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Calculating the net benefit of admitting
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Abstract

This paper calculates the net benefit of admitting immigrants under the defined-return-ratio pay-as-you-go pension system, considering the assimilation costs the next generation whose parents are from abroad must pay as additional costs. As a result, no matter how many immigrants come, the host country can get the positive net benefits through the defined-return-ratio pension system. This result is quietly different from those in Jinno (2011, 2013).

Keywords: Immigrants, the defined-return-ratio, the net benefits.

1 Introduction

We develop an overlapping-generations model based on Razin and Sadka (1999) to consider the effects of the assimilation costs like Jinno (2011, 2013). The assimilation costs are the costs the first child generation of the immigrants must pay to assimilate themselves into the host country, which in a sense, implies a loss of the productivity per capita of the next generation. The different point in this paper from Jinno (2011, 2013) is the finance method of pension where the rate of return from pension contribution is already defined a certain level and constant; we call this system as the RDC system in this paper (The [R]ate of return from the pension contribution is [D]efined [C]onstant).

Basically the future generation suffers from the higher burden when the fertility rate is low under the RDC system because the fewer future generation must finance the constant rate of returns from pension contributions. Thus, on the contrary, accepting immigrants under the RDC system generally could decrease the burden of the working generation because it means an increase in the number of working people who share the burden of pension finance and thus, the pension contribution per capita would be lower than without any immigrants. A decrease in the pension contribution would also means a decrease in the future burden of the pension finance and the size of pension itself, which also implies an improvement in the future welfare.

However, in this paper, the assimilation costs are also considered, thus, accepting immigrant under the RDC system does not directly imply an improvement of the welfare of the future native generations because the assimilation costs decreases the productivity of the next generation per capita. We would like to check whether accepting immigrants could increase the welfare of the native residents or not considering the assimilation costs under the RDC system.

In Jinno (2013), a defined-benefit pension system (DB system) and a defined contribution pension system (DC system) are compared considering the assimilation costs. One of some results is that the net benefits for native residents caused by permitting a small (large) number of immigrants under the DB system becomes higher (lower) than that under the DC system in certain practical situations. What is more, according to Jinno (2011, 2013), under both of the DB system and the DC system the net benefit for native residents from accepting immigrants does not become positive. However, this paper shows that even if the assimilation costs are considered, the net benefits for native residents under the RDC system always becomes positive without any conditions¹.

2 The Model

An overlapping-generation model based on Razin and Sadka (1999) is used considering the assimilation cost. Individuals live for two periods, namely the working period and the retired period. First of all in the working period, they decide

¹There are so many papers considering the effects of admitting immigrants; Lagos and Lacomba (2005) analyze when the optimal period of being retired should be, Kemnitz(2003, 2006, 2008) does how the effects of admitting immigrants are when the unemployment is considered, Casarico and Devillanova (2003) does how the optimal choices on education should be, and the effects of admitting immigrants in various sectors are considered Shimasawa and Oguro (2011) and Saito, Kato, and Takeda(2017). These papers do not consider the assimilation costs which have much influences on the future generations.

whether to be a skilled worker or an unskilled worker. Innate higher (lower) ability individuals study less (more) time and thus only innate higher ability individuals can be skilled workers whose productivity are higher than unskilled workers. The rest of them with lower abilities become unskilled workers without any loss of working time. Compare with their peers, immigrants offspring require more time to become skilled workers because of the language and culture barriers they must overcome. We call this extra burden immigrants offsprings must pay to be skilled workers the assimilation costs. During the retired period, they consume all the returns from their savings and pension benefits.

In this paper, the effects of accepting immigrants for the net benefits for the native residents are analyzed under the RDC pension system where the rate of pension benefits is constant. Thus, the pension contribution may be decided endogenously to balance the pension budget. Here, immigration occurs only in the first period. The subscript n is used to denote native residents and m to denote immigrants.

2.1 Immigration

In period zero, the country receives young unskilled immigrants without any capital. Although they can not become skilled workers, their offspring can with more extra burden than the native residents. We assume that the innate abilities of the offspring are distributed similarly and that the fertility rate is identical in both groups in line with the literature (Razin and Sadka 1999; Krieger 2003).

2.1.1 Individual ability and behavior

Individuals can become skilled workers by investing e units of time in school, which ranges from 0 to 1, represents the innate ability to acquire skills. They will work for the remaining time, denoted by $1 - e$ when they become skilled workers. This assumption implies that a lower e individual is more capable and needs less time to acquire skills². For simplicity, the difference in labor output between skilled and unskilled workers is the productivity of labor supply: while the productivity of skilled worker is one unit for each unit of working time, an unskilled worker provides only $q < 1$ units of effective labor for each unit of working time. Individuals can be unskilled workers without any loss of time, which implies the working time of unskilled workers becomes one while it depends on the innate ability of individuals who become skilled workers.

When becoming skilled workers, the offspring of immigrants have to spend additional time learning the language, culture, and so on. This additional time is the cost of assimilation, φ . This cost is the same value for all immigrant offspring, but the continuity of the assimilation cost is assumed for only one period, which implies that the second-and-further offspring of immigrants can be skilled workers without any assimilation costs³.

²The innate ability, e , ranges from 0 to 1. The cumulative distribution function of ability e is the same between the native and the immigrant individuals and denoted by $G(\cdot)$, where $G(e_x)$, ($x = n$ or m) refers to the number of individuals with an innate ability parameter that is below or equal to e_x . For simplicity, during the initial analysis of the economy, the number of native individuals born in period zero is normalized to one, that is $G(1) = 1$.

³OECD (2016) "PISA 2015 Results" shows that the difference in the performance is smaller between second-generation immigrant students and non-immigrant students than that between first-generation immigrant students and non-immigrant students. Thus, we assume the burden

In the first period, each individual decides whether to acquire skills to become a skilled worker, after which he or she then works; bears $1 + g$ offspring (the same for all individuals); consumes a single all-purpose good; and saves for his or her retirement. In the second period (i.e., during retirement), each individual consumes his or her pension and all the returns from savings that were accumulated in the previous period.

We would like to compare the results in this paper with those in Jinno (2011, 2013). Thus, we also assume a small country with free access to international capital markets, which implies the rate of return from saving (r) and the wage rate (w) are also fixed independent of the level of migration.

The income of a native-born i -th smart individual who decides to acquire skills through schooling is represented by $(1 - e_n^i) w (1 - \theta(M))$, where $\theta(M)$ represents the endogenous pension contribution (tax) rate which depends on the number of immigrants. Since the immigrant offspring requires additional time to develop his or her skills (i.e., the assimilation cost), the income of those who work as skilled workers is represented by $(1 - e_m^i - \varphi) w (1 - \theta(M))$. The income of unskilled workers for the natives and the immigrants is represent as $qw(1 - \theta(M))$.

Individuals choose to become skilled as long as the income of skilled workers is higher than that of unskilled workers. Further, there exists a cutoff levels for innate ability e_x^* , where both incomes are equal, which are,

$$(1 - e_n^*) w (1 - \theta(M)) = qw(1 - \theta(M)) \quad (1)$$

$$(1 - e_m^* - \varphi) w (1 - \theta(M)) = qw(1 - \theta(M)) \quad (2)$$

where $\theta(M)$ represents the endogenous pension contribution rate when M immigrants are admitted.

Individuals whose innate ability e_x is lower than e_x^* will decide to be skilled workers, whereas those whose innate ability exceeds the cutoff levels will decide to be unskilled workers. Based on equations (1) and (2), we have

$$e_n^* = 1 - q \quad (3)$$

$$e_m^* = 1 - q - \varphi \quad (4)$$

Since the productivities of unskilled native and immigrant workers are assumed to be identical, based on equations (3) and (4), the relationship between the cutoff levels for innate ability for native and immigrant workers is written as $e_m^* = e_n^* - \varphi$, which shows that the cutoff level for innate ability for immigrant workers, e_m^* that is, the level at which they decide whether to become a skilled or an unskilled worker, is less than that for native individuals. Hence, the number of skilled immigrant workers is found to be less than that of skilled native workers. If $\varphi \geq 1 - q$ holds, all immigrant offspring will become unskilled workers. Thus, we assume that $\varphi < 1 - q$.

2.2 Maximization Problems

The preference to the consumption is the same between the natives and the immigrants. They get utility from the consumptions in the working period (c) and of assimilating themselves into the host country lasts for a period.

in the retired period (d). The utility function is assumed as Cobb-Douglas utility function like $u(c, d) = \alpha \log c + \beta \log d$, where α (β) is the preference to the consumption in the working period (in the retired period). The sum of the returns from savings and pension benefits is consumed in the retirement period. There is no bequest for simplification. Individuals face the intertemporal budget constraint

$$c(e_x^i) + \frac{d(e_x^i)}{1+r} = (1 - \theta(M)) W(e_x^i) + \frac{b(e_x^i)}{1+r} \equiv I(e_x^i) \quad (5)$$

where $W(e_x^i)$ represents the pre-pension-contribution-paid incomes, $b(e_x^i)$ the pension benefits, and $I(e_x^i)$ the present valued and after-pension-contribution-paid total income. The pre-pension-contribution-paid incomes for the natives become

$$W(e_n^i) = \begin{cases} (1 - e_n^i) w & e_n^i < e_n^* \\ qw & e_n^i \geq e_n^* \end{cases}$$

. On the contrary the pre-pension-contribution-paid incomes for the immigrants become

$$W(e_m^i) = \begin{cases} (1 - e_m^i - \varphi) w & e_m^i < e_m^* \\ qw & e_m^i \geq e_m^* \end{cases}$$

The pre-pension-contribution-paid incomes for the native and immigrant unskilled worker become equal.

Maximizing utility with respect to consumptions and savings subject to the intertemporal budget constrain yields the optimal consumptions and savings:

$$s^*(e_x^i) = \frac{\beta}{(\alpha + \beta)} (1 - \theta_t) W(e_x^i) - \frac{\alpha}{(\alpha + \beta)} \frac{b(e_x^i)}{(1+r)} \quad (6)$$

$$c^*(e_x^i) = \frac{\alpha}{(\alpha + \beta)} I(e_x^i; t) \quad (7)$$

$$d^*(e_x^i) = \frac{\beta}{(\alpha + \beta)} (1+r) I(e_x^i; t) \quad (8)$$

2.3 Labor Supply

In 0 period, there are N unit of the natives and M unit of the unskilled immigrants. The labor supply of skilled native resident workers with the i -th ability is denoted by $(1 - e_n^i)$, while that of unskilled native resident and immigrant workers is denoted by q . The aggregate supply of effective labor in period zero is given by

$$L_0 = \left(\int_0^{e_n^*} (1 - e) dG \right) N + q [1 - G(e_n^*)] N + qM. \quad (9)$$

The first term on the right-hand side of the equation refers to the effective labor supply of skilled native resident workers, while the second term refers to the effective labor supply of unskilled native resident workers. Since it is assumed

that immigrants participate in production as unskilled workers in period zero, the effective labor supply of immigrant workers is represented as the third term on the right-hand side of the equation. Further, we define $E_n^* \equiv \int_0^{e_n^*} (1 - e) dG + q[1 - G(e)]$ and rewrite equation (9) using this definition as

$$L_0 = E_n^* N + qM \quad (10)$$

In the first period, the total effective labor supply becomes

$$L_1 = (1 + g) (E_n^* N + E_m^* M) \quad (11)$$

where $E_m^* \equiv \int_0^{e_m^*} (1 - e - \varphi) dG + q[1 - G(e_m^*)]$ which represents the effective labor force supplied by offspring whose parents are immigrants.

After the first period, offspring of immigrants are perfectly assimilated into the host country. Thus the assimilation costs disappear after that. Thus, the total effective labor supplies after the second period become

$$L_t = (1 + g)^t (N + M) E_n^* \quad (12)$$

2.4 Pension as the RDC system

Pension system is financed as pay-as-you-go way under the RDC system. Retirees receive pension benefits proportional to the contribution they paid in the working period. The rate of pension benefits to contribution $\left(z = \frac{b(e_x^i)}{\theta(M)W(e_x^i)} \right)$ is constant over periods. The pension contribution is endogenously determined to equalize total pension premium revenue with total pension benefits expenditure. Admitting immigrants affects native residents through the change in the contribution rate in the period in which they arrive.

The superscript *no* is used to denote the value when no immigrants is admitted and the superscript *im* is used to denote the value when M immigrants are admitted.

In the zero period, when the host country accept M unskilled immigrants, the budget equilibrium in pension system becomes

$$z \frac{1}{(1 + g)} \theta_{-1}^{no} E_n^* N w = \theta_0^{im} (E_n^* N + qM) w \quad (13)$$

The left side of equation (13) represents the sum of the pension benefits the retirees receive which equals the sum of the contributions the retirees paid in the working period multiplied by the constant rate of returns from the pension contributions. The right side of equation (13) represents the sum of the pension contributions the working generation must pay. Solving equation (13) with respect to the pension contribution rate, θ_0^{im} , we get $\theta_0^{im} = z \frac{1}{(1 + g)} \theta_{-1}^{no} \frac{E_n^* N}{(E_n^* N + qM)}$.

When the host country does not accept any immigrants from abroad, the budget equilibrium in pension system becomes

$$z \frac{1}{(1 + g)} \theta_{-1}^{no} E_n^* N w = \theta_0^{no} E_n^* N w \quad (14)$$

Solving equation (13) with respect to the pension contribution rate, θ_0^{no} , we get $\theta_0^{no} = z \frac{1}{(1+g)} \theta_{-1}^{no}$. Regardless of whether accepting immigrants or not, the pension benefits in the period zero are the same. On the pension contribution rate, we get $\theta_0^{im} = \frac{E_n^* N}{(E_n^* N + qM)} z \frac{1}{(1+g)} \theta_{-1}^{no} = \frac{E_n^* N}{(E_n^* N + qM)} \theta_0^{no}$, thus $\theta_0^{im} < \theta_0^{no}$. The pension contribution rate when immigrants are admitted becomes lower than that when no immigrants is admitted. The burden of the working generation in the period zero becomes lower when immigrants are admitted.

We define the sum of the pension contribution paid by the t -th generation native residents as P_t . When the difference between when immigrants are admitted and when no immigrants is admitted is denoted with superscript *Net*, we get $P_0^{Net} = P_0^{no} - P_0^{im}$, which can be calculated as

$$\begin{aligned} P_0^{Net} &= z \frac{1}{(1+g)} \theta_{-1}^{no} E_n^* N w - z \frac{1}{(1+g)} \theta_{-1}^{no} \frac{E_n^* N}{(E_n^* N + qM)} E_n^* N w \\ P_0^{Net} &= z \frac{1}{(1+g)} \theta_{-1}^{no} E_n^* N w \left[\frac{qM}{(E_n^* N + qM)} \right] > 0 \end{aligned} \quad (15)$$

. The value of equation (15) represents the net benefits in the period zero when immigrants are admitted.

Next, the pension contributions paid by the 1-th generation who are working in the period zero and become retired in the 1-th period becomes lower, thus the pension benefits becomes also lower in the one period because the rate of return from pension contribution is constant under the RDC system. This is a negative effect from admitting immigrants. On the contrary, the pension burden for the 2-nd generation also becomes lower while the offspring of the immigrants are less effective than the native offspring. These two effects are positive and negative. We can put together these effects as follows. We define the sum of the pension benefits the retirees get as B_t . The difference in B_t between $(B_1^{im} = z \left(\frac{z}{1+g} \right) \left(\frac{E_n^* N}{E_n^* N + qM} \right) \theta_{-1}^{no} E_n^* N w)$ when immigrants are admitted and $(B_1^{no} = z \left(\frac{z}{1+g} \right) \theta_{-1}^{no} E_n^* N w)$ when no immigrants is admitted becomes

$$B_1^{Net} = z \left(\frac{z}{1+g} \right) \left(\frac{qM}{E_n^* N + qM} \right) \theta_{-1}^{no} E_n^* N w \quad (16)$$

. The value of equation (16) is the net decrease in the pension benefits caused by admitting immigrants. Thus this is the negative effect. On the contrary, the pension contribution receives the positive and the negative effects from admitting immigrants. After some calculation, we get the net difference in the pension contribution as

$$P_1^{Net} = z \left(\frac{z}{1+g} \right) \left(\frac{E_m^* M}{E_n^* N + E_m^* M} \right) \theta_{-1}^{no} E_n^* N w \quad (17)$$

which represents a net decrease in the pension contribution, which implies a decrease in the burden of the native residents even if the productivities of the offspring of immigrants caused by the assimilation costs are lower than the native residents. Thus, the value of equation (17) is positive effect from admitting immigrants. Using equations (16) and (17), the net benefits from admitting immigrants in the period one becomes

$$NB_1 = z \left(\frac{z}{1+g} \right) \left(\frac{E_n^* N (E_m^* - q) M}{(E_n^* N + qM) (E_n^* N + E_m^* M)} \right) \theta_{-1}^{no} E_n^* N w > 0 \quad (18)$$

After the period two, we can calculate the net benefits from admitting immigrants. All effects are put together in table 1. Table 1 shows that the total net benefits for the native residents from admitting immigrants become positive without any conditions. According to Jinno (2011) and Jinno (2013), the net benefits under the DC pension system is $\frac{((1+M)A - (E_n^* - E_m^*))M}{(1+m)A}$ and the net benefits under the DB pension system is $\frac{AE_m^* - q(E_n^* - E_m^*)}{q(1+r)E_n^* E_m^*}$, where $A \equiv q(1+r)/(1+g)$. These values under the DC and the DB pension systems become positive or negative depending on the number of immigrants and the value of the assimilation costs. On the other hand, the net benefits under the RDC pension system always becomes positive. Thus, the RDC pension system may be the best pension system of the three when countries accept immigrants.

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Period	Benefit	Loss	Net Benefits
zero	$z \frac{\theta_{-1}^{no}}{(1+g)} w \left(1 - \frac{E_n^* N}{(E_n^* N + qM)} \right)$	0	$z \frac{\theta_{-1}^{no}}{(1+g)} w \left(\frac{qM}{(E_n^* N + qM)} \right)$
one	$z \left(\frac{z}{1+g} \right) \left(\frac{E_m^* M}{E_n^* N + E_m^* M} \right) \theta_{-1}^{no} E_n^* N w$	$z \left(\frac{z}{1+g} \right) \left(\frac{qM}{E_n^* N + qM} \right) \theta_{-1}^{no} E_n^* N w$	$z \left(\frac{z}{1+g} \right) \left(\frac{E_n^* N (E_m^* - q) M}{(E_n^* N + qM)(E_n^* N + E_m^* M)} \right) \theta_{-1}^{no} E_n^* N w$
two	$z^2 \left(\frac{z}{1+g} \right) \left(\frac{M}{1+M} \right) \theta_{-1}^{no} E_n^* N w$	$z^2 \left(\frac{z}{1+g} \right) \left(\frac{E_m^* M}{E_n^* N + E_m^* M} \right) \theta_{-1}^{no} E_n^* N w$	$z^2 \left(\frac{z}{1+g} \right) \left(\frac{(E_n^* N - E_m^*) M}{(1+M)(E_n^* N + E_m^* M)} \right) \theta_{-1}^{no} E_n^* N w$
three	$z^3 \left(\frac{z}{1+g} \right) \left(\frac{M}{1+M} \right) \theta_{-1}^{no} E_n^* N w$	$z^3 \left(\frac{z}{1+g} \right) \left(\frac{M}{1+M} \right) \theta_{-1}^{no} E_n^* N w$	0
total	$z \frac{\theta_{-1}^{no}}{(1+g)} w \left(\frac{qM}{(E_n^* N + qM)} + z \frac{E_n^* N (E_m^* - q) M}{(E_n^* N + qM)(E_n^* N + E_m^* M)} E_n^* N + z^2 \frac{(E_n^* N - E_m^*) M}{(1+M)(E_n^* N + E_m^* M)} E_n^* N \right) > 0$		

Table 1: List of benefits and losses from admitting immigrants

3 Discussion

Many advanced countries face the problem that the fertility rate is low under the pay-as-go pension system, thus want to change the pension system from the pay-as-you-go pension system to the funded pension system. The last subsection shows that the net benefits from accepting immigrants under the RDC pension system always becomes positive. We can use the net benefits caused by admitting immigrants as financial funds to compensate the double burden to change the pension system from the pay-as-you-go to the funded pension system. In this subsection, we would like to calculate how many immigrants should be admitted to finance the double burden the people must pay in the period when the pension system is changed.

3.1 Scenario of admitting immigrants

The last section shows that admitting immigrants under the RDC pension system can decrease the pension contribution rate to balance the pension premium revenue with the pension benefit expenditure. However, the rate of pension contribution and the rate of pension benefits to contribution are also constant. We admit M immigrants in the period of *zero* and in the period of 3 when the net benefits of admitting immigrants becomes *zero*, the pension system will be changed from the pay-as-you-go to the funded pension system.

For individuals, both of the pension contribution rate and the rate of pension benefits to contribution are constant, thus the pension benefits also are constant. The budget of pension system is different from the last section. The revenue of the pension system is $\theta_0^{no} (E_n^* N + qM) w$. The extra revenue of admitting immigrants: (ER) becomes $ER_0 = \theta_0^{no} (E_n^* N + qM) w - z \frac{1}{(1+g)} \theta_{-1}^{no} E_n^* N w$ which shows the extra revenue caused by admitting immigrants in the period 0. θ_0^{no} is $z \frac{1}{(1+g)} \theta_{-1}^{no}$. Thus,

$$ER_0 = z \frac{1}{(1+g)} \theta_{-1}^{no} qMw$$

which is the net funds from admitting immigrants in the period *zero*

In the period of *one*, while the necessary pension revenue is $z^2 \frac{1}{(1+g)} \theta_{-1}^{no} (E_n^* N + qM) w$, the real pension revenue is $z^2 \frac{1}{(1+g)} \theta_{-1}^{no} (E_n^* N + E_m^* M) w$. Thus, the financial funds in the period of *one* becomes

$$ER_1 = z^2 \frac{1}{(1+g)} \theta_{-1}^{no} (E_m^* - q) Mw$$

which is the net funds from admitting immigrants in the period *one*.

In the period of *two*, while the necessary pension revenue is $z^3 \frac{1}{(1+g)} \theta_{-1}^{no} (E_n^* N + E_m^* M) w$, the real pension revenue is $z^3 \frac{1}{(1+g)} \theta_{-1}^{no} E_n^* (N + M) w$. Thus, the financial funds in the period of *two* becomes

$$ER_2 = z^3 \frac{1}{(1+g)} \theta_{-1}^{no} (E_n^* - E_m^*) Mw$$

which is the net funds from admitting immigrants in the period *two*. Thus, the present value of the total financial funds caused by admitting immigrants in the period *three*, (ER^T), becomes

$$ER^T = z \frac{1}{(1+g)} \theta_{-1}^{n_o} q M w R \{qR^2 + z(E_m^* - q)R + z^2(E_n^* - E_m^*)\} \quad (19)$$

The necessary pension benefits the second generation will get in the period of *three*, (NPB^T), becomes

$$NPB^T = z^4 \frac{1}{(1+g)} \theta_{-1}^{n_o} E_n^* (N + M) w \quad (20)$$

NPB^T is the double burden the working generation must pay when the pension system is changed from the pay-as-you-go to the funded pension system. If ER^T can finance all the NPB^T , from the period of *three* the pension system will be perfectly changed to be the funded pension system.

In the case of $R = z$, ER^T becomes $z^4 \frac{1}{(1+g)} \theta_{-1}^{n_o} E_n^* M w$, which can finance only $\frac{M}{N+M}$ of NPB^T . $R = z$ means there is no risk premium on the pension system. Thus, if there is no risk premium on the pension system, the admitting countries can not get enough financial funds to change the pension system no matter how many immigrants will be admitted.

In the case of $R > z$, there is a possibility of full changing the pension system from the pay-as-you-go to the funded pension system without extra burdens. $R > z$ means that there is risk premiums on the pension system. In this case, according to the equations of (19) and (20), we can calculate the necessary number of immigrants to finance $\lambda\%$ of the double burden as follows:

$$M_\lambda^* = \frac{\lambda z^3 E_n^* N}{R(qR^2 + z(E_m^* - q)R + z^2(E_n^* - E_m^*)) - \lambda z^3 E_n^*}. \quad (21)$$

We also have $\frac{dM_\lambda^*}{d\lambda} = \frac{z^3 [R(qR^2 + z(E_m^* - q)R + z^2(E_n^* - E_m^*)) - \lambda z^3 E_n^*] E_n^* N}{[R(qR^2 + z(E_m^* - q)R + z^2(E_n^* - E_m^*)) - \lambda z^3 E_n^*]^2} > 0$. An increase in the value of λ needs more immigrants. If $\lambda = 1$, the equation becomes

$$M^* = \frac{z^3 E_n^* N}{(R - z)(qR^2 + zE_m^* R + z^2 E_n^*)}. \quad (22)$$

We have $\frac{dM^*}{d\varphi} = \frac{dM^*}{dE_m^*} \frac{dE_m^*}{d\varphi} > 0$. Thus, an increases in the assimilation costs

increases the necessary number of immigrants we should admit to finance the double burdens to change the pension system.

3.2 Numerical Example

In this subsection, we would like to show a simple numerical example. The assumptions we use to calculate the numerical example are as follows. We substitute the average wage of the labor for the productivity of the native labor. The productivity of the immigrant labor considering the assimilation costs supposed to be less than that of the natives, however, the exact productivity of the immigrants can not be estimated. Thus, the productivity is assumed that $E_m^* = \alpha E_n^*$ where $\alpha < 1$. α is used for the sake of convenience. The total sum of cash earnings of establishment with 5 or more employees for all industries in "Monthly Labour Survey" in August 2019 is used as the proxy variable of the

Parameters	Values
E_n^*	276,296
$E_m^* = 0.8E_n^*$	221,036.8
N	50.9 million
$R = 1.02^{30}$	2.427
$z = 1.005^{30}$	1.161
q	133,548

Table 2: Parameters

λ	0.1	0.3	0.5	0.8	1.0
M^* (million)	0.83	2.58	4.46	7.53	9.78
M^*/N (%)	1.64	5.07	8.75	14.78	19.18

Table 3: Necessary number of immigrants

productivity of the native: E_n^* . We also use the average wage of technical interns as the productivity of the unskilled labor of the immigrants: 133,548 per month in "Kigyō niokeru gaikoku-jin ginō jissū-sei no ukeire nikansuru chōsa" by the Japan Institute for Labour Policy and Training (2016). The number of the labor is 51.0 million people according to "Monthly Labour Survey" in August 2019. The interest rate per year is assumed 1.02. The rate of return for pension is assumed 1.005. The number of year for one generation is 30. The all variables we use to calculate the numerical example are put together in Table 2.

We can calculate the necessary number of immigrants to finance $\lambda\%$ of the double burden the third generation must pay using the parameters in Table 2. The necessary numbers of immigrants are put together in Table 3. According to the Table 3, we accept only less than 20 % of the native labor to get the fund of the double burden because there is the net benefits of accepting immigrants which can be multiplied by a risk premium on the pension: $R > z$.

4 Summary and remaining issues

We have analyzed the effects of admitting immigrants under the RDC pension system considering the assimilation costs like Jinno (2011, 2013). Jinno (2013) shows that the net benefits from admitting immigrants becomes higher under the defined benefits pension system than under the defined contributions pension system when the number of immigrants is small. However, the net benefits can be negative under the both pension system, while this paper shows that it always be positive under the RDC pension system without any conditions even if the assimilation costs are considered. We should pay attention to this point.

In this paper, the net benefits of admitting immigrants under the RDC pension system always become positive. Thus we wonder if the net benefits could be used as funds to transform from the pay-as-you-go pension system which is not difficult to be sustainable to the funded pension system which can be sustainable when the fertility rate is low. We show if Japan accept only less than 20 % of the native labor to get the fund of the double burden because

there is the net benefits of accepting immigrants which can be multiplied by a risk premium on the pension using simple numerical example.

Admitting immigrants may raise the unemployment rate which may decrease the welfare of the native residents This issue is also remain to be solved.

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