Differential Fertility, Intergenerational Mobility and the Process of Economic Development

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Abstract

This paper analyzes the interactions among population dynamics with differential fertility, intergenerational mobility, income inequality and economic development in an overlapping generations framework. Population dynamics with differential fertility between the educated and the uneducated has two effects on the economy: the direct effect on the educated share through changing in population size of the economy as whole, and the indirect effect on the educated share through decreasing/increasing transfer per child. When population growth increases sufficiently, the mobility and income inequality exhibit cyclical behavior due to rapidly decreasing transfer per child and increasing population size. In contrast, when population growth decreases sufficiently, the mobility and income inequality monotonically approach steady state, and the economy has two steady states: low steady state with high population growth and income inequality, and high steady state with low population growth and income inequality. As a result, this paper shows that population dynamics plays crucial role in the transitional dynamics of mobility and economic development.

JEL classifications: I24, I25, J13, J62
Keywords: Differential the fertility, Intergenerational mobility, Economic development

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1. Introduction

Interest in intergenerational mobility, along with income inequality, is growing in a wide range of fields, including sociology, demographics and economics. Intergenerational mobility is important not only in considering intergenerational equity and efficiency but also in analyzing the process of economic development. The relationship between fertility and intergenerational mobility has also long been argued in many fields: sociology, demographics, social biology and economics. If parents limit family size, i.e. parents decrease the number of children, then intergenerational transfers per child, e.g., educational investment and bequest for children increase and therefore intergenerational upward-mobility will increase. This hypothesis is called "family size limitation hypothesis" in literature of intergenerational mobility in sociology.[1] Van Bavel (2006) and Van Bavel et al. (2011) find that family size limitation has strong effect on intergenerational mobility in Belgium. Lindahl (2008) also show that increase in fertility prevents the upward-mobility from the rich (or the educated) to the poor (or the uneducated) in Sweden. Although many empirical studies have indicated the strong relationship between fertility and intergenerational mobility, there are few theoretical studies explaining these interactions. In contrast, we analyze the effects of population dynamics with differential fertility between the educated and the uneducated on intergenerational mobility, income inequality and economic development and shows their interactions.

The seminal work by Maoz and Moav (1999) indicate the incentive of income inequality for acquiring education and analyze the relationship intergenerational mobility, income inequality and economic growth in simple overlapping generations framework. They show the mobility monotonically increases with decreasing income inequality as the economy grows. Hence, as empirical studies have indicated, they demonstrate that the mobility is negatively correlated with income inequality. In contrast, Nakamura and Murayama (2011) focus on the education cost in Maoz and Moav (1999) model. Assuming general cost function, they show that the behavior of education cost plays crucial role in the transitional dynamics of the mobility. Owen and Weil (1999) and Galo and Zeira (1993) analyze the dynamics of mobility in an economy with imperfect capital market. Galor and Zeira (1993) show upward mobility is hindered by high borrowing costs; as a result, multiple equilibria emerge in the economy. Iyigun (1999) and Davies et al. (2005) discuss that the education system, public or private, is an important factor in determining the upward-mobility. Davies et al. (2005) show that public education system more encourages the

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[1] In economics, an idea similar to "family limitation hypothesis" is known as "Quality and Quantity hypothesis" in the literature of human capital accumulation. Early works on Becker and Lewis (1973) and Barro and Becker (1989) show the trade-off between child-quality and child-quantity. De la Croix and Doepke (2003) analyze the effects of differential mortality among individuals on inequality and economic development in quality-quantity model. These studies analyze human capital accumulation, but not intergenerational mobility.

mobility than private education system. Also, Bernasconi and Profeta (2012) Uchida (2018) analyze the role of private and/or public education sysetem on the mobility in the economy with the mismatch of talents. In this frame work, Bernasconi and Profeta (2012) show that expanding public education decreases the mismatch and therefore encourages the mobility, while Uchida (2018) indicate that the transitional dynamics of the mobility depend on political power between the educated and the uneducated incorporating private education and voting on public education into Bernasoni and Profeta (2012). Hassler et al. (2007) and Murayama (2018) analyze the effects of fiscal policy on the mobility and income inequality. Hassler et al. (2007) show that educational subsidies make a difference in intergenerational mobility and income inequality around each countries. Introducing governmental cash transfers into Maoz and Moav (1999) model, Murayama (2018) show that larger transfers to children with higher ability encourage upward mobility and growth if the economy has low income inequality.

Many previous theoretical studies analyze intergenerational mobility and income inequality by focusing on education system, education cost, liquidity constraint and fiscal policies. To our knowledge, few theoretical papers analyze the relationship between population dynamics and intergenerational mobility. An exception is Aso and Nakamura (2020). Aso and Nakamura (2020) analyze the effect of population growth on the mobility. However, they assume exogenous fertility difference among workers and exogenous worker's wages, and therefore they do not adequately analyze the effects of population dynamics with economic development on the mobility and income inequality. In fact, we show that population dynamics with differential fertility plays crucial role in the transitional dynamics of mobility, income inequality and economic development and hence (not shown in Aso and Nakamura; 2020) various transitional dynamics of the economy appears in this paper.

The purpose of this paper is to analyze the effect of population dynamics on the mobility, income inequality and economic development and show their interactions. For the purpose of our analysis, we extend the Maoz and Moav (1999) model by incorporating the (endogenous) differential fertility.[2] Our results are summarized as follows. (i) differential fertility between the educated and the uneducated depend crucially on the elasticity of substitution between consumption, transfer for children and the number of children. (ii) Population dynamics has two effect on the economy: the direct effect on the educated share through changing population size of the economy as a whole, and the indirect effect on the educated share through increasing/decreasing transfer per child. (iii) When the elasticity of substitution is enough smaller than unity, i.e., the fertility of the educated is enough larger than that of the uneducated and

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[2] Maoz and Moav (1999) analyze the transitional dynamics of intergenerational mobility, income inequality and economic development in simple framework. Thus, we can clearly show the effects of population dynamics with (endogenous) differential fertility on the mobility and economic development using Maoz and Moav model.

therefore population growth increases sufficiently, the mobility and income inequality exhibit cyclical behavior due to rapidly decreasing population effect and transfer per child. (iv) In contrast, when the elasticity of substitution is enough larger than unity, i.e., the fertility of the uneducated is enough larger than that of the educated and therefore population growth decreases sufficiently, the mobility and income inequality monotonically approach steady state and the economy may have two stable steady state: low steady state with high population growth and income inequality, and high steady state with low population growth and income inequality.

In fact, these various motions of intergenerational mobility have been observed in developed countries. Bratberg et al. (2007) indicate monotonous increase in Norway, while Aaronson and Mazumber (2008) show that the mobility has changed non-monotonous motion in the USA. This paper supplies one explanation and contribution to their findings. In addition, this study discusses the relationship between demographic transition and intergenerational mobility in our model.

The rest of this paper is organized as follows. Section 2 sets up the model. Section 3 analyzes the transitional dynamics of the economy. Section 4 examines the transitional dynamics of the economy using numerical analysis. Section 5 provides the discussion in our model. Section 6 concludes the paper.

2. The model

Consider the competitive equilibrium of an overlapping generations economy with endogenous fertility. Each individual lives for two periods: childhood and adulthood. In the first period, she does not work, receives a transfer from her parent. It is used for consumption and possible education. In the second period, she chooses the number of children, works and divides her income between own consumption, child cost and transfer for her children.

2.1 Technology and factor prices

We assume that aggregate output in period t is characterized by the following constant returns to scale production function.

$$Y_t = AE_t^{1-\alpha}U_t^{\alpha}, \qquad A > 0, \quad 0 < \alpha < 1 \tag{1}$$

where E_t is the number of educated workers, and U_t is the number of uneducated workers. Adult population in period t is $N_t = E_t + U_t$. Therefore, per capita output becomes

$$y_t = A\lambda_t^{1-\alpha}(1-\lambda_t)^{\alpha}, \tag{2}$$

where $\lambda_t = E_t/N_t$ is the population share of the educated. Similarly, $(1-\lambda_t) = U_t/N_t$ is the population share of the uneducated worker. Hence, we have following in equilibrium:

$$w_t^e = (1-\alpha)A\left(\frac{1-\lambda_t}{\lambda_t}\right)^{\alpha} \tag{3}$$

$$w_t^u = \alpha A\left(\frac{1-\lambda_t}{\lambda_t}\right)^{\alpha-1} \tag{4}$$

where subscripts e and u denote "educated" and "uneducated," respectively. Hence, the wage inequality becomes:

$$\frac{w_t^e}{w_t^u} = \frac{1-\alpha}{\alpha}\left(\frac{1-\lambda_t}{\lambda_t}\right). \tag{5}$$

To ensure that $w_t^e > w_t^u$, we assume that $\lambda_t < \hat{\lambda} \equiv 1-\alpha$. From (5), income inequality shrinks as λ_t increases.

2.2 Individuals

Individuals derive utility from consumption c_t of an individual born in period t, consumption c_{t+1} in period $t+1$, transfer for children b_{t+1} and their number of children n_{t+1} in period $t+1$. The preference is assumed to be represented by CES (Constant Elasticity of Substitution) type utility function:

$$u_t = c_t\left\{\left[(c_{t+1})^{\beta}(b_{t+1})^{1-\beta}\right]^{\frac{\sigma-1}{\sigma}} + (n_{t+1})^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}}, \tag{6}$$

where $0 < \beta < 1$ and $\sigma > 0$. σ stands for the elasticity of substitution between $(c_{t+1})^{\beta}(b_{t+1})^{1-\beta}$ and n_{t+1}. To focus on the role of σ, we assume that the marginal rate of substitution between consumption and transfer is independent demand for children n_{t+1}.[3]

Let h_t^i and w_{t+1}^i denote the education cost of individual i, born in period t and her wage in period $t+1$. Assuming no capital market in the economy, if she acquires education, her budget constraints are

$$c_t^i + h_t^i = x_t^i, \qquad c_{t+1}^i + b_{t+1}^i + \gamma n_{t+1}^i w_{t+1}^e = w_{t+1}^e, \tag{7.a}$$

where $i \in \{e, u\}$. $x_t^i = b_t^i/n_t^i$ is transfer per child and $\gamma n_{t+1}^i w_{t+1}^i$ is non-negligible cost for rearing children, where $0 < \gamma < 1$. We assume that $n_{t+1}^i < 1/\gamma$ always holds in the following. If she does not acquire education, her budget constraint become

$$c_t^i = x_t^i, \qquad c_{t+1}^i + b_{t+1}^i + \gamma n_{t+1}^i w_{t+1}^u = w_{t+1}^u, \tag{7.b}$$

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[3] With $\sigma \to 1$, the utility function becomes the Cobb-Douglas function. This type of utility function is used in Borck (2011) and Nakamura (2018).

Since we assume that there is no capital market, the utility maximization problem can be solved backwards in two stages. First, each individual considers optimal allocation in the second period. Then, each individual decides whether to acquire education in the first period.

Suppose that the utility maximization problem of the second period. We can solve the utility maximization of second period in two steps. At the first step, each individual determines optimal consumption c_{t+1}^{i} and transfer for children b_{t+1}^{i} assuming that the number of children n_{t+1}^{i} has been optimally chosen.

$$\max_{c_{t+1}^{i},b_{t+1}^{i}} \left(c_{t+1}^{i}\right)^{\beta}\left(b_{t+1}^{i}\right)^{1-\beta} \qquad \textit{subject to } \; c_{t+1}^{i}+b_{t+1}^{i}+\gamma n_{t+1}^{i}w_{t+1}^{i}=w_{t+1}^{i},$$

Hence, her optimal consumption and transfer for children become

$$c_{t+1}^{i}=\beta\left(w_{t+1}^{i}-\gamma n_{t+1}^{i}w_{t+1}^{i}\right), \tag{8}$$

$$b_{t+1}^{i}=(1-\beta)\left(w_{t+1}^{i}-\gamma n_{t+1}^{i}w_{t+1}^{i}\right). \tag{9}$$

Substituting (8) and (9) into the utility function of the second period, we can represent it as a function of only n_{t+1}^{i} as follows.

$$v\left(n_{t+1}^{i}\right)=\left[\tilde{\beta}\left(w_{t+1}^{i}-\gamma n_{t+1}^{i}w_{t+1}^{i}\right)^{\frac{\sigma-1}{\sigma}}+\left(n_{t+1}^{i}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{10}$$

where $\tilde{\beta}=\left[\beta^{\beta}(1-\beta)^{1-\beta}\right]^{\frac{\sigma-1}{\sigma}}$. At the second step, each individual chooses the number of children n_{t+1}^{i} so as to maximize (10). Hence,

$$\max_{n_{t+1}^{i}}\left[\tilde{\beta}\left(w_{t+1}^{i}-\gamma n_{t+1}^{i}w_{t+1}^{i}\right)^{\frac{\sigma-1}{\sigma}}+\left(n_{t+1}^{i}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}.$$

The first-order condition is

$$n_{t+1}^{i}=\left(\tilde{\beta}\gamma w_{t+1}^{i}\right)^{-\sigma}\left[w_{t+1}^{i}-\gamma n_{t+1}^{i}w_{t+1}^{i}\right]. \tag{11}$$

From (11), we obtain the following:

$$n_{t+1}^{i}=\frac{w_{t+1}^{i}}{\gamma w_{t+1}^{i}+\left(\tilde{\beta}\gamma w_{t+1}^{i}\right)^{\sigma}}. \tag{12}$$

From (9) and (12), transfer per child become

$$x_t^i = \frac{b_t^i}{n_t^i} = (1-\beta)\left(\tilde{\beta}\gamma w_t^i\right)^{\sigma}. \tag{13}$$

Differentiating (12) with respect to w_{t+1}^i, we obtain

$$\frac{\partial n_{t+1}^i}{\partial w_{t+1}^i} = \frac{\left(\tilde{\beta}\gamma w_{t+1}^i\right)^{\sigma}(1-\sigma)}{\left[\gamma w_{t+1}^i + \left(\tilde{\beta}\gamma w_{t+1}^i\right)^{\sigma}\right]^2} \gtreqless 0 \ \Leftrightarrow \ \sigma \lesseqgtr 1\,. \tag{14}$$

When wage rises, whether the number of children increases or decreases depend on the elasticity of substitution σ. Since c_{t+1}^i, b_{t+1}^i and n_{t+1}^i is normal goods, rise in wage increases the number of children due to the income effect. On the other hand, rise in wage has the substitution effect due to the substitution between consumption, bequest and the number of children. A rise in wage increases the rearing-child cost and therefore each individual substitutes the number of children by own consumption and transfer for children.[4] If $\sigma < 1$, the fertility increases with wage since the income effect dominates the substitution effect. In contrast, if $\sigma > 1$, the fertility decreases with income since substitution effect dominates income effect. Hence, we have following the proposition.

Proposition 1 Differential fertility between the educated and the uneducated depends on the elasticity of substitution σ. When $\sigma < 1$, since the income effect is dominant, the fertility of the educated is larger than that of the uneducated. When $\sigma > 1$, since the substitution effect is dominant, the fertility of the uneducated is larger than that of the educated. When $\sigma = 1$, since the income effect and the substitution effect are offset, the fertility of the educated equals that of the uneducated, that is, there is no differential fertility between income classes.

2.3 Education choice

In this subsection, we consider individual's education choice in first period. Substituting (12) into (10), we obtain indirect utility function in second period:

$$z\left(w_{t+1}^i\right) = \left\{\tilde{\beta}\left[w_{t+1}^i - \gamma n^i\left(w_{t+1}^i\right) w_{t+1}^i\right]^{\frac{\sigma-1}{\sigma}} + \left[n^i\left(w_{t+1}^i\right)\right]^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}}. \tag{15}$$

If the utility derived from investing in education is higher than or equal to the utility derived from not investing in education, then individual i will acquire education. Thus, if the following holds, then individual i acquires education.

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[4] When $\sigma = 1$, the the fertility between the educated and the uneducated is the same $n_t^i = n_t = 1/2\gamma$ and become constant over time.

$$\left(x_t^i - h_t^i\right) z(w_{t+1}^e) \geq x_t^i\, z(w_{t+1}^u).$$

or

$$h_t^i \leq x_t^i \left[1 - \frac{z(w_{t+1}^u)}{z(w_{t+1}^e)}\right]. \tag{16}$$

Hence, we have the following critical value of education cost $\hat{h}_t^i$ for individual i :

$$\hat{h}_t^i = x_t^i \left[1 - \frac{z(w_{t+1}^u)}{z(w_{t+1}^e)}\right], \tag{17}$$

where $\partial z\left(w_{t+1}^i\right)/\partial w_{t+1}^i > 0$ and $z(w_{t+1}^e) > z(w_{t+1}^u)$. As can be seen from (18), the wage inequality w_{t+1}^e/w_{t+1}^u plays an important role in education choice. The higher w_{t+1}^e/w_{t+1}^u is, the higher $z(w_{t+1}^e)/z(w_{t+1}^u)$ is, and therefore the larger the incentive to acquire education becomes. In addition, transfer per child x_t^i is also crucial factor for education choice. It is funding source for educational investment and therefore individuals with larger x_t^i has a higher critical value for education cost.

Suppose that $\hat{h}_t^e$ $(\hat{h}_t^u)$ is the critical value of educational cost for the individual born to an educated (uneducated) worker to acquire education. Then, from (13) and (17),

$$\hat{h}_t^e = (1-\beta)\left(\tilde{\beta}\gamma w_t^e\right)^{\sigma} \left[1 - \frac{z(w_{t+1}^u)}{z(w_{t+1}^e)}\right], \tag{18}$$

$$\hat{h}_t^u = (1-\beta)\left(\tilde{\beta}\gamma w_t^u\right)^{\sigma} \left[1 - \frac{z(w_{t+1}^u)}{z(w_{t+1}^e)}\right]. \tag{19}$$

2.4 Education cost among individuals

Following Maoz and Moav (1999), the following cost is assumed to be incurred for the education of individual i in period t.

$$h_t^i = \theta^i c(\overline{w}_t) = \theta^i (a + b\overline{w}_t), \tag{20}$$

where θ^i is a parameter representing individual $i's$ ability to learn; the higher the ability, the lower is the value of θ^i. We further assume that θ^i is uniformly distributed in the interval $\left(\underline{\theta}, \overline{\theta}\right)$, regardless of the ability and class of the parents in any period. Hence, h_t^i is also uniformly distributed in the interval $\left(\underline{h}_t, \overline{h}_t\right)$, where $\underline{h}_t = \underline{\theta}(a + b\overline{w}_t)$ and $\overline{h}_t = \overline{\theta}(a + b\overline{w}_t)$.

2.5 Population growth

Adult population in period $t+1$ is $N_{t+1} = E_{t+1} + U_{t+1}$ or $N_{t+1} = n_t^e E_t + n_t^u U_t$. Therefore, the population growth as a whole in period t; $\bar{n}_t$ becomes

$$\bar{n}_t = \bar{n}(\lambda_t) = n_t^e \lambda_t + n_t^u (1 - \lambda_t). \tag{21}$$

When the educated share λ_t increases, the behavior of population growth $\bar{n}(\lambda_t)$ can be expressed as follows.

$$\frac{\partial \bar{n}(\lambda_t)}{\partial \lambda_t} = \left[(1 - \lambda_t) \frac{\partial n_t^u}{\partial w_t^u} \frac{\partial w_t^u}{\partial \lambda_t} - \lambda_t \frac{\partial n_t^e}{\partial w_t^e} \frac{\partial w_t^e}{\partial \lambda_t} \right] + (n_t^e - n_t^u) \tag{22}$$

The first term in the right-hand side presents increase/decreases in the fertility through changing in wage among income groups. On the other hand, the second term in the right-hand side presents changing in population growth through intergenerational mobility. From (14), the following holds:

$$(1 - \lambda_t) \frac{\partial n_t^u}{\partial w_t^u} \frac{\partial w_t^u}{\partial \lambda_t} - \lambda_t \frac{\partial n_t^e}{\partial w_t^e} \frac{\partial w_t^e}{\partial \lambda_t} \gtreqless 0 \iff \sigma \lesseqgtr 1, \tag{23}$$

$$n_t^e - n_t^u \gtreqless 0 \iff \sigma \lesseqgtr 1.$$

Thus, the dynamics of population growth $\bar{n}(\lambda_t)$ becomes

$$\frac{\partial \bar{n}(\lambda_t)}{\partial \lambda_t} \gtreqless 0 \iff \sigma \lesseqgtr 1 \tag{24}$$

As can be seen from (24), the elasticity of substitution σ plays crucial role in dynamics of population growth. Thus, we have the following proposition.

Proposition 2 The dynamics of population growth depends on the elasticity of substitution σ. When $\sigma < 1 (\sigma > 1)$, since the fertility of the educated is larger (smaller) than that of the uneducated, population growth increases (decreases) with increase in the educated share. When $\sigma = 1$, population growth is constant over since the fertility between income classes is the same.

3. The dynamics of the model

In this section, we analyze the dynamics of intergenerational mobility, income inequality and population growth. At the first step, we show intergenerational mobility in this model.

3.1 Intergenerational mobility

The dynamics of E_t can be expressed as

$$E_{t+1} = n_t^e E_t \, p_t + n_t^u U_t \, q_t \,, \tag{25}$$

where

$$p_t = \frac{\hat{h}_t^e - \underline{h}_t}{\overline{h}_t - \underline{h}_t}, \qquad\qquad q_t = \frac{\hat{h}_t^u - \underline{h}_t}{\overline{h}_t - \underline{h}_t}, \tag{26}$$

where $\hat{h}_t^i(E_t, E_{t+1})$, and hence, $p_t = p(E_t, E_{t+1})$ and $q_t = q(E_t, E_{t+1})$. The first term in the right-hand side of (25) represents individuals born to the educated acquiring education, while the second term represents individuals born to the uneducated acquiring education. The dynamics of the educated share λ_t is obtained by dividing both sides by the total working population in period $t+1$; $N_{t+1} = \bar{n}(\lambda_t) N_t$.

$$\lambda_{t+1} = \lambda_t \, \frac{n^e(\lambda_t)}{\bar{n}(\lambda_t)} \, p_t + (1 - \lambda_t) \, \frac{n^u(\lambda_t)}{\bar{n}(\lambda_t)} \, q_t \,, \tag{27}$$

where $\hat{h}_t^i(\lambda_t, \lambda_{t+1})$, and hence $p_t = p(\lambda_t, \lambda_{t+1})$, $q_t = q(\lambda_t, \lambda_{t+1})$. Hence, λ_{t+1} represents as implicit function of λ_t, i.e., $\lambda_{t+1} = \psi(\lambda_t)$. From (27), if $\lambda_t = 0$, then we get *trivial* steady state, $\lambda_{t+1} = 0$. Eq. (27) can be rewritten as follows:

$$\lambda_{t+1} - \lambda_t = \underbrace{(1 - \lambda_t) \, \frac{n^u(\lambda_t)}{\bar{n}(\lambda_t)} \, q_t}_{\equiv UM_t} - \underbrace{\left[\lambda_t - \lambda_t \, \frac{n^e(\lambda_t)}{\bar{n}(\lambda_t)} \, p_t\right]}_{\equiv GDM_t}, \tag{28}$$

where UM_t represents *upward-mobility* and GDM_t represents *gross downward-mobility*. In our model, upward-mobility means that individuals born to an uneducated parent become educated workers, while gross downward-mobility means that how much individuals born to educated parent increases or decreases compared to educated workers of parents' generation. Upward-mobility equals to gross downward-mobility in steady state; $UM_t = GDM_t$. Moreover, we can rewrite (28) as follows.

$$\lambda_{t+1} - \lambda_t = \underbrace{(1 - \lambda_t) \, \frac{n^u(\lambda_t)}{\bar{n}(\lambda_t)} \, q_t}_{\equiv UM_t} - \underbrace{\lambda_t (1 - p_t) \frac{n^e(\lambda_t)}{\bar{n}(\lambda_t)}}_{\equiv DM_t} - \underbrace{\lambda_t \left[1 - \frac{n^e(\lambda_t)}{\bar{n}(\lambda_t)}\right]}_{\equiv PE_t}, \tag{29}$$

where DM_t is *(net) downward-mobility* and PE_t is *population effect*. Downward-mobility means that individuals born to an educated parent become uneducated workers. On the other hand, population effect represents the effect of increase or decrease in population growth on the educated share in the economy. Increase (decrease) in population growth, regardless of the mobility, directly decreases (increases) the educated share through increasing (decreasing) in population size of the economy as a whole. We call the effect "*Population effect*" and define the sum of the downward-mobility and the population effect as the *gross downward-mobility* in this paper. When $\sigma < 1$ ($\sigma > 1$), i.e., the fertility of the educated is larger (smaller) than population growth of the economy as a whole; the sign of population effect is negative (positive); $PE_t < 0$ ($PE_t > 0$).[5] It implies that λ_{t+1} is larger (smaller) than λ_t since $n_t^e > (<) \bar{n}(\lambda_t)$. Hence, the larger difference between the fertility of the educated and population growth facilitates larger population effect, while population effect is smaller when the difference is small.

If there is no upward-mobility and downward-mobility ($q_t \leq 0$ and $p_t \geq 1$) in the economy, then the educated share in period $t+1$; λ_{t+1} depends on only the population effect.[6] Therefore, the dynamics of λ_t becomes:

$$\lambda_{t+1} = \lambda_t \frac{n^e(\lambda_t)}{\bar{n}(\lambda_t)}. \tag{30}$$

Whether the educated share in period $t+1$; λ_{t+1} increases or decreases compared to the educated share in period t; λ_t depends on whether the fertility of the educated is larger than population growth. Therefore, if $q_t \leq 0$ and $p_t \geq 1$, then

$$\lambda_{t+1} \gtreqless \lambda_t \iff n^e(\lambda_t) \gtreqless \bar{n}(\lambda_t) \iff \sigma \lesseqgtr 1. \tag{31}$$

When $\sigma > 1$, if there is no upward-mobility in the economy, then $0 < n^e(\lambda_t)/\bar{n}(\lambda_t) < 1$ and $\lambda_{t+1} < \lambda_t\, n^e(\lambda_t)/\bar{n}(\lambda_t)$, and therefore the economy falls into a poverty trap ($\lambda_{t+1} = \lambda_t = 0$) due to negative population effect. In this paper, poverty trap refers to a state in which the educated share decreases and approaches $\lambda_t = 0$ in the long run.[7] In contrast, when $\sigma < 1$, the economy does not fall into poverty trap due to positive population effect. In summary, we have the following proposition.

Proposition 3 When $\sigma < 1$, since the fertility of the educated is larger than population growth, the economy does not fall into poverty trap due to population growth even without upward-mobility, i.e. *positive* population effect. In contrast, when $\sigma > 1$ and there is no upward-mobility, since the fertility of the population growth is larger than the fertility of the educated, the economy falls into poverty trap due to *negative* population effect.

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[5] If $n^e(\lambda_t) \gtreqless \bar{n}(\lambda_t)$, then $n^e(\lambda_t)/\bar{n}(\lambda_t) \gtreqless 1$ and therefore $PE_t \lesseqgtr 0$.

[6] If there is no population effect, i.e., there is no differential fertility between the educated and the uneducated (30) is, then $\lambda_{t+1} = \lambda_t$.

[7] In section 4, we illustrate an example of poverty trap using numerical analysis.

Proposition 3 implies that population dynamics is crucial factor in the dynamics of economy at the early stage of economic development which has no upward-mobility and downward-mobility. Next, we analyze the economy that has both upward-mobility and downward-mobility. Taking into account $\overline{h}_t = \overline{\theta}c(\overline{w}_t)$, $\underline{h}_t = \underline{\theta}c(\overline{w}_t)$, $\overline{w}_t = \lambda_t w_t^e + (1-\lambda_t)w_t^u = y_t$, (18) and (19), Eq. (27) can be written as follows:

$$\lambda_{t+1} = \frac{f(\lambda_{t+1})}{(\overline{\theta}-\underline{\theta})}\frac{\overline{b}(\lambda_t)}{c(y_t)\overline{n}(\lambda_t)} - \frac{\underline{\theta}}{\overline{\theta}-\underline{\theta}}, \tag{32}$$

where $f(\lambda_{t+1}) = [1 - z(w_{t+1}^u)/z(w_{t+1}^e)]$ and $\overline{b}(\lambda_t) = \lambda_t b^e(\lambda_t) + (1-\lambda_t)b^u(\lambda_t)$ is average of transfer for children.

3.2 Transitional dynamics

The dynamics of λ_t is given by (32). We can express Eq. (32) as an implicit function.

$$G(\lambda_{t+1}, \lambda_t) = \lambda_{t+1} - \frac{f(\lambda_{t+1})}{(\overline{\theta}-\underline{\theta})}\frac{\overline{b}(\lambda_t)}{c(y_t)\overline{n}(\lambda_t)} + \frac{\underline{\theta}}{\overline{\theta}-\underline{\theta}} = 0 \tag{33}$$

Investigating Eq. (33), we can see the dynamic behavior of intergenerational mobility, income inequality, population dynamics and the process of economic development. Eq. (33) holds that (i) $G(\lambda_{t+1}, \lambda_t)$ is continuous in both λ_t and λ_{t+1}, (ii) for each λ_t there exists λ_{t+1} such that $G(\lambda_{t+1}, \lambda_t) = 0$, and (iii) $\partial G(\lambda_{t+1}, \lambda_t)/\partial \lambda_{t+1} > 0$ for all $\lambda_{t+1} \in [0, \hat{\lambda}]$. Hence, Eq. (33) holds implicit function theorem. Totally differentiating Eq. (33),

$$G_1 d\lambda_{t+1} = G_2 d\lambda_t, \tag{34}$$

where

$$G_1 = 1 - \frac{f'(\lambda_{t+1})}{(\overline{\theta}-\underline{\theta})}\frac{\overline{b}(\lambda_t)}{c(y_t)\,\overline{n}(\lambda_t)} > 0, \tag{35}$$

and $f'(\lambda_{t+1}) < 0$, and therefore $G_1 > 0$,

$$G_2 = \frac{f(\lambda_{t+1})}{(\overline{\theta}-\underline{\theta})}\frac{\varepsilon_t^{\overline{b}} - \varepsilon_t^c - \varepsilon_t^{\overline{n}}}{c(y_t)\,\overline{n}(\lambda_t)\,\lambda_t/\,\overline{b}(\lambda_t)}, \tag{36}$$

and

$$\varepsilon_t^{\overline{b}} = \frac{\partial \overline{b}(\lambda_t)/\overline{b}(\lambda_t)}{\partial \lambda_t/\lambda_t} = \overline{b}'(\lambda_t)\,\frac{\lambda_t}{\overline{b}(\lambda_t)} > 0, \tag{37}$$

$$\varepsilon_t^c = \frac{\partial c(y_t)/c(y_t)}{\partial \lambda_t/\lambda_t} = c'(y_t)\, y_t' \,\frac{\lambda_t}{c(y_t)} > 0, \tag{38}$$

$$\varepsilon_t^{\bar{n}} = \frac{\partial \bar{n}(\lambda_t)/\bar{n}(\lambda_t)}{\partial \lambda_t/\lambda_t} = \bar{n}'(\lambda_t)\,\frac{\lambda_t}{\bar{n}(\lambda_t)} \gtreqless 0 \quad \Leftrightarrow \quad \sigma \lesseqgtr 1. \tag{39}$$

$\varepsilon_t^{\bar{b}}$ represents the elasticity of average of transfer per child with respect to the share of the educated in period t, ε_t^c represents the elasticity of education cost with respect to the share of the educated in period t and $\varepsilon_t^{\bar{n}}$ represents the elasticity of population growth with respect to the share of the educated in period t. Since $G_1 > 0$, the transitional dynamics of λ_t depend on sign of G_2, i.e., the sum of each elasticities. Hence,

$$sign\left[\frac{d\lambda_{t+1}}{d\lambda_t}\right] = sign\left[\frac{G_2}{G_1}\right] = sign\left[\varepsilon_t^{\bar{b}} - \varepsilon_t^c - \varepsilon_t^{\bar{n}}\right]. \tag{40}$$

As can be seen from (40), ε_t^c discourages the mobility since increase in education cost deteriorates upward-mobility. In particular, $\varepsilon_t^{\bar{n}}$ and $\varepsilon_t^{\bar{b}}$ plays an important role in the transitional dynamics of λ_t. Population growth directly influences the educated share through changing in population size of the economy as a whole, i.e., population effect. On the other hand, increasing in population growth as whole of economy decreases average transfer per child and therefore discourages largely the mobility. When population growth decreases, the opposite holds. If population growth decreases sufficiently; $\varepsilon_t^{\bar{n}} \ll 0$, then $G_2/G_1 = \left[\varepsilon_t^{\bar{b}} - \varepsilon_t^c - \varepsilon_t^{\bar{n}}\right] > 0$; and hence Eq. (32) is upwards-sloping in the $(\lambda_t, \lambda_{t+1})$ plane. In contrast, if population growth increases sufficiently; $\varepsilon_t^{\bar{n}} \gg 0$, then $G_2/G_1 = \left[\varepsilon_t^{\bar{b}} - \varepsilon_t^c - \varepsilon_t^{\bar{n}}\right] < 0$; and hence, Eq. (32) is downwards-sloping in the $(\lambda_t, \lambda_{t+1})$ plane. As a result, the following proposition holds about the transitional dynamics of intergenerational mobility, income inequality and population growth.[8]

Proposition 4 The transitional dynamics of intergenerational mobility depends on the sum of each elasticities: the elasticity of average of transfer per child with respect to the share of the educated, the elasticity of education cost with respect to the share of the educated and the elasticity of population growth with respect to the share of the educated. In particular, population dynamics has two effects on intergenerational mobility: the direct effect on the educated share through changing in population size of the economy as whole, and the indirect effect on the educated share

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[8] When the fertility between income classes is the same, population growth does not influence the basic dynamics of the economy. Then, the transitional dynamics of the mobility depends on income share of education cost.

through decreasing/increasing transfer per child. When the fertility of the uneducated is greatly larger than that of the educated, that is, population growth decreases sufficiently with increase in the share of the educated, the mobility monotonically increases and therefore income inequality decreases. In contrast, when the fertility of the educated is greatly larger than that of the uneducated, that is, population growth increases sufficiently with increase in the share of the educated, the mobility and income inequality exhibit cyclical behavior.

Proposition 4 implies that population dynamics is crucial factor for the transitional dynamics of economy at the mature stage of economic development which has upward-mobility and downward-mobility. It influences the mobility through two effects: population effect and dilution effect, which is the effect of increase/decrease in average transfer per child. As can be shown in Proposition 2, Proposition 3 and Proposition 4, population dynamics with differential fertility between the educated and the uneducated is interdependent with the process of economic development.

4. Numerical analysis

As the analysis in the previous section has shown, the transitional dynamics of mobility and income inequality depend crucially on population dynamics with differential fertility. In this section, we use numerical analysis to illustrate the Proposition 3 and Proposition 4, and show the transitional dynamics of the economy with population dynamics. We take the parameter values to satisfy $n_t^i < 1/\gamma$. The parameters are given by $A = 50,\ \alpha = 0.5,\ \beta = 0.5,\ \gamma = 0.3,\ a = 0.03$, $b = 0.02$, and $\overline{\theta} = 5,\ \underline{\theta} = 1$ following Maoz and Moav (1999) and Nakamura (2018).

4.1 Constant population growth $(\boldsymbol{\sigma = 1})$

When $\sigma = 1$, the fertility between income classes is the same, that is, population growth is constant over time. We analyze the case as the *benchmark case*. Fig.1 demonstrates the transitional dynamics of λ_t in the case of constant population growth. When the fertility between income classes is the same, the population growth does not influence the basic dynamics of λ_t. In other words, there is no population dynamics in the economy and therefore economy has not population effect and dilution effect in Fig.1.

In Fig.1(a), the educated share monotonically increases with upward mobility and therefore income inequality decreases. If initial educated share is sufficiently small due to high education cost ($a = 0.1$ and $b = 0.03$), as can be shown in Fig.1 (b), the educated share does not change in the long run since there is no population effect.[9] However, if initial the educated

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[9] This situation is considered as *poverty trap*, which has no both upward-mobility and downward-

share is large sufficiently, then the educated share increases with upward mobility. This increase in turn decreases the income share of education cost and hence it further encourages upward-mobility. As a result, with decreasing income inequality, the mobility approaches toward steady state with constant population growth. In other words, if there is no differential fertility among income classes, the transitional dynamics of the mobility and income inequality follow Maoz and Moav (1999).

Numerical result 1 When $\sigma = 1$, the economy has no population effect since the fertility between the educated and the uneducated is the same. If the education cost is high and hence upward-mobility does not occur at an early stage of the economy, the economy does not develop at all since there is no population effect. In contrast, if the upward-mobility occurs, the mobility and income inequality monotonically approach steady state.

[Insert Fig.1 about here]

4.2 Increasing population growth $(\boldsymbol{\sigma} < \mathbf{1})$

Next, suppose that $\sigma < 1$. When $\sigma < 1$, the fertility of the educated is larger than that of the uneducated, and therefore population growth increases with increase in the educated share. Increase in population growth discourages the mobility due to decreasing (*positive*) population effect and transfer per child.

Fig.2 illustrates the transitional dynamics of mobility when $\sigma < 1$. As can be shown in Fig.2 (a), when the elasticity of substitution is small sufficiently, that is, population growth greatly increases with increase in the educates share, the mobility exhibits cyclical behavior. The intuition behind the cyclical behavior is as follows. When λ_t is small, education cost is also small. Although the educated have many children, many children born to the educated acquire education since educated worker's wages is high and therefore children born to the educated receive a large enough transfer. On the other hand, the gross downward-mobility is negative. It implies that the educated share increases due to population effect. In addition, since the elasticity of substitution is small, the fertility of the uneducated is also small. This encourages upward-mobility since each individual born to the uneducated receives larger transfer per child. In summary, when λ_t is small, the educated share greatly increases due to population effect and upward-mobility. This sharp increase in λ_t, in turn, increases education cost and population growth. Increase in population discourages the mobility growth due to decreasing average transfer and population effect. As a result, λ_t in next period decreases due to increase in population growth and education

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mobility.

cost. These observations explain the cyclical behavior.

Fig.2(b) and Fig.2(c) demonstrates the dynamics of $\sigma = 0.75$ and $\sigma = 0.95$, respectively. As the elasticity of substitution approaches unity, the direct and indirect effect of population growth on the mobility is smaller. Thus, when $\sigma = 0.75$, the fluctuations of economy are smaller. When value of the elasticity of substitution is almost unity ($\sigma = 0.95$), the mobility monotonically increases since differential fertility between income classes is quite small, that is, the effects of population dynamics on the mobility is sufficiently small. As can indicated by Fig.2, the educated share in steady state get larger as the elasticity of substitution approaches unity since the effects of population dynamics: population effect and dilution effect are smaller.

Finally, as shown in Proposition 3, we illustrate that the economy does not fall into poverty trap when $\sigma < 1$. Fig.2(d) demonstrates that education cost is high sufficiently ($a = 0.1$ and $b = 0.03$), and therefore there is no upward mobility at an early stage of economy. In contrast to Fig.1 (b) in the case of $\sigma = 1$, the economy develops due to population effect without falling into the poverty trap. We summarize the results below.

Numerical result 2 When the elasticity of substitution is smaller than unity, as the elasticity approaches toward unity, the transitional dynamics of mobility changes from non-monotonous behavior to monotonous behavior, and the educated share in steady state is larger since the effects of population dynamics on the mobility is smaller. In addition, even if the education cost is sufficiently high and hence upward-mobility does not occur at an early stage of the economy, the economy does not fall into poverty trap due to population effect.

[Insert Fig.2 about here]

4.3 Decreasing population growth ($\sigma > 1$)

When $\sigma > 1$, the fertility of the uneducated is larger than that of the educated, and therefore population growth decreases with increase in the educated share. Decrease in population growth encourages the mobility due to decreasing (*negative*) population effect for the educated share and increasing transfer per child.

Fig.3 illustrates the transitional dynamics of mobility when $\sigma = 1.3$ and $\sigma = 1.6$, respectively. When $\sigma > 1$, the gross downward-mobility is positive value since the value of population effect is positive. Thus, if the upward-mobility does not occur at an early stage of the economy, in other words, if $\lambda_t < \lambda_1^*$, then the economy falls into poverty trap; $\psi(\lambda_t) = 0$ as can be indicted by Proposiiton 2. If $\lambda_t > \lambda_1^*$, the educated share increases and hence the economy converges to λ_2^* with decreasing income inequality and population growth.

When $\sigma > 1$, since increase in σ increases the fertility of the uneducated and (negative) population effect and hence discourages the upward mobility and facilitates downward-mobility, it promotes the economy into poverty trap as can be shown in Fig.3(b). In addition, when the larger $\sigma > 1$, the fertility of uneducated rapidly decreases with increase in uneducated worker's wage due to increase in the educated share and hence it sharply increases the upward-mobility. Hence, when the larger $\sigma > 1$, the educated share increases in both low-equilibrium λ_1^* and high-equilibrium λ_2^* and therefore population growth and income inequality are lower in both equilibriums. In summary, we have the following the result.

Numerical result 3 When the elasticity of substitution is larger than unity, the economy has three equilibrium: the stable high-equilibrium with low population growth and income inequality, the unstable low-equilibrium with high population growth and income inequality and poverty trap equilibrium, where the educated share is zero in the long run. As the elasticity of substitution increases, when the elasticity of substitution is larger than unity, an increase in the elasticity of substitution promotes the economy into poverty trap, while the educated share in the equilibrium increases due to rapid decrease in the fertility of the uneducated with income, and therefore population growth and income inequality decrease in the equilibrium.

[Insert Fig.3 about here]

Next, we analyze the economy that the upward mobility occurs at an early of the economy, that is, the economy does not fall into poverty trap. A policy to get out of poverty trap is for the government to reduce the education cost for individuals with high ability. To simplify the analysis, we assume that the government reduce the education cost for individuals with high ability $\underline{h}_t$ to encourage the upward-mobility.

As can be shown in Fig.4 compared to Fig.3, upward-mobility occurs at an early of the economy due to lower $\underline{h}_t$, and therefore the economy develops without falling into poverty trap.[10] Hence, it is important that the government supports individual with high ability so that the economy does not fall into the poverty trap. However, even if the government implements such a policy, the economy may converge to stable low-equilibrium, that is, the economy may fall into development trap, when σ is significantly large ($\sigma = 2.3$) as can be illustrated in Fig.5. when σ is large significantly, multiple steady states appear in the economy: the high steady state with low population growth and small income inequality and the low steady state with high population growth and income inequality.

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[10] In order to set lower $\underline{h}_t$, we take $\underline{\theta} = 0.5$ in Fig.4 and 5.

[Insert Fig.4 about here]

The intuition behind the multiple steady states appear as follows. Although high ability children born to the uneducated become the educated since $\underline{h}_t$ is low, the larger fertility of the uneducated discourages the upward-mobility due to larger σ. In addition, larger population effect increases significantly gross downward-mobility due to low the fertility of the educated and high population growth. As a result, the economy falls into development trap since gross downward mobility is larger than upward-mobility. As the economy develops sufficiently, the fertility of the uneducated sharply decreases with increase in wage and therefore the transfer per child sharply increases. Increase in them encourages upward-mobility. In contrast, since the difference between the fertility of the educated and population growth shrinks with economic development, population growth effect decreases and therefore gross downward-mobility also decreases. As a result, the economy develops and converges the high steady state since upward-mobility is larger than gross downward-mobility. Hence, the rapid change in population dynamics with economic development generates the multiple steady states. In summary, we have the following the result.

Numerical result 4 If the government implements a policy to support individuals with high ability, then the economy can get out of the poverty trap. However, when the elasticity of substitution is larger significantly, the economy has low steady state with higher population growth and income inequality and high steady state with lower population growth and income inequality due to rapid change in population dynamics.

[Insert Fig.5 about here]

5. Discussion

As can be indicated by Doepke and Tertilt (2016), fertility choice and population dynamics play important role for economic development. In this paper, they influence the mobility through changing transfer per child which is funding source for education and population size of the economy as a whole. This result is consistent with empirical studies: Lindahl (2008), Van Bavel (2006) and Van Bavel et al. (2011). In addition, this paper shows cyclical motion of mobility in the economy with increasing population growth, while monotonous motion in the economy with decreasing population growth. In fact, Aaronson and Mazumder (2008) find cyclical motion of mobility in the USA which has experienced rapid increase and decrease in population growth from 1950 to 2000. In contrast, using Norwegian cohorts born in 1950, 1955 and 1960, Bratberg et al. (2007) indicate that the mobility has monotonically increased in Norway which has experienced decreasing population growth.[11]

This paper indicates that population dynamics is crucial factor in determining the transitional dynamics of mobility and economic development. Thus far, we consider the dynamics of mobility in economy with increase or decrease in population growth, i.e., $\sigma < 1$ or $\sigma > 1$. However, as cab be show in Liao (2011), developed countries have experienced demographic transition, and it influences the growth path through increase or decrease in population growth. Our model can explain the mechanism of demographic transition from the reversal of the fertility of the educated and the uneducated, i.e., from $\sigma < 1$ to $\sigma > 1$. At the early stage of economic development such as in $\sigma < 1$, the fertility of the educated is larger than that of the uneducated and hence population growth with increase in the educated share. When individual prefer quantity to quality of children, i.e., the substitution elasticity shifts from $\sigma < 1$ to $\sigma > 1$, the fertility of the uneducated becomes larger than that of the educated. Hence, the population growth decreases with economic development at the mature stage of economic development. In fact, this is consistent with empirical evidences indicated by Skirbekk (2008) and Drive et al. (2014). They find that the rich have more children in a less developed economy, while the poor have more children in advanced economy. As the result, he indicates that the positive relationships between income and fertility is changed to the negative as the economy develops. Demographic transition influences the behavior of mobility in our model since it depends on population dynamics, i.e., increase or decrease population growth. If demographic transition appears in the economy, then the two motions of mobility: cyclical motion and monotonous motion may be observed in our model. It implies that the various motions of mobility and population dynamics may appear in the process of economic development.

6. Conclusions

As many empirical studies indicate, the fertility influences significantly intergenerational mobility. Incorporating population dynamics with (endogenous) differential fertility into overlapping generations model, this paper analyzes the effect of population dynamics on the mobility, income inequality and economic development and shows their interactions. Population dynamics has two effect on the economy: the direct effect on the educated share through changing population size of the economy as a whole, and the indirect effect on the educated share through increasing/decreasing transfer per child. When the elasticity of substitution is enough larger than

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[11] In short period from 1950 to 1956, population growth increased from 1.30% to 1.76% in USA, but decreased to 1.14% in 2000. Between 1950 and 2000, population growth decreased from 1.09% to 0.53% in Norway. (Source: United Nations, World Population Prospects 2019).

unity, i.e., the fertility of the educated is enough larger than that of the uneducated and therefore population growth increases sufficiently, the mobility and income inequality exhibit cyclical behavior due to rapidly decreasing population effect and transfer per child. In contrast, when the elasticity of substitution is enough smaller than unity, i.e., the fertility of the uneducated is enough larger than that of the educated and therefore population growth decreases sufficiently, the mobility and income inequality monotonically approach steady state and the economy may have two steady state: low steady state with high population growth and income inequality, and high steady state with low population growth and income inequality. As a result, this paper shows that population dynamics with differential fertility plays crucial role in transitional dynamics of mobility and economic development.

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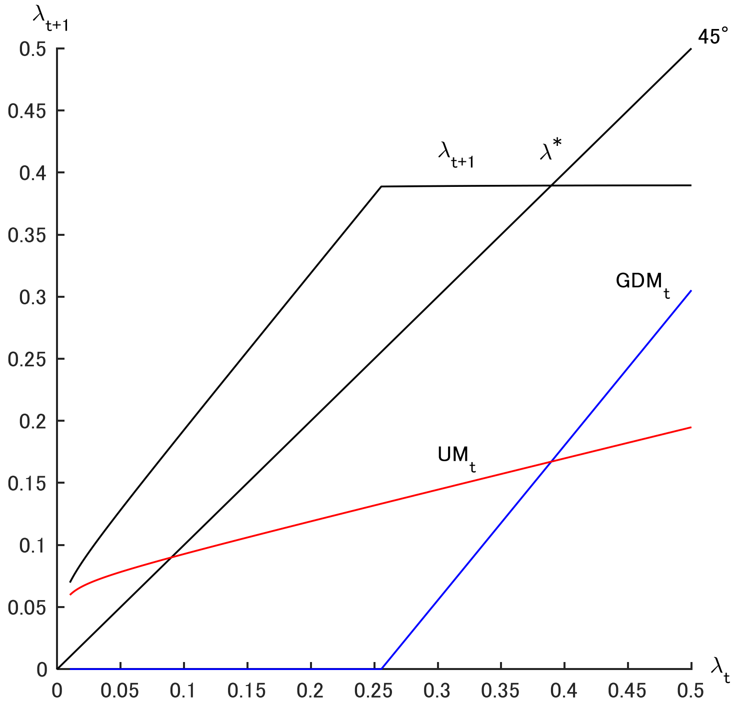
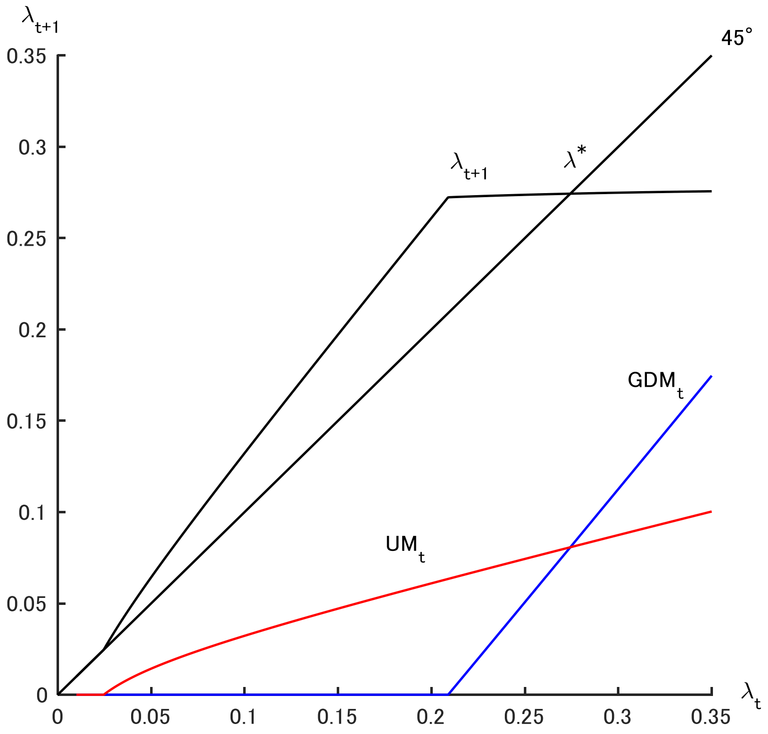
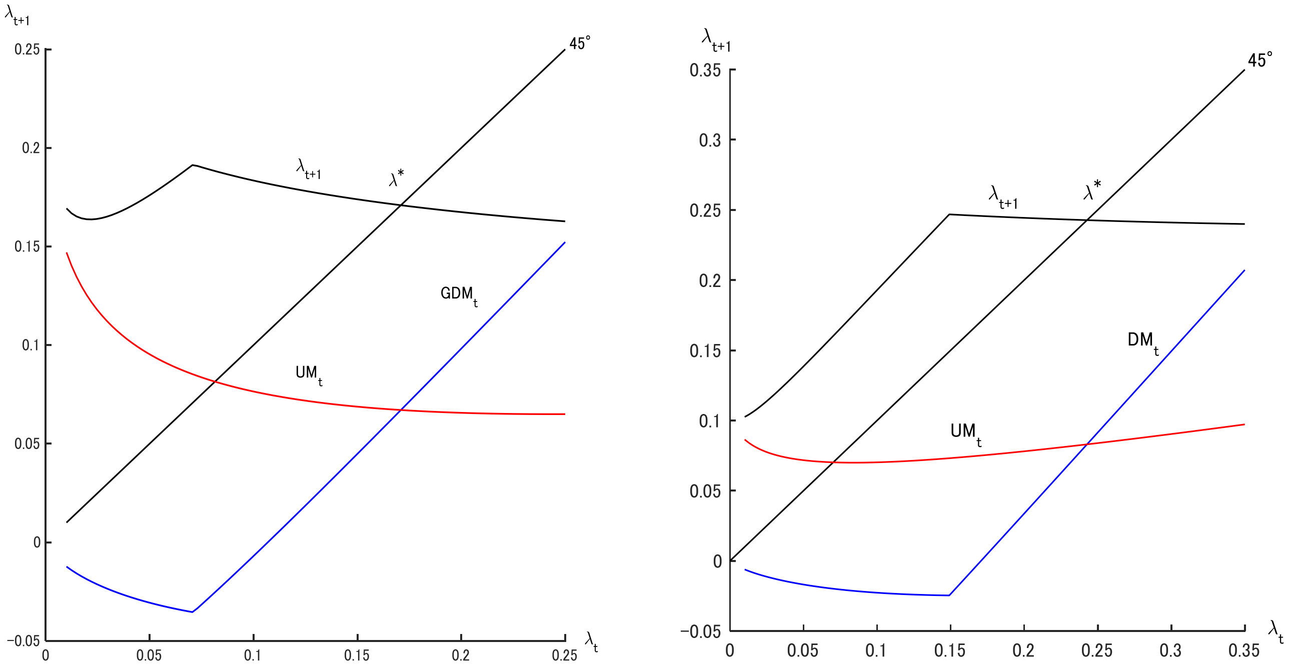


(a) $a = 0.03,\ b = 0.02,\ (\lambda^* = 0.3895,\ n^* = 1.667)$



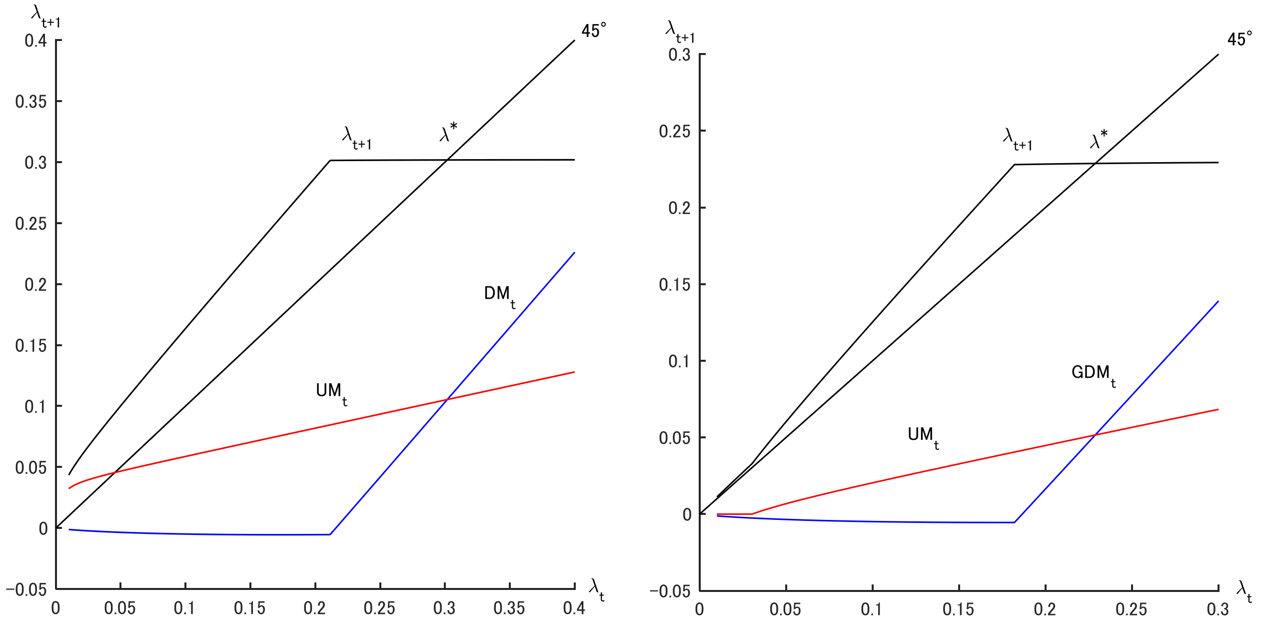
(b) $a = 0.1,\ b = 0.03,\ (\lambda^* = 0.2743,\ n^* = 1.667)$

Fig.1 The transitional dynamics of λ_t when $\sigma = 1$



(a) $\boldsymbol{\sigma = 0.5}$, $\boldsymbol{(\lambda^* = 0.17092,\ \bar{n}^* = 1.9872}$

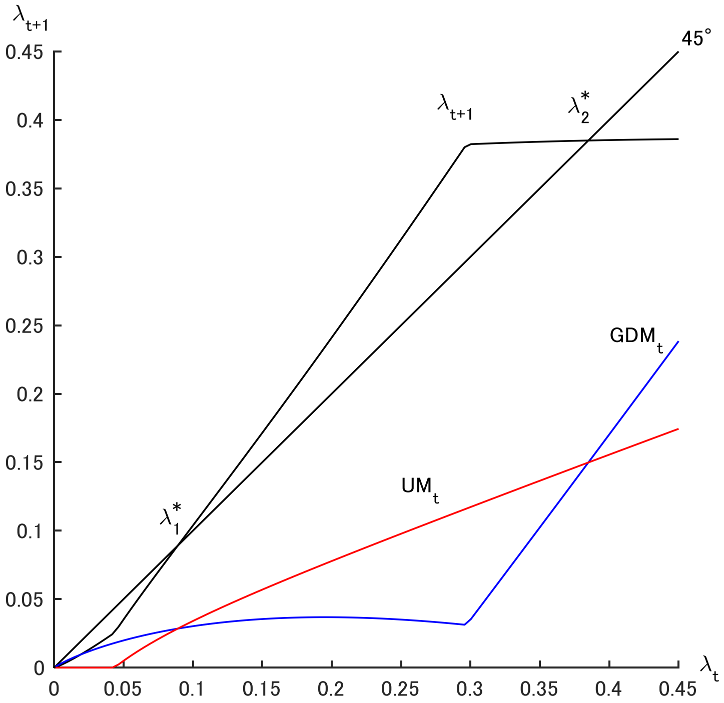
(b) $\boldsymbol{\sigma = 0.75}$, $\boldsymbol{(\lambda^* = 0.2427,\ \bar{n}^* = 1.8790)}$



(c) $\boldsymbol{\sigma = 0.95}$, $\boldsymbol{(\lambda^* = 0.3017\ \ \bar{n}^* = 1.7148)}$

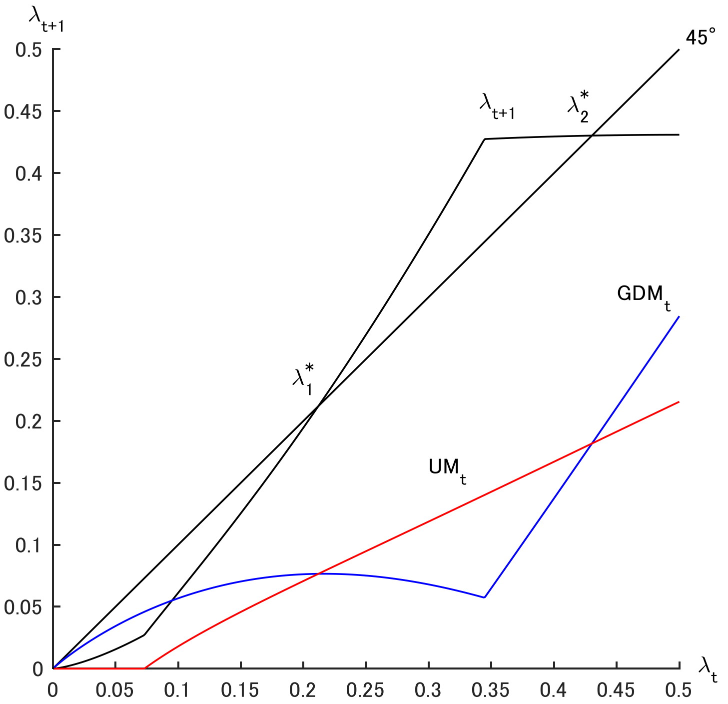
(d) $\boldsymbol{\sigma = 0.95}$, $\boldsymbol{(\lambda^* = 0.3017\ \ \bar{n}^* = 1.7148)}$

Fig.2 The transitional dynamics of $\boldsymbol{\lambda_t}$ when $\boldsymbol{\sigma < 1}$



(a) $\boldsymbol{\sigma = 1.3}$,

$(\boldsymbol{\lambda_1^* = 0.0891}$, $\boldsymbol{\bar{n}_1^* = 1.5775}$, $\boldsymbol{\lambda_2^* = 0.3849}$, $\boldsymbol{\bar{n}_2^* = 1.3538})$



(b) $\boldsymbol{\sigma = 1.6}$,

$(\boldsymbol{\lambda_1^* = 0.2129}$, $\boldsymbol{\bar{n}_1^* = 1.2185}$, $\boldsymbol{\lambda_2^* = 0.4303}$, $\boldsymbol{\bar{n}_2^* = 1.0477})$

Fig.3 The transitional dynamics of $\boldsymbol{\lambda_t}$ when $\boldsymbol{\sigma > 1}$

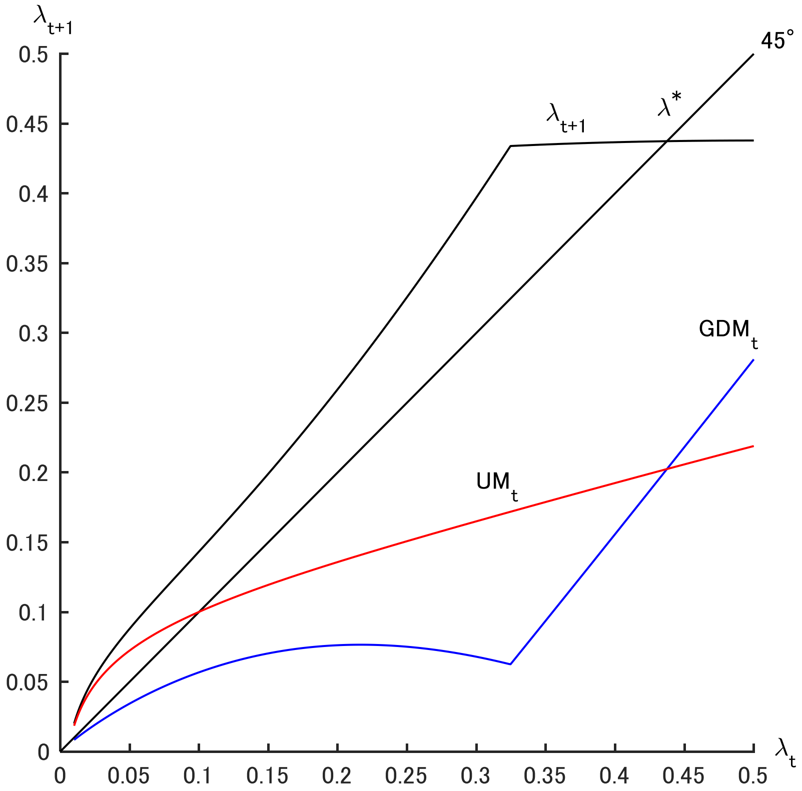
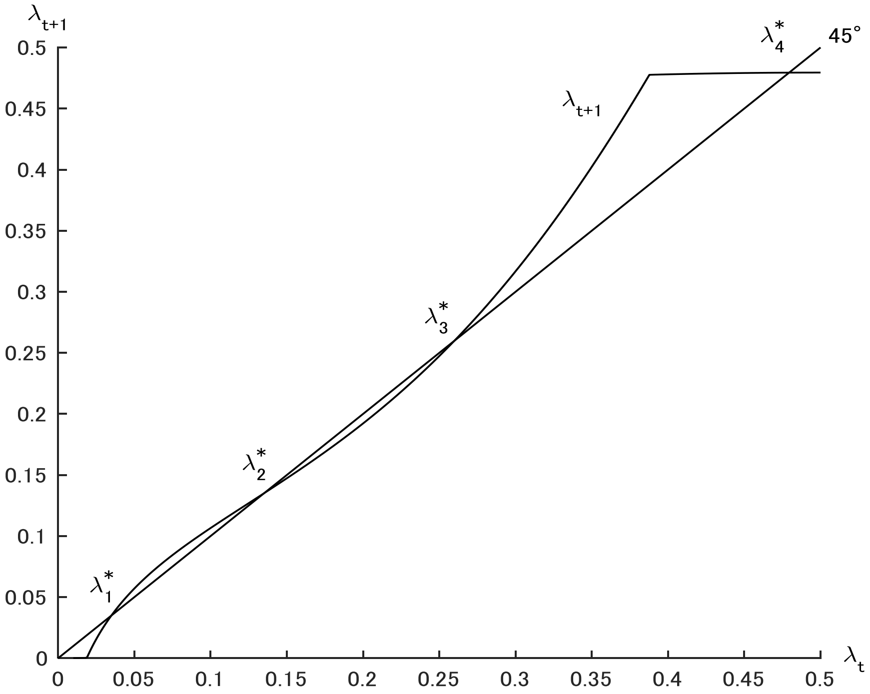
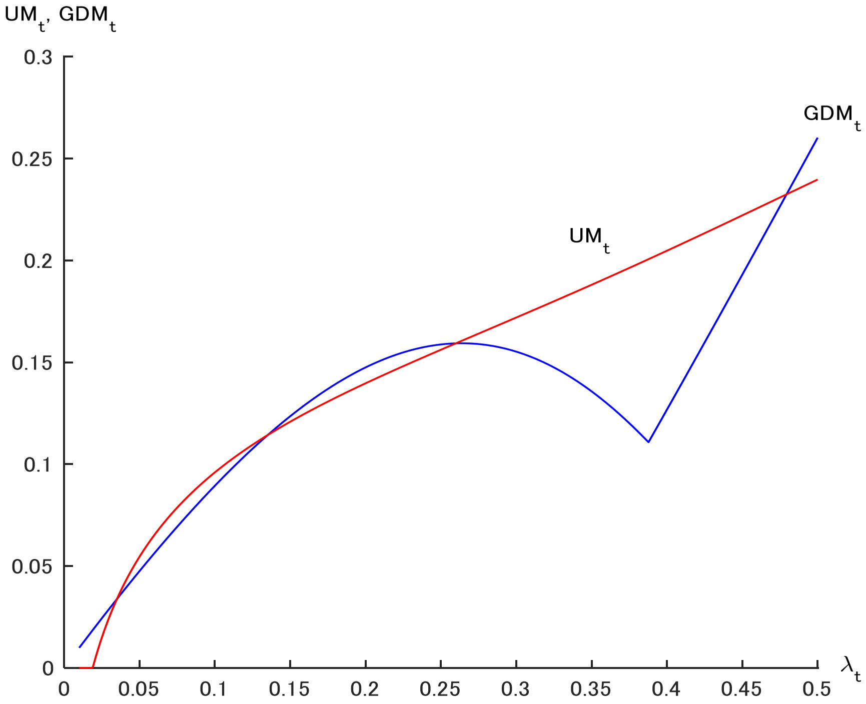


Fig.4 The transitional dynamics of $\boldsymbol{\lambda_t}$ when $\boldsymbol{\sigma = 1.6}$,
$(\boldsymbol{\lambda^* = 0.4374}, \quad \boldsymbol{\bar{n}^* = 1.0409})$



(a) Transitional dynamics of $\boldsymbol{\lambda_t}$



(b) $\boldsymbol{UM_t}$, $\boldsymbol{GDM_t}$

Fig.5 The transitional dynamics of $\boldsymbol{\lambda_t}$ when $\boldsymbol{\sigma = 2.3}$

$$\begin{pmatrix} \boldsymbol{\lambda_1^* = 0.0349,\ \bar{n}_1^* = 1.9571,\ \lambda_2^* = 0.1351,\ \bar{n}_2^* = 1.1036,} \\ \boldsymbol{\lambda_3^* = 0.2598,\ \bar{n}_3^* = 0.7175,\ \lambda_4^* = 0.4794,\ \bar{n}_4^* = 0.5084} \end{pmatrix}$$