Escaping the middle income trap and getting economic growth: How does FDI can help the host country?

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Escaping the middle income trap and getting economic growth: How does FDI can help the host country?

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Abstract

The paper investigates the country receiving FDI's optimal strategy in an optimal growth context. First, if the multinational enterprise has high productivity or the entry cost is high, no domestic firm enters the new industry. Still, the host economy's investment stock converges to a higher steady state than that of the closed economy. Second, if the old sector is strong enough and the domestic firm's productivity is high, the foreign firm will be dominated, even eliminated by the domestic one. Third, we show that if the host country invests in R&D, its economy may grow without bounds. In this case, FDI helps the host country only at the first stages of its development process. We present empirical evidence that supports our theoretical findings.

Keywords: Optimal growth, FDI, MNE, R&D, fixed cost.
JEL Classifications: E2, F23, F4, O3, O4.

1 Introduction

Over the past few decades, opening-up to the global economy and attracting foreign direct investment (FDI) are significant policy priorities in developing countries for promoting their economic growth. The main argument is that multinational enterprises (MNEs) would boost investment, bring new technologies, (management) skills, and generate FDI spillovers on domestic firms. However, the effects of FDI on the host country's development is far from clear.

At the micro-level, MNEs are expected to generate spillovers either to their domestic competitors in the same industry (horizontal spillovers) or upstream and downstream local firms (vertical spillovers). Concerning vertical FDI, empirical evidences show positive spillovers from downstream FDI firms (particularly joint venture FDI firms) to domestic input suppliers, but negative spillovers from upstream FDI firms to

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downstream domestic producers.[1] Moreover, a large body of literature provides strong evidence of mixed results concerning horizontal spillovers from FDI.[2][3]

At the macro-level, the empirical literature finds that the effect of FDI on the host country's economic growth is relatively weak (Carkovic and Levine, 2005). More precisely, whether this effect is significant or not depends on local conditions such as the host country's human capital (Borensztein et al., 1998; Li and Liu, 2005) and the development of local financial markets (Alfaro et al., 2004, 2010).

The previous conflicting results on the effects of FDI motivate us to address fundamental questions: What is the optimal strategy of a country receiving FDI? How does FDI can help the host country to escape the middle-income trap and potentially get economic growth in the long run? Our paper aims to investigate these questions by using both theoretical and empirical approaches. We first introduce FDI in optimal growth models and use them to study the optimal allocation of the host country receiving FDI. We then provide empirical evidence supporting our theoretical findings.

Let us briefly describe the ingredients of our optimal growth models. The host country is assumed to be a small open economy with three goods: consumption, physical capital, and new good. These commodities are freely tradable with the rest of the world. There are two agents (a representative consumer of the host country and an MNE) and two production sectors (a traditional industry producing the consumption good and a new industry fabricating the new good). Assume that only domestic firms in the host country produce the consumption good using physical capital as the sole input. By contrast, producing the new good requires physical capital and so-called *specific labor* (or skilled labor). In the beginning, there is the sole MNE in the new sector (i.e., FDI takes place.) However, a domestic firm can enter this sector and produce the new good only if it holds a critical level of specific labor. This threshold represents a setup cost. By contrast, the MNE does not have to pay that fixed cost, thanks to its parent firm's support.[4]

Our main contribution is twofold. First, we explore conditions under which the host country should (or should not) invest in the new industry where the MNE has

[1]For more discussions on vertical FDI spillover, see Javorcik (2004), Newman et al. (2015), Lu et al. (2017) for the case of Lithuania, Vietnam, China, respectively, and Gorodnichenko et al. (2014) for 17 transition countries.

[2]Indeed, there are negative or nil impacts of horizontal FDI on domestic firms in developing countries as, for example, Morocco (Haddad and Harrison, 1993), Uruguay (Kokko et al., 1996), Eastern Europe countries (Jude, 2012), Vietnam (Newman et al., 2015). By contrast, evidence of positive horizontal spillovers from FDI in developed countries is reported in Ruane and Ugur (2005) for Ireland, Haskel et al. (2007) for the UK, or Keller and Yeaple (2009) for the US.

[3]See Blomstrom and Kokko (1998); Greenaway and Gorg (2004); Crespo and Fontoura (2007) for more complete reviews of FDI spillovers, and Meyer and Sinani (2009), Iršová and Havránek (2013) for meta-analyses.

[4]Our assumption on the setup cost is in line with several studies in the literature. Indeed, Smith (1987) and Markusen (1995) pointed out that a potential domestic firm has to pay a firm-specific fixed cost to enter a new industry. By contrast, the MNE has a plant in its home country where it has already invested in that specific cost. Hence, this firm does not suffer such expenditure in producing in the host country (Smith, 1987). In another context, Fosfuri et al. (2001) indicated that a domestic firm might access new technologies thanks to worker mobility who initially worked for the MNE. To this end, the domestic firm has to pay a fixed cost that one interprets as an absorptive capability. In our framework, the host country must have at least a critical number of skilled workers to set up the production process.

been well installed. We prove that if the host country has a low initial resource or the setup cost is high, or the FDI spillover effect is insufficient, no domestic firm can operate in the new industry and produce the new good regardless of its total factor productivity (TFP hereafter). Once these necessary conditions hold, the host country produces the new good if (and only if) the potential domestic firm in the new industry has a high TFP. Moreover, the domestic firm can dominate and even eliminate the MNE (in the sense that the MNE stops its production in the host country) if its TFP is high enough. Our empirical investigation in Vietnamese manufacturing industries (during the period 2000-2016) strongly supports our theoretical findings.

Our finding contributes to explain why horizontal FDI's impact on domestic firms may be insignificant or positive, as reported in empirical studies. We show that whether this impact is positive depends not only on the local conditions (resource, human capital, ...) but also on time. Indeed, in the beginning, there is no domestic firm that can operate in the industry where MNE has been well installed. However, the impact turns out to be positive after some periods. Our point about the role of timing on domestic firms' development is in line with empirical investigations of Merlevede et al. (2014) and references therein. Indeed, Merlevede et al. (2014), by using firm-level data from a panel of Romanian manufacturing firms during 1996-2005, find that the effect of foreign entry is initially negative but will be positive for a longer time.

Our second contribution is to investigate the interplay between FDI, R&D, and growth in an endogenous growth context. We substantiate that with the presence of FDI spillovers and by investing in R&D, some host countries can avoid the middle-income trap and reach a higher economic growth level. Furthermore, if local circumstances in terms of investment efficiency are good enough, a host developing country can obtain unbounded growth and catch up with developed economies. It is interesting to notice that a country may get economic growth in the long run even it does not receive FDI. Our empirical investigation on 52 developing countries over 1996-2018 is likely to support our theoretical results. Countries having invested in R&D seem to exhibit higher economic growth.

Our result leads to an interesting implication: Consider a low-income country so that the country cannot immediately invest in R&D and new technology. If the leverage of new technology is high enough or the country has potential in R&D, the optimal strategy of the country should be as follows:

- Stage 1: the country should train specific workers.
- Stage 2: specific workers will work for the MNE to get a favorable salary and improve the country's income and capital stock.
- Stage 3: once the country's resource is high enough, it should focus on R&D to create new technology that increases the domestic firms' TFP. Thanks to this, its economy may have sustainable growth in the long run.

The existing literature provides some theoretical models to study the effect of FDI on growth. Looking back to history, Findlay (1978) attempts to study the role of FDI in a dynamic framework by assuming that the sequences of domestic and foreign firms' capital stocks are determined by a continuous time dynamical system.[5] A key insight in Findlay (1978) is his assumption of 'contagion' effect: the level of efficiency of domestic firms depends on (but it is lower than) that of the advanced part of the world. Wang (1990) develops this idea by assuming that there is technology diffusion: the host country's human capital stock is an increasing function of the ratio of foreign investment to domestically owned capital. By using this modeling of FDI and a two-country model with free capital mobility and exogenous propensities to save, Wang (1990) shows that opening to FDI has beneficial implications for the host country.

Notice that in Wang (1990), the propensity to save is fixed. Other papers considers models with endogenous saving rate. In a continuous time model with a continuum of varieties of capital goods,[6] Borensztein et al. (1998) model FDI as the fraction of varieties produced by foreign firms in the total varieties of products. Under specific setups (Cobb-Douglas production and CRRA utility functions), they compute the rate of growth in the steady state equilibrium, which is an increasing function of the fraction of varieties produced by foreign firms in the total varieties of products. Berthélemy and Démurger (2000) extend Borensztein et al. (1998)'s model by endogenizing the numbers of varieties produced by domestic and foreign firms. As in Borensztein et al. (1998), Berthélemy and Démurger (2000) focus on the steady state equilibrium and compute the growth rate of the host country in the case of Cobb-Douglas production and CRRA utility functions. Using a continuous time product variety-based endogenous growth, Alfaro et al. (2010) study the role of local financial markets in enabling FDI to promote growth through backward linkages.[7] Alfaro et al. (2010) focus on the balanced growth path and their calibration shows that an increase in FDI leads to higher growth rates in financially developed countries compared to those observed in financially poorly developed ones.

Our model is distinct from the above papers in two ways. First, we introduce FDI and study its effect in infinite-horizon optimal growth frameworks. Second, we study the global dynamics of the optimal paths and provide the qualitative analyses without restrictions on the utility function. By the way, we are able to figure out the optimal strategy of the host country receiving FDI.

Our paper also contributes to the literature on optimal growth with thresholds (see Azariadis and Drazen, 1990; Bruno et al., 2009; Le Van et al., 2010, 2016 among others) and increasing returns (see Romer, 1986; Jones and Manuelli, 1990; Kamihigashi and Roy, 2007 among others). Our added value is to show the role of FDI. We point out that FDI may partially contribute to the capital accumulation of the host country and hence enable the country to overcome the threshold at the first stage of its development process. However, whether a host country can obtain a sustainable growth in the long run does not depend on FDI but local conditions (mainly its innovation capacity, and the efficiency of its investment in R&D). In a technical point of view, our analysis is far from trivial because of the presence of both domestic and foreign firms. For instance, the method used in Bruno et al. (2009), Le Van et al. (2010) cannot be directly applied

[5]This system's parameters include domestic and foreign firms' technological efficiency that are exogenous.

[6]For this kind of growth models, see Romer (1990), Grossman and Helpman (1991).

[7]In Alfaro et al. (2010), the development level of the local financial market is modeled by the difference between the instantaneous borrowing rate and the lending rate.

in our model (see Section 3).

The paper is structured as follows. Section 2 introduces an optimal growth model with FDI to study conditions under which the host country should invest in a new industry. We also provide empirical evidence from Vietnamese manufacturing industries. Section 3 investigates the interplay between FDI, R&D, and economic growth of the host country by using an endogenous growth model and a cross-country empirical analysis. Section 4 conclude. The Appendix section reports technical proofs and further information on the data used in this research.

2 FDI versus new industry in optimal growth

2.1 Benchmark model

Let us start with a benchmark model in which there is a small open economy with three goods: a consumption, a capital, and a new goods. The consumption good is taken as numéraire. The price (in terms of consumption good) of physical capital is exogenous and denoted by p. In each period, there is a MNE in the considered country (called hereafter a *host country*). Its produces the new good by using two inputs: physical capital and specific labor. This sector is refereed to a *new industry* (or *new sector*). In developing countries, this sector may be 'Computer and Peripheral equipment manufacturing', 'Electrical Equipment manufacturing', 'Radio, Television and Communication equipment manufacturing, etc.

At each date t, the foreign firm (without market power) chooses $K_{e,t}$ units of physical capital and $L^D_{e,t}$ units of specific labor in order to maximize its profit:

$$(F_t): \quad \pi_{e,t} = \max_{K_{e,t}, L^D_{e,t} \geq 0} \Big[p_n F^e_t(K_{e,t}, L^D_{e,t}) - pK_{e,t} - w_t L^D_{e,t} \Big] \tag{1}$$

where p_n is the exogenous price (in term of consumption good) of new good.

Assumption 1. *We assume that* $F^e_t(K_{e,t}, L^D_{e,t}) = A_e K^{\alpha_e}_{e,t}(L^D_{e,t})^{1-\alpha_e}$.

There is a representative agent in the host country. She decides the allocation of resources to maximize the intertemporal welfare of the whole population. If the country uses $K_{c,t+1}$ units of physical capital at date t, it can get $A_c K^{\alpha}_{c,t+1}$ units of consumption good at date $t+1$, where $\alpha \in (0,1)$ and A_c represents the TFP.[8] Otherwise, if the host country invests H_{t+1} units of consumption good in training specific labor at date t, there will be $A_h H^{\alpha_h}_{t+1}$ units of specific labors at date $t+1$. Specific labor works for the MNE to get a wage w_t (in term of consumption good), which is endogenous and determined by the market clearing condition given in Definition 1 below.

Thus, the representative agent solves:

$$(P_1): \quad \max_{\left(c_t, K_{c,t}, L_{e,t}, H_t\right)_{t=0}^{+\infty}} \Big[\sum_{t=0}^{+\infty} \beta^t u(c_t) \Big] \tag{2a}$$

$$\text{subject to} \quad c_t + pK_{c,t+1} + H_{t+1} \leq A_c K^{\alpha}_{c,t} + w_t L_{e,t} \tag{2b}$$

$$L_{e,t} \leq A_h H^{\alpha_h}_t \tag{2c}$$

[8]For the sake of simplicity, we assume here that the depreciation rate of physical capital equals 1.

where $\beta \in (0,1)$ is a discount factor and $K_{c,0}, L_{e,0}$ are given. We assume that the utility function u is in C^1, strictly increasing, concave, and $u'(0) = \infty$.

We provide a formal definition of equilibrium.

Definition 1. *An intertemporal equilibrium is a list $(c_t, K_t, H_t, L_{e,t}, L^D_{e,t}, K^D_{e,t}, w_t)_{t=0}^{\infty}$ such that*

(i) Given $(w_t)_{t=0}^{\infty}$, $(c_t, K_t, H_t, L_{e,t})_{t=0}^{\infty}$ is a solution of the problem (P_1).

(ii) Given w_t, $(L^D_{e,t}, K^D_{e,t})$ is a solution of the problem (F_t).

(iii) Labor market clears: $L^D_{e,t} = L_{e,t}$.

Remark 1. *In the absence of the MNE, we recover the closed economy. In this case, the problem (P_1) becomes the standard Ramsey optimal growth model with the budget constraint: $c_t + pK_{c,t+1} \leq A_c K^{\alpha}_{c,t}$ $\forall t$. We can prove that $\lim_{t\to\infty} S_t = S_a$, where S_a is defined by $S_a^{1-\alpha} = \alpha\beta A_c/p^{\alpha}$.*

At equilibrium, we have $L^D_{e,t} = L_{e,t} > 0$. Hence, the first order conditions of the problem (F_t) imply that, for every t:

$$w_t = w := \Big(\alpha_e^{\alpha_e}(1-\alpha_e)^{1-\alpha_e}\frac{p_n A_e}{p^{\alpha_e}}\Big)^{\frac{1}{1-\alpha_e}}. \tag{3}$$

Thus, wage w depends not only on the foreign firm TFP but also on the prices of physical capital and new good.

Denote $S_t = pK_{c,t} + H_t$ the total saving of the host country. We define the function $F(S)$ by:

$$F(S) \equiv \max_{pK_c + H \leq S, K_c \geq 0, H \geq 0} \{A_c K_c^{\alpha} + wA_h H^{\alpha_h}\}. \tag{4}$$

Thus, $F(S)$ is strictly increasing, strictly concave, smooth and satisfies Inada condition $F'(0) = \infty$. Moreover, given S, there is a unique pair (K_c, H) with $pK_c + H = S_t$ such that $F(S) = A_c K_c^{\alpha} + wA_h H^{\alpha_h}$.

The problem (P_1) can be rewritten as follows:

$$(P_1') : \qquad \max_{(c_t, S_{t+1})_{t=0}^{+\infty}} \Big[\sum_{t=0}^{+\infty} \beta^t u(c_t)\Big] \text{ subject to } c_t + S_{t+1} \leq F(S_t) \tag{5}$$

We are now ready to state the main result in this section.

Proposition 1. *Under above specifications, there is a unique equilibrium. In equilibrium, we have,*

$$w_t = w := \Big(\alpha^{\alpha}(1-\alpha)^{1-\alpha}\frac{p_n A_e}{p^{\alpha}}\Big)^{\frac{1}{1-\alpha}} \qquad \forall t$$

and S_t converges to S_b defined by $\beta F'(S_b) = 1$. Moreover, S_b increases in A_c, w, A_h, and $S_b > S_a$.

In a particular case where $\alpha = \alpha_h$, the value S_b can be explicitly computed by:

$$S_b^{1-\alpha} = \alpha\beta A \text{ where } A \equiv \Big(\big(\frac{A_c}{p^{\alpha}}\big)^{\frac{1}{1-\alpha}} + (wA_h)^{\frac{1}{1-\alpha}}\Big)^{1-\alpha}. \tag{6}$$

Proof. See Appendix B.1. □

Proposition 1 is likely to support a positive impact of FDI on the host economic growth. The property $S_b > S_a$ means that with the presence of FDI, the economy's investment stock converges to a steady state which is higher than that of a closed economy. Moreover, the steady state level S_b is increasing in the TFP of domestic firms, wage, as well as the TFP of foreign firms. It implies that the effect of FDI on the steady state output depends on both FDI and the host country's circumstances. This is consistent with several empirical studies mentioned in Introduction.

2.2 Enjoying FDI or investing in a new industry?

In Section 2.1, we assume that only the MNE produces in the new sector and the host country can enjoy the associated payroll to have higher economic growth. However, it is interesting to investigate whether or not the host country invests in the new industry. To this end, we suppose that the host country may create a domestic firm in the new industry. While the problem (F_t) of MNE remains the same, the representative agent solves the following dynamic growth problem:

$$(P): \qquad \max_{\left(c_t, K_{c,t}, K_{d,t}, L_{d,t}, L_{e,t}, H_t\right)_{t=0}^{+\infty}} \left[\sum_{t=0}^{+\infty} \beta^t u(c_t)\right] \tag{7}$$

subject to, for every $t \geq 0$.

$$0 \leq K_{c,t}, K_{d,t}, L_{d,t}, L_{e,t}, H_t \tag{8a}$$

$$c_t + S_{t+1} \leq A_c K_{c,t}^{\alpha} + w_t L_{e,t} + p_n F_t^d(K_{d,t}, L_{d,t}) \tag{8b}$$

$$S_{t+1} = p(K_{c,t+1} + K_{d,t+1}) + H_{t+1} \tag{8c}$$

$$L_{e,t} \leq A_h H_t^{\alpha_h} \tag{8d}$$

$$L_{d,t} \leq \left(A_h H_t^{\alpha_h} - L_{e,t}\right) + Spillovers(A_e, L_{e,t}, S_t). \tag{8e}$$

where $K_{c,0}, L_{e,0} > 0$ are given, and $K_{d,0} = L_{d,0} = 0$. Let us denote $X_0 \equiv A_c K_{c,0}^{\alpha} + w_0 L_{e,0}$.

The definition of equilibrium is similar to Definition 1. The labor market clearing condition is $L_{e,t}^D = L_{e,t}$. Therefore, the wage is always given by Equation (3).

Constraints (8a-8c) are standard. Let us explain (8d) and (8e). We assume that if the MNE uses $L_{e,t}$ units of specific labor, it can generate $Spillovers(A_e, L_{e,t}, S_t)$ units of specific labor for the host economy. Then, there are $\left(A_h H_t^{\alpha_h} - L_{e,t}\right) + Spillovers(A_e, L_{e,t}, S_t)$ units of labor being available for the domestic firm. This is represented by (8e). We assume that FDI spillovers have the following specification:

$$Spillovers(A_e, L_{e,t}, S_t) = \frac{BA_e}{1 + S_t} L_{e,t} \tag{9}$$

Equation (9) means that each unit of specific labor hired by the MNE can generate $\frac{BA_e}{1+S_t}$ units of specific labor. FDI spillovers through labor turnover occur when a domestic firm can hire former multinational specific labor. Thus, the higher units of specific labor $L_{e,t}$, the higher FDI spillovers. However, the latter decreases in the host country's development level S_t and increases in the MNE productivity A_e. Besides, parameter B represents either the absorbability of specific labor or learning by doing effects.[9]

We assume that the domestic firm has the following production function:

$$F_t^d(K_{d,t}, L_{d,t}) = \begin{cases} A_d K_{d,t}^{\alpha_d} \big((L_{d,t} - \bar{L})^+\big)^{1-\alpha_d} & \text{if } Y_{d,s} = 0 \quad \forall s \leq t-1 \\ A_d K_{d,t}^{\alpha_d} L_{d,t}^{1-\alpha_d} & \text{otherwise.} \end{cases} \tag{10}$$

This setup means that the domestic firm needs to make an initial investment to enter the new industry. We model this investment by the fixed cost $\bar{L}$ that represents a minimum number of specific labor needed to ensure the functionality of the production process. Once the domestic firm enters the new industry, it no longer needs to pay this cost. By contrast, the MNE made that investment in the home country and did not pay this cost again to produce in the host country.

2.2.1 Static analysis

Let's firstly explore the static analysis by solving a general equilibrium model at each date. Given S, the representative agent maximizes the following problem:

$$(G_S): \qquad G(S) = \max_{K_c, K_d, L_d, L_e, H \geq 0} A_c K_c^{\alpha} + wL_e + p_n A_d K_d^{\alpha_d} \big((L_d - \bar{L})^+\big)^{1-\alpha_d} \tag{11a}$$

$$\text{subject to: } p(K_c + K_d) + H \leq S \tag{11b}$$

$$L_e \leq A_h H^{\alpha_h} \tag{11c}$$

$$L_d \leq (A_h H^{\alpha_h} - L_e) + \frac{BA_e}{1+S} L_e. \tag{11d}$$

where wage is given in Equation (3). Notice that $G(S)$ is the national income. It is easy to see that the function G is increasing in S. Moreover, $G(S) \geq F(S)$, $\forall S$.

Denote $Y_d := F^d(K_d, L_d)$ and $Y_e := F^e(K_e, L_e)$.

Lemma 1. *At optimal, we have:*

(i) If $\frac{BA_e}{1+S} > 1$, then $L_e = A_h H^{\alpha_h}$.

(ii) If $\max(\frac{BA_e}{1+S}, 1) A_h S^{\alpha_h} \leq \bar{L}$, then $Y_d = 0$.

Proof. See Appendix B.1. □

The first point of Lemma 1 indicates that if FDI spillovers are high enough, all specific labor trained by the host country will work for the foreign firm to get a high amount of payroll. The host country can then benefit from these spillovers to create a domestic firm in the new industry. The second point shows that when the entry cost is high, a (poor) host country is unable to invest in the new industry.

The following result shows the role of productivity.

[9] Fosfuri et al. (2001) prove, through a static model, that labor turnover can be a channel of FDI spillovers. Furthermore, the degree of such spillovers is increasing with the absorbability of domestic firms. Evidence from Brazil supports heterogeneous impacts of spillovers through labor mobility Poole (2013). Higher-skilled former foreign firms' workers have a better ability to transfer information, and so do higher-skilled incumbent workers to absorb information. However, Crespo and Fontoura (2007); Meyer and Sinani (2009) argue that the higher the host country's development level, the less FDI spillovers.

Lemma 2. *Let S be given. Assume that $BA_e < 1$ and there exists s such that $A_h S^{\alpha_h} > A_h s^{\alpha_h} > \bar{L}$. There exists $\bar{A}_d$ depending on S such that for every $A_d \geq \bar{A}_d$, we have $Y_d > 0$.*

Proof. See Appendix B.1. □

Lemma 2 implies that with low FDI spillovers (i.e., $BA_e < 1$), the host country still enables to invest in the new sector if its resource S is high enough ($A_h S^{\alpha_h} > \bar{L}$) or the domestic firm is efficient enough (i.e., its TFP is high).

2.2.2 Global dynamic analysis

We now investigate the dynamic analysis of equilibrium. More precisely, we are interested in the evolution of allocations $S_t, K_{c,t}, K_{d,t}, H_t, L_{e,t}, L_{d,t}$ as well as the aggregate output. First of all, we have that:

Lemma 3. *The optimal path $(S_t)_t$ is monotonic. Moreover, (S_t) does not converge to zero.*

Proof. See Appendix B.1. □

The optimal path is monotonic because the function $G(\cdot)$ is continuous, strictly increasing and $G(0) = 0$. It cannot converge to zero because $G(.)$ satisfies Inada's condition (indeed, when S is small enough, we have $G(S) = F(S)$ and hence $G'(0) = F'(0) = \infty$).

Let us study the convergence of optimal growth paths. Define the sequence (x_t) as $x_0 = X_0, x_{t+1} = F(x_t)$, where the function F is given in (4). Denote x^* and $\bar{S}$ be uniquely defined by:

$$F(x^*) = x^* \text{ and } \bar{S} := \max\{X_0, x^*\}. \tag{12}$$

Notice that x^* and $\bar{S}$ depend on (i) the productivity A_c and capital elasticity α of the consumption good sector, (ii) the efficiency of specific labor training A_h, α_h, (iii) wage w,[10] but not on the TFP A_d of the potential domestic firm in the new sector. We also observe that $F(x) \leq F(x^*) = x^*$ for every $x \leq x^*$ and $F(x) \leq x$ for every $x \geq x^*$.[11] At equilibrium, it is easy to prove that $S_t \leq x_t \leq \bar{S}\ \forall t$.[12] By consequence, we obtain the following result:

Proposition 2 (middle income trap)**.** *Assume that* $\max(BA_e, 1)A_h\bar{S}^{\alpha_h} \leq \bar{L}$. *Then we have $Y_{d,t} = 0$ for every t. In this case, S_t converges to S_b ($\lim\limits_{t\to\infty} S_t = S_b$), where S_b is defined in Proposition 1.*

Proof. See Appendix B.1. □

Proposition 2 indicates that no domestic firm can be created in a new industry if:

[10] If $\alpha_h = \alpha$, we can explicitly compute that $x^* = (\frac{A_c}{p^\alpha})^{\frac{1}{1-\alpha}} + (wA_h)^{\frac{1}{1-\alpha}}$.

[11] Indeed, if $x < x^*$, then $F(x) < F(x^*) = x^*$. If $x > x^*$, then $\frac{F(x)}{x} \leq \frac{F(x^*)}{x^*} = 1$ since F is concave.

[12] It is obvious that $S_t \leq x_t \forall t$. We prove $x_t \leq \bar{S}\ \forall t$ by induction argument. First, we see that $x_0 \leq \bar{S}$. Second, assume that $x_s \leq \bar{S}\ \forall s \leq t$. If $X_0 \leq x^*$, then $x_t \leq \bar{S} = x^*$, then $x_{t+1} = F(x_t) \leq F(x^*) = x^* = \bar{S}$. If $X_0 > x^*$, then $x_t \leq \bar{S} = X_0$ and hence $x_{t+1} = F(x_t) = F(x_0) \leq x_1 \leq \bar{S}$.

(i) The host country has a low initial endowment X_0.

(ii) The consumption good sector TFP A_c is low.

(iii) The training sector has a low productivity A_h.

Interestingly, this result holds whatever the level of the TFP A_d of the potential domestic firm.

The following result provides sufficient conditions under which domestic firms produce in the new industry.

Proposition 3 (Emergence of domestic firms in the new sector)**.** *Assume that*

$$\max\left(\frac{BA_e}{1+S_0}, 1\right) A_h X_0^{\alpha_h} < \bar{L} \tag{13}$$

$$\max\left(\frac{BA_e}{1+S_b}, 1\right) A_h S_b^{\alpha_h} > \bar{L}. \tag{14}$$

There exists $\bar{A}_d > 0$ such that for each $A_d > \bar{A}_d$, there exists a date $t_d > 1$ such that $Y_{d,t} = 0$ for every $t < t_d$ and $Y_{d,t_d} > 0$.

Proof. See Appendix B.1. □

Let us explain conditions (13) and (14). Condition (13) means that the fixed cost $\bar{L}$ is high with respect to the host country's initial resource X_0 so that the host country is not able to produce in the new industry at the initial date. Condition (14) means that the host country may overcome the fixed cost $\bar{L}$ if the steady state S_b of the country in the benchmark model is high enough.[13]

Proposition 3 indicates that under conditions (13) and (14), the country should then invest in the new industry if (and only if) the productivity A_d of the entrant firm is high enough.

Propositions 2 and 3 offer an explanation for the absence or insignificant impact of horizontal FDI spillovers as reported in several empirical studies.[14] Indeed, Propositions 2 and 3 indicate that if the local conditions are not sufficiently good, there is no horizontal FDI spillovers (in the sense that no domestic firm can operate in the new industry). However, we may have a positive spillovers after a finite period of time (in the sense that $Y_{d,t} > 0 \ \forall t \geq t_d$ in Proposition 3). Indeed, it takes time for the MNE to improve its involvement in the host country by hiring more local employees ($L_{e,t}$), and so generating more FDI spillovers ($\frac{BA_e}{1+S_t} L_{e,t}$).

Our point about the role of time since foreign entry in the development of domestic firms is in line with empirical investigation of Merlevede et al. (2014). Indeed, Merlevede et al. (2014) use firm-level data form a panel of Romanian manufacturing firms during 1996-2005 and find that MNEs initially negatively affect local competitors' productivity. However, the effect turns out to be permanently positive for a longer time.

[13] Notice that (13) and (14) are satisfied if, for instance, $BA_e < 1$ and $A_h X_0^{\alpha_h} < \bar{L} < A_h S_n^{\alpha_h}$.

[14] See Blomstrom and Kokko (1998); Greenaway and Gorg (2004); Crespo and Fontoura (2007) for more detailed reviews.

Proposition 3 leads to an interesting implication for a low development country having high productivity of both old sectors (A_c) and modern sectors (A_d). However, the latter sectors are underdeveloped owing to high fixed costs. Hence, the country optimal development strategy is the following:

- First, the country should attract both FDI and train specific workers for the modern sectors.
- Then, those workers work for MNEs (that have located in the modern industries) to get a high salary and high-skill knowledge (through learning by doing effects or specific training) to improve the country's GNP.
- Once its GNP reaches a critical threshold, the country can cover the fixed costs, and new domestic firms can further enter the new industries.

Not only helping the host country to develop new industries, FDI also affects its economic growth. Besides, the domestic entrant firm can even eliminate the MNE as we state in the following proposition.

Proposition 4 (growth and convergence)**.** *Assume that*

$$BA_e < 1, \quad X_0 < S_a, \quad A_h S_a^{\alpha} < \bar{L} < A_h S_b^{\alpha} \tag{15}$$

then there exists $A^* > 0$ and $t^* > 1$ such that, for each $A_d > A^*$, we have:

(i) $Y_{d,t} = 0, Y_{e,t} > 0$ for every $t < t^*$ and $Y_{d,t} > 0, Y_{e,t} = 0$ for every $t \geq t^*$.

(ii) $\lim_{t\to\infty} c_t = c$, $\lim_{t\to\infty} S_t = S_c$, $\lim_{t\to\infty} K_{c,t} = K_c$, $\lim_{t\to\infty} K_{d,t} = K_d$, $\lim_{t\to\infty} H_t = H$.

Moreover, $S_c > S_b$.

Proof. See Appendix B.1. □

In Proposition 4, we assume that FDI spillovers are not so high (in the sense that $BA_e < 1$). Condition $X_0 < S_a$ and $A_h S_a^{\alpha} < \bar{S}$ means that in the absence of FDI, the host country cannot invest in the new industry, regardless of the entrant firm TFP A_d. Condition $A_h S_b^{\alpha} > \bar{L}$ means that the steady state S_b of the economy with FDI and without domestic firms in the new industry can cover the fixed cost.

According to Proposition 4, the country receiving FDI would invest in the new sector if the domestic firm's TFP in this sector is high enough. Moreover, the MNE can be eliminated ($Y_{e,t} = 0 \ \forall t \geq t^*$).[15] Moreover, we obtain the convergence of allocations in the long run. Particularly, the total savings S_t and the income $G(S_t)$ converges to a higher value than the values S_b and $F(S_b)$ in the benchmark setup where only the MNE produces in the new industry.

Proposition 4 is also related to Markusen and Venables (1999) who show that FDI may contribute to the creation of local industrial sectors. There are two differences between Markusen and Venables (1999) and our paper: (1) Markusen and Venables (1999) provide a static partial equilibrium model while we consider an infinite-horizon growth model, (2) Markusen and Venables (1999) assume that there are two imperfectly competitive industries producing consumption and intermediate goods (multinational and foreign firms produce consumption good) while we consider perfect competition.

[15] Point (i) of Proposition 4 is indeed complementary to Proposition 3 by justifying a dominance of the domestic entrant firm on the MNE

2.3 Case of Vietnamese manufacturing industries

This subsection aims to provide some empirical evidence from Vietnamese manufacturing industries. More precisely, we investigate the role of productivity in creating and developing a new industry and how the former may affect the competition between foreign and domestic firms. In this analysis, the new sector refers to "Computer and Peripheral equipment manufacturing", "Electrical equipment manufacturing", or "Radio, Television and Communication equipment manufacturing".

2.3.1 Data and estimation strategy

We rely on the data conducted from the Vietnamese Enterprises Survey between 2000 and 2016. It is an annual and one of the biggest surveys organized by the General Statistics Office of Vietnam since 2000. Each wave gathers different information on the firm characteristics and activities as tax identification, legal status, turnover, capital stock, payroll, raw materials cost, investment, and so forth.

Note that there are a representative MNE and a potential domestic firm in the new sector in the above framework. Since the data are at the firm level, we should create a new database at the industrial level to make data compatible with our theoretical framework. To this end, except for the firm TFP, we only need to sum up all related firms (all domestic firms together, and all foreign firms together) in an industry to get the aggregate level.

When it comes to TFP, the aggregate level is computed as a share-weighted average of firm productivity, according to Melitz and Polanec (2015):

$$\Omega_{it} = \sum s_{it} * \omega_{it} \tag{16}$$

where Ω_{it}, ω_{it} are the aggregate and firm TFPs, respectively and s_{it} is the market share of firm i at time t, $\sum s_{it} = 1$. There are many potential candidates for the weight share s_{it} as employment shares or output market shares. Given our interest in competition between firms, output market shares is used to compute the aggregate productivity.[16]

To obtain $\Omega \equiv (\Omega_{it})$, we should firstly compute the firm TFP, $\omega \equiv (\omega_{it})$. However, getting such productivity can face some econometric issues because it is associated with estimating firm production function form where biased problems can arise. In what follows, we mention some econometric issues associated with such estimation and provide solutions.

Let's start with the firm production function in Cobb-Douglas form:

$$Y_{it} = A_{it} K_{it}^{\beta_k} L_{it}^{\beta_l}$$

where Y_{it} is the firm's output at time t, and K_{it}, L_{it} its capital stock and number of employees, respectively. The econometric form (in log) of this production function can be represented as:

$$y_{it} = \beta_k k_{it} + \beta_l l_{it} + \omega_{it} + \varepsilon_{it} \tag{17}$$

[16] Notice that employment shares are the most commonly used, together with the firm TFP to get aggregate productivity in the topic of job reallocation.

where $\log A_{it} = \omega_{it} + \varepsilon_{it}$ and other lower cases are the logarithm form; ω_{it} represents an unobserved productivity shock that is observed by the firm's owner but unknown by the econometricians and ε_{it}, the idd (independent and identically distributed) error terms.

One of the main issues of estimating Equation (17) is the presence of the unobserved productivity ω_{it}. It makes usual estimators as Fixed Effects in panel data or OLS (Ordinary Least Square) unsuitable. To deal with this problem, Olley and Pakes (1996) and Levinsohn and Petrin (2003) suggest a semi-parametric method including two-step estimation through which the first stage is to estimate the parameters of inputs (labor) and the second stage is to estimate the coefficient for capital. The authors propose using a proxy to control for unobserved productivity shock (investment in Olley and Pakes, 1996 and materials in Levinsohn and Petrin, 2003). However, It should be noteworthy that these estimators, being relevant to control for the endogeneity bias, represent two main limits. The first is associated with unconditional input demands that may lead to a functional dependence problem. The second limit is related to the error terms' correlation at the moment.

Taking all the above issues into account, Wooldridge (2009) and Ackerberg et al. (2015) are likely to suggest, for instance, the most proper technic to estimate the firm production function. In this section, Equation (17) is performed by using the GMM (Generalized Method of Moments) proposed by Wooldridge (2009).[17]

2.3.2 Empirical results

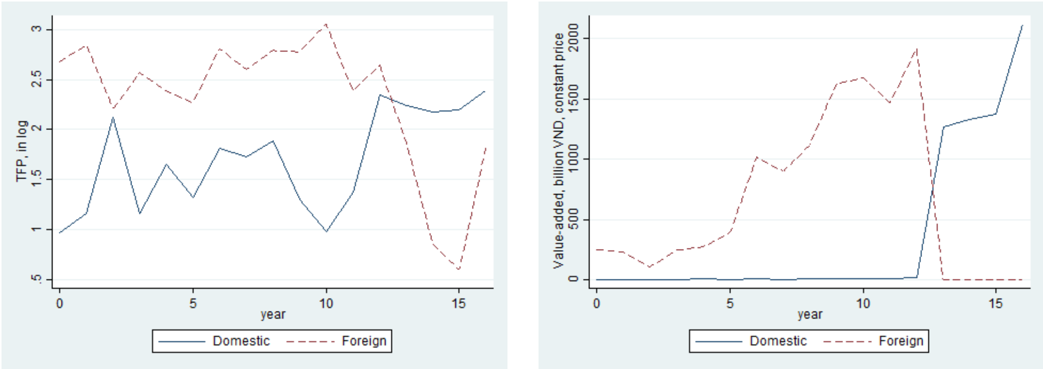
Using the GMM estimator proposed by Wooldridge (2009), we can first estimate the firm's production function and then compute its TFP. Table B1 in Appendix B.3 reports the estimated results and Table B2, descriptive statistics of interested variables in each selected industry. We observe that over the period 200-2016, a foreign firm is, on average, bigger and has a higher TFP than its domestic counterpart. Also, the former has a greater production output than the latter does. Nevertheless, domestic firms are more numerous than foreign ones. These statements hold regardless of the considered industry.

Using the firm-level data, we compute the aggregate production and TFP for domestic and foreign firms at the industrial-level. We then show the evolution of these variables in Figures 1-3.

Figures 1-3 are likely to support our above theoretical findings. Indeed, taking a look at the 'Computer and Peripheral equipment manufacturing' in Figure 1, we state that over the period 2000-12, domestic firms' productivity is low compared to that of foreign competitors. As a consequence, domestic production (measured by its value-added) remains very small. However, once its TFP becomes higher since 2013, there is a huge increase in domestic production. Thus, these findings seem to support our theoretical findings in Propositions 3 and 4. On the one side, the small level of domestic production between 2000-12 and the low level of its TFP seem to match with the statement $Y_{d,t} = 0$ for $t < t^*$ in Proposition 3. On the other side, the increase in domestic production when its productivity is high since 2013 appears to connect to the case $Y_{d,t} > 0$ for $t \geq t^*$ of this position.

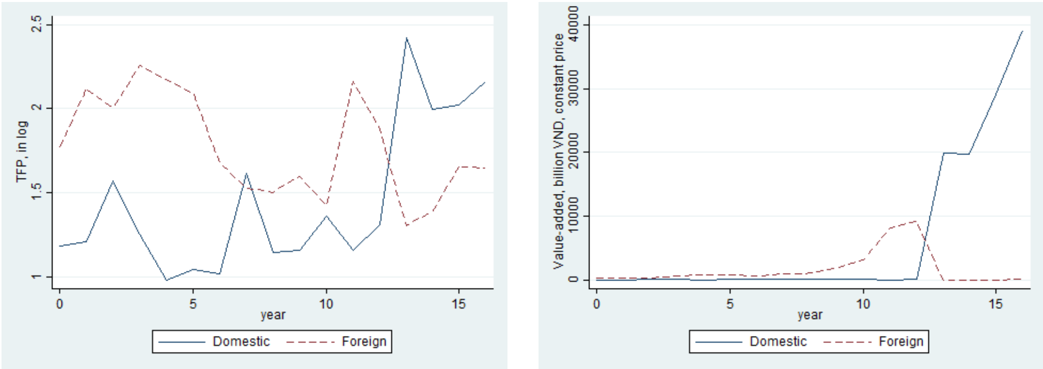
[17]Please refer to Appendix B.2 for a detailed explanation of this method.

Figure 1: Computer and Peripheral equipment manufacturing



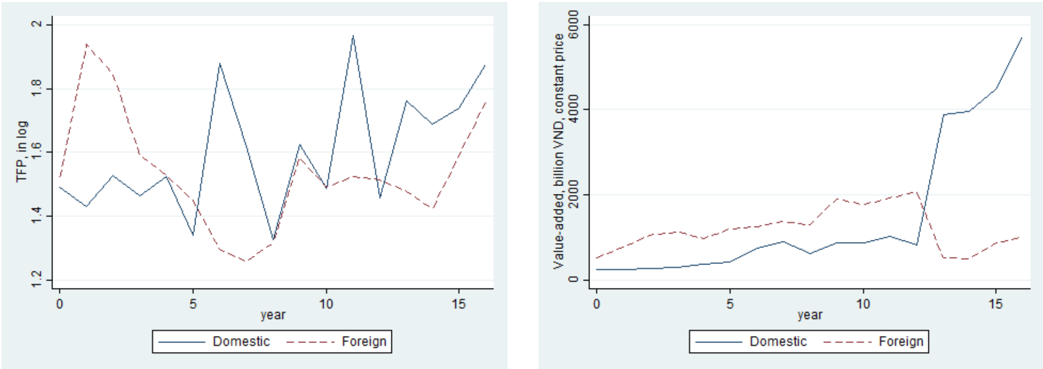
(a) Aggregate productivity (b) Aggregate production

Figure 2: Radio, Television, and Communication equipment



(a) Aggregate productivity (b) Aggregate production

Figure 3: Electrical equipment manufacturing



(a) Aggregate productivity (b) Aggregate production

Moreover, Figure 1 also displays a decrease and convergence to zero of foreign production. Hence, domestic firms seem to dominate and eliminate their foreign counterparts in the competition (i.e., conditions $Y_{d,t} > 0$ and $Y_{e,t} = 0$ in Proposition 4).

The findings in 'Computer and Peripheral equipment manufacturing' are likely to hold in the other two industries. More precisely, Figure 2 indicates that the domestic production in 'Radio, Television, and Communication equipment' remains very low for the period 2000-12. Meanwhile, aggregate domestic productivity is also small. However, such productivity is higher since 2013, leading to an increase in domestic production. Moreover, the latter are so higher than the foreign production that domestic firms tend to bring out their foreign competitors. Hence, these findings are what we state in Propositions 3-4. When it comes to 'Electrical equipment manufacturing,' Figure 3 reports the same phenomenon: a low domestic production firstly associated with a small TFP then a high and overtaking of the former on the foreign production.

3 FDI, R&D, and endogenous growth

Notice that in Section 2, the firm productivity is exogenous. With this assumption, the host country can suffer a middle income trap when domestic firms have low productivity, as mentioned in Proposition 2. This point leads to a natural question: How a host country could avoid such a middle income trap, and get sustainable growth in the long-run? We address this question by endogenizing the TFP of domestic firms and studying the host country's optimal strategy in an endogenous growth model.

While the MNE problem remains the same as in Equation (1), the host country can invest in R&D to get a new technology, which improves the productivity of a sector. Without loss of generality, we assume that this investment is taken place in the old sector. The social planner solves the dynamic growth problem below:

$$(PN): \qquad \max_{\left(c_t, K_{c,t}, H_t, N_t, L_{e,t}\right)_{t=0}^{+\infty}} \Big[\sum_{t=0}^{+\infty} \beta^t u(c_t)\Big] \tag{18}$$

subject to

$$0 \leq c_t, K_{c,t}, H_t, L_{e,t}, N_t \tag{19a}$$

$$c_t + S_{t+1} \leq \Big(A_c + a(bN_t^{\sigma} - \bar{x})^+\Big) K_{c,t}^{\alpha} + w_t L_{e,t} \tag{19b}$$

$$S_{t+1} = pK_{c,t+1} + N_{t+1} + H_{t+1} \tag{19c}$$

$$L_{e,t} \leq A_h H_t^{\alpha_h}. \tag{19d}$$

for every $t \geq 1$, where, at initial date, $K_{c,0}, L_{e,0} > 0$ are given, $N_0 = 0$. Let us denote $X_0 \equiv A_c K_{c,0}^{\alpha} + w_0 L_{e,0}$.

Indeed, the host country has three investment choices on each date. The first choice is to use H_t units of the consumption good to train $A_h H_t^{\alpha_h}$ units of specific labor. The latter works for the MNE to get a total wage of $w_t A_h H_t^{\alpha_h}$ units of the consumption good. The second choice is to buy $K_{c,t}$ units of physical capital to produce $A_c K_{c,t}^{\alpha}$ units of the consumption good. The third choice is to invest in R&D to create new technology (denoted by bN_t^{σ} of technology where b represents the efficiency of the research process) by expanding an amount of N_t units of the consumption good. The new technology can improve the old sector's productivity but only if the amount of investment in R&D exceeds a critical threshold such that $bN_t^{\sigma} > \bar{x}$, where $\bar{x} > 0$

represents a fixed cost. In this case, the productivity goes up to $A_c+a(bN_t^\sigma-\bar{x})$ where the parameter a indicates the efficiency or the leverage of the new technology.[18]

We assume that $a\bar{x}>A_c$, i.e., the fixed cost $\bar{x}$ is not too low.

The definition of equilibrium is similar to Definition 1. The labor market clearing condition is $L_{e,t}^D=L_{e,t}$. By consequence, the wage is given by Equation (3).

We now analyze the properties of equilibrium. By using Equation (3), the problem (PN) can be rewritten as follows:

$$(PN'):\quad \max_{(c_t,S_{t+1})_{t=0}^{+\infty}}\Big[\sum_{t=0}^{+\infty}\beta^t u(c_t)\Big] \text{ subject to: } c_t,S_t\geq 0,\quad c_t+S_{t+1}\leq G(S_t) \tag{20}$$

for any $t\geq 1$ and $c_0+S_1\leq X_0\equiv A_cK_{c,0}^\alpha+w_0L_{e,0}$, where $G(S)$ is defined by

$$(G_S): G(S)\equiv \max_{K_c,N,H}\Big\{g(K_c,N,H): pK_c+N+H\leq S; K_c,N,H\geq 0\Big\} \tag{21a}$$

$$\text{where } g(K_c,N,H)\equiv\Big(A_c+a(bN^\sigma-\bar{x})^+\Big)K_c^\alpha+wA_hH^{\alpha_h}. \tag{21b}$$

Notice that the function $G(\cdot)$ is continuous, strictly increasing and $G(0)=0$. However, it may be non-concave and non-smooth.

3.1 Static analysis

In this subsection, given S, we study the optimization problem (G_S). First, it is easy to see that this problem has an solution. However, since the objective function is not concave, the uniqueness of solutions may not be ensured.

We now provide some properties of solution of the problem (G_S).

Lemma 4. *(i) If $bS^\sigma\leq\bar{x}$ then $N=0$ for any a.*

(ii) If $bS^\sigma>\bar{x}$ and $\Big[A_c+a\Big(\big(b^{\frac{1}{\sigma}}\frac{S}{2}+\frac{\bar{x}^{\frac{1}{\sigma}}}{2}\big)^\sigma-\bar{x}\Big)\Big]\frac{1}{p^\alpha}\Big(\frac{S}{2}-\frac{\bar{x}^{\frac{1}{\sigma}}}{2b^{\frac{1}{\sigma}}}\Big)^\alpha>F(S)$, then $N>0$.

Proof. See Appendix C.1. □

Point (i) of Lemma 4 indicates that if either the efficiency of the research process or the initial resource is low or the fixed cost is high, the host country may not invest in R&D. Besides, point (ii) implies that the country invests in R&D when a and b are high enough (because $F(S)$ depends neither on a nor b).

If $\sigma+\alpha\geq 1$ and we fix all parameters excepted S, the condition in point (ii) is satisfied when S is high enough. It implies that the host country will invest in R&D once it is rich enough, as stated in the following lemma:

Lemma 5. *There exists a unique S^* such that: (i) $G(S)-F(S)=0$ for $S\leq S^*$, and (ii) $G(S)>F(S)$ and $N>0$ for $S>S^*$.*

Notice that $b(S^*)^\sigma-\bar{x}>0$. Moreover, we have $G'(S)=F'(S)$ if $S<S^*$, and $G'(S)=G_0'(S)>F'(S)$ if $S>S^*$. At $S=S^*$, the left derivative is $F'(S^*)$ and the right derivative is $G_0'(S^*)$.

[18] To introduce R&D, we can also write, for example, $A_c+\gamma((N_t-N^*)^+)^\sigma$ instead of $A_c+a(bN^\sigma-\bar{x})^+$. However, the main results have similar insights.

Proof. See Appendix C.1. □

Lemma 5 plays a crucial role in our analysis. It is also in line with Lemma 3 in Bruno et al. (2009). However, notice that the method used in Bruno et al. (2009), Le Van et al. (2010) cannot be directly applied in our model.[19]

Let us provide a sketch of our proof. First, we introduce functions g_0 and G_0

$$g_0(K_c, N, H) \equiv \big(A_c + a(bN^\sigma - \bar{x})\big)K_c^\alpha + wA_hH^{\alpha_h}$$
$$G_0(S) = \max\{g_0(K_c, N, H) : pK_c + N + H \le S; K_c, N, H \ge 0; bN^\sigma \ge \bar{x}\}$$

Observe that $G_0(S) \le G(S)$. More importantly, we have that

$$G(S) - F(S) = \max\{F(S), G_0(S)\} - F(S) = \max\{0, G_0(S) - F(S)\} \quad (22)$$

Second, we prove that $G_0(S) - F(S)$ is strictly increasing in S. The value S^* is in fact the unique solution of the equation $G_0(S^*) = F(S^*)$.

3.2 Global dynamic analysis

In this subsection, we explore the dynamics of equilibrium. Like Lemma 3, we have the following result.

Lemma 6. *The optimal path $(S_t)_t$ is monotonic. Moreover, S_t does not converge to zero.*

We then show the middle income trap which is similar to Proposition 2.

Proposition 5 (middle income trap). *If $X_0 \equiv A_cK_{c,0}^\alpha + w_0L_{e,0} \le x^*$ and $b(x^*)^\sigma \le \bar{x}$, where x^* is defined by (12), then $N_t = 0$ for any t. In this case, we have $\lim_{t\to\infty} S_t = S_b$.*

This result indicates that when the host country has both a low initial resource and a weak research process efficiency, it never invests in R&D ($N_t = 0$ for $\forall t$). In this case, both investment S_t and the output are bounded from above. More precisely, S_t converges to the same value S_b (defined in Proposition 1) as in the benchmark economy.

We now study the case under which the economy may grow without bound.

Proposition 6 (convergence and growth with increasing return to scale). *Assume that $\alpha + \sigma \ge 1$, $\alpha_h + \frac{1}{\alpha} \ge 2$, and $a\bar{x} - A_c \ge 0$, and*

$$\beta \min\Big(F'(S^*), \Gamma(a, b, \bar{x})\Big) > 1 \quad (23)$$

$$\textit{where } \Gamma(a, b, \bar{x}) \equiv \frac{\left(\frac{\alpha A_c}{p\sigma}\right)^\alpha \bar{x}^{\frac{-(1-\alpha)(1-\sigma)}{\sigma}} a^{1-\alpha} b^{\frac{1-\alpha}{\sigma}}}{\left(1 + \frac{\alpha}{\sigma} + \left(\frac{\alpha_h wA_h(p\sigma)^\alpha}{\sigma(\alpha A_c)^\alpha}\right)^{\frac{1}{1-\alpha_h}} \frac{1}{a^{\frac{1-\alpha}{1-\alpha_h}} b^{\frac{\alpha_h-\alpha}{\sigma(1-\alpha_h)}}}\right)^\alpha} \quad (24)$$

Then, for any level of initial resource, we have $\lim_{t\to\infty} S_t = \infty$. Moreover,

$$\lim_{t\to\infty} \frac{N_t}{S_t} = \frac{\sigma}{\alpha + \sigma}, \quad \lim_{t\to\infty} \frac{pK_{c,t}}{S_t} = \frac{\alpha}{\alpha + \sigma}, \quad \lim_{t\to\infty} \frac{H_t}{S_t} = 0. \quad (25)$$

[19]Indeed, their method relies on the set B defined on page 291 of Bruno et al. (2009). In our model with FDI and $\alpha \neq \alpha_h$, this trick no longer works.

Proof. See Appendix C.1. □

Condition (23) ensures that the productivity of function G is high enough (in the sense that $\beta D^+G(S) > 1\ \forall S > 0$, where $D^+G(S)$ is the Dini derivative of function G). This happens if a and b are high enough because the function $\Gamma(a, b, \bar{x})$ is strictly increasing in a and b.

Notice that the conditions given in Proposition 6 do not depend on the initial resource $X_0 \equiv A_c K_{c,0}^\alpha + w_0 L_{e,0}$ which is less than x^*. So, our theoretical results lead to an interesting implication: Consider a low-income country characterized by condition $bX_0^\alpha < \bar{x}$. According to Lemma 4, we have $N_1 = 0$, i.e., the country cannot immediately improve the local firm TFP. Now, suppose that the leverage of new technology a is high enough and conditions in Proposition 6 hold. In this case, the country obtains a sustainable growth (in the sense that $\lim_{t\to\infty} S_t = \infty$). According to point (ii) of Lemma 4, there is a date t_0 along the optimal path such that the country should focus on R&D from date t_0 on (i.e., $N_t = 0\ \forall t \leq t_0$ and $N_t > 0$ for any $t > t_0$). Therefore, the optimal strategy of the country should be as follows.

- First, the country should train specific workers.

- Second, specific workers will work for the MNE to improve the country's income.

- Third, once the country's resource is high enough, it should focus on R&D to create new technology that increases the country's TFP. Hence, its economy may grow faster and converge to a high-income country.

Proposition 6 is consistent with Propositions 3, 4. The main difference is that by investing in R&D, the economy may grow without bound (Proposition 6). In contrast, with only FDI, the economy's capital stock in Propositions 3 or 4 is uniformly bounded from above.

Proposition 6 is related to the economic growth literature with increasing return to scale (Romer, 1986; Jones and Manuelli, 1990; Bruno et al., 2009; Le Van et al., 2010). Our main contribution is to introduce and study the role of FDI in growth models. In our model, FDI only helps the host country at the first stages of its development process. However, the property $\lim_{t\to\infty} S_t = \infty$ and condition (25) indicate that in the long run, when the host country's resource is high enough, it should focus on domestic investment in physical capital and R&D but not on FDI.

Remark 2. *It is interesting to note that conditions in Proposition 6 can be satisfied even if $A_e = w = 0$. In other words, a host country may get economic growth in the long run even in the absence of FDI. The key factors for such growth are the efficiency of investment in R&D (parameter b), the new technology's leverage on the firm TFP (parameter a), and increasing return to scale.*

In the case of decreasing return to scale, the capital stock may converge to a finite steady state, which is higher than that of an economy described in Proposition 1. Formally, we have the following result.

Proposition 7 (decreasing return to scale)**.** *Let X_0 be such that $X_0 < S_b$. Assume that $\alpha + \sigma < 1$. The optimal path (S_t) increasingly converges to a finite value $S_d \geq S_b$. Moreover, $S_d > S_b$ if a and b are high enough.*

Proof. See Appendix C.1. □

On the relationship FDI-growth. So far, we have provided several theoretical results, particularly Propositions 1, 4, 6, to show the role of FDI on the host country's economic growth. In general, the host country benefits from FDI. More importantly, the effect of FDI on growth depends not only on the nature of FDI but, more importantly, on the circumstances of the host country (initial resources, domestic firms' TFP, education system, efficiency of R&D process, ...). Indeed, look back at Proposition 1, if the host country only focuses on FDI, the steady state S_b, that is higher than the steady state of the closed economy, is increasing in the local conditions (the domestic TFP A_c, the efficiency of the training process A_h) and the TFP of the MNE. However, according to Proposition 4, if the country invests in the new industry, then the steady state S_c in this case will be higher than S_b. If the country focuses on R&D and the local conditions are good enough, the host country may get a sustainable growth in the long run (Proposition 6) even the country does not receive FDI.

Our point concerning the conditional impact of FDI on economic growth, our theoretical results is supported by several empirical studies. For instance, Borensztein et al. (1998), Berthélemy and Démurger (2000), Li and Liu (2005) show that the higher the host country's human capital stock, the higher the FDI impact on economic growth.

3.3 Empirical evidence in developing countries

This subsection aims to provide empirical evidence supporting our theoretical findings in Section 3. We are particularly interested in how R&D expenditure affects the economic growth rate. To this end, we rely on a database of 52 developing countries over the period 1996-2018.[20] We focus on four variables: GDP per cap, PPP (constant 2017 international US$), Gross fixed capital formation (GFCF, as % of GDP), inward FDI stock (as % of GDP), and investment in R&D (as % of GDP).[21]

3.3.1 Methodology

Consistently with our theoretical framework, the economic output of a country i, expressed by its GDP per cap (in log) (s), in year t can be written as:

$$gdp_{i,t} = \beta_{1,i}k_{i,t} + \beta_{2,i}fdi_{i,t} + \beta_{3,i}rd_{i,t} + \mu_i + \epsilon_{i,t} \tag{26}$$

where the lower cases express the log value of GDP per cap, investment in physical capital (as % of GDP),[22] inward FDI stock (as % of GDP), rd investment in R&D (as % of GDP), respectively, and μ represents the fixed effects.

The estimation target of Equation (26) is to study both the short-run and long-run causalities of different covariates on GDP per cap. To this end, the dynamic fixed effect (DFE) estimator *à la Pesaran* (Pesaran et al., 1999) is likely to be relevant

[20]According to the World Bank, developing countries include three groups of countries: Upper-Middle income economies (those with a GNI per capita between $3,997 and $12,375), Lower-Middle income economies (those with a GNI per capita between $1,026 and $3,996), and Low-income economies (those with a GNI per capita of $1,025 or less)

[21]See Appendix C.2 for the descriptive statistics of interested variables and the list of countries.

[22]$k_{i,t} = GFCF_{i,t} - rd_{i,t}$

for several reasons. First, this estimator can handle the endogeneity issue associated with a dynamic data model. Second, both short-run and long-run coefficients can be estimated. Third, the method provides a negative error correction term (ECT), and once it is statistically significant, there exists a long-run causality among variables. Fourth, this estimator restricts the coefficients of the cointegrating vector, the speed of adjustment coefficient, and the short-run coefficients to be equal across panels.

If the variables are $I(1)$ and cointegrated, then the error term is $I(0)$ for all i. The associated dynamic panel specification of (26) is:

$$\begin{aligned} gdp_{i,t} =& \lambda_i gdp_{i,t-1} + \beta_{10,i} k_{i,t} + \beta_{11,i} k_{i,t-1} + \beta_{20,i} fdi_{i,t} + \beta_{21,i} fdi_{i,t-1} \\ &+ \beta_{30,i} rd_{i,t} + \beta_{31,i} rd_{i,t-1} + \mu_i + \epsilon_{i,t} \end{aligned} \tag{27}$$

from where the error correction equation is given by:

$$\begin{aligned} \Delta gdp_{i,t} =& \phi_i \Big(gdp_{i,t-1} - \theta_{0,i} - \theta_{1,i} k_{i,t-1} - \theta_{2,i} fdi_{i,t-1} - \theta_{3,i} rd_{i,t-1} \Big) \\ &- \beta_{11,i} \Delta k_{i,t} - \beta_{21,i} \Delta fdi_{i,t} - \beta_{31,i} \Delta rd_{i,t} + \epsilon_{i,t} \end{aligned} \tag{28}$$

where:

$$\theta_{0,i} = \frac{\mu_i}{1-\lambda_i}, \quad \theta_{1,i} = \frac{\beta_{10,i} + \beta_{11,i}}{1-\lambda_i}, \quad \theta_{2,i} = \frac{\beta_{20,i} + \beta_{21,i}}{1-\lambda_i}, \quad \theta_{3,i} = \frac{\beta_{30,i} + \beta_{31,i}}{1-\lambda_i}$$

and $\phi_i = -(1-\lambda_i)$.

In Equation (28), parameter $\theta_{0,i}$ represents fixed-specific effects, and $\theta_{1,i}, \theta_{2,i}$, and $\theta_{3,i}$) give the long-run dynamic effect. By contrast, $\beta_{11,i}, \beta_{21,i}$, and $\beta_{31,i}$ imply the dynamic of short-term impacts. Lastly, ϕ_i is a coefficient of the ECT to illustrate the speed of adjustment back to the long-term stability after a short-run vibration. This factor furthermore should be negative and statistically significant to confirm the existence of a long-run relationship between different covariates and GDP per cap.

3.3.2 Empirical findings

As we have discussed above, Equation (28) is performed by the DFE estimator with a cluster on the country identification.[23] The estimating results are displayed in Table 1.

[23]In general, the *xtpmg* command in Stata allows us to perform the pooled mean group, mean group, and DFE estimators. However, the sole DFE estimator works owning to insufficient observations for the two other methods

Table 1: Impacts of Investment in physical capital, R&D, and FDI on GDP per cap

	Full sample		UMI economies		LMI economies		Low income economies	
VARIABLES	LR	SR	LR	SR	LR	SR	LR	SR
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Speed of adjustment (ECT)	-0.045***		-0.050***		-0.029**		-0.059	
	(0.006)		(0.007)		(0.009)		(0.041)	
Investment in R&D (as % of GDP)	0.606*	-0.050+	0.605*	-0.055+	0.958	0.031	-0.235	-0.019
	(0.242)	(0.029)	(0.246)	(0.030)	(1.438)	(0.058)	(0.669)	(0.065)
Physical capital (as % of GDP)	0.029**	0.002	0.032*	0.004*	0.073	0.003	-0.004	0.004+
	(0.011)	(0.002)	(0.015)	(0.002)	(0.049)	(0.002)	(0.026)	(0.002)
FDI (as % of GDP)	0.459**	-0.086	0.644**	-0.285***	0.479	-0.025*	0.807	-0.161
	(0.143)	(0.062)	(0.243)	(0.052)	(0.402)	(0.012)	(1.102)	(0.161)
Constant		0.398***		0.446***		0.218*		0.463
		(0.052)		(0.066)		(0.091)		(0.286)
Numbers of country	52	52	31	31	15	15	6	6
$N \times T$	681	681	457	457	166	166	58	58

Standard errors in parentheses, *** p<0.001, ** p<0.01, * p<0.05, + p<0.1 LR: Long-run estimation, SR: Short-run estimation
UMI: Upper-Middle income LMI: Lower-Middle income

Columns 1-2 report the short-run and long-run estimations of the full sample. The ETC coefficient is negative and statically significant at 0.1%, confirming a long-term relationship between variables. Take a look at the short-run estimation (column 2), investment in R&D and FDI have a negative impact on GDP per capita while the inverse is valid for the effect of GFCF. However, notice that only investment in R&D matters in the short-term (at a 10% level of significance). It implies that in the short-run period, the share of investment in R&D over GDP of a country augments by 1%, its GDP per capita would decrease by 0.05%, *ceteris paribus*. Very interestingly, investment in R&D turns out to positively and significantly impact GDP per capita in the long-term. A 1% increase in investment in R&D (as % of GDP) would improve the GDP per capita by 0.6%. Likewise, the impacts of inward FDI stock and GFCF on GDP per capita are positive and significant. These results are likely to support our above theoretical findings that it takes time such that an investment in R&D has a real impact on a country's economic development. Also, FDI contributes to this development but in the long-term rather than in the short-term.

To have more in-depth insights about the role of the development level and the efficiency in investment in R&D, Equation (28) is performed by different sub-samples: UMI (Upper-Middle income) economies, LMI (Lower-Middle income) economies, and Low-income economies. The short-run and long-run estimations are respectively represented in columns 3-4 (UMI economies), 5-6 (LMI economies), and 7-8 (Low-income economies) of Table 1. Yet, the long-run effect of different covariates is only significant for UMI economies and becomes insignificant for LMI and Low-income economies. Notice that compared to LMI and low-income economies, UMI economies exhibit higher investment in R&D (on average, 0.5% of GDP) while inward FDI stock is not the highest (see Table C1 in Appendix).

These empirical results are consistent with our theoretical findings in two ways. On the one hand, to benefit from FDI, the host country should reach some development level. On the other hand, there would be some threshold in terms of development or investment in R&D's efficiency such that R&D investment has a real impact on the economy.

4 Conclusion

We have investigated the nexus between FDI, R&D, and growth of a host country by using infinite-horizon optimal growth models. According to our results, the very question does not rely on whether or not developing countries should attract inward FDI, but instead on how they implement policies to benefit from FDI spillovers. We have proved that FDI can act as a catalyst, helping a host developing country to avoid a middle income trap and potentially attain a higher growth rate. However, to reach sustainable economic growth in the long run, the host country should focus on domestic investment and R&D.

Appendix

A The optimal growth theory: a preliminary

For a pedagogical purpose, we present general results showing the property of optimal growth paths in models without the concavity of production functions. Although there is a vast literature on the optimal growth theory, some results in this section are new, and they are used in the present paper.[24]

We now introduce a formal optimal growth model. There is one agent who maximizes her intertemporal

$$(P): \max_{\left(c_t, S_{t+1}\right)_{t=0}^{+\infty}} \Big[\sum_{t=0}^{+\infty} \beta^t u(c_t)\Big] \tag{A.1}$$

$$\text{subject to: } c_t + S_{t+1} \le f(S_t), \quad c_t, S_{t+1} \ge 0, \tag{A.2}$$

where S_0 is given. For short, we write (x_t) instead of $(x_t)_{t=0}^{\infty}$, where x_t is a vector.[25]

A path (c_t, S_t) is feasible if it satisfies (A.2) for every t. A capital path (S_t) is feasible if there exists a consumption path (c_t) such that (c_t, S_t) is a feasible path. A path (c_t, S_t) (or capital path (S_t)) is from S if $S_0 = S$.

A path (c_t, S_t) is optimal from S if it solves problem (P) with $S_0 = S$. A path (c_t, S_t) (resp. capital path (S_t)) is stationary if $c_t = c$ and $S_t = S$ for every t. A pair (c, S) is a steady state if the stationary path (c_t, S_t) with $c_t = c$ and $S_t = S$ is optimal. A capital stock $S \ge 0$ is a steady state if (c,S) is a steady state for some $c \ge 0$.

We require standard assumptions which are maintained throughout this paper.

Assumption (H1): *u is in C^1, strictly increasing and concave and $u'(0) = \infty$.*

Assumption (H2): *f is strictly increasing and $f(0) \ge 0$.*[26]

Assumption (H3):*For every $S > 0$, there exists a feasible path (c_t, S_t) from S such that $\sum_{t=0}^{\infty} \beta^t u(c_t) > -\infty$. We also have $\sum_{t=0}^{\infty} \beta^t u(f^t(S)) < \infty$, where f^t is defined by $f^1 = f, f^{t+1} = f(f^t)$.*

The last assumption require that the utility function is well defined and finite.

Let denote $v(S_0)$ be the value function of the problem (P). We have the Bellman equation $v(S_0) = \max\limits_{0 \le S \le f(S_0)} \{u(f(S_0) - S) + \beta v(S)\}$. By using this Bellman equation and the argument in Amir (1996), we obtain that:

Lemma 7. *Every optimal capital path is monotonic. By consequence, if an optimal path is bounded from above, then it converges.*

Following Kamihigashi and Roy (2007), we have Euler condition in the form of inequality

[24]Let us mention some papers which are very closed to ours. Dechert and Nishimura (1983) give a complete characterization of optimal growth paths in a model with convex-concave technologies. Hung et al. (2009) studies an optimal growth model where the aggregate production function is maximum of concave technologies. Majumdar and Mitra (1982), Kamihigashi and Roy (2007) study non-smooth, non-convex models. Jones and Manuelli (1990) work with increasing return to scale technologies.

[25]Some studies replace constraint $S_{t+1} \ge 0$ by $S_{t+1} \ge r(S_t)$. The reader is referred to Kamihigashi and Roy (2007), Dimaria et al. (2002), Chapter 5 of Le Van and Dana (2003) for discrete time model, and Romer (1986) for continuous time model.

[26]When $f(0) = 0$, the function $f(S)$ can be interpreted as a gross return of investment S. However, in general, $f(S)$ can contain initial endowment and/or gross interest rate. This setting is general enough to cover concave-convex or convex-concave production functions. It will also be useful when we consider a non-differentiable function, for example, a function in which there is a threshold.

Lemma 8. *Let (c_t, S_t) be an optimal path, we have for every $t \geq 0$*

$$\beta u'(c_{t+1})D^- f(S_{t+1}) \geq u'(c_t) \geq \beta u'(c_{t+1})D^+ f(S_{t+1}). \tag{A.3}$$

where the Dini derivatives of function f are defined by $D^+ f(x) = \limsup\limits_{\epsilon \downarrow 0} \frac{f(x+\epsilon)-f(x)}{\epsilon}$ and $D^- f(x) = \liminf\limits_{\epsilon \downarrow 0} \frac{f(x)-f(x-\epsilon)}{\epsilon}$.

Proof. Since $u'(0) = \infty$, we have $c_t > 0$ for every $t \geq 0$. For each $t \geq 0$. we consider path (c'_s, S'_s) as follows

$$\begin{aligned} c'_s &= c_s \quad \forall s \in \{t, t+1\}, \quad S'_s = S_s \quad \forall s \neq t+1 & \text{(A.4)} \\ c'_t &= c_t - \epsilon, \quad S'_{t+1} = S_{t+1} + \epsilon & \text{(A.5)} \\ c'_{t+1} &= c_{t+1} + f(S_{t+1} + \epsilon) - f(S_{t+1}). & \text{(A.6)} \end{aligned}$$

For $\epsilon > 0$ small enough, the path (c'_s, S'_s) is feasible. Therefore,

$$u(c_t - \epsilon) + \beta u(c_{t+1} + f(S_{t+1} + \epsilon) - f(S_{t+1})) \leq u(c_t) + \beta u(c_{t+1}).$$

Let ϵ tend to 0, we obtain the right inequality of (A.3). By using the similar argument, we can prove the left inequality of (A.3). $\square$

Corollary 1. *If $S > 0$ is a steady state, then we have $\beta D^- f(S) \geq 1 \geq \beta D^+ f(S)$.*

Let us start our exposition with the following result which provides a condition under which the optimal capital path cannot converge to zero. The idea is that if productivity is high enough at original, then we will produce.

Proposition A 1. *Assume that there exists $\underline{x} > 0$ such that $\beta D^+ f(S) > 1$ for every $0 \leq S \leq \underline{x}$, then no optimal capital path converges to zero.*

Proof. Since $u'(0) = \infty$, we have $c_t > 0$ for every t, and so is S_t. By Euler inequality, we get that $u'(c_t) > \beta u'(c_{t+1})D^+ f(S_{t+1})$. According budget constraint, we have $\lim\limits_{t \to +\infty} c_t = f(0)$.
Case 1: $f(0) = 0$. We have $\lim\limits_{t \to +\infty} c_t = 0$. Since $\lim\limits_{t \to +\infty} S_t = 0$, there exists t_0 such that $\beta D^+ f(S_{t+1}) > 1$ for every $t \geq t_0$. Consequently, $c_t \leq c_{t+1}$ for every $t \geq t_0$. Contradiction to the fact that $\lim\limits_{t \to +\infty} c_t = 0$.
Case 2: $f(0) > 0$. We have $\lim\limits_{t \to \infty} \dfrac{u'(c_t)}{u'(c_{t+1})} = 1$. Euler inequality implies that $\limsup\limits_{t \to +\infty} \beta D^+ f(S_t) \leq 1$, contradiction! $\square$

Proposition A 2. *Assume that there exists $x_0 \geq x_1 > 0$ and a function $g : \mathbb{R}^+ \to \mathbb{R}^+$ strictly increasing such that*

(i) $f(x) \leq g(x)$ for every $x \leq x_0$.

(ii) $x \leq g(x) \leq g(x_1) = x_1$ for every $x \leq x_1$, and $g(x) \leq x$ for every $x \geq x_1$.

We have every optimal growth from $S_0 \leq x_1$ is bounded above by x_1.

Proof. We have $c_0 + S_1 \leq f(S_0) \leq f(x_1) \leq G(x_1) = x_1$. Thus, $S_1 \leq x_1$ and $f(S_1) \leq f(x_1) \leq x_1$. By induction argument, we get $f(S_t) \leq x_1$ for every t. $\square$

The intuition of Proposition 2 is the following: If the return function f is dominated by a function g with which the optimal path is bounded above, then the optimal path associated with function f is also bounded above. When $f(x) = g(x) = Ax^\alpha + (1-\delta)x$, with δ is the depreciation rate, $A > 0$ is TFP, $\alpha \in (0,1)$, we recover the standard Ramsey model.

Proposition 2 also complements Proposition 4.1 in Kamihigashi and Roy (2007) because our result covers the following function while Proposition 4.1 in Kamihigashi and Roy (2007) does not: $f(x) = Ax^\alpha$, if $x \leq a$ and $= (A + x - a)x^\alpha$ if $x \geq a$ where $A > 0$, $a \geq 0$ and $\alpha \in (0,1)$.

We end this section by presenting a condition for unbounded growth with arbitrary initial capital stock S_0. This result is a consequence of Proposition 4.6 in Kamihigashi and Roy (2007).

Proposition A 3. *Assume that $\beta D^+ f(x) > 1$ for every $x > 0$. Then every optimal capital path goes to infinity.*

B Appendix for Section 2

B.1 Theoretical framework

Proof of Proposition 1. The problem (P_1) can be rewritten as follows

$$(P_1') : \quad \max_{\left(c_t, S_{t+1}\right)_{t=0}^{+\infty}} \left[\sum_{t=0}^{+\infty} \beta^t u(c_t)\right] \text{ subject to } c_t + S_{t+1} \leq F(S_t), c_t, S_{t+1} \geq 0, \tag{B.1}$$

where S_0 is given and the function F is defined by

$$F(S) := \max_{K_c, H, L_e \geq 0} A_c k_c^\alpha + wL_e \text{ subject to: } \quad pK_c + H \leq S, L_e \leq A_h H^{\alpha_h}.$$

We see that F is continuous, strictly increasing (notice that if $\alpha = \alpha_h$, then $F(S) = AS^\alpha$). According to Lemma 7, S_t is monotonic. Since $\alpha < 1$ and $\alpha_h < 1$, we can prove that S_t is bounded from above. Hence, as in the standard Ramsey model, there exists the limit $\lim_{t\to\infty} S_t \equiv S_b$. We now check that $S_b > S_a$. Indeed, we have $\beta F'(S_b) = 1 = \beta \frac{\alpha A_c S_a^{\alpha-1}}{p^\alpha}$. It is easy to prove that $F'(X) > \frac{\alpha A_c X^{\alpha-1}}{p^\alpha}$ $\forall X > 0$. So, $S_b > S_a$. □

Proof of Lemma 1. Point 1. Assume that $\frac{BA_e}{1+S} > 1$. If $L_e < A_h H^{\alpha_h}$, then consider $L_e' = L_e + \epsilon$ such that $L_e' < A_h H^{\alpha_h}$ and $L_d' \equiv (A_h H^{\alpha_h} - L_e') + \frac{BA_e}{1+S} L_e'$. Then, $L_d' > L_d$, $L_e' > L_e$ and the allocation $(K_c, K_d, L_d', L_e', H)$ strictly dominates the allocation (K_c, K_d, L_d, L_e, H), a contradiction!

Point 2. If $\frac{BA_e}{1+S} \leq 1$, we have $L_d \leq A_h H^{\alpha_h} \leq A_h S^{\alpha_h} \leq \bar{L}$. By consequence, $Y_d = 0$.

If $\frac{BA_e}{1+S} \geq 1$, we have

$$\begin{aligned}
L_d &\leq A_h H^{\alpha_h} - L_e + \frac{BA_e}{1+S} L_e \\
&\leq A_h H^\alpha + (\frac{BA_e}{1+S} - 1) A_h H^{\alpha_h} = \frac{BA_e}{1+S} A_h H^{\alpha_h} \leq \frac{BA_e}{1+S} A_h S^{\alpha_h} \leq \bar{L}.
\end{aligned}$$

This implies that $Y_d = 0$.

□

Proof of Lemma 2. First, we claim that there exists a threshold A_1 such that $Y_d > 0$ for every $A_d > A_1$. Indeed, when $Y_d = 0$, $G(S)$ does not depend on A_d. Take $s < S$ such that $A_h s^{\alpha_h} > \bar{L}$. Since $A_h s^{\alpha_h} > \bar{L}$, we can choose $H < s < S$ such that $A_h H^{\alpha_h} > \bar{L}$ and $K_d = (s - H)/p$. Then we choose $L_d = A_h H^{\alpha_h}$ and hence $L_d > \bar{L}$. With these choices, we can see that $G(S)$ tends continuously to infinity when A_d tends to infinity. This contradicts the optimality of allocation.

We now consider $A_d > A_1$, we have $Y_d > 0$. Suppose that $Y_e > 0$. We can compute that

$$w = \Big(\alpha_e^{\alpha_e}(1-\alpha_e)^{1-\alpha_e}\frac{p_n A_e}{p^{\alpha_e}}\Big)^{\frac{1}{1-\alpha_e}}. \tag{B.2}$$

Denote $\lambda, \lambda_1, \lambda_2, \lambda_l$ Lagrange multipliers associated to constraints (11b), (11c), (11d), and $L_e \geq 0$ respectively. We have

$$\begin{aligned}
K_c: &\quad \alpha A_c K_c^{\alpha-1} = \lambda p &\text{(B.3)}\\
K_d: &\quad \alpha_d p_n A_d K_d^{\alpha_d-1}(L_d - \bar{L})^{1-\alpha_d} = \lambda p &\text{(B.4)}\\
L_d: &\quad (1-\alpha_d) p_n A_d K_d^{\alpha_d}(L_d - \bar{L})^{-\alpha_d} = \lambda_2 &\text{(B.5)}\\
H: &\quad \lambda = (\lambda_1 + \lambda_2)\alpha_h A_h H^{\alpha_h - 1} &\text{(B.6)}\\
L_e: &\quad w = \lambda_1 + (1 - \frac{BA_e}{1+S})\lambda_2 \geq (1 - \frac{BA_e}{1+S})\lambda_2. &\text{(B.7)}
\end{aligned}$$

FOCs of K_d, L_d imply that

$$\frac{p_n A_d}{p^{\alpha_d}}\alpha_d^{\alpha_d}(1-\alpha_d)^{1-\alpha_d} = \lambda^{\alpha_d}\lambda_2^{1-\alpha_d} \leq \lambda^{\alpha_d}\Big(\frac{w}{1 - \frac{BA_e}{1+S}}\Big)^{1-\alpha_d}$$

where the last inequality is from (B.7). Combining this with (B.2), we obtain

$$\lambda^{\alpha_d} \geq (1 - \frac{BA_e}{1+S})^{1-\alpha_d}\frac{p_n A_d}{p^{\alpha_d}}\alpha_d^{\alpha_d}(1-\alpha_d)^{1-\alpha_d}\Big(\alpha_e^{\alpha_e}(1-\alpha_e)^{1-\alpha_e}\frac{p_n A_e}{p^{\alpha_e}}\Big)^{\frac{-(1-\alpha_d)}{1-\alpha_e}}.$$

or equivalently $\lambda \geq D$, where

$$D \equiv \Big((1 - \frac{BA_e}{1+S})^{1-\alpha_d}\frac{p_n A_d}{p^{\alpha_d}}\alpha_d^{\alpha_d}(1-\alpha_d)^{1-\alpha_d}\Big(\alpha_e^{\alpha_e}(1-\alpha_e)^{1-\alpha_e}\frac{p_n A_e}{p^{\alpha_e}}\Big)^{\frac{-(1-\alpha_d)}{1-\alpha_e}}\Big)^{\frac{1}{\alpha_d}}.$$

Notice that

$$D \geq D_1 \equiv \Big((1 - BA_e)^{1-\alpha_d}\frac{p_n A_d}{p^{\alpha_d}}\alpha_d^{\alpha_d}(1-\alpha_d)^{1-\alpha_d}\Big(\alpha_e^{\alpha_e}(1-\alpha_e)^{1-\alpha_e}\frac{p_n A_e}{p^{\alpha_e}}\Big)^{\frac{-(1-\alpha_d)}{1-\alpha_e}}\Big)^{\frac{1}{\alpha_d}}.$$

Condition (B.7) implies that $w \geq (\lambda_1 + \lambda_2)(1 - \frac{BA_e}{1+S})$. Therefore, condition (B.6) implies that

$$H^{1-\alpha_h} = \alpha_h A_h \frac{\lambda_1 + \lambda_2}{\lambda} \leq \alpha_h A_h \frac{w}{1 - \frac{BA_e}{1+S}}\frac{1}{D} \leq \alpha_h A_h \frac{w}{1 - BA_e}\frac{1}{D_1}$$

In other words, we have $H \leq \bar{H}$, where $\bar{H}$ is defined by

$$\bar{H}^{1-\alpha} = \alpha_h A_h \frac{w}{1 - BA_e}\frac{1}{D_1}.$$

Observe that $\bar{H}$ depends on A_d. We write $\bar{H} = \bar{H}(A_d)$. We define the threshold $\bar{A}$ by $\bar{L} = A_h\big(\bar{H}(\bar{A})\big)^{\alpha}$. Notice that $\bar{A}$ does not depend on S.

If we choose $A_d \geq \max(A_1, \bar{A})$, then $\bar{L} < L_d \leq A_h H^{\alpha} \leq A_h \bar{H}^{\alpha}$, a contradiction. As a result, $Y_e = 0$. □

Proof of Lemma 3. One can see that the function $G(\cdot)$ is continuous, strictly increasing and $G(0) = 0$. Hence, according to Lemma 7 in Appendix A, the optimal path $(S_t)_t$ is monotonic. Moreover, since $G(\cdot)$ satisfies conditions in Proposition A1 in Appendix A, S_t does not converge to zero. $\square$

Proof of Proposition 2. We assume that $\max(BA_e, 1)A_h\bar{S}^{\alpha_h} \leq \bar{L}$.

As in the proof of Lemma 1, we have, for every t,

$$L_{d,t} \leq \max\Big(\frac{BA_e}{1+S_t}, 1\Big)A_hS_t^{\alpha_h} \leq \max(BA_e, 1)A_hS_t^{\alpha_h}$$

Recall that $S_t \leq x_t \leq \bar{S}$ $\forall t$. So, we have $L_{d,t} \leq \max(BA_e, 1)A_h\bar{S}^{\alpha} \leq \bar{L}$ $\forall t$. Therefore, $Y_{d,t} = 0$ $\forall t$.

$\square$

Proof of Proposition 3. Assume that $Y_{d,t} = 0$ for every t, the welfare of the country W does not depend on A_d and we have that $\lim_{t\to\infty} S_t = S_b$. By consequence, there exists t such that $\max(\frac{BA_e}{1+S_t}, 1)A_hS_t^{\alpha_h} > \bar{L}$.

If $\frac{BA_e}{1+S_t} < 1$. Let $L'_{e,t} = 0$ and $L'_{d,t} = A_h(H'_t)^{\alpha_h}$. Choose H'_t closed to S_t such that $\max(\frac{BA_e}{1+S_t}, 1)A_h(H'_t)^{\alpha_h} > \bar{L}$. Let A_d be high enough, the new welfare of the country (with this allocation) will be higher than W. This violates the optimality of the country's choice.

If $\frac{BA_e}{1+S_t} \geq 1$. Let $L'_{e,t} = A_hH_t'^{\alpha_h}$ and $L'_{d,t} = \frac{BA_e}{1+S_t}A_hH_t^{\alpha_h}$. Choose H'_t is closed to S_t such that $\max(\frac{BA_e}{1+S_t}, 1)A_h(H'_t)^{\alpha_h} > \bar{L}$. Let A_d be high enough, the new welfare of the country will be higher than W. This violates the optimality of the country's choice.

The above arguments imply that there is a date t_0 such that $Y_{d,t_0} > 0$. Then t_d will be determined by $t_d = \inf\{t_0 : Y_{d,t_0} > 0\}$.

It is easy to see that $t_d > 1$ because $\max(\frac{BA_e}{1+S_0}, 1)A_hX_0^{\alpha_h} < \bar{L}$. $\square$

Proof of Proposition 4. Under condition (15), assumptions in Proposition 3 are satisfied. Therefore, there exist A_1 and t_1 such that $Y_{d,t_1} > 0$. As a result, for every $t > t_1$, we have

$$F_t^d(K_{d,t}, L_{d,t}) = A_dK_{d,t}^{\alpha}L_{d,t}^{1-\alpha}.$$

According to Lemma 2, we can choose $A^* > A_1$ such that $Y_{d,t} > 0$, $Y_{e,t} = 0$ for every $A_d \geq A^*$. The timing t^* is then determined by $t^* = t_1 + 1$.

We now write the social planner's problem from date t^*

$$\max_{\left(c_t, S_{t+1}\right)} \Big[\sum_{t=t^*}^{+\infty}\beta^tu(c_t)\Big] \text{ subject to } c_t + S_{t+1} \leq F_1(S_t) \tag{B.8}$$

where F_1 is defined by

$$F_1(S) \quad = \quad \max\Big\{A_cK_c^{\alpha} + p_nA_dK_d^{\alpha_d}(A_hH^{\alpha_h})^{1-\alpha_d} : p(K_c + K_d) + H \leq S; K_c, K_d, H \geq 0\Big\}$$

It is easy to see that $F_1(S)$ is continuous, strictly increasing, and it is dominated by a decreasing return to scale function. According to Proposition A2, the sequence S_t is bounded from above. Moreover, Lemma 7 and the assumption $X_0 < S_a$ imply that S_t is increasing in t. Hence, the sequence S_t converges and so do $(c_t, K_{c,t}, K_{d,t}, H_t)$. Denote $S_c \equiv \lim_{t\to\infty} S_t = S_c$.

We can choose A_d high enough such that $F'_1(x) - F'(x) > 0$ for any x in an interval containing S_c and S_b (note that $F'_1(x) - F'(x)$ may be negative for some $x > 0$). By consequence, we obtain $S_c > S_b$ at the steady state. $\square$

B.2 TFP estimation's methodology

We use the GMM estimator proposed by Wooldridge (2009) to estimate the firm's TFP. According to the author, materials (also called intermediate inputs) are used as a proxy to control for unobserved productivity shocks. Hence, productivity ω_{it} can be represented as:

$$\omega_{it} = \omega(k_{it}, m_{it}) \tag{B.9}$$

where m_{it} is intermediate inputs.

At the beginning, assume that

$$E\left(\varepsilon_{it} \mid l_{it}, k_{it}, m_{it}\right) = 0 \quad t = 1, \ldots, T \tag{B.10}$$

then we have the following regression function:

$$E\left(y_{it} \mid l_{it}, k_{it}, m_{it}\right) = \beta_0 + \beta_l l_{it} + \beta_k k_{it} + \omega(k_{it}, m_{it}) = \beta_l l_{it} + f(k_{it}, m_{it}) \tag{B.11}$$

where $f(k_{it}, m_{it}) = \beta_0 + \beta_k k_{it} + \omega(k_{it}, m_{it})$.

The estimation procedure takes place in two stages. The first is to estimate parameter β_l and the second β_k. To identify β_l, we need three assumptions. The first is on ε_{it}:

$$E\left(\varepsilon_{it} \mid l_{it}, k_{it}, m_{it}, l_{it-1}, k_{it-1}, m_{it-1}, \ldots, l_{i1}, k_{i1}, m_{i1}\right) = 0 \quad t = 1, \ldots, T$$

The second assumption is to restrict the dynamic in the productivity process:

$$E\left(\omega_{it} \mid \omega_{it-1}, \ldots, \omega_{i1}\right) = E\left(\omega_{it} \mid \omega_{it-1}\right) \quad t = 2, \ldots, T$$

The third assumption is that k_{it} is uncorrelated with the productivity shock (τ) defined as follows:

$$\tau_{it} = \omega_{it} - E\left(\omega_{it} \mid \omega_{it-1}\right)$$

In the second stage, the conditional expectation applied to find β_k depends upon (k_{it-1}, m_{it-1}). Therefore, τ_{it} must be uncorrelated with (k_{it-1}, m_{it-1}) and then a sufficient condition could be formulated as:

$$E\left(\omega_{it} \mid l_{it}, k_{it}, m_{it}, l_{it-1}, k_{it-1}, m_{it-1}, \ldots, l_{i1}, k_{i1}, m_{i1}\right) = E\left(\omega_{it} \mid \omega_{it-1}\right) = f\left[\omega\left(k_{it-1}, m_{it-1}\right)\right]$$

Notice that components of l_{it} are allowed to be associated with τ_{it}. Then the production function can be driven as:

$$y_{it} = \beta_0 + \beta_k k_{it} + \beta_l l_{it} + f\left[\omega\left(k_{it-1}, m_{it-1}\right)\right] + \tau_{it} + \varepsilon_{it}$$

Hence, to find β_k and β_l, two functions are derived:

$$\begin{aligned} y_{it} &= \beta_0 + \beta_k k_{it} + \beta_l l_{it} + \omega\left(k_{it}, m_{it}\right) + \varepsilon_{it} \quad \forall t = 1, \ldots, T \\ y_{it} &= \beta_0 + \beta_k k_{it} + \beta_l l_{it} + f\left[\omega\left(k_{it-1}, m_{it-1}\right)\right] + u_{it} \quad \forall t = 2, \ldots, T \end{aligned}$$

where $u_{it} \equiv \tau_{it} + \varepsilon_{it}$. The orthogonal conditions are stated as:

$$E\left(u_{it} \mid k_{it}, l_{it-1}, k_{it-1}, m_{it-1}, \ldots, l_{i1}, k_{i1}, m_{i1}\right) = 0 \quad t = 2, \ldots, T$$

Estimating β_k and β_l requires investigating the unknown function $f(.)$ and $\omega(.)$ and Wooldridge (2009) proposes that:

$$\omega\left(k_{it}, m_{it}\right) = \gamma_0 + c\left(k_{it}, m_{it}\right)\gamma$$

and $f(.)$ can be approximately explained by a polynomial in ω

$$f(\omega) = \rho_0 + \rho_1\omega + \cdots \rho_n\omega^n$$

from where the production function can be rewritten as:

$$y_{it} = \zeta_0 + \beta_k k_{it} + \beta_l l_{it} + c_{it}\gamma + \varepsilon_{it} \quad t = 1, \ldots, T \tag{B.12}$$

and

$$y_{it} = \alpha_0 + \beta_k k_{it} + \beta_l l_{it} + \rho_1(c_{i1}\gamma) + \cdots \rho_n(c_{it-1}\gamma)^n + u_{it} \quad t = 2, \ldots, T \tag{B.13}$$

where $\zeta_0 = \beta_0 + \gamma_0$ and $\alpha_0 = \zeta_0 + \rho_0$.

According to Wooldridge (2009), the GMM is performed to estimate Regressions (B.12)-(B.13). Once β_k and β_l are estimated, the firm's TFP (in log) is obtained by:

$$\omega_{it} = y_{it} - \beta_k k_{it} - \beta_l l_{it} \tag{B.14}$$

from where we can obtain the aggregate productivity given in Equation (16).

B.3 Case of Vietnamese manufacturing industries: Estimation results

Table B1: Estimation of the firm production function

	(1)	(2)	(3)
VARIABLES	lnva	lnva	lnva
lnl	0.658***	0.610***	0.645***
	(0.065)	(0.014)	(0.018)
lnk	0.316***	0.402***	0.400***
	(0.079)	(0.023)	(0.037)
Observations	213	4,689	2,774
Number of groups	218	2,547	1,683

Note: Standard errors in parentheses
*** p<0.001, ** p<0.01, * p<0.05, + p<0.1
(1) Computer and Peripheral equipment manufacturing
(2) Electrical equipment manufacturing
(3) Radio, television and communication equipment manufacturing

Table B2: Descriptive statistics at the firm level

Variable	Mean	Std. Dev.	Min	Max
Computer and Peripheral equipment manufacturing				
Full sample				
Added-value (in log)	6.77	2.94	-0.904	13.71
Physical capital (in log)	8.59	2.67	4.13	14.69
Labor (in log)	4.00	2.38	0	10.06
TFP	1.42	0.98	- 3.64	4.51
Number of observations	453			
Domestic firms				
Added-value (in log)	5.87	2.48	-0.904	13.36
Physical capital (in log)	7.72	2.22	4.12	14.67
Labor (in log)	3.26	1.998	0	10.04
TFP	1.28	0.91	-3.64	4.51
Number of observations	342			
Foreign firms				
Added-value (in log)	9.53	2.49	1.99	13.71
Physical capital (in log)	11.27	2.10	6.35	14.69
Labor (in log)	6.27	1.99	0.69	10.06
TFP	1.84	1.08	-2.32	3.82
Number of observations	111			
Electrical equipment manufacturing				
Full sample				
Added-value (in log)	6.14	2.29	-3.23	13.12
Physical capital (in log)	8.31	2.05	-1.51	14.03
Labor (in log)	3.52	1.67	0	9.33
TFP	0.64	0.98	- 5.87	4.29
Number of observations	7,854			
Domestic firms				
Added-value (in log)	5.79	2.18	-3.23	13.11
Physical capital (in log)	7.96	1.97	-1.51	14.02
Labor (in log)	3.29	1.57	0	9.33
TFP	0.59	0.96	-5.81	4.12
Number of observations	6,033			
Foreign firms				
Added-value (in log)	7.29	2.26	-0.395	12.57
Physical capital (in log)	9.46	1.86	4.31	13.42
Labor (in log)	4.35	1.75	0	9.104
TFP	0.83	1.03	-5.87	4.29
Number of observations	1,821			
Radio, television and communication equipment manufacturing				
Full sample				
Added-value (in log)	6.92	2.53	-1.63	16.5
Physical capital (in log)	8.61	2.27	-0.53	17.30
Labor (in log)	4.27	1.91	0	11.07
TFP	0.73	1.03	- 7.33	4.53
Number of observations	4,759			
Domestic firms				
Added-value (in log)	6.58	2.51	-1.63	16.50
Physical capital (in log)	8.24	2.24	-0.53	17.30
Labor (in log)	4.02	1.89	0	11.07
TFP	0.69	1.01	-7.34	4.53
Number of observations	3,616			
Foreign firms				
Added-value (in log)	8.05	2.24	-0.92	15.34
Physical capital (in log)	9.81	1.90	4.75	16.47
Labor (in log)	5.08	1.74	0	10.24
TFP	0.85	1.07	-5.94	3.29
Number of observations	1,143			

C Appendix for Section 3

C.1 Theoretical framework

Proof of Lemma 4. Let $x := bS^\sigma - \bar{x}$. Since $x > 0$, there exists $\alpha_n \in (0,1)$ such that $bS^\sigma \alpha_n^\sigma = \bar{x}$. Define K_c, N, H by

$$N = (\alpha_n + \epsilon)S, \quad pK_c = \epsilon S, \quad H = 0 \tag{C.1}$$

where $\epsilon > 0$ such that $\alpha_n + 2\epsilon = 1$ (so that $N + pK_c = S$). Precisely, $\epsilon = \frac{1}{2}\Big(1 - (\frac{\bar{x}}{bS^\sigma})^{\frac{1}{\sigma}}\Big)$. With such N, K_c, we have $bN^\sigma > \bar{x}$ and so

$$\begin{aligned} g(K_c, N, H) &= \Big[A_c + a(bN^\sigma - \bar{x})\Big]K_c^\alpha && \text{(C.2)}\\ &= \Big[A_c + a\Big(b\big(\frac{S}{2} + \frac{\bar{x}^{\frac{1}{\sigma}}}{2b^{\frac{1}{\sigma}}}\big)^\sigma - \bar{x}\Big)\Big]\frac{1}{p^\alpha}\Big(\frac{S}{2} - \frac{\bar{x}^{\frac{1}{\sigma}}}{2b^{\frac{1}{\sigma}}}\Big)^\alpha && \text{(C.3)}\\ &= \Big[A_c + a\Big(\big(b^{\frac{1}{\sigma}}\frac{S}{2} + \frac{\bar{x}^{\frac{1}{\sigma}}}{2}\big)^\sigma - \bar{x}\Big)\Big]\frac{1}{p^\alpha}\Big(\frac{S}{2} - \frac{\bar{x}^{\frac{1}{\sigma}}}{2b^{\frac{1}{\sigma}}}\Big)^\alpha && \text{(C.4)} \end{aligned}$$

$g(K_c, N, H)$ is increasing in a and b. It will be higher than $F(S)$ when a and b are high enough because $F(S)$ does not depend on (a, b).

□

Proof of Lemma 5. We need an intermediate step.

Claim 1. *Assume that $a\bar{x} > A_c$. Denote $N^* \equiv (\bar{x}/b)^{1/\sigma}$, $x^* \equiv \big(\frac{\alpha+\sigma}{\sigma} - \frac{\alpha}{\sigma}\frac{a\bar{x}-A_c}{a\bar{x}}\big)N^*$ and*

$$G_1(x) = \max\{\big(A_c + a(bN^\sigma - \bar{x})\big)K_c^\alpha : K_c, N \geq 0, pK_c + N \leq x, bN^\sigma \geq \bar{x}\}.$$

We have that $x^* \geq N^*$. The function G_1 is well-defined on the interval $[N^*, \infty)$. On this interval, G_1 is strictly increasing, continuously differentiable and $G_1'(x) > \frac{\alpha A x^{\alpha-1}}{p^\alpha}$. $G_1(x) - A_c x^\alpha/p^\alpha$ is strictly increasing in x and there exists a unique x^* such that $G_1(x^*) = A_c(x^*)^\alpha/p^\alpha$. Moreover, $b(x^*)^\sigma - \bar{x} > 0$.

Proof of Claim 1. If $x \leq N^*$, then $G_1(x) = 0$.

Consider the case $x > N^*$. Let (K_c, N) be an optimal point. It is clear that $K_c > 0$ and $N < x$.

If $N \leq N^*$, then $N = N^*$, $G_1(x) = A_c(x - N^*)^\alpha/p^\alpha$ and $G_2(x) = A_c x^\alpha/p^\alpha$.

If $N \in (N^*, x)$ at optimal, we write FOCs

$$\sigma ab N^{\sigma-1}K_c^\alpha = \lambda \tag{C.5}$$

$$\big(A_c + a(bN^\sigma - \bar{x})\big)\alpha K_c^{\alpha-1} = p\lambda = p\sigma abN^{\sigma-1}K_c^\alpha. \tag{C.6}$$

It follows that $\big(A_c + a(bN^\sigma - \bar{x})\big)\alpha = \sigma abN^{\sigma-1}pK_c$ or equivalently

$$\sigma ab\frac{x - N}{N} + \frac{\alpha(a\bar{x} - A_c)}{N^\sigma} - \alpha ab = 0. \tag{C.7}$$

The left hand side (LHS) is strictly decreasing in N because $a\bar{x} - A_c > 0$. When $N = x$, the LHS equals $\frac{\alpha(a\bar{x}-A_c)}{x^\sigma} - \alpha ab < 0$ because $x > N^*$. When $N = N^*$, the LHS equals

$$LHS(N^*) \equiv \sigma ab\frac{x - N^*}{N^*} + \frac{\alpha(a\bar{x} - A_c)}{(N^*)^\sigma} - \alpha ab. \tag{C.8}$$

Observe that $LHS(N^*) \geq 0$ if and only if

$$x \geq x^* \equiv \frac{\alpha+\sigma}{\sigma}N^* - \frac{\alpha}{\sigma}N^*\frac{a\bar{x}-A_c}{a\bar{x}} = N^* + \frac{\alpha(N^*)^{1-\sigma}}{\sigma ab}\Big(\alpha ab(N^*)^\sigma - a\bar{x} + A_c\Big).$$

Therefore, we get that:

1. If $x \leq x^*$, then $N = N^*$ and $G_1(x) = A_c(x-N^*)^\alpha/p^\alpha$. In this case, we have

$$G_1'(x) = \frac{\alpha A_c(x-N^*)^{\alpha-1}}{p^\alpha} > \frac{\alpha A x^{\alpha-1}}{p^\alpha}$$

   because $N^* > 0$ and $\alpha - 1 < 0$.

2. If $x > x^*$, then the equation (C.7) has a unique N_x in the interval (N^*, x). The optimal point (K_c, N) is given by $N = N_x$ and $pK_c + N = S$. Moreover, when x increases, we have N_x, $x - N_x$ and $\frac{x-N_x}{N_x}$ increase.

   We have $G_1(x) = \big(A_c + a(bN^\sigma - \bar{x})\big)K_c^\alpha$ where N is uniquely given by (C.7) and $pK_c = S - N$. By computing directly or using the envelop theorem, we have

$$G_1'(x) = \big(A_c + a(bN^\sigma - \bar{x})\big)\alpha K_c^{\alpha-1} > \frac{\alpha A x^{\alpha-1}}{p^\alpha} \tag{C.9}$$

   because $bN^\sigma - \bar{x} > 0$ and $pK_c < S$.

3. When x tends to x^*, we have N_x tends to N^* and therefore $G_1'(x) = \big(A_c + a(bN^\sigma - \bar{x})\big)\alpha K_c^{\alpha-1}$ tends to $\frac{\alpha A_c(x-N^*)^{\alpha-1}}{p^\alpha}$.

   To sum up, the function G_1 is continuously differentiable and $G_1'(x) > \frac{\alpha A x^{\alpha-1}}{p^\alpha}$.

□

We now prove Lemma 5. Observe that

$$G_0(S) \equiv \max_{K_c,N,H}\Big\{G_1(x) + wA_hH^{\alpha_h} : x + H \leq S; x, H \geq 0\Big\}. \tag{C.10}$$

Let (x_g, H_g) be an optimal point. Since G_1 is differentiable, we have the FOC

$$G_1'(x_g) - \alpha_h w A_h (S - x_g)^{\alpha_h - 1} = 0.$$

Let $(x, S-x)$ be the unique pair such that $A_c(\frac{x}{p})^\alpha + wA_hH^{\alpha_h} = F(S)$. Then, we have $\frac{\alpha A x^{\alpha-1}}{p^\alpha} - \alpha_h w A_h(S-x)^{\alpha_h-1} = 0$.

Since $G_1'(x_g) > \frac{\alpha A(x_g)^{\alpha-1}}{p^\alpha}$, we have $0 > \frac{\alpha A(x_g)^{\alpha-1}}{p^\alpha} - \alpha_h w A_h(S-x_g)^{\alpha_h-1}$. Therefore, we have $x_g > x$ and hence $H_g < H$. It follows that

$$G_0'(S) = \alpha_h w A_h H_g^{\alpha_h-1} > \alpha_h w A_h H_g^{\alpha_h-1} = F'(S). \tag{C.11}$$

So, $G_0(S) - F(S)$ is strictly increasing.

When S is small enough, $G_0(S) - F(S)$ is negative. When S is high enough, $G_0(S) - F(S)$ is positive (see, for example, point (ii) of Lemma 4). So, there exists a unique S^* such that $G_0(S^*) - F(S^*) = 0$.

According to (22) and $G(S) \geq G_0(S)\ \forall S$, we have $G(S) - F(S) = 0\ \forall S \leq S^*$, and $G(S) - F(S) > 0\ \forall S > S^*$.

□

Proof of Proposition 5. The proof is similar to that of Proposition 2. We will prove, by induction argument, that $b\bar{x}_t^\sigma \leq \bar{x}$ and $S_t \leq x^* \ \forall t \geq 1$.

When $t = 1$. We have $N_1 \leq S_1 \leq X_0 \leq x^*$, So, $bN_1^\sigma \leq b\bar{S}_t^\sigma \leq \bar{x}$.

Assume that $b\bar{x}_t^\sigma \leq \bar{x}$ and $S_t \leq x^* \ \forall t \leq T$. This implies that $N_T = 0$, because otherwise we can reduce N_T and augment $K_{c,T}$ in order to get a higher utility, which is a contradiction.

Since $N_T = 0$, we have that $G(S_T) = F(S_T)$. Since $S_T \leq x^*$, we have $F(S_T) \leq F(x^*) = x^*$.

Hence, $S_{T+1} \leq G(S_T) \leq x^*$ and therefore $b\bar{x}_{T+1}^\sigma \leq bS_{T+1}^\sigma \leq b(x^*)^\sigma \leq \bar{x}$. We have finished our proof.

□

Proof of Proposition 6. We need intermediate steps.

Lemma 9. *Assume that $\alpha + \sigma \geq 1$. For any solution K_c, N, H of the problem (G_S), there exists the following limits*

$$\lim_{S\to\infty} \theta_c = \frac{\alpha}{\alpha+\sigma}, \lim_{S\to\infty} \theta_n = \frac{\sigma}{\alpha+\sigma}, \lim_{S\to\infty} \theta_h = 0. \tag{C.12}$$

Proof of Lemma 9. Observe that, when S is high enough, we have $bN^\sigma - \bar{x} > 0$ at optimal. It is also to see that $\theta_c, \theta_h > 0$. By consequence, we can write FOCs for the problem (G') as follows (we have FOCs even the objective function is not concave):

$$\alpha_h w A_h S^{\alpha_h} \theta_h^{\alpha_h - 1} = \lambda \tag{C.13}$$

$$\Big(A_c + a(bS^\sigma\theta_n^\sigma - \bar{x})^+\Big)\frac{\alpha}{p^\alpha}\theta_c^{\alpha-1}S^\alpha = \lambda \tag{C.14}$$

$$ab\sigma S^{\sigma+\alpha}\theta_n^{\sigma-1}\big(\frac{\theta_c}{p}\big)^\alpha = \lambda \tag{C.15}$$

where λ is the multiplier associated to the constraint $\theta_c + \theta_n + \theta_h \leq 1$. The first and the third equations imply that

$$\frac{\alpha_h w A_h p^\alpha}{ab\sigma} = S^{\sigma+\alpha-\alpha_h}\theta_n^{\sigma-1}\theta_c^\alpha\theta_h^{1-\alpha_h} = (S\theta_n)^{\sigma-1}(S\theta_c)^\alpha(S\theta_h)^{1-\alpha_h} \tag{C.16}$$

while the second and third conditions imply that

$$\Big(A_c + a(bS^\sigma\theta_n^\sigma - \bar{x})^+\Big)\alpha = ab\sigma S^\sigma\theta_n^{\sigma-1}\theta_c. \tag{C.17}$$

By consequence, we obtain

$$\theta_c = \frac{\alpha}{\sigma}\theta_n + \frac{\alpha\theta_n^{1-\sigma}(A_c - a\bar{x})}{ab\sigma S^\sigma} \tag{C.18}$$

$$\text{or equivalently } \frac{S\theta_c}{S\theta_n} = \frac{\alpha}{\sigma} + \frac{\alpha(A_c - a\bar{x})}{\sigma ab(S\theta_n)^\sigma}. \tag{C.19}$$

From this, we get $\lim_{S\to\infty}(\frac{\sigma\theta_c}{\alpha\theta_n} - 1)\theta_n^\sigma = 0$. By combining this with the fact that $\sigma \leq 1$, we obtain $\lim_{S\to\infty}(\theta_c - \frac{\alpha}{\sigma}\theta_n) = 0$.

Notice that $b(S\theta_n)^\sigma > N$ for S high enough.

We will prove that when S tends to infinity, $S\theta_h$ is bounded from above, and hence $\lim_{S\to\infty}\theta_h = 0$. To do so, we firstly prove that $\liminf_{S\to\infty}\frac{(S\theta_c)^\alpha}{(S\theta_n)^{1-\sigma}} > 0$. Indeed, according to (C.19), we have

$$\frac{(S\theta_c)^\alpha}{(S\theta_n)^{1-\sigma}} = (S\theta_n)^{\alpha+\sigma-1}\Big(\frac{\alpha}{\sigma} + \frac{\alpha(A_c - a\bar{x})}{\sigma ab(S\theta_n)^\sigma}\Big)^\alpha \tag{C.20}$$

Suppose that there is a sequence (S_k) tends to infinity such that $\lim_{k\to\infty} \frac{(S_k\theta_c)^\alpha}{(S_k\theta_n)^{1-\sigma}} = 0$. Notice that

$$\frac{(S\theta_c)^\alpha}{(S\theta_n)^{1-\sigma}} = \frac{1}{(S\theta_n)^{(1-\sigma)(1-\alpha)}}\Big(\frac{\alpha}{\sigma ab}\big[A_c + a(b(S\theta_n)^\sigma - \bar{x})\big]\Big)^\alpha \geq \frac{1}{(S\theta_n)^{(1-\sigma)(1-\alpha)}}\Big(\frac{\alpha}{\sigma ab}A_c\Big)^\alpha$$

for any S high enough, which implies that $\lim_{k\to\infty} S_k\theta_n = \infty$. However, this will follow that

$$\frac{(S_k\theta_c)^\alpha}{(S_k\theta_n)^{1-\sigma}} = (S_k\theta_n)^{\alpha+\sigma-1}\Big(\frac{\alpha}{\sigma} + \frac{\alpha(A_c - a\bar{x})}{\sigma ab(S_k\theta_n)^\sigma}\Big)^\alpha \tag{C.21}$$

is bounded away from zero (because $\alpha + \sigma \geq 1$), a contradiction.

So, we get that $\liminf_{S\to\infty} \frac{(S\theta_c)^\alpha}{(S\theta_n)^{1-\sigma}} > 0$. By combining this with (C.16), we have that $S\theta_h$ is bounded from above and hence $\lim_{S\to\infty} \theta_h = 0$. Combining with (C.18), we obtain (C.12).

□

Lemma 10. *Assume that $a\bar{x} - A_c \geq 0$. We have*

$$D^+G(S) = \limsup_{\epsilon\downarrow 0} \frac{G(S+\epsilon) - G(S)}{\epsilon} \geq \min\Big(F'(S^*), \Gamma(a,b,\bar{x})\Big) \tag{C.22}$$

$$\textit{where } \Gamma(a,b,\bar{x}) \equiv \frac{\big(\frac{\alpha A_c}{p\sigma}\big)^\alpha \bar{x}^{\frac{-(1-\alpha)(1-\sigma)}{\sigma}} a^{1-\alpha} b^{\frac{1-\alpha}{\sigma}}}{\Big(1 + \frac{\alpha}{\sigma} + \Big(\frac{\alpha_h w A_h (p\sigma)^\alpha}{\sigma(\alpha A_c)^\alpha}\Big)^{\frac{1}{1-\alpha_h}} \frac{1}{a^{\frac{1-\alpha}{1-\alpha_h}} b^{\frac{\alpha_h-\alpha}{\sigma(1-\alpha_h)}}}\Big)^\alpha} \tag{C.23}$$

By consequence, when a and b are high enough and $\alpha_h + \frac{1}{\alpha} \geq 2$, we have $\beta D^+G(S) > 1$ $\forall S > 0$.

Proof. **Part 1**. We prove (C.22). Let $S > S^*$. Consider the function

$$(G_0): \quad G_0(S) \equiv \max_{K_c,N,H} \Big\{\big(A_c + a(bN^\sigma - \bar{x})\big)K_c^\alpha + wA_hH^{\alpha_h}\Big\} \tag{C.24a}$$

$$\text{subject to: } pK_c + N + H \leq S, \quad bN^\sigma \geq \bar{x} \text{ and } K_c, N, H \geq 0. \tag{C.24b}$$

When $S > S^*$, we have $G(S) = G_0(S)$ and $bN^\sigma > \bar{x}$ at optimal. We will quantify $G_0'(S)$.

Let λ be the multiplier associated to the constraint $pK_c + N + H \leq S$, we have FOCs

$$(abN^\sigma - (a\bar{x} - A_c))\alpha K_c^{\alpha-1} = p\lambda \tag{C.25}$$

$$ab\sigma N^{\sigma-1}K_c^\alpha = \lambda \tag{C.26}$$

$$\alpha_h w A_h H^{\alpha_h-1} = \lambda. \tag{C.27}$$

FOCs imply that $\alpha(abN^\sigma - (a\bar{x} - A_c)) = pab\sigma N^{\sigma-1}K_c$ and hence

$$\frac{\alpha}{\sigma}N \geq pK_c = \frac{\alpha}{\sigma}N\Big(1 - \frac{a\bar{x} - A_c}{abN^\sigma}\Big) > N\frac{\alpha A_c}{\sigma a\bar{x}} \tag{C.28}$$

because $a\bar{x} - A_c \geq 0$ and $bN^\sigma \geq \bar{x}$.

Since $ab\sigma N^{\sigma-1}K_c^\alpha = \alpha_h w A_h H^{\alpha_h-1}$, we have

$$\frac{H^{1-\alpha_h}}{N^{1-\alpha_h}} = \frac{\alpha_h w A_h}{ab\sigma} N^{1-\sigma-(1-\alpha_h)} K_c^{-\alpha} \tag{C.29}$$

$$\leq \frac{\alpha_h w A_h}{ab\sigma} N^{1-\sigma-(1-\alpha_h)} \Big(N\frac{\alpha A_c}{p\sigma a\bar{x}}\Big)^{-\alpha} = \frac{\alpha_h w A_h}{ab\sigma}\Big(\frac{p\sigma a\bar{x}}{\alpha A_c}\Big)^\alpha N^{-(\alpha+\sigma-\alpha_h)} \tag{C.30}$$

$$\leq \frac{\alpha_h w A_h}{ab\sigma}\Big(\frac{p\sigma a\bar{x}}{\alpha A_c}\Big)^\alpha (\frac{\bar{x}}{b})^{-\frac{\alpha+\sigma-\alpha_h}{\sigma}} = \frac{\alpha_h w A_h (p\sigma)^\alpha}{\sigma(\alpha A_c)^\alpha} \frac{1}{a^{1-\alpha} b^{\frac{\alpha_h-\alpha}{\sigma}}} \tag{C.31}$$

Thus,

$$H \leq N\Big(\frac{\alpha_h w A_h (p\sigma)^\alpha}{\sigma(\alpha A_c)^\alpha}\Big)^{\frac{1}{1-\alpha_h}} \frac{1}{a^{\frac{1-\alpha}{1-\alpha_h}} b^{\frac{\alpha_h-\alpha}{\sigma(1-\alpha_h)}}} \tag{C.32}$$

Since $S = N + pK_c + H$, we have

$$S \leq N + N\frac{\alpha}{\sigma} + N\Big(\frac{\alpha_h w A_h (p\sigma)^\alpha}{\sigma(\alpha A_c)^\alpha}\Big)^{\frac{1}{1-\alpha_h}} \frac{1}{a^{\frac{1-\alpha}{1-\alpha_h}} b^{\frac{\alpha_h-\alpha}{\sigma(1-\alpha_h)}}} \tag{C.33}$$

which implies that

$$N\Big(1 + \frac{\alpha}{\sigma} + \Big(\frac{\alpha_h w A_h (p\sigma)^\alpha}{\sigma(\alpha A_c)^\alpha}\Big)^{\frac{1}{1-\alpha_h}} \frac{1}{a^{\frac{1-\alpha}{1-\alpha_h}} b^{\frac{\alpha_h-\alpha}{\sigma(1-\alpha_h)}}}\Big) \geq S \geq \Big(\frac{\bar{x}}{b}\Big)^{\frac{1}{\sigma}} \tag{C.34}$$

Denote $d \equiv a\bar{x} - A_c \geq 0$. We have

$$\begin{aligned} G_0(S) &= (abN^\sigma - d)K_c^\alpha + wA_h H^{\alpha_h} \\ G_0'(S) &= (abN^\sigma - d)\alpha K_c^{\alpha-1} K_c'(S) + \sigma ab N^{\sigma-1} K_c^\alpha N'(S) + \alpha_h w A_h H^{\alpha_h - 1} H'(S) \\ &= \sigma ab N^{\sigma-1} K_c^\alpha \end{aligned}$$

because $pK_c'(S) + N'(S) + H'(S) = 1$.

By combining this with $K_c \geq \frac{\alpha A_c}{p\sigma a\bar{x}} N$ and $\sigma < 1$, we have

$$\begin{aligned} G_0'(S) &= abN^{\sigma-1}K_c^\alpha \geq abN^{\sigma+\alpha-1}\Big(\frac{\alpha A_c}{p\sigma a\bar{x}}\Big)^\alpha && \text{(C.36)} \\ &\geq \Big(\frac{\alpha A_c}{p\sigma\bar{x}}\Big)^\alpha a^{1-\alpha} b \Big(\frac{\bar{x}}{b}\Big)^{\frac{\sigma+\alpha-1}{\sigma}} \frac{1}{\Big(1 + \frac{\alpha}{\sigma} + \Big(\frac{\alpha_h w A_h (p\sigma)^\alpha}{\sigma(\alpha A_c)^\alpha}\Big)^{\frac{1}{1-\alpha_h}} \frac{1}{a^{\frac{1-\alpha}{1-\alpha_h}} b^{\frac{\alpha_h-\alpha}{\sigma(1-\alpha_h)}}}\Big)^\alpha} && \text{(C.37)} \\ &= \frac{\Big(\frac{\alpha A_c}{p\sigma}\Big)^\alpha \bar{x}^{\frac{-(1-\alpha)(1-\sigma)}{\sigma}} a^{1-\alpha} b^{\frac{1-\alpha}{\sigma}}}{\Big(1 + \frac{\alpha}{\sigma} + \Big(\frac{\alpha_h w A_h (p\sigma)^\alpha}{\sigma(\alpha A_c)^\alpha}\Big)^{\frac{1}{1-\alpha_h}} \frac{1}{a^{\frac{1-\alpha}{1-\alpha_h}} b^{\frac{\alpha_h-\alpha}{\sigma(1-\alpha_h)}}}\Big)^\alpha} \equiv \Gamma(a, b, \bar{x}) && \text{(C.38)} \end{aligned}$$

At point S^*, the right Dini derivative of G is

$$D^+G(S^*) = \limsup_{\epsilon \downarrow 0} \frac{G(S^*+\epsilon) - G(S^*)}{\epsilon} \geq \limsup_{\epsilon \downarrow 0} \frac{F(S^*+\epsilon) - F(S^*)}{\epsilon} = F'(S^*). \tag{C.39}$$

When $S < S^*$, we have $G(S) = F(S)$ and hence $G'(S) = F'(S) \geq F'(S^*)$ because F' is decreasing.

Part 2. We prove that, when a and b are high enough and $\alpha_h + \frac{1}{\alpha} \geq 2$, we have $\beta D^+G(S) > 1 \; \forall S > 0$.

Observe that $\Gamma(a, b, \bar{x})$ is increasing in a and $\beta\Gamma(a, b, \bar{x}) > 1$ when a is high enough.

When $\alpha_h + \frac{1}{\alpha} \geq 2$, we have $\frac{1-\alpha}{\sigma} + \frac{\alpha_h - \alpha}{\sigma(1-\alpha_h)} \geq 0$ and therefore $\Gamma(a, b, \bar{x})$ is strictly increasing in b. In this case, it is easy to see that $\beta\Gamma(a, b, \bar{x}) > 1$ when b is high enough.

We now prove that $\beta F'(S^*) > 1$ when a or b is high enough. As in proof of point (ii) of Lemma 4, we have that: If $bS^\sigma > \bar{x}$, then

$$G(S) \geq \Big[A_c + a\Big(\big(b^{\frac{1}{\sigma}}\frac{S}{2} + \frac{\bar{x}^{\frac{1}{\sigma}}}{2}\big)^\sigma - \bar{x}\Big)\Big]\frac{1}{p^\alpha}\Big(\frac{S}{2} - \frac{\bar{x}^{\frac{1}{\sigma}}}{2b^{\frac{1}{\sigma}}}\Big)^\alpha > 0 \tag{C.40}$$

At point S^* which depends on a and b, we have $F(S^*) = G(S^*)$ and hence

$$\Big[A_c + a\Big(\big(b^{\frac{1}{\sigma}}\frac{S^*}{2} + \frac{\bar{x}^{\frac{1}{\sigma}}}{2}\big)^{\sigma} - \bar{x}\Big)\Big]\frac{1}{p^{\alpha}}\Big(\frac{S^*}{2} - \frac{\bar{x}^{\frac{1}{\sigma}}}{2b^{\frac{1}{\sigma}}}\Big)^{\alpha} \leq F(S^*) \leq \max(A_c(S^*)^{\alpha}, w_h A_h S^{\alpha_h}).$$

We prove that S^* tends to zero when a or b goes to infinity. Indeed, let, for example, b tend to infinity. If $\liminf_{b\to\infty} S^* > 0$, by using the property $\alpha + \sigma \geq 1 > \max(\alpha, \alpha_h)$, we get that

$$\Big[A_c + a\Big(\big(b^{\frac{1}{\sigma}}\frac{S^*}{2} + \frac{\bar{x}^{\frac{1}{\sigma}}}{2}\big)^{\sigma} - \bar{x}\Big)\Big]\frac{1}{p^{\alpha}}\Big(\frac{S^*}{2} - \frac{\bar{x}^{\frac{1}{\sigma}}}{2b^{\frac{1}{\sigma}}}\Big)^{\alpha} > \max(A_c(S^*)^{\alpha}, w_h A_h S^{\alpha_h}).$$

where b is high enough, a contradiction.

So, when a or b is high enough, S^* is low enough and hence $\beta F'(S^*) > 1$ since $F'(0) = \infty$.

□

We are now ready to prove Proposition 6.

According to Lemma 10, we have $\beta D^+G(S) > 1$ $\forall S > 0$ when a and b are high enough. Applying Proposition A3, we have $\lim_{t\to\infty} S_t = \infty$. According to Lemma 9, we obtain point 2 of Proposition 6.

□

Proof of Proposition 7. We observe that

$$G(S) \leq (A_c + abS^{\sigma})\frac{1}{p^{\alpha}}S^{\alpha} + wA_hS^{\alpha_h} \leq \begin{cases} \frac{A_c+ab}{p^{\alpha}} + wA_h \text{ if } S \leq 1 \\ \Big(\frac{A_c+ab}{p^{\alpha}} + wA_h\Big)S^{\max(\alpha+\sigma,\alpha_h)} \text{ if } S \geq 1. \end{cases}$$

By using $\max(\alpha+\sigma, \alpha_h) < 1$, it is easy to prove that S_t is bounded from above (see Proposition A2). Since it is monotonic, it must converge to a finite value, say S_d. According to Corollary 1, we have $\beta D^-G(S_d) \geq 1 \geq \beta D^+G(S_d)$.

If $S_d < S^*$, then G is differentiable at S_d and $\beta G'(S_d) = 1 = \beta F'(S_b)$ which in turn implies that $S_d = S_b$.

If $S_d > S^*$, then G is differentiable at S_d and $\beta F'(S_b) = 1 = \beta G'(S_d) > \beta F'(S_d)$ which in turn implies that $S_d > S_b$ (because $F'(S)$ is decreasing).

If $S_d = S^*$, then we have $\beta F'(S_d) = \beta D^-G(S_d) \geq 1 \geq \beta D^+G(S_d) \geq \beta F'(S_d)$. So, $S = S^* = S_b$.

To sum up, we have $S_d \geq S_b$. Since $X_0 < S_b$, we have $S_1 < S_b \leq S$. Hence S_t is increasing in t. Moreover, when a and b are high enough, we have $S_b > S^*$. In this case, we must have $S_d > S_b$.

□

C.2 Evidence from developing countries: Descriptive statistics

To create variables at constant prices, we extract GDP per capita (PPP, at 2017 international US$), GDP at current US$, and Gross fixed capital formation as % of GDP from the World Development Indicators (WDI, `https://databank.worldbank.org`) database, inward FDI stock at current US$ from the United National Conference on Trade and Development (UNCTAD, `https://unctadstat.unctad.org`) database, and investment in R&D (as % of GDP) from the UNESCO Institute for Statistics (UIS, `http://data.uis.unesco.org`).

Descriptive statistics of covariates used in this study are shown in Table C1, and Table 1, list of countries.

Table C1: Descriptive statistics

Variable	Mean	Std. Dev.	Min	Max
Full sample				
Investment in R&D (as % of GDP)	0.442	0.363	0.015	2.145
Physical capital (as % of GDP)	22.513	6.479	6.225	57.412
Inward FDI stock (as % of GDP)	0.349	0.307	0	2.513
UMI economies				
Investment in R&D (as % of GDP)	0.506	0.391	0.015	2.145
Physical capital (as % of GDP)	22.665	6.594	9.301	57.412
Inward FDI stock (as % of GDP)	0.324	0.224	0	1.204
LMI economies				
Investment in R&D (as % of GDP)	0.353	0.269	0.034	1.113
Physical capital (as % of GDP)	22.611	6.210	12.673	48.182
Inward FDI stock (as % of GDP)	0.483	0.463	0.067	2.513
Low income economies				
Investment in R&D (as % of GDP)	0.176	0.098	0.015	0.476
Physical capital (as % of GDP)	20.980	6.208	6.225	37.657
Inward FDI stock (as % of GDP)	0.154	0.132	0.007	0.491

Table C2: Liste of developing countries

ARG - Argentina	GTM - Guatemala	NIC - Nicaragua
ARM - Armenia	HND - Honduras	PAK - Pakistan
AZE - Azerbaijan	IDN - Indonesia	PER - Peru
BLR - Belarus	KAZ - Kazakhstan	RUS - Russian Federation
BOL - Bolivia	KGZ - Kyrgyz Republic	MYS - Malaysia
BRA - Brazil	MAR - Morocco	TUR - Turkey
BWA - Botswana	MDG - Madagascar	THA - Thailand
BFA - Burkina Faso	JAM - Jamaica	PRY - Paraguay
BDI - Burundi	NPL - Nepal	PHL - Philippines
BGR - Bulgaria	MDA - Moldova	TUN - Tunisia
BIH - Bosnia and Herzegovina	ROU - Romania	UGA - Uganda
CHN - China	MKD - North Macedonia	LKA - Sri Lanka
CRI - Costa Rica	MEX - Mexico	VEN - Venezuela
ECU - Ecuador	MNG - Mongolia	MDV - Maldives
SLV - El Salvador	IRQ - Iraq	SRB - Serbia
GAB - Gabon	MNE - Montenegro	UKR - Ukraine
GEO - Georgia	MUS - Mauritius	ZAF - South Africa
TJK - Tajikistan	UZB - Uzbekistan	

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