2008, from an average of less than $10 billion per day during the week of September 4, 2008 (see Garbade *et al.* [10]). Unlike earlier episodes, fails during the crisis involved securities across the entire yield curve and other classes of assets such as agency debt securities and agency mortgage-backed securities (see Garbade and Keane [11]).

To mitigate sellers' incentives to fail, two types of novel interventions have been introduced since the financial crisis: a *fails charge* and an *interest reset*. The fails charge has been implemented since May 1, 2009, by the Treasury Market Practices Group for U.S. Treasury securities. It allows a buyer to claim monetary compensation from a seller when the seller fails at the starting leg. Garbade *et al.* [10] document that fails averaged about $14.4 billion per day during the first four months of 2009, but only $4.2 billion per day after the implementation of the fails charge until July 2010. The interest reset has been included as an optional supplementary condition in the Global Master Repurchase Agreement since 2011. It requires the repo interest to be reset to zero until the fail is cured, if a seller fails at the starting leg of a negative interest repo. This is currently an optional condition, and applicable only for negative interest repos (see ICMA [17, 16]).

In this study, we construct a search theoretic model of repos wherein the sellers' incentives to fail and the endogenous determination of the repo interest are both explicitly incorporated. In our baseline model, described in Section 2, the sellers' benefits from repos are stochastic and not known until the starting leg. Repos are assumed to be incomplete contacts in the sense that future realization of the sellers' benefits from repos cannot be included in the terms of the repos. Agents cannot commit to delivery of the collateral security. If sellers receive a low realization of the benefits from repos, they choose to fail at the starting leg. In the equilibrium, too many sellers choose to fail relative to the social optimum.

In this framework, we show that both the fails charge and interest reset attain the socially optimal outcome. These interventions improve efficiency by lowering the fraction of sellers failing at the starting leg and making it easier for buyers to find their counterparties. We provide a complete characterization of the optimal interventions. As is intuitive, the optimal fails charge equals the lost benefit of a failed buyer. The optimal fails charge equals the lost benefit of a failed buyer. The optimal reset interest is such that the lost benefit for a failed buyer equals the difference between the reset interest and the original repo interest. The result for the optimal reset suggests that, in the baseline environment, the zero reset for negative interest repos does not achieve the social optimum.

We then explore three extensions of the framework in Section 5 wherein we incorporate some important aspects of reality. In the first extension, we study the case in which agents take leveraged positions using trading opportunities outside the repo pair. We show that the optimal fails charge becomes higher as the outside price falls faster over time, while the optimal reset interest does not even depend on the outside prices.

In the second extension, we study the case wherein sellers default at the closing leg with some probability. We show that the optimal reset interest becomes lower as sellers default with higher probability, while the result on the optimal fails charge depends crucially on the repo interest.

In the third extension, we study the case wherein sellers are allowed to make late delivery. We provide a rationale for the fact that, in reality, the fails charge is imposed when a fail is cured. In the second (default) and third (late delivery) extensions, we provide a rationale for the zero reset, which is in contrast to the result in the baseline environment.

This study is particularly relevant to understand the extraordinary volume of fails during the recent financial crisis as well as those in other episodes and the workings of the novel interventions implemented recently. In various extensions that are relevant to the existing literature outlined below, we show that the optimal fails charge and optimal interest reset are differently affected by fundamental variables such as asset prices outside a repo pair and the probability of sellers' default. This implication emphasizes that, when the interventions are implemented, a policymaker needs to carefully distinguish between the two.

Related Literature. Repo markets have attracted growing attention, especially since "run-on-repo" discussions on the global financial crisis. Using data from a high-quality dealer in the bilateral repo market, Gorton and Metrick [12] argue that the crisis was caused by a run on the repo market. Copeland, Martin, and Walker [4] show that repo haircuts were surprisingly stable in the market for most other cash borrowers, although there was a sharp decline in the tri-party repo funding of Lehman Brothers in September 2008. Using data from the tri-party repo market, Krishnamurthy, Nagel, and Orlov [19] suggest that the run on repo backed by private-sector collateral was not central to the collapse of short-term funding in aggregate. Gorton, Metrick, and Ross [13] note that significant details of the run remain shrouded because many of the providers of repo finance, especially those in the bilateral repo market, are unregulated cash pools.

This study fits in a body of literature that uses search theoretic models to describe the repo markets; it is motivated by the fact that repo markets are over-the-counter markets (see Choudhry [3]). Duffie, Gârleanu, and Pedersen [6] show that repo premia on particular securities ("repo specials") are larger when inventories are larger and interest-rate volatility is higher. Vayanos and Weill [26] explain why just-issued bonds ("on-the-run") trade at lower yields than previously issued bonds, when short positions can be established in a repo market. Tomura [23] shows that a need for a repo arises from an investor's short investment horizon. Our model shares basic structures with their models, such as search and bargaining frictions. Adding to the literature, our study sheds light on the workings of interventions on fails by explicitly incorporating sellers' incentives to fail at the starting leg and the endogenous determination of the repo interest.

Our first extension on leverage is related to the literature on leverage and re-use (also referred to as "rehypothecation") of collateral assets in the context of repo markets. Park and Kahn [22] show that rehypothecation incurs deadweight cost by misallocating assets among agents when a cash borrower defaults. Bottazi, Luque, and Páscoa [2] hint that default and fails have important consequences on rehypothecation. Our leverage extension adds to the literature insights on how the optimal fail interventions are characterized when agents are allowed to take leveraged positions.

In line with existing literature on default, we study a seller's default in our second extension. Gottardi, Maurin, and Monnet [14] emphasize that the punishment for default may exceed the future market value of the collateral because of the recourse nature of repos. Infante [15] emphasize that cash borrowers are directly exposed to an intermediary dealer's default because they risk losing their collateral. Valderrama [25] shows how a liquidity shock to a cash lender propagates in the market even if the cash lender remains solvent in all states of nature. Nuño and Thomas [21] explain the observed fluctuations in intermediary leverage and real economic activity through the lens of bank default risk, limited liability, and moral hazard. Donaldson and Micheler

[5] show that a decrease in credit frictions cause an increase in systemic risk arising from default in credit chains. We contribute to the literature by investigating how the optimal fail interventions depend on the possibility of a seller's default.

Organization. The remainder of the paper is organized as follows. In Section 2, we define the baseline environment. In Section 3, we characterize the equilibrium with no interventions and the social optimum in the baseline environment. In Section 4, we characterize the optimal fails charge and the optimal reset interest in the baseline environment. In Section 5, we study three extensions of the model. We provide the details of the proofs in the Appendix.

2 Environment

There are three periods: $t = 0, 1, 2$. There is a continuum of *buyers* whose measure is exogenously given. There is a continuum of potential *sellers* who can endogenously enter the repo market by paying entry cost $k > 0$. The ratio of the measure of buyers to that of sellers is denoted by θ. Each seller holds one unit of a *security*, which pays out value $v > 0$ at the end of period 2. For simplicity, we assume that agents can hold at most one unit of the security, following Duffie, Gârleanu, and Pedersen [6]; Vayanos and Weill [26]; Tomura [23]; and Infante [15] among others. "One unit" can be interpreted as the unit that a buyer wants to trade and that a counterparty seller agrees to trade. Each agent has a linear utility function. Each agent is assumed to have a sufficient amount of cash to cover transactions.

In period 0, buyers and sellers are randomly matched into pairs. The probability of a buyer finding a seller, denoted as $\zeta(\theta)$, and that of a seller finding a buyer, denoted as $\eta(\theta)$, depends on the buyer-to-seller ratio θ. The function $\zeta : \mathbb{R}_{++} \to [0, 1]$ is smooth and strictly decreasing, and the function $\eta : \mathbb{R}_{++} \to [0, 1]$ is smooth and strictly increasing. Because the probability of finding a partner multiplied by the measure of a party must equal the measure of matched pairs, the two functions must satisfy $\eta(\theta) = \theta\zeta(\theta)$. We assume that $\lim_{\theta\to 0+}\eta(\theta) = 0$ and $\lim_{\theta\to\infty}\eta(\theta) = 1$. This is a generalized version of a matching technology in Antinolfi *et al.* [1]. The pairwise-meeting structure here is meant to capture the over-the-counter nature of the repo markets (see Choudhry [3]).

In period 0, each pair of a buyer and a seller determines the terms of a repurchase agreement (or a *repo*). A repo consists of a sequence of transactions, as illustrated in Figure 1. In period 1, which we call the *starting leg*, the buyer purchases the security from the seller for price p_1. In period 2, which we call the *closing leg*, the seller repurchases the security from the buyer for price p_2 if the buyer purchased it in period 1. Notice that they determine the purchase price and the repurchase price in period 0 (prior to the starting leg). Following Trejos and Wright [24], we suppose that they split the expected gains from a repo proportionally (à la Kalai [18]) with the buyer's bargaining power $\sigma \in (0, 1)$. The difference between the repurchase price and the purchase price (i.e., $p_2 - p_1$) is called the *repo interest*. The repo interest is normally expressed as a percentage, called a repo rate, but the expression here is qualitatively equivalent.

If a buyer and a seller make a transaction at the starting leg, the buyer uses the security and the seller uses cash at the end of period 1. If a buyer holds the security at the end of period 1, he enjoys net benefit x. If a seller holds cash at the end of period 1, she enjoys net benefit $\tilde{y}$, which is a random variable realized at the end of period 0. We suppose that a repo is an incomplete contract in the sense that the realization of $\tilde{y}$ cannot be included in the terms of a repo (see ICMA [16]). We assume that $\tilde{y}$ is an i.i.d. uniform random variable on $[y_l, y_h]$ across different sellers. This can be thought of as a version of the assumption in Vayanos and Weill [26] that the motive for an asset sale is some idiosyncratic shock. The net benefits x and $\tilde{y}$ can be interpreted as benefits such as supplying their clients with the security, covering leveraged positions, and hedging derivatives (see ICMA [17]).

We suppose that agents cannot commit to whether they deliver the security or cash.

Following the market practice, we suppose that, when an agent fails to deliver the security or cash, the seller must pay the repo interest to the buyer. Most importantly, when a seller fails to deliver the security at the starting leg, she remains obliged to pay the repo interest, as illustrated in Figure 2 (see Fleming and Garbade [8]). In other words, by paying the repo interest, the seller must compensate the buyer for not being able to use the security at the end of period 1.

Our main focus is what Garbade *et al.* [10] call *strategic fails*, wherein sellers intentionally fail to deliver the security. Although there are other reasons for failing, such as miscommunication and operational problems (see ICMA [17]), we do not model them explicitly. In practice, fails are not unusual and are generally not viewed as events of contractual default, as Fleming and Garbade [9] document. When a fail occurs, repo participants usually choose to negotiate a solution before declaring a default because it is very costly to put a cash borrower into default; it is also considered to be the last resort (see ICMA [17]). Because we are interested in sellers' incentives to fail, we leave our analysis of default to Section 5.2.

Throughout the study, we make the following assumptions about parameters.

Assumption 1. *(i)* $x > 0$, *(ii)* $y_h > 0 > y_l$, *(iii)* $y_h + y_l > 0$, *and (iv)* $-x > y_l$.

Intuitively, the implications of these assumptions are as follows: (*i*) guarantees that, in equilibrium, each buyer has an incentive to deliver cash at the starting leg. This allows us to focus on the incentives of the seller side. (*ii*) implies that, in equilibrium, some sellers choose to fail to deliver the security at the starting leg. This makes our analysis about the sellers' incentives to fail non-trivial. (*iii*) implies that sellers derive positive expected gains from a repo. This guarantees a positive measure of potential sellers to enter the repo market in equilibrium. (*iv*) guarantees that an efficient outcome is physically feasible under the optimal interventions. Even without this assumption, some level of interventions is socially beneficial, although the efficient outcome cannot be attained.

3 Equilibrium and Optimum

In this section, we characterize the equilibrium with no interventions and the social optimum. We show that the equilibrium with no interventions is inefficient because too many sellers fail at the starting leg relative to the social optimum.

3.1 Equilibrium

We write down the equilibrium conditions under no interventions.

Consider the incentives at the closing leg. If a buyer delivers the security at the closing leg, he receives an amount p_2 of cash. If he fails to deliver the security at the closing leg, he receives the repo interest $p_2 - p_1$ and obtains a monetary value v from the security. Hence, the buyer has an incentive to deliver the security at the closing leg if and only if

$$p_2 \geq p_2 - p_1 + v. \tag{1}$$

If a seller delivers cash at the closing leg, she pays an amount p_2 of cash and obtains a monetary value v from the security. If she fails to deliver cash at the closing leg, she pays the repo interest $p_2 - p_1$. Hence, the seller has an incentive to deliver cash at the closing leg if and only if

$$-p_2 + v \geq -(p_2 - p_1). \tag{2}$$

Consider the incentives at the starting leg, given that the incentives at the closing leg are satisfied. If a buyer delivers cash at the starting leg, he pays p_1 at the starting leg, obtains a net benefit x, and receives p_2 at the closing leg. If he chooses not to purchase the security at the starting leg, he receives the repo interest $p_2 - p_1$ at the closing leg. Hence, the buyer has an incentive to deliver cash at the starting leg if and only if

$$-p_1 + x + p_2 \geq p_2 - p_1. \tag{3}$$

If a seller delivers the security at the starting leg, she receives p_1 at the starting leg, obtains a realized value y of the random net benefit $\tilde{y}$, and pays p_2 at the closing leg. If she chooses not to sell the security, she pays the repo interest $p_2 - p_1$ at the closing leg and obtains a monetary value v from the security. Hence, the seller has an incentive to deliver the security at the starting leg if and only if

$$p_1 + y - p_2 + v \geq -(p_2 - p_1) + v. \tag{4}$$

In period 0, a buyer–seller pair determines the repurchase price p_2 by proportional bargaining. The buyer takes a fraction $\sigma \in (0,1)$ of net expected gains from a repo and the seller takes the remaining fraction $1-\sigma$.

Let $\bar{y}$ be the cutoff value of y that satisfies (4) with equality. It is clear that $\bar{y}$ is a key variable to measure efficiency and is zero in the equilibrium with no interventions. Let $\mu_h := \text{Prob}(\tilde{y} \geq \bar{y})$ (resp. $\mu_l = 1 - \mu_h$) be the probability of sellers not failing (resp. sellers failing) at the starting leg. Let $\tilde{y}^e := \mathbb{E}[\tilde{y}|\tilde{y} \geq \bar{y}]$ be the expectation of $\tilde{y}$ conditional on the event that sellers deliver the security at the starting leg. Let $\pi := \mu_h(x + \tilde{y}^e)$ be the expected gains from a repo.

If the seller delivers the security at the starting leg, the buyer obtains $-p_1 + x + p_2$. If the seller fails to deliver the security at the starting leg, the buyer obtains $p_2 - p_1$. Hence, the buyer's proportional bargaining is given by

$$\mu_h(-p_1 + x + p_2) + \mu_l(p_2 - p_1) = \sigma\pi. \tag{5}$$

If the seller delivers the security at the starting leg, the seller obtains $p_1 + \tilde{y}^e - p_2 + v$. If the seller fails to deliver the security at the starting leg, the seller obtains $-(p_2 - p_1) + v$. The seller's outside option is v. Hence, the seller's proportional bargaining is given by

$$\mu_h(p_1 + \tilde{y}^e - p_2 + v) + \mu_l(-p_2 + p_1 + v) - v = (1-\sigma)\pi. \tag{6}$$

There is a continuum of potential sellers who can enter the repo market by paying the entry cost. If a seller finds a buyer, she obtains $\mu_h(p_1 + \tilde{y}^e - p_2 + v) + \mu_l(-p_2 + p_1 + v)$. If a seller does not find a buyer, she obtains v. Because the sellers' net gains from entry must equal zero, we obtain

$$\eta(\theta)\left[\mu_h(p_1 + \tilde{y}^e - p_2 + v) + \mu_l(-p_2 + p_1 + v)\right] + \left[1 - \eta(\theta)\right]v - v = k. \tag{7}$$

Definition 1. *An equilibrium is a tuple* $(p_1, p_2, \theta, \bar{y})$ *that satisfies (1-7).*

Proposition 1. *An equilibrium exists if and only if the expected gains from a repo to the seller are larger than the cost of entry, that is,*

$$k < (1-\sigma)\pi. \tag{8}$$

When there are no interventions, we obtain $\bar{y} = 0$. In this case, the expected gains from a repo can be written in terms of fundamentals as

$$\pi = \frac{y_h}{y_h - y_l}\left(x + \frac{y_h}{2}\right).$$

Proof. Substituting (6) into (7), we obtain

$$\eta(\theta) = \frac{k}{(1-\sigma)\pi}. \tag{9}$$

The right-hand side of (9) is less than 1 if and only if (8) holds. □

3.2 Social Optimum

When (7) is satisfied, the sellers' net expected gains from entry are zero. Hence, the social welfare, denoted as W, simply equals the buyers' expected gains, that is,

$$W = \zeta(\theta)\left[\mu_h(-p_1 + x + p_2) + \mu_l(-p_1 + p_2)\right]. \tag{10}$$

Definition 2. *The social optimum is a tuple* $(p_1, p_2, \theta, \bar{y})$ *that maximize* W *subject to (5-7).*

That is, the social optimum is what is socially optimal subject to the search friction, bargaining friction, and entry cost, but not the incentive constraints of the agents. The following proposition summarizes the social optimum.

Proposition 2. *At the social optimum, (i) the expected gains from a repo are maximized, (ii) the buyer-to-seller ratio is minimized, and (iii) the fraction of sellers failing to deliver the security at the starting leg is lower than in the equilibrium with no interventions.*

Proof. To prove the proposition, we write the social welfare in terms of $\bar{y}$. Substituting (5) into (10) and using $\eta(\theta) = \theta\zeta(\theta)$ and (9), we obtain

$$W = \frac{\sigma}{1-\sigma} \times \frac{k}{\theta}.$$

The only endogenous variable in this expression is the buyer-to-seller ratio θ. The social welfare is maximized when θ is minimized. From (9), θ is minimized when π is maximized.

The expected gains π from a repo are expressed in terms of $\bar{y}$ as

$$\pi = \frac{y_h - \bar{y}}{y_h - y_l} \left(x + \frac{y_h + \bar{y}}{2} \right). \tag{11}$$

Let $\bar{y}^*$ be the value of $\bar{y}$ at the social optimum. From (11), we obtain $\bar{y}^* = -x < 0$. This implies that $\bar{y}$ is higher in the equilibrium with no interventions than at the social optimum. The fraction μ_l of sellers failing at the starting leg depends positively on $\bar{y}$. □

We show in Section 4 that optimal interventions attain $\bar{y} = -x$ in the equilibrium. In other words, they attain the highest expected gains π from a repo. Hence, from Proposition 1, the equilibrium with an optimal intervention exists if the equilibrium with no interventions exists.

4 Policy Analysis

In this section, we study two interventions: a *fails charge* and an *interest reset*. This study sheds light on the workings of these interventions in the model in which the sellers' incentives to fail and the endogenous determination of the repo interest are both explicitly considered. We show that the social optimum is attained in the equilibrium with the optimal interventions. We provide complete characterizations of both the optimal fails charge and the optimal reset interest.

4.1 Fails Charge

The Treasury Market Practices Group introduced a fails charge for U.S. Treasuries on May 1, 2009. Garbade *et al* [10] argue that the fails charge is important for two reasons. First, it mitigates an important dysfunctionality in the repo market of significance to the Federal Reserve in its execution of monetary policy. Second, it exemplifies the value of cooperation between the public and private sectors in responding to altered market conditions.

Motivated by this intervention, we suppose that a seller is required to pay a compensation charge, denoted as $c > 0$, to a buyer when the seller fails to deliver the security at the starting leg. Hence, the buyer obtains $p_2 - p_1 + c$ and the seller obtains $-(p_2 - p_1) - c$. We do not impose a penalty on any other deviation from a repo. This is without loss of generality because $\bar{y}$ is solely determined by the seller's incentive at the starting leg.

Let c^* be the optimal fails charge. It is characterized as follows.

Proposition 3. *The optimal fails charge equals the lost benefit for a failed buyer, that is,*

$$c^* = x.$$

The essential part of the proof is to realize that the seller's incentive (4) to deliver the security at the starting leg is changed as

$$p_1 + y - p_2 + v \geq -(p_2 - p_1) + v - c.$$

Hence, the cutoff value for the seller between failing and not failing at the starting leg is

$$\bar{y} = -c.$$

All the other incentives (1-3) remain unchanged. As in Section 3.2, we obtain $\bar{y}^* = -x$ at the social optimum. Hence, the optimal fails charge is $c^* = x$.

4.2 Interest Reset

The International Capital Market Association has recommended that, when a seller fails to deliver a security at the starting leg of a negative interest repo, the repo interest is automatically reset to zero until the fail is cured. This recommendation has been included as an optional supplementary condition in the Global Master Repurchase Agreement since 2011 (see ICMA [17, 16]).

Motivated by this intervention, we assume that, if a seller fails to deliver the security at the starting leg, the repo interest is forced to be reset to a certain level, denoted as $r \in \mathbb{R}$. Hence, the buyer obtains r and the seller obtains $-r$. Notice that in reality, this interest reset is only applicable for negative interest repos, when both the buyer and the seller agree to sign, and the reset interest is always zero. To characterize the optimal reset interest, we do not restrict our attention to the zero reset.

The optimal reset interest is characterized as follows. Let r^* be the optimal reset interest and $(p_2 - p_1)^*$ be the repo interest at the social optimum.

Proposition 4. *The optimal reset interest is such that the lost benefit for a failed buyer is compensated by the difference between the reset interest and the original repo interest, that is,*

$$x = r^* - (p_2 - p_1)^*. \tag{12}$$

Let π^* be the expected gains from a repo at the social optimum. Then, (12) is equivalent to the optimal reset interest being equal to the expected gains from a repo to a buyer, that is,

$$r^* = \sigma\pi^*,$$

which is always positive. This implies that, although it is in a right direction, the zero reset (i.e., $r = 0$), recommended by ICMA [17] particularly for negative interest repos, does not achieve the social optimum in the baseline environment. This conclusion is overturned when we introduce sellers' default in Section 5.2 or late delivery in Section 5.3. These extensions provide a rationale for the zero reset recommendation.

The essential part of the proof is to realize that the seller's incentive (4) to deliver the security at the starting leg is changed as

$$p_1 + y - p_2 + v \geq -r + v.$$

Hence, the cutoff value for the seller between failing and not failing is

$$\bar{y} = p_2 - p_1 - r.$$

All the other incentives (1-3) remain unchanged. As in Section 3.2, we obtain $\bar{y}^* = -x$ at the social optimum. Hence, the optimal reset interest satisfies (12).

5 Extensions

In this section, we study several extensions of the baseline environment, in which some important aspects of reality, such as leverage, default, and late delivery, are incorporated. As is outlined in the Introduction, these aspects have been extensively studied in existing models, but for different purposes. This study identifies how the optimal interventions are affected in different manners by fundamental variables introduced in these extensions.

In Subsection 5.1, we suppose that agents take leveraged positions using trading opportunities outside the repo pair. We show that the optimal fails charge becomes higher as the outside price falls faster over time, while the optimal reset interest does not even depend on the outside prices. In Subsection 5.2, we suppose that sellers default at the closing leg with some probability. We show that the optimal reset interest becomes lower as sellers default with higher probability, while the result on the optimal fails charge depends crucially on the repo interest. In Subsection 5.3, we suppose that there exists a middle leg at which sellers failing at the starting leg can still make late delivery. We provide a rationale for the fact that, in reality, the fails charge is imposed when a fail is cured. In the second (default) and third (late delivery) extensions, we provide a rationale for the zero interest reset, which stands in contrast to Proposition 4.

5.1 Leveraged Positions

In a repo, one party can borrow cash to finance a long position in a security, and the counterparty can borrow the security to establish a short position (see ICMA [17]). Some studies emphasize the importance of short selling as, for example, a source of increased liquidity in the market (see Vayanos and Weill [26]). The purpose of this extension is to study how leveraged transactions affect the terms of repos and the optimal interventions.

We suppose that sellers are not endowed with a unit of the security, unlike in the baseline environment. Other than participation in the repo market, both buyers and sellers face some outside trading opportunities. In period 1, each seller (resp. buyer) has an opportunity to buy the security from (resp. sell the security to) an outside opportunity for price $\hat{p}_1$. In period 2, each seller (resp. buyer) has an opportunity to sell the security to (resp. buy the security from) an outside opportunity for price $\hat{p}_2$. Implicitly, we suppose that both buyers and sellers want only to take leveraged positions. In other words, we focus on their incentives to participate in both the repo market and the outside opportunity simultaneously, but not in only one of them. We suppose that the outside prices $\hat{p}_1$ and $\hat{p}_2$ are exogenously given and deterministic. We do not explicitly model the pricing mechanism in the outside trading opportunities. We reinterpret x and $\tilde{y}$ as net benefits from taking leveraged positions. These benefits do not include capital gains from outside price changes. For example, a buyer wants to buy it in the repo market in order to supply his client with the security. The environment is otherwise the same as in Section 2.

The difference between the current market value of an asset and the purchase price of the asset in a repo is called a *haircut*. In this environment, the haircut is defined as

$\hat{p}_1 - p_1$. The equilibrium with no interventions is characterized as follows.

Proposition 5. *Suppose that agents are allowed to take leveraged positions. Then, as the outside price falls faster over time, (i) more sellers fail at the starting leg, (ii) the haircut becomes higher, and (iii) the social welfare becomes lower.*

To understand the proposition, consider a seller's incentive at the starting leg. If the seller purchases the security from the outside opportunity and sells it at the starting leg of a repo, she obtains $p_1 - \hat{p}_1$. This is the construction of a leveraged long position. If the seller repurchases the security at the closing leg of a repo and sells it to the outside opportunity, she obtains $\hat{p}_2 - p_2$. This is the settlement of the leveraged long position. The seller has an incentive to construct a leveraged long position at the starting leg if and only if

$$p_1 - \hat{p}_1 + y - p_2 + \hat{p}_2 \geq -(p_2 - p_1).$$

Hence, the cutoff value for the seller between failing and not failing at the starting leg is

$$\bar{y} = -(\hat{p}_2 - \hat{p}_1). \tag{13}$$

This implies that, as $\hat{p}_1 - \hat{p}_2$ becomes higher, more sellers fail at the starting leg.

The buyer has an incentive to construct a short-selling position at the starting leg if and only if

$$-p_1 + \hat{p}_1 + x + p_2 - \hat{p}_2 \geq p_2 - p_1.$$

Hence, we obtain $x \geq \hat{p}_2 - \hat{p}_1$. From (13), we obtain $\bar{y} \geq -x$. We can show that, as in the baseline environment, we obtain $\bar{y}^* = -x$ at the social optimum. Hence, it is always the case that $\bar{y} \geq \bar{y}^*$. This implies that, as $\hat{p}_1 - \hat{p}_2$ becomes higher, the social welfare only becomes lower.

From agents' incentives at the closing leg, the haircut in the equilibrium is $\hat{p}_1 - \hat{p}_2$. Hence, as $\hat{p}_1 - \hat{p}_2$ becomes higher, the haircut becomes higher. Similar results about the haircut are obtained in, for example, Gottardi, Maurin, and Monnet [14]; Infante [15];

and Park and Kahn [22]. In this environment, although the haircut is bounded from below because $\hat{p}_1 - \hat{p}_2 \geq -x$, it can be negative if $\hat{p}_2 > \hat{p}_1$. In reality, there are always non-negative haircuts in repo markets. This is partly because we do not consider the potential loss of the collateral value owing to such factors as price volatility and the cost of liquidating the collateral asset.

5.1.1 Fails Charge

In this environment, the optimal fails charge is characterized as follows.

Proposition 6. *Suppose that agents are allowed to take leveraged positions. Then, the optimal fails charge is* $c^* = x - (\hat{p}_2 - \hat{p}_1)$. *As the outside price falls faster over time, the optimal fails charge becomes higher.*

With a fails charge, we obtain $\bar{y} = -(\hat{p}_2 - \hat{p}_1) - c$. Hence, the optimal fails charge is $c^* = x - (\hat{p}_2 - \hat{p}_1)$. That is, the optimal fails charge equals the lost benefit for a failed buyer. In this environment, the lost benefit for a failed buyer is not only x, but also the capital gain from the outside price fall, that is, $\hat{p}_1 - \hat{p}_2$. The intuition is simple. As the outside price falls faster, more sellers fail at the starting leg. Hence, a higher fails charge should be imposed to prevent too many sellers from failing.

5.1.2 Interest Reset

In this environment, the optimal reset interest is characterized as follows.

Proposition 7. *Suppose that agents are allowed to take leveraged positions. Then, the optimal reset interest is such that* $x - (\hat{p}_2 - \hat{p}_1) = r^* - (p_2 - p_1)^*$. *The optimal reset interest does not depend on the outside prices.*

With the interest reset, we obtain $\bar{y} = p_2 - p_1 - r - (\hat{p}_2 - \hat{p}_1)$. Hence, the optimal reset interest is such that $x - (\hat{p}_2 - \hat{p}_1) = r^* - (p_2 - p_1)^*$. This is equivalent to saying that, just as in the baseline environment, the optimal reset interest equals the expected gains from a repo to a buyer, that is, $r^* = \sigma\pi^*$. This implies that, unlike the optimal fails charge, the optimal reset interest does not depend on the outside prices. This is because the repo rate reacts perfectly as the outside price changes over time. Indeed, the buyer's proportional bargaining implies that

$$p_2 - p_1 = \hat{p}_2 - \hat{p}_1 + \frac{1}{\mu_h}(\sigma\pi - \mu_h x - \mu_l r). \tag{14}$$

In other words, if the outside price falls over time, the repo interest falls by exactly the same amount.

Of course, the implication above is overemphasized to the extent that we assume that *all* agents take leveraged positions. In the baseline environment, the buyer's proportional bargaining implies that

$$p_2 - p_1 = \frac{1}{\mu_h}(\sigma\pi - \mu_h x). \tag{15}$$

Hence, if there is an idiosyncratic shock to a buyer prior to the starting leg whose realization determines whether he needs to take a leveraged position (as in this section) or he simply has sufficiently large cash holdings (as in the baseline environment), then the repo interest is a convex combination between (14), which perfectly reacts to the outside prices, and (15), which is independent of the outside prices.

5.2 Sellers' Default

Placing a counterparty into default is a serious step, which has significant market implications (see ICMA [17]). For example, Donaldson and Micheler [5] emphasize the importance of default as a source of systemic risk. The purpose of this extension is to study how the possibility of sellers' exogenous default affects sellers' strategic fails.

We suppose that sellers who undertake the transaction at the starting leg invest the cash in a risky investment opportunity. The return of the investment opportunity is realized at the end of period 1. With an exogenous probability $\alpha \in (0, 1)$, the return is so small that sellers can pay neither the repurchase price nor the repo interest. In other words, sellers default at the closing leg with probability α. If a seller defaults, she incurs the cost of default, denoted as $\gamma > 0$. Gottardi *et at.* [14] make a similar assumption for the recourse nature of repos. The environment is otherwise the same as in Section 2.

The following Proposition summarizes the effects of default.

Proposition 8. *Suppose that sellers default at the closing leg with probability α, which is sufficiently close to zero. Then, as sellers default with higher probability, more sellers fail at the starting leg and the social welfare becomes lower if the cost of default is larger than the repo interest, that is,*

$$\gamma > \lim_{\alpha \to 0} (p_2 - p_1). \tag{16}$$

To understand the proposition, consider a seller's incentive to deliver the security at the starting leg. If the seller does not default, she obtains $p_1 + y - p_2 + v$. If the seller defaults, she obtains $p_1 + y - \gamma$. Hence, the seller has an incentive to deliver the security at the starting leg if and only if

$$(1 - \alpha)(p_1 + y - p_2 + v) + \alpha(p_1 + y - \gamma) \geq -(p_2 - p_1) + v.$$

Hence, the cutoff value for the seller between failing and not failing at the starting leg is

$$\bar{y} = -\alpha(p_2 - p_1) + \alpha\gamma. \tag{17}$$

Although the repo interest depends on α, we can show that if α is sufficiently close to zero and if (16) holds, $\bar{y}$ is strictly increasing in α.

In this environment, the expected gains from a repo are given by

$$\pi = \mu_h(x + \tilde{y}^e - \alpha\gamma). \tag{18}$$

When α rises, the expected gains from a repo decrease because of the $\alpha\gamma$ term in (18). This is a direct effect of default on social welfare. In addition, as α becomes higher, more sellers fail at the starting leg because of (17). This is an indirect effect of default on social welfare.

From (18), we obtain $\bar{y}^* = -x + \alpha\gamma$ at the social optimum. In other words, $\bar{y}^*$ is strictly increasing in α. This is because, as sellers default with higher probability, it becomes less profitable for them to enter the repo market. If we did not tolerate sellers failing at the starting leg to some extent, there would be too few sellers in the repo market. This would harm buyers by reducing the probability of them finding a repo seller to trade with.

Both $\bar{y}$ and $\bar{y}^*$ are strictly increasing in α. Hence, it is not so obvious whether the equilibrium gets further away from the social optimum, as sellers default with higher probability. We can show that this is indeed the case.

Unlike in the baseline environment, (17) shows that there exists a negative relationship between the repo interest and the fraction of sellers failing at the starting leg. This explains the mechanism of failure by sellers better because, in reality, it is known that the number of fails by sellers tends to increase when the repo interest falls (see Fleming and Garbade [7]).

5.2.1 Fails Charge

In this environment, the optimal fails charge is characterized as follows.

Proposition 9. *Suppose that sellers default at the closing leg with probability α, which is sufficiently close to zero. Then, the optimal fails charge is such that $c^* = x - \alpha(p_2 - p_1)^*$. The optimal fails charge becomes lower as sellers default with higher probability if and only if the repo interest is positive, that is,* $\lim_{\alpha \to 0}(p_2 - p_1)^* > 0$.

With a fails charge, we obtain $\bar{y} = -\alpha(p_2 - p_1) + \alpha\gamma - c$. Hence, the optimal fails charge is such that $c^* = x - \alpha(p_2 - p_1)^*$. The lost benefit for a failed buyer is not only x, but also the repo interest that the failing seller cannot pay because of her default. Hence, unlike in the baseline environment, the optimal fails charge depends on the repo interest. In this way, whether the optimal fails charge is increasing or decreasing in α crucially depends on the repo interest.

5.2.2 Interest Reset

In this environment, the optimal reset interest is characterized as follows.

Proposition 10. *Suppose that sellers default at the closing leg with probability α, which is sufficiently close to zero. Then, the optimal reset interest is such that $x = r^* - (1 - \alpha)(p_2 - p_1)^* + \alpha\gamma$. The optimal reset interest becomes lower as sellers default with higher probability.*

With interest reset, we obtain

$$\bar{y} = (1 - \alpha)(p_2 - p_1) - r. \tag{19}$$

Hence, the optimal reset interest is such that $x = r^* - (1 - \alpha)(p_2 - p_1)^* + \alpha\gamma$. Equation (19) shows that there exists a positive relationship between the repo interest and the fraction of sellers failing at the starting leg. This effect is not expected to be too large in reality, because the interest reset is currently only an optional condition of repos (see ICMA [16]).

Let μ_h^* be the probability of sellers not failing at the social optimum. The optimal reset interest is strictly decreasing in α. In particular, a positive reset interest is socially optimal if and only if the expected gains from a repo to a buyer are larger than the expected cost of default for a seller, that is,

$$\sigma\pi^* > \mu_h^*\alpha\gamma.$$

The zero reset interest is socially optimal when the expected gains from a repo equal the expected cost of default. This extension rationalizes the zero reset interest when sellers default with sufficiently high probability.

5.3 Late Delivery

When a seller fails to deliver a security at the starting leg, the repo remains in force unless the buyer decides to terminate it. Hence, it is possible that the seller makes late delivery between the original purchase date and the repurchase date (see ICMA [17]). The purpose of this extension is to study circumstances in which a seller does not deliver the security on time but may deliver the security any time between the original purchase date and the repurchase date.

We suppose that there are four periods: $t = 0, 1, 2, 3$. In period 0, a buyer and a seller sign a repo contract wherein the promised purchase date is period 1 and the repurchase date is period 3. We call period 1 the starting leg, period 2 the middle leg, and period 3 the closing leg. The buyer buys the security from the seller at the starting leg for price p_1. If the seller fails to deliver the security at the starting leg, no transaction takes place, but the seller can still deliver the security at the middle leg. Following the market practice, we suppose that the purchase price at the middle leg is still p_1.

At the end of period 1, the buyer enjoys net benefit $x_1 > 0$ from holding the security and the seller enjoys net benefit $\tilde{y}_1$ from holding cash. We assume that $\tilde{y}_1$ is an i.i.d. uniform random variable on $[y_l, y_h]$ across different sellers. At the end of period 2, the buyer enjoys net benefit $x_2 > 0$ from holding the security and the seller enjoys net benefit $y_2 > 0$ from holding cash. We assume that y_2 is deterministic. The environment is otherwise the same as in Section 2.

Importantly, even if the seller does not deliver the security on time at the starting leg, there is still a possibility that a buyer can benefit from receiving the security late at the middle leg. Our main focus is to study whether a seller delivers the security on time at the starting leg or she delivers the security late at the middle leg. The repo interest is defined as $p_3 - p_1$.

The assumption that y_2 is positive guarantees that, in the equilibrium, the sellers who fail to deliver the security at the starting leg choose to deliver late at the middle leg. This allows us to focus on sellers' fails at the starting leg. The equilibrium with no interventions and the social optimum is characterized as follows.

Proposition 11. *Suppose that sellers are allowed to make late delivery. Then, the fraction of sellers failing at the starting leg is higher in the equilibrium with no interventions than at the social optimum by the lost benefit for a failed buyer from late delivery.*

To observe this, consider a seller's incentive to deliver the security at the starting leg. When she does not deliver the security at the starting leg, she has two options. If she delivers the security at the middle leg, she obtains $p_1 + y_2 - p_3 + v$. If she does not deliver the security until the closing leg, she obtains $-(p_3 - p_1) + v$. Hence, the seller has an incentive to deliver the security at the starting leg if and only if

$$p_1 + y_1 + y_2 - p_3 + v \geq \max\{p_1 + y_2 - p_3 + v, -(p_3 - p_1) + v\}. \qquad (20)$$

Let $\bar{y}_1$ be the cutoff value that satisfies (20) with equality and $\bar{y}_1^*$ be part of the social optimum. Because we assume $y_2 > 0$, the seller prefers late delivery to a fail at the middle leg, that is, $p_1 + y_2 - p_3 + v > -(p_3 - p_1) + v$. Hence, we obtain $\bar{y}_1 = 0$ from (20). Meanwhile, we obtain $\bar{y}_1^* = -x_1$ at the social optimum, because the expected gains from a repo are given by

$$\pi = \frac{y_h - \bar{y}_1}{y_h - y_l}\left(x_1 + \frac{y_h + \bar{y}_1}{2}\right) + x_2 + y_2.$$

We study the optimal interventions. In this environment, sellers do not have an incentive to fail at the middle leg without any penalty. Hence, the penalty on fails at the middle leg (i.e., no delivery at all) can be anything as long as it is larger than or equal to the penalty on fails at the starting leg (i.e., late delivery at the middle leg). Without loss of generality, we impose the same penalty on fails at the middle leg as that on fails at the starting leg. Let c be the fails charge at the starting leg and r be the reset interest at the starting leg.

5.3.1 Fails Charge

In this environment, the optimal fails charge is characterized as follows.

Proposition 12. *Suppose that sellers are allowed to make late delivery. Then, the optimal fails charge equals the lost benefit for a failed buyer from late delivery, that is,* $c^* = x_1$.

With a fails charge, we obtain $\bar{y}_1 = -c$. Hence, the optimal fails charge is $c^* = x_1$. It is important to note that $c^* \neq x_1 + x_2$. An important implication of this finding is that the optimal fails charge depends on the duration of a fail. In reality, the fails charge is imposed *when a fail is cured, not when a fail occurs.* The proposition rationalizes this fact.

5.3.2 Repo Interest Reset

In this environment, the optimal reset interest is characterized as follows.

Proposition 13. *Suppose that sellers are allowed to make late delivery. Then, the optimal reset interest is such that the lost benefit for a failed buyer from late delivery is compensated by the difference between the reset interest and the original repo interest, that is,* $x_1 = r^* - (p_3 - p_1)^*$.

With interest reset, we obtain $\bar{y}_1 = p_3 - p_1 - r$. Hence, the optimal reset interest is such that $x_1 = r^* - (p_3 - p_1)^*$. This is equivalent to the optimal reset interest being equal to the expected gains from a repo to a buyer minus the net benefit to the buyer from late delivery, that is,

$$r^* = \sigma\pi^* - x_2.$$

Unlike in the baseline environment, the optimal reset interest can be negative if, for example, the bargaining power σ of the buyer is sufficiently close to zero and the net benefit x_2 from late delivery for the buyer is sufficiently large.

6 Conclusions

We develop a search theoretic model of repos wherein the sellers' incentives to strategically fail and the endogenous determination of the repo interest are both explicitly incorporated. In the framework, we study two types of interventions: a fails charge and an interest reset. We show that if they are implemented at the optimal level, both interventions achieve the efficient outcome. These interventions improve efficiency by lowering the fraction of sellers failing at the starting leg and making it easier for buyers to find their counterparties. We provide a complete characterization of the optimal interventions. The optimal fails charge equals the lost benefit for a failed buyer. The optimal reset interest is such that the lost benefit for a failed buyer equals the difference between the reset interest and the original repo interest. The result for the optimal reset suggests that the zero reset for negative interest repos does not achieve the social optimum in the baseline environment.

In three extensions, we study leveraged transactions, sellers' default, and late delivery. The results suggest that the nature of the optimal fails charge and that of the optimal interest reset are very different. In the first (leverage) extension, we show that the optimal fails charge becomes higher as the outside price falls faster over time, while the optimal reset interest does not even depend on the outside prices. In the second (default) extension, the optimal reset interest becomes lower as sellers default with higher probability, while the result on the optimal fails charge depends crucially on the repo interest. In the third (late delivery) extension, we provide a rationale for the fact that, in reality, the fails charge is imposed when a fail is cured. In the second and third extensions, we provide a rationale for the zero reset under some parameter conditions, which is in contrast to the result obtained in the baseline environment.

This study is particularly relevant to understand the extraordinary volume of fails during the recent financial crisis as well as those in other episodes and the workings of the novel interventions implemented recently. In various setups that are relevant to the existing literature, we show that the optimal fails charge and the optimal interest reset are affected in very different manners by fundamental variables such as asset prices outside a repo pair and the probability of sellers' default. This emphasizes that, when the interventions are implemented, a policymaker needs to carefully distinguish between the two. In this study, we focus on the microstructure of the repo market. The importance of the sellers' incentives to fail and their implications on the optimal interventions under various setups leave an open question. How would dysfunctionality in the repo markets arising from strategic fails propagate to other markets, in particular, to the real sectors or the entire economy? Such investigation would be an interesting direction for future research and would call for a more macroeconomic-oriented model with a flavor of the methodology developed here.

Appendix: Proofs

Proof of Proposition 3. The buyer's proportional bargaining is given by

$$\mu_h(-p_1 + x + p_2) + \mu_l(-p_1 + p_2 + c) = \sigma\pi.$$

The seller's proportional bargaining is given by

$$\mu_h(p_1 + y - p_2 + v) + \mu_l(p_1 - p_2 - c) - v = (1-\sigma)\pi.$$

The zero profit condition for sellers is given by

$$\eta(\theta)\left[\mu_h(p_1 + y - p_2 + v) + \mu_l(p_1 - p_2 - c)\right] + \left[1 - \eta(\theta)\right] v - v = k.$$

The social welfare is given by

$$W = \zeta(\theta)\left[\mu_h(-p_1 + x + p_2) + \mu_l(-p_1 + p_2 + c)\right].$$

In the same way as we obtain the proof of Proposition 2, we obtain the same expressions as (9), (11), and

$$W = \frac{\sigma}{1-\sigma} \times \frac{k}{\theta}. \tag{21}$$

This implies that we obtain $\bar{y}^* = -x$ at the social optimum. Because we obtain $\bar{y} = -c$, the optimal fails charge is $c^* = x$. □

Proof of Proposition 4. The buyer's proportional bargaining is given by

$$\mu_h(-p_1 + x + p_2) + \mu_l r = \sigma\pi.$$

The seller's proportional bargaining is given by

$$\mu_h(p_1 + y - p_2 + v) + \mu_l(-r) - v = (1 - \sigma)\pi.$$

The zero profit condition of sellers is given by

$$\eta(\theta)\left[\mu_h(p_1 + y - p_2 + v) + \mu_l(-r)\right] + \left[1 - \eta(\theta)\right] v - v = k.$$

The social welfare is given by

$$W = \zeta(\theta)\left[\mu_h(-p_1 + x + p_2) + \mu_l r\right].$$

In the same way as we obtain the proof of Proposition 2, we obtain the same expressions as (9), (11), and (21). This implies that we obtain $\bar{y}^* = -x$ at the social optimum. Because we obtain $\bar{y} = p_2 - p_1 - r$, the optimal reset interest satisfies $x = r^* - (p_2 - p_1)^*$. □

Proof of Proposition 5. The proof has two parts: In Part 1, we show that $p_1 = \hat{p}_2$. In Part 2, we show that $\bar{y}^* = -x$ at the social optimum.

Part 1. We show that $p_1 = \hat{p}_2$. We consider incentives at the closing leg. The buyer has an incentive to purchase the security from the outside opportunity and sell it at the closing leg of a repo if and only if

$$p_2 - \hat{p}_2 \geq p_2 - p_1.$$

The seller has an incentive to purchase the security at the closing leg of a repo and sell it to the outside opportunity if and only if

$$-p_2 + \hat{p}_2 \geq -(p_2 - p_1).$$

They together imply that $p_1 = \hat{p}_2$.

Part 2. We show that $\bar{y}^* = -x$ at the social optimum. The buyer's proportional bargaining is given by

$$\mu_h(-p_1 + \hat{p}_1 + x + p_2 - \hat{p}_2) + \mu_l(p_2 - p_1) = \sigma\pi.$$

The seller's proportional bargaining is given by

$$\mu_h(p_1 - \hat{p}_1 + \tilde{y}^e - p_2 + \hat{p}_2) + \mu_l(p_1 - p_2) = (1 - \sigma)\pi.$$

The zero profit condition of sellers is given by

$$\eta(\theta)\left[\mu_h(p_1 - \hat{p}_1 + \tilde{y}^e - p_2 + \hat{p}_2) + \mu_l(p_1 - p_2)\right] = k.$$

The social welfare is given by

$$W = \zeta(\theta)\left[\mu_h(-p_1 + \hat{p}_1 + x + p_2 - \hat{p}_2) + \mu_l(p_2 - p_1)\right].$$

In the same way that we obtain the proof of Proposition 2, we obtain the same expressions as (9), (11), and (21). This implies that we obtain $\bar{y}^* = -x$ at the social optimum. □

Proof of Proposition 6. We derive only the cutoff $\bar{y}$. The seller has an incentive to purchase the security from the outside opportunity and sell it at the starting leg of a repo if and only if

$$p_1 - \hat{p}_1 + y - p_2 + \hat{p}_2 \geq -(p_2 - p_1) - c.$$

From this, we obtain

$$\bar{y} = -(\hat{p}_2 - \hat{p}_1) - c,$$

which completes the proof. □

Proof of Proposition 7. We derive only the cutoff $\bar{y}$. The seller has an incentive to purchase the security from the outside opportunity and sell it at the starting leg of a repo if and only if

$$p_1 - \hat{p}_1 + y - p_2 + \hat{p}_2 \geq -r.$$

From this, we obtain

$$\bar{y} = p_2 - p_1 - r - (\hat{p}_2 - \hat{p}_1),$$

which completes the proof. □

Proof of Proposition 8. The proof has three parts. In Part 1, we derive the same expression as (9) and (21), except that π has a different expression. This implies that social welfare is strictly decreasing in α if π is strictly decreasing in α. In Part 2, we show that π is strictly decreasing in α if $\bar{y}$ is strictly increasing in α. In Part 3, we show that $\bar{y}$ is strictly increasing in α.

Part 1. We write down the equilibrium conditions for this environment. The incentive constrains at the closing leg are the same as (1) and (2). The buyer has an incentive to deliver cash at the starting leg if and only if

$$(1-\alpha)(-p_1 + x + p_2) + \alpha(-p_1 + x + v) \geq p_2 - p_1.$$

The cutoff value for the seller between failing and not failing at the starting leg is given by (17).

The expected gains π from a repo are given by (18). The buyer's proportional bargaining is given by

$$\mu_h \left\{(1-\alpha)(-p_1 + x + p_2) + \alpha(-p_1 + x + v)\right\} + \mu_l(p_2 - p_1) = \sigma\pi. \tag{22}$$

The seller's proportional bargaining is given by

$$\begin{aligned}\mu_h \left\{(1-\alpha)(p_1 + \tilde{y}^e - p_2 + v) + \alpha(p_1 + \tilde{y}^e - \gamma)\right\} + \mu_l(v - p_2 + p_1) - v \\ = (1-\sigma)\pi.\end{aligned}$$

The zero profit condition of sellers is given by

$$\begin{aligned}\eta(\theta)\left[\mu_h \left\{(1-\alpha)(p_1 + \tilde{y}^e - p_2 + v) + \alpha(p_1 + \tilde{y}^e - \gamma)\right\} + \mu_l(v - p_2 + p_1)\right] \\ +[1-\eta(\theta)]v - v = k.\end{aligned}$$

The social welfare is given by

$$W = \zeta(\theta)\left[\mu_h \left\{(1-\alpha)(-p_1 + x + p_2) + \alpha(-p_1 + x + v)\right\} + \mu_l(p_2 - p_1)\right].$$

In the same way that we obtain the proof of Proposition 2, we obtain the same expressions as (9) and (21).

Part 2. We show that π is strictly decreasing in α. We suppose that α is sufficiently close to zero. We obtain

$$\lim_{\alpha \to 0} \frac{\partial \pi}{\partial \alpha} = -\frac{1}{y_h - y_l}\left(x \lim_{\alpha \to 0} \frac{\partial \bar{y}}{\partial \alpha} + y_h \gamma\right),$$

which implies that π is strictly decreasing in α if $\bar{y}$ is strictly increasing in α.

Part 3. We show that $\bar{y}$ is strictly increasing in α. From (22), we obtain

$$p_2 - p_1 = \frac{1}{1-\alpha}(\sigma\pi - \mu_h x - \sigma\alpha\gamma).$$

Substituting it into (17) yields the following equation for $\bar{y}$:

$$\left(1 - \alpha\frac{y_h - \bar{y}}{y_h - y_l}\right)(\bar{y} - \alpha\gamma) + \alpha\frac{y_h - \bar{y}}{y_h - y_l}\left\{\sigma\frac{y_h + \bar{y}}{2} - (1-\sigma)x - \sigma\alpha\gamma\right\} = 0. \tag{23}$$

Define a function F by

$$F(\hat{y}, \alpha) = \left(1 - \alpha\frac{y_h - \bar{y}}{y_h - y_l}\right)(\hat{y} - \alpha\gamma) + \alpha\frac{y_h - \hat{y}}{y_h - y_l}\left\{\sigma\frac{y_h + \hat{y}}{2} - (1-\sigma)x - \sigma\alpha\gamma\right\}.$$

Equation (23) is equivalent to $F(\bar{y}, \alpha) = 0$.

To show that $\bar{y}$ is strictly increasing in α, we show that (i) F is strictly increasing in $\hat{y}$ and (ii) F is strictly decreasing in α at $\hat{y} = \bar{y}$. First, we obtain

$$\lim_{\alpha \to 0} \frac{\partial F}{\partial \hat{y}}(\hat{y}, \alpha) = 1,$$

which implies that F is strictly increasing in $\hat{y}$. Second, because (16) holds, we obtain

$$\lim_{\alpha \to 0} \frac{\partial F}{\partial \alpha}(\bar{y}, \alpha) = -\gamma + \lim_{\alpha \to 0}(p_2 - p_1) < 0,$$

which implies that F is strictly decreasing in α at $\hat{y} = \bar{y}$. Hence, $\bar{y}$ is strictly increasing in α. □

Proof of Proposition 9. We show that the optimal fails charge is strictly decreasing in α if and only if $\lim_{\alpha\to 0}(p_2 - p_1)^* > 0$. The seller has an incentive to deliver the security at the starting leg if and only if

$$(1-\alpha)(p_1 + y - p_2 + v) + \alpha(p_1 + y - \gamma) \geq -(p_2 - p_1) + v - c.$$

Hence, the cutoff value for the seller between failing and not failing at the starting leg is

$$\bar{y} = -\alpha(p_2 - p_1) + \alpha\gamma - c.$$

This implies that the optimal fails charge is given by

$$c^* = x - \alpha(p_2 - p_1)^*. \tag{24}$$

The buyer's proportional bargaining is given by

$$\mu_h\{(1-\alpha)(-p_1 + x + p_2) + \alpha(-p_1 + x + v)\} + \mu_l(p_2 - p_1 + c) = \sigma\pi.$$

From this, we obtain

$$p_2 - p_1 = \frac{1}{\mu_h(1-\alpha) + \mu_l}(\sigma\pi - \mu_h x - \mu_l c). \tag{25}$$

From (24) and (25), we obtain

$$c^* = \frac{1}{1-\alpha}\left\{x - \alpha\sigma\frac{y_h + x - \alpha\gamma}{y_h - y_l}\left(x + \frac{y_h - x + \alpha\gamma}{2} - \alpha\gamma\right)\right\}.$$

From this, we obtain

$$\lim_{\alpha\to 0}\frac{\partial c^*}{\partial \alpha} = -\lim_{\alpha\to 0}(p_2 - p_1)^*,$$

which completes the proof. □

Proof of Proposition 10. We show that the optimal reset interest is strictly increasing in α. The seller has an incentive to deliver the security at the starting leg if and only if

$$(1-\alpha)(p_1 + y - p_2 + v) + \alpha(p_1 + y) \geq -r + v.$$

Hence, the cutoff value for the seller between failing and not failing at the starting leg is

$$\bar{y} = (1-\alpha)(p_2 - p_1) - r.$$

This implies that the optimal reset interest is such that

$$x = r^* - (1-\alpha)(p_2 - p_1)^* + \alpha\gamma. \tag{26}$$

The buyer's proportional bargaining is given by

$$\mu_h\{(1-\alpha)(-p_1 + x + p_2) + \alpha(-p_1 + x + v)\} + \mu_l r = \sigma\pi.$$

From this, we obtain

$$p_2 - p_1 = \frac{1}{\mu_h(1-\alpha)}\left(\sigma\pi - \mu_h x - \mu_l r\right). \tag{27}$$

From (26) and (27), we obtain

$$r^* = \frac{y_h + x - \alpha\gamma}{2(y_h - y_l)}\left\{\sigma(y_h + x) - (2+\sigma)\alpha\gamma\right\}.$$

From this, we obtain

$$\lim_{\alpha\to 0}\frac{\partial r^*}{\partial \alpha} = -(1+\sigma)\frac{y_h + x}{y_h - y_l}\gamma < 0,$$

which completes the proof. □

Proof of Proposition 11. We show that $\bar{y}_1^* = -x_1$ at the social optimum. The buyer's proportional bargaining is given by

$$\mu_h(-p_1 + x_1 + x_2 + p_3) + \mu_l(-p_1 + x_2 + p_3) = \sigma\pi.$$

The seller's proportional bargaining is given by

$$\mu_h(p_1 + \tilde{y}_1^e + y_2 - p_3 + v) + \mu_l(p_1 + y_2 - p_3 + v) - v = (1-\sigma)\pi.$$

The zero profit condition of sellers is given by

$$\eta(\theta)\left[\mu_h(p_1 + \tilde{y}_1^e + y_2 - p_3 + v) + \mu_l(p_1 + y_2 - p_3 + v)\right] + \left[1 - \eta(\theta)\right]v - v = k.$$

The social welfare is given by

$$W = \zeta(\theta)\left[\mu_h(-p_1 + x_1 + x_2 + p_3) + \mu_l(-p_1 + x_2 + p_3)\right].$$

In the same way that we obtain the proof of Proposition 2, we obtain the same expressions as (9) and (21) with a different expression for π. The expected gains π from a repo are

$$\pi = \frac{y_h - \bar{y}_1}{y_h - y_l}\left(x_1 + \frac{y_h + \bar{y}_1}{2}\right) + x_2 + y_2.$$

Hence, we obtain $\bar{y}^* = -x_1$. □

Proof of Proposition 12. We derive only the cutoff $\bar{y}_1$. The seller has an incentive to deliver the security at the starting leg if and only if

$$p_1 + y_1 + y_2 - p_3 + v \geq \max\{p_1 + y_2 - p_3 + v - c, -(p_3 - p_1) + v - c\}.$$

From this, we obtain

$$\bar{y}_1 = -c,$$

which completes the proof. □

Proof of Proposition 13. We derive only the cutoff $\bar{y}_1$. The seller has an incentive to deliver the security at the starting leg if and only if

$$p_1 + y_1 + y_2 - p_3 + v \geq \max\{y_2 - r + v, -r + v\}.$$

From this, we obtain

$$\bar{y}_1 = -r + (p_3 - p_1),$$

which completes the proof. □

References

[1] Antinolfi, Gaetano, Francesca Carapella, Charles M. Kahn, Antoine Martin, David C. Mills, Jr., and Ed Nosal (2015) "Repos, Fire Sales, and Bankruptcy Policy," *Review of Economic Dynamics* 18, pp. 21–31.

[2] Bottazi, Jean-Marc, Jaime Luque, and Mário R. Páscoa (2012) "Securities Market Theory: Possession, Repo and Rehypothecation," *Journal of Economic Theory* 147, pp. 477–500.

[3] Choudhry, Moorad (2010) *The Repo Handbook*, 2nd ed., Butterworth-Heinemann, Oxford, United Kingdom.

[4] Copeland, Adam, Antoine Martin, and Michael Walker (2014) "Repo Runs: Evidence from the Tri-Party Repo Market," *Journal of Finance* 6, pp. 2343–2380.

[5] Donaldson, Jason Roderick, and Eva Micheler (2018) "Resaleable Debt and Systemic Risk," *Journal of Financial Economics* 127, pp. 485–504.

[6] Duffie, Darrell, Nicolae Gârleanu, and Lasse H. Pedersen (2007) "Valuation in Over-the-Counter Markets," *Review of Financial Studies* 20, pp. 1865–1900.

[7] Fleming, Michael J., and Kenneth D. Garbade (2002) "When the Back Office Moved to the Front Burner: Settlement Fails in the Treasury Market after 9/11," *Economic Policy Review* 8, pp. 35–57.

[8] Fleming, Michael J., and Kenneth D. Garbade (2004) "Repurchase Agreements with Negative Interest Rates," *Current Issues in Economics and Finance* 10, pp. 1–7.

[9] Fleming, Michael J., and Kenneth D. Garbade (2005) "Explaining Settlement Fails," *Current Issues in Economics and Finance* 11, pp. 1–7.

[10] Garbade, Kenneth D., Frank M. Keane, Lorie Logan, Amanda Stokes, and Jennifer Wolgemuth (2010) "The Introduction of the TMPG Fails Charge for U.S. Treasury Securities," *Economic Policy Review* 16, pp. 45–71.

[11] Garbade, Kenneth D., and Frank M. Keane (2017) "The Treasury Market Practices Group: Creation and Early Initiatives," Federal Reserve Bank of New York Staff Reports No. 822.

[12] Gorton, Gary, and Andrew Metrick (2012) "Securitized Banking and the Run on Repo," *Journal of Financial Economics* 104, pp. 425–451.

[13] Gorton, Gary, Andrew Metrick, and Chase P. Ross (2020) "Who Ran on Repo?" *AEA Papers and Proceedings* 110, pp. 487–492.

[14] Gottardi, Piero, Vincent Maurin, and Cyril Monnet (2019) "A Theory of Repurchase Agreements, Collateral Re-Use, and Repo Intermediation," *Review of Economic Dynamics* 33, pp. 30–56.

[15] Infante, Sebastian (2019) "Liquidity Windfalls: The Consequences of Repo Rehypothecation," *Journal of Financial Economics* 133, pp. 42–63.

[16] International Capital Market Association (2013) "2011 Global Master Repurchase Agreement," June 2013.

[17] International Capital Market Association (2020) "A Guide to Best Practice in the European Repo Market," September 2020.

[18] Kalai, Ehud (1977) "Proportional Solutions to Bargaining Situations: Interpersonal Utility Comparisons," *Econometrica* 45, pp. 1623–1630.

[19] Krishnamurthy, Arvind, Stefan Nagel, and Dmitry Orlov (2014) "Sizing Up Repo," *Journal of Finance* 69, pp. 2381–2417.

[20] Martynova, Natalya, Lev Ratnovski, and Razvan Vlahu (2020) "Bank Profitability, Leverage Constraints, and Risk-Taking," *Journal of Financial Intermediation* 44, pp. 1–19.

[21] Nuño, Galo, and Carlos Thomas (2017) "Bank Leverage Cycles," *American Economic Journal: Macroeconomics* 9, pp. 32–72.

[22] Park, Hyejin, and Charles M. Kahn (2019) "Collateral, Rehypothecation, and Efficiency," *Journal of Financial Intermediation* 39, pp. 34–46.

[23] Tomura, Hajime (2016) "Investment Horizon and Repo in the Over-the-Counter Market," *Journal of Money, Credit and Banking* 48, pp. 145–164.

[24] Trejos, Alberto, and Randall Wright (2016) "Search-Based Models of Money and Finance: An Integrated Approach," *Journal of Economic Theory* 164, pp. 10–31.

[25] Valderrama, Laura (2015) "Macroprudential Regulation under Repo Funding," *Journal of Financial Intermediation* 24, pp. 178–199.

[26] Vayanos, Dimitri, and Pierre-Olivier Weill (2008) "A Search-Based Theory of the On-the-Run Phenomenon," *Journal of Finance* 63, pp. 1361–1398.