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Random Encounters and Information Diffusion about Product Quality*

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Abstract

This paper explores how social interactions among consumers shape markets. In a two-country model, consumers meet and exchange information about the quality of the goods. As information spreads, the demands evolve, affecting the prices and quantities manufactured by profit-maximizing firms. We show that market prices with informational frictions reach the duopoly price with full information, at the limit. However, this convergence can take two different paths depending on the size asymmetry between countries. In particular, when countries are of very different sizes, the single market does not immediately turn into a duopoly and monopoly prices may persist for several periods. Hence, the price-reducing trade effects may take longer to appear. In view of an intense globalization process, understanding how social meetings affect market outcomes is critical for understanding the performance of international economic integration.

Keywords: *Consumer Encounters, Information Diffusion, Country Size, Product Quality.*

JEL Classification: D42, D43, D83, F15, L13.

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1 Introduction

People are on the move. Various international agreements, such as the European Union, the establishment of the Euro, and the Schengen Area agreement, have massively reduced the cost of mobility, intensifying globalization. This process has been accompanied by a substantial reduction in international transportation costs for commodities. Citizens located in different countries now meet much more frequently, sharing personal consumption experiences. Accordingly, meetings have become an essential vehicle of diffusion of information about goods, prices, and quality. This vehicle and its price effects are the main topics of this paper.

We explore how interpersonal non-market interactions affect market quantities and prices. How do domestic markets metamorphose into a single market? If there is a convergence of prices, what paths are involved and how rapid is the convergence?

To answer these questions, we build a two-country model with two vertically differentiated goods (Gabszwewicz and Thisse, 1979) and the presence of informational frictions. The countries have different population densities, and each country has a national market on which a domestic firm sells a domestic good. As long as these markets remain strictly national, consumers in each country remain ignorant regarding the foreign commodity's existence and its features. There is no mutual influence between their respective markets: each national firm is a monopolist on its national market. An international agreement is signed that reduces people's costs of being mobile. This agreement determines the birth and outgrowth of mutual interactions among agents in the two asymmetrically sized countries. Mutual experiences of consumption habits are exchanged progressively and according to the number and frequency of meetings. Inspired by Lazear (1995, 1999), we model meetings in a random encounter model. This implies that information about the goods does not spread automatically to all consumers.

As consumers get informed, the exchange of consumption experiences reinforces the process of competition between the national and the foreign substitute. This process magnifies competition between the products, transforming two national monopolies into a single duopoly market with vertically differentiated products. We investigate how prices change along the sequence of equilibria generated by the dynamics of international interactions.

The main results of our analysis show that the market prices tend to the duopoly solution with full information, at the limit. The intuition behind this result relates to information diffusion. As consumers become informed, the market power of firms reduces, and when all consumers are informed about both goods the single market becomes a duopoly. Surprisingly, this convergence can take two different paths. When countries are of similar sizes, interpersonal meetings between consumers from each country are relatively frequent and a significant share of the population in each country becomes aware of the other country's good. Therefore, competition between national goods intensifies relatively

quickly in the common market. Market evolution from monopoly to duopoly occurs in the first period, while prices take time to adjust to their full information level.

In contrast, when countries are of very different sizes, consumer meetings across different countries are rare; thus, information diffusion regarding the foreign commodity's existence is considerably slower. As a consequence, we show that an interior duopoly equilibrium may not exist for a limited number of periods. Hence, the expected price-reducing trade effects may take longer to appear depending on the size asymmetry between countries. However, we also show that a finite period exists in which informed consumers are sufficiently numerous to make the interior duopoly equilibrium appear. Learning about the goods finally leads to the establishment of the single market. Finally, we analyse the properties of duopoly prices and quantities in the presence of informational frictions, highlighting some novel properties.

Our work combines the literature about markets with the growing research trend on the role played by social interactions in market shifts. The contribution to prior literature is twofold. We firstly propose an elegant model of vertical differentiation with informational frictions that evolve dynamically to the state of full information. This model is relatively simple, but it allows a deep understanding of how non-market forces may affect market formation.

Secondly, we contribute to an extensive literature on the effects of globalization on markets, highlighting population size asymmetry. In fact, our model allows us to pin down how size asymmetry between countries affects the price convergence path when citizens meet. In an influential paper, Alesina, Spolaore, and Wacziarg (2005) argue that our understanding of economic performance and the history of international economic integration can be greatly improved by bringing country size to the forefront of the analysis of prices and growth. The authors use country size to document how trade affects a country's growth, showing that the size of countries shapes trade intensity.

The article is set out as follows. In the following subsection, we situate our paper in the related literature. Section 2 provides a description of the model. In Section 3, we develop the multi-period market solution. Section 4 unveils some interesting properties of the duopoly outcome with informational frictions, and finally, Section 5 presents the study's conclusions.

1.1 Related literature

Our paper brings together three theoretical tools: *(i)* a vertical differentiation model, *(ii)* open economies and trade, and finally, *(iii)* a dynamic setting to capture the evolution of a particular feature in a given population.

Firstly, we contribute to the industrial organization literature as we revisit the classical model of vertically differentiated markets in industrial organization, introducing informational frictions. More precisely, before information fully spreads to reach all consumers, there are three different market

segments according to the consumers' information set. The uninformed consumers who know their domestic good but are ignorant of the foreign one and the informed consumers who know both goods. In this novel setting, the firm behaves as a monopolist for uninformed consumers and as a duopolist for the informed ones. We call this new market configuration a duopoly with informational frictions. Prior literature on vertically differentiated markets focuses on market interactions, whereas we focus on non-market interactions. For instance, Caminal and Vives (1999) analyse price dynamics in a duopoly where consumers learn about good quality differentials. In that paper, market shares aggregate consumers' dispersed and private information about the quality of products. Hence, past market shares act as a signal of product quality. The authors find that despite price wars, consumers learn slowly and convergence to full information is also slow. Tarola and Zanaj (2020) use a similar setting of two countries and vertical product differentiation to investigate how home bias shapes the market solution. Vettas (1998) investigates the endogenous diffusion of information along both sides of the market: firms and consumers. The diffusion of new firms' entry when consumers learn about the new goods through past consumption experience follows an S-shaped diffusion path. Our paper contributes to this literature by providing a simple model in which meetings are combined with strategic pricing to assess the type and pace of convergence from monopolies to a duopoly after the opening up of movement between two countries.

The existing literature on international trade, both theoretical and empirical, is huge. Papers aim to quantify and test the empirical relevance of trade theories using, for instance, numerical general equilibrium models calibrated with real-world data (Mercenier and Schmitt, 1996). These papers have played an essential role in the Canada–U.S. free-trade agreement, the North American Free Trade Agreement (NAFTA), and the European Single Market. However, the importance of country size in the theoretical contributions has been neglected. The importance of country size in trade is well documented in empirical papers showing that trade has a more substantial impact on market competitiveness for small economies. For instance, Hong Kong and Singapore are small open-to-trade countries that do not have a competition policy authority. Trade has relatively greater effects on these economies and acts as a disciplinary mechanism. Moreover, Hoekman et al. (2001), find evidence that country size negatively influences the effect trade has on market prices. Finally, Novy (2013) shows that trade is more sensitive to trade agreements if the exporting country only provides a small share of the destination country's imports. If two large countries like the USA and Germany are engaged in strong trade relations, a trade agreement change will affect trade flows less than if the USA and Iceland were engaged in trade. The reason is the relative population size of Iceland. The intensity of trade between the two larger countries is higher than that between the USA and Iceland. Our paper is also related to the existing literature on trade and quality. For instance, Motta and Thisse (1993) extend the vertical differentiation model to two countries with two firms to analyse the effects of quality standards in autarky and free trade.

Herguera et al. (2000) study the effect of quantity restrictions on a vertically differentiated model. To the best of our knowledge, we are the first in this literature of international duopolies with vertical differentiation to investigate single-market formation.

Finally, we contribute to the literature exploring the evolution of specific traits in two different population groups. Lazear (1995, 1999) investigates the evolution of norms (e.g. spoken language) via interpersonal meetings. Bisin and Verdier (2001) analyse the intergenerational transmission of norms by considering meetings within and outside the family.[1] These papers concentrate on the dynamic process in the absence of any explicitly modelled market mechanism. Our contribution is that we introduce the dynamic process in a duopoly competition. By doing so, we model a dynamic process of demand evolution and solve for market outcomes.

2 The model

Consider a two-country-two-good model, where country $i = 1$ produces good 1 and country $i = 2$ produces good 2. Heterogeneous consumers in each country are indexed by θ and uniformly distributed over the interval $[0, 1]$. The parameter θ captures the consumers' heterogeneous willingness to pay for the good: the higher is θ, the higher the utility obtained when consuming the good.

At time $t = 0$, in each country, consumers are only aware of the domestic good, regardless of whether or not (s)he actually consumes the good. In this pre-agreement period, each consumer can either buy one unit of the domestic commodity or not buy anything at all. Formally, a consumer's utility is given by

$$U(\theta) = \begin{cases} \theta u_i - p_i \text{ if buying variant } i \\ 0 \text{ if refraining from buying} \end{cases} \quad (1)$$

where u_i denotes the quality of the domestic variant and p_i its market price.[2]

At period $t = 1$, the two governments decide to sign an agreement (i.e. free movement of citizens and goods, similar to the EU agreements) that opens the two countries to unrestricted citizen circulation, beyond international trade.[3] Starting from period 1, consumers have the chance to meet, at each further period, either a domestic or a foreign consumer and share his/her knowledge about the goods. We assume that these social interactions arise for various reasons (work, friendship, schooling, romantic

[1] The interested reader may find an extensive coverage of these models in Bowles (2004).

[2] For simplicity, we assume that at $t = 0$ all consumers know precisely the quality of their national good. Alternatively, we could assume that consumers hold an evaluation about the quality of the domestic good and learn the intrinsic qualities u_1 and u_2 only when they get informed about both goods. It can be shown that the main results of our model, remain unchanged. We thank an anonymous referee for pointing out this alternative setting.

[3] The reasons for this agreement are kept exogenous to the model.

exchanges, or simply vacations). Whatever the reason, when two consumers meet we assume that they exchange information about the goods they know about. Only then will some consumers become acquainted to *both* goods, and they acknowledge them as vertically differentiated in accordance with

$$U(\theta) = \begin{cases} \theta u_1 - p_1 \text{ if buying variant 1} \\ \theta u_2 - p_2 \text{ if buying variant 2} \\ 0 \text{ if refraining from buying} \end{cases} \tag{2}$$

It follows that information about both goods spreads with frictions: not every consumer learns about the quality of the two goods immediately. Accordingly, the exchangeability of information creates two groups of consumers: *(i)* consumers only knowing the domestic good; and *(ii)* consumers informed about both goods.[4]

Let $s \in (0,1)$ denote the positive fraction of consumers living in country *one* and $(1-s)$ that of country *two*. Accordingly, the population of country *one* is s and that of country two is $(1-s)$. We assume, for simplicity, that both goods are made available in both markets with zero transportation costs. Without loss of generality, we assume that good 1 produced by country *one* is of lower quality than good 2 produced by country *two*, namely $u_2 > u_1$. For simplicity, we assume zero costs of production.[5]

We now proceed by providing the market solution for autarky and for the scenario of full information.

2.1 Monopoly solution in autarky

At $t = 0$, countries live in a regime of *autarky*. Populations do not mix, and, hence, they only purchase the domestic good at the monopoly price. Encounters and information diffusion will start putting pressure on the two monopolies from period 1. The market is endogenously uncovered, and consumer θ_i^M, who is indifferent in regards to buying and not buying the good in country $i = 1, 2$, is located at

$$\theta_i^M = \frac{p_i^M}{u_i}, i = 1, 2.$$

The demand functions of each firm operating in autarky at the initial period $t = 0$ are then, respectively:

$$D_1^M = s\left(1 - \theta_1^M\right) \text{ and } D_2^M = (1-s)\left(1 - \theta_2^M\right).$$

Firms maximize their profits by setting monopoly prices in each country as,

$$p_1^M = \frac{u_1}{2} \text{ and } p_2^M = \frac{u_2}{2}, \tag{3}$$

[4]Information exchange can convey a piece of persuasive information to consumers in the vein of a persuasive advertisement. Through communication with foreigners, consumers may get information about how valuable each good is.

[5]Introducing a production cost dependent on product quality would make the analysis more cumbersome without improving the model intuitions.

covering half of their domestic markets as a result

$$D_1^M = \frac{s}{2} \text{ and } D_2^M = \frac{(1-s)}{2}, \tag{4}$$

and gaining monopoly profits equal to

$$\Pi_1^M = \frac{s \cdot u_1}{4} \text{ and } \Pi_2^M = \frac{(1-s) \cdot u_2}{4}. \tag{5}$$

2.2 Duopoly solution under full information

After the international agreement, if everyone living in country i *would instantaneously meet everyone* from country j, then all consumers would immediately become fully informed about the existence of the two goods. Hence, the marginal consumer $\theta_2(p_1, p_2)$ choosing between good 1 and good 2 and the marginal consumer $\theta_1(p_1)$ choosing between consuming good 1 or refraining from consuming at all are given by:

$$\theta_2(p_1, p_2) = \frac{p_2 - p_1}{u_2 - u_1} \text{ and } \theta_1(p_1) = \frac{p_1}{u_1}. \tag{6}$$

Then, with perfectly informed consumers, the demand functions for goods one $D_1(p_1, p_2)$ and two $D_2(p_1, p_2)$ are, respectively:

$$D_1(p_1, p_2) = \frac{p_2 - p_1}{u_2 - u_1} - \frac{p_1}{u_1} \quad \text{and} \quad D_2(p_1, p_2) = 1 - \frac{p_2 - p_1}{u_2 - u_1},$$

thus yielding, in turn, the following equilibrium prices:

$$p_1^* = u_1 \cdot \frac{u_2 - u_1}{4u_2 - u_1} \quad \text{and} \quad p_2^* = 2u_2 \cdot \frac{u_2 - u_1}{4u_2 - u_1}, \tag{7}$$

demands

$$D_1^* = \frac{u_2}{4u_2 - u_1} \quad \text{and} \quad D_2^* = \frac{2u_2}{4u_2 - u_1}, \tag{8}$$

and profits

$$\Pi_1^* = \frac{u_1 u_2 (u_2 - u_1)}{(4u_2 - u_1)^2} \quad \text{and} \quad \Pi_2^* = \frac{4u_2^2 (u_2 - u_1)}{(4u_2 - u_1)^2}. \tag{9}$$

However, since not *all consumers* meet at once, when the two countries open their markets, at every period there may exist consumers with different levels of knowledge about the two goods. In the next section, we analyse how the demands in each country evolve over time.

2.3 Market solution with informational frictions

We now describe a vertically differentiated model with informational frictions. The international agreement that opens the countries to exchanges creates two types of consumers according to the information they hold: *uninformed* and *informed* consumers:

Definition 1. *Uninformed consumers are only aware of the domestic good but ignore the existence and/or quality of the foreign good.*

Uninformed consumers populate both countries. In country one, uninformed consumers – whose mass is denoted by U_1 – are uninformed about good 2. Similarly, in country two, consumers of mass U_2 are uninformed about good 1.

Definition 2. *Informed consumers become acquainted with the existence of the foreign good by socially interacting with a foreign consumer or a domestic one who has already met a foreign consumer..*

Informed consumers also exist in both countries. Their total mass is denoted by I. Since the world population is normalized to one, it directly follows that $I + U_1 + U_2 = 1$. Importantly, note that the size of these market segments changes over time as information about goods diffuses. In this section, we focus on identifying the market solution for any arbitrary partition of consumers into *uninformed* (about goods 2 and 1) and *informed*, i.e. any (U_1, U_2, I) such that $U_1 \cap U_2 \cap I = \emptyset$ and $U_1 \cup U_2 \cup I = 1$. In turn, in the next section we specify a process of information diffusion, thus illustrating in more detail how the evolution of consumers' information affects the market equilibrium.

Henceforth, for simplicity, we assume not only that firm quality is ranked by increasing order (i.e. $u_2 > u_1 > 0$) but also that the difference between neighbouring qualities is equal to u, with $u > 0$. This is obtained by simply setting $u_0 = 0$, $u_1 = u$ and $u_2 = 2u$. Thus, when the quality gap u increases, this moves variant 1 up from zero quality as well as variant 2 away from variant 1. This assumption improves the readability of the paper without qualitatively altering the study' results.[6]

Thus, the market demand functions of each firm are given by:

$$\begin{aligned} D_1(p_1, p_2) &= U_1\left(1 - \frac{p_1}{u}\right) + I\left(\frac{p_2 - p_1}{u} - \frac{p_1}{u}\right), \\ D_2(p_1, p_2) &= U_2\left(1 - \frac{p_2}{2u}\right) + I\left(1 - \frac{p_2 - p_1}{u}\right), \end{aligned} \tag{10}$$

highlighting the fact that uninformed consumers buy the domestic good whereas the informed ones are portioned between the two goods in accordance with their willingness to pay. Every firm in country $i = 1, 2$ sets it price to maximize its profit:

$$\begin{aligned} \Pi_1(p_1, p_2) &= p_1 \cdot U_1\left(1 - \frac{p_1}{u}\right) + p_1 \cdot I\left(\frac{p_2 - p_1}{u} - \frac{p_1}{u}\right) \\ \Pi_2(p_1, p_2) &= p_2 \cdot U_2\left(1 - \frac{p_2}{2u}\right) + p_2 \cdot I\left(1 - \frac{p_2 - p_1}{u}\right), \end{aligned} \tag{11}$$

[6] This is a rather common simplifying assumption (see, for instance, Gabszewicz and Thisse, 1980 and Gabszewicz et al., 2016). The authors can provide to interested readers the solution of the model in the absence of this assumption.

thus yielding the following Nash equilibrium prices as a function of the consumer information partition:

$$\begin{aligned} p_1^*(U_1, U_2, I) &= u \cdot \frac{(I + U_2 + 2U_1) I + U_1 U_2}{(7I + 4U_1 + 4U_2) I + 2U_1 U_2} \\ p_2^*(U_1, U_2, I) &= u \cdot \frac{(4I + 4U_2 + 3U_1) I + 2U_1 U_2}{(7I + 4U_1 + 4U_2) I + 2U_1 U_2}, \end{aligned} \tag{12}$$

Substituting these prices in (6), we obtain $\theta_1^*(U_1, U_2, I)$ and $\theta_2^*(U_1, U_2, I)$. Note that for the existence of a vertically differentiated product duopoly equilibrium, two conditions are required. The first is

$$p_2^*(U_1, U_2, I) > p_1^*(U_1, U_2, I) > 0,$$

which by (12), holds as $U_1, U_2 \in (0, 1)$ and $I = 1 - U_1 - U_2$, and the second is

$$1 > \theta_2^*(U_1, U_2, I) > \theta_1^*(U_1, U_2, I) > 0$$

which can be written as

$$1 > \frac{p_2^*(U_1, U_2, I) - p_1^*(U_1, U_2, I)}{u} > \frac{p_1^*(U_1, U_2, I)}{u} > 0. \tag{13}$$

Straightforward calculations show that while the first and the last inequalities in (13) hold for $U_1, U_2 \in (0, 1)$ and $I = 1 - U_1 - U_2$, the intermediate condition – required for good 1 to remain on sale in the duopoly market – only holds for $U_1 < 2/3$. This is illustrated in the next lemma.

Lemma 1. *For the low-quality firm to sell its product in a duopoly with informational frictions, i.e. for* $\theta_2^*(U_1, U_2, I) > \theta_1^*(U_1, U_2, I)$ to arise, *the mass of uninformed consumers in country one cannot be too large:* $U_1 < 2/3$.

Proof. Using (12) and substituting the constraint $I = 1 - U_1 - U_2$ we obtain that

$$\theta_1^*(U_1, U_2) - \theta_1^*(U_1, U_2) = \frac{p_2^*(U_1, U_2) - p_1^*(U_1, U_2)}{u} - \frac{p_1^*(U_1, U_2)}{u} > 0 \Leftrightarrow (2 - 3U_1)(1 - U_1 + U_2) > 0$$

which for $U_1, U_2 \in (0, 1)$ requires $U_1 < 2/3$. ■

The above result indicates that when the share of consumers uninformed about the high-quality good (living in country one) is too high, firm 1 can rely on a very large captive share of consumers. This endows this firm with very high market power, providing this firm with an incentive to set the price too high (compared to that of the high-quality firm) to attract informed consumers and to remain competitive in the international market. The next proposition adds more findings regarding the effect of information on the equilibrium prices and demand in the international duopoly with frictions.

Proposition 1 *In a duopoly with informational frictions, equilibrium prices monotonically increase with the mass of uninformed consumers in both countries, whereas they decrease with the mass of informed ones.*

Proof. See Appendix A.1. ■

For the equilibrium demands, we obtain

Proposition 2 *In a duopoly with informational frictions, equilibrium demand decreases with the mass of uninformed foreign consumers if a firm's own equilibrium price is more sensitive to this change than the rival firm's price, i.e.,* $\partial D_i^*/\partial U_j < 0$ *if* $\partial p_i^*/\partial U_j > \partial p_j^*/\partial U_j$ *for* $i = 1, 2$ *and* $j \neq i$*. The impact of the mass of uninformed domestic consumers and informed consumers on equilibrium demand is ambiguous and depends on the countries' relative sizes.*

Proof. See Appendix A.2. ■

To sum up, since prices behave as strategic complements, a larger set of consumers uninformed about one good increases the monopoly power of the selling firm, causing a rise in its price as well as in that of its rival. This, in turn, can increase or decrease the demand for each good depending on the effect that information diffusion has on the price gap $p_2^* - p_1^*$. This and other effects can be analysed in detail and require a more specific model of information transmission to yield sharper predictions. In the next sections, we aim to investigate these specific effects in a multi-period setting, where the masses of informed and uninformed people evolve over time.

3 Multi-period setting

In the previous section, we did not specify how the partition of informed and uninformed consumers originates and evolves over time as an effect of the process of information diffusion. We now introduce a simple dynamic process of information transmission. Specifically, we assume that increased mobility yields social interactions and encounters between people from the two countries that diffuse information about markets and goods. In particular, we assume that an international agreement to open the two countries to trade and citizen mobility increases international personal meetings. In fact, if two consumers from different countries meet, they exchange information about their consumption habits. It follows that consumers of country i get to know the properties of the other country's good, i.e. the quality difference u between the two goods. Accordingly, consumers can decide to change their consumption habits and consume the foreign good. Once acquainted with the foreign good, these consumers may become 'ambassadors' of the foreign good among their co-nationals. In contrast, when two uninformed consumers who are co-nationals meet, there is no information diffusion about the foreign good. This

iterative process of information diffusion has distinctive effects on market competition and, thus, on the prices set by the firms.

More specifically, social interaction among consumers of the two countries brings two important consequences: *(i)* the two goods become available in both countries at negligible trade costs; *(ii)* informed consumers originating from social interactions modify the demand function of each firm.

3.1 Knowledge Transmission over Time

Formally, the process of knowledge transmission among consumers is inspired by Lazear (1995, 1999). In each period, every consumer randomly meets one consumer who can either be a foreign or a domestic consumer. Given the fraction of consumers s and $(1-s)$ in each country C_i $(i = 1, 2)$, the probability that a consumer from country one meets another domestic consumer at period 1 and thus remains uninformed about the existence of the other good is simply given by

$$\Pr\left\{(i \in C_1) \cap (j \in C_1)\right\}_{t=1} = s^2.$$

Similarly, the probability that an individual from country two meets a domestic consumer, thus remaining uninformed about country one's good, is given by

$$\Pr\left\{(i \in C_2) \cap (j \in C_2)\right\}_{t=1} = (1-s)^2.$$

Thus, the probability that the consumers of the two countries become *informed* in period 1, is simply given by

$$\Pr\left\{(i \in C_1) \cap (j \in C_2)\right\}_{t=1} = 1 - s^2 - (1-s)^2 = 2s\,(1-s)\,.$$

A similar knowledge transmission process occurs in all subsequent periods $t \in \mathbb{N}$ with one new feature. The informed domestic consumers are now ambassadors for the foreign good in the domestic market. Accordingly, from period 2 onwards, in the domestic market information about foreign goods is transmitted by foreign consumers and informed domestic inhabitants. In what follows, we analyse how the sets of informed and uninformed consumers in each country evolve over time. The population dynamics of these two subsets defines the demand for each good.

In particular, we can denote the *uninformed* consumers living in country i at time t as $U_i(t)$ for $i = 1, 2$, and similarly, the set of consumers *becoming informed* (necessarily about both goods) at every time t as $I(t)$. It is easy to see that the set of consumers uninformed about good two (located in *country*

1) progresses geometrically as follows:

$$\begin{aligned}
U_1(0) &= s \\
U_1(1) &= \Pr\{(i \in U_1(0)) \cap (j \in U_1(0))\} = s \cdot s = s^2 \\
U_1(2) &= \Pr\{(i \in U_1(1)) \cap (j \in U_1(1))\} = s^2 \cdot s^2 = s^4 \\
U_1(3) &= \Pr\{(i \in U_1(2)) \cap (j \in U_1(2))\} = s^4 \cdot s^4 = s^8 \\
&... \\
U_1(t) &= \Pr\{(i \in U_1(t-1)) \cap (j \in U_1(t-1))\} = s^{2^{(t-1)}} \cdot s^{2^{(t-1)}} = s^{2^t}.
\end{aligned}$$

Thus, the greater the size of country one (the lower that of country two), the higher will be the number of periods needed for all people uninformed about good 1 to become informed. Analogously, the group of consumers uninformed about good one (residing in country two) progresses geometrically as follows:

$$\begin{aligned}
U_2(0) &= (1-s) \\
U_2(1) &= \Pr\{(i \in U_2(0)) \cap (j \in U_2(0))\} = (1-s) \cdot (1-s) = (1-s)^2 \\
U_2(2) &= \Pr\{(i \in U_2(1)) \cap (j \in U_2(1))\} = (1-s)^2 \cdot (1-s)^2 = (1-s)^4 \\
U_2(3) &= \Pr\{(i \in U_2(2)) \cap (j \in U_2(2))\} = (1-s)^4 \cdot (1-s)^4 = (1-s)^8 \\
&... \\
U_2(t) &= \Pr\{(i \in U_2(t-1)) \cap (j \in U_2(t-1))\} = (1-s)^{2^{(t-1)}} \cdot (1-s)^{2^{(t-1)}} = (1-s)^{2^t}.
\end{aligned}$$

As before, the greater the size of country two (the smaller that of country one), the faster the decrease in people uninformed about good 1 will be over time.

Using the dynamics of the above functions $U_1(t)$ and $U_2(t)$, it immediately follows that the dynamics of consumers informed about both goods in every period t is simply given by

$$I(t) = 1 - U_1(t) - U_2(t) = 1 - s^{2^t} - (1-s)^{2^t}. \tag{14}$$

Thus, what matters for the diffusion of information and the progression over time of the set of informed agents $I(t)$ is that the size of the populations in the two countries is not too asymmetric.

Figure 1 plots the dynamics of the set of informed agents for different sizes of the two countries and for $t = 0, 1, 2, ..., 10$.

It can be noted that when the two countries have exactly the same size (*dashed line,* $s = 0.5$), the set of informed people grows faster than in the asymmetric cases (*dotted line,* $s = 0.75$, and *continuous line*, $s = 0.01$), covering a large portion of the total population of the two countries in fewer periods.

Over time, the set of informed agents converges to 1, whereas the two sets of uninformed consumers disappear. It is also important to keep in mind that the information diffusion process takes into account

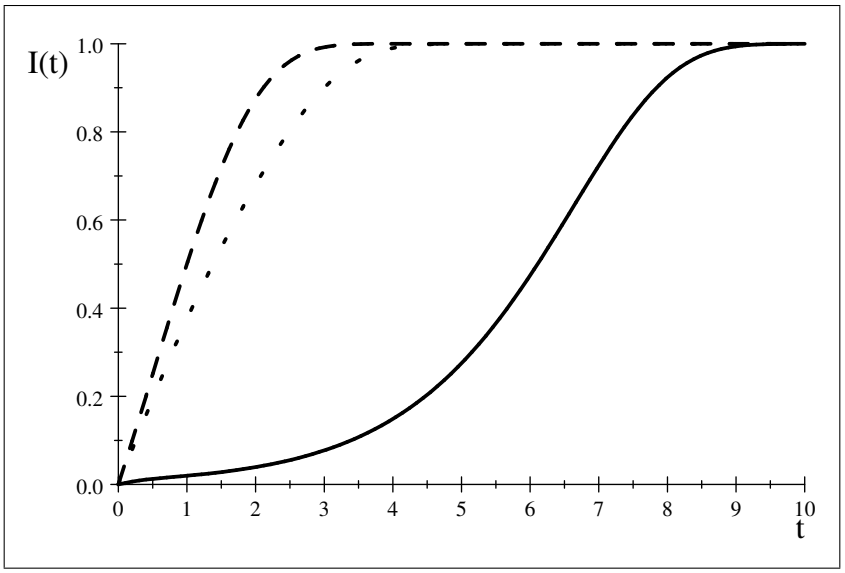

Figure 1: Dynamics of $I(t)$.

information transmission from all informed consumers regardless of their country of origin. In particular, starting from period $t = 1$ a consumer in country i, $i = 1, 2$, can learn about product $j \neq i$ either if she meets a consumer from country j or if she meets an informed consumer from her own country.

3.2 Equilibrium Market Solution

Using the information diffusion process described above, we are now able to express the demand functions of firms in (10) at every period $t \in \mathbb{N}$ by simply using the time-variant sets of uninformed $U_i(t)$ and informed people $I(t)$ illustrated above. The demand function for the two goods at period t are:

$$D_1(p_1(t), p_2(t)) = s^{2^t}\left(1 - \frac{p_1(t)}{u}\right) + \left(1 - s^{2^t} - (1-s)^{2^t}\right)\left(\frac{p_2(t) - p_1(t)}{u} - \frac{p_1(t)}{u}\right), \tag{15}$$

$$D_2(p_1(t), p_2(t)) = (1-s)^{2^t}\left(1 - \frac{p_2(t)}{2u}\right) + \left(1 - s^{2^t} - (1-s)^{2^t}\right)\left(1 - \frac{p_2(t) - p_1(t)}{u}\right). \tag{16}$$

The per-period profit functions of firms are, therefore,

$$\Pi_1(p_1(t), p_2(t)) = p_1(t) \cdot D_1(p_1(t), p_2(t)),$$

$$\Pi_2(p_1(t), p_2(t)) = p_2(t) \cdot D_2(p_1(t), p_2(t)).$$

In every period t, every firm sets its price to maximize its per-period payoff. In what we can denote as a *duopoly with informational frictions*, the equilibrium prices correspond to expressions (12) evaluated

according to the specific information dynamics presented above:

$$\begin{aligned} p_1^*(t) &= u \cdot \frac{1-(1-s)^{2^t}-s^{2^{t+1}}}{7-10\,(1-s)^{2^t}-10s^{2^t}+8s^{2^t}\,(1-s)^{2^t}+3\,(1-s)^{2^{t+1}}+3s^{2^{t+1}}} \\ p_2^*(t) &= u \cdot \frac{4-4\,(1-s)^{2^t}-5s^{2^t}+s^{2^{t+1}}+3s^{2^t}\,(1-s)^{2^t}}{7-10\,(1-s)^{2^t}-10s^{2^t}+8s^{2^t}\,(1-s)^{2^t}+3\,(1-s)^{2^{t+1}}+3s^{2^{t+1}}}, \end{aligned} \tag{17}$$

where it is easy to check that $p_2^*(t) > p_1^*(t)$. The marginal consumers (6) evaluated at the equilibrium prices (17) are, therefore, located at:

$$\begin{aligned} \theta_2\left(p_1^*(t), p_2^*(t)\right) &= \frac{3-3\,(1-s)^{2^t}-5s^{2^t}+3s^{2^t}\,(1-s)^{2^t}+2s^{2^{t+1}}}{7-10\,(1-s)^{2^t}-10s^{2^t}+8s^{2^t}\,(1-s)^{2^t}+3\,(1-s)^{2^{t+1}}+3s^{2^{t+1}}} < 1 \\ \theta_1\left(p_1^*(t), p_2^*(t)\right) &= \frac{1-(1-s)^{2^t}-s^{2^{t+1}}}{7-10\,(1-s)^{2^t}-10s^{2^t}+8s^{2^t}\,(1-s)^{2^t}+3\,(1-s)^{2^{t+1}}+3s^{2^{t+1}}} < 1 \end{aligned} \tag{18}$$

where it can be checked that $\theta_2^*(t) - \theta_1^*(t) > 0$ if and only if

$$s < \overline{s}(t) \equiv 2^{\frac{1}{2^t}} \cdot 3^{-\frac{1}{2^t}} \in (0,1)\,. \tag{19}$$

The above condition guarantees that $U_1 = s^{2^t} < 2/3$ and, therefore, that the solution of a *duopoly with informational frictions* exists. Note that since $\overline{s}(t)$ is monotonically increasing in t, if the constraint (19) holds at the initial period $t = 1$, i.e. $s < \overline{s}(1) = 0.8165$, it holds, *a fortiori,* in any other future period t.

3.2.1 The duopoly equilibrium with informational frictions

To investigate the evolution of the market solution from the duopoly with frictions to the duopoly with full information, we can initially focus – for illustrative purpose – on the equilibrium outcome of period 1.

At period $t = 1$, the demand function $D_1((p_1(1), p_2(1))$ for good 1 is equal to

$$D_1((p_1(1), p_2(1)) = s^2\left(1 - \frac{p_1(1)}{u}\right) + 2s(1-s)\left(\frac{p_2(1)-p_1(1)}{u} - \frac{p_1(1)}{u}\right), \tag{20}$$

where the first element $s^2\,(1 - p_1/u)$ is the demand of *uninformed consumers* in country one while $2s(1-s)\left[(p_2 - p_1)/u - p_1/u\right]$ is the total mass of *informed consumers* in both countries who are demanding good 1. *Mutatis mutandis*, the demand function for good 2 in period $t = 1$ is equal to:

$$D_2((p_1(1), p_2(1))) = (1-s)^2\left(1 - \frac{p_2(1)}{2u}\right) + 2s(1-s)\left(1 - \frac{p_2(1)-p_1(1)}{u}\right)$$

Firms set prices to maximize their profits. Using expressions (12), Nash equilibrium prices $p_1^*(1)$ and $p_2^*(1)$ immediately obtain as:

$$p_1^*(1) = \frac{1}{2}\frac{u\,(2+s(1+s))}{4+7s\,(1-s)} \text{ and } p_2^*(1) = \frac{u\,(4+s\,(1-2s))}{4+7s\,(1-s)}. \tag{21}$$

The corresponding marginal consumers are given by:

$$\theta_2^*(1) = \frac{(s+1)}{2}\frac{6-5s}{4+7s\,(1-s)} \text{ and } \theta_1^*(1) = \frac{1}{2}\frac{s\,(s+1)+2}{4+7s\,(1-s)}. \tag{22}$$

For the solution to be a duopoly with vertical differentiation, we need

$$1 > \frac{p_2^*(1) - p_1^*(1)}{u} > \frac{p_1^*(1)}{u} > 0$$

or

$$u > p_2^*(1) > 2p_1^*(1) > 0$$

which, in this case, simply writes as

$$1 > \frac{4+s\,(1-2s)}{4+7s\,(1-s)} > \frac{2+s(1+s)}{4+7s\,(1-s)} > 0.$$

While the first and the last inequalities always hold, it is easy to see that the second one ($p_2^* > 2p_1^*$) requires a size of country 1 (coinciding with the mass of consumers initially *uninformed about good 2*) of $s < \overline{s}(t) < 2^{\frac{1}{2}} \cdot 3^{-\frac{1}{2}} = 0.8165$.

It is straightforward to extend this result to every period t using the threshold $\overline{s}(t)$ obtained above in (19). This is stated in the following proposition:

Proposition 3 *If the mass of uninformed consumers in country one, $U_1(t)$, is not too large – namely $s \in (0, \overline{s}(t))$– then at every period t there exists a unique noncooperative duopoly equilibrium with informational frictions with prices given by (17).*

Proof. See Appendix A.3. ■

We discuss the case of $s > \overline{s}(t)$ in the next section.

3.2.2 The non-existence of a duopoly equilibrium with frictions

The analysis of the duopoly equilibrium prices has revealed that the international duopoly equilibrium may arise in every period t if $s \in (0, s(t))$, namely if the size of the population in the country that produces the low-quality good is not too large. When country one is very large and country two is very small, openness does not translate into a relatively quick diffusion of the information about the goods

and, therefore, competition does not constrain the behaviour of the two initial monopolies. Indeed, when $s > s(t)$, this leads to rare encounters with foreigners for the consumer living in the large country. If the asymmetry of population s is too high such that $s > s(t)$, then $\left[p_2^*(1) - p_1^*(1)\right]/u \leq p_1^*(1)/u$. More specifically, when the country producing the low-quality good is too large, the two prices $p_1^*(t)$ and $p_2^*(t)$ become too close and condition $p_2^*(t) > 2p_1^*(t)$ is not satisfied. This drives the position of the marginal consumer θ_1^* to coincide with θ_2^*. Accordingly, no informed consumer is willing to buy good 1 and they prefer to buy good 2, whereas only consumers uninformed about good 2 continue to buy good 1.

Accordingly, when $s \geq \overline{s}(t)$, using expression (18) and (19), at a duopoly candidate equilibrium $(p_1(t), p_2(t))$ the active consumers of country two – whether informed or uninformed – will buy good 2, while the consumers of country one will prefer to buy their own good only when uninformed about the foreign good. Since the total mass of consumers informed and uninformed about good 1 is $I + U_2 = \left[1 - s^{2^t} - (1-s)^{2^t}\right] + (1-s)^{2^t} = 1 - s^{2^t}$, the demand functions at every period t become

$$\begin{aligned} D_1(t) &= s^{2^t}\left(1 - \frac{p_1(t)}{u}\right), \\ D_2(t) &= \left(1 - s^{2^t}\right)\left(1 - \frac{p_2(t)}{2u}\right) \end{aligned}$$

with maximizing prices obtained as in (3) and demands and profits given, respectively, by

$$D_1(p_1^M(t)) = \frac{s^{2^t}}{2} \quad \text{and} \quad D_2(p_2^M(t)) = \frac{1 - s^{2^t}}{2}$$

and

$$\Pi_1(p_1^M(t)) = s^{2^t} \cdot \frac{u}{4}, \;\; \Pi_2(p_2^M(t)) = \left(1 - s^{2^t}\right) \cdot \frac{u}{2}.$$

We summarize these results below.[7]

Proposition 4 *At any given period t, when the low-quality country is very large (i.e. for $s > \overline{s}(t)$), an international duopoly equilibrium does not exist and prices are set by the two firms behaving as monopolists, with $\left(p_1^M(t), p_2^M(t)\right) = (u/2, u)$.*

3.2.3 Convergence to full information duopoly

We have now fully elucidated the two types of market solutions that can arise at every period t. One can naturally ask: How does the market solution evolve? Is there a convergence towards the full information duopoly? Are there reversals? Namely, can we see a monopoly solution reversing into an international duopoly with frictions or vice versa?

[7] Note that this market solution recalls the *quasi monopoly* equilibrium arising after a collusive agreement breaks in d'Aspremont and Gabszewicz (1980).

In this section, we briefly show that the convergence of the dynamic setting is not path dependent. Regardless of the different paths the market solution can take, at some future period when information has spread and all consumers know the existence of both goods, the market necessarily metamorphoses into a *duopoly with full information* and no reversal from the international duopoly with full information to captive autarky or to a duopoly with informational frictions can take place.

We study convergence starting from the two different market solutions that can appear at period 1. Assume that the economy starts with a duopoly with frictions in period 1. Then, we can easily prove that in the domain of values of s for which the duopoly equilibrium with informational frictions exists, we obtain that duopoly equilibrium prices $p_1^*(t)$ and $p_2^*(t)$ are decreasing in t and converge over time to their counterparts p_1^* and p_2^* in a duopoly with vertically differentiated goods and fully informed agents, namely

$$\lim_{t\to\infty} p_1^*(t) = p_1^* \text{ and } \lim_{t\to\infty} p_2^*(t) = p_2^*.$$

Let us now consider the equilibrium corresponding to the solution described in Proposition 4. In view of this proposition, one may wonder whether the monopoly prices can persist over time. Actually, our model reveals that the knowledge transmission among the two countries ultimately plays a balancing role in the market. As time goes by and the mass of informed consumers in both countries increases, the number of consumers living in the large country who are only purchasing the domestic good progressively shrinks, thus driving the price gap (that would occur under duopoly) once again within a reasonable range. Hence, as information diffuses, it puts pressure on the monopoly firms, which at a certain period are 'obliged' to play a duopoly. From that period onwards, a duopoly market equilibrium arises. Indeed, we show that:

Proposition 5 *The non-existence of a duopoly equilibrium with informational frictions and the persistence of monopoly prices may only last for a finite number of periods, after which the market transforms into a duopoly.*

Proof. See Appendix A.4. ■

4 Properties of the duopoly with frictions

This section explores some peculiar features of the duopoly equilibrium in the presence of informational frictions. We focus on the impact of country size asymmetry on prices, demand, and profits over time.

4.1 The role of size asymmetry on prices and demands

Two properties of the profit-maximizing prices (17) are worth noting. First, both prices are non-monotone and convex functions of parameter s. Given u, the convexity of prices is due to the frequency

of meetings implied by the two countries' relative sizes. Prices are high for very small or very large s because this implies a small chance of meeting foreign consumers and ultimately getting informed. Thus, for high size asymmetry, firms do not incur a dramatic reduction of domestic demand. In contrast, when countries' populations are similar (which occurs for intermediate values of s), consumers meet frequently. Accordingly, equilibrium prices become substantially smaller already in the first period of social interaction. In Figure 2 below, we plot duopoly equilibrium prices $p_1^*(t)$ (in black) and $p_2^*(t)$ (in red) in periods 1 (continuous) and 2 (dashed). In period 1, when $s > \bar{s}(1) = 0.8135$, the duopoly equilibrium does not exist (by Proposition 4) and both firm charge monopoly prices (straight lines) that are insensitive to the size of countries. The same occurs at period two for $s > \bar{s}(2) = 0.9036$.

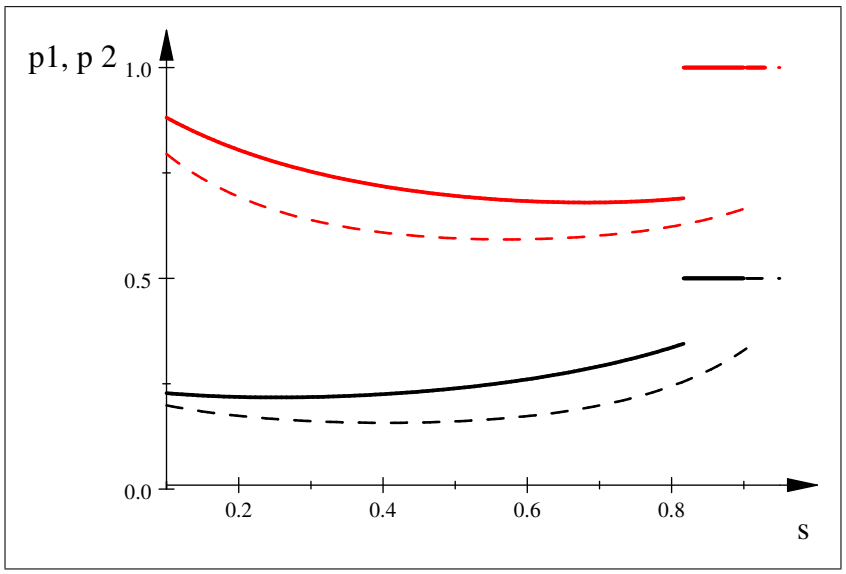

Figure 2: Duopoly equilibrium prices

In Figure 3 below, we plot the dynamics of the price gap $p_2^*(t) - p_1^*(t)$ over time for $t = 1, 2, ..10$ and $u = 1$.[8] The price gap can either fall (for $s = 0.2$ dotted line) or rise (for $s = 0.8$ continuous line) over time until reaching, in both cases, the price gap of the full information duopoly (green line), namely $p_2^* - p_1^* = 4u/7 - u/7 = 3u/7$. It is possible to identify a threshold in s, for every period, so that the price gap increases rather than decreases over time. At period 1, this threshold is $s(1) = 0.61604$, whereas at $t = 2$, is equal to $s(2) = 0.53989$, and so on. This threshold tends to 0.5 as t grows larger.

When countries have a similar size ($s = 0.5$, dashed line), the price gap remains relatively steady over time. Interestingly, the dynamics of the price gap $p_2^*(t) - p_1^*(t)$ over time are determined by which country (either the largest or the smallest) is producing which quality of good. If the small country produces the low-quality good, the price gap falls over time because many consumers become aware of

[8]Note that a change in the quality gap u only shifts the graphs up or down, without altering the dynamics of the represented variables.

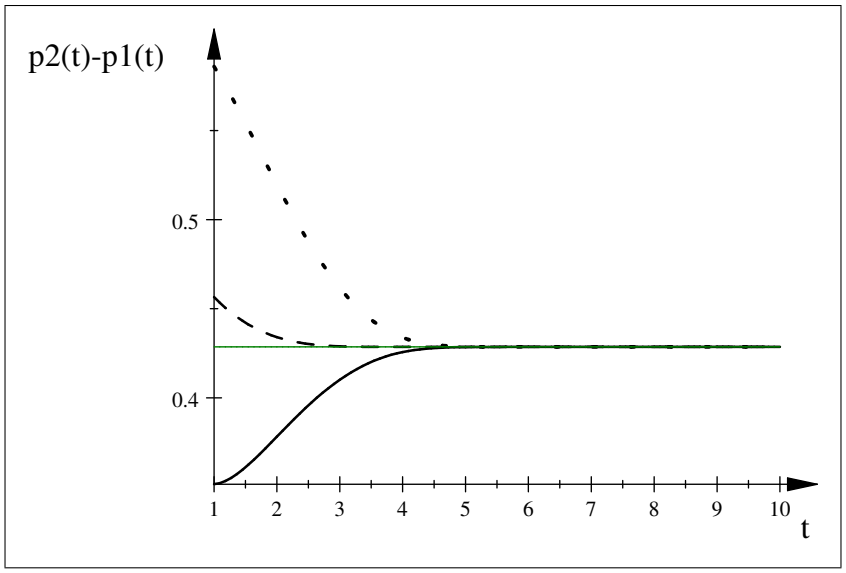

Figure 3: Duopoly equilibrium price gap $p_2(t) - p_1(t)$ over time.

the low-quality good, squeezing down the price of good 2 more than that of commodity 1. In contrast, when the larger country produces the low-quality variant, the price gap rises over time, simply because a large population quickly becomes acquainted with the high-quality good.

We turn now to the role played by country asymmetry in the equilibrium demand. Proposition 1 showed that the effect of information diffusion on equilibrium demand is not trivial. The firms' demands at a duopoly equilibrium with frictions are

$$\begin{aligned} D_1(t) &= \frac{\left(2-2\left(1-s\right)^{2^t}-s^{2^t}\right)\left(1-\left(1-s\right)^{2^t}-s^{2^{t+1}}\right)}{7+3\left(\left(1-s\right)^{2^{t+1}}+s^{2^{t+1}}\right)+8\left(1-s\right)^{2^t}s^{2^t}-10\left(\left(1-s\right)^{2^t}+s^{2^t}\right)} \\ D_2(t) &= \frac{1}{2}\frac{\left(4-4\left(1-s\right)^{2^t}-5s^{2^t}+s^{2^{t+1}}+3s^{2^t}\left(1-s\right)^{2^t}\right)\left(2-\left(1-s\right)^{2^t}-2s^{2^t}\right)}{7+3\left(\left(1-s\right)^{2^{t+1}}+s^{2^{t+1}}\right)+8s^{2^t}\left(1-s\right)^{2^t}-10\left(\left(1-s\right)^{2^t}+s^{2^t}\right)} \end{aligned} \tag{23}$$

Graphical inspection of the equilibrium demand for the high-quality variant $D_2(t)$ shows that demand increases over time, regardless of the relative size of the two countries. In contrast, the demand for the low-quality good $D_1(t)$ increases (resp. decreases) with time if the size of country one is small (resp. large). Indeed, we know from Proposition 1 that the duopoly prices of both goods invariably decline with information diffusion. However, while competition is always favourable to the high-quality firm, it boosts the demand for the low-quality product only if the mass of captive consumers in country one is small. In this case, the loss in the monopoly power of firm 1 is more than offset by the demand increase from informed consumers living in the larger foreign market of country 2. In Figures 4A and 4B, we

show the dynamics of firm 1 and firm 2 equilibrium demand, respectively, for $s = 0.8$ (dotted), $s = 0.5$ (dashed), and $s = 0.2$ (continuous) for $t = 1, 2, ..., 10$. The demands converge to the full information demand $D_1^* = 2/7$ and $D_2^* = 4/7$ (green lines). As shown in the figure below, the higher demand gain for firm 1 happens for $s = 0.2$ (continuous line), where the price gap increases over time, thus relaxing firm competition — which is beneficial for both firms. To summarize the equilibrium price gap $p_2^*(t) - p_1^*(t)$ decreases (resp. increases) with time when country one is relatively small (resp. large). In addition, the equilibrium demand for the high-quality good $D_2(t)$ always increases over time. By contrast, the demand for the low-quality good $D_1(t)$ increases (resp. decreases) with time when country one is relatively small (resp. large).

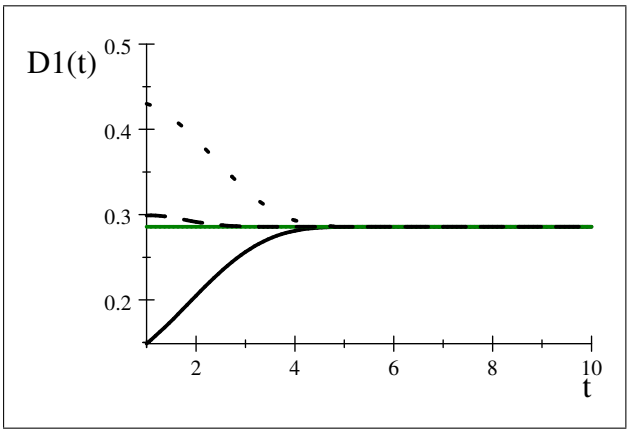

Figure 4A - Firm 1 duopoly equilibrium demand over time.

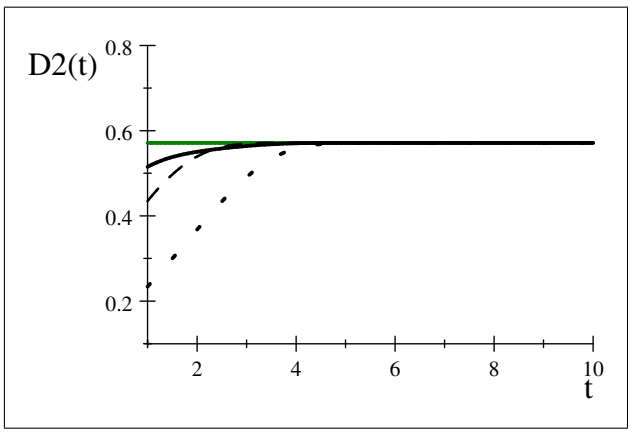

Figure 4B - Firm 2 duopoly equilibrium demand over time.

4.2 The role of social interactions in profitability

We turn finally to the profits of the two firms. Social interactions have an ambiguous overall effect on firm profits because exchanges determine two contrasting effects. On the one hand, encounters may enlarge markets served for each firm. This is the *market expansion effect*. Some consumers in country one will meet consumers of country two and start consuming good 2, enlarging the market share of good 2. However, some country two consumers will also meet country one consumers and some may start consuming good 1, increasing the demand for good 1. How much additional demand firms win depends a priori on the intensity of meetings (namely, on s).

On the other hand, there is a *competition effect* because firms face a foreign competitor due to encounters. Indeed, information diffusion eventually transforms the two initial monopolies into a duopoly market. Likewise, the intensity of competition depends on the intensity of interactions, which are ultimately determined by the size asymmetry between countries, given u.

For illustration, in Figures $5A$ and $5B$ we depict the profit of firm 1 and firm 2 for $t = 0, 1, 2, ..., 10$ and $u = 1$. When country one is large ($s = 0.8$, dotted line), its advantage in gaining new foreign consumers is more than offset by the loss in domestic consumers: the profit of firm 1 decreases. Precisely the opposite occurs for firm 2, whose profit increases. When country one is relatively small ($s = 0.2$, continues line), information diffusion is favourable to firm 1 and disadvantageous to firm 2 from period 1 onwards. Note that the profit of firm 2 (resp. firm 1) always increases (resp. decreases) with meetings when we move from period 0 to period 1, irrespective of s. This property is briefly proven in Proposition $A1$ in Appendix $A.5$.

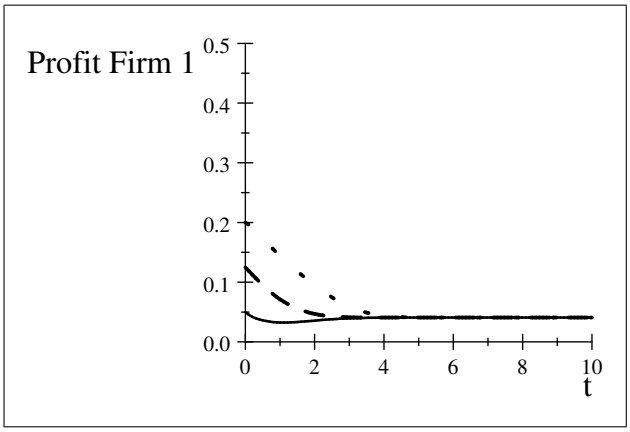

Figure 5A - Firm 1 profit over time

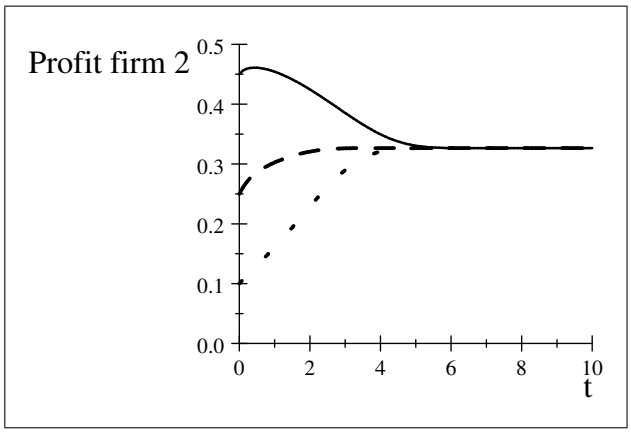

Figure 5B - Firm 2 profit over time

Summing up, under a duopoly with informational frictions, the equilibrium profits of firm 1 (resp. firm 2) increase (resp. decrease) from period $t = 1$ onwards if the relative population size of country one is small (resp. large). The opposite occurs if country one is relatively large. The profits of both firms invariably converge to full information profits (9) as $t \to \infty$.

To conclude, information diffusion and international competition can be good but also bad news for firms. These findings greatly depend on the share of captive consumers that each firm either gains or loses in view of social meetings. Our results suggest that when countries have very heterogeneous population sizes and produce different-quality products, unanimous agreements regarding agents' mobility are not easy to agree upon without specific side-payments.

5 Concluding Remarks

Opening markets to foreign consumers may involve a transition period when informational frictions exist such that local consumers learn about the existence of international varieties only through interacting with those with such knowledge. We build a novel multi-period market setting with vertical differentiation to explore how prices change along the sequence of equilibria generated by individual interactions

through time. In such an environment, social interactions upon opening markets internationally can act as a catalyst for the demand for foreign goods in local markets. We examine the implications of such social interactions for market competition between vertically differentiated goods, each produced by a local monopoly, and the formation of a full information duopoly environment.

We prove that while convergence to a full information duopoly does take place, it can occur following different paths depending on the size asymmetry between the countries. For symmetric countries, social interactions will be equally intense, alleviating informational frictions. This will generate fiercer competition between local monopolies, leading to faster convergence to the international duopoly, although prices take time to converge to their full information counterparts.

If, however, countries are asymmetric in terms of population size, the informational frictions may favour the monopoly located in the larger country due to a more significant frequency of social interactions promoting its variant. When size asymmetry is such that there is still an interior equilibrium, we show that demand for the high-quality product increases and demand for the low-quality product decreases over time until they reach their full information levels. When the country producing the high-quality (low-quality) product is larger, our model predicts that the price gap between varieties is higher (lower) than that under full information and decreases (increases) over time (to reach its full information level).

Nonetheless, when asymmetry in country size is sufficiently large—such that initially there is no interior equilibrium—we show that monopoly pricing can be sustained only for a finite number of periods. This result is informative for the trade literature in that it suggests that open trade is an essential instrument to discipline firms' market power, reducing prices. When countries have very asymmetric sizes, mobility and openness may take time to be beneficial to consumers by reducing consumers' prices.

One last remark is in order. In our setting, the transmission of information only occurs between consumers holding different information sets. However, the information could evidently be transmitted in alternative ways. For instance, advertising could reveal the existence of a product and its quality. Furthermore, firms could use prices in order to diffuse information about their goods. By lowering their prices, they could attract a more extensive set of new consumers. This would relate our paper to the recent literature in economics and management that studies 'market seeding' and information transmission through 'consumers/ambassadors' (Hinz et al., 2011, Groeger and Buttle, 2013). We leave this and other related issues to future research.

Appendix

A.1. Proof of Proposition 1

Since prices are strategic complements, using expressions (12) and the fact that $I = 1-U_1-U_2$, standard comparative statics (see, for instance, Vives 2000, chapter 6) at the interior duopoly equilibrium yield:

$$\begin{aligned}\text{sign}\frac{\partial p_1^*}{\partial U_1} &= \text{sign}\frac{\partial^2 \Pi_1}{\partial p_1 \partial U_1} \equiv \text{sign}\left(1 - \frac{p_2 - p_1}{u} + \frac{p_1}{u}\right) > 0,\\ \text{sign}\frac{\partial p_2^*}{\partial U_2} &= \text{sign}\frac{\partial^2 \Pi_2}{\partial p_2 \partial U_2} \equiv \text{sign}\left(\frac{p_2 - p_1}{u}\right) > 0,\end{aligned}$$

for any profile $p = (p_1, p_2)$ of interior prices (i.e. such that $1 > \theta_2 > \theta_1$). Again, plugging equality $I = 1 - U_1 - U_2$ into (12), and applying standard comparative statics technique yields

$$\begin{aligned}\text{sign}\frac{\partial p_1^*}{\partial U_2} &= \text{sign}\frac{\partial^2 \Pi_1}{\partial p_1 \partial U_2} = \text{sign}\left(\frac{4p_1^* - p_2^*}{u}\right)\\ &= \text{sign}\left[\frac{U_1\left(5 - 3U_2 - 5U_1\right)}{7 + 3\left(U_1^2 + U_2^2\right) + 8U_1U_2 - 10\left(U_1 + U_2\right)}\right] > 0,\end{aligned}$$

as the denominator of the fraction above is positive (given that p_1^* and p_2^* are positive at the interior equilibrium) and the numerator is positive as well, since $U_1\left(5 - 3U_2 - 5U_1\right) > U_1\left(5 - 5\left(U_1 + U_2\right)\right) > 0$ for $U_1 + U_2 < 1$. Similarly,

$$\begin{aligned}\text{sign}\frac{\partial p_2^*}{\partial U_1} &= \text{sign}\frac{\partial^2 \Pi_2}{\partial p_2 \partial U_1} = \text{sign}\left(\frac{2p_2^* - p_1^*}{u}\right) - 1\\ &= \text{sign}\left[\frac{U_2\left(3 - 2U_1 - 3U_2\right)}{7 + 3\left(U_1^2 + U_2^2\right) + 8U_1U_2 - 10\left(U_1 + U_2\right)}\right] > 0.\end{aligned}$$

Finally, using the fact that $I = 1 - U_1 - U_2$ both results above jointly imply that

$$\frac{\partial p_i^*}{\partial I} < 0 \text{ for } i = 1, 2.$$

A.2. Proof of Proposition 2

Looking at the demand for the two good (10), the effect of a rise in the mass of consumer uninformed about a given good i is

$$\frac{\partial D_1\left(p_1\left(U_2\right), p_2\left(U_2\right), U_2\right)}{\partial U_2} = -\frac{U_1}{u}\frac{\partial p_1}{\partial U_2} - \left(\frac{p_2 - p_1}{u} - \frac{p_1}{u}\right) + \frac{I}{u}\left(\frac{\partial p_2}{\partial U_2} - 2\frac{\partial p_1}{\partial U_2}\right) < 0$$

which, since $\partial p_i/\partial U_j > 0$ and $\partial p_i/\partial U_i > 0$ for $i, j = 1, 2$ and $j \neq i$, is surely satisfied for $2 \cdot \partial p_1/\partial U_2 > \partial p_2/\partial U_2$. Similarly,

$$\frac{\partial D_2\left(p_1\left(U_1\right), p_2\left(U_1\right), U_1\right)}{\partial U_1} = -\frac{U_2}{2u}\frac{\partial p_2}{\partial U_1} - \left(1 - \frac{p_2 - p_1}{u}\right) + \frac{I}{u}\left(-\frac{\partial p_2}{\partial U_1} + \frac{\partial p_1}{\partial U_1}\right) < 0$$

if $\partial p_2/\partial U_1 > \partial p_1/\partial U_1$. Finally,

$$\frac{\partial D_1\left(p_1\left(U_1\right), p_2\left(U_1\right), U_1\right)}{\partial U_1} = \left(1 - \frac{p_1}{u}\right) - \frac{U_1}{u}\frac{\partial p_1}{\partial U_1} - \left(\frac{p_2 - p_1}{u} - \frac{p_1}{u}\right) + \frac{I}{u}\left(\frac{\partial p_2}{\partial U_1} - 2 \cdot \frac{\partial p_1}{\partial U_1}\right)$$

and

$$\frac{\partial D_2\left(p_1\left(U_2\right), p_2\left(U_2\right), U_2\right)}{\partial U_2} = \left(1 - \frac{p_1}{2u}\right) - \frac{U_2}{2u}\frac{\partial p_2}{\partial U_2} - \left(1 - \frac{p_2 - p_1}{u}\right) + \frac{I}{u}\left(\frac{\partial p_1}{\partial U_2} - \frac{\partial p_2}{\partial U_2}\right),$$

which imply that the demand for each good can either increase of decrease with the mass of consumers uninformed about that good. As a result, the effect of an increase in the number of informed consumers on the demand for both firms is ambiguous and specifically depends on the relative size of the two countries.

A.3. Proof of Proposition 3

We know that if in every period $s(t) \in (0, \overline{s}(t))$, then there is room for both firms to trade their products in the international duopoly market at the maximizing prices (17). We need to prove that this pair of prices is the unique noncooperative Nash equilibrium of the multi-stage setting where firms are assumed to maximize their profits at every period. This can be proved by first noticing that no firm can strategically influence the information diffusion with its price and, hence, its profit-maximizing choice is to play its best response at every period t, namely

$$\begin{aligned} \widetilde{p}_1(p_2) &= \arg\max_{p_i}\ \Pi_1(p_1(t), p_2(t)) \equiv \frac{s^{2^t} \cdot u + p_2\left(1 - s^{2^t} - (1-s)^{2^t}\right)}{4 - 4\left(1-s\right)^{2^t} - 2s^{2^t}}, \\ \widetilde{p}_2(p_1) &= \arg\max_{p_2}\ \Pi_2(p_1(t), p_2(t)) \equiv \frac{(1 - s^{2^t}) \cdot u + p_1\left(1 - s^{2^t} - (1-s)^{2^t}\right)}{2 - \left(1-s\right)^{2^t} - 2s^{2^t}}. \end{aligned} \tag{24}$$

Since

$$\begin{aligned} \frac{\partial^2 \Pi_1}{\partial p_1^2} + \frac{\partial^2 \Pi_1}{\partial p_1 \partial p_2} &= -\frac{3 - 3\left(1-s\right)^{2^t} - s^{2^t}}{u} < 0, \\ &\text{and} \\ \frac{\partial^2 \Pi_2}{\partial p_2^2} + \frac{\partial^2 \Pi_2}{\partial p_2 \partial p_1} &= -\frac{1 - s^{2^t}}{u} < 0 \end{aligned}$$

both firms' best replies (24) are *contractions* (see, for instance, Vives, 2000, p.47), and thus, if $s < \overline{s}(t)$, this suffices for the pair of Nash equilibrium prices (17) to be the unique *noncooperative* Nash equilibrium at every period t. Notice that this equilibrium is also subgame perfect.

A.4. Proof of Proposition 5

Let us first consider the solution illustrated in Proposition 4. The mass of uninformed consumers served by the firms in these outcomes are $U_1(t) = s^{2^t}$ and $U_2(t) = (1-s)^{2^t}$. It is easy to see that $\lim_{t\to\infty} s^{2^t} = \lim_{t\to\infty} (1-s)^{2^t} = 0$. As time goes by, the uninformed market segment disappears and the market solution necessarily returns to a full information duopoly. Furthermore, the move towards the duopoly equilibrium occurs, by definition, when the value of the marginal consumer is

$$\theta_2^*(p_1(t), p_2(t)) - \theta_1^*(p_1(t), p_2(t)) = p_2^*(t) - 2p_1^*(t) > 0$$

which we know occurs when $U_1(t) < 2/3$, which translates from (19) into condition $s < \overline{s}(t) \equiv 2^{\frac{1}{2^t}} \cdot 3^{-\frac{1}{2^t}} \in (0,1)$. Therefore, whatever the existing asymmetry between the populations of the two countries, there always exists a finite period for which $U_1(t)$ becomes smaller than two thirds of the whole population of the two countries, i.e. $s^{2^t} < 2/3$. Since $s \in (0,1)$, s^{2^t} is decreasing in t and, therefore, for any $s \in (\overline{s}(t), 1)$ there exists a value of t such that $s^{2^t} < 2/3$. For instance, let us assume that country one is disproportionately large, with $s = 0.99 > \overline{s}(1)$. Therefore, at period 1 the price solution will not be that of a duopoly with informational frictions but rather a monopolistic one. However, since $U_1(t) = (0.99)^{2^t} = 2/3$ is solved for $t \simeq 5.3343$, it follows that $\overline{s}(6) > 0.99 > \overline{s}(5)$, hence implying that at period $t = 6$ the monopoly market will turn into a duopoly equilibrium with frictions. The same exercise can be replicated for any size of country 1 $s \in (0,1)$ and, therefore, although the number of periods needed to return to the duopoly increases more and more as s approaches 1, this number is always finite if $s \neq 1$. This concludes the proof.

A.5. Proposition A1 and its proof

Proposition A1. In the first period, market opening is always profitable for firm 2 and unprofitable for firm 1, *namely* $\Pi_2(1) - \Pi_2(0) > 0$ and $\Pi_1(1) - \Pi_1(0) < 0$.

Proof. Using (17) and (23), we easily obtain

$$\begin{aligned} \text{sign } [\Pi_1(1) - \Pi_1(0)] &= \text{sign } \left[\frac{1}{4}\frac{s(4-3s)(s+s^2+2)^2}{(7s^2-4-7s)^2} - \frac{1}{4}s\right] \\ &= \text{sign } \left[s\left(54s+3s^2-37\right)-52\right] < 0 \text{ for } s \in (0,1). \end{aligned}$$

Similarly,

$$\text{sign } [\Pi_2(1) - \Pi_2(0)] = \text{sign } \left[\frac{1}{2}\frac{s^2(1-s)(4s+1)(3s^2+16-15s)}{(-7s+7s^2-4)^2}\right]$$

which in turn is equal to the sign$[3s^2 + 16 - 15s] > 0$ for $s \in (0, 1)$. ■

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