Endogenous fertility and unemployment -Considering the effects of immigrants through school system

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-Considering the effects of immigrants through school system-[†]

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Abstract

We consider the effects of admitting immigrants with the burden of giving schooling to the native and the immigrant children. It may sound paradoxical; this model shows admitting immigrants may improve the welfare of the native when the necessary number of educators the immigrant children need is sufficiently high. What is more, admitting immigrants also improves the employment rate of the native.

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1. Introduction

IOM (2019) says that the number of international migrants is estimated to be almost 272 million globally, which is about 3.5% of the world's population. This average figure remains a very small percentage, but the distribution of international migrants is different from country to country and region to region.

IOM (2019) also shows that shares of international migrants in 2019 were highest in Oceania, North America, and Europe, respectively, 21 per cent, 16 per cent and 11 per cent of the total population and that Asia experience the most remarkable growth from 2000 to 2019, at 69 per cent (around 34 million people in absolute terms). These current situations implies that international migrants play the important role in the advanced countries mainly. In this paper, we focus on the effects of admitting international immigrants on the host country.

The model in this paper considers the effects of admitting immigrants on the native welfare paying attention to the schooling burden. In the literature, there are so many papers considering the effect of admitting immigrants on accumulation of the native human capital through education system or on the wages, the unemployment through the labor market.

Recently, Basten (2019), Fulanetto and Robstad (2019), and Esposito, Collignon, and Scicchitano (2020) and so on show that there are positive effects of admitting international immigrants on the employment rate of the native[1].

On the effects of admitting immigrants on education, for example, Speciale (2012) considers the impact of immigration on public education expenditures in EU-15 countries, which shows an increase in foreign population is found to have a small negative effect on public education expenditures. On the other hand, Mavisakalyan (2011) shows an increase in the share of immigrant population raises private school enrollment across countries by leading to a decrease in the share of public education spending with an instrument constructed from gravity model estimates. Hunt (2017) also examines the impact of immigration the high school completion of natives in the United States and shows a positive net effect.

Albornoz et al. (2018 takes it o consideration structure of education where student effort and talent interact with parental and teachers' investments as well as school system resources. Albornoz, et al. (2018) find that immigrant children perform better if their parents faced higher emigration costs. However, they do not consider the burden of

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[1] On the other hand, Borjas (2003), Edo (2017), and Dustmann et al (2017) and so on show there are negative effects of admitting immigrants on the employment rate of the native.

schooling like financing wages of educators as well as a decrease in the labor force caused by an additional increase in the number of educators caused by admitting immigrants. This paper makes it clear that why admitting immigrants improves not only the employment of the native by considering the schooling burden but also the welfare of the native when the burden of schooling is sufficiently heavy, which may sound paradoxical.

This paper is organized as follows. Section 2 provides the model and the final section concludes the paper.

2. The model

An overlapping generations model is applied, where individuals live for three periods: the childhood, the working, and the retirement period. Children need someone who educate and rear themselves, (educator). Individuals in the working period is endowed with one unit of labour which is inelastically supplied to consumption firm or education sector if they are employed. If unemployed, individuals receive an unemployment benefit. According to the expected income, individuals decide the amount of consumption, saving and the number of children. When old, he/she is retired. To investigate the effects of admitting immigrants through the education system on the social welfare for the native, we do not consider the pension system.

2.1 Immigration

In period zero, working immigrants, λ rate of the native working people, enter the country without capital[2].

$$\boldsymbol{N_t^{IM} = \lambda N_t^N} \tag{1}$$

where N_t^{IM}: the number of immigrants in period t and N_t^N: the number of the native working people in period t. The superscript IM is used to denote immigrants and N is used to denote native residents. Thus, the population of t-th generation including the immigrant becomes $N_t^T = N_t^N + N_t^{IM} = (1+\lambda)N_t^N$.

Immigrants in the working period need to spend a certain unit $\big(\rho \in (0,1)\big)$ of labour time to adjust themselves to the host country. The endowment of time for immigrant entering the host country becomes less than one unit. $\mathrm{P} \equiv (1-\rho)$ is the workable time.

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[2] Like Razin and Sadka (2000), Kemnitz (2003), Jinno (2011) and so on, we should consider the distribution in the skill level. Muysken et al. (2015) also shows medium-skill immigration decreases low-skilled unemployment under the flexible regime with general equilibrium framework considered. However, this paper does not pay attention to this point because we would like to consider the burden of schooling clearly and for simplicity of calculation.

2.2 Child-Rearing and Education

Native (immigrant) children need a certain number of educators to be reared and educated, h^N (h^{IM}). Children of immigrant also need a certain extra number of people who bring up and educate themselves more than the native need. The relationship between the numbers of educators per native and immigrant children becomes

$$\boldsymbol{h^N < h^{IM} = qh^N}, \tag{2}$$

where $q > 1$. The total number of educators the native and the immigrant children need in the period t, H_t, becomes

$$\boldsymbol{H_t = h^N n_t^N + h^{IM} n_t^{IM}}, \tag{3}$$

where n_t^N (n_t^{IM}) is the number of children per the native (the immigrant).

Thanks to additional educators, they can fully exert their abilities as well as the natives when being adult[3]. For simplicity, the children of immigrant become the native in the t+1 period. Thus, the transition of the population of generation including the immigrant becomes

$$\boldsymbol{N_{t+1}^N = n_t^N N_t^N + n_t^{IM} N_t^{IM}}. \tag{4}$$

2.3 Labor market

There are two labor market, consumption sector and education sector. For simplicity, the native as well as the immigrant workers in the consumption sector face the common unemployment rate,

$$\boldsymbol{u_t^E = \frac{Un_t^N}{E_t^N} = \frac{Un_t^{IM}}{E_t^{IM}}} \tag{5}$$

where u_t^E: the ratio of unemployed workers to employed workers, Un_t^X: the number of unemployed workers attributed to $X \in (N\ or\ IM)$, and E_t^X: the number of employed workers attributed to X. The superscript E is used to denote the variable is common among the native and the immigrant. The ratio of the employed to the population of t-th generation attributed to X is

$$\boldsymbol{\epsilon_t^X = \frac{E_t^X}{N_t^X}}. \tag{6}$$

Using equation (1), (5) and (6), the number of unemployed native (immigrant) workers becomes

$$\boldsymbol{Un_t^N = u_t^E \epsilon_t^N N_t^N}, \tag{7-a}$$

[3] This is a very strong assumption. PISA (2018) shows the scores of immigrant children are less than those of the natives, even though they receive some additional more support from educators than native children do in some host countries. Thus, when this strong assumption is relaxed, the results of this paper may change and need to be discussed.

$$\boldsymbol{Un_t^{IM} = u_t^E \epsilon_t^{IM} N_t^{IM}}$$
$$\boldsymbol{= u_t^E \epsilon_t^{IM} \lambda N_t^N}. \qquad (7-b)$$

In the education sector, only the native are employed because they have to bring up and educate the immigrant children as well as the native children to learn not only the host language, culture, and so on. Labor in the education sector do not face unemployment. The native can be employed or unemployed in the consumption sector and can be employed in the education sector. The constitution of the native labor becomes

$$\boldsymbol{N_t^N = E_t^N + Un_t^N + H_t^N} \qquad (8)$$

Using equation from (6) to (8), the number of the educator in period t becomes

$$\boldsymbol{H_t^N = (1-(1-u_t^E)\epsilon_t^N)N_t^N}. \qquad (9)$$

Substituting equations from (7) to (9) into equation (6), we get the employment rate in the consumption sector for the native (the immigrant) in period t

$$\boldsymbol{\epsilon_t^N = \frac{1-(n_t^N + \lambda q n_t^{IM})h^N}{1+u_t^E}}, \qquad (10-a)$$

$$\boldsymbol{\epsilon_t^{IM} = \frac{1}{1+u_t^E}}. \qquad (10-b)$$

Using equation (10), the relationship in the employment rate in the consumption sector between the native and the immigrant becomes

$$\boldsymbol{\epsilon_t^{IM} = \frac{1}{1-(n_t^N + \lambda q n_t^{IM})h^N}\epsilon_t^N}. \qquad (11)$$

On the other hand, while the native can fully supply one unit of time, the immigrant only supply $(1-\rho)$ unit of time. Considering the workable time for the native and the immigrant, labor forces supplied by the native and the immigrant in the consumption sector respectively become $L_t^N = E_t^N = \epsilon_t^N N_t^N$ and $L_t^{IM} = \mathrm{P}E_t^{IM} = \mathrm{P}\epsilon_t^{IM} N_t^{IM} = \lambda \mathrm{P}\epsilon_t^{IM} N_t^N$. Thus, the total labor force supplied by the native and the immigrant, $L_t^L = L_t^N + L_t^{IM}$, becomes

$$\boldsymbol{L_t^L = \left(1 + \frac{\lambda \mathrm{P}}{1-(n_t^N + \lambda q n_t^{IM})h^N}\right)\epsilon_t^N N_t^N}, \qquad (12)$$

where unemployment workers are considered. If all of workers are employed, the total labor force supplied by the native and the immigrant becomes

$$\boldsymbol{L_t^F = (1+u_t^E)\left(1 + \frac{\lambda \mathrm{P}}{1-(n_t^N + \lambda q n_t^{IM})h^N}\right)\epsilon_t^N N_t^N}. \qquad (13)$$

2.4 Firms in the consumption sector

The production function is $Y_t = (K_t^T)^\delta (L_t^T)^{(1-\delta)}$, where $\delta \in (0,1)$. Y_t, K_t, and L_t respectively denote the output produced, capital, and labor. The variables attached by the superscript of T are total number of variables.

We assume that the wage in the consumption sector at period t, w_t^L, is set as

$$\boldsymbol{w_t^L = \mu w_t^F, \mu > 1} \tag{14}$$

where μ is the mark-up rate, which bring about unemployment workers, and w_t^F is the wage rate when all of workers in the consumption sector are fully employed[4].

All of capital at the end of each period are assumed to be depreciated in one period. According to profit maximization, we have

$$\boldsymbol{w_t^L = (1-\delta)(k_t^N)^{\delta}\left(\frac{L_t^T}{N_t^N}\right)^{-\delta}}, \tag{15-a}$$

$$\boldsymbol{1 + r_t^L = \delta(k_t^N)^{\delta-1}\left(\frac{L_t^T}{N_t^N}\right)^{1-\delta}} \tag{15-b}$$

where $k_t^N \equiv \frac{K_t^T}{N_t^N}$, when there are unemployed workers in consumption sector. While the native supply whole one unit of time, the immigrant can only P unit of time. Thus, the income of the immigrant becomes $\mathrm{P}w_t^m$.

On the other hand, we also have solutions according to profit maximization when all of workers in the consumption sector are employed:

$$\boldsymbol{w_t^F = (1-\delta)(k_t^N)^{\delta}\left(\frac{L_t^F}{N_t^T}\right)^{-\delta}}, \tag{16-a}$$

$$\boldsymbol{1 + r_t^F = \delta(k_t^N)^{\delta-1}\left(\frac{L_t^F}{N_t^T}\right)^{1-\delta}}. \tag{16-a}$$

Using equation from (14) to (16), the endogenously calculated common unemployment rate in consumption sector becomes constant in this model:

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[4] To effectively, we should take it into consideration the theoretical foundation of why the wage rate is set over that under full employment. In this paper, we consider it important to pay the relation between the effects of admitting immigrants under considering the school burdens and social welfare. Thus, the model is simplified as possible as we can that the theoretical foundation is not considered in this paper, which is also one of the serious problems left for us to be solved.

$$\mathbf{1}+\boldsymbol{u}^{\boldsymbol{E}^*}=\boldsymbol{\mu}^{\frac{\mathbf{1}}{\boldsymbol{\delta}}} \tag{17}$$

According to equation (17), an increase in the mark-up rate raises the unemployment rate. We also get the endogenously derived employment rate of the immigrant

$$\boldsymbol{\epsilon}_t^{\boldsymbol{IM}^*}=\frac{\mathbf{1}}{\mathbf{1}+\boldsymbol{u}^{\boldsymbol{E}^*}}=\boldsymbol{\mu}^{-\frac{\mathbf{1}}{\boldsymbol{\delta}}}. \tag{$10-b^*$}$$

2.5 The government

The government endogenously imposes common income tax, τ_t^E, on the native and the immigrant workers and the unemployment to finance the unemployment benefits, b_t^X, which is set to be

$$\boldsymbol{b}_t^{\boldsymbol{N}}=\boldsymbol{\varphi}\boldsymbol{w}_t^{\boldsymbol{F}}\ , \tag{$18-a$}$$

$$\boldsymbol{b}_t^{\boldsymbol{IM}}=\boldsymbol{\varphi}\mathbf{P}\boldsymbol{w}_t^{\boldsymbol{F}}\ , \tag{19 – b}$$

where φ is the replacement rate $(\varphi\in(0,1))$[5]. Note that the unemployment benefits between the native and the immigrant, which implies that the unemployment benefits are proportional to the taxes they have to pay for the unemployment benefits. The budget constraint on the income tax becomes

$$\boldsymbol{\tau}_t^{\boldsymbol{E}}\left(\boldsymbol{w}_t^{\boldsymbol{L}}\boldsymbol{E}_t^{\boldsymbol{N}}+\mathbf{P}\boldsymbol{w}_t^{\boldsymbol{L}}\boldsymbol{E}_t^{\boldsymbol{IM}}+\boldsymbol{w}_t^{\boldsymbol{edu}}\boldsymbol{H}_t^{\boldsymbol{N}}+\boldsymbol{b}_t^{\boldsymbol{N}}\boldsymbol{Un}_t^{\boldsymbol{N}}+\boldsymbol{b}_t^{\boldsymbol{IM}}\boldsymbol{Un}_t^{\boldsymbol{IM}}\right)$$
$$=\boldsymbol{b}_t^{\boldsymbol{N}}\boldsymbol{Un}_t^{\boldsymbol{N}}+\boldsymbol{b}_t^{\boldsymbol{IM}}\boldsymbol{Un}_t^{\boldsymbol{IM}} \tag{19}$$

where w_t^{edu} is the wage rate of educators.

Workers in the consumption sector face the expected income: $\left(\frac{1}{1+u_t^E}\right)w_t^L+\left(\frac{u_t^E}{1+u_t^E}\right)b_t^N$ for the native and $\left(\frac{1}{1+u_t^E}\right)\mathrm{P}w_t^L+\left(\frac{u_t^E}{1+u_t^E}\right)b_t^{IM}$ for the immigrant. Using equation (18), the expected incomes become

$$\boldsymbol{In}_t^{\boldsymbol{N_W}}=\boldsymbol{\eta}^*\boldsymbol{w}_t^{\boldsymbol{L}}, \tag{$20-a$}$$

$$\boldsymbol{In}_t^{\boldsymbol{IM}}=\boldsymbol{\eta}^*\mathbf{P}\boldsymbol{w}_t^{\boldsymbol{L}}, \tag{$20-b$}$$

where $\eta^*\equiv\left(\frac{1}{1+u^{E^*}}\right)\left(1+\frac{\varphi}{\mu}u^{E^*}\right)$. Some calculation leads to $\eta^*<1$.

The government also collects education expense per child, z_t, to finance the wage of educator, w_t^{edu}, which is the same as the expected income in consumption sector:

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[5] It may be strange that the government imposes income tax on unemployment benefit. However, this assumption is for calculation simplicity. In a sense, the income tax on the unemployment benefit just implies a decrease in the benefit level.

$$\boldsymbol{w_t^{edu} = \eta^* w_t^L}. \tag{21}$$

The budget constraint for the education expenses becomes

$$\boldsymbol{z_t(n_t^N N_t^N + n_t^{IM} N_t^{IM}) = w_t^{edu} H_t}. \tag{22}$$

The deterministic wage of educator becomes less than that of an employed worker in consumption sector because of $\eta^* < 1$. The incomes of educators, $In_t^{N_H}$, are equal the expected incomes in consumption sector: $In_t^{N_H} = In_t^{N_W} = In_t^N$ where In_t^N is the native common income. We also get the relation: $In_t^{IM} = \mathrm{P} In_t^N$.

2.6 Individuals

Individuals live in three periods (child, working and old) and obtain the utility from consumption in the last two periods and the number of children. In the child period, they spend whole time on schooling but do not consume no goods, which implies behavior in the child period do not have impact on their utilities. In the working period, they work when employed in consumption sector or education sector or be unemployed in consumption sector. Thus, they obtain wages or unemployment benefits. In the old period, they consume the savings.

The utility function is

$$\boldsymbol{U_t^X = \alpha \ln c_t^X + \beta \ln d_{t+1}^X + \gamma \ln n_t^X} \tag{23}$$

where $\alpha + \beta + \gamma = 1$ and $X = N\ or\ IM$. The expected budget constraints for native or immigrant individuals are

$$\boldsymbol{c_t^X + z_t^E n_t^X + s_t^X = (1 - \tau_t^E) In_t^X} \tag{24 - a}$$

$$\boldsymbol{d_{t+1}^X = (1 + r_{t+1}) s_t^X}. \tag{24 - b}$$

Individuals chose the optimal amount of consumption and savings and number of children in the working period to maximize the whole utility. Some calculations lead to the optimal solutions:

$$\boldsymbol{s_t^X = \beta\,(1 - \tau_t^E) In_t^X} \tag{25 - a}$$

$$\boldsymbol{n_t^X = \frac{\gamma}{z_t}(1 - \tau_t^E)\, In_t^X} \tag{25 - b}$$

$$\boldsymbol{c_t^X = \alpha\,(1 - \tau_t^E) In_t^X} \tag{25 - c}$$

$$\boldsymbol{d_{t+1}^X = (1 + r_{t+1})\beta(1 - \tau_t^E)\, In_t^X} \tag{25 - d}$$

Thus, we have following relations in the optimal solutions for the native and the immigrant: $\boldsymbol{s_t^{IM} = \mathrm{P} s_t^N}$, $\boldsymbol{n_t^{IM} = \mathrm{P} n_t^N}$, $\boldsymbol{c_t^{IM} = \mathrm{P} c_t^N}$ and $\boldsymbol{d_t^{IM} = \mathrm{P} d_t^N}$.

2.7 Welfare for the native

By substituting equation (6), (7), (9), (18) and (21) into equation (19), we have the following relation between the disposable income rate and the employment rate:

$$\mathbf{1}-\boldsymbol{\tau}_t^E=\frac{(\mathbf{1}+\boldsymbol{\lambda}\mathbf{P})\boldsymbol{\eta}^*-\left(\boldsymbol{\epsilon}_t^N+\boldsymbol{\lambda}\mathbf{P}\boldsymbol{\epsilon}_t^{IM^*}\right)\frac{\boldsymbol{\varphi}}{\boldsymbol{\mu}}\boldsymbol{u}^{E^*}}{(\mathbf{1}-\boldsymbol{\lambda}\mathbf{P})\boldsymbol{\eta}^*} \tag{26}$$

By substituting equation (2), (3), (7), (21) and the relation of $n_t^{IM}=\mathrm{P}n_t^N$ into equation (22), we have the endogenously derived education expense:

$$\boldsymbol{z}_t^{E^*}=\left(\frac{\mathbf{1}+\boldsymbol{q}\boldsymbol{\lambda}\mathbf{P}}{\mathbf{1}+\boldsymbol{\lambda}\mathbf{P}}\right)\boldsymbol{h}^N\boldsymbol{\eta}^*\boldsymbol{w}_t^L \tag{27}$$

We can also get the endogenously derived employment rate:

$$\boldsymbol{\epsilon}_t^{N^*}=\frac{(\mathbf{1}-\boldsymbol{\gamma})\boldsymbol{\eta}^*-\boldsymbol{\gamma}\boldsymbol{\lambda}\mathbf{P}\boldsymbol{\epsilon}_t^{IM^*}}{\mathbf{1}+(\mathbf{1}-\boldsymbol{\gamma})\frac{\boldsymbol{\varphi}}{\boldsymbol{\mu}}\boldsymbol{u}^{E^*}} \tag{28}$$

by substituting the equations from (25) to (27) into equation (10). We finally have the endogenously derived common disposable income rate and the optimal number of children for the native:

$$\mathbf{1}-\boldsymbol{\tau}_t^{E^*}=\frac{\mathbf{1}}{\mathbf{1}+(\mathbf{1}-\boldsymbol{\gamma})\frac{\boldsymbol{\varphi}}{\boldsymbol{\mu}}\boldsymbol{u}^{E^*}}, \tag{29}$$

$$\boldsymbol{n}_t^{N^*}=\frac{\boldsymbol{\gamma}(\mathbf{1}+\boldsymbol{\lambda}\mathbf{P})}{(\mathbf{1}+\boldsymbol{q}\boldsymbol{\lambda}\mathbf{P})\left(\mathbf{1}+(\mathbf{1}-\boldsymbol{\gamma})\frac{\boldsymbol{\varphi}}{\boldsymbol{\mu}}\boldsymbol{u}^{E^*}\right)} \tag{30}$$

by substituting equation (28) into equations (25) and (26).

Some calculation leads to the endogenously derived optimal solutions by using the equation (20), (25), (29) and (30). We can also get the endogenously derived wage rate and the interest rate in period t when there are some unemployed workers:

$$\boldsymbol{w}_t^L=(\mathbf{1}-\boldsymbol{\delta})(\boldsymbol{k}_t^N)^{\boldsymbol{\delta}}\left(\frac{(\mathbf{1}-\boldsymbol{\gamma})(\mathbf{1}+\boldsymbol{\lambda}\mathbf{P})\boldsymbol{\eta}^*}{\left(\mathbf{1}+(\mathbf{1}-\boldsymbol{\gamma})\frac{\boldsymbol{\varphi}}{\boldsymbol{\mu}}\boldsymbol{u}^{E^*}\right)}\right)^{-\boldsymbol{\delta}}, \tag{31}$$

$$\mathbf{1}+\boldsymbol{r}_t^L=\boldsymbol{\delta}(\boldsymbol{k}_t^N)^{\boldsymbol{\delta}-\mathbf{1}}\left(\frac{(\mathbf{1}-\boldsymbol{\gamma})(\mathbf{1}+\boldsymbol{\lambda}\mathbf{P})\boldsymbol{\eta}^*}{\left(\mathbf{1}+(\mathbf{1}-\boldsymbol{\gamma})\frac{\boldsymbol{\varphi}}{\boldsymbol{\mu}}\boldsymbol{u}^{E^*}\right)}\right)^{\mathbf{1}-\boldsymbol{\delta}}. \tag{32}$$

In period t+1, the number of t+1-th generation native is $N_{t+1}=n_t^{N^*}N_t^N+n_t^{IM^*}N_t^{IM}$[6]. The relation in the number of children between the native and the immigrant is $n_t^{IM^*}=\mathrm{P}n_t^{N^*}$. Thus, the number of t+1-th generation is $N_{t+1}=(1+\lambda\mathrm{P})n_t^{N^*}N_t^N$. We also have

[6] In this paper, we assume that the children of the immigrants can exert their productivity as well as the native children and they are assimilated in the host country owing to the additional educators.

the amount of capital in the t+1 period: $K_{t+1} = (1 + \lambda \mathrm{P}) s_t^{N^*} N_t^N$. The capital-labor ratio in period t+1 is $k_{t+1}^T = \frac{s_t^{N^*}}{n_t^{N^*}}$ which does not depend on the whether the immigrant are admitted or not. Thus, there is not any influence caused by admitting immigrants in period t after period t+1.

We define the social welfare function of the native as

$$\boldsymbol{W_t^N = \sigma^{t-1} \sum_{t=0}^{\infty} U_{t-1}^N} \tag{33}$$

where $\sigma \in (0, 1)$, the discount rate.

By substituting the endogenously derived optimal solutions and equations (31) and (32) into the (23), we get the indirect utility function for the native:

$$\boldsymbol{U_t^N = \ln\left(\left(1 - \tau_t^{E^*}\right) In_t^N\right) + \beta \ln\left(1 + r_{t+1}^L\right) - \gamma \ln\left(z_t^{E^*}\right) + D} \tag{34}$$

where $D = \alpha \ln(\boldsymbol{\alpha}) + \beta \ln(\boldsymbol{\beta}) + \boldsymbol{\gamma}\, \alpha \ln(\boldsymbol{\gamma})$.

By differentiating equation (33) with respect to λ and evaluating it with $\lambda = 0$, we get

$$\left.\frac{dW_t^N}{d\lambda}\right|_{\lambda=0} = -\frac{\beta(1-\delta)\mathrm{P}}{\sigma} - (1+\gamma)(1+\delta \mathrm{P}) + \gamma(q-1)\mathrm{P}. \tag{35}$$

Thus, we get a proposition:

Proposition

Admitting immigrants improve the native welfare when the necessary number of educators is sufficiently high.

Proof:

$\left.\frac{dW_t^N}{d\lambda}\right|_{\lambda=0} \gtrless 0$ if $(q-1) \gtrless \frac{\beta(1-\delta)}{\sigma\gamma} + \frac{(1+\gamma)(1+\delta \mathrm{P})}{\gamma \mathrm{P}}$. Q.E.D.

We consider a numerical example by taking the following exogenous parameters(see Table 1). The value of $\frac{\beta(1-\delta)}{\sigma\gamma} + \frac{(1+\gamma)(1+\delta \mathrm{P})}{\gamma \mathrm{P}}$ becomes 3.48. Thus, if q(the difference in the necessary number of educators between the native and the immigrant) is over 4.48, the social welfare of the native improves by admitting immigrants.

Table 1 Exogenous parameters

σ	α	β	γ	δ	ρ

0.99	0.3	0.1	0.6	0.25	0.05

We also have following two lemmas.

Lemma 1

The unemployment rate of the immigrant becomes lower than that of the native.

Proof:

We have $\frac{Un_t^{N^*}}{N_t^N} = u_t^{E^*}\epsilon_t^{N^*}$, $\frac{Un_t^{IM^*}}{N_t^{IM}} = u_t^{E^*}\epsilon_t^{IM^*}$ and equation (11). Because $\frac{\mathbf{1}}{\mathbf{1}-(\boldsymbol{n_t^N}+\boldsymbol{\lambda q n_t^{IM}})\boldsymbol{h^N}}$ is less than 1, $\frac{Un_t^{N^*}}{N_t^N} > \frac{Un_t^{IM^*}}{N_t^{IM}}$. Q.E.D.

Lemma 2

The employment rate in the consumption sector become lower by admitting immigrants.

Proof:

By differentiating equation (28) with respect to λ and evaluating it with $\lambda = 0$, we get $\epsilon_t^{N^*}\Big|_{\lambda=0} = \frac{(1-\gamma)\eta_t^*}{1+(1-\gamma)\frac{\varphi}{\mu}u_t^{E^*}}$ which is higher than $\epsilon_t^{N^*} = \frac{(1-\gamma)\eta_t^*-\left(\frac{1}{1+u_t^{E^*}}\right)}{1+(1-\gamma)\frac{\varphi}{\mu}u_t^{E^*}}$. Thus, the number of the employed becomes higher if immigrants are admitted. Q.E.D.

3. Concluding Remarks

We consider the effects of admitting immigrants with the burden of giving schooling to the native and the immigrant children. It may sound paradoxical; this model shows admitting immigrants may improve the welfare of the native when the necessary number of educators the immigrant children need is sufficiently high. What is more, admitting immigrants also improves the employment rate of the native.

However, in this model, the pension system and accumulation of human capital are not considered. So many advanced countries face the serious aging and shrinking population problem where the burden of working generation is very heavy because of the pension system. It remains to be seen whether admitting immigrant may improve the burden of the working generation with the pension system considered or let the native increase accumulation of human capital.

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