Endogenous lifetime, intergenerational mobility and economic development

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Abstract

This paper analyzes the interactions among lifetime, intergenerational mobility and economic development in overlapping generations framework. In addition, we explain the mechanism that causes two motions of intergenerational mobility, monotonous motion and cyclical motion, which are observed in empirical studies. We show that these motions of mobility depend crucially on lifetime. We assume that lifetime increases through health effect with economic development. Increasing lifetime encourages incentives of education investment while decreasing transfer, which is the funding source for education. If lifetime slowly increases sufficiently, mobility monotonically increases while income inequality decreases. However, if lifetime increases rapidly with economic development, the economy exhibits cyclical and even chaotic behavior. Even if we consider that differential lifetime between the educated and the uneducated, we get two motions of mobility: monotonous and cyclical motion.

JEL classifications: I15, I24, J62

Keywords: Endogenous lifetime, Intergenerational mobility, Economic development, income inequality

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1. Introduction

Two motions of intergenerational mobility have been shown in empirical studies. While Jin et al. (2019) find that mobility has non-monotonically changed in China, Bratberg et al. (2007) show the mobility has monotonically increased in Norway. However, few theoretical studies explain the difference between these motions. The seminal work by Maoz and Moav (1999) provides a simple, but useful framework in analyzing the relationship between income inequality and intergenerational mobility. They show that the economy monotonically approaches the steady state with a decrease in income inequality between the educated and uneducated. Galor and Tsiddon (1997) analyze the effect of technological progress on mobility, income inequality, and economic growth. Iyigun (1999) and Davies et al. (2005) discuss that the type of education system—public or private—is an important factor in determining upward-mobility. Fan and Zhang (2013) show the economy converges to a unique equilibrium under the private education system while multiple equilibria may exist under the public education system. As Owen and Weil (1998) also points out, parental support or self-financing, with or without a liquidity constraint, is also an issue to be analyzed. Galor and Zeira (1993) focus on the imperfect capital market. They indicate that upward mobility is hindered by high borrowing costs; as a result, multiple equilibria emerge in the economy. Using the Maoz and Moav model, Murayama (2019) analyzes how government transfers affect intergenerational mobility and growth. He shows that larger transfers to children with higher ability foster upward mobility and growth if the economy has low income inequality.

Many previous studies on intergenerational mobility show monotonous motion of mobility. In contrast, we show various motions of mobility and income inequality focusing on endogenous lifetime. As has been indicated by many studies and historical data, economic development increases lifetime by improving nutrition and sanitation. Similarly, lifetime is general important in determining economic growth (e.g., Cervellati and Sunde 2005; Chen 2010; Fanti and Gori 2014).[1] Lifetime affects the mobility and economic development through changes in household economic behavior, such as savings and educational investment. This suggests that

[1] In empirical studies, Acemoglu and Johnson (2007) show the negative relationship between lifetime and per capita GDP. In contrast, Cervellati and Sunde (2011) show that the negative relationship between lifetime and per capita GDP is changed with demographic transition.

lifetime, intergenerational mobility, and economic development have a high degree of interdependence. Lifetime is expected to play a crucial role in the mobility if they are considered in the context of economic growth.[2]

In this study, we analyze the effects on the mobility, income inequality, and economic development incorporating endogenous lifetime into the model of Maoz and Moav (1999).[3] Assuming that an individual's surviving rate depends on health status, which improves with economic development, we show that the transitional dynamics of an economy depends on lifetime. If the increase in lifetime is sufficiently small, then the mobility and income inequality monotonically converge toward steady state, as in Maoz and Moav (1999). In contrast, if lifetime rapidly increases, then the economy exhibits cyclical behavior and even chaos in the economy.

In fact, in China which have experienced cyclical motion, life expectancy rose far more rapidly than that of Norway which have experienced monotonous motion. Between 1960 and 2015, life expectancy increased from 42.4 to 74.6 years in China, while Norway's life expectancy slowly increased from 71.4 to 80.5 (Source: OECD Health Statistics 2019). Hence, this paper indicates that the increase in lifetime may be one of the causes of various motions of the mobility that has been observed in developed countries.

The rest of this paper is organized as follows. Section 2 sets up the model. Section 3 analyzes the transitional dynamics of the economy. Section 4 expands the model with differential lifetime Section 5 concludes the paper.

2. The model

Consider the competitive equilibrium of an overlapping generations economy. Each individual lives potentially for three periods, that is, "*childhood*", "*young adulthood,*" and "*old adulthood.*" While individuals certainly live during the second period, survival into old adulthood is uncertain.

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[2] In fact, demographic variables, such as fertility and lifetime, have a significant impact on the mobility and income inequality. Aso and Nakamura (2020) show that the fertility difference between the educated and uneducated plays a crucial role in the transitional dynamics of mobility. In addition, Strulik (2018) also explain the relationships between lifetime and social class.

[3] Maoz and Moav (1999) analyze the transitional dynamics of intergenerational mobility, income inequality and economic development in simple framework. Thus, we can clearly show the effects of endogenous lifetime on the transitional dynamics of economy by incorporating it into Maoz and Moav (1999).

2.1 Production and factor prices

Following Owen and Weil (1998), we assume that aggregate output in period t is characterized by the following production function:

$$Y_t = AK_t^{\alpha} E_t^{(1-\alpha)(1-\beta)} U_t^{(1-\alpha)\beta}, \qquad A > 0, 0 < \alpha < 1, 0 < \beta < 1, \tag{1}$$

where K_t is the physical capital, E_t is the number of educated workers, and U_t is the number of uneducated workers. The total number of workers is normalized to unity and each supplies one unit of labor. Then, $E_t + U_t = 1$; therefore, the above production function can be written as per capita income; $Y_t = AK_t^{\alpha} E_t^{(1-\alpha)(1-\beta)} (1 - E_t)^{(1-\alpha)\beta}$.

To focus on human capital accumulation, as with Owen and Weil (1998), we assume that this model economy has a small open capital market, despite labor not being internationally mobile. Hence, the marginal product of physical capital is determined by the world interest rate $\bar{r}$. Hence, we have the following in the equilibrium:

$$w_t^e = (1-\beta)\Theta A \left(\frac{1-E_t}{E_t}\right)^{\beta}, \tag{2}$$

$$w_t^u = \beta\Theta A \left(\frac{1-E_t}{E_t}\right)^{\beta-1}, \tag{3}$$

where $\Theta = (1-\alpha)(\alpha/\bar{r})^{\alpha/1-\alpha}$; the subscripts e and u denote "educated" and "uneducated," respectively. Thus, the wage inequality becomes:

$$\frac{w_t^e}{w_t^u} = \frac{1-\beta}{\beta}\left(\frac{1-E_t}{E_t}\right). \tag{4}$$

To ensure that $w_t^e > w_t^u$, we assume that $E_t < 1 - \beta$.

2.2 Individuals

As a child, who does not work, the individual receives a transfer from her parent. It is used for consumption and possible education. When young, she works, and divides her income among consumption, savings, and a transfer for her children, regardless of the survival status during old age.[4] She faces a survival probability from young to old adulthood. If she survives to old age, she retires and only consumes. The preference of individual i, born in period t, is expressed by the following expected lifetime utility function:

[4] The modeling of transfer follows Zhang et al. (2001).

$$v_t^i = logc_t^i + logc_{t+1}^i + logx_{t+1}^i + \pi_{t+1} logc_{t+2}^i, \tag{5}$$

where $i \in \{e, u\}$; c_t^i is consumption in period t, c_{t+1}^i is consumption in period $t+1$, x_{t+1}^i is the transfer per child in period $t+1$, π_{t+1} is the survival probability in period $t+1$ and c_{t+2}^i is consumption in period $t+2$.

Let h_t^i denote the education cost of individual i, born in period t. As with Maoz and Moav (1999), we assume an imperfect capital market that a child cannot access. Hence, individual uses up all the transfers from parents during childhood. A surviving individual will receive not only her own past savings plus interest, but also the return from mutual funds since we assume a perfect annuities market. Thus, individual i's budget constraint becomes

$$c_t^i + \eta^i h_t^i = x_t^i, \qquad \eta^i = \begin{cases} 1 & \text{if individual } i \text{ acquires education} \\ 0 & \text{if otherwise} \end{cases} \tag{6.a}$$

$$w_{t+1}^i = c_{t+1}^i + x_{t+1}^i + s_{t+1}^i, \quad \frac{R}{\pi_{t+1}} s_{t+1}^i = c_{t+2}^i, \tag{6.b}$$

where $R = 1 + \bar{r}$. Since we assume an imperfect capital market, the utility maximization in the periods of adulthood is formulated as follows:

$$\max_{c_{t+1}^i, x_{t+1}^i, c_{t+2}^i} logc_{t+1}^i + logx_{t+1}^i + \pi_{t+1} logc_{t+2}^i,$$

$$subject\ to\ c_{t+1}^i + x_{t+1}^i + \frac{\pi_{t+1}}{R} c_{t+2}^i = w_{t+1}^i.$$

The optimal consumption, transfer in period $t+1$, and optimal consumption in period $t+2$ respectively become,

$$c_{t+1}^i = x_{t+1}^i = \frac{w_{t+1}^i}{2 + \pi_{t+1}}, \qquad c_{t+2}^i = \frac{R\, w_{t+1}^i}{2 + \pi_{t+1}}. \tag{7}$$

Hence, the indirect utility function in adulthood is:

$$z(w_{t+1}^i) = 2log\left[\frac{w_{t+1}^i}{2 + \pi_{t+1}}\right] + \pi_{t+1} log\left[\frac{Rw_{t+1}^i}{2 + \pi_{t+1}}\right]. \tag{8}$$

If the utility derived from investing in education is higher than or equal to the utility derived from not investing in education, then individual i will acquire education. Thus we have,

$$log(x_t^i - h_t^i) + z(w_{t+1}^e) \geq logx_t^i + z(w_{t+1}^u), \tag{9}$$

From (9), we have the following critical value of education cost $\hat{h}_t^i$ for individual i :

$$\hat{h}_t^i = x_t^i\left[1-\frac{z(w_{t+1}^u)}{z(w_{t+1}^e)}\right] = x_t^i\left[1-\left(\frac{w_{t+1}^u}{w_{t+1}^e}\right)^{2+\pi_{t+1}}\right]. \tag{10}$$

She acquires education if $h_t^i \leq \hat{h}_t^i$, and vice versa. As can be seen from (10), in addition to the wage inequality w_{t+1}^e/w_{t+1}^u and the transfer x_t^i, the surviving rate π_{t+1} also plays an important role in education choice. The higher the value of π_{t+1}, the larger the incentive to acquire education since lifetime returns to education investment increases. From x_t^i and (10),

$$\hat{h}_t^e = \frac{w_t^e}{2+\pi_t}\left[1-\left(\frac{w_{t+1}^u}{w_{t+1}^e}\right)^{2+\pi_{t+1}}\right], \qquad \hat{h}_t^u = \frac{w_t^u}{2+\pi_t}\left[1-\left(\frac{w_{t+1}^u}{w_{t+1}^e}\right)^{2+\pi_{t+1}}\right]. \tag{11}$$

2.3 Education cost among individuals

Following Maoz and Moav (1999), the following cost is assumed to be incurred for the education of individual i in period t.

$$h_t^i = \theta^i c(\overline{w}_t) = \theta^i(a + b\overline{w}_t), \tag{12}$$

where $\overline{w} = E_t w_t^e + (1-E_t)w_t^u$ is a weighted average of educated and uneducated wages, θ^i is a parameter representing individual $i's$ ability to learn; the higher the ability, the lower is the value of θ^i. We further assume that θ^i is uniformly distributed in the interval $(\underline{\theta}, \overline{\theta})$, regardless of the ability and class of the parents in any period. Hence, h_t^i is also uniformly distributed in the interval $(\underline{h}_t, \overline{h}_t)$, where $\underline{h}_t = \underline{\theta}(a + b\overline{w}_t)$ and $\overline{h}_t = \overline{\theta}(a + b\overline{w}_t)$.

2.4 Endogenous lifetime

We assume that the probability of surviving π_t depends on health status H_t; this relationship is represented as follows:

$$\pi_t = \pi(H_t) = \frac{\underline{\pi} + \overline{\pi}H_t^\delta}{1 + H_t^\delta}, \tag{13}$$

where $\delta > 0$; $0 < \overline{\pi} \leq 1$; $0 < \underline{\pi} < \overline{\pi}$; $\pi(0) = \underline{\pi} > 0$; $\pi'(H) > 0$; $lim_{H\to\infty}\pi(H) = \overline{\pi} \leq 1$; $\pi''(H) < 0$ if $\delta \leq 1$ and $\pi''(H) \gtreqless 0$, for any $H \lesseqgtr \overline{H} \equiv [(\delta-1)/(1+\delta)]^{1/\delta}$ if $\delta > 1$.

The parameter δ represents how an additional unit of health investment is transformed into greater lifetime through health technology. If $\delta \leq 1$, π_t is a concave function. If $\delta > 1$, π_t is a S-shaped function, which means threshold effects exist (see Fanti and Gori, 2014). In other words, when $\delta < 1$, there is a relatively slow increase in economic development. In contrast, if $\delta > 1$, lifetime suddenly and rapidly increases with economic development owing to the sudden effect. Hence, a larger δ increases the speed of converges from $\underline{\pi}$ to $\overline{\pi}$. Similar to Chen (2010), we assume that the health status H_t is determined by economic development, i.e., per capita income Y_t.

$$H_t = H(y_t) = \phi Y_t, \qquad \phi > 0, \tag{14}$$

where ϕ represents health productivity.

3. Dynamics of the model

In this section, we show the dynamics of the economy. Intergenerational mobility can be expressed in two ways—upward-mobility (UM_t) and downward-mobility (DM_t). In our model, upward-mobility means that individuals born to an uneducated parent become educated adults, while downward-mobility means that individuals born to an educated parent become uneducated adults. The dynamics of E_t can therefore be expressed as:

$$E_{t+1} - E_t = \underbrace{(1 - E_t)\frac{\hat{h}_t^u - \underline{h}_t}{\overline{h}_t - \underline{h}_t}}_{\equiv UM_t} - \underbrace{E_t\frac{\overline{h}_t - \hat{h}_t^e}{\overline{h}_t - \underline{h}_t}}_{\equiv DM_t} \tag{15}$$

or

$$E_{t+1} = (1 - E_t)\frac{\hat{h}_t^u - \underline{h}_t}{\overline{h}_t - \underline{h}_t} + E_t\frac{\hat{h}_t^e - \underline{h}_t}{\overline{h}_t - \underline{h}_t} \tag{16}$$

Taking into account $\overline{h}_t = \overline{\theta}c(\overline{w}_t)$, $\underline{h}_t = \underline{\theta}c(\overline{w}_t)$ and Eq. (11), Eq. (16) can be written as follows:

$$E_{t+1} = \frac{1}{[2 + \pi(Y_t)](\overline{\theta} - \underline{\theta})}\frac{f(E_{t+1})}{s(\overline{w}_t)} - \frac{\underline{\theta}}{\overline{\theta} - \underline{\theta}}, \tag{17}$$

where $f(E_{t+1}) = \left[1 - (w_{t+1}^u/w_{t+1}^e)^{2+\pi(Y_{t+1})}\right]$ and $s(\overline{w}_t) = c(\overline{w}_t)/\overline{w}_t$ represents the average income share of education cost.

Investigating Eq. (17), we can see the dynamic behavior of intergenerational mobility, inequality, and economic development. Totally differentiating Eq. (17),

$$G_1 dE_{t+1} = G_2 dE_t, \tag{18}$$

where

$$G_1 = 1 - \frac{1}{[2 + \pi(Y_t)](\overline{\theta} - \underline{\theta})}\frac{f'(E_{t+1})}{s(\overline{w}_t)}, \tag{19}$$

$$G_2 = -\frac{\varepsilon_t^s + [\pi(Y_t)/2 + \pi(Y_t)]\varepsilon_t^\pi}{[2 + \pi(Y_t)](\overline{\theta} - \underline{\theta})} \frac{f(E_{t+1})}{s(\overline{w}_t)\, E_t} \gtreqless 0, \tag{20}$$

and

$$\varepsilon_t^s = \frac{\partial s(\overline{w}_t)/s(\overline{w}_t)}{\partial E_t/E_t} = s'(\overline{w}_t)\, \overline{w}_t' \frac{E_t}{s(\overline{w}_t)} < 0, \tag{21}$$

$$\varepsilon_t^\pi = \frac{\partial \pi(Y_t)/\pi(Y_t)}{\partial E_t/E_t} = \pi'(Y_t) Y_t' \frac{E_t}{\pi(Y_t)} > 0. \tag{22}$$

ε_t^s and ε_t^π represent the elasticity of education cost share with respect to the share of the educated in period t and the elasticity of surviving rate with respect to the share of the educated in period t, respectively. An increase in surviving rate π_t decreases the transfer that is the funding source of education investment, and, therefore, discourages intergenerational mobility.

Since $G_1 > 0$, as is evident from Eq. (21) and (22), the transitional dynamics of intergenerational mobility depends on ε_t^s and ε_t^π. Hence, we have

$$sign\left[\frac{dE_{t+1}}{dE_t}\right] = sign\left[\frac{G_2}{G_1}\right] = -sign\left[\varepsilon_t^s + \frac{\pi(Y_t)}{2 + \pi(Y_t)}\varepsilon_t^\pi\right]. \tag{23}$$

As educated workers increase, that is, the economy grows, the education cost share decreases, and $\varepsilon_t^s < 0$. In other words, the average wage increases more than the education cost. This reduction in education cost share encourages the mobility. On the other hand, lifetime increases with economic development, and $\varepsilon_t^\pi > 0$. This increase in lifetime decreases transfer from parents to children caused by higher savings, and, therefore discourages mobility. If increase in lifetime is sufficiently small, then $G_2/G_1 = -\left[\varepsilon_t^s + \frac{\pi(Y_t)}{2+\pi(Y_t)}\varepsilon_t^\pi\right] > 0$; hence Eq. (18) is upwards-sloping in the (E_t, E_{t+1}) plane. In contrast, if increase in lifetime is sufficiently large, then $G_2/G_1 = -\left[\varepsilon_t^s + \frac{\pi(Y_t)}{2+\pi(Y_t)}\varepsilon_t^\pi\right] < 0$; hence, Eq. (18) is downwards-sloping in the (E_t, E_{t+1}) plane. Thus, we have the following proposition.[5]

Proposition The transitional dynamics of intergenerational mobility depends on two effects: the positive effect of a decrease in education cost share and the negative effect of an increase in lifetime. When the former is dominant, that is, increase in lifetime is sufficiently small, the mobility and income inequality monotonically approach the steady state, as in Maoz and Moav

[5] If surviving rate is an exogenous value, the transitional dynamics of mobility depends only on the education cost share as in Maoz and Moav (1999).

(1999). In contrast, when the latter is dominant, that is, the increase in lifetime is sufficiently large, the mobility and income inequality exhibit cyclical behavior.

We show numerical examples of the Proposition. We take the parameter values $A = 12$, $\alpha = 0.3$, $\beta = 0.5$, $\overline{r} = 0.05$, $\overline{\theta} = 5$, $\underline{\theta} = 1$, $\overline{\pi} = 0.95$, $\underline{\pi} = 0.3$, and $\phi = 0.1$.[6] Fig.1 shows the transitional dynamics of intergenerational mobility. Since increase in lifetime is sufficiently small, Eq. (18) is upwards-sloping in the (E_t, E_{t+1}) plane in Fig. 1(a). Thus, the mobility and lifetime monotonically increase toward the steady state, while income inequality decreases. In contrast, Fig. 1 (b) shows that Eq. (18) is downwards-sloping in the (E_t, E_{t+1}) plane since the increase in lifetime is sufficiently large. Hence, the mobility monotonically approaches economic development initially, and then exhibits cyclical behavior around the steady state.

This cyclical behavior of the mobility can be interpreted as follows. Suppose that the educated E_t is low, and hence education cost and lifetime are also low. Then, as the educated increases, the wage of an uneducated worker increases and upward-mobility occurs. Hence, the educated E_t monotonically increases owing to upward-mobility. When E_t exceeds a certain threshold, that is, economy is sufficiently developed, the lifetime increases rapidly. This sharp increase in lifetime raises the incentive for educational investment, even as it greatly decreases transfer. Since this decrease in transfer is dominant, an increase in lifetime impedes mobility. Whether E_t increases or decreases in the next period depends on the positive effect of the decline in education cost share and the negative effect of increase in lifetime with economic development. Since the latter is larger than the former, E_t decreases in the next period in Fig. 1 (b); this, in turn, decreases the lifetime and education cost, and, therefore, E_t increases in the following period. This observation explains the cyclical behavior of economy.

[Insert Fig.1 around here]

Numerical Result 1 Increase in lifetime with economic development plays a crucial role in the transitional dynamics of mobility. If the increase in lifetime with economic development is

[6] Except for A and ϕ, the parameters follow Fanti and Gori (2014), Maoz and Moav (1999), and Owen and Weil (1998).

quite small, the mobility approaches toward steady state, as in Moaz and Moav (1999). In contrast, if increase in lifetime with economic development is sufficiently large, the mobility exhibits cyclical behavior around the steady state.

In particular, if the decrease in education cost share is quite small and lifetime increases more suddenly and rapidly, the fluctuation is greater and even chaotic as can be shown in Fig. 2.[7] Since δ is greater, the lifetime increases more rapidly, and, then, greatly decreases the transfer. This rapid decrease in transfer deteriorate upward-mobility, while it promotes downward-mobility. Hence, larger δ generates larger fluctuations of economy and a chaotic equilibrium appears in the economy.

[Insert Fig.2 around here]

Numerical Result 2 If the decrease in education cost share is sufficiently small and lifetime increases rapidly, that is, the negative effect of an increase in lifetime is much larger than the positive effect of a decrease in the income share of education cost, the mobility, income inequality, and lifetime exhibit larger fluctuations and even a chaotic equilibrium.

4. Expansion: Differential lifetime

In this section, we analyze the dynamics of economy with differential lifetime between the educated and the uneducated. As can be indicated by OECD (2017), the educated workers have a longer lifetime than the uneducated workers. For simplicity, we assume that the surviving rate of the educated is enough high, i.e., $\pi_t^e = \pi^e = \bar{\pi}$. Hence, only lifetime of uneducated workers increases with economic development, which depends on Eq. (14) and (15), i.e., $\pi^u(Y_t)$.

From optimal allocation, we get the indirect utility function in adulthood of the educated and the uneducated.

[7] In addition to δ, the parameters of education cost a and b is also important for determining transitional dynamics of mobility. The smaller the value of a (larger b) is, the larger the value of ε_t^S. It implies that income share of education cost decreases more slowly, and therefore the cyclical behavior exhibits in Fig. 2.

$$z\left(w_{t+1}^{i}\right) = 2log\left[\frac{w_{t+1}^{i}}{2+\pi_t^i}\right] + \pi^e log\left[\frac{Rw_{t+1}^{i}}{2+\pi_t^i}\right]. \tag{24}$$

Thus, we have the following critical value of education cost $\hat{h}_t^i$ for individual i:

$$\hat{h}_t^i = x_t^i\left[1 - \underbrace{\frac{(2+\pi^e)^{2+\pi^e}}{(2+\pi_{t+1}^u)^{2+\pi_{t+1}^u}}}_{(*1)}\left(\frac{w_{t+1}^u}{w_{t+1}^e}\right)^2 \underbrace{\frac{(Rw_{t+1}^u)^{\pi_{t+1}^u}}{(Rw_{t+1}^e)^{\pi^e}}}_{(*2)}\right]. \tag{25}$$

From Eq.(25), in addition to w_{t+1}^e/w_{t+1}^u and x_t^i, differential lifetime between the educated and the uneducated π^e/π_{t+1}^u have two effects of incentives for acquiring education. Term (*1) presents decreasing the incentives by larger differential lifetime. In contrast, term (*2) presents that larger differential lifetime increases the incentives through the increase in lifetime returns to education investment. Hence, the effects of differential lifetime on the incentives for acquiring education is ambiguous.

The dynamics of E_t can therefore be expressed as:

$$E_{t+1} = \frac{\overbrace{\bar{b}(E_t,\pi^e,\pi_t^u)}^{(*3)}}{(\bar{\theta}-\underline{\theta})}\frac{\overbrace{f(E_{t+1},\pi^e,\pi_{t+1}^u)}^{(*5)}}{\underbrace{c(\overline{w}_t)}_{(*4)}} - \frac{\underline{\theta}}{\bar{\theta}-\underline{\theta}}, \tag{26}$$

where $\bar{b}(E_t,\bar{\pi},\pi_t^u)$ represents a weighted average of educated and uneducated transfer for children, $\bar{b}(E_t,\pi^e,\pi_t^u) = E_t b_t^e(w_t^e,\pi^e) + (1-E_t)b_t^u(w_t^u,\pi_t^u)$. The dynamics of E_t depends on terms (*3), (*4) and (*5). Term (*3) presents changes in average transfer, while Term (*4) and (*5) present the effects increase in education cost and change in the incentives for acquiring education through income inequality and differential lifetime on the mobility.

Using the same parameter values in Section 4; $A = 12$, $\alpha = 0.3$, $\beta = 0.5$, $\bar{r} = 0.05$, $\bar{\theta} = 5$, $\underline{\theta} = 1$, $\bar{\pi} = 0.95$, $\underline{\pi} = 0.3$, and $\phi = 0.1$, we show examples of dynamics of E_t. As can be seen from Fig. 3 and Fig. 4, we get the main results even if we consider differential lifetime.[8] As a result, although differential lifetime generates the incentive for acquiring education, it does not play a crucial role in the main results of this paper. In other words, decrease in upward-mobility by increasing in lifetime of the uneducated is an important factor in the dynamics of E_t.

[8] Since the lifetime of the educated is constant in section 4, increase in lifetime is smaller than section 3, i.e., ε_t^{π} is small. Hence, to get the same result of chaos equilibrium as section3, we set up the larger value of δ, b, and the smaller value of a. In other words, the existence of chaos equilibrium requires a more rapid increase in lifetime and a smaller decrease in education cost share in differential lifetime model .

[Insert Fig.3 and 4 around here]

Numerical Result 3 Even if we suppose that differential lifetime, i.e., the educated workers have a longer lifetime than the uneducated workers, we get two motions of mobility: monotonous motion and cyclical motion and even a chaotic equilibrium.

5. Conclusions

This paper studies the impacts of endogenous lifetime on intergenerational mobility and economic development in overlapping generations framework. Increase in lifetime has two effects on the mobility: the positive effect through increasing incentive of educational investment and the negative effect through decreasing transfer for children. These effects of lifetime play crucial role in determining transitional dynamics of economy.

Two motions of intergenerational mobility, monotonous and cyclical, have been shown in empirical studies. This paper explains the two motions caused by the lifetime mechanism. We show that the transitional dynamics of mobility depends on lifetime. When an increase in lifetime is quite small, the mobility and income inequality monotonically approach the steady state with economic development, as in Maoz and Moav (1999). In contrast, when an increase in lifetime is sufficiently large, the economy exhibits cyclical and chaotic behavior. In fact, China where lifetime increased rapidly, has experienced a cyclical motion of mobility, while monotonous motion has been observed in Norway where lifetime slowly increased.

In future research, we can analyze the positive effect of an increase in lifetime on economic development by incorporating health capital into the model. Considering this will make it more interesting to explore interactions. In addition, available evidence is limited and it is necessary to explore the various factors that cause different motions in each country.

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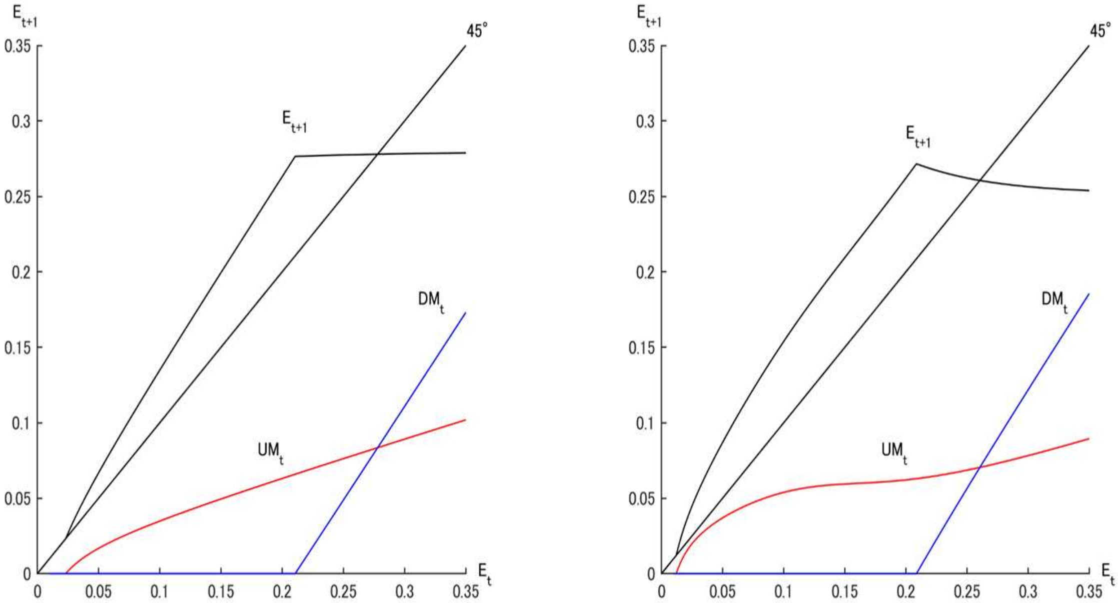


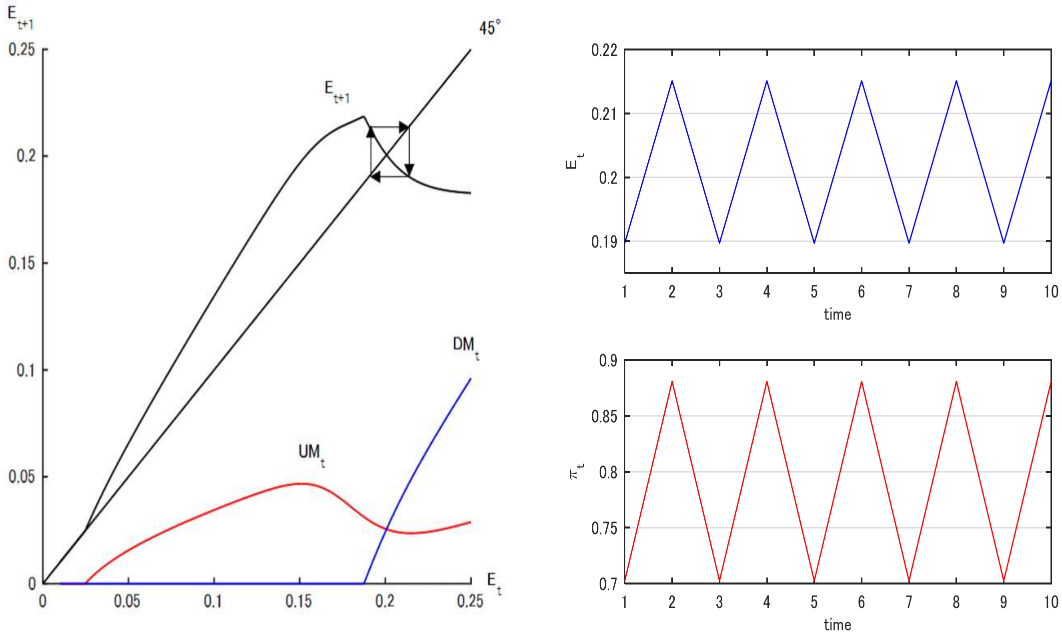
(a) $\boldsymbol{\delta = 0.75}$, $\boldsymbol{\varepsilon_t^s + \varepsilon_t^\pi < 0}$

$(E^* = 0.2780,\ \pi^* = 0.6429)$

(b) $\boldsymbol{\delta = 10}$, $\boldsymbol{\varepsilon_t^s + \varepsilon_t^\pi > 0}$

$(E^* = 0.2605,\ \pi^* = 0.8072)$

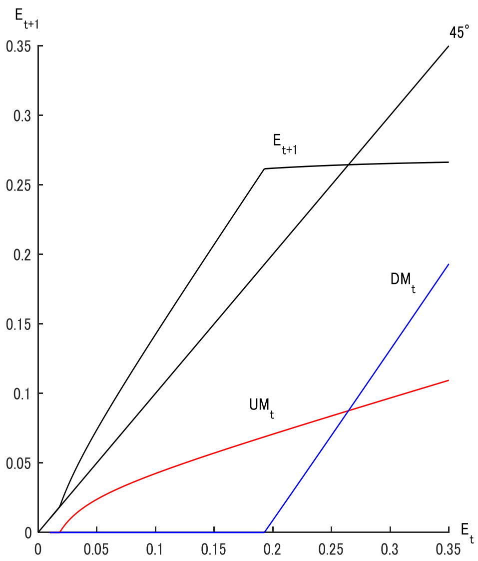
Fig.1 Transitional dynamics of $\boldsymbol{E_t}$ $(\boldsymbol{a = 0.12},\ \boldsymbol{b = 0.15})$



(a) Transitional dynamics of $\boldsymbol{E_t}$

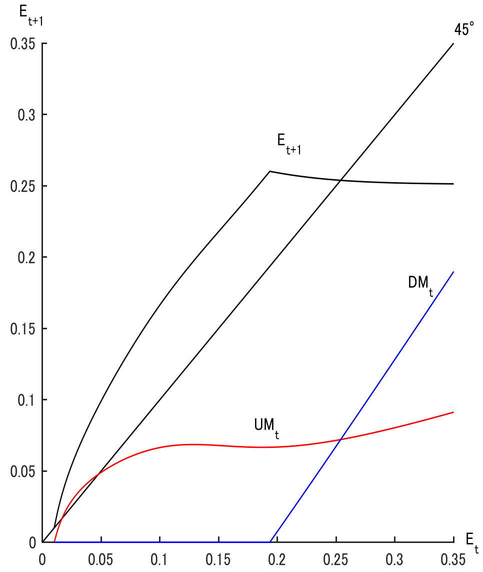
(b) Fluctuations in $\boldsymbol{E_t}$ and $\boldsymbol{\pi_t}$

Fig.2 Chaos equilibrium ($\boldsymbol{\delta = 35}$, $\boldsymbol{a = 0.1}$, $\boldsymbol{b = 0.18}$)



(a) $\boldsymbol{\delta = 0.75}$

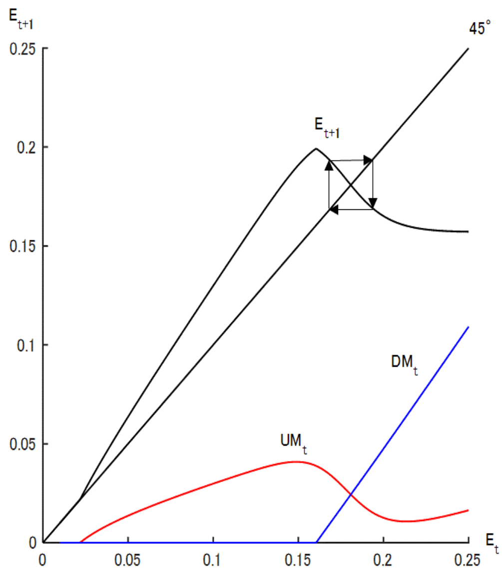
($E^* = 0.2646$, $\pi^{u*} = 0.6410$)



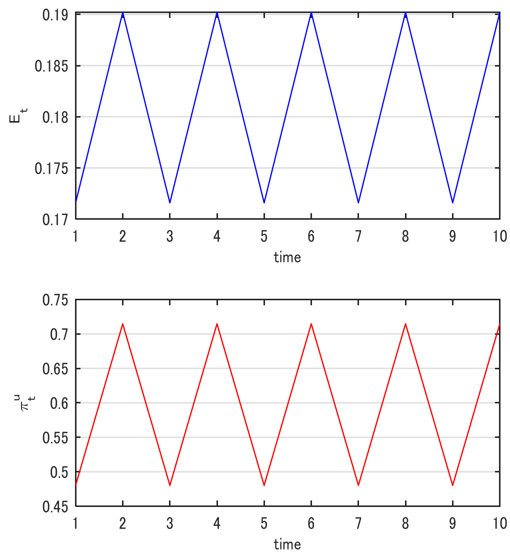
(b) $\boldsymbol{\delta = 10}$

($\boldsymbol{E^* = 0.2538}$, $\boldsymbol{\pi^{u*} = 0.7975}$)

Fig.3 Transitional dynamics of $\boldsymbol{E_t}$ ($\boldsymbol{a = 0.12}$, $\boldsymbol{b = 0.15}$)



(a) Transitional dynamics of $\boldsymbol{E_t}$



(b) Fluctuations in $\boldsymbol{E_t}$ and $\boldsymbol{\pi^u_t}$

Fig.4 Chaos equilibrium ($\boldsymbol{\delta = 38}$, $\boldsymbol{a = 0.05}$, $\boldsymbol{b = 0.2}$)